

# Financing Development: The Role of Long-term Debt <sup>\*</sup>

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## Abstract

We study the effect of costly external finance on the allocation of resources and economic development in a general equilibrium model where entrepreneurs vary in their productivity, financial assets, and debt. In addition to short-term borrowing subject to collateral constraints, producers are allowed to take on risky, illiquid loans that help mitigate their financing constraints. In this environment, we quantitatively assess how the costs of long-term borrowing and consumption smoothing motives jointly affect aggregate output and productivity in economies with underdeveloped financial markets. Specifically, risky, illiquid debt increases the consumption volatility of producers already facing uninsurable productivity risk. This tends to reallocate production away from poor, highly productive entrepreneurs and increase resource misallocation. We study a variety of different costs of borrowing in our framework. One important ingredient is a random access to external finance. In periods without access, borrowers cannot actively adjust their debt but must continue with existing principal payments or default. Default is costly as it leads to a loss of assets. When the expected period without access to the debt markets rises from 4 to 10 years, both aggregate output and productivity falls substantially.

*Keywords:* entrepreneurship, long-term debt, misallocation, aggregate productivity

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# 1 Introduction

A large literature documents substantial dispersions of firms' marginal products in developing economies, suggesting an inefficient allocation of productive resources.<sup>1</sup> Financial frictions are commonly viewed to be an important source of resource misallocation; cross-country differences in levels of financial development have been found to be quantitatively significant for explaining the differences in aggregate productivity.<sup>2</sup> In addition, individual producers tend to rely more on long-term external borrowing as a country's financial and legal systems improve.<sup>3</sup> Despite this, the role of long-term debt is mostly absent in the recent macroeconomic studies of financial frictions and economic development. In this paper, we evaluate the quantitative implications of long-term debt in a model of entrepreneurship and market incompleteness.

Our emphasis on long-term debt is motivated by the evidence on business lending across countries. In particular, as shown in Figure 1, it is clear that corporate bond issuance and its maturity, and firms with bank credit all increase in the extent of financial development.<sup>4</sup> These positive relationships indicate that businesses in developing economies are not easily accessible to capital markets with long-term financing instruments, such as corporate bonds and commercial bank loans, when compared to those in developed economies. In turn, the limited financing options disproportionately affect individuals at the micro level; highly productive firms are

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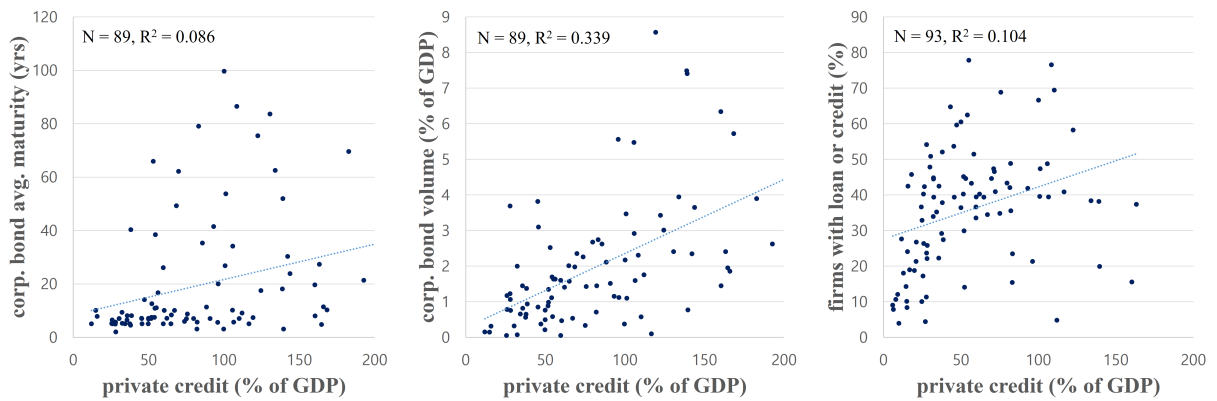
<sup>1</sup>Restuccia and Rogerson (2008) quantitatively show that idiosyncratic distortions among heterogeneous establishments lead to large losses in aggregate total factor productivity (TFP). Hsieh and Klenow (2009) empirically measure the gaps in marginal products of labor and capital in developing economies and compare them with those observed in the US.

<sup>2</sup>King and Levine (1993) use aggregate measures of domestic credit for distinguishing the levels of financial development across countries. Banerjee and Duflo (2005) emphasize that production heterogeneity and factor allocation are the main source of resource misallocation at the micro level. Among others, Buera, Kaboski, and Shin (2011) and Buera and Shin (2013) examine the aggregate effects of collateral constraints in a model of heterogeneous entrepreneurs and occupational choices. Chen, Habib, and Zhu (2023) focus on the role of managerial inputs in amplifying the productivity losses due to financial frictions.

<sup>3</sup>Demirguc-Kunt and Maksimovic (1999) identify country-specific determinants of corporate capital structure and find that firms in developing economies are less reliant on long-term debt mainly due to weak financial and legal systems. Beck, Demirguc-Kunt, and Maksimovic (2008) argue that financial and institutional reforms can be more beneficial for small businesses by facilitating bank finance.

<sup>4</sup>We use the ratio of domestic private credit to GDP as a proxy of financial development. The regression coefficients are positive and significant with their p-values below 0.01. This finding remains robust when we instead use a broader measure of financial development, the Financial Development Index provided by IMF.

Figure 1 : Financial Development and Business Lending



Note: Data is from World Development Indicators and Global Financial Development in World Bank. *private credit* is the domestic credit to private sectors divided by GDP, *corp. bond avg. maturity* is the volume-weighted average maturity of corporate bonds issued by non-financial entities, *corp. bond volume* is the new corporate bond issuance by non-financial entities in relative to GDP, and *firms with loan or credit* is the share of firms with a bank loan or line of credit. Each observation corresponds to the most recent value of a country between 2015 to 2021.

particularly prevented from undertaking large-scale investment. This not only slows down firm growth, as emphasized by Cooley and Quadrini (2001), but also worsens the allocative efficiency in economies with underdeveloped financial markets as shown by Buera and Shin (2013). We complement these existing works of production heterogeneity and financial frictions by studying the role of long-term borrowing for businesses from the perspective of financial development.

We build a model with heterogeneous entrepreneurs facing uninsurable idiosyncratic shocks to their productivity and limited access to financial markets. Entrepreneurs hire labor and rent capital to produce homogeneous final output. Capital is rented subject to collateral constraints and entrepreneurs' wealth serves as collateral as in Buera, Kaboski, and Shin (2011, 2021). Importantly, we introduce long-term debt as an additional financing option for entrepreneurs in the model; business loans are provided by competitive lenders at discount prices and they are repaid continuously until the maturity of debt contracts.

The introduction of long-term debt not only generates a rich financial structure in the model, but also involves mixed effects on resource allocation across producers. First, when compared to the case only with collateral constraints, entrepreneurs are able to borrow externally more than

that allowed by their collateral. This mitigates financing constraints such that more resources can be directed toward productive businesses, which improves allocative efficiency of resources. Debt financing, on the other hand, also requires repeated payments of principal and interest over time. Since these payments are non-contingent, while entrepreneurs' earnings are risky, they have an incentive to refinance or default before their existing debt matures. This raises the costs of external borrowing, which in turn deters entry into entrepreneurship and restricts incumbents' excessive debt financing. Both the reduction in the number of entrepreneurs and in the levels of their borrowing may worsen the allocative efficiency, thereby reducing aggregate productivity. Given these offsetting effects, our model environment offers an opportunity for us to quantify the macroeconomic impact of easing entrepreneurs' access to long-term debt markets, in addition to addressing the cross-country evidence on financing patterns.

We discipline our model economy to be consistent with both aggregate and micro-level moments in the US data. We then show that productivity losses arising from financial frictions can be substantially amplified when fewer entrepreneurs are allowed to take on long-term debt. Specifically, we vary the probability of gaining access to discount loan markets. When the probability falls from 25 to 10 percent, our model predicts a 6.2 percent decline in aggregate productivity. At the same time, the number of entrepreneurs engaged in production falls by more than 25 percent, significantly reducing aggregate output in the long run. It follows that an economy-wide financial reform may have large aggregate impacts in developing countries, when implemented to expand financial inclusion and reduce idiosyncratic risk of individual businesses.<sup>5</sup>

Issuing long-term debt incurs payments of principal and interest over time in the presence of idiosyncratic risk in the model. Specifically, entrepreneurs face uninsurable shocks to their productivity while their individual access to debt markets is stochastic. This in turn implies that highly leveraged entrepreneurs anticipate a rise in the volatility of consumption. When they're

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<sup>5</sup>Our approach is in line with Dabla-Norris et al. (2021). They consider different measures of financial development in terms of breadth, depth, and efficiency, and show that different policies should be implemented conditional on those measures of financial development.

unable to refinance debt, negative productivity shocks increase their debt payments relative to income. Consequently, the liquidity of long-term debt has sharp implications for the risk of borrowing and thus for entrepreneurs' willingness to take on debt.

Economies with illiquid debt markets, resulting from infrequent access to refinancing, feature higher costs of borrowing for entrepreneurs. This discourages external financing and leaves highly productivity but poor entrepreneurs with insufficient capital. The allocation of resources suffers. Alternatively, when entrepreneurs with access to refinancing experience an unexpected rise in debt-to-income ratio, they smooth consumption by paying off debt using financial assets. This reduces their collateral. Across entrepreneurs, this weakens the correlation between productivity and scale, increasing misallocation. Further, diminishing marginal utility of consumption leads to the welfare cost of debt repayment rising nonlinearly with decreases in the average duration of debt market access. Thus, as discussed by Buera and Shin (2011), the persistence of idiosyncratic risk plays a crucial role in determining aggregate outcomes in the presence of market incompleteness. In our model, this risk is not only from real shocks, but also from financial shocks.

Lastly, the costs and availability of long-term debt affects the value of entry. When the chances of long-term debt financing fall for startups, potential entrepreneurs are reluctant to pay the cost of entry and become active in production. Less entry leads to a decline in the number of producers in our model economy. The number of workers rises as a result, even though factor prices decrease in equilibrium. This further lowers aggregate productivity over and beyond the effects of long-term debt on active entrepreneurs.<sup>6</sup>

The rest of the paper is organized as follows. We first discuss the relevant literature below. Section 2 describes the model environment, and Section 3 provides the quantitative results from the calibrated model. Section 4 concludes.

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<sup>6</sup>Given decreasing-returns-to-scale production technologies, the number of production units is a determinant of measured TFP, as illustrated by Jo and Senga (2019) and Khan, Senga, and Thomas (2022).

**Related Literature** This paper is closely related to the vast literature on resource misallocation and economic development. Following Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), this area has focused on examining the sources of micro-level distortions in the presence of production heterogeneity. The recent literature has emphasized that financial frictions are one important source of misallocation, which is consistent with the evidence on different levels of financial development across countries. To quantify the implications of financial frictions in existing models, it is common to assume collateral constraints that limit entrepreneurs' borrowing within a given period.<sup>7</sup> In contrast, our paper particularly focuses on long-term illiquid debt and incorporates it into an otherwise standard model.<sup>8</sup> This allows us to not only examine the significance of long-term financing in quantifying aggregate productivity, but also address the observed differences in corporate debt structure between developed and developing economies.<sup>9</sup>

Our work is also related to recent quantitative studies of default risk in models with production heterogeneity. Khan, Sengal, and Thomas (2022) study business cycles in a model of heterogeneous firms with individual default risk, while Ottonello and Winberry (2020) highlight on the role of firms with low default risk in propagating monetary shocks. In the presence of firm default risk, Jo et al. (2024) additionally consider differences in health status across households and study aggregate dynamics during and after a pandemic. Further, Arellano, Bai, and Zhang (2018) investigate the role of financial frictions in explaining cross-country differences of firm growth patterns.<sup>10</sup> In contrast to these papers, we develop a model of heterogeneous entrepreneurs with

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<sup>7</sup>See the excellent survey by Buera, Kaboski, and Shin (2015). Recent applications of the standard framework include Ranasinghe and Restuccia (2018) in which weak legal institutions amplify the adverse impacts of financial frictions in developing economies.

<sup>8</sup>Albuquerque and Hopenhayn (2004) consider a dynamic contracting problem under limited commitment and characterize the patterns of firm growth in their model. Cole, Greenwood, and Sanchez (2016) introduce persistent information asymmetry and costly state verification into a dynamic contracting problem and firms' decision on technology adoption.

<sup>9</sup>Consistent with Demircug-Kunt and Maksimovic (1999), Booth et al. (2001) confirms the importance of country-specific factors such as financial institutions in determining firms' capital structure. Schmukler and Vesperoni (2006) find that accessing international financial markets allows firms to use more long-term debt and extend their debt maturity. Relatedly, Benmelech and Dvir (2013) examine the East Asian financial crisis and show that the prevalence of short-term debt is a symptom of weak financial institutions in emerging economies.

<sup>10</sup>They document variations in firm leverage and sales growth across countries that differ in their levels of financial development. As in our work, their model features both debt financing and default in the presence of production

long-term debt and focus on aggregate outcomes across economies with different levels of financial development.<sup>11</sup>

## 2 Model

Consider an economy with a continuum of heterogeneous individuals, competitive financial intermediaries, and a government. Individuals are infinitely-lived and distinguished by their occupations, worker and entrepreneur. These individuals maximize lifetime utility given persistent idiosyncratic shocks. Entrepreneurs make decisions of production, saving, and borrowing in the presence of financial frictions. Time is continuous and markets are perfectly competitive.

### 2.1 Individuals

Both workers and entrepreneurs are risk-averse, and their preference is represented by a utility function,

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \quad \sigma \geq 0,$$

where  $c$  is the non-durable consumption and  $\sigma$  is the risk aversion parameter. Given idiosyncratic shocks, they accumulate wealth in liquid assets  $a$  that earn risk-free interest rate of  $r$ . The individuals are subject to taxes conditional on their occupation, either on labor income or profits. Let  $g(\cdot)$  denote the entire distribution of individuals in the economy.

**Permanent Workers** There is a fixed share  $s_w$  of individuals who are permanently working without operating their businesses. Leisure has no value, so they supply labor inelastically given the unit time endowment at each instant. Let  $\varepsilon$  be an individual worker's labor productivity heterogeneity. In contrast, we focus on quantifying the productivity losses in an economy with long-term debt.

<sup>11</sup>Our approach is also in line with Karabarounis and Macnamara (2021) who focus on the difference between private and public firms. In their model, only public firms are allowed to issue bonds that mature stochastically as in Gomes and Schmid (2021).

drawn from a time-invariant distribution. We assume that shocks to labor productivity arrive at the Poisson rate of  $\lambda_\varepsilon$ , and the transition probability of  $\varepsilon$  is given by  $\pi_{\varepsilon\varepsilon'}$ .<sup>12</sup> A worker is then identified by its individual holding of liquid asset holding  $a$  and labor productivity  $\varepsilon$ . The worker's problem can be recursively written by

$$\begin{aligned} \rho v^w(a, \varepsilon; g) &= \max_c \left[ u(c) + \frac{\partial v^w(a, \varepsilon; g)}{\partial a} \dot{a} + \sum \pi_{\varepsilon\varepsilon'} v^w(a, \varepsilon'; g) \right] & (1) \\ \dot{a} &= \varepsilon w(g) + r(g)a - c - T(w\varepsilon, r(g)a) \\ a &\geq \underline{a}, \end{aligned}$$

where  $v^w$  is the worker's value,  $\rho$  is the rate of time preference and  $T(\cdot)$  summarizes the taxes on labor and interest income earned by the worker. We denote the wage rate and interest rate as functions of the endogenous aggregate state  $g$ , by assuming stationary equilibrium of the model to be described later.

**Entrepreneurs** The rest of individuals are entrepreneurs, and they are different in their exogenous productivity  $z$  for operating a production unit.<sup>13</sup> We assume that  $z$  is drawn from a time-invariant distribution and the individual productivity shocks follow a Poisson process with the intensity of  $\lambda_z$  and the transition probability of  $\pi_{zz'}$ . Given  $z$ , an entrepreneur is able to hire labor  $l$  and rent capital  $k$  to produce homogeneous output goods. The production technology exhibits decreasing-returns-to-scale (DRS),  $y = zk^\alpha l^\nu$  with  $\alpha, \nu > 0$  and  $\alpha + \nu < 1$ . Operating a production process requires payments of fixed costs  $\xi(z)$  in units of output, and the amount of capital input is restricted by the entrepreneurs' financial wealth in terms of  $a$ . The latter is in the

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<sup>12</sup>To be specific,  $\varepsilon$  and  $\lambda_\varepsilon$  summarize labor productivity and its shock intensity distinguished by employment status;  $(\varepsilon_w, \lambda_w)$  and  $(\varepsilon_u, \lambda_u)$  respectively for employed and unemployed workers. That is, an employed worker with  $\varepsilon_w$  is subject to labor productivity shocks with the intensity of  $\lambda_w$  while remaining employed. He is also exposed to unemployment shocks that arrive at the rate of  $\lambda_u$ , and his productivity is then denoted by  $\varepsilon_u$  which determines the level of unemployment benefits.

<sup>13</sup>The entrepreneurs' productivity represents inalienable or innate skills that are required for managing business and monitoring production process. It captures the heterogeneity in managerial ability or intangible organizational practices as documented by Bloom and Van Reenen (2007).

form a collateral constraint,  $k \leq \gamma a$ , where  $\gamma$  represents the asset tangibility. Each unit of capital is rented at the competitive price  $r + \delta$ , where  $\delta$  is the depreciation rate. We define the earnings of an entrepreneur with  $(a, z)$  as below.

$$\pi(a, z; g) = \max_{k, l} \left[ zk^{\alpha} l^{\nu} - w(g)l - (r(g) + \delta)k - \xi(z) \right], \quad k \leq \gamma a$$

An important feature of our model is that intertemporal borrowings are made in the form of long-term debt and that entrepreneurs are allowed to default or refinance before maturity. A new debt contract therefore involves a loan price schedule that depends on an entrepreneur's individual state and choices of saving and borrowing. In this way, entrepreneurs may hold wealth in liquid assets simultaneously with outstanding long-term debt.

Specifically, we assume that entrepreneurs randomly gain or lose access to discount loan markets with probability  $\lambda_b$ .<sup>14</sup> Let  $b$  be the existing debt level of an entrepreneur. He is required to pay the interest and a fixed fraction  $\theta$  of the principal at each instant of time.<sup>15</sup> Otherwise, the entrepreneur is allowed to default and exit, which incurs a one-time utility cost  $\xi_d$  for securing a fraction  $\chi$  of his liquid asset holding. In addition, default negatively affects credit history, preventing bankrupt entrepreneurs from immediately re-entering and starting businesses. This default flag is removed stochastically at the rate of  $\lambda_d$ .

In case of refinancing given the access to the loan markets, on the other hand, entrepreneurs are required to fully repay the remaining balance and to choose a lower level of debt. Further, a new debt issuance of  $b'$  is subject to a proportional cost  $\xi_b$  in units of output. Due to default risk, the discount loan price  $q$  depends on an entrepreneur's current productivity  $z$ , and his choices of saving and borrowing  $(a', b')$ . Thus, borrowing is costly, worsening the extent of resource

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<sup>14</sup>When an entrepreneur does not have the option to borrow, his productivity is  $\psi z$  where  $\psi < 1$ . This reduces the incentive for expanding production scale when the collateral constraint binds. Due to debt repayment schedule, in addition, the borrower also takes into account of the possibility of losing the access to loan markets in the future.

<sup>15</sup>When the value of  $\theta$  is large, our model instead allows for short-term debt financing with default risk, analogous to the heterogeneous firm model of Khan, Senga, and Thomas (2022). In this paper, however, we focus on examining the implications of long-term debt given the cross-country evidence on financing patterns and economic development.

misallocation across producers.

Entrepreneurs may also quit running businesses after fully repaying the outstanding debt. This type of exit occurs either exogenously with probability  $\lambda_{10}$  or endogenously by shutting down their production units. Regardless, once they exit, entrepreneurs remain inactive and supply labor as interim workers given their productivity  $z$ . In order to start a new business, on the other hand, an inactive entrepreneur has to pay a fixed entry cost  $\kappa(z)$  in units of output. Given a borrowing opportunity and without a default flag, he is allowed to finance a fraction  $\gamma_k$  of the entry cost using debt.

Given the above description of entrepreneurs' state and decisions, we use an indicator  $o$  of their production and borrowing status. The entrepreneur is then identified by his productivity  $z$ , liquid asset holding  $a$ , existing long-term debt level  $b$ , and the indicator  $o$ . An individual with  $\omega \equiv (a, b, z, o)$  solves the following problem.

$$v(\omega_t; g_t) = \max_{\{c_t, (k_t, l_t)\}_{o \in [1,2]}\}_{t \geq 0, \tau}} \left\{ \mathbb{E}_0 \int_0^\tau u(c_t) dt + e^{-\rho\tau} v^*(\omega_\tau; g_\tau) \right\}, \quad (2)$$

$$v^*(\omega_\tau; g_\tau) = \begin{cases} \max\{v^r, v^x, v^b\} & \text{if } o = 1 \\ \max\{v^x, v^b\} & \text{if } o = 2 \\ v^e & \text{if } o = 3, \end{cases}$$

$$(a_0, b_0, z_0, o_0) = (a, b, z, o),$$

where the indicator  $o$  is described in the following table.

$o$	production and borrowing status
1	active with borrowing opportunity
2	active without borrowing opportunity
3	inactive
4	inactive with default flag

The entrepreneur chooses the optimal sequence of consumption, and conditional on being

active, capital and labor for production. Without a default flag, he also optimizes the stopping time for discrete choices, which brings a large shift in his asset position, occupation, or credit history. At the stopping date  $\tau$ , the entrepreneur has the following options: (i) refinancing debt and continue, (ii) exiting after repaying debt, (iii) defaulting and exit, and (iv) starting a business. These choices correspond to the values  $v^r$ ,  $v^x$ ,  $v^b$ , and  $v^e$  respectively as summarized by the table below.<sup>16</sup> The maximum of the values in each case then leads to the stopping value  $v^*$  in (2).

	option at $\tau$	value	$a'$	$b'$
(i)	refinance and continue	$v^r$	$a_\tau - b_\tau + qb'$	$b' \leq b$
(ii)	exit after repaying debt	$v^x$	$a_\tau - b_\tau$	0
(iii)	default and exit	$v^b$	$\chi a_\tau$	0
(iv)	start a business	$v^e$	$a_\tau + qb'$	$b' \leq \gamma_k \kappa(z)$

Prior to stopping, the Hamilton-Jacobi-Bellman (HJB) equation for an active entrepreneur's problem with  $(a, b, z, o; g)$  is given by,

$$\rho v(a, b, z, o; g) = \max_{c, (k, l)_{o \in [1, 2]}} \left[ u(c) + \frac{\partial v(a, b, z, o; g)}{\partial a} \dot{a} + \sum \pi_{zz'} v(a, b, z', o; g) \right. \\ \left. + \mathbf{1}_{o \in [1, 2]} \left( \frac{\partial v(a, b, z, o; g)}{\partial b} \dot{b} \right. \right. \\ \left. \left. - \lambda_{10} (v(a, b, z, o; g) - v(a - b - \xi_{b0}, 0, z, 3; g)) \right. \right. \\ \left. \left. - \lambda_b (v(a, b, z, o; g) - v(a, b, z, o'; g)) \right) \right] \\ \dot{a} = \begin{cases} \pi(a, z; g) + r(g)a - c - (r(g) + \theta)b - T(\cdot), & \text{if } o \in [1, 2] \\ zw(g) + r(g)a - c - T(\cdot), & \text{if } o \in [3, 4] \end{cases} \\ \dot{b} = -\theta b \\ v(a, b, z, o; g) \geq v^*(a, b, z, o; g),$$

where the value of  $(o, o')$  is either (1,2) or (2,1). In the above, the RHS term with the indicator

<sup>16</sup>We abstract from fixed or transaction costs in individual decisions in the table for brevity.

$\mathbf{1}_{o \in [1,2]}$  involves two exogenous cases, involuntary exit and borrowing opportunity. With probability  $\lambda_{10}$ , the entrepreneur is determined to exit after a full repayment of the existing debt, and his assets become  $a - b - \xi_{b0}$  where  $\xi_{b0}$  is the debt adjustment cost. Hence, the difference  $v(a, b, z, o; g) - v(a - b - \xi_{b0}, 0, z, 3; g)$  captures the change in the value of an exiting entrepreneur in the HJB equation. Next, given the random access of probability  $\lambda_b$ ,  $v(a, b, z, o; g) - v(a, b, z, o'; g)$  is the change in the entrepreneur's value with a debt adjustment or new borrowing.

Lastly, we define the values for discrete choices at the stopping time  $\tau$  which, given their availability, determines the optimal value  $v^*$ .

### 1. Refinancing

$$v^r(a_\tau, b_\tau, z_\tau, o_\tau; g) = \max_{a', b'} v(a', b', z_\tau, o_\tau; g)$$

$$a' = a_\tau - b_\tau + q(a', b', z_\tau, o_\tau; g)b' - \xi_b b' - \xi_{b0}$$

$$b' \leq b_\tau$$

### 2. Exiting

$$v^x(a_\tau, b_\tau, z_\tau, o_\tau; g) = v(a_\tau - b_\tau - \xi_{b0}, 0, z_\tau, 3; g)$$

### 3. Defaulting

$$v^b(a_\tau, b_\tau, z_\tau, o_\tau; g) = v(\chi a_\tau, 0, z_\tau, 4; g) - \xi_d$$

#### 4. Entering

$$\begin{aligned}
v^e(a_\tau, 0, z_\tau, 3; g) &= \max_{a', b'} v(a', b', z_\tau, 2; g) \\
a' &= a_\tau + q(a', b', z_\tau, 2; g)b' - \xi_b b' - \xi_{b0} - \kappa(z) \\
b' &\leq \gamma_\kappa \kappa(z)
\end{aligned}$$

## 2.2 Financial Intermediaries

There are numerous financial intermediaries operating in the loan markets for active entrepreneurs. These intermediaries are risk-neutral and take short-term deposits from individuals as the source for their loan provision. For a new loan offered, the intermediaries competitively value the individual default risk of borrowers conditional on their current productivity and holding of debt and assets. This leads to a discount loan price schedule  $q(a, b, z, o; g)$  that eventually determines the loan size delivered to a borrower. When the probability of future default is non-zero,  $q(a, b, z, o; g)$  is less than the risk-free loan price, reflecting the associated risk premium.

Each instant after a loan of  $b_0$  is provided, the borrower has to pay the risk-free interest rate  $r_t(g)$  and a fraction  $\theta$  of the outstanding debt  $b_t$ . As the loan matures at the rate of  $\theta$ , hence, the financial intermediaries discount the value of a loan by  $r_t + \theta$ . Since they earn zero expected profit from each loan, the discounted value of debt at its issuance has to be equal to the expected sum of cash flows. Then the loan price function is given by

$$q(a_0, b_0, z_0, o_0; g_0)b_0 = \mathbb{E} \left[ \mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s + \theta) ds} (r_t + \theta)b_0 dt + e^{-\int_0^\tau r_s ds} b(a_\tau, b_\tau, z_\tau, o_\tau; g) \right].$$

Notice that the above pricing equation applies only to the borrowers with their individual state at the non-default region. Otherwise, he would have immediately defaulted and hence there is no loan provision.

In case of an entrepreneur's default, he is allowed to retain  $\chi a$  of assets after paying a utility

cost. In turn, the intermediaries recover  $\min[(1 - \chi)a, b]$ . Applying the Feynman-Kac formula, the loan price equation can be written as a partial differential equation.

$$\begin{aligned}
& (\theta + r(g))q(a, b, z, o; g) \\
&= \theta + r(g) + \frac{\partial q(a, b, z, o; g)}{\partial a} \dot{a} + \frac{\partial q(a, b, z, o; g)}{\partial b} \dot{b} \\
&+ \sum \pi_{zz'} q(a, b, z', o; g) - \lambda_{10}(q(a, b, z, o; g) - q(a - b - \xi_{b0}, 0, z, 3; g)) \\
&- \lambda_b(q(a, b, z, o; g) - q(a, b, z, o'; g)) \quad \text{for } t \in [0, \tau), \\
& q(a, b, z, o; g) = \frac{\min[(1 - \chi)a, b]}{b} \quad \text{for } t = \tau
\end{aligned} \tag{3}$$

$$\tag{4}$$

The loan price only exists for active entrepreneurs with  $o \in \{1, 2\}$  where the value of  $(o, o')$  is either (1,2) or (2,1).

### 2.3 Government

The government collects taxes from individual workers and entrepreneurs; taxes are imposed on their labor income or profits in addition to all interest and dividend income. We assume that a fraction of the tax revenue is spent on unemployment benefits, and the rest is used for government consumption that has no value to the individuals.

### 2.4 Recursive Equilibrium

A recursive competitive equilibrium of the economy is a set of functions that solve the problems of individuals and financial intermediaries and clear the markets for assets, debt, labor, and output goods. It is described by the following conditions in which the individual distribution is separated by occupation,  $g^w$  for workers and  $g^e$  for entrepreneurs.

1. Households optimize. Given prices  $\{r, w, q\}$ ,  $v^w$  solves (1), and  $v$  solves (2).
2. The debt discount price  $q$  satisfies (3) and (4) for financial intermediaries to earn zero profits.

3. Asset market clears,

$$\begin{aligned} \int a g^w(a, \varepsilon) d[a \times \varepsilon] + \int (a + b) g^e(a, b, z, o) d[a \times b \times z \times o] \\ = \int_{o \in [1,2]} k g^e(a, b, z, o) d[a \times b \times z \times o]. \end{aligned}$$

4. Labor market clears,

$$\begin{aligned} \int \varepsilon g^w(a, \varepsilon) d[a \times \varepsilon] + \int_{o \in [3,4]} z g^e(a, b, z, o) d[a \times b \times z \times o] \\ = \int_{o \in [1,2]} l g^e(a, b, z, o) d[a \times b \times z \times o]. \end{aligned}$$

5. The government budget constraint holds.

## 3 Quantitative Results

### 3.1 Calibration

We calibrate the model at annual frequency and reproduce the key empirical moments in the US aggregate and micro-level data. Given a subset of exogenously fixed parameters, we choose the values of the remaining parameters at the stationary equilibrium of the model. In particular, due to the heterogeneity across both workers and production units, our model delivers an empirically-consistent individual earnings distribution. Table 1 reports the parameter values by distinguishing the fixed and fitted parameters in our model, and Table 2 compares the model generated moments with their empirical counterparts.

**Preference and Productivity** As standard in the literature, we set the values of risk aversion parameter  $\sigma$  at 2 and that of subjective discount rate  $\rho$  at 0.055. We fix the population share of permanent workers at 50 percent, so that the share of active entrepreneurs is about 10 percent in

Table 1 : Parameter Values

Fixed Parameters		
	Value	Description
$\rho$	0.055	subjective discount rate
$\sigma$	2.0	risk aversion
$\nu$	0.6	labor share, production
$\delta$	0.07	depreciation rate
$s_w$	0.5	population share of permanent workers
$\tau_1$	0.181	tax progressivity
$\bar{\varepsilon}$	0.5	mean worker productivity
$\underline{\varepsilon}$	0.4	unemployment benefit ratio
$\lambda_{\varepsilon 10}$	0.238	job separation rate
$\lambda_{\varepsilon 01}$	3.113	job finding rate
$\bar{z}$	1.9	mean entrepreneur productivity
$\lambda_d$	0.2	shock intensity, default flag removal
Fitted Parameters		
	Value	Description
$\alpha$	0.285	capital share, production
$\tau_0$	0.91	income tax factor
$\eta_{\varepsilon}$	1.0	Pareto shape, worker productivity
$\lambda_{\varepsilon}$	0.1	shock intensity, worker productivity
$\eta_z$	2.00	Pareto shape, entrepreneur productivity
$\lambda_z$	0.2	shock intensity, entrepreneur productivity
$\psi$	0.80	reduced productivity with no borrowing
$\kappa_z$	-8, -2	upper and lower bounds of entry cost
$\xi$	0.05	fixed cost of operation
$\lambda_b$	0.25	frequency, borrowing availability shock
$\lambda_{10}$	0.005	frequency, exit shock
$\theta$	0.05	debt amortization rate
$\gamma$	1.5	collateral constraint
$\gamma_{\kappa}$	0.75	fraction of borrowing for entry cost
$\xi_b$	0.01	proportional cost for debt issuance
$\xi_d$	0.025	utility cost of default
$\chi$	0.4	retention rate of asset upon default

Table 2 : Calibration Targets and Model Moments

Description	Data	Model	Source
real interest rate	0.04	0.04	-
Capital-Output ratio	2.4	1.8	BEA
Labor share ( $wn/y$ )	0.6	0.6	BLS
Tax-Output ratio	0.16	0.18	CBO
Exit rate	8.7%	9.0%	BDS
Bankruptcy rate	3.0%	4.0%	Ottonello and Winberry (2020)
Aggregate leverage	37.2%	32.2%	Flow of Funds
Share of small firms without debt	30%	24%	SBCS
Share of earnings in top 20%	65.0%	66.1%	SCF
Share of earnings in bottom 60%	14.4%	16.0%	SCF
Share of entrepreneurs in top 20% of wealth dist.	26.0%	24.0%	BEA, Cagetti and De Nardi (2006)
Share of entrepreneurs in top 1% of wealth dist.	62.0%	57.0%	BEA, Cagetti and De Nardi (2006)
Share of active entrepreneur	11.1%	9.7%	Cagetti and De Nardi (2006)
Share of firms that hire 1-99	90.25%	97.1%	BDS

our model.<sup>17</sup>

We assume that individual productivity is drawn from a Pareto distribution, each for workers and entrepreneurs. At each instant of time, an idiosyncratic shock arrives at a constant intensity and individuals get a new draw of productivity. We set the values of the intensity and the Pareto-shape parameters,  $(\lambda_\varepsilon, \lambda_z)$  and  $(\eta_\varepsilon, \eta_z)$ , to jointly match the observed heterogeneity across households and firms in the US data. Specifically, we target the earnings distribution in the Survey of Consumer Finance (SCF).

For entrepreneurial productivity, we discretize the support of  $z$  with 3 values such that the mean productivity  $\bar{z}$  is 1.9.<sup>18</sup> When an entrepreneur loses the access to the loan markets and hence is unable to adjust debt, his productivity falls by 20 percent ( $\psi = 0.80$ ).

For workers' labor productivity  $\varepsilon$ , we use 20 evenly-spaced grid points that are augmented with an additional point below and above. These two boundary points respectively target the income of the top 1 percent and the unemployed households. Since workers experience unem-

<sup>17</sup>This value is within the range of the empirical entrepreneurship rate, 7.6 to 16.7 percent, reported in Cagetti and De Nardi (2006).

<sup>18</sup>Each grid point of  $z$  corresponds to the population share of entrepreneur-type households, respectively for 24, 46, and 25 percent.

ployment shocks, we choose the values of the shock intensities for job separation  $\lambda_{\varepsilon_{10}}$  and job finding  $\lambda_{\varepsilon_{01}}$  to imply the average job duration of 4.2 years and unemployment duration of 16.7 weeks as reported by the Bureau of Labor Statistics.

**Tax and Government** Our specification of income tax function is taken from Heathcote et al. (2017). That is, given the taxable income  $y$ , we assume  $T(y) = y - \tau_0 y^{1-\tau_1}$  where  $\tau_1$  determines the degree of tax progressivity. We set  $\tau_1$  at 0.181 as estimated by Heathcote et al., and choose the value of  $\tau_0$  to closely match the average ratio of tax revenue to output reported by the Congressional Budget Office (CBO). The calibrated value of  $\tau_0$  is 0.91 and the model implied tax revenue-output ratio is roughly 0.18.

Since the above tax function also implies transfers when an individual's taxable income is relatively low, we assume that the government spending is the difference between its tax revenue collected and the transfer payments. For the unemployment benefits, in addition, we set the value of  $\underline{\varepsilon}$  such that the unemployed workers get 40 percent of the median income.

**Production and Firm Dynamics** Following Khan and Thomas (2013), we fix the values of the labor share in the production function at 0.6 and the depreciation rate of capital at 0.07. We then calibrate the value of capital share parameter  $\alpha$ , so that the model generates a plausible ratio of aggregate capital to output.

We assume that entry costs  $\kappa(z)$  vary with entrepreneurs' exogenous productivity. In particular, highly productive entrants have to bear relatively larger costs, and hence their entry is discouraged.<sup>19</sup> This specification allows us to target the relative employment size of young firms in the BDS. Similarly, we further assume that the fixed operation cost  $\xi(z)$  rises nonlinearly in individual productivity,  $\xi(z) = \xi \cdot z^{\frac{1}{1-\alpha-\nu}}$ . Since the cost governs how quickly entrepreneurs accumulate wealth and escape their borrowing constraints, we set the value of the common factor  $\xi$  such that the firm size distribution is sufficiently skewed.

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<sup>19</sup>We assume  $\kappa(z) = \exp(\kappa_z) \cdot \pi^*(z)$  where  $\pi^*$  is the profit of entrepreneurs that are not financially constrained at the steady state. The value of  $\kappa_z$  also depends on  $z$ , and we choose its lower and upper bounds.

Notice that our model features both involuntary exit from exogenous shocks and endogenous choices of business closure or default. We calibrate the frequency of exogenous exit  $\lambda_{10}$  given the entry and operation costs, so that the total exit rate in the model is consistent with the annual firm exit rate of 8.7 percent in the BDS.

**Borrowing and Default** To jointly calibrate the model parameters related to external borrowing, we mainly target the debt distribution moments of firms in the Small Business Credit Survey (SBCS) and the aggregate leverage ratio of non-financial businesses in the Flow of Funds.<sup>20</sup> The resulting interest rate is about 4 percent in equilibrium.

First, we set the collateral constraint parameter  $\gamma$  at 1.5. Next, recall that entrepreneurs in our model stochastically gain or lose their access to the loan markets. In addition to collateral constraints, this shock affects the value of entry and the speed of firm growth, since entrepreneurs' key motive of external borrowing is to increase their production scale. We set the frequency of such borrowing availability shock  $\lambda_b$  such that the average duration of borrowing or no-borrowing status is 4 years. This allows us to generate realistic patterns of firm dynamics in both real and financial variables in our model.

For a new debt  $b'$ , entrepreneurs are responsible for its issuance cost of  $\xi_b b'$  in units of output, and we calibrate the cost parameters  $\xi_b = 0.01$ . The amortization rate of existing debt  $\theta$  is set to be 0.05, implying a half-life of 10 years when borrowers do not refinance or fully repay the loan. The average ratio of leverage is about 0.32 in our model, fairly close to the US aggregate moment. In addition, the population share of active entrepreneurs with or without debt in the model is comparable to the empirical moment in the SBCS.

The utility cost of default  $\xi_d$  is 0.025, and the retention rate of assets upon default  $\chi$  is set at 0.40. The removal rate of default flag  $\lambda_d$  is set at 0.20, so that the credit history lasts for about 5 years.<sup>21</sup> In our model, the resulting default rate is 4 percent, slightly higher than the mean

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<sup>20</sup>SBCS reports the financial information of US firms with less than 500 employees. The sample hence naturally corresponds to entrepreneurs in our calibrated model.

<sup>21</sup>D'Erasmus and Corbae (2021) estimate that the corporate debt recovery rate is about 40 percent in the US.

bankruptcy rate in the data. Lastly, the fraction of entry costs eligible for borrowing  $\gamma_k$  is set at 0.75, so that at least 25 percent of the entry cost must be self-financed.

### 3.2 Aggregate Results

Using the calibrated model, we conduct counterfactual experiments and compare the results. We particularly focus on the availability of long-term debt in economies with relatively under-developed financial markets when compared to the US. Hence, in our counterfactuals, we adjust the relevant parameter values to restrict entrepreneurs' access to debt financing, and solve for the corresponding equilibrium of the model. This allows us to quantitatively how financing constraints for long-term borrowing and consumption smoothing motive jointly affect aggregate output and productivity in the long run.

First, Table 3 presents the aggregate results when entrepreneurs' status of external borrowing changes less frequently and hence debt is more illiquid. Specifically, we reduce the value of  $\lambda_b$  respectively to 0.125 (C1) and 0.10 (C2), while holding other parameter values fixed.<sup>22</sup> In these counterfactual economies, hence, an entrepreneur currently with a binding collateral constraint but without the access to loan markets may remain financially constrained relatively longer.<sup>23</sup>

Comparing the first and third columns of the table (baseline and C2), in particular, the aggregates in the model substantially change when the average duration of each borrowing status increases from 4 years to 10 years. Aggregate output decreases by 2.4 percent, while real interest rate and wage rate fall by more than 7 percent. This is mainly because the entrepreneurs' decisions of entry, exit, and borrowing are crucially affected by the availability of long-term debt as the value of  $\lambda_b$  decreases. Due to fewer opportunities of using debt for their potential growth, entrepreneurs with relatively low productivity or insufficient wealth stop operating their busi-

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<sup>22</sup>When the value of  $\lambda_b$  is 0, the model collapses to a version of Buera, Kaboski, and Shin (2011) or Buera and Shin (2013). On the other hand, a rise in  $\lambda_b$  corresponds to financial development in terms of breadth and depth as defined by Dabal-Norris et al. (2021).

<sup>23</sup>Buera, Moll, and Shin (2011) provides a cautious tale that targeted subsidies may negatively affect aggregate productivity in the long run when such policies are hard to alter over time.

Table 3 : Relative Changes to Baseline Model 1

	baseline	C1	C2
shock frequency $\lambda_b$	0.25	0.125	0.10
real interest rate	100	94.38	92.56
real wage rate	100	93.75	91.16
output	100	98.88	97.61
capital	100	101.17	99.93
employment	100	105.46	107.07
measured TFP	100	95.45	93.71
aggregate leverage (pp)	-	-11.71	-14.73
default rate (pp)	-	-3.24	-3.62
share of entrepreneurs (pp)	-	-2.76	-3.58

nesses and become wage earners. Remaining producers also have to rely on their own savings to raise their capital given collateral constraints. Further, there are relatively fewer entrants since entrepreneurs are less likely to finance their entry costs even with reduced factor prices. As a result, both the population share of entrepreneurs and the aggregate leverage ratio decline drastically from those in the baseline economy. The extent of resource misallocation hence worsens significantly, reducing the measured TFP by 6.4 percent.

Moreover, entrepreneurs in the counterfactual economies are less willing to take on large debt although they are allowed to issue or adjust debt. This is because of their consumption smoothing motive against idiosyncratic risks. That is, given the lower levels of consumption and income in a distorted economy with weak financial markets, individuals are on average at the region of relatively higher levels of marginal utility. Since the debt contracts in our model involve continued payments of principal and interests, entrepreneurs may need to reduce their consumption for such payments following a negative shock in the future. The corresponding utility costs become larger as consumption decreases. Hence, entrepreneurs are required to save to pay for the entry costs, and upon entry, to rely relatively more on self-financing. As a result, both aggregate leverage and default rate declines as only highly productive and wealthy entrepreneurs decide to take on small amount of debt. This further reduces the allocative efficiency among producers in

Table 4 : Relative Changes to Baseline Model 2

	baseline	C3	C4	C5	C6
acquiring prob. $\lambda_{21}$	0.25	0.20	0.125	0.25	0.25
losing prob. $\lambda_{12}$	0.25	0.25	0.25	0.20	0.125
real interest rate	100	98.28	96.37	100.83	103.39
real wage rate	100	97.02	90.73	101.30	103.69
output	100	98.54	94.97	101.49	104.31
capital	100	99.77	98.21	101.46	104.66
employment	100	101.57	104.66	100.19	100.60
measured TFP	100	97.69	92.88	100.95	102.60
aggregate leverage (pp)	-	-3.64	-9.84	-0.40	-1.09
default rate (pp)	-	-1.63	-3.44	-0.20	-1.31
share of entrepreneurs (pp)	-	-0.80	-2.34	-0.09	-0.31

the counterfactual economies with a relatively lower probability of accessing debt markets.

We also confirm the above result is robust to asymmetric changes in debt adjustment frequency  $\lambda_b$ . In Table 4, the counterfactual economies (C3 to C6) have lower values of  $\lambda_{21}$  ( $\lambda_{12}$ ), the probability of acquiring (losing) the access to long-term debt markets while holding the other probability at the baseline level ( $\lambda_{21} = \lambda_{12} = \lambda_b = 0.25$ ). For instance, consider reducing the probability of gaining the access while the average duration of remaining at  $o = 1$  borrowing status is 4 years (C3 and C4). In other words, when it gets harder for entrepreneurs to issue a new debt or adjust their existing debt level, we observe substantial declines in aggregate output and factor prices in the long run. As discussed earlier, relatively more producers are subject to static borrowing with collateral constraints instead of long-term debt, further distorting the allocation of resources. The loss in aggregate productivity is about 7 percent when the value of  $\lambda_{21}$  is half of that in the baseline economy.

To understand the aggregate results more clearly, we look at the comparative statics at the disaggregate level. In particular, we highlight the mechanism of long-term debt and consumption smoothing motive by distinguishing entrepreneurs based on their borrowing status at a given instant. Let *type-1 entrepreneurs* be those currently active with the access to the loan markets. In contrast, *type-2 entrepreneurs* are active but without the chance of adjusting their debt.

Table 5 : Changes by Entrepreneur Type

shock frequency $\lambda_b$	0.25	0.20	0.05
rel. share of type-1	0.42	0.37	0.13
mTFP, type-1	0.64	0.62	0.51
mTFP, type-2	0.56	0.56	0.53
share of constrained type-1	0.63	0.64	0.80
share of constrained type-2	0.29	0.31	0.54
$k/k^*$ , type-1	0.74	0.73	0.60
$k/k^*$ , type-2	0.83	0.82	0.72

Table 5 reports the average changes in each type of entrepreneurs when we lower the value of  $\lambda_b$  from the calibrated model. Since the entering entrepreneurs do not have access to borrowing (type-2), it is intuitive that the relative share of type-1 entrepreneurs falls as the borrowing opportunity becomes less frequent. Since the capital choice is only limited by their wealth, the share of entrepreneurs with binding collateral constraints substantially increases in both types. This makes the actual choice of capital further away from the optimal level, and the distribution of capital and debt becomes more dispersed among producers. Thus, the productivity of each type endogenously falls as the implied duration of borrowing status increases. Since the number of type-2 entrepreneurs rather increases, their productivity falls relatively less than that of type-1.

In sum, we find the nontrivial role of long-term debt and borrowing opportunity in explaining the differences in aggregate and entrepreneurial productivity. In particular, as the active entrepreneurs with existing debt have to smooth their consumption over time against uninsurable risks, the incentive for taking a large amount of loan is endogenously discouraged in our model economy. While the availability of long-term debt is typically understood as productivity improving, we show the potential pitfalls in implementing financial reforms of easing entrepreneurs' access to borrowing in developing economies.

### 3.3 Costly Borrowing with Default Risk

As described in Section 2, the entrepreneurs in the model can use long-term debt to mitigate the collateral constraint. While our model features the collateral constraint as most papers in development literature, allowing entrepreneurs to have long-term debt works differently. In this section, we contrast a one-asset model such as Buera and Shin (2011) and a simplified version of our model (two-asset model). The key difference is how entrepreneurs use external financing. In one asset model, the parameter for collateral constraint  $\gamma$ , ( $k \leq \gamma a$ ) is two and entrepreneurs use within-period credit for production. In two-asset model,  $\gamma$  is one and entrepreneurs use long-term debt and are allowed to default.

The one-asset model assumes that the within-period credit is defaultable but it assumes that the default happens randomly and does not depend on borrowers' states.<sup>24</sup> Also, no one actually defaults, and credit terms are identical to all. This friction is backward looking. How much potential you have as an entrepreneur matters less under this friction but how much wealth you accumulated matters more. It may exaggerate the effects of selection by wealth; resource is allocated to wealthy entrepreneurs, not to productive entrepreneurs.

In contrast, in the two-asset model, entrepreneurs use long-term debt rather than within-period credit and the default choices depend on their states. Terms of credit are affected by default choices and more productive entrepreneurs can borrow at cheaper prices in an equilibrium. This friction is forward looking. Also, unlike  $\gamma$  determines the degree of friction in the one-asset model, the degree of friction are combination of many aspects of the debt contract (e.g., amortization rate, cost of default, and debt adjustment cost).

Table 6 compares aggregate quantities of one and two asset model. As mentioned above, in

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<sup>24</sup>Consider an entrepreneur with asset  $a$  who demands a loan  $a_i$ . The entrepreneur transforms her wealth-on-hand  $a + a_i$  into capital  $k = a + a_i$  without cost, which is used as collateral for the loan. The entrepreneur can default and once she chooses to do so, she loses the collateral. Importantly, this model assumes that the liquidation value of collateral is uncertain at the time of writing the contract. With probability  $1 - \frac{1}{\gamma}$  intermediaries recover the full value, but with probability  $\frac{1}{\gamma}$  they recover nothing. Thus to break even, intermediaries restrict the amount of loan to  $a_i < (1 - \frac{1}{\gamma})k$ .

Table 6 Aggregate quantities of one- and two-asset model

	Assets	Debt	K	L	Active	Default	Y
One asset (GE)	0.81	-	0.81	0.83	75	-	0.96
Two asset (PE)	2.62	-1.40	1.82	1.58	57	37	1.84
Two asset (PE, $\theta = 0.5$ )	0.65	-0.09	0.35	0.34	39	53	0.40
Two asset (PE, costly def.)	5.30	-6.70	1.41	1.25	52	40	1.44
Two asset (GE)	1.78	-0.79	0.99	0.86	44	48	1.04

Note: GE is general equilibrium and PE is partial equilibrium. Two-asset PE quantities are computed using one-asset GE prices. ‘Active’ and ‘Default’ is share of active entrepreneur and inactive entrepreneur with default flags.  $\theta$  is amortization rate and its value is 0.05 except the case with  $\theta = 0.5$ . In costly default case, utility cost of default rises from 1 to 10, retention rate falls from 0.1 to 0.01, and average duration with default record increases from 6 years to 12 years.

the two-asset model, more productive entrepreneurs can use cheaper loans and they demand more loans. Therefore, the degree of distortion of resource allocation is lower than one-asset model and the output is 91% larger in the two-asset model (PE). Several features of the two-asset model affect resource allocation. We tried two additional versions of the two-asset model: a version with i) faster amortization and ii) costly default. For the faster amortization case, entrepreneurs are required to pay 50% of the remaining principal at each instant instead of 5%. For the costly default case, we raise the utility cost of default from 1 to 10, lower the retention rate from 0.1 to 0.01, and increase the average duration with default record from 6 years to 12 years. When the default is costly, entrepreneurs avoid defaults thus the loan price becomes cheaper than the benchmark case. As a result, entrepreneurs use more debt. However, once entrepreneurs default they keep the default flag longer. This reduces entry and lower output. In the case of faster amortization, most entrepreneurs choose not to use debt as the payment requirement lowers their disposable income excessively and output is less than half of the one-asset model.

## 4 Conclusion

Models of economic and financial development address how costly external finance leads to a misallocation of resources across firms, thereby lowering aggregate total factor productivity. Until now, the literature has largely ignored the explicit modeling of debt. While retaining the assumption that a borrower's assets constrain the quantity of capital they may rent for production, we introduce long-term, illiquid, risky debt into a heterogeneous-agent model.

Debt is illiquid in our model for several reasons. First, adjusting debt involves transaction costs. Second, borrowers suffer idiosyncratic shocks that prevent them from taking new loans or paying off existing debt. When they are unable to actively adjust debt, producers either continue to make principal payments or default.

Risks of long-term debt amplify the misallocation we see from ordinary collateral constraints. Entrepreneurs' ability to rent capital is now not only limited by their assets but also their willingness to increase the volatility of their income. We find explicit modeling of risky debt and studying varying costs of using debt markets, substantially adds to the channels through which financial development affects GDP. Poorer entrepreneurs attempting to smooth consumption are reluctant to take on large, non-contingent debt that is required to augment their collateral and reach more efficient levels of production. In the aggregate, the share of production undertaken by financially constrained entrepreneurs falls relative to an economy with only collateral constraints. This increases the impact of the level of financial development on real economic activity.

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