

Financial Crisis Cycles

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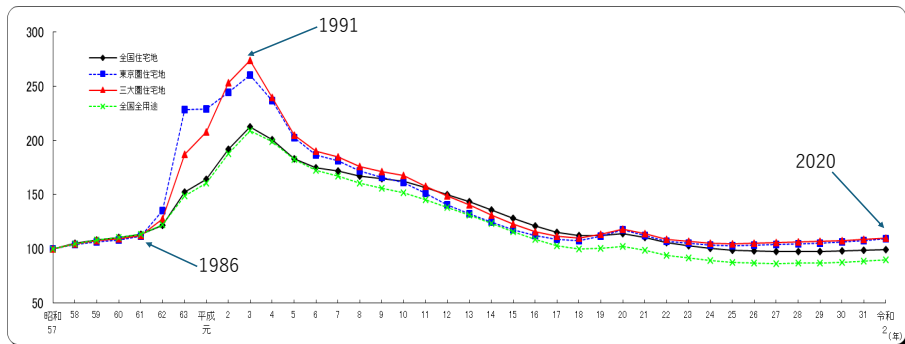
June 2026

Motivations

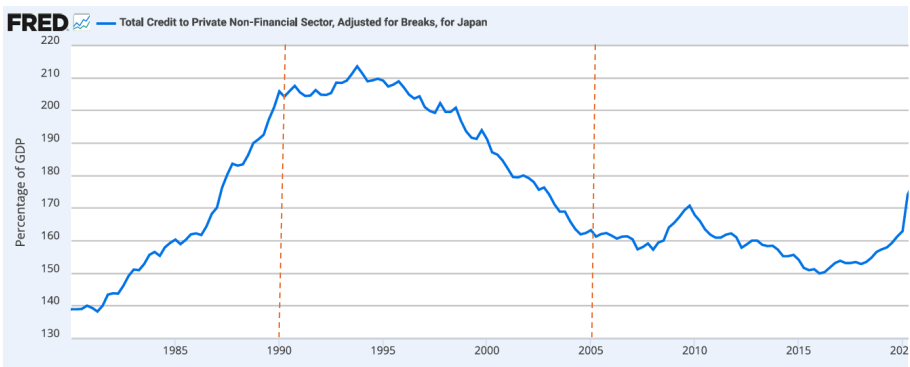
- Construct a simple theoretical model of financial crises to:
 - Account for empirical regularities
 - Provide new perspectives and policy implications
- Empirical regularities in financial crises
 - 1 **Booms** in asset prices and credit
 - 2 Subsequent **busts** in asset prices
 - 3 **Deep and persistent declines** in productivity

Japan: Land Prices before/after the Bubble Collapse

Index (1982=100)

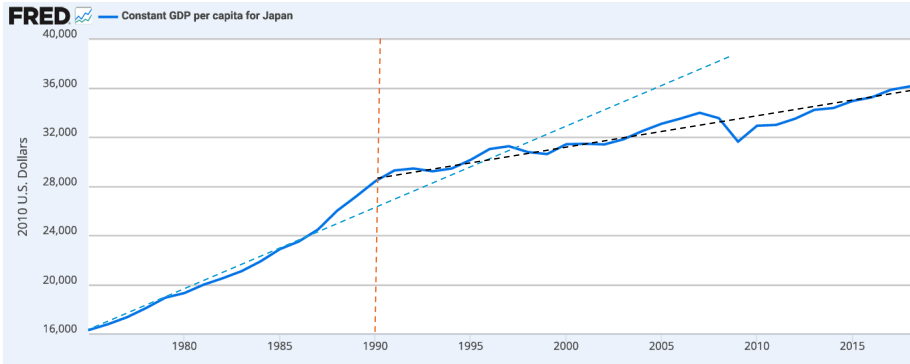


Japan: Total Credit to Private Non-Financial Sector



Source: Bank for International Settlements via FRED®

Japan: GDP per capita before/after the Bubble Collapse



Source: World Bank via FRED®

Model Mechanism

A simple dynamic model of a financial crisis

1 Risk-shifting bubbles

- Firms purchase assets from banks using credit.
- Optimism drives up asset prices.
- Credit-financed asset purchases amplify the boom.

2 Corporate debt overhang

- After the burst, excessive debt remains.
- High leverage disincentivizes effort, suppressing productivity.

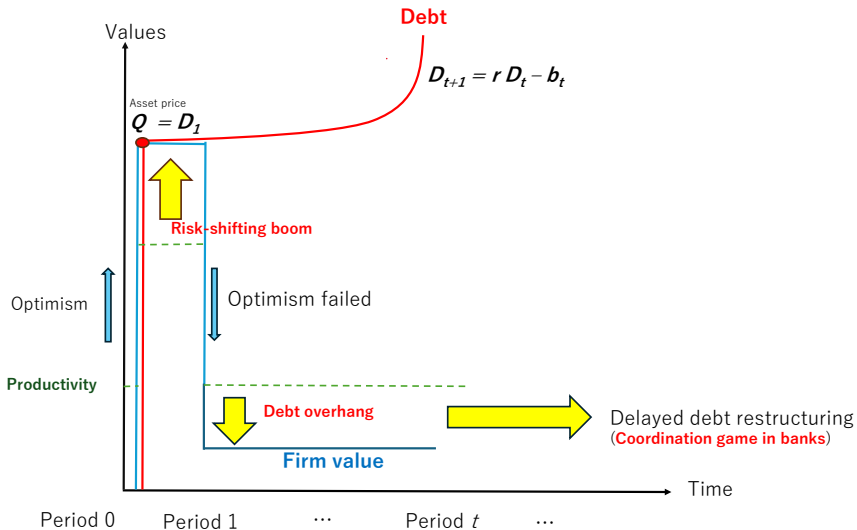
3 Coordination game in a bank

- Bank knows: Debt restructuring could restore the productivity immediately.
- Conflicts of interest within the bank regarding debt restructuring
- Coordination failure prevents restructuring when debt is high.
⇒ High debt levels trap banks into rolling over inefficient loans.

- **Key Takeaway:** Post-crisis stagnation can emerge from **borrower balance sheet problems.**

Dynamics of debt and firm value

Firm purchases asset from bank on credit



Why do we need a new model?

- **Shift in focus from lenders to borrowers**

- **Existing literature:** Emphasizes financial frictions (e.g., financial accelerators) and bank-side policies (e.g., macroprudential regulations, monetary policy)
- **This paper:** Highlights the significance of borrower balance sheets and borrower-side policies (e.g., debt restructuring/forgiveness, balance sheet rehabilitation)

Policy implications

- Ex-post policy responses to crisis:
 - Bank subsidies (= Bank recapitalization conditional on debt restructuring)
 - Borrower subsidies (= Fiscal policy to stimulate aggregate demand)
- Main results:
 - **Bank recapitalization outperforms borrower bailouts in resolving debt overhang**
 - **Efficiency of bank subsidies:**

Bank subsidies, contingent on debt restructuring, entail lower fiscal costs than borrower subsidies
 - **Potential pitfalls of borrower subsidies:**

If amount is not exceptionally large, borrower subsidies can **hinder** debt restructuring (⇒ Fiscal stimulus inadvertently impede productivity recovery)

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Literature: Empirical regularities

- **Credit-fueled asset price booms** may lead to financial crises followed by deep and persistent recessions:
 - **Jordà, Schularick, and Taylor (2015)**, Greenwood, Hanson, Shleifer and Sørensen (2022)
- Financial crisis followed by persistent **productivity slowdown**
 - Duval et al. (2020), **Hayashi and Prescott (2002)**, Kehoe and Prescott (2002)
 - Zombie lending: Caballero, Hoshi and Kashyap (2008)
- Corporate-credit booms as an indicator of financial crises
 - Greenwood et al. (2022), Jordá et al. (2022), Ivashina et al. (2024), Kornejew et al. (2024).

Our model: Integrated account for asset price, credit and productivity

Literature: Theoretical ingredients

- Risk shifting booms of asset prices
 - Allen and Gale (2000), Allen, Barlevy and Gale (2022)
- Debt overhang
 - Sachs (1988), Krugman (1988), Lamont (1995), Occhino and Pescatori (2015), Kobayashi, Nakajima and Takahashi (2022)
 - Empirics: Honda, Ono, Uesugi and Yasuda (2024)
 - Borrowing constraints for long-term debt
 - Albuquerque and Hopenhayn (2004), Kovrijnykh and Szentes (2007), Aguiar, Amador and Gopinath (2009)

Our model: New attempt to combine these theories

Literature: Financial crisis

- Source of inefficiency \Rightarrow **Our model: Debt overhang** (No borrowing constraints)
 - Pecuniary externality that tightens the borrowing constraint: Benigno et al. (2023); Bianchi (2011, 2016); Bianchi and Mendoza (2010); Gertler, Kiyotaki, and Queralto (2012); Jeanne and Korinek (2020); Lorenzoni (2008)
 - Coordination failure: Diamond and Dybvig (1983); Gertler and Kiyotaki (2015); Keister (2016)
- Bailout policy \Rightarrow **Our model: Bank subsidies are more efficient**
 - Bianchi (2016); Chari and Kehoe (2016); Green (2010); **Haavio, Ripatti and Takaloa (2025)**; Keister (2016)

Literature: Zombie lending

- **Zombie lending:** Bank loans with distorted incentives to non-viable firms
 - Japan: Peek and Rosengren (2005), Caballero, Hoshi and Kashyap (2008)
 - Acharya, Lenzu and Wang (2024) and references therein
- **Zombie firms are intrinsically unproductive and exert negative congestion externalities.** Should be eliminated. (CHK 2008)
- Becker and Ivashina (2022): Inefficient bankruptcy procedures amplify zombie lending

Our model: Debt reduction restores the efficiency of zombie firms

- Nakamura and Fukuda (2013): Zombie firms in the 1990s revived in the 2000s

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Theoretical Ingredients

- **Risk-shifting asset boom** (Allen and Gale 2000; Allen, Barlevy and Gale 2022)
 - Firms buy the risky asset by borrowed money and can default on the debt
 - Borrowers bid up the ex-ante asset price by shifting the risk to banks
- **Debt overhang** (Sachs 1988; Krugman 1988; Kobayashi, Nakajima and Takahashi 2022)
 - Firms can produce output from the risky asset
 - When the lenders take all, borrowers do not expend efforts

Intuition
- **Coordination game on debt restructuring**
 - Debt restructuring restores borrower productivity and improves bank payoffs
 - Bank managers and stakeholders in the bank engage in coordination games
 - Coordination failure arises when debt is large \Rightarrow delayed debt restructuring

Interest rates in the model of dynamic debt overhang

- Debt dynamics:

$$D_{t+1} = r_t D_t - b_t, \quad \text{without debt restructuring.}$$

- **Determination of the interest rate r_t ?**

- D_t grows over time
- The value D_t cannot be fully repaid

- **We follow Allen and Gale (2000)**

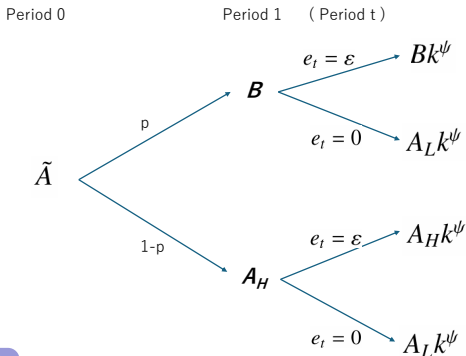
- Introducing safe assets
- Banks cannot distinguish safe and risky loans
⇒ Loan rates are determined by the risk-free rate,

$$r_t = \frac{1}{\beta}$$

- Banks suffer losses from risky loans due to information asymmetry and inferior technology

Structure of the model (without debt restructuring)

- Time periods: $t = 0, 1, 2, \dots$
- Banks and firms with measure one, respectively
- Risky asset: land, $k = 1$ (fixed supply).
- Production technology: $y = \tilde{A}k^\psi$, where $A_L < A_H \ll B$



Risk-shifting bubble

- Effort e_t is disutility for the firm

Structure of the model (without debt restructuring)

● Period 0

- Optimism about land productivity prevails
- Firms purchase k with debt $D_1 = Qk$

● Period 1

- Productivity of k is revealed = Optimism fails or is realized.
- Productivity is invariant from period 1 onward.
- Productions take place, $y_1 = Ak^\psi$.
- Repayment of debt $b_1 = r_1^L D_1 - D_2$.

● Period $t (\geq 2)$

- Productions take place, $y_t = Ak^\psi$.
- Repayment of debt $b_t = r_t^L D_t - D_{t+1}$.

Firm's problem

- We omit safe assets, x_t . We set $r = \beta^{-1}$.

Full version

- **Period 0:**

$$V_0 = \max_{D_1, k} \beta \mathbb{E}_{\tilde{A}} [V(D_1; \tilde{A}, k)], \quad (1)$$

$$\text{s.t. } D_1 = Qk.$$

- **Period $t (\geq 1)$:** \tilde{A} is revealed to be $A \in \{A_H, B\}$.

$$V(D; A, k) = \max_e c - e + \beta V(D_{+1}; A, k), \quad (2)$$

$$\text{s.t. } c + b(e) = y(e, A, k),$$

$$D_{+1} = rD - b(e),$$

Bank's problem

- We omit safe assets, x_t . We set $r = \beta^{-1}$.

Full version

- Period 0:**

$$\max_{k \in [0,1]} \beta \mathbb{E}_{\tilde{A}}[d(D_1, e_1; \tilde{A}, k)], \quad (3)$$

$$\text{s.t. } D_1 = Qk,$$

$$e_1 = e(D_1; \tilde{A}, k),$$

- Period $t (\geq 1)$:**

$$d(D, e; A, k) = \max_b b + \beta d(D_{+1}, e_{+1}; A, k), \quad (4)$$

$$\text{s.t. } 0 \leq b \leq y(e, A, k),$$

$$0 \leq b \leq rD,$$

$$D_{+1} = rD - b,$$

$$e_{+1} = e(D_{+1}; A, k).$$

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Equilibrium without debt restructuring

- Debt restructuring is prohibited \Rightarrow Debt grows with $D_{t+1} = rD_t - b_t$
- First, we describe **debt overhang** in period t for $t = 1, 2, \dots$.
- Then, we go back to **risk-shifting bubble** in period 0.

Debt overhang (Intuition)

- Consider a one-period model, in which a firm owes debt D .
- With additional effort e , the firm can produce

$$y = \begin{cases} y_L \equiv A_L k^\psi & \text{for } e = 0, \\ y_H \equiv A_H k^\psi & \text{for } e = \varepsilon, \end{cases}$$

where the bank can seize y and $y_L < y_H \ll D$.

- Firm's options:
 - $e = 0 \Rightarrow V = \max\{0, y_L - D\} - e = 0$
 - $e = \varepsilon \Rightarrow V = \max\{0, y_H - D\} - e = -\varepsilon < 0$
 - In equilibrium, $e = 0$ is chosen and **the productivity is low**: $A_L (< A_H)$.
- This intuition applies to dynamic settings.

Equilibrium

Definition

- Markov perfect equilibrium
- the set of $Q, k, D, \{V(D; A, k), d(D; A, k)\}, \{e(D; A, k), b(D; A, k)\}$ that satisfies the following conditions:
 - Given Q, k and D_1 solve (1) and (3), and the market clears, $k = 1$.
 - Given that \tilde{A} is revealed to be A , the policy functions solve (2) and (4), respectively, yielding the corresponding value functions.
 - Debt D evolves according to $D_{+1} = rD - b$.

Equilibrium without debt restructuring

Proposition

The Markov perfect equilibrium is uniquely given by

$$\text{Firm value: } V(D; A, k) = \begin{cases} V_{\max} - rD, & \text{if } rD \leq V_{\max}, \\ 0, & \text{if } rD > V_{\max}, \end{cases} \quad (5)$$

$$\text{Bank value: } d(D; A, k) = \begin{cases} rD, & \text{if } rD \leq V_{\max}, \\ d_L, & \text{if } rD > V_{\max}, \end{cases} \quad (6)$$

$$\text{Production: } e(D; A, k) = \begin{cases} \varepsilon, & \text{if } rD \leq V_{\max}, \\ 0, & \text{if } rD > V_{\max}, \end{cases} \quad (7)$$

where

$$V_{\max} = \frac{y_H - \varepsilon}{1 - \beta} \quad \text{and} \quad d_L = \frac{y_L}{1 - \beta}.$$

In the case where $rD > V_{\max}$, $b(D; A, k) = y(D; A, k) = y_L$. The law of motion for D is given by $D_{+1} = rD - b(D; A, k)$.

Note on the equilibrium

- Firm chooses e , taking the bank's choice $b(e)$ as given
- **In the case where $rD > V_{\max}$, debt D grows indefinitely according to $D_{+1} = rD - b$, and the state variable D is no longer a payoff-relevant state variable.**

Risk-shifting bubble in period 0

Production technology

- Firm problem in period 0:

$$\max_{D_1=Qk} \mathbb{E}_{\tilde{A}}[\max\{\beta V_{\max} - \beta r D_1, 0\}] = \max_k p \left[\beta \frac{Bk^\psi - \varepsilon}{1 - \beta} - Qk \right],$$

assuming that **the firm defaults when** $\tilde{A} = A_H$.

- FOC implies

$$D_1 = Q = \frac{\beta\psi}{1 - \beta} B, \quad (8)$$

- Fundamental value Q^F :

$$Q^F = \frac{\beta\psi}{1 - \beta} \{pB + (1 - p)A_H\} \ll Q.$$

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Internal bargaining over debt restructuring

Reducing debt to $D_{\max} = \beta V_{\max}$ maximizes bank's payoff:

- If $D > D_{\max}$, the payoff is $d_L = y_L(1 - \beta)^{-1}$.
- If $D = D_{\max}$, the payoff is $rD_{\max} = (y_H - \varepsilon)(1 - \beta)^{-1}$.

Why not banks immediately reduce debt?

⇒ **Conflict among stakeholders in the bank:**

Debt restructuring is implemented only upon agreement in both games

- **Coordination game between bank managers**

- Concession (C) vs. No concession (N) game between bank managers (BM1, BM2) ⇒ **If both choose N, the game continues**

- **Nash bargaining**

- Following the coordination game, BMs must secure consent from n stakeholders, where $n = \phi D_1$ ⇒ **Larger debt volume lowers BMs' payoffs**

Internal bargaining over debt restructuring

• Coordination game in a bank

- BM1 and BM2 compete for a larger share of the surplus.
- Share is α ($< \frac{1}{2}$) for C and $1 - \alpha$ for N. Share is $\frac{1}{2}$ if [C, C] or [N, N].
- Concession \Rightarrow **game is settled** \Rightarrow **debt restructuring**
- If both BMs choose N, **game continues** \Rightarrow **debt restructuring delayed**

Table: Coordination game between two bank managers

BM1 / BM2	C (Concession)	N (No concession)
C	$(\frac{1}{2}(V_{\max} - nq), \frac{1}{2}(V_{\max} - nq))$	$(\alpha(V_{\max} - nq), (1 - \alpha)(V_{\max} - nq))$
N	$((1 - \alpha)(V_{\max} - nq), \alpha(V_{\max} - nq))$	$(\frac{1}{2}y_L + \beta\pi^{BM1}, \frac{1}{2}y_L + \beta\pi^{BM2})$

• Nash bargaining bw bank managers and stakeholders \Rightarrow Decide q

Nash bargaining and coordination game

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Equilibrium with debt restructuring

- Define a threshold D^* , where

$$D^* = \frac{1}{\phi\xi} \left\{ 1 - \frac{(1-\beta)(\frac{1}{2} - \alpha)y_L}{\alpha(y_H - \varepsilon) - (\beta\frac{1}{2} + (1-\beta)\alpha)y_L} \right\}. \quad (9)$$

Proposition

For $D_1 > D^*$, the strategy profile $[N^\infty, N^\infty]$ is the unique SPE.

Debt restructuring never occurs if $D_1 > D^$.*

For $D_1 \leq D^*$, strategy C is chosen in period 1 in any SPE.

Debt restructuring occurs immediately in period 1 if $D_1 \leq D^$.*

- For $D_1 \leq D^*$, the strategy profile $[C^\infty, N^\infty]$ and $[N^\infty, C^\infty]$ are SPEs. There may exist other SPEs.
- We assumed away the mixed-strategy equilibrium.**

Why is D_1 relevant?

- D_1 affects the payoffs through the number of stakeholders, $n = \phi D_1$.
- Given the opponent's choice of N , the player chooses N , if

$$\underbrace{\frac{\frac{1}{2}y_L}{1-\beta}}_{\text{payoff of N}} > \underbrace{\alpha(\Pi - nq)}_{\text{payoff of C}}, \quad \text{where } \Pi = \frac{y_H - \varepsilon}{1-\beta} \quad (10)$$

- Larger $n \Rightarrow$ Smaller payoff of choosing C
- Condition (10) is rewritten as

$$n > n^*$$

which is rewritten as

$$D_1 > D^*$$

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Policy response to debt overhang

- Ex-post crisis management = Policy response to debt overhang
- Policy objective:
Productivity recovery by debt restructuring ($D \rightarrow D_{\max}$), where
 $D > D^* > D_{\max}$
- Subsidy to firms S^F ,
representing fiscal policy that stimulates aggregate demand
- Subsidy to banks S^B ,
representing bank recapitalization, conditional on debt restructuring

Firm subsidy: Ineffectiveness

- Subsidy S^F to firm in period 1.
- Firm pays S^F to bank and the remaining debt becomes $rD_1 - S^F$.
- **BMs divide S^F in the coordination game.**
 - BM1 and BM2 divide S^F equally if they choose [N,N].
 - If [C, N], BM1 takes αS^F and BM2 takes $(1 - \alpha)S^F$, where $\alpha < \frac{1}{2}$.
- Condition (10) implies that **condition for BMs to choose C** is

$$\underbrace{\frac{\frac{1}{2}y_L}{1-\beta} + \frac{1}{2}S^F}_{\text{payoff of N}} \leq \underbrace{\alpha(\Pi + S^F - n(S^F)q)}_{\text{payoff of C}}, \quad \text{where } n(S^F) = \phi(D_1 - r^{-1}S^F),$$

which is rewritten as

$$\frac{\frac{1}{2}y_L}{1-\beta} + \left\{ \frac{1}{2} - \alpha(1 + \beta\phi\xi(\Pi - \pi)) \right\} S_F \leq \alpha(\Pi - nq), \quad \text{where } n = \phi D_1.$$

Firm subsidy: Ineffectiveness

- We have $\frac{\frac{1}{2}y_L}{1-\beta} > \alpha(\Pi - nq)$ for $S^F = 0$
- **Condition for BMs to choose C is never satisfied** for any $S^F > 0$, if

$$\frac{1}{2} - \alpha \left(1 + \frac{\beta \phi \xi A_H}{1 - \beta} \right) > 0,$$

which holds as $\phi \xi \ll O(1)$.

- ⇒ **Direct subsidy to borrowers does not make bank managers choose C**
- ⇒ No debt restructuring
- ⇒ No productivity recovery **as long as D_1 is not fully repayable.**

Firm subsidy for no default

- Firm subsidy cannot force bank to implement debt restructuring.
- Debt restructuring is necessary only if D_1 is not repayable.
- **Suppose $S^F(D_1)$ makes D_1 fully repayable: $rD_1 \leq rD_{\max} + S^F(D_1)$**
 - Then, debt overhang never arises.
 - Productivity stays high: $A_H (> A_L)$.

$$S^F(D_1) = r(D_1 - D_{\max}).$$

- **Required amount for productivity recovery is $S^F(D_1)$**

Bank subsidy

- Subsidy S^B to bank, **conditional on debt restructuring** in period 1
- Condition (10) implies that condition to choose C is

$$\underbrace{\frac{\frac{1}{2}y_L}{1-\beta}}_{\text{payoff of N}} \leq \underbrace{\alpha(\Pi(S^B) - nq)}_{\text{payoff of C}}, \quad \text{where } \Pi(S^B) = \frac{y_H - \varepsilon}{1-\beta} + S^B$$

This **condition for BM to choose C is satisfied** if

$$D_1 \leq D^*(S^B),$$

where

$$D^*(S^B) \equiv \frac{1}{\phi\xi} \left\{ 1 - \frac{(1-\beta)(\frac{1}{2} - \alpha)y_L}{\alpha[y_H + (1-\beta)S^B - \varepsilon] - [\frac{1}{2}\beta + \alpha(1-\beta)]y_L} \right\}. \quad (11)$$

- Required amount $S^B(D_1)$: **Solution to** $D_1 = D^*(S^B)$

Bank subsidy

- Required amount of bank subsidy for debt restructuring:

$$S^B(D_1) = \frac{\{\frac{1}{2} - [\frac{1}{2}\beta + \alpha(1 - \beta)]\phi\xi D_1\}y_L}{\alpha(1 - \phi\xi D_1)(1 - \beta)} - \frac{y_H - \varepsilon}{1 - \beta}.$$

- Is $S^B(D_1)$ smaller than the gap of debt $r(D_1 - D^*)$?

- Condition $S^B(D_1) < r(D_1 - D^*)$ is written as

$$\left(\frac{1}{2} - \alpha\right)y_L < G(D_1),$$

where $G(D)$ is a quadratic function.

- This condition is rewritten as

$$D^* < D_1 \quad \left(< \bar{D} \approx \frac{1}{\phi\xi} - O(1) \right).$$

- For $D_1 > D^*$,

required bank subsidy is smaller than gap of debt: $S^B(D_1) < r(D_1 - D^*)$

Comparison: Firm vs. Bank subsidies

Suppose $D_1 > D^* > D_{\max}$.

- **Firm subsidy:** Required for no-default and productivity recovery

$$S^F(D_1) = r(D_1 - D_{\max}).$$

- **Bank subsidy:** Required for debt restructuring (DR) and recovery

$$S^B(D_1) < r(D_1 - D^*) < S^F(D_1).$$

- **Bank subsidies achieve economic recovery at a lower fiscal cost.**

Intuition:

- Efficiency requires subsidies to be **contingent on debt restructuring (DR)**.
- Bank subsidies can be made contingent as banks are the primary decision-makers.
- Firm subsidies cannot, thereby distorting bank manager incentives.

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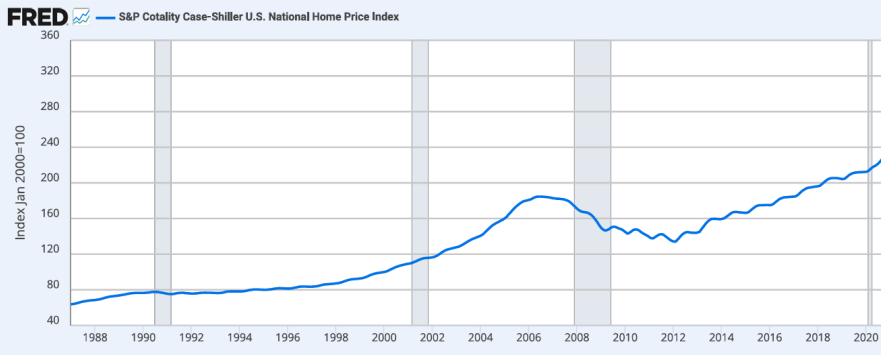
Conclusion

- Risk shifting boom, debt overhang, and coordination game can replicate empirical regularities, i.e., boom, bust and persistent productivity declines.
- We do **not need financial frictions (e.g., borrowing constraint)** to explain stagnation in the aftermath of financial crises.
- When debt is small, voluntary debt restructuring is implemented. When debt is large, debt restructuring is delayed due to coordination failure in banks.
⇒ **A larger asset boom leads to a more persistent recession**
- Bank recapitalization or bank subsidies, conditional on debt restructuring, is more efficient than direct bailouts of borrowers
 - **Bank subsidies achieve economic recovery at a lower fiscal cost than borrower subsidies.**
 - If not extremely large, **borrower subsidies hinder debt restructuring:**
Fiscal policy (demand stimulus) may impede economic recovery if the problem is debt overhang.

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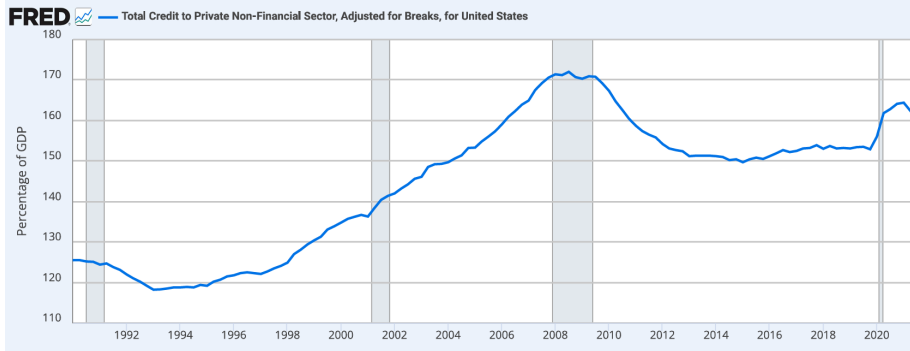
US Home Price after the Global Financial Crisis

S&P Cotality Case-Shiller U.S. National Home Price Index (CSUSHPinsa) | FRED | St. Louis Fed



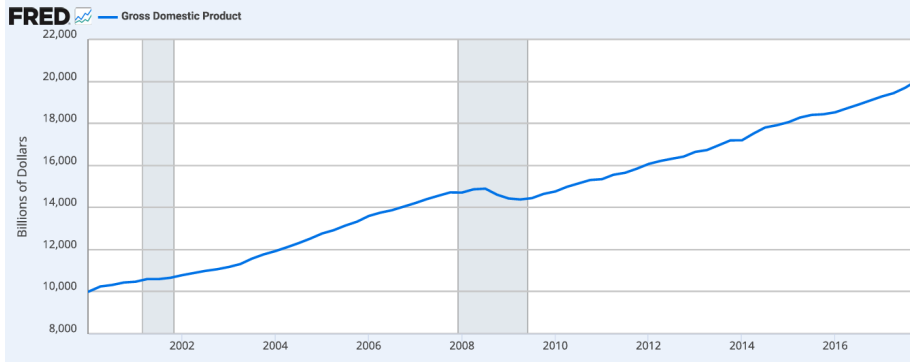
Source: S&P Dow Jones Indices LLC via FRED®

US Credit to Private Non-Financial Sector



Source: Bank for International Settlements via FRED®

US GDP after the Global Financial Crisis



Source: U.S. Bureau of Economic Analysis via FRED®

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Interest rates: $r = \beta^{-1}$ (Intuition)

- Bank initially owns safe assets, x , and risky asset (land) k .
- Firm purchases safe assets on credit: $D^S = x$, where D^S is safe loan.
- Firm also purchases land on credit: $D = Qk$, where D is risky loan.
- **Assumption 1: Banks cannot distinguish D^S and D . The loan rates for D^S and D are identical, r^L . Banks cannot operate the assets but only sell them to the firms.**
- Safe loan rate should be $r_{t+1}^L = r_{t+1}^S$, where $r_{t+1}^S = f'(x_{t+1})$ is the marginal product of safe assets.
- In equilibrium, the return rate of safe assets equals the time-discount factor:
 $r_{t+1}^L = r_{t+1}^S = \beta^{-1}$ and $x_t = x^*$ where $f'(x^*) = \beta^{-1}$.

Interest rates: $r^L = r^S = \beta^{-1}$ and $x_t = x^*$, where $f'(x^*) = \beta^{-1}$

- Show $r^L = r^S$.
 - Suppose $r^L < r^S$. Then firm's value $V \rightarrow \infty$ by $x \rightarrow \infty$. But it contradicts $r^S = f'(x) \rightarrow 0$ as $x \rightarrow \infty$
 - Suppose $r^L > r^S$. Then firms optimally choose $x = 0$ as it minimizes D_{+1} . But it contradicts $r^S = f'(x) \rightarrow \infty$ as $x \rightarrow 0$.
 - Thus, $r^L = r^S \equiv r$.
- Show $r = \beta^{-1}$.
 - Bank's profit from x is $\beta r x - x = (\beta r - 1)x$.
 - If $r > \beta^{-1}$, then banks choose $x \rightarrow \infty$, which contradicts $r = f'(x) \rightarrow 0$.
 - If $r < \beta^{-1}$, then banks choose $x = 0$, which contradicts $r = f'(x) \rightarrow \infty$ as $x \rightarrow 0$
 - Thus, $r = r^L = r^S = \beta^{-1}$
- **In equilibrium, $x_t = x^*$, where $f'(x^*) = \beta^{-1}$, and $r_t = f'(x^*) = \beta^{-1}$ for all t .**

Interest rates: Note on the risky debt

- D_{t+1} is risky: **Banks incur the loss as D_{t+1} is not fully paid.**
 - The loss is an unavoidable cost of the inability to distinguish D_{t+1}^S and D_{t+1} .
 - Banks have no alternative means of storing value other than lending to firms.
 - Banks cannot directly manage the safe assets x_{t+1} or land k . Only firms have the technology to operate them.

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Firm's problem (Full version)

- **Assumption 2:** Firm can default and walk away if $V < 0$.

- **Period 0:**

$$V_0 = \max_{D_1, D_1^S, x_1, k} \beta \mathbb{E}_{\tilde{A}}[V(D_1, D_1^S, x_1; \tilde{A}, k)],$$

$$\text{s.t. } D_1 = Qk,$$

$$D_1^S = x_1.$$

- **Period t (≥ 1):** \tilde{A} is revealed to be $A \in \{A_H, B\}$.

$$V(D, D^S, x; A, k) = \max_{D_{+1}, x_{+1}, e, c} c - e + \beta V(D_{+1}, D_{+1}^S, x_{+1}; A, k),$$

$$\text{s.t. } c + b(e) = y(e, A, k) + \max\{r^S x - r^L D^S, 0\},$$

$$D_{+1} = r^L D - b(e) + \max\{r^L D^S - r^S x, 0\},$$

$$D_{+1}^S = x_{+1}.$$

Bank's problem (Full version)

- **Period 0:**

$$\max_{k \in [0,1], x_1 \geq 0, D_1, D_1^S} \beta \mathbb{E}_{\tilde{A}} [V^B(D_1, D_1^S, x_1, e_1; \tilde{A}, k)] - x_1,$$

$$\text{s.t. } D_1 = Qk,$$

$$D_1^S = x_1,$$

$$e_1 = e(D_1, D_1^S, x_1; \tilde{A}, k),$$

- **Period $t (\geq 1)$:**

$$V^B(D, D^S, x, e; A, k) = \max_{b, x_{t+1} \geq 0} b + \min\{r^L D^S, r^S x\} - x_{t+1} + \beta V^B(D_{t+1}, D_{t+1}^S, x_{t+1}, e_{t+1}; A, k),$$

$$\text{s.t. } 0 \leq b \leq y(e, A, k) + \max\{r^S x - r^L D^S, 0\},$$

$$0 \leq b \leq r^L D + \max\{r^L D^S - r^S x, 0\},$$

$$D_{t+1} = r^L D - b + \max\{r^L D^S - r^S x, 0\},$$

$$D_{t+1}^S = x_{t+1},$$

$$e_{t+1} = e(D_{t+1}, D_{t+1}^S, x_{t+1}; A, k).$$

Outline of proof

- In the case where $rD \leq V_{\max}$, $\Rightarrow V = V_{\max} - rD$ and $d = rD$.
- In the case where $rD > V_{\max}$
 - Assumption 2: Firm can walk away if $V < 0$. $\Rightarrow V \geq 0$ in equilibrium
 - $d = rD$ is not compatible with $V \geq 0$ $\Rightarrow d < rD$ and $V \geq 0$ in equilibrium
 - $V > 0$ is not compatible with $d < rD$ $\Rightarrow d < rD$ and $V = 0$ in equilibrium
 - $b < y$ is not compatible with $d < rD$ $\Rightarrow b = y$ in equilibrium
 - $e = \varepsilon$ **is not compatible with $b = y$ and $V = 0$** $\Rightarrow e = 0$ in equilibrium
 - In equilibrium, $e = 0$ for all t , and $V = 0$ and $d = d_L < rD$.

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Nash Bargaining with stakeholders

- Bank managers (BM1 and BM2) and n stakeholders, where $n = \phi D_1$
- Π : total payoff for BMs of debt restructuring (= **value of agreement**)
- π : total payoff for BMs of not implementing debt restructuring in the current period (= **value of disagreement**)
- ξ : bargaining power of one stakeholder
- Nash bargaining problem:

$$\max_{q_i} \left(\Pi - \pi - \sum_{i=1}^n q_i \right)^{1-n\xi} \prod_{i=1}^n q_i^\xi,$$

- **BMs need to pay q to the stakeholders**

$$q = \xi(\Pi - \pi),$$

- **Total payoff for BMs:**

$$\Pi - nq = (1 - n\xi)\Pi + n\xi\pi. \tag{12}$$

Coordination game between bank managers

- Bank managers (BM1 and BM2) play game each period $t = 1, 2, 3, \dots$
- Concession (C) / No concession (N)
- If BM1 and/or BM2 **choose C, game ends** \Rightarrow **Debt restructuring**
 - Total payoff: $\Pi - nq$
 - If $[C, N]$, BM1 obtains α ($< \frac{1}{2}$) and BM2 obtains $1 - \alpha$ of the total payoff
- If both BM1 and BM2 **choose N, game continues** \Rightarrow **No debt restructuring**
 - D_t grows to $D_{t+1} = rD_t - b_t$
 - Total payoff: $\pi = \pi^{BM1} + \pi^{BM2}$

Table: Coordination game between two bank managers

BM1 / BM2	C (Concession)	N (No concession)
C	$(\frac{1}{2}(\Pi - nq), \frac{1}{2}(\Pi - nq))$	$(\alpha(\Pi - nq), (1 - \alpha)(\Pi - nq))$
N	$((1 - \alpha)(\Pi - nq), \alpha(\Pi - nq))$	(π^{BM1}, π^{BM2})

Note on the coordination game

- Only if $[N, N]$, the game continues.
- Possible history is $h^t = [N, N]^{t-1}$
- Strategy $\Sigma = \{\sigma_1, \sigma_2, \dots, \sigma_t, \dots\}$ means that the player chooses σ_t only if the history is $[N, N]^{t-1}$
- The payoffs of disagreement π, π^{BM1} , and π^{BM2} depend on the **(expected) equilibrium path** that is determined by the bank managers' choices of strategy Σ and Σ' .
- We define
 - the strategy $C^\infty \equiv \{C, C, \dots, C, \dots\}$,
 - the strategy $N^\infty \equiv \{N, N, \dots, N, \dots\}$