

Financing Development: The Role of Long-term Debt

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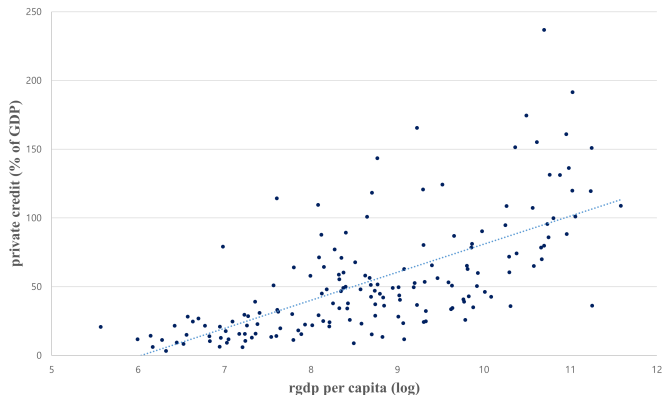
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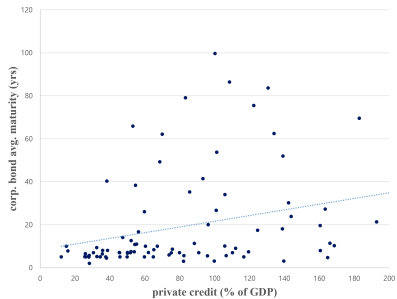
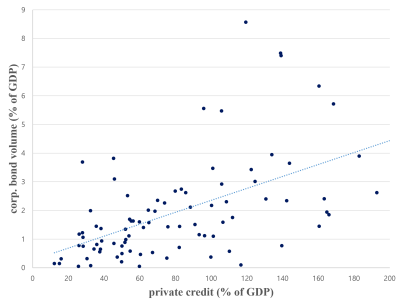
Motivation: Financial Development

“Financial deepening” as a robust relationship between domestic credit and output across countries



Motivation: Financial Development

Financial development and business lending in capital markets



▶ Access to Loans

Questions

- What are the macroeconomic implications of allowing producers to externally finance with risky, illiquid debt?
- How much does financial development in terms of access to debt financing affect aggregate productivity?
- How does idiosyncratic risk interact with saving, borrowing, and production decisions?

This Paper

- A macro-development model with long-term illiquid debt
 - Heterogeneous entrepreneurs with idiosyncratic risk
 - Costly borrowing with non-zero probability of default
- Quantitative investigation of the role of debt financing
 - Individual decisions of consumption, saving, production, entry, exit, and borrowing
 - Allocative efficiency across producers in the presence of financial frictions and default risk
 - Access to discount debt markets as an additional channel of financial development and resource misallocation

Preview of Results and Mechanism

- Substantial losses in aggregate productivity due to limited borrowing opportunities
 - Measured TFP falls by 5 to 7 percent, when access probability to debt market decreases from 0.25 to 0.125
 - Fewer entrepreneurs by 25 percent, engaged in production with insufficient capital
- Reluctance to take on large debt in the presence of idiosyncratic risk
 - Decisions of debt issuance, refinancing, and repayment are functions of individual state and uninsurable risk
 - Risk-averse borrowers maintain low levels of leverage on costly debt, deterring business growth

Related Papers

- **Resource misallocation and macro development**
Restuccia and Rogerson (2008), Hsieh and Klenow (2009, 2015)
- **Production heterogeneity and financial development**
Buera, Kaboski, and Shin (2011, 2015, 2021), Buera and Shin (2013), Chen, Habib, and Zhu (2023)
- **Heterogeneous firms with default risk**
Arellano, Bai, and Zhang (2018), Khan, Senga, and Thomas (2022), Ottonello and Winberry (2020), Jo et al. (2024)

Model

Model Overview

- A continuous-time model of heterogeneous entrepreneurs
 - Individuals with idiosyncratic shocks
 - Permanent workers
 - Entrepreneurs, either active or inactive
 - Competitive financial intermediaries
 - Government with balanced budget
- Financial frictions for entrepreneurs' external borrowing
 - Instantaneous borrowing with collateral constraints
 - Long-term debt financing subject to default risk
 - Stochastic access to discount debt markets

Individuals, Distribution, and Prices

- A unit measure of infinitely-lived individuals
 - CRRA utility from consumption; subjective discount rate ρ
 - Idiosyncratic shocks to productivity; ϵ for workers and z for entrepreneurs
 - Permanent workers; population share of s_w
 - Production by active entrepreneurs; others as interim workers
 - Endogenous exit (default) and entry, along with exogenous exit
 - Progressive income/profit tax $T(\cdot)$
- Competitive prices given the distribution of individuals, $g(\cdot)$
 - Risk-free interest rate $r(g)$, real wage rate $w(g)$, and discount debt price schedule $q(\cdot, g)$

Workers' Problem

- Given (a, ϵ) , a permanent worker solves the following HJB equation

$$\rho v^w(a, \epsilon) = \max_c \left[u(c) + \frac{\partial v^w(a, \epsilon)}{\partial a} \dot{a} + \sum \pi_{\epsilon\epsilon'} v^w(a, \epsilon') \right]$$

$$\dot{a} = \epsilon w + ra - c - T$$

$$a \geq \underline{a},$$

where $\pi_{\epsilon\epsilon'}$ is the transition probability of Markov chain for ϵ

Production and Entrepreneur Status

- Define an active producer's earnings with (a, z)

$$\pi(a, z) = \max_{k, l} \left[zk^\alpha l^\nu - wl - (r + \delta)k - \xi(z) \right]$$
$$k \leq \gamma a$$

- Short-term secured borrowing for capital, given liquid asset a
- Fixed operation cost $\xi(z)$ varying with productivity
- An indicator o of production activity and access to debt markets

o	entrepreneur status
1	active with debt financing opportunity
2	active without debt financing opportunity
3	inactive, interim worker
4	inactive with default flag, interim worker

Long-term Debt

- Access to debt markets
 - Debt issuance or refinancing only at $o = 1$, and partial borrowing for entry cost with $o = 3$
 - At $o = 2$, lower individual productivity ψz with $\psi < 1$
 - With probability λ_b , gain/lose access; (o, o') as an additional element of idiosyncratic risk
 - Fixed adjustment cost ξ_{b0} of existing debt, and proportional cost ξ_b for debt issuance
- Debt issuance, repayment, and default
 - Given the access, allowed to issue discount debt b' at competitive price $q(a', b', z, o)$
 - For existing b , repay $(r + \theta)b$ at each instant where θ is a fraction of principal
 - In case of refinancing, only $b' \leq b$ at corresponding $q(\cdot)$
 - Upon default, χa is forfeited while incurring utility cost of ξ_d
 - Default flag disappears at the rate of λ_d

Stopping Options

- At $\tau \geq 0$, an active entrepreneur with $(a_\tau, b_\tau, z_\tau, o_\tau)$ has 4 options, each corresponding to a value and choices of (a', b') (in the absence of associated costs)

	option	value	a'	b'
(i)	refinance and continue	v^r	$a_\tau - b_\tau + qb'$	$b' \leq b$
(ii)	exit after repaying debt	v^x	$a_\tau - b_\tau$	0
(iii)	default and exit	v^b	χa_τ	0
(iv)	start a business	v^e	$a_\tau + qb'$	$b' \leq \gamma_k \kappa(z)$

Entrepreneurs' Problem

- From $t = 0$, an individual with (a, b, z, o) solves

$$v(a_t, b_t, z_t, o_t) = \max_{\{c_t, (k_t, l_t)_{o \in \{1, 2\}}, \tau\}} \mathbb{E}_0 \left[\int_0^\tau u(c_t) dt + e^{-\rho\tau} v^*(a_\tau, b_\tau, z_\tau, o_\tau) \right]$$

$$(a_0, b_0, z_0, o_0) = (a, b, z, o)$$

$$v^*(a_\tau, b_\tau, z_\tau, o_\tau) = \begin{cases} \max\{v^r, v^x, v^b\} & , o = 1 \\ \max\{v^x, v^b\} & , o = 2 \\ v^e & , o = 3, \end{cases}$$

Entrepreneurs' Problem

- Following Moll (2016), the problem can be written using an HJB equation

$$\rho v(a, b, z, o) = \max_{c, (k, l)_{o \in \{1, 2\}}} \left[u(c) + \frac{\partial v(a, b, z, o)}{\partial a} \dot{a} + \sum \pi_{zz'} v(a, b, z', o) \right. \\ \left. + \mathbf{1}_{o \in \{1, 2\}} \left(\frac{\partial v(a, b, z, o)}{\partial b} \dot{b} \right. \right. \\ \left. \left. - \lambda_{10} (v(a, b, z, o) - v(a - b - \xi_{b0}, 0, z, 3)) \right. \right. \\ \left. \left. - \lambda_b (v(a, b, z, o) - v(a, b, z, o')) \right) \right] \\ \dot{a} = \begin{cases} \pi(a, z) + ra - c - (r + \theta)b - T & , o \in \{1, 2\} \\ zw + ra - c - T & , o \in \{3, 4\} \end{cases} \\ \dot{b} = -\theta b \\ v(a, b, z, o) \geq v^*(a, b, z, o),$$

where (o, o') is either $(1, 2)$ or $(2, 1)$

Values for Discrete Choices

- At the stopping time $\tau \geq 0$, values for possible choices

(i) Refinance and continue

$$v^r(a_\tau, b_\tau, z_\tau, o_\tau) = \max_{a', b'} v(a', b', z_\tau, o_\tau)$$

$$a' = a_\tau - b_\tau + q(a', b', z_\tau, o_\tau)b' - \xi_b b' - \xi_{b0}$$

$$b' \leq b_\tau$$

(ii) Exit after paying debt

$$v^x(a_\tau, b_\tau, z_\tau, o_\tau) = v(a - b - \xi_{b0}, 0, z_\tau, 3)$$

(iii) Default and exit

$$v^b(a_\tau, b_\tau, z_\tau, o_\tau) = v(\chi a_\tau, 0, z_\tau, 4) - \xi_d$$

Values for Discrete Choices

- At the stopping time $\tau \geq 0$, values for possible choices
- (iv) Entry with borrowing

$$v^e(a_\tau, 0, z_\tau, 3) = \max_{a', b'} v(a', b', z_\tau, 2)$$

$$a' = a_\tau + q(a', b', z_\tau, 2)b' - \xi_b b' - \xi_{b0} - \kappa(z)$$

$$b' \leq \gamma_k \kappa(z)$$

Debt Price Schedule

- Risk-neutral financial intermediaries
 - Take short-term deposits as the source of loan provision
 - Competitive valuation of individual default risk
 - Discount value of debt at issuance is equal to the expected sum of repayments
- For a loan b_0 offered to an active borrower at non-default region,

$$q(a_0, b_0, z_0, o_0)b_0 = \mathbb{E}_0 \left[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s + \theta) ds} (r_t + \theta) b_0 dt + e^{-\int_0^\tau r_s ds} b(a_\tau, b_\tau, z_\tau, o_\tau) \right]$$

Debt Price Schedule

- The corresponding PDE for debt price

$$\begin{aligned}(\theta + r)q(a, b, z, o) &= \theta + r + \frac{\partial q(a, b, z, o)}{\partial a} \dot{a} + \frac{\partial q(a, b, z, o)}{\partial b} \dot{b} \\ &+ \sum \pi_{zz'} q(a, b, z', o) - \lambda_{10}(q(a, b, z, o) - q(a - b - \xi_{b0}, 0, z, 3)) \\ &- \lambda_b(q(a, b, z, o) - q(a, b, z, o')) \quad \text{for } t \in [0, \tau),\end{aligned}$$

$$q(a, b, z, o) = \frac{\min[(1 - \chi)a, b]}{b} \quad \text{for } t = \tau$$

Quantitative Results

Parameter Values

Fixed Parameters		
	Value	Description
ρ	0.055	subjective discount rate
σ	2.00	risk aversion
ν	0.60	labor share, production
δ	0.07	depreciation rate
τ_1	0.181	tax progressivity
λ_d	0.20	shock intensity, default flag removal
s_w	0.50	population share of permanent workers

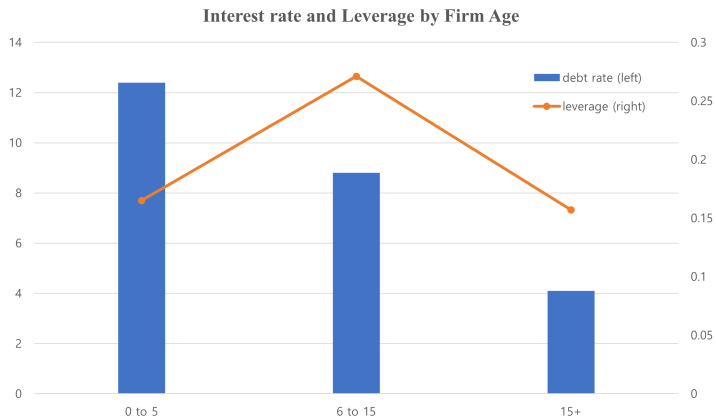
Parameter Values

Fitted Parameters		
	Value	Description
α	0.285	capital share, production
ψ	0.80	productivity loss without access
γ	1.50	collateral constraints
θ	0.05	debt amortization rate
χ	0.40	debt recovery rate upon default
τ_0	0.92	income tax factor
γ_k	0.75	borrowing for entry cost
λ_ϵ	0.10	shock intensity, worker productivity
λ_z	0.20	shock intensity, entrepreneur productivity
λ_b	0.25	debt market access rate
λ_{10}	0.05	exogenous exit rate
ξ_b	0.01	debt issuance cost, proportional
ξ_{b0}	0.00	debt adjustment cost, fixed
ξ_d	0.025	utility cost of default

Baseline Moments

Description	Data	Model	Source
real interest rate	0.04	0.04	-
real wage rate	-	1.66	-
capital-output ratio	2.40	1.84	BEA
labor share	0.60	0.60	BLS
exit rate	0.08	0.09	BDS
bankruptcy rate	0.03	0.04	OW (2020)
share of entrepreneurs	0.11	0.10	CD (2006)
aggregate leverage	0.37	0.32	Flow of Funds
share without debt	0.30	0.24	SBCS
earnings share at top 20%	0.65	0.66	SCF
earnings share at bottom 60%	0.14	0.16	SCF
share of entrepreneurs at wealth top 20%	0.26	0.40	CD (2006)
share of entrepreneurs at wealth top 1%	0.62	0.95	CD (2006)

Costly Borrowing by Firm Age



Aggregate Effects of Financial Frictions

	baseline	full access	frictionless
real interest rate	100	115.61	154.43
real wage rate	100	114.88	122.72
output	100	113.79	135.26
capital	100	118.09	158.75
employment	100	99.05	110.22
measured TFP	100	109.14	111.85
aggregate leverage (pp)	-	3.62	9.80
default rate (pp)	-	0.45	5.61
share of entrepreneurs (pp)	-	0.39	-6.10

Counterfactuals

- Relative changes from the baseline model by varying λ_b
 - "Financial deepening" as λ_b increases; active debt issuance via corporate bond markets
 - Developing economies with low λ_b ; longer duration with or without access to debt markets
- Alternatively, consider asymmetric probability between acquiring and losing access, λ_{21} and λ_{12}
 - Lower (higher) λ_{12} (λ_{21}) representing financial development

Counterfactuals

Relative changes from the baseline model with $\lambda_b = 0.25$

	baseline	C1	C2
λ_b	0.25	0.125	0.10
real interest rate	100	94.38	92.56
real wage rate	100	93.75	91.16
output	100	98.88	97.61
capital	100	101.17	99.93
employment	100	105.46	107.07
measured TFP	100	95.45	93.71
aggregate leverage (pp)	-	-11.71	-14.73
default rate (pp)	-	-3.24	-3.62
share of entrepreneurs (pp)	-	-2.76	-3.58

Counterfactuals

Relative changes from the baseline model with $\lambda_b = 0.25$

	baseline	C3	C4	C5	C6
λ_{21}	0.25	0.20	0.125	0.25	0.25
λ_{12}	0.25	0.25	0.25	0.20	0.125
real interest rate	100	98.28	96.37	100.83	103.39
real wage rate	100	97.02	90.73	101.30	103.69
output	100	98.54	94.97	101.49	104.31
capital	100	99.77	98.21	101.46	104.66
employment	100	101.57	104.66	100.19	100.60
measured TFP	100	97.69	92.88	100.95	102.60
aggregate leverage (pp)	-	-3.64	-9.84	-0.40	-1.09
default rate (pp)	-	-1.63	-3.44	-0.20	-1.31
share of entrepreneurs (pp)	-	-0.80	-2.34	-0.09	-0.31

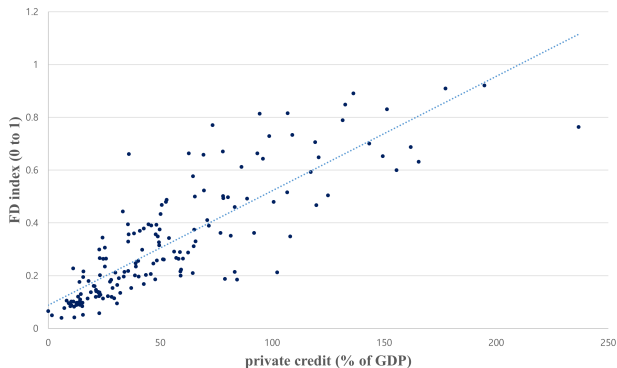
Concluding Remarks

Takeaways

- A quantitative macro-development framework
 - Idiosyncratic risk, production heterogeneity, and financial frictions with long-term illiquid debt
 - A new channel of misallocation addressing cross-country differences in aggregate productivity
- Evidence on debt financing and capital market access in developing economies
 - Substantial losses in allocative efficiency due to limited access to long-term financing

Extra Slides

Domestic Credit and Financial Development



◀ Return

Evidence: Access to External Borrowing

Share of firms with loans or bank credit

