Aggregate Productivity with Heterogeneous Agents

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How to Aggregate with Heterogeneous Agents

- Individuals disagree over how to rank economy-wide allocations.
- When the economy changes, there are winners and losers.
- Basic question: how should we define the aggregate value of a change?
- We define it by the surplus — factor endowments left over — if winners compensate losers.
- This measures the amount of factor savings after we restore the same welfare levels, so we call it the change in **aggregate productivity**.
- Every rep. agent welfare result can be translated into a result about aggregate productivity with het. agents — Hulten, misallocation formulas, gains from trade formulas, etc.
- We characterize aggregate productivity in terms of observables (expenditure shares, price elasticities, no Pareto weights, cardinal utilities, etc.)

Relation to Prior Work

- We build on the approach by Allais (1978) and Debreu (1951).
- Related to traditional cost-benefit, which measures aggregate value of a change by:

  How much money is left over if winners compensate losers?
- Usual implementation, sum of comp. variations/real GDP, won't answer question correctly.
- These can be positive even when winners **cannot** compensate losers.
- Cost benefit also assumes costless transfers are available, which may not be the case.
- We provide a GE counterpart to cost benefit.

Difference to Social Welfare Functions

- The dominant approach in many literatures is to use a SWF.
- Although our definition is not a SWF, it can accommodate SWF-like analysis.
- If agents' preferences over economy-wide allocations are all the same, then our measure is the change in that commonly-held SWF in TFP-equivalent terms.
- More generally, our measure automatically accounts for equity concerns of agents in the model without requiring that they all agree.
- No need for analyst to impose externally specified Pareto weights or cardinal utility functions.
- For most of talk, focus on case with selfish preferences only; altruism at end if time.

Research Agenda

- This paper is a general framework paper part of broader agenda.
- Our broader research agenda applies this analysis to different settings:
  - *“Misallocation due to Incomplete Markets”* (in closed & open economies)
  - *“Aggregate Welfare with Discrete Choice Across Places and Jobs”*
  - *“An Approximate General Theory of Second Best”* (optimal policy to max agg prod)
  - *“Monetary Policy without Redistributive Concerns”* (in HANK environments)

Selection of Other Related Papers

- Policy analysis without SWFs:

  Auerbach, Kotlikoff & Skinner (1983); Benabou (2002); Farhi & Werning (2012); Schulz, Tsyvinski & Werquin (2023); Aguiar, Amador, Arellano (2024).

- Aggregate Welfare in Heterogeneous Agent Models:

  Heathcote, Storesletten, Violante (2008); Conesa, Kitao, & Krueger (2009); Davila, Hong, Krusell, & R. Rull (2012); Antras, de Gortari & Itskhoki (2017); Kim & Vogel (2020); Boar and Midrigan (2022); Galle, Rodriguez-Clare & Yi (2023); Aguiar, Itskhoki & Mukhin (2024); Redding & Rossi-Hansberg (2025) .

- Decompositions of Social Welfare Functions:

  Bhandari, Evans, Golosov & Sargent (2021); Davila & Schaab (2023, 2024); Donald, Fukui & Miyauchi (2024).

- Disaggregated Production:

  Hulten (1978); Hsieh-Klenow (2009); Gabaix (2011); Acemoglu, Carvalho, Tahbaz-Salehi, Ozdaglar (2012); Baqaee-Farhi (2019, 2020); Liu (2019); Bigio-La’O (2020); Buera-Trachter (2024).

Economic Environment: Arrow-Debreu with Distortive Wedges

- Households $h \in H$ with preferences $\succeq_h$ over consumption vectors $\boldsymbol{c}_h$.
- There are N goods and F factor endowments.
- Producer of good i minimizes costs given general neoclassical production function:

$$y_i = z_i G_i \left(\{ y_{ij} \}, Z \{ l_{if} \} \right).$$

  where z_i is i's productivity and Z is total factor productivity.
- Distorting implicit or explicit bilateral tax, denoted by $\boldsymbol{\mu}$, so $p_i = \mu_i \times mc_i$. Revenues rebated.
- Standard resource constraints for goods and primary factor endowments.
- Environment is very general and flexible
  (e.g. production networks, incomplete markets, nominal rigidities, international trade, etc.)

Definition of Aggregate Productivity given Lump Sum Transfers

- Index techn. and wedges by t; let $t = 0$ be status quo and $t > 0$ counterfactual of interest.
- So $\boldsymbol{z}(t)$ and $\boldsymbol{\mu}(t)$ are some counterfactual technologies and distortions.
- Let $\mathcal{C}(t, Z)$ be set of allocations supported as equil. given $(\boldsymbol{z}(t), \boldsymbol{\mu}(t), Z)$ and some transfers.
- $A(t)$ is maximum reduction in Z at t needed to make everyone indifferent to initial allocation.

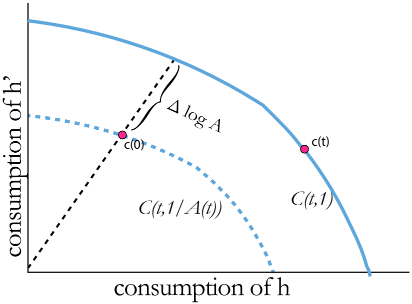
$$A(t) \equiv \max \left\{ Z \in \mathbb{R} : \text{there is } \boldsymbol{c} \in \mathcal{C}(t, 1/Z) \text{ and } \boldsymbol{c}_h \succeq_h \boldsymbol{c}_h(0) \text{ for every } h \right\}.$$

- If $A(t) > A(0) = 1$, there exists a potential Pareto improvement.

  e.g. if $A(t) = 1.1$, it is possible to discard 10% of every factor and keep everyone indifferent.

Graphical illustration

- Shock pushes frontier but in equilibrium, one agent is worse off.
  After transfers, goods left over, so $A > 1$.



- No stance on who gets the left over resources.

Nesting Other Aggregate Measures

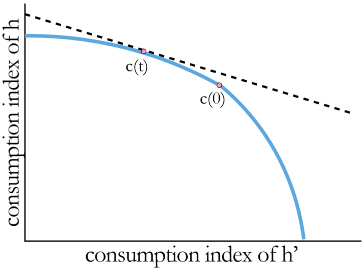
- Start by considering benchmark where $A(t)$ coincides with existing popular measures.
- If everyone has identical, homothetic preferences & faces same prices, then

$$A(t) = \text{}^{\text{consumption equivalent}}_{\text{for positive rep agent}}(t) = \text{real GDP}(t) = {}^{\text{income relative}}_{\text{to compensating income}}(t).$$

- But lots of examples violate one of these assumptions (e.g. non-homothetic, incomplete markets, different tastes).
- Outside of these assumptions, other measures go haywire (violate Pareto criterion)!
- Rest of the talk focuses on characterizing $A(t)$ when these conditions do not hold.

Sum of Compensating Variations with Different Preferences

- Consider transfer from h' to h with different preferences.



$$\frac{\text{income at (t)}}{\text{compensating income at (t)}} > 1.$$

- After winners compensate losers, they have money left over (Boadway 1974). But, unfeasible.
- But $A(t) = 1$. This economy is Pareto efficient.

Sum of Compensating Variations with Different Prices

- Suppose there is unit endowment of consumption good.
- Agent 1 pays higher markup μ_1 than agent 2 who pays μ_2.
- A lump-sum transfer from agent 2 to agent 1 raises one's spending share $\Delta\omega_1$, and

$$\Delta \log \frac{\text{income at (t)}}{\text{compensating income at (t)}} \approx \bar{\mu} \left[\frac{\mu_1 - \mu_2}{\mu_1 \mu_2} \right] \Delta\omega_1,$$

- One unit of 1's consumption is worth $\mu_1/\mu_2 \geq 1$ units of agent 2's consumption.
- But $A(t) = 1$. This economy is Pareto efficient.

Compensated Representative Agent

- In general, aggregate productivity can be computed directly using its definition.
- But in many cases, easier solve for a dual problem.
- *Equilibrium with compensated rep agent* is equilibrium with the same technologies & wedges as the original economy, but with a rep agent that has preferences

$$U(\boldsymbol{c}) = \min\left\{\tilde{u}_1(\boldsymbol{c}_1), \ldots, \tilde{u}_H(\boldsymbol{c}_H)\right\},$$

  where $\tilde{u}_h(\boldsymbol{c}_h)$ is h's individual consumption-equivalents (invariant to monotone transforms).
- Agent h's consumption-equivalent $\tilde{u}_h(\boldsymbol{c}_h)$ solves

$$u_h\left(\frac{\mathbf{c}_h}{\tilde{u}_h}\right) = u_h(\mathbf{c}_h(0))$$

Computing $A(t)$ via Equilibrium with Representative Agent

- **Theorem**

  Given some mild conditions

  $$A(t) = U(\boldsymbol{c}^{comp}(t)) \qquad \text{for every } t,$$

  at $t = 0$, prices and quantities in equil. with comp. rep agent the same as decentralized equil:

  $$\boldsymbol{p}^{\text{comp}}(0) = \boldsymbol{p}(0), \qquad \boldsymbol{c}^{\text{comp}}(0) = \boldsymbol{c}(0).$$

- So we can use representative agent results to study $A(t)$ in heterogenous agent economies.

  (e.g. Hulten; Baqaee-Farhi; Arkolakis-Costinot-Rodriguez-Clare; Harberger; Hsieh-Klenow, etc.)

Effect of Productivity Shocks in Competitive Economy

Hulten's Theorem (1978)

With perfect competition $\boldsymbol{\mu} = 1$, in response to $\Delta \log z_i$, to a first-order

$$\Delta \log U(\boldsymbol{c}^{comp}) \approx \frac{sales_i}{GDP} \times \Delta \log z_i \equiv \lambda_i \Delta \log z_i$$

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- Under perfect competition, $A(t)$ obeys Hulten with arbitrary heterogenous agents.

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Beyond Hulten's Theorem, Baqaee & Farhi (2019)

With perfect competition, in response to $\Delta \log z_i$, to a second-order

$$\Delta \log A \approx \lambda_i \Delta \log z_i + \frac{1}{2} \Delta \lambda_i^{\text{comp}} \Delta \log z_i .$$

where $\Delta \lambda_i^{\text{comp}}$ is change in sales share with as-if rep agent (paper characterizes $\Delta \lambda^{\text{comp}}$).

Porting Arkolakis et al. (2012) for Gains from Trade

- Example application: Losses from autarky with heterogeneous agents.

- With one agent, then Arkolakis et al. (2012) show losses from autarky are:

$$\Delta \log A = \frac{\log \overbrace{s_h}^{\text{share of spending on domestic}}}{\underbrace{\theta_h - 1}_{\text{trade elasticity}}}.$$

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$$\Delta \log A \approx \underbrace{\mathbb{E}_\omega \left[\frac{\log s_h}{\theta_h - 1} \right]}_{\text{first order}} - \underbrace{\frac{1}{2} Var_\omega \left[\frac{\log s_h}{\theta_h - 1} \right]}_{\text{second order}}.$$

where ω is cross-sectional distribution of spending.

- Loss greater with heterog. import shares. More exposed agents need more compensation.

Porting Misallocation Results

- Use theorem to characterize losses from distortions.

Harberger's Triangles (Harberger, 1971; Baqaee & Farhi, 2020)
Suppose we eliminate all wedges, to a second-order:

$$\Delta \log A = \Delta \log U(\boldsymbol{c}^{comp}) \approx -\sum_i \frac{sales_i}{GDP} \times \underbrace{\left(\frac{1}{2} \log \mu_i \Delta \log q_i^{comp}\right)}_{\substack{\text{area of} \\ \text{deadweight loss triangle}}},$$

where Δq_i^{comp} is first-order change in quantity of i in equilibrium with compensated rep agent.

- In paper we characterize Δq_i^{comp} in terms of expenditure shares and price elasticities.

Example: Misallocation with Heterogeneous Agents

- Suppose

$$u_h(\boldsymbol{c}_h) = \left[\sum_i c_{hi}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

- Household h pays markup μ_{hi} on good i. Goods made from labor endowment.
- If there is a single agent, as in Hsieh-Klenow (2009), then:

$$\Delta \log A \approx \frac{\theta}{2} Var\left[\log \boldsymbol{\mu}\right].$$

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- With heterogeneous agents, misallocation depends on average **conditional** variance:

$$\Delta \log A \approx \frac{\theta}{2} \mathbb{E}_{\omega}[Var\left[\log \boldsymbol{\mu}_h | h\right]].$$

- No comparison of average markups across individuals — those are purely redistributive.

- Companion paper uses this idea to measure *misallocation due to incomplete markets*.

Aggregate Productivity with Costly Redistribution

- Recall that

  $A(t)$ **is how much we can reduce TFP to make everyone indifferent given lump-sum transfers.**

- But we can handle cases without lump-sum too.

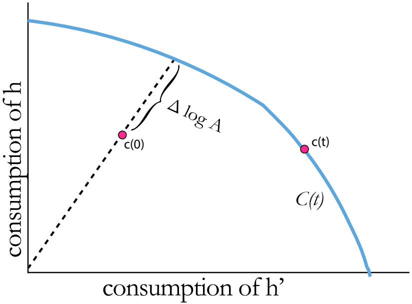
  $A^{\text{costly}}(t)$ **is how much we can reduce TFP to make everyone indifferent given costly redistribution.**

- Let $\mathcal{C}^{\text{costly}}(t, Z)$ be set of allocations given $(\boldsymbol{z}(t), \boldsymbol{\mu}(t), Z)$, and some transfers & taxes in $\mathcal{T}$.
- $A^{\text{costly}}(t)$ uses $\mathcal{C}^{\text{costly}}(t, Z)$ in place of $\mathcal{C}(t, Z)$.

Aggregate Productivity with Costly Redistribution

- Graphical illustration:

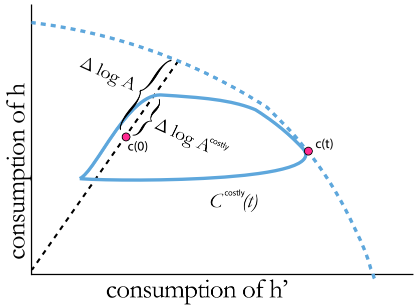
  shock pushes out frontier but in equilibrium, one agent worse off.
  after transfers, goods left over, so $\Delta \log A > 0$.



Aggregate Productivity with Costly Redistribution

- Graphical illustration:

  if only distortionary labor tax available, compensating loser causes misallocation.



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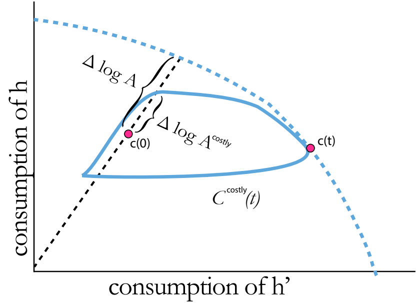

- In the paper, we show

$$\Delta \log A^{\text{costly}} \approx \Delta \log A - \text{deadweight loss triangles from redistribution.}$$

ACR Example Revisited with Distortionary Taxes

- Consider again the gains from trade relative to autarky with heterogeneous import shares.
- Now agents have preferences over consumption bundle (of domestic & foreign) and leisure.
- Substitution elast. between cons-leisure is ρ, all work same number of hours in status-quo.
- Suppose compensations must be financed by linear labor income tax.
- The losses from autarky are:

$$\Delta \log A^{\text{linear tax}} \approx \Delta \log A^{\text{lump-sum}} - \underbrace{\frac{1}{2}\rho \Omega_c (1-\Omega_c)(d \log \tau^*)^2}_{\text{2nd order losses from distorting taxes}},$$

  with two agents, this is $d \log \tau^* = \frac{\omega_h}{\theta - 1}\left[\log s_{h'd} - \log s_{hd}\right] > 0$.

- Losses from limited redist. growing in ρ and heterogeneous exposure and falling in θ.

Quantitative Example: Impact of Rise of China on the US

- Off-the-shelf quant. trade model, (Baqaee-Farhi, 2024) — 10 regions & 30 industries, IO.
- Low-, medium-, high-skill labor & capital in each industry. Each factor-industry is an agent.
- China grows from 5% of world GDP to 20% (2008 to 2023). What is effect on the US?
- Compensations among US agents from lump-sum transfers or from import tariffs.

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	National labor market	Industry-level labor market
Available Instruments	$\Delta \log A$	$\Delta \log A$
Lump-sum transfers	0.010	0.008
Tariffs with targeted rebates		
Tariffs with untargeted rebates		

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- Change in aggregate productivity depends on:
  1. How heterogeneous is exposure of workers to the shock (in this case, flexibility of labor market).
  2. How distortionary is redistributive tool.

Summary

- Household heterogeneity central to much of modern macro.
- Yet, still want aggregate measures that summarize welfare-relevant outcomes & guide policy.
- Cost-benefit in general equilibrium with policy limits on transfers.
- Counterfactual question about observables, no Pareto weights or cardinal utility.
- Characterized in terms of supply and demand curves.
- Lots of open questions to explore with this type of measure.

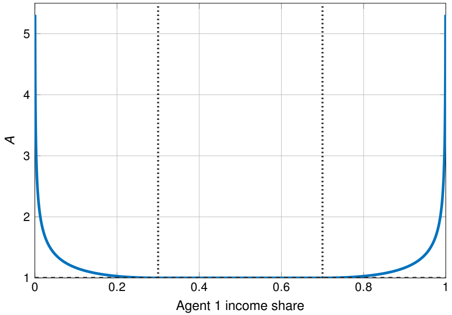
Appendix Slides

Altruism and Misallocation

- Consider two agents with altruistic preferences:

$$u_i(c_i, c_{-i}) = (1-\beta)\log c_i + \beta \log c_{-i}.$$

- Resource constraint: $c_1 + c_2 = 1$. Status quo: $c_1 = z_1$, $c_2 = 1 - z_1$.
- Misallocation, $A > 1$, only if inequality is large enough relative to altruism: $z_1 \notin [\beta, 1-\beta]$.



- If $\beta = 1/2$, then A is misallocation according to a utilitarian SWF.

Formal Economic Environment

- Households $h \in \{1, ..., H\}$ with preferences $\succeq_h$ over consumption vectors $\boldsymbol{c}_h \in \mathbb{R}^N$:

$$\max u_h(\boldsymbol{c}_h) \text{ such that } \sum_i p_i c_{hi} \leq \sum_f \omega_{hf} w_f L_f + T_h.$$

- Producer of good i minimizes costs given general neoclassical production function:

$$\min \sum_j p_j y_{ij} + \sum_f w_f l_{if}, \text{ such that } y_i = z_i G_i \left(\{y_{ij}\}, Z \{l_{if}\} \right).$$

Z is total factor-augmenting productivity.

- Wedges μ_i on i, explicit (e.g. tax/markup) or implicit (e.g. borrowing constraint).

$$p_i = \mu_i mc_i.$$

Formal Economic Environment

- Resource constraint for goods and factors:

$$\sum_j y_{ji} + \sum_h c_{hi} \leq y_i, \qquad \text{and} \qquad \sum_i l_{if} \leq z_f L_f.$$

- Aggregate budget balance.

$$\sum_h T_h = \sum_i p_i y_i \left(1 - \mu_i^{-1}\right).$$

- Decentralized equilibrium with wedges:

  Utility maximization, cost minimization, $p_i = \mu_i mc_i$, budget balance, & resource constraints.

Formal Definitions of Other Aggregate Measures

- Chain-weighted real GDP:

$$\log \text{Real GDP}(t) = \int_0^t \sum_i \frac{p_i(s)c_i(s)}{\sum_{i'} p_{i'}(s)c_{i'}(s)} \frac{d \log c_i(s)}{ds} ds.$$

- Sum of compensating variations (Kaldor-Hicks or cost-benefit analysis)

$$\frac{\text{income at (t)}}{\text{compensating income at (t)}} = \frac{\sum_h e_h(\boldsymbol{p}(t), u_h(t))}{\sum_h e_h(\boldsymbol{p}(t), u_h(0))}.$$

- Consumption-equivalent variation for positive representative agent (if it exists)

$$u^{RA}\left(\boldsymbol{c}^{RA}(t)/A^{RA}(t)\right) = u^{RA}\left(\boldsymbol{c}^{RA}(0)\right).$$