# Financial Crisis Cycles

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**Abstract**

This paper develops a theoretical model of financial crisis cycles in which credit-driven asset price booms are followed by busts and prolonged recessions. The accumulation of private debt during booms induces a debt overhang that persistently impairs aggregate productivity following a crash. When this debt burden is substantial, creditor coordination failures delay necessary debt restructuring, leading to a protracted decline in productivity. We show that bank recapitalization or subsidies, provided conditional on debt restructuring, are more efficient than unconditional subsidies for borrowing firms, achieving economic recovery at a lower fiscal cost. Furthermore, we demonstrate that borrower subsidies, or expansionary fiscal policy in general, can inadvertently prolong stagnation by discouraging banks from restructuring debt, unless the subsidy is exceptionally large.

## 1 Introduction

We construct and analyze a simple theoretical model of financial crises that accounts for key empirical regularities and provides a new perspective on both the crisis mechanism and its associated policy implications. A defining empirical regularity of financial crises is that they are typically preceded by booms in asset prices and credit; these booms subsequently burst, leading to deep and persistent declines in productivity (see, e.g., Jordà, Schularick, and Taylor, 2015).

The core mechanism of a financial crisis in our model unfolds as follows: First, an asset price boom emerges from optimistic expectations regarding, for instance, new technologies. During this period, firms finance asset purchases through increased leverage, which further bids up asset prices. Once these expectations prove unfounded, asset prices collapse sharply, leaving borrowers with substantial debt overhang. This overhang, in turn, impairs productive efficiency by discouraging borrowers from exerting effort. Although banks recognize that debt restructuring is essential to restore the productivity of

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distressed borrowers, they fail to implement such measures in a timely manner due to coordination failures when the debt is sufficiently large. This is because the restructuring of larger debt necessitates coordination among a larger number of stakeholders on the lender side. This delay in restructuring ultimately results in protracted stagnation. We formalize this mechanism by integrating risk-shifting bubbles (Allen and Gale, 2000), borrower debt overhang (Lamont, 1995), and creditor coordination games. A notable contribution of our framework is that it accounts for post-crisis stagnation as emerging from borrowers' balance sheet problems, rather than from standard financial frictions, such as borrowing constraints or pecuniary externalities that tighten those constraints. This distinction is important because the existing literature, which emphasizes these standard frictions, tends to focus on problems and policy measures aimed at the financial sector—such as macroprudential regulation or monetary policy—often overlooking the balance sheet problems of borrowers. Our model demonstrates that resolving borrower debt overhang is the key to economic recovery in the aftermath of a crisis.

Therefore, our framework yields distinct policy implications for ex-post crisis management. In our model, a debt overhang distorts borrower incentives and reduces productivity after a crash. To restore productivity through debt resolution, we find that bank recapitalization or subsidies contingent on restructuring are more cost-effective than direct bailouts of borrowers. Furthermore, borrower subsidies can be counterproductive: if the subsidy amounts are insufficient, they may actually discourage debt restructuring by exacerbating distortions in bank managers' decision-making. This is because borrower subsidies increase bank managers' gains from postponing debt restructuring relatively more than those from implementing it right away. Moreover, if we interpret borrower subsidies as a form of expansionary fiscal policy aimed at boosting the incomes of borrowers in general, this result implies that such policy may inadvertently impede productivity recovery in the aftermath of a financial crisis. In other words, fiscal expansion can hinder economic recovery should the fundamental cause of the productivity decline be a debt overhang. These results provide critical insights for the design of crisis management policies.

The remainder of the paper is organized as follows: Section 2 reviews the related literature. Section 3 presents the baseline model under the assumption that banks are prohibited from restructuring nonperforming loans. Section 4 demonstrates that the equilibrium path characterizes a financial crisis cycle. Section 5 extends the model to incorporate debt restructuring through costly internal bargaining. Section 6 analyzes the equilibrium and shows that debt restructuring occurs only when the debt level is small. Section 7 discusses policy implications, and Section 8 concludes.

# 2 Literature

## 2.1 Empirical Regularities

Our model provides an integrated account of the empirical regularities characterizing the financial crisis. Jordà, Schularick, and Taylor (2015), analyzing 140 years of data across 17 countries, show that asset-price booms accompanied by credit expansions tend to culminate in financial crises, followed by deep and protracted recessions. Similarly, Greenwood, Hanson, Shleifer, and Sørensen (2022) find that rapid growth in private credit and asset prices serves as a robust predictor of financial crises. Several studies demonstrate that credit booms alone can predict crises.[1] Schularick and Taylor (2012) and Verner (2019) report that credit booms frequently precede financial crises. Furthermore, Giroud and Mueller (2021) find that firm leverage booms predict boom-bust cycles in employment. Krishnamurthy and Muir (2024) document that pre-crisis periods are characterized by high credit growth and abnormally low credit spreads, typically followed by severe and protracted recessions. Corporate debt appears to be a crucial driver of these dynamics. While much of the literature since the Global Financial Crisis (GFC) has focused on household debt, recent empirical work emphasizes the role of corporate debt buildups (Greenwood et al., 2022; Jordà et al., 2022; Sever, 2023; Ivashina et al., 2024). For instance, Ivashina et al. (2024) find that corporate debt accounts for the majority of nonperforming loans post-crisis and predicts slower recovery in the average country. They argue that their findings differ from Jordà et al. (2022), who find corporate debt to be a weaker predictor, due to differences in data coverage; specifically, Ivashina et al. (2024) focus on bank loans, whereas Jordà et al. (2022) include corporate bonds, which appear less correlated with crises. The institutional environment also plays a pivotal role. Kornejew, Lian, Ma, Ottonello, and Perez (2024) document that business credit booms lead to severe output declines in environments with inefficient bankruptcy systems. While they also develop a model of non-fundamental credit booms, our framework differs in that we explicitly incorporate asset prices and analyze policy interventions aimed at enhancing debt restructuring. Similarly, Müller and Verner (2023) distinguish between "good" and "bad" credit booms, noting that the latter are often concentrated in the non-tradable sector.[2]

Finally, our work relates to the literature on post-crisis productivity. Financial crises

[1] While long-run credit deepening is associated with higher long-term economic growth (King and Levine, 1993), short-run credit booms often lead to crises.

[2] Regarding the drivers of these booms, Verner (2019) and Justiniano, Primiceri, and Tambalotti (2019) argue that pre-crisis expansions are primarily driven by credit-supply shocks rather than credit-demand shocks. Mian, Sufi, and Verner (2017) also show that credit-supply shocks induce consumption booms and persistently lower subsequent GDP growth.

are well-known to be followed by persistent productivity slowdowns (Adler et al., 2017). Duval, Hong, and Timmer (2020) argue that financial frictions were a primary driver of the productivity slump during the Great Recession. This is consistent with the literature on decade-long recessions in the 20th century, where deep and persistent productivity declines are identified as the major cause (Hayashi and Prescott, 2002; Kehoe and Prescott, 2002).

## 2.2 Theoretical ingredients

We integrate the theories of risk-shifting asset booms and debt overhang to characterize the key features of financial crises.

**Risk-shifting asset booms:** This study relates to the literature on risk-shifting asset bubbles, as theoretically analyzed by Allen and Gale (2000) and Allen, Barlevy, and Gale (2022). These studies demonstrate that asset-price booms can be driven by risk-shifting incentives among investors who leverage their purchases. In their models, however, the costs of default are exogenous, serving as the primary source of ex-post inefficiency. Consequently, there are no policy measures available to mitigate such inefficiency. In contrast, in our model, the inefficiency is endogenously determined, and ex-post debt reduction can effectively alleviate it.

**Debt overhang:** Our analysis also contributes to the broad literature on debt overhang. Following Kobayashi, Nakajima, and Takahashi (2023), debt overhang can be categorized into two distinct types: the first arising from a lack of borrower commitment, and the second from a lack of lender commitment. This paper focuses on the latter, which focuses on inefficiencies arising when lenders cannot credibly commit to rewarding borrower effort. When debt levels exceed borrower revenues, lenders possess the legal right to seize all proceeds. Anticipating this, borrowers refrain from exerting effort, leading to productive inefficiency. We emphasize the second type because it aligns with the experiences of Japan's "lost decades," which were characterized by a persistent contraction in demand. The first type of debt overhang represents a form of borrowing constraint, as analyzed by Albuquerque and Hopenhayn (2004), Kovrijnykh and Szentes (2007), and Aguiar, Amador, and Gopinath (2009). The second type is discussed in the macroeconomic literature by Sachs (1988), Krugman (1988), Lamont (1995), Occhino and Pescatori (2015), and Kobayashi et al. (2023). Empirical evidence for debt overhang is provided by Honda, Ono, Uesugi, and Yasuda (2024), who study the turnaround of small and medium-sized enterprises in Japan.

## 2.3 Theoretical studies on financial crises and policy responses

This paper is related to the extensive literature on financial crises and policy interventions. We distinguish our model from existing studies across three key dimensions: the source of inefficiencies, the propagation of inefficiencies, and the design of policy interventions. First, regarding the source of inefficiency, the literature primarily emphasizes pecuniary externalities arising from borrowing constraints (e.g., Benigno et al., 2023; Bianchi, 2011, 2016; Bianchi and Mendoza, 2010; Gertler, Kiyotaki, and Queralto, 2012; Jeanne and Korinek, 2020; Lorenzoni, 2008) or coordination failures, such as bank runs (Diamond and Dybvig, 1983; Gertler and Kiyotaki, 2015; Keister, 2016; Keister and Narasiman, 2016). This paper emphasizes debt overhang as the source of inefficiency. Second, concerning the propagation of inefficiencies, many existing models feature consumption misallocations (Bianchi, 2011; Chari and Kehoe, 2016; Farhi, Golosov, and Tsyvinski, 2009; Keister, 2016) or inefficient production due to a credit crunch (Bianchi, 2016; Bianchi and Mendoza, 2010; Gertler, Kiyotaki, and Queralto, 2012; Lorenzoni, 2008), whereas we focus on the coordination game that delays debt resolution as a mechanism of persistence. Third, regarding policy interventions, the existing literature primarily focuses on time inconsistency—specifically, the trade-off between ex-ante incentives and ex-post efficiency induced by bailout policies (Bianchi, 2016; Chari and Kehoe, 2016; Farhi and Tirole 2012; Green, 2010; Keister, 2016; Keister and Narasiman, 2016). In our model, we compare bank recapitalization and direct bailout of the borrowers as crisis management policies to resolve debt problems. Haavio, Ripatti, and Takalo (2025) and Homar and van Wijnbergen (2017) also compare bank recapitalization and borrower bailouts, while their analyses focus on resolving credit crunch and banks' risk-shifting to the government, whereas our model focuses on borrowers' debt overhang. Finally, our paper relates to the literature on secular stagnation (e.g., Rachel and Summers, 2019; Eggertsson, Mehrotra, and Robbins, 2019).

## 2.4 Zombie Lending

Our theory is closely related to the growing literature on zombie lending and evergreening in the aftermath of financial crises. Zombie lending refers to credit extension to non-viable or distressed firms driven by distorted bank incentives. The pioneering work of Peek and Rosengren (2005) and Caballero, Hoshi, and Kashyap (2008) documented the proliferation of zombie lending in Japan during the 1990s (see also Sekine, Kobayashi, and Saita (2003) for an early account of "forbearance lending"). More recently, Acharya, Lenzu, and Wang (2023) and the references therein analyze models of zombie lending, emphasizing that accommodative government policies can distort bank incentives and induce lending practices that lead to persistent stagnation. Our model complements the existing literature in three key respects. First, while much of the literature focuses on distortionary policies, in our model, internal coordination failure in a bank is sufficient to generate persistent debt overhang and protracted stagnation. Second, whereas Acharya et al. (2021) imply that the removal of distortionary policy is welfare-improving, our framework suggests that active policy intervention is necessary to resolve debt overhang. Third, while most studies assume that zombie firms are intrinsically inefficient and that their exit improves productivity by mitigating congestion, our model implies that such firms can restore efficiency if their debt burden is reduced. The empirical findings of Nakamura and Fukuda (2013) provide support for our theory; they report that a significant portion of Japanese zombie firms in the non-tradable sector eventually recovered and became productive in the 2000s, suggesting they were not intrinsically unproductive. Furthermore, Becker and Ivashina (2022) and Kornejew et al. (2024) demonstrate, both empirically and theoretically, that inefficient bankruptcy procedures exacerbate the costs of zombie lending. These results imply that debt reduction through restructuring or bankruptcy can enhance efficiency, further supporting our argument regarding the welfare-improving effects of debt resolution.

# 3 Model without debt restructuring

Time is discrete and continues from 0 to infinity: $t = 0, 1, 2, \cdots$. There are a unit mass of banks and a unit mass of firms. There exist two assets in the economy: land $k_t$ and safe assets $x_t$. Land is a fixed-supply asset, the total amount of which is a unit mass ($k_t = 1$). In the initial period 0, each bank owns one unit of land so that all land is owned by banks. Safe assets are variable supply, the total amount of which is $x_t$, where $x_t$ is an endogenous variable. In every period, banks can generate safe assets from consumption goods on a one-to-one basis. The safe assets represent the general capital in the economy as in Allen and Gale (2000). While firms produce consumption goods using safe assets and land as detailed below, banks lack the necessary productive technologies to utilize these inputs. Consequently, banks are limited to selling their holdings of safe assets and land to firms on credit. The following assumption formalizes this setup:

**Assumption 1.** Banks cannot produce goods from safe assets or land. Rather, banks can only sell these assets to firms on credit or hold them passively yielding zero output. Safe assets depreciate completely in each period, whereas land is not subject to depreciation.

**Production using safe assets:** Similarly to Allen and Gale (2000), safe assets can be interpreted as capital goods leased to the general corporate sector, operating independently of the specific banks and firms analyzed in this paper. In period $t$, a firm purchases $x_{t+1}$ units of the safe assets from a bank on credit. Thus, the firm borrows $D^S_{t+1} = x_{t+1}$ to buy $x_{t+1}$ units of the assets. The loan rate $r^L_{t+1}$ is determined in equilibrium as described below. The firm that has acquired the safe assets $x_{t+1}$ immediately leases them out to the general corporate sector where the consumption goods are produced using a constant-returns-to-scale (CRS) technology, $F(x, l)$, where $l$ is the labor input. We assume for simplicity that the labor supply in this economy is unity, $l = 1$. The firm who leased $x_{t+1}$ will have the return $r^S_{t+1} x_{t+1}$ in period $t + 1$, where the rate of return $r^S_{t+1}$ is determined in the market as described below. The general corporate sector maximizes the profit:

$$F(x_{t+1}, l_{t+1}) - r^S_{t+1} x_{t+1} - w_{t+1} l_{t+1},$$

where $w_{t+1}$ and $l_{t+1}$ are the wage rate and the labor input in period $t + 1$. Define $f(x) \equiv F(x, 1)$. In the competitive equilibrium where $l = 1$, $x$ and $w$ are given by

$$r^S_{t+1} = f'(x_{t+1})$$

and $w = f(x) - f'(x)x$. Equation $r^S_{t+1} = f'(x_{t+1})$ can be interpreted as an equation that determines $r^S_{t+1}$ from the aggregate amount of $x_{t+1}$ in the market. The production function $f(x)$ satisfies $f'(x) > 0$ and $f''(x) < 0$ for all $x > 0$, $\lim_{x \to 0} f'(x) = +\infty$, and $\lim_{x \to \infty} f'(x) = 0$.

**Production using land:** Each bank owns one unit of land $k = 1$ in period 0. Banks sell land $k$ to firms on credit so that a firm purchases $k$ units of land by borrowing the loan $D_1 = Qk$ from a bank, where Q is the price of the land in period 0 in terms of consumer goods. As firms pay the debt $D_1$ gradually over time, there may be remaining debt $D_{t+1}$ in period $t$ that evolves at the loan rate $r^L_t$. Firms can produce consumption goods $y(e_t, \tilde{A}, k)$ using land $k$, where $e_t$ is additional input of effort, where the firms can choose whether $e_t = 0$ or $e_t = \varepsilon > 0$. The value of $e_t$, which is either 0 or $\varepsilon$, is the consumption equivalence of disutility that the firm suffers from expending the effort.

$$y(e_t, \tilde{A}, k) = \begin{cases} y_L \equiv A_L k^{\psi} & \text{for } e_t = 0, \\ y_H \equiv \tilde{A} k^{\psi} & \text{for } e_t = \varepsilon, \end{cases} \tag{1}$$

where $0 < \psi < 1$ and $\tilde{A}$ is a macroeconomic random variable in period 0, which is revealed in period 1. The value of $\tilde{A}$ is the following, which is common for all firms and time-invariant from period 1 on:

$$\tilde{A} = \begin{cases} B & \text{with probability } p, \\ A_H & \text{with probability } 1 - p. \end{cases}$$

We assume

$$A_L < A_H \ll B,$$
$$y_L < y_H - \varepsilon \quad \text{at } \tilde{A} = A_H \quad \text{and} \quad k = 1.$$

Incorporating production technology and market structure of Dixit-Stiglitz monopolistic competition allows for an analysis of aggregate demand externalities arising from debt overhang, as in Lamont (1995). For expositional simplicity, however, we defer this analysis to Appendix A.

**Purchase of safe assets:** Banks produce safe assets $x_{t+1}$ in period $t$. We assume that the consumption goods produced in period $t$ are all consumed or rotten when the safe assets $x_{t+1}$ put up for sale. So firms cannot pay the unconsumed goods for $x_{t+1}$, but need to borrow a bank loan $D^S_{t+1} = x_{t+1}$ to purchase $x_{t+1}$ from the bank.[3]

**Loan rates:** We assume that banks cannot distinguish between loans intended for the purchase of land $D_{t+1}$, and those for safe assets $D^S_{t+1}$.[4] This setup implies the following assumption regarding the loan rates:

**Assumption 2.** The loan rates for $D_{t+1}$ and $D^S_{t+1}$ are identical and denoted by $r^L_{t+1}$.

**Firm's problem:** Denote the firm's value in period $t$ by $V(D_t, D^S_t, x_t; \tilde{A}, k)$. Note that the land input $k$ is constant from period 0, while its productivity $\tilde{A}$ is constant from period 1 when $\tilde{A} = B$ or $\tilde{A} = A_H$ is revealed. The firm purchases land in period 0 and operates it in producing the consumer goods from period 1 on. We formulate the following assumption concerning the firm's participation constraint.

**Assumption 3.** The firm can cease operations and default on its debt if its anticipated value is negative; in contrast, it fulfills its repayment obligations specified by the bank as long as its value remains non-negative. In the event of default, the firm incurs no penalty and the bank recovers zero value.

In period 0, a firm solves

$$\begin{aligned} V_0 = &\max_{D_1, D^S_1, x_1, k} \beta \mathbb{E}_{\tilde{A}}[V(D_1, D^S_1, x_1; \tilde{A}, k)], \\ &\text{s.t. } D_1 = Qk, \\ &\qquad D^S_1 = x_1. \end{aligned}$$

[3]The constraint that $D^S_{t+1} = x_{t+1}$ is equivalent to imposing the following constraint

$$c = y(e, A, k) + \max\{r^S x - r^L D^S,\ 0\} - b(e)$$

on the firm's problem, which says that all remaining consumption goods after paying for input costs (the RHS of the above equation) must be consumed and cannot be used to purchase safe assets, $x_{t+1}$.

[4]Specifically, the bank provides a single loan, which the firm allocates between $D_{t+1}$ and $D^S_{t+1}$ after borrowing.

In period $t$, where $t \geq 1$ and $\tilde{A}$ has been revealed to be $A \in \{A_H, B\}$ in period 1, a firm solves

$$
\begin{aligned}
V(D, D^S, x; A, k) = \max_{D^S_{+1}, x_{+1}, e, c} \quad & c - e + \beta V(D_{+1}, D^S_{+1}, x_{+1}; A, k), \\
\text{s.t. } & c + b(e) = y(e, A, k) + \max\{r^S x - r^L D^S,\ 0\}, \\
& D_{+1} = r^L D - b(e) + \max\{r^L D^S - r^S x,\ 0\}, \\
& D^S_{+1} = x_{+1},
\end{aligned}
$$

where $c$ is consumption, which can be interpreted as the dividend to the firm owner. The first constraint is the resource constraint in which the profits from safe assets ($r^S x - r^L D^S$) is added to the available resources when it is positive. When it is negative, the second constraint, which is the law of motion for debt $D_t$, states that the unpaid portion of $r^L_t D^S_t$, i.e., $r^L_t D^S_t - r^S_t x_t$, is capitalized into the subsequent period's debt $D_{t+1}$. The variable $b(e)$ is taken as given by the firm and chosen by the bank who solves the following problem.

**Bank's problem:** The value of the bank is denoted by $V^B$. Here we make a crucial assumption:

**Assumption 4.** In every period $t$ ($\geq 1$), the bank decides the amount to be repaid $b$ after observing the borrower's choice of production $e \in \{0, \varepsilon\}$ in that period.

In period 0, a bank solves

$$
\begin{aligned}
\max_{k \in [0,1], x_1 \geq 0, D_1, D^S_1,} \quad & \beta \mathbb{E}_{\tilde{A}}[V^B(D_1, D^S_1, x_1, e_1; \tilde{A}, k)] - x_1, \\
\text{s.t. } \quad & D_1 = Qk, \\
& D^S_1 = x_1, \\
& e_1 = e(D_1, D^S_1, x_1; \tilde{A}, k),
\end{aligned}
$$

where $e(D, D^S, x; \tilde{A}, k)$ is the equilibrium policy function for the firm. In period $t$, where $t \geq 1$ and $\tilde{A}$ has been revealed to be $A \in \{A_H, B\}$ in period 1, the bank solves

$$
\begin{aligned}
V^B(D, D^S, x, e; A, k) = \max_{b, x_{+1} \geq 0} \quad & b + \min\{r^L D^S, r^S x\} - x_{+1} + \beta V^B(D_{+1}, D^S_{+1}, x_{+1}, e_{+1}; A, k), \\
\text{s.t. } & 0 \leq b \leq y(e, A, k) + \max\{r^S x - r^L D^S,\ 0\}, \\
& 0 \leq b \leq r^L D + \max\{r^L D^S - r^S x,\ 0\}, \\
& D_{+1} = r^L D - b + \max\{r^L D^S - r^S x,\ 0\}, \\
& D^S_{+1} = x_{t+1}, \\
& e_{+1} = e(D_{+1}, D^S_{+1}, x_{+1}; A, k).
\end{aligned}
$$

The solution $b = b(e)$ is taken as given by the firms. Note that $b(e)$ is bounded from above.[5] [6]

**Determination of the interest rates $r_t^S = r_t^L$:** We can show that $r_t^S = r_t^L = \frac{1}{\beta}$ with reasoning similar to that of Allen and Gale (2000). First, we show that $r_t^S = r_t^L$ for $t \geq 1$ by analyzing the firm's problem. Since $D^S = x$, the firm's problem can be rewritten as

$$
\begin{aligned}
V(D, D^S, x; A, k) = &\max_{x_{+1}, e, c} c - e + \beta V(D_{+1}, D_{+1}^S, x_{+1}; A, k)], \\
&\text{s.t. } c + b(e) = y(e, A, k) + \max\{(r^S - r^L)x, 0\}, \\
&\quad D_{+1} = r^L D - b(e) + \max\{(r^L - r^S)x, 0\}, .
\end{aligned}
$$

Suppose that $r^S > r^L$. Then, the firm can make $c = +\infty$ by choosing $x = +\infty$ in period $t-1$ for $t \geq 1$, because $b(e)$ and $y(e, k)$ are bounded from above and $x$ is not bounded. But $x = +\infty$ and $r^S > r^L \geq 0$ contradict the fact that $r^S = f'(x)$ and $f'(\infty) = 0$. Therefore, $r^S \leq r^L$ must hold. Now suppose that $r^S < r^L$. In this case, the firm can minimize $D$ by setting $x = 0$ in period $t - 1$. The firms choose optimally $x = 0$, while the result that $r^S < r^L$ and $x = 0$ contradicts the fact that $r^S = f'(x)$ and $f'(0) = +\infty$. Therefore, $r^S = r^L$. In what follows, we denote

$$
r_t \equiv r_t^L = r_t^S.
$$

Note that firms are indifferent to the choice of $x_t$ when $r_t^S = r_t^L$ for all $t$, as the firm value remains independent of this decision.

**Determination of the interest rate $r_t = \frac{1}{\beta}$:** Define $d(D, x, e; A, k) \equiv V^B - rx$. Since $r^L = r^S$ and $x = D^S$, the bank's problem can be rewritten as

$$
\begin{aligned}
d(D, x, e; A, k) = &\max_{b, x_{+1} \geq 0} b + (\beta r_{+1} - 1)x_{+1} + \beta d(D_{+1}, x_{+1}, e_{+1}; A, k), \\
\text{s.t. } &b \leq y(e, A, k), \\
&0 \leq b \leq rD, \\
&D_{+1} = rD - b, \\
&e_{+1} = e(D_{+1}, D_{+1}^S, x_{+1}; A, k).
\end{aligned}
$$

The bank's problem in period 0 is $\max_{k, x_1, D_1} \beta \mathbb{E}_{\tilde{A}}[d(Qk, x_1, e_1; \tilde{A}, k)] + (\beta r_1 - 1)x_1$. Now, suppose that $\beta r > 1$. Then, the bank could make $d(D, x, e; A, k) = +\infty$ by setting

[5] If $r^S \geq r^L$, $b \leq r^L D$ as $x = D^S$. If $r^L \geq r^S$, $b \leq y(e, A, k)$. Thus, it is the case that $b \leq \max\{r^L D, y(e, k)\}$.

[6] Note also that firms choose the demand for the safe and risky assets ($x_{+1}^d$ and $k^d$), banks choose the supply of them ($x_{+1}^s$ and $k^s$), and they are equal in equilibrium ($x_{+1} = x_{+1}^d = x_{+1}^s$ and $k = k^d = k^s$).

$x_{+1} = +\infty$. But since $r = f'(x)$, $x_{+1} = \infty$ implies that $r = f'(\infty) = 0$, which contradicts $\beta r > 1$. Therefore, $\beta r \leq 1$ must hold. Now suppose that $\beta r < 1$. Then, the bank would optimally choose $x_{+1} = 0$. In this case, $r_{+1} = f'(x_{+1}) = f'(0) = +\infty$, which contradicts $\beta r_{+1} < 1$. Thus, it must be the case that $r_t (= r_t^S = r_t^L) = \frac{1}{\beta}$ for all $t \geq 1$.

Given that $r_t = \frac{1}{\beta}$ for all $t$, both firms and banks are indifferent to the choice of $x_t$, as values of firms and banks are independent of $x_t$. Thus, the value of $x_t$ is given as $x_t = x^*$, where $x^*$ is the solution to $f'(x) = \frac{1}{\beta}$. Except that the choice of $x_t$ determines the interest rate $r_t$, it is irrelevant to the equilibrium path of the other variables. Thus, given that $x_t = x^*$ and $r_t = \frac{1}{\beta}$, the problems for firms and banks can be rewritten as follows.

**Simplified firm's problem:** In period 0, a firm solves

$$\begin{aligned} V_0 = &\max_{D_1, k} \beta \mathbb{E}_{\tilde{A}}[V(D_1; \tilde{A}, k)], \\ &\text{s.t. } D_1 = Qk. \end{aligned} \tag{2}$$

In period $t$, where $t \geq 1$ and $\tilde{A}$ has been revealed to be $A \in \{A_H, B\}$ in period 1, a firm solves

$$\begin{aligned} V(D; A, k) = &\max_{e} c - e + \beta V(D_{+1}; A, k), \\ &\text{s.t. } c + b(e) = y(e, A, k), \\ &\quad D_{+1} = rD - b(e), \end{aligned} \tag{3}$$

where $b(e)$ is the solution to (5).

**Simplified bank's problem:** In period 0, a bank solves

$$\begin{aligned} &\max_{k \in [0,1]} \beta \mathbb{E}_{\tilde{A}}[d(D_1, e_1; \tilde{A}, k)], \\ &\text{s.t. } D_1 = Qk, \\ &\quad e_1 = e(D_1; \tilde{A}, k), \end{aligned} \tag{4}$$

where $e(D; A, k)$ is the equilibrium policy function for the firm. In period $t$ $(\geq 1)$, a bank solves the following problem, given that $e$ is already decided by the firm and $\tilde{A}$ has been revealed to be $A \in \{A_H, B\}$ in period 1:

$$\begin{aligned} d(D, e; A, k) = &\max_{b} b + \beta d(D_{+1}, e_{+1}; A, k), \\ &\text{s.t. } 0 \leq b \leq y(e, A, k), \\ &\quad 0 \leq b \leq rD, \\ &\quad D_{+1} = rD - b, \\ &\quad e_{+1} = e(D_{+1}; A, k). \end{aligned} \tag{5}$$

## 4 Equilibrium without debt restructuring

In period 0, firms and banks expect that $\tilde{A}$ will be $B$ in period 1 with probability $p$ and $A_H$ with probability $1-p$. We are interested in the case where $\tilde{A}$ turns out to be $A_H$ in period 1, and we will show that the overly optimistic expectation $B(\gg A_H)$ generates asset-price boom due to the risk-shifting effect demonstrated by Allen and Gale (2000).

**Definition of equilibrium:** The equilibrium concept in this paper is the Markov perfect equilibrium. The equilibrium is the set of price $Q$, allocation $k$, debt $D$, value functions $\{V(D;A,k),\ d(D;A,k)\}$, the policy function for firms $e(D;A,k)$ and the policy function for banks $b(D;A,k)$ that satisfies the following conditions:

- Given price $Q$, the allocation $k$ and debt $D_1$ solve (2) and (4), and the market clears, $k=1$.
- Given that $\tilde{A}$ is revealed to be $A$, the policy functions solve (3) and (5), respectively, yielding the corresponding value functions. Note that $b(D;A,k)=b(D,e(D;A,k);A,k)$, where $b(D,e;A,k)$ is the solution to (5).
- Debt $D$ evolves according to $D_{+1}=rD-b(D;A,k)$.

The following proposition states that the equilibrium is unique, and it is inefficient when the debt $D_t$ is sufficiently large.

**Proposition 1.** *The Markov perfect equilibrium is uniquely given by*

$$V(D;A,k)=\begin{cases} V_{\max}-rD, & \textit{if } rD\le V_{\max},\\ 0, & \textit{if } rD>V_{\max},\end{cases} \tag{6}$$

$$d(D;A,k)=\begin{cases} rD, & \textit{if } rD\le V_{\max},\\ d_L, & \textit{if } rD>V_{\max},\end{cases} \tag{7}$$

$$e(D;A,k)=\begin{cases} \varepsilon, & \textit{if } rD\le V_{\max},\\ 0, & \textit{if } rD>V_{\max},\end{cases} \tag{8}$$

*where*

$$\begin{aligned} V_{\max} &= \frac{y_H-\varepsilon}{1-\beta},\\ y_H &= Ak^{\psi},\\ d_L &= \frac{y_L}{1-\beta},\\ y_L &= A_L k^{\psi}. \end{aligned}$$

*In the case where $rD\le V_{\max}$, the value of $b(D;A,k)\in[0,\ \min\{rD,\ y(D;A,k)\}]$ is determined arbitrarily, on the condition that $rD_{+1}\le V_{\max}$ whenever $rD\le V_{\max}$. Here,*

*$y(D; A, k)$ is defined as $y(e(D; A, k),\ A,\ k)$. In the case where $rD > V_{\max}$,*

$$b(D; A, k) = y(D; A, k) = y_L.$$

*The law of motion for $D$ is given by*

$$D_{+1} = rD - b(D; A, k).$$

*Note that the law of motion for $D$ implies that $D_t$ grows explosively once the condition $rD > V_{\max}$ is satisfied. The state variable $D_t$ is no longer a payoff relevant state variable, once it satisfies $rD_t > V_{\max}$.*

*Proof.* The maximum value that the firm can generate is $V(0; A, k) = V_{\max}$, where $V_{\max} = \frac{Ak^{\psi}-\varepsilon}{1-\beta}$, as of the beginning of period $t$ $(\geq 1)$, given the productivity of land $A \in \{A_H, B\}$ has been revealed in period 1. If $rD_t \leq V_{\max}$ then the debt $rD_t$ is fully repayable from period $t$ onward.

Now, we can prove the proposition in the case where $rD \leq V_{\max}$.

**Claim 1.** In the case where $rD \leq V_{\max}$, the value function for the firm is $V_{\max} - rD$, that for the bank is $rD$, and $e = \varepsilon$.

*Proof.* (Proof of Claim 1) In the case where $rD \leq V_{\max}$, the maximum repayment of the debt is constrained from above $(rD)$. In this case, suppose $e = 0$ in some period $t$. Then, the firm value can be increased by choosing $e = \varepsilon$ without changing the bank's value. Thus, $e = \varepsilon$ must hold if $rD \leq V_{\max}$. This gives the value functions for $rD \leq V_{\max}$. □

Next we consider the case where $rD > V_{\max}$.

- Suppose that the bank's recovery value $d$ satisfies $d = rD$ in equilibrium for $rD > V_{\max}$. Since $rD > V_{\max}$, the bank will take more than $V_{\max}$, leading to the firm value negative: $V < 0$. But it cannot be an equilibrium because if the firm chooses $e_t = 0$ unconditionally for all $t \geq 1$, then it can make $V \geq 0$. This is because the firm produces $y_L$ without additional cost and pays $b \leq y_L$. Therefore, in equilibrium, $V \geq 0$, which implies that $d \leq V_{\max} - V < rD$. Thus, we have $d < rD$ and $V \geq 0$ in equilibrium for $rD > V_{\max}$.

- Now, suppose that $V > 0$ and $d < rD$ in equilibrium. It is impossible as the bank can strictly increase $d$ by marginally decreasing $V$ (or increasing $b$). Thus, it must be the case that if $rD > V_{\max}$, then $V = 0$ and $d < rD$ in equilibrium.

- Suppose that $b < y(e, A, k)$ in some $t$ in equilibrium in which $d < rD$. It is impossible because the bank can increase $d$ by increasing $b$ to $b = y(e, A, k)$, without changing the firm's action from $t + 1$ on. (In what follows we show the outline of the proof of the statement that the bank can increase $b_t$ without changing the firm's action from $t+1$ on. Suppose the bank changes $b_t$ to $b_t + \delta$, where $\delta$ is an infinitesimally small positive number. Then the bank can keep all constraints nonbinding in the bank's problem (5). Then, the bank can keep $b(e)$ unchanged on the premise that $e_{t+s+1} = \tilde{e}(D_{t+s+1}; A, k)$, where $\tilde{e}(D_{t+s+1}; A, k) = e(D_{t+s+1} - \beta^{-s}\delta; A, k)$. Then the firm's problem can be made identical to the original one and so the solutions to the firm's problem. ) Thus, in equilibrium where $d < rD$, we have $b = y(e, A, k)$.

- Suppose that the firm chooses $e_t = \varepsilon$ in some period $t$ in equilibrium where $d < rD$ and $V = 0$. It is impossible because in the equilibrium where $d < rD$, the bank chooses $b = y(e, A, k)$ for all $t$. Given this, if the firm chooses $e_t = \varepsilon$ in some $t$, $V \leq -\varepsilon < 0$ holds, which contradicts $V = 0$.

- We have shown that for $rD > V_{\max}$, the equilibrium is uniquely given by $e_t = 0$ for all $t$, $V = 0$ and $d = d_L < rD$.

□

**Risk-shifting bubble of asset price in period 0:** Firm's problem in period 0 is written as

$$\max_{D_1 = Qk} \beta \mathbb{E}_{\tilde{A}}[V(D_1; \tilde{A}, k)] = \max_{D_1 = Qk} \mathbb{E}_{\tilde{A}}[\max\{\beta V_{\max} - \beta r D,\ 0\}] = \max_k p \left[\beta \frac{Bk^{\psi} - \varepsilon}{1 - \beta} - Qk\right].$$

The last equality is due to the assumption that the equilibrium price of land is so high that the firm falls into the debt overhang state ($rD_1 > V_{\max}$) where $V = 0$, if $\tilde{A} = A_H$. The FOC with respect to $k$ at $k = 1$ implies that

$$D_1 = Q = \frac{\beta\psi}{1 - \beta} B, \tag{9}$$

whereas the fundamental price of the asset should be $Q^F = \frac{\beta\psi}{1-\beta}\{pB + (1-p)A_H\}$. Thus the market price of the asset $Q$ is much higher than the fundamental price $Q^F$, due to the risk shifting effect (as in Allen and Gale (2000)). We assume that $\psi B < B - \varepsilon$ to ensure that $D_1$ is repayable when the productivity parameter $\tilde{A}$ turns out to be $B$. We also assume that $\psi B > A_H - \varepsilon$ to ensure that $rD_1 > V_{\max}$ when $\tilde{A}$ turns out to be $A_H$.

## 5 Model with debt restructuring

If $D > D_{\max} \equiv \frac{1}{r} V_{\max} = \frac{\beta(y_H - \varepsilon)}{1-\beta}$, the social surplus is $d + V = d_L = \frac{y_L}{1-\beta}$, while if $D \leq D_{\max}$, it is $V_{\max} = \frac{y_H - \varepsilon}{1-\beta}$. Since $V_{\max}$ is greater than $d_L$, it is obvious that when $D$ is greater than $D_{\max}$, the debt relief, which is to reduce $D$ to $D_{\max}$, improves the social surplus and increases the value of the bank from $d_L$ to $V_{\max}$. In the baseline model in the previous section, we prohibited the debt relief by banks. In this section, we modify the model so that banks can choose whether to provide debt restructuring.

**Proposition 2.** *Suppose that when $D_t > D_{\max}$ in period $t$, $rD_t$ is reduced to $rD_{\max}$ exogenously with a certain probability. In this case, equilibrium production is still given by (8).*

*Proof.* (Outline) The same reasoning as in the proof of Proposition 1 holds in the case where there exists an exogenous probability with which $D$ ($> D_{\max}$) is replaced by $D_{\max}$. The reason why the same reasoning applies is that the reduction in debt to $D_{\max}$ does not change the firm value $V = 0$. Thus, the possibility of debt reduction does not affect the behavior of firms. Since the firm value $V$ remains zero by reducing the debt from $D$ to $D_{\max}$, the value function $V(D; A, k)$ remains the same as Proposition 1. □

## 5.1 Bank decision-making on debt restructuring

When $D_t > D_{\max}$, the bank is better off by reducing $D_t$ to $D_{\max}$. But there are the following frictions in the bank's decision making on debt restructuring. There are two bank managers, BM1 and BM2, who play the coordination game (described below). Given that BM1 and BM2 achieve the agreement on debt relief in the coordination game, they need to obtain the consent from stakeholders by solving the Nash bargaining (described below). A bank can conduct debt restructuring only if both the coordination game and the Nash bargaining are settled by agreement.

First, we describe the Nash bargaining between bank managers and stakeholders, and then, secondly, the coordination game between BM1 and BM2.

**Nash Bargaining with stakeholders:** After the bank managers, BM1 and BM2, agreed on the debt restructuring, they need to obtain the consent from $n$ stakeholders within the bank, where

$$n = \phi D_1,$$

where $D_1$ is the initial amount of the loan originated in period 0. We assume that the number of stakeholders increases with the size of the initial loan $D_1$, since it is natural to assume that larger loans involve more decision-makers within the bank. The bank managers and the stakeholders within the bank need to maximize the following generalized Nash product, taking $\Pi$ and $\pi$ as given, where $\Pi$ is the bank's value after debt restructuring ($\Pi = \frac{y_H - \varepsilon}{1-\beta}$) and $\pi$ is the bank's value in the case where debt restructuring is not conducted in the current period:

$$\max_{q_i} \left( \Pi - \pi - \sum_{i=1}^{n} q_i \right)^{1-n\xi} \prod_{i=1}^{n} q_i^{\xi},$$

where $q_i$ is the payment to $i$-th stakeholder, where $i = 1, 2, \cdots, n$ and $\xi$ is the bargaining power of $i$-th stakeholder. The values of $\Pi$ and $\pi$ are endogenously determined in equilibrium. The expression $\Pi - \sum_{i=1}^{n} q_i$ represents the joint payoff for BM1 and BM2 upon reaching an agreement in the Nash bargaining with the stakeholders, while $\pi$ denotes their disagreement payoff. We consider that there is a upper bound for $D_1$ such that the Nash bargaining is well defined:

$$D_1 < \frac{1}{\phi\xi}.$$

The symmetric solution where $q_i = q$ is given by

$$q = \xi(\Pi - \pi),$$

implying that

$$\Pi - nq = (1 - n\xi)\Pi + n\xi\pi. \tag{10}$$

**Coordination game between bank managers:** Anticipating the above Nash bargaining solution, BM1 and BM2 play the coordination game in which each player chooses either concession (C) or no concession (N). If either BM1 or BM2 makes a concession, debt restructuring is implemented and the total surplus for the bank managers becomes $\Pi - nq$. If both BM1 and BM2 choose the action C, they share the surplus equally so that each of them receives $\frac{1}{2}(\Pi - nq)$. In the case where BM1 makes a concession and BM2 does not, debt restructuring is implemented: however, BM1's share will be smaller than that of BM2. We formalize this by introducing a parameter $\alpha \in (0, \frac{1}{2})$ so that BM1 receives $\alpha(\Pi - nq)$ and BM2 receives $(1 - \alpha)(\Pi - nq)$. In the case where both players choose no concession, debt restructuring is not implemented, and BM1 and BM2 receive $\pi^{BM1}$ and $\pi^{BM2}$ respectively, where $\pi^{BM1} + \pi^{BM2} = \pi$. The values of $\pi^{BM1}$, $\pi^{BM2}$, and $\pi$ are given endogenously in equilibrium. Here we assume the following:

**Assumption 5.** When both BM1 and BM2 choose the same action N in the current period, they share the output in the current period ($y_L$) equally.

Note that when both players choose the action N, the coordination game continues in the next period as debt restructuring is not yet implemented in the current period. Thus, $\pi^{BM1}$ can be written as

$$\pi^{BM1} = \frac{1}{2}y_L + \beta \times \{\text{Value that BM1 obtains in the next period in equilibrium}\}.$$

The following Table 1 is the payoff matrix for BM1 and BM2. In this table, $(X, Y)$ indicates that BM1 receives the payoff X and BM2 receives Y, respectively.

Table 1: Coordination game between two bank managers

| BM1 / BM2 | C (Concession) | N (No concession) |
|---|---|---|
| C | $(\frac{1}{2}(\Pi - nq),\ \frac{1}{2}(\Pi - nq))$ | $(\alpha(\Pi - nq),\ (1-\alpha)(\Pi - nq))$ |
| N | $((1-\alpha)(\Pi - nq),\ \alpha(\Pi - nq))$ | $(\pi^{BM1},\ \pi^{BM2})$ |

# 6 Equilibrium with debt restructuring

In this section, we consider pure strategy equilibrium of the game. We characterize mixed strategy equilibrium in Appendix B.

Here, we define several notations. Each bank manager chooses an action $a_t$ from the set $\{C, N\}$ in every period $t = 1, 2, \ldots$ as long as the game continues. Let $[a_t, a'_t]$ denote a profile of the actions chosen in period $t$, where BM1 chooses $a_t$ and BM2 chooses $a'_t$, where $a_t, a'_t \in \{C, N\}$. Since the game ends at the time when (at least) one of the bank managers chooses $C$, we can restrict our attention to the set of histories of the form: $h^t \equiv [N, N]^{t-1}$, for $t = 1, 2, \ldots$, where $[N, N]^{t-1}$ denotes that both BM1 and BM2 choose N in all previous periods $1, 2, \cdots, t-1$, and $h^1 = [N, N]^0 \equiv \emptyset$ is the initial history. Let $H \equiv \bigcup_{t=1}^{\infty} h^t$ be the set of all histories relevant for this game. Subgames are indexed by histories $h^t \in H$. For example, subgame $h^t$ is the one that starts after history $h^t = [N, N]^{t-1}$. Since there is only one history $h^t = [N, N]^{t-1}$ for each period $t$, we will use the terms "period $t$" and "history $h^t$" interchangeably.

A strategy of a bank manager is a function $\Sigma : H \to \{C, N\}$. Given that $H$ has a particular form here, a strategy $\Sigma$ can be expressed as $\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_t, \cdots\}$, where $\sigma_t \in \{C, N\}$ denotes the action taken in period $t$ in history $h^t = [N, N]^{t-1}$. In particular, we denote by $C^\infty$ the strategy $\{C, C, C, \cdots\}$, which means $\sigma_t = C$ for all $t$, i.e., in all histories $h^t$. Similarly, $N^\infty$ denotes the strategy $\{N, N, N, \cdots\}$, which means $\sigma_t = N$ for all $t$. We denote by $[\Sigma, \Sigma']$ a strategy profile where BM1 chooses strategy $\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_t, \cdots\}$ and BM2 chooses strategy $\Sigma' = \{\sigma'_1, \sigma'_2, \cdots, \sigma'_t, \cdots\}$.

**Goal of this section:** We will show in Propositions 4 and 6 that $[N^\infty, N^\infty]$ is a subgame perfect equilibrium (SPE) iff $D_1 > D^*$, and $[C^\infty, N^\infty]$ is an SPE iff $D_1 \leq D^*$, where

$$D^* = \frac{1}{\phi\xi}\left\{1 - \frac{(1-\beta)(\frac{1}{2}-\alpha)y_L}{\alpha(y_H-\varepsilon) - (\beta\frac{1}{2} + (1-\beta)\alpha)y_L}\right\}. \tag{11}$$

Note that the debt $D_1$ in the above conditions represents the amount in period 1, which subsequently evolves over time according to the interest rate $r = \beta^{-1}$ and the repayment $b_t$. The level of debt in period 1, $D_1$, matters because it determines the number of stakeholders $n$. Debt levels $D_t$ for $t \geq 2$ are not relevant for the outcome of this game. Based on this result, we will show in Propositions 7 and 8 that, iff $D_1 > D^*$, $[N^\infty, N^\infty]$ is the unique SPE and debt restructuring never takes place, and that, iff $D_1 \leq D^*$, debt restructuring takes place in period 1 immediately after the bubble bursts and debt overhang emerges.

**Relation between $D^*$ and payoffs, $y_H - \varepsilon$ and $y_L$:** Before going into the details of the analysis, we specify the scale of the threshold $D^*$. In this model, we restrict our attention to the case where $D_1$ satisfies

$$0 < D_1 < \frac{1}{\phi\xi}.$$

We can show the following lemma about the relation between threshold $D^*$ and payoffs, $y_H - \varepsilon$ and $y_L$.

**Lemma 3.** *For a parameter $x \in [0,1]$, $D^* = (1-x)\frac{1}{\phi\xi}$ iff*

$$y_H - \varepsilon = \left[1 + \left(\beta + \frac{1-\beta}{x}\right)\left(\frac{1}{2\alpha} - 1\right)\right] y_L.$$

This lemma implies, for example, that $D^* = 0$ if $y_H - \varepsilon = \frac{1}{2\alpha} y_L$, $D^* \le \frac{1}{2\phi\xi}$ if $y_H - \varepsilon \le \alpha^{-1}(\beta\frac{1}{2} + (1-\beta)(1-\alpha))y_L$, and $D^* \le \frac{1}{\phi\xi}$ if $y_H - \varepsilon \le +\infty$.

### 6.1 Preliminaries

Here are some preliminary results.

**[C, C] is not played in any SPE:** Suppose that a strategy profile $[\Sigma, \Sigma']$ constitutes an SPE, where $\Sigma = \{\sigma_1, \sigma_2, \ldots\}$ and $\Sigma' = \{\sigma'_1, \sigma'_2, \ldots\}$. We ask whether it is possible to have $\sigma_t = \sigma'_t = C$ for any $t$. The answer is no, as we see in the following claim.

**Claim 2.** Any strategy profile in which both BM1 and BM2 choose $C$ in some period $t$ (in history $h^t$) is not an SPE.

We prove this claim only for $t = 1$ ($h^1$). Exactly the same argument can be used to establish the fact for a subgame starting at any $h^t \in H$. In fact, the proof simply follows from the above Table 1.

Suppose that there exists an SPE $[\Sigma, \Sigma']$, in which $\sigma_1 = \sigma'_1 = C$. Therefore, $\Sigma = \{C, \sigma_2, \sigma_3, \cdots\}$ and $\Sigma' = \{C, \sigma'_2, \sigma'_3, \cdots\}$. When the two players follow $[\Sigma, \Sigma']$, (10) implies that the payoff of BM1 is written as

$$\frac{1}{2}(\Pi - nq) = \frac{1}{2}\{(1-n\xi)\Pi + n\xi\pi_1\},$$

where $\pi_1$ is the total payoff for BM1 and BM2 when both of them choose $N$ in period 1, that is, the payoff they would earn in subgame $h^2$. (In that subgame, they follow the continuation strategy $\Sigma(h^2) = \{\sigma_2, \sigma_3, \cdots\}$ and $\Sigma'(h^2) = \{\sigma'_2, \sigma'_3, \cdots\}$.)

Now suppose that BM1 makes the following deviation $\Sigma^{(d)} = \{N, \sigma_2, \sigma_3, \cdots\}$, which means that BM1 changes $\sigma_1$ from $C$ to $N$, and do not change $\sigma_t$ for all $t \ge 2$. With the strategy profile $[\Sigma^{(d)}, \Sigma']$, BM1's payoff becomes

$$(1-\alpha)(\Pi - nq^{(d)}) = (1-\alpha)\{(1-n\xi)\Pi + n\xi\pi_1^{(d)}\}.$$

Here, $\pi_1^{(d)}$ is the total payoff for BM1 and BM2 in subgame $h^2$, so that $\pi_1^{(d)} = \pi_1$. Since $1 - \alpha > \frac{1}{2}$ and $\pi_1^{(d)} = \pi_1$, we have shown that BM1 obtains strictly larger payoff by choosing $\Sigma^{(d)}$ than by choosing $\Sigma$, which implies Claim 2. □

**Upper and lower bounds of $\pi^{(d)}$:** Consider any strategy profiles $[\Sigma, \Sigma']$ such that $\sigma_1 = \sigma'_1 = N$, that is, both BM1 and BM2 choose $N$ in period 1 (the continuation strategies from $h^2$ are arbitrary). We denote by $\pi^{(d)}$ the joint payoff for BM1 and BM2 associated with that strategy profile. The superscript $(d)$ is placed because we typically use $\pi^{(d)}$ to evaluate the payoff from a deviation strategy. We can derive bounds on $\pi^{(d)}$ as shown in the following claim.

**Claim 3.** Let $\pi^{(d)}$ be the joint payoff for BM1 and BM2 for any strategy profiles of the form $[\Sigma, \Sigma'] = [\{N, \sigma_2, \sigma_3, \cdots\}, \{N, \sigma'_2, \sigma'_3, \cdots\}]$. Then the following inequalities hold true:

$$\pi_{\min} \leq \pi^{(d)} \leq \pi_{\max}, \tag{12}$$

where

$$\pi_{\min} \equiv \frac{y_L}{1-\beta}, \tag{13}$$

$$\pi_{\max} \equiv \frac{y_L + \beta(1-n\xi)\Pi}{1-\beta n\xi} \tag{14}$$

*Proof.* Let $[\Sigma, \Sigma'] = [\{N, \sigma_2, \sigma_3, \cdots\}, \{N, \sigma'_2, \sigma'_3, \cdots\}]$ be given. With the continuation strategy profile $[\Sigma(h^2), \Sigma'(h^2)]$, we can compute the joint payoff for BM1 and BM2 *if* debt restructuring were achieved in period 1. It is given by

$$\Pi - nq^{(d)} = (1-n\xi)\Pi + n\xi\pi^{(d)} > \pi^{(d)},$$

because the Nash bargaining between bank managers (BM1 and BM2) and stakeholders is to divide the surplus $\Pi - \pi^{(d)}$ and bank managers get $(1-n\xi)(\Pi - \pi^{(d)}) > 0$. Since the reservation value is $\pi^{(d)}$, the bank managers obtain $\Pi - nq = (1-n\xi)(\Pi - \pi^{(d)}) + \pi^{(d)} = (1-n\xi)\Pi + n\xi\pi^{(d)} > \pi^{(d)}$.

As a (potential) deviation from $[\Sigma, \Sigma']$ in history $h^t = [N,N]^{t-1}$, consider a continuation strategy profile $[\hat{\Sigma}(h^t), \hat{\Sigma}'(h^t)] = [\{\hat{\sigma}_t, \hat{\sigma}_{t+1}, \cdots\}, \{\hat{\sigma}'_t, \hat{\sigma}'_{t+1}, \cdots\}]$, where $\hat{\sigma}_t = \hat{\sigma}'_t = N$, $\hat{\sigma}_{t+j} = \sigma_{t+j}$ and $\hat{\sigma}'_{t+j} = \sigma'_{t+j}$ for all $j \geq 1$. Let $\pi_t^{(d)}$ denote the joint payoff for BM1 and BM2 in subgame $h^t$ associated with strategy profile $[\hat{\Sigma}(h^t), \hat{\Sigma}'(h^t)]$. It must satisfy

$$\pi_t^{(d)} = y_L + \beta V_{t+1}^{(d)},$$

where $V_{t+1}^{(d)}$ is the joint payoff for BM1 and BM2 in subgame $h^{t+1} = [N,N]^t$. Since $V_{t+1}^{(d)} \in \{\pi_{t+1}^{(d)}, \Pi - nq_{t+1}^{(d)}\}$ and $\pi_{t+1}^{(d)} < \Pi - nq_{t+1}^{(d)} = (1-n\xi)\Pi + n\xi\pi_{t+1}^{(d)}$, we have

$$\pi_1^{(d)} \leq y_L + \beta\{(1-n\xi)\Pi + n\xi\pi_2^{(d)}\} \leq \cdots \leq \frac{y_L + \beta(1-n\xi)\Pi}{1-\beta n\xi},$$

because $\lim_{t\to\infty}(\beta n\xi)^t \pi_{t+1}^{(d)} = 0$, as $\pi_{t+1}^{(d)} \leq \frac{y_H}{1-\beta} < +\infty$. Similarly, we have

$$\pi_1^{(d)} \geq y_L + \beta\pi_2^{(d)} \geq \cdots \geq \frac{y_L}{1-\beta}.$$

This completes the proof. □

### 6.2 Condition for $[N^\infty, N^\infty]$ to be an SPE

We consider whether the strategy profile $[N^\infty, N^\infty]$, where both BM1 and BM2 choose N for all periods, constitutes an SPE. For this purpose, let us consider a deviation by BM1. For any deviation, there exists $m \in \{1, 2, \cdots,\}$ such that BM1 chooses $N$ in histories $h^1, h^2, \cdots, h^m$, and she chooses $C$ in history $h^{m+1}$ for the first time. As Assumption 5 implies, the payoff for BM1 from this deviation is

$$(1+\beta+\cdots+\beta^{m-1})\frac{1}{2}y_L + \beta^m\alpha(\Pi - nq_{m+1}^{(d)}) = \frac{1-\beta^m}{1-\beta}\frac{1}{2}y_L + \beta^m\alpha\{(1-n\xi)\Pi + n\xi\pi_{m+1}^{(d)}\},$$

whereas the payoff of not deviating from $[N^\infty, N^\infty]$ is

$$\frac{\frac{1}{2}y_L}{1-\beta}.$$

Therefore, comparing the original path and the (particular) deviation, the condition for BM1 not to choose the deviation is $\frac{1}{2(1-\beta)}y_L > \frac{1-\beta^m}{1-\beta}\frac{1}{2}y_L + \beta^m\alpha\{(1-n\xi)\Pi + n\xi\pi_{m+1}^{(d)}\}$, which is rewritten as $\frac{1}{2(1-\beta)}y_L > \alpha\{(1-n\xi)\Pi + n\xi\pi_{m+1}^{(d)}\}$. Since $\pi_{m+1}^{(d)} \leq \pi_{\max}$, the sufficient condition for the above inequality is

$$\frac{\frac{1}{2}y_L}{1-\beta} > \alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\} = \alpha\{(1-n\xi)\Pi + n\xi\frac{y_L + \beta(1-n\xi)\Pi}{1-\beta n\xi}\}. \tag{15}$$

Using $n = \phi D_1$, we can rewrite this sufficient condition as

$$D_1 > D^*,$$

where $D^*$ is defined by (11). This sufficient condition applies to any deviation path. Therefore, as long as $D_1 > D^*$, BM1 is better off staying in $[N^\infty, N^\infty]$ than choosing a deviation in *any* history. Therefore, we have proved the following claim.

**Claim 4.** If $D_1 > D^*$, the strategy profile $[N^\infty, N^\infty]$ is an SPE.

Next, we consider whether $[N^\infty, N^\infty]$ is an SPE when $D_1 \leq D^*$. The condition $D_1 \leq D^*$ can be rewritten as a reverse of (15):

$$\frac{\frac{1}{2}y_L}{1-\beta} \leq \alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\}.$$

Suppose that BM1 deviates by choosing $C^\infty = \{C, C, \cdots\}$ instead of $N^\infty = \{N, N, \cdots\}$. Given that BM2 still chooses $N^\infty$, we can calculate $\pi_t^{(d)}$ the total payoff in period $t$ for bank managers when both choose N in histories $h^1, h^2, \cdots, h^t$, along with this deviation $C^\infty(h^{t+1})$. Then, we have

$$\pi_t^{(d)} = y_L + \beta\{(1-n\xi)\Pi + n\xi\pi_{t+1}^{(d)}\},$$

which implies that $\pi_t^{(d)} = \pi_{\max}$. Therefore, BM1's payoff from deviation is $\alpha[(1-n\xi)\Pi + n\xi\pi_1^{(d)}] = \alpha[(1-n\xi)\Pi + n\xi\pi_{\max}]$. The condition $D_1 \leq D^*$ implies that the deviation $[C^\infty, N^\infty]$ gives a higher payoff for BM1 than $[N^\infty, N^\infty]$. Thus, we have proved the following claim.

**Claim 5.** If $D_1 \leq D^*$, then $[N^\infty, N^\infty]$ is not an SPE.

These two claims imply the following proposition.

**Proposition 4.** *The strategy profile $[N^\infty, N^\infty]$ is an SPE if and only if $D_1 > D^*$.*

### 6.3 Condition for $[C^\infty, N^\infty]$ to be an SPE

We consider whether the strategy profile $[C^\infty, N^\infty]$, where BM1 chooses C and BM2 chooses N in all histories $h^t = [N, N]^{t-1}$, $t \geq 1$, constitutes an SPE. We first consider the deviation by BM1, and then consider the deviation by BM2, secondly.

**Deviation by BM1:** The payoff for BM1 in $[C^\infty, N^\infty]$ is $\alpha(\Pi - nq) = \alpha\{(1-n\xi)\Pi + n\xi\pi\}$, where $\pi$ is the total payoff for BM1 and BM2 in period 1 from $[\{N, C^\infty\}, N^\infty]$. Since $\pi_t = y_L + \beta\{(1-n\xi)\Pi + n\xi\pi_{t+1}\}$, we have $\pi = \pi_{\max}$ in $[C^\infty, N^\infty]$. Therefore, the payoff of BM1 in $[C^\infty, N^\infty]$ is

$$\alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\}.$$

Denote by $\pi_t^{(d)BM1}$ the payoff in subgame $h^t$ for BM1 associated with a particular deviation. For any deviation, there exists $m \in \{1, 2, \cdots\}$ such that BM1 chooses $N$ in histories $h^1, h^2, \cdots, h^m$, and she chooses $C$ in history $h^{m+1}$ for the first time. Then we have

$$\pi_1^{(d)BM1} = (1 + \beta + \cdots + \beta^{m-1})\frac{1}{2}y_L + \beta^m \alpha\{(1-n\xi)\Pi + n\xi\pi_{m+1}^{(d)}\},$$

where $\pi_{m+1}^{(d)}$ is the total payoff for BM1 and BM2 along with the deviation path. Thus,

$$\pi_1^{(d)BM1} \leq \frac{1-\beta^m}{1-\beta}\frac{1}{2}y_L + \beta^m \alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\}.$$

Suppose that

$$\alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\} < \pi_1^{(d)BM1}. \tag{16}$$

Then we have

$$\alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\} < \frac{1-\beta^m}{1-\beta}\frac{1}{2}y_L + \beta^m \alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\},$$

which is rewritten as

$$\alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\} < \frac{\frac{1}{2}y_L}{1-\beta},$$

This is, in turn, rewritten as $D_1 > D^*$. Thus, we have proved that if (16) holds then $D_1 > D^*$. The contraposition is that if $D_1 \leq D^*$, then

$$\alpha\{(1-n\xi)\Pi + n\xi\pi_{\max}\} \geq \pi_1^{(d)BM1},$$

for the particular deviation. Since this reasoning holds for any deviations, we have proved the following claim.

**Claim 6.** Suppose that $D_1 \leq D^*$, and that BM2 chooses $N^\infty$. Then BM1 is better off choosing $C^\infty$ than choosing any other strategies.

As we saw in (15), if $D_1 > D^*$, the payoff for BM1 of choosing $C^\infty$ is smaller than that of choosing $N^\infty$, given that BM2 chooses $N^\infty$. Thus, it is better for BM1 to deviate from $C^\infty$. We have proved the following lemma.

**Lemma 5.** *If $D_1 > D^*$, then $[C^\infty, N^\infty]$ is not an SPE. If $D_1 \leq D^*$, there is no profitable deviation from $[C^\infty, N^\infty]$ for BM1.*

**Deviation by BM2:** Suppose that BM2 deviates from $N^\infty$, given that BM1 chooses $C^\infty$. First, consider any deviation from $N^\infty$, $\hat{\Sigma}'$, with $\hat{\sigma}'_1 = N$. The payoff for BM2 from this deviation is

$$(1-\alpha)\{(1-n\xi)\Pi + n\xi\pi_1^{(d)}\}.$$

Since $\pi_1^{(d)} \leq \pi_{\max}$, we have

$$(1-\alpha)\{(1-n\xi)\Pi + n\xi\pi_1^{(d)}\} \leq (1-\alpha)\{(1-n\xi)\Pi + n\xi\pi_{\max}\},$$

where the right-hand side is the payoff for BM2 of not deviating. Therefore, BM2 does not choose any deviation strategy in which $N$ is chosen in period 1.

Next, consider a deviation strategy for BM2 of the form: $\hat{\Sigma}'$ with $\hat{\sigma}'_1 = C$. The payoff for BM2 from such a strategy is

$$\frac{1}{2}\{(1-n\xi)\Pi + n\xi\pi_1^{(d)}\},$$

which satisfies

$$\frac{1}{2}\{(1-n\xi)\Pi + n\xi\pi_1^{(d)}\} \leq \frac{1}{2}\{(1-n\xi)\Pi + n\xi\pi_{\max}\} < (1-\alpha)\{(1-n\xi)\Pi + n\xi\pi_{\max}\},$$

where the righ-hand side is the payoff for BM2 from not deviating. Thus, BM2 has no gain from deviations of any form. We have proved the following claim.

**Claim 7.** Given that BM1 chooses $C^\infty$, it is optimal for BM2 to choose $N^\infty$.

This claim and Lemma 5 imply the following proposition.

**Proposition 6.** *The strategy profile $[C^\infty, N^\infty]$ is an SPE if and only if $D_1 \leq D^*$.*

### 6.4 Condition for debt restructuring

In this subsection, we examine the existence of SPEs other than $[C^\infty, N^\infty]$ and $[N^\infty, C^\infty]$ for $D_1 \leq D^*$, or $[N^\infty, N^\infty]$ for $D_1 > D^*$.

Any SPE falls into exactly one of the following three categories, depending on the bank managers' (BMs') actions in period 1:

- Both BM1 and BM2 choose $C$;
- One BM chooses $C$ while the other chooses $N$;
- Both BMs choose $N$.

Here, Claim 2 implies that the first case where both BM1 and BM2 choose $C$ in period 1 is not an SPE. It suffices to examine the remaining two cases. Note also that given the simple structure of our model, what we can say about period 1 can be extended to any other histories straightforwardly.

### 6.4.1 The case where $D_1 > D^*$

When $D_1 > D^*$, we have proved that $[N^\infty, N^\infty]$ is an SPE.

**Can $[C, N]$ be an equilibrium outcome?:** Suppose that a strategy profile $[\Sigma, \Sigma'] = [\{\sigma_1, \sigma_2, \cdots\}, \{\sigma'_1, \sigma'_2, \cdots\}]$ with $\sigma_1 = C$ and $\sigma'_1 = N$ is an SPE. The payoff for BM1 of staying in the equilibrium is

$$\alpha\{(1 - n\xi)\Pi + n\xi\pi_1\},$$

where $\pi_1$ is the total payoff for BM1 and BM2 from $[\{N, \sigma_2, \sigma_3, \cdots\}, \{N, \sigma'_2, \sigma'_3, \cdots\}]$. By Claim 3, we know that $\pi_1 \leq \pi_{\max}$.

Now, we consider the specific deviation in which BM1 chooses the strategy profile $\Sigma^{(d)BM1} = \{N, C, C, C, \cdots\}$, i.e., $N$ is chosen in period 1 and $C$ is chosen for all subsequent histories $h^t$, $t \geq 2$. The payoff for BM1 in period 1 of this deviation $\pi_1^{(d)BM1}$ satisfies

$$\pi_1^{(d)BM1} \geq \frac{1}{2}y_L + \beta\alpha[(1 - n\xi)\Pi + n\xi\pi_2^{(d)}],$$

where $\pi_t^{(d)}$ is the joint (continuation) payoff for bank managers in subgame $h^t$ when the two managers follow the continuation strategies $[\{N, \Sigma^{(d)BM1}(h^{t+1})\}, \{N, \Sigma'(h^{t+1})\}]$. The joint continuation payoff $\pi_t^{(d)}$ satisfies $\pi_t^{(d)} = y_L + \beta\{(1 - n\xi)\Pi + n\xi\pi_{t+1}^{(d)}\}$, which implies that

$$\pi_t^{(d)} = \pi_{\max}.$$

The condition for BM1 not to deviate from the original strategy $\Sigma$ is

$$\alpha[(1 - n\xi)\Pi + n\xi\pi_1] \geq \pi_1^{(d)BM1}.$$

The necessary condition for this is $\alpha[(1-n\xi)\Pi+n\xi\pi_{\max}] \geq \frac{1}{2}y_L+\beta\alpha[(1-n\xi)\Pi+n\xi\pi_{\max}]$, which is equivalent to $D_1 \leq D^*$. This result contradicts the assumption $D_1 > D^*$. We have thus proved the following claim:

**Claim 8.** Suppose that $D_1 > D^*$. Then there does not exist any SPE in which one bank manager chooses $C$ and the other chooses $N$ in period 1 for any history $h^t$, $t \geq 1$.

**Can $[N, N]$ be an equilibrium outcome?** We already know that when $D_1 > D^*$, $[N^\infty, N^\infty]$ is an SPE. Here, we ask whether or not there exists any other SPE, where $[N, N]$ is chosen in period 1. Suppose that there exists an SPE strategy profile $[\Sigma, \Sigma'] = [\{\sigma_1, \sigma_2, \cdots\}, \{\sigma'_1, \sigma'_2, \cdots\}]$ with $\sigma_1 = N$ and $\sigma'_1 = N$. Since $[\Sigma, \Sigma'] \neq [N^\infty, N^\infty]$, there must exist $m \geq 1$ such that both bank managers choose $N$ for histories $h^1, h^2, \cdots, h^m$, and one bank manager chooses $C$ in history $h^{m+1}$ for the first time. To be specific, let us assume BM1 chooses $C$ in history $h^{m+1}$. The payoff for BM1 with the strategy profile $[\Sigma, \Sigma']$ is

$$(1 + \beta + \cdots + \beta^{m-1})\frac{1}{2}y_L + \beta^m \alpha[(1 - n\xi)\Pi + n\xi\pi_{m+1}].$$

Consider the strategy $\Sigma^{(d)} = N^\infty$ as a specific deviation by BM1. If BM2's strategy is indeed $\Sigma' = N^\infty$, then BM1 obtains $\frac{\frac{1}{2}y_L}{1-\beta}$ by deviating to $\Sigma^{(d)} = N^\infty$. Otherwise, if BM2 chooses $C$ in history $h^{m'}$, then BM1's payoff from the deviation becomes

$$(1 + \beta + \cdots + \beta^{m'})\frac{1}{2}y_L + \beta^{m'}(1 - \alpha)[(1 - n\xi)\Pi + n\xi\pi^{(d)}_{m'+1}].$$

Now we know that

$$(1 - n\xi)\Pi + n\xi\pi^{(d)}_{m'+1} \geq \pi^{(d)}_{m'+1} \geq \pi_{\min} = \frac{y_L}{1 - \beta}.$$

This inequality implies that

$$(1 - \alpha)[(1 - n\xi)\Pi + n\xi\pi^{(d)}_{m'+1}] \geq (1 - \alpha)\pi_{\min} > \frac{1}{2}\pi_{\min} = \frac{\frac{1}{2}y_L}{1 - \beta}.$$

Therefore, BM1's deviation payoff is no less than

$$\frac{\frac{1}{2}y_L}{1 - \beta},$$

which must not be greater than BM1's equilibrium payoff, i.e.,

$$\frac{1 - \beta^m}{1 - \beta}\frac{1}{2}y_L + \beta^m\alpha[(1 - n\xi)\Pi + n\xi\pi_{m+1}] \geq \frac{\frac{1}{2}y_L}{1 - \beta},$$

This inequality holds only if

$$\alpha[(1 - n\xi)\Pi + n\xi\pi_{\max}] \geq \frac{\frac{1}{2}y_L}{1 - \beta},$$

which is equivalent to $D_1 \leq D^*$. This result contradicts the assumption that $D_1 > D^*$. Thus, we have proved the following claim.

**Claim 9.** Suppose that $D_1 > D^*$. Then $[N^\infty, N^\infty]$ is the only SPE in which both BM1 and BM2 choose $N$ in period 1.

The above two claims imply the following proposition.

**Proposition 7.** *Suppose that $D_1 > D^*$. Then the strategy profile $[N^\infty, N^\infty]$ is the unique SPE. Thus, debt restructuring never occurs if $D_1 > D^*$.*

### 6.4.2 The case where $D_1 \leq D^*$

When $D_1 \leq D^*$, we have proved that $[C^\infty, N^\infty]$ is an SPE.

**Can $[N, N]$ be an equilibrium outcome?** Suppose that there exists an SPE strategy profile $[\Sigma, \Sigma'] = [\{\sigma_1, \sigma_2, \cdots\}, \{\sigma'_1, \sigma'_2, \cdots\}]$ with $\sigma_t = \sigma'_t = N$ for $t = 1, \ldots, m$ and $[\sigma_{m+1}, \sigma'_{m+1}] \neq [N, N]$. By Claim 2 we know that $[\sigma_{m+1}, \sigma'_{m+1}] \neq [C, C]$. Without loss of generality, let us assume that $[\sigma_{m+1}, \sigma'_{m+1}] = [C, N]$. The equilibrium payoff of BM1 is then

$$\frac{1-\beta^m}{1-\beta}\frac{1}{2}y_L + \beta^m \alpha[(1-n\xi)\Pi + n\xi\pi_{m+1}].$$

Now suppose that BM1 deviates from the equilibrium strategy to choose $\Sigma^{(d)} = C^\infty$. In this case, the deviation payoff for BM1 is

$$\alpha[(1-n\xi)\Pi + n\xi\pi_{\max}].$$

Then the condition for no deviation is

$$\frac{1-\beta^m}{1-\beta}\frac{1}{2}y_L + \beta^m \alpha[(1-n\xi)\Pi + n\xi\pi_{m+1}] > \alpha[(1-n\xi)\Pi + n\xi\pi_{\max}].$$

Since $\pi_{m+1} \leq \pi_{\max}$, a necessary condition for no deviation is

$$\frac{\frac{1}{2}y_L}{1-\beta} > \alpha[(1-n\xi)\Pi + n\xi\pi_{\max}],$$

which is equivalent to $D_1 > D^*$. This condition contradicts the assumption that $D_1 \leq D^*$. Thus, the following claim holds true.

**Claim 10.** Suppose that $D_1 \leq D^*$. Then there does not exist an SPE in which both BM1 and BM2 choose $N$ in period 1.

**Can $[C, N]$ be an equilibrium outcome?** When $D \leq D^*$, is $[C^\infty, N^\infty]$ the unique SPE in which BM1 chooses $C$ and BM2 chooses $N$ in period 1? There might exist other SPEs with such a property. However, it follows from the above arguments that when $D_1 \leq D^*$, every SPE necessarily involves one BM choosing $C$ and the other choosing $N$ in period 1. Thus the result is summarized as follows.

**Proposition 8.** *For $D_1 \leq D^*$, any SPE involves one bank manager choosing $C$ in period 1. Consequently, debt restructuring occurs in period 1, immediately after the bubble collapse and the emergence of debt overhang, provided that $D_1 \leq D^*$.*

# 7 Policy implications

We have shown that when the debt exceeds $D_{\max}$, borrower's production becomes inefficient, that is, the production net of effort cost is $y_H - \varepsilon = A_H k^\psi - \varepsilon$ with $k = 1$ if

$D_1 = Q \leq D_{\max}$, and the production is $y_L = A_L k^{\psi}$ with $k = 1$ if $D_1 > D_{\max}$. This model implies that reducing the debt burden for the borrower will restore efficient production ($y_H - \varepsilon$). Given the finding in the previous section that banks voluntarily restructure debt to $D_{\max}$ as long as $D_1 \leq D^*$, this section focuses on the case where

$$D_1 > D^*, \tag{17}$$

and compare the following two policy options:

- Subsidy to the borrowing firms, $S_t^F$.
- Subsidy to banks, $S_t^B$, conditional on the restructuring of nonperforming loans from $D_t$ to $D_{\max}$.

This section demonstrates that bank subsidies are more efficient than firm subsidies, as a smaller fiscal outlay to banks can more effectively alleviate debt overhang and restore firm productivity. Conversely, we show that firm subsidies carry a potential pitfall: an insufficient subsidy may inadvertently hinder the debt restructuring.

**V-shaped recovery or secular stagnation?** Here we provide a short note on the tradeoff between the V-shaped recovery and the L-shaped recession (the secular stagnation). In policy debate on the policy response after the financial crisis, it is often argued that the government needs to choose whether to quickly liquidate over-indebted zombie firms or to gradually liquidate them. The premise is that the quick liquidation of a large number of zombie firms induces a large short-term decline and quick recovery of the economy (the V-shaped recovery), whereas the gradual liquidation of zombie firms induces a shallow decline of the economy that continues for an extended period of time (the L-shaped stagnation). See, for example, Acharya, Lenzu and Wang (2023). Both V-shaped recovery and L-shaped stagnation entail significant economic costs, as they involve the inevitable liquidation of over-indebted borrowers. In contrast, our model suggests that debt restructuring facilitates a recovery devoid of such inefficiencies. Debt reduction from $D_t$ ($> D_{\max}$) to $D_{\max}$ makes production recover from $y_L$ to $y_H - \varepsilon$ without any cost associated with liquidation of the borrowers.[7]

## 7.1 Subsidy to borrowers

In the state where $D_1 > D_{\max}$ and production is going to be inefficient ($y_L$), one policy option to restore efficient production $y_H - \varepsilon$ is a direct subsidy to the borrowing firms.

[7]Debt reduction $D_t - D_{\max}$ would be recognized as an extraordinary loss in corporate accounting, but does not lead to an actual decrease in production. Gross Domestic Product (GDP) does not decrease with debt reduction, but increases as firms produce $y_H - \varepsilon$ rather than $y_L$.

Suppose that the government gives subsidy $S_1^F$ in period 1, after productivity is revealed to be $A_H$.

In this case, the borrowing firm pays $S^F$ to the bank immediately and the remaining debt becomes $rD_1 - S^F$. Then, the number of stakeholders in the bank becomes

$$n(S^F) = \phi(D_1 - r^{-1}S^F).$$

The amount $S^F$ that is paid to the bank is divided by two bank managers (BM1 and BM2) in the coordination game as follows.

- If both BM1 and BM2 choose the strategy N, they divide $S^F$ equally.
- If BM1 chooses C and BM2 chooses N, BM1 takes $\alpha S^F$ and BM2 takes $(1-\alpha)S^F$.

Arguments similar to those of the proof of Proposition 8 in Section 6.4.2 imply that the condition for a BM to choose C is

$$\underbrace{\frac{\frac{1}{2}y_L}{1-\beta} + \frac{1}{2}S^F}_{\text{payoff for choosing N}} \leq \underbrace{\alpha(\Pi + S^F - n(S^F)q)}_{\text{payoff for choosing C}}, \tag{18}$$

where the left-hand side is the payoff for choosing N, and the right-hand side is the payoff for choosing C, given that the opponent chooses N. This condition is equal to $D_1 \leq D^*$ if $S^F = 0$. Condition (18) is rewritten as

$$\frac{\frac{1}{2}y_L}{1-\beta} + \left\{\frac{1}{2} - \alpha\left(1 + \beta\phi\xi(\Pi - \pi)\right)\right\} S_F \leq \alpha(\Pi - nq), \quad \text{where } \ n = \phi D_1.$$

In this section, we have assumed that $D_1 > D^*$ or $\frac{\frac{1}{2}y_L}{1-\beta} > \alpha(\Pi - nq)$. This assumption and the above inequality lead to the following claim.

**Claim 11.** Given that $D_1 > D^*$, condition (18) is never satisfied with any nonnegative value of $S^F$ if

$$\frac{1}{2} - \alpha\left(1 + \frac{\beta\phi\xi A_H}{1-\beta}\right) > 0. \tag{19}$$

Inequality (19) is plausible because $\phi\xi < O(1)$ and $\alpha < \frac{1}{2}$. The intuition behind this claim is as follows. While $S^F$ increases the payoffs for both options N and C, it disproportionately benefits the former ($\frac{1}{2}S^F$) relative to the latter ($\alpha S^F$). This outcome runs counter to the policy's intended objective, which is to incentivize the choice of C by making its payoff larger than that of N.

This claim implies that direct subsidies to borrowers disincentivize bank managers from choosing C, thereby impeding debt restructuring. Alternatively, the government could utilize firm subsidies to render restructuring unnecessary by ensuring the full repayment of $D_1$. Suppose the government provides a subsidy $S^F(D_1)$ such that:

$$rD_1 = rD_{\max} + S^F(D_1).$$

Under this scheme, the debt $D_1$ becomes fully repayable, effectively preventing the emergence of debt overhang. We have proved the following lemma.

**Lemma 9.** *A firm subsidy $S^F$ provided in period 1 can successfully restore productivity $A_H$ only if it equals or exceeds $S^F(D_1)$, defined as:*

$$S^F(D_1) = r(D_1 - D_{\max}). \tag{20}$$

*Conversely, if $S^F < S^F(D_1)$, the intervention becomes counterproductive; it impedes debt restructuring and prevents the recovery of firm productivity.*

This highlights a critical non-monotonicity in firm subsidies: they are either costly or detrimental, supporting the relative efficiency of bank subsidies, which we analyze below. One caveat deserves mention. We assume that subsidies are financed via lump-sum taxes to neutralize any distortionary effects of taxation. This simplification is justified because our analysis focuses on the relative scale of $S^F$ versus $S^B$. Insofar as tax distortions apply equally to both schemes, omitting them does not alter the fundamental comparison between the two policies.

**Interpretation of fiscal policy:** If we interpret borrower subsidies as a form of expansionary fiscal policy aimed at boosting the incomes of borrowers in general, this result implies that such expansionary policy may inadvertently impede productivity recovery in the aftermath of a financial crisis. In other words, fiscal expansion can hinder economic recovery if the fundamental cause of the productivity decline is a debt overhang.

## 7.2 Subsidy to banks

In this subsection, we consider the bank subsidy $S_t^B$, which the government pays to the bank that restructured the loan $D_1$ to $D_{\max}$ in period 1. We assumed that the government cannot give subsidies to the bank managers (BM1 or BM2) or the stakeholders within the bank directly conditional on their respective actions. This assumption seems plausible because the games among bank managers and stakeholders take place inside the bank and are neither observable nor verifiable to the government.

Without a bank subsidy, the total surplus that the bank obtains by reducing debt $rD_1$ to $rD_{\max}$ is $\Pi = \frac{y_H - \varepsilon}{1-\beta}$, which will increase to $\Pi + S_t^B = \frac{\{y_H + (1-\beta)S_t^B\} - \varepsilon}{1-\beta}$ by receiving the bank subsidy. Therefore, we can interpret that the bank subsidy $S^B$ changes $y_H$ to $y_H + (1-\beta)S$. We define $D^*(S^B)$ by changing $y_H$ to $y_H + (1-\beta)S^B$ in the definition of $D^*$:

$$D^*(S^B) \equiv \frac{1}{\phi\xi}\left\{1 - \frac{(1-\beta)(\frac{1}{2}-\alpha)y_L}{\alpha[y_H + (1-\beta)S^B - \varepsilon] - [\frac{1}{2}\beta + \alpha(1-\beta)]y_L}\right\}. \tag{21}$$

In the economy where the government provides bank subsidies $S^B$, Proposition 8 implies that if $D_1 \leq D^*(S^B)$ then debt restructuring is conducted in period 1 and the economy immediately recovers. We define $S^B(D_1)$ as the solution to $D_1 = D^*(S^B)$, which is written as

$$S^B(D_1) = \frac{\{\frac{1}{2} - [\frac{1}{2}\beta + \alpha(1-\beta)]\phi\xi D_1\}y_L}{\alpha(1-\phi\xi D_1)(1-\beta)} - \frac{y_H - \varepsilon}{1-\beta}. \tag{22}$$

This is the threshold level of bank subsidy that ensures that the bank opts for debt restructuring.

**Bank subsidy is more efficient than borrower subsidy:** Now we characterize the condition for the bank subsidy (22) in period 1 being smaller than the firm subsidy. First, we clarify the condition for $S^B(D_1) < rD_1 - rD^*$, which is rewritten as

$$\left(\frac{1}{2} - \alpha\right) y_L < G(D_1),$$

where $G(D)$ is a following quadratic function of $D$:

$$G(D) = \alpha \left[ rD + \frac{y_H - \varepsilon}{1-\beta} - \left\{ \beta\frac{1}{2} + (1-\beta)\alpha \right\} \frac{y_L}{\alpha(1-\beta)} - rD^* \right] (1 - \phi\xi D).$$

We can easily confirm

$$G(D^*) = \left(\frac{1}{2} - \alpha\right) y_L. \tag{23}$$

We denote the solutions to $G(D) = \left(\frac{1}{2} - \alpha\right) y_L$ by $\underline{D}$ and $\overline{D}$, where $\underline{D} < \overline{D}$. The above result (23) implies that $\underline{D} = D^*$, which is obvious because $S_t^B = 0$ by definition in the case where $D_1 = D^*$. We have shown that the necessary amount of bank subsidy $S^B(D_1)$ is smaller than $r(D_1 - D^*)$, if $D_1$ satisfies

$$D^* < D_1 < \overline{D}.$$

It is straightforward that $S^B(D_1) < S^F(D_1)$ for $D_1 \in (D^*, \overline{D})$, because

$$S^B(D_1) < r(D_1 - D^*) < r(D_1 - D_{\max}) = S^F(D_1)$$

for $D_1 \in (D^*, \overline{D})$. To characterize the magnitude of the threshold $\overline{D}$, we first observe that $\phi\xi$ is of an order smaller than $O(1)$. It follows that $\overline{D}$ can be expressed as $\frac{1}{\phi\xi} - O(1)$, where $O(1)$ term is approximately $\frac{y_H - \varepsilon}{1-\beta} - (\beta\frac{1}{2\alpha} + 1 - \beta)\frac{y_L}{1-\beta}$. Since $\overline{D}$ approaches the upper bound $\frac{1}{\phi\xi}$, the condition $D^* < D_1 < \overline{D}$ holds for any plausible value of $D_1$. We have shown the following proposition.

**Proposition 10.** *We assume that the pure strategy equilibrium is selected in the coordination game between the bank managers, when both pure and mixed strategy equilibria* *exist. The amount of bank subsidy necessary to make the productivity recover is* (22)*. For* $D_1 \in (D^*, \overline{D}]$*, the bank subsidy is a more efficient policy than the firm subsidy, as the required bank subsidy* (22) *is smaller than the corresponding firm subsidy* (20)*. However, in the extreme case where* $\overline{D} < D_1 < \frac{1}{\phi\xi}$*, this relationship may be reversed.*

This result implies that the bank subsidy is more efficient than the borrower subsidy in most plausible scenarios. The bank subsidy in this model can be interpreted as *bank recapitalization,* contingent on restructuring nonperforming loans. Our findings thus suggest that bank recapitalization aimed at incentivizing debt restructuring constitutes a more efficient policy response to financial crises than direct bailouts of over-indebted borrowers.

**Intuition for the efficiency of bank subsidies:** The relative efficiency of bank subsidies can be understood as follows. Since the objective of these subsidies is to facilitate debt restructuring and restore productivity, their effectiveness relies on being contingent upon the restructuring process. In our framework, bank subsidies can be directly conditioned on debt restructuring because banks are the primary agents responsible for deciding and implementing these operations. In contrast, subsidies to firms cannot be tied to restructuring; more importantly, such unconditional transfers exacerbate distortions in bank managers' incentives, thereby hindering the restructuring process.

**Note on the mixed strategy equilibrium:** Thus far, we have restricted our analysis to pure strategy equilibria in the coordination game between bank managers. We discuss the policy implications of the mixed strategy equilibrium in Appendix B. There, we argue that a comprehensive comparison between bank and firm subsidies requires explicitly accounting for their respective financing costs, including the distortions arising from taxation.

**Bank subsidy does not induce moral hazard of borrowers:** We can show that the bank subsidy does not induce the ex-ante moral hazard of the borrowers.

**Lemma 11.** *The announcement in period 0 of the bank subsidy provided in period 1 or later does not affect the equilibrium prices and allocations in period 0.*

The economic intuition for the reason why the bank subsidy does not intrinsically induce borrowers' moral hazard is the following: The bank subsidy is paid in the state where $V = 0$. It affects only the ex-post payoffs of banks, leaving the ex-post payoffs of firms unchanged ($V = 0$). On the other hand, the ex-ante equilibrium prices and allocations are determined by the firms' actions. Since the firms' payoffs remain unchanged at $V = 0$ regardless of whether the bank subsidy is anticipated, their actions in period 0 are unaffected by the bank subsidy.

# 8 Conclusion

This paper has demonstrated that a framework incorporating risk-shifting asset bubbles, borrower debt overhang, and creditor coordination games can replicate key empirical regularities of financial crises—specifically, booms in asset prices and credit followed by a bust and persistent productivity declines. Our analysis reveals that when debt levels are modest, voluntary restructuring occurs, and the social optimum is achieved without intervention. In contrast, when debt is elevated, debt restructuring is delayed due to coordination failure among creditors. This finding is consistent with empirical evidence that larger credit-driven asset booms tend to precede more protracted recessions. A notable contribution of our model is that it explains post-crisis stagnation without invoking standard financial frictions, such as credit crunches or tightening of borrowing constraints. This point is noteworthy because the existing literature that focuses on credit frictions primarily emphasizes policies targeting the banking sector, whereas our model underscores the necessity of policies aimed at repairing borrower balance sheets.

Therefore, the policy implications of our model diverge from the existing literature. To resolve the inefficiencies of debt overhang, we find that bank recapitalization or subsidies, contingent on debt restructuring, is more efficient than direct bailouts of borrowers. This efficiency stems from the fact that bank subsidies achieve economic recovery at a lower fiscal cost. Furthermore, we find that borrower subsidies can be counterproductive: if the subsidy amount is not exceptionally large, they may actually hinder debt restructuring by exacerbating distortions in bank decision-making. Consequently, if borrower subsidies are interpreted as a form of expansionary fiscal policy aimed at boosting borrower incomes, this result implies that such expansionary policy may inadvertently impede productivity recovery in the aftermath of a financial crisis. In other words, fiscal expansion can hinder economic recovery if the fundamental cause of the productivity decline is a debt overhang. These insights provide critical considerations for the design of future crisis management policies.

# Appendix A: Demand externality

As we note in the paragraph of production using land in Section 3, we can extend the model to incorporate monopolistic competition following the Dixit-Stiglitz framework. In this extended model, we have the demand externality that hampers debt restructuring and amplifies stagnation. The results derived in this appendix yield the following intuitive predictions regarding financial crises:

- When a credit-fueled asset price bubble bursts and the resulting debt overhang is confined to narrow market segments, debt restructuring tends to occur promptly.

Consequently, the ensuing economic stagnation is likely to be short-lived.

- When the bust is economy-wide and debt overhang pervades the entire market, banks often lack the incentive to implement restructuring. This can lead the economy into a trap of protracted stagnation.

To derive this result, we compare the case of the aggregate shock where all firms have debt overhang and the case of the idiosyncratic shock where only one firm has debt overhang.

**Production technology:** We assume the following regarding the production technology.

- The risky asset is heterogeneous and each bank $i$ initially owns one unit of asset $i$, where the productivity parameter $A_i$ can be different from that of asset $j$ for $j \neq i$. The price of the asset $i$, $Q_i$, can be different from that of other assets, $Q_j$, where $j \neq i$. There is a competition among buyers (=firms) who push up $Q_i$. The buyer of asset $i$ is denoted by firm $i$.

- The value of productivity parameter $A_i$ takes on either $B$ or $A_H$, where $A_H < B$, and $B$ represents optimism.

- The output of asset $i$ (=firm $i$) is $y_i$ and it is aggregated by

$$Y = \left( \int_0^1 y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

  where $y_i = A_i k = A_i$ for $k = 1$. The revenue for firm $i$ is $p_i y_i = Y^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}}$. In the case where $A_j = \bar{A}$ for all $j \neq i$, the revenue of firm $i$ is $\bar{A}^{\frac{1}{\sigma}} A_i^{\frac{\sigma-1}{\sigma}}$.

**Productivity shocks:** Productivity $A_i$ is either $A_H$ or $B$, i.e., $A_i \in \{A_H, B\}$. We consider the two cases for productivity shocks:

- Aggregate shock is that all assets have the same stochastic productivity so that $A_i$ for all $i$ is uncertain in period 0 and is revealed in period 1 to be either $A_i = A_H$ for all $i$ or $A_i = B$ for all $i$.

- Idiosyncratic shock is that the particular asset $i$ has stochastic productivity $A_i \in \{A_H, B\}$ so that $A_i$ is uncertain in period 0 and is revealed in period 1, while all the other assets $j (\neq i)$ have the non-stochastic productivity $A_j = A_H$ so that it is publicly known from period 0 onward that $A_j = A_H$ for all $j (\neq i)$.

**Idiosyncratic shock case:** We consider the case where $A_i$ for a particular $i$ is revealed to be $A_H$ and only firm $i$ has the debt overhang. In this case, there is uncertainty only for asset $i$, while the productivity of the other assets is known in advance. Consequently, only firm $i$ has debt overhang, and the other firms do not have debt overhang and they produce $y_H = A_H$ from period 1 onward. Only bank $i$ faces whether to implement debt restructuring. Since $Y = A_H$ in the idiosyncratic shock case, if debt restructuring is implemented, firm $i$'s revenue $p_i y_i$ is $y_H$; and if debt overhang continues, firm $i$'s revenue is $y_L$, where

$$y_H = A_H,$$
$$y_L = A_H^{\frac{1}{\sigma}} A_L^{\frac{\sigma-1}{\sigma}}.$$

**Aggregate shock case:** We consider the case where $A_i$ is revealed to be $A_H$ for all $i$. We assume that bank $i$ considers debt restructuring, on the premise that all other banks do not implement the debt restructuring. If bank $i$ implements debt restructuring, firm $i$'s revenue $p_i y_i$ is $\tilde{y}_H$, while it is $\tilde{y}_L$ if debt overhang continues, where

$$\tilde{y}_H = A_L^{\frac{1}{\sigma}} A_H^{\frac{\sigma-1}{\sigma}} = \omega y_H,$$
$$\tilde{y}_L = A_L = \omega y_L,$$
$$\omega = (A_L/A_H)^{\frac{1}{\sigma}} < 1,$$

since $Y = A_L$, when all other banks do not implement debt restructuring.

**Thresholds of debt restructuring:** In Section 6, we show that if $D_1 < D^*$, [C, N] or [N, C] is realized and debt restructuring takes place immediately in period 1; and if $D_1 > D^*$, [N, N] is realized and debt overhang continues. In the extended model in this appendix, we can consider the threshold for both the idiosyncratic-shock case and the aggregate-shock case. We denote by $D^*$ the threshold in the idiosyncratic-shock case, where bank $i$ implements the debt restructuring if $D_1 \leq D^*$. Similarly, we denote by $\tilde{D}^*$ the threshold for the aggregate-shock case. The same arguments as in Section 6 apply, and we have

$$D^* = \frac{1}{\phi\xi}\left\{1 - \frac{(1-\beta)(\frac{1}{2}-\alpha)y_L}{\alpha(y_H - \varepsilon) - (\beta\frac{1}{2} + (1-\beta)\alpha)y_L}\right\},$$
$$\tilde{D}^* = \frac{1}{\phi\xi}\left\{1 - \frac{(1-\beta)(\frac{1}{2}-\alpha)y_L}{\alpha(y_H - \frac{\varepsilon}{\omega}) - (\beta\frac{1}{2} + (1-\beta)\alpha)y_L}\right\}.$$

Since $\omega < 1$, we have $\tilde{D}^* < D^*$. Suppose $D_1$ satisfies $\tilde{D}^* < D_1 < D^*$. In this case,

- if the debt overhang $D_1$ is generated in the idiosyncratic-shock case, debt restructuring ensues, resulting in the restoration of the firm's productivity, provided that $D_1 < D^*$,

- if the debt overhang $D_1$ is generated in the aggregate-shock case, debt restructuring is not implemented, and inefficient production continues, as $D_1 > \tilde{D}^*$.

Again, we note that the aggregate-shock case above analyzes the decision on debt restructuring of one bank, on the premise that the other banks do not conduct debt restructuring. The difference between the idiosyncratic-shock case and the aggregate-shock case in the above results is due to the demand externality.

This difference may explain why the resolution of bad debt problems tends to be more protracted when they arise from a systemic crisis rather than an idiosyncratic shock.

## Appendix B: Mixed strategy equilibrium of debt restructuring

In Section 6, we analyzed the pure-strategy equilibrium in the coordination game between bank managers on debt restructuring. In this appendix, we show that the coordination game has a mixed-strategy equilibrium if $D_1$ is smaller than a certain threshold value. For $D_1 \leq D^*$, both $[C^\infty, N^\infty]$ and $[N^\infty, C^\infty]$ are the SPEs. In this region, we may be able to find a concession probability $p$, with which both bank managers choose $C$ in every history $h^t = [N, N]^{t-1}$ in the mixed-strategy equilibrium. In this appendix we will show that there is a threshold $D^{**}$ ($< D^*$) such that the concession probability $p$ exists if $D_1 < D^{**}$. See condition (29).

We define $d(p)$ as a joint payoff of BM1 and BM2 when both of them choose N in the mixed strategy equilibrium. In Table 1, we set $\Pi = V_{\max} = \frac{y_H - \varepsilon}{1-\beta}$, $\pi^{BM1} = \pi^{BM2} = \frac{1}{2}\pi$ and $\pi = d(p)$. As we saw $\Pi - nq = (1 - n\xi)\Pi + n\xi\pi$, $d(p)$ is given by

$$\begin{aligned} d(p) &= y_L + \beta[(1-p)^2 d(p) + \{1 - (1-p)^2\}\{(1-n\xi)V_{\max} + n\xi d(p)\}] \\ &= \frac{y_L + \beta\{1-(1-p)^2\}(1-n\xi)V_{\max}}{1 - \beta\{n\xi + (1-n\xi)(1-p)^2\}}. \end{aligned} \tag{24}$$

The value of $p$ is determined to make the payoffs of choosing $C$ and $N$ equal when the opponent chooses $C$ with probability $p$. The payoff of choosing $C$ is

$$\frac{p}{2}(\Pi - nq) + (1-p)\alpha(\Pi - nq).$$

The payoff of choosing $N$ is

$$p(1-\alpha)(\Pi - nq) + (1-p)\frac{1}{2}d(p).$$

Equalizing the above two, we have the equation to decide $p$, that is,

$$p = \frac{\alpha(\Pi - nq) - \frac{1}{2}d(p)}{\frac{1}{2}(\Pi - nq) - \frac{1}{2}d(p)},$$

which is rewritten as

$$p = H(p), \tag{25}$$

where

$$H(p) = 2\alpha - \frac{(1-2\alpha)d(p)}{(1-n\xi)(V_{\max} - d(p))}. \tag{26}$$

We have the following results:

- $H(p)$ is a decreasing function of $p$. So, there exists a unique solution to (25).
- As $D_1$ increases, $H(p)$ decreases. So, the equilibrium value of $p$, which is the solution to (25), decreases as $D_1$ increases.
- As $V_{\max}$ increases, $H(p)$ increases. So, the equilibrium $p$, which is the solution to (25), increases as $V_{\max}$ increases.
- The bank subsidy, conditional on implementing debt restructuring, that increases $V_{\max}$ is effective in increasing the equilibrium value of $p$, which is the probability of debt restructuring.
- The firm subsidy reduces $p$ if parameter $\phi$ is sufficiently small, implying that the firm subsidy hinders debt restructuring.

These results are given in the following claims.

**Claim 12.** $H(p)$ is a decreasing function of $p$.

*Proof.* By definition, it is obvious that $H(p)$ decreases as $d(p)$ increases. By differentiating (24), we have

$$\{1 - \beta n\xi - \beta(1-p)^2(1-n\xi)\}d'(p) = 2\beta(1-p)(1-n\xi)(V_{\max} - d(p)). \tag{27}$$

As $p < 1$ and $V_{\max} > d(p)$, the above equation implies that $d'(p) > 0$. Therefore, we have shown that $H(p)$ decreases as $p$ increases.

□

**Claim 13.** We assume that the following condition holds true:

$$2\alpha\{(1-n\xi)(y_H - \varepsilon) + n\xi y_L\} - y_L > 0. \tag{28}$$

The equilibrium value of $p$, which solves $p = H(p)$ exists uniquely.

*Proof.* We have

$$H(0) = 2\alpha - \frac{1-2\alpha}{1-n\xi}\frac{y_L}{y_H-\varepsilon-y_L} = \frac{2\alpha\{(1-n\xi)(y_H-\varepsilon)+n\xi y_L\}-y_L}{(1-n\xi)(y_H-\varepsilon-y_L)}.$$

It is straightforward that $H(0) < 2\alpha < 1$. Given that (28) holds, we have $H(0) > 0$. Next,

$$H(1) = 2\alpha - \frac{(1-2\alpha)}{1-n\xi}\frac{(1-\beta n\xi)V_{\max}-X}{X} = 2\alpha - \frac{(1-2\alpha)}{X}[\frac{y_L}{1-n\xi}+\beta V_{\max}] < 2\alpha < 1,$$

where $X \equiv (1-\beta)V_{\max} - y_L$. Thus, we have that $0 < H(0) < 1$, $H(1) < 1$ and $H(p)$ is decreasing in $p$. It follows that $p = H(p)$ has a unique solution that satisfies $0 < p < 1$. □

Since $n = \phi D_1$, condition (28) can be rewritten as

$$D_1 < D^{**}, \tag{29}$$

where

$$D^{**} = \frac{1}{\phi\xi}\left(\frac{\alpha(y_H-\varepsilon)-\frac{1}{2}y_L}{\alpha(y_H-\varepsilon-y_L)}\right).$$

It is easily confirmed that $D^{**} < D^*$, as $\alpha < \frac{1}{2}$ and

$$D^* = \frac{1}{\phi\xi}\left(\frac{\alpha(y_H-\varepsilon)-\frac{1}{2}y_L}{\alpha(y_H-\varepsilon)-\{\beta\frac{1}{2}+(1-\beta)\alpha\}y_L}\right).$$

We have shown that if $D_1 < D^{**}$ then there exists a unique mixed-strategy equilibrium.

**Claim 14.** $H(p)$ decreases as debt $D_1$ increases.

*Proof.* We can rewrite

$$d(p) = V_{\max} - \frac{X}{1-\beta+Z(1-n\xi)},$$

where $X = (1-\beta)V_{\max} - y_L$ and $Z = \beta\{1-(1-p)^2\}$. Then we have

$$J \equiv \frac{d(p)}{(1-n\xi)(V_{\max}-d(p))} = ZV_{\max}X^{-1} + \frac{y_L}{(1-n\xi)X}.$$

$J$ increases as $n$ increases. Since $H(p) = 2\alpha - (1-2\alpha)J$, it is obvious that $H(p)$ decreases as $n = \phi D_1$ increases. □

**Claim 15.** $H(p)$ increases as $V_{\max}$ increases.

*Proof.* We can write $d(p) = a + bV_{\max}$, where

$$a = \frac{y_L}{1-\beta\{n\xi+(1-n\xi)(1-p)^2\}} \quad \text{and} \quad b = \frac{\beta\{1-(1-p)^2\}(1-n\xi)}{1-\beta\{n\xi+(1-n\xi)(1-p)^2\}}.$$

Then,

$$H(p) = 2\alpha - \frac{1-2\alpha}{1-n\xi}g(V_{\max}),$$

where

$$g(V) = \frac{a+bV}{-a+(1-b)V} = -1 + \frac{1}{-\frac{a}{V}+1-b},$$

which implies that $g(V)$ decreases as $V$ increases. Therefore, we have proved that if $V_{\max}$ increases, $g(V_{\max})$ decreases, leading to an increase in $H(p)$. □

**Bank subsidy:** Claims 12–15 straightforwardly imply the following lemma.

**Lemma 12.** *The bank subsidy that increases $V_{\max}$ increases $p$.*

We have shown that bank subsidy is effective in facilitating debt restructuring by increasing the concession probability $p$.

**Firm subsidy:** Now suppose that firm subsidy $S^F$ is given to the firm unconditionally and the firm pays $S^F$ to the bank as debt repayment. In this case, the bank manager's payoff of choosing C is

$$\frac{p}{2}(\Pi - nq + S^F) + (1-p)\alpha(\Pi - nq + S^F),$$

and the payoff of choosing N is

$$p(1-\alpha)(\Pi - nq + S^F) + (1-p)\frac{1}{2}(d(p) + S^F).$$

The condition that these two payoffs are equal in equilibrium gives the value of $p$. The condition is rewritten as:

$$p = H(p, S^F), \tag{30}$$

where

$$H(p, S^F) = 2\alpha - \frac{(1-2\alpha)(d(p)+S^F)}{(1-n\xi+\phi\xi\beta S^F)(V_{\max}-d(p))} = 2\alpha - \frac{(1-2\alpha)d(p)}{(1-n\xi)(V_{\max}-d(p))}\frac{1+\frac{1}{d(p)}S^F}{1+\frac{\phi\xi\beta}{1-n\xi}S^F}.$$

This equation implies that $\frac{\partial}{\partial S^F}H(p,S^F) < 0$ if

$$1 > \frac{\phi\xi\beta}{1-n\xi}d(p),$$

which is satisfied if

$$1 > \frac{\phi\xi\beta}{1-n\xi}\frac{y_L + \beta(1-n\xi)V_{\max}}{1-\beta n\xi}.$$

This condition is satisfied for a sufficiently small $\phi$. In this case, the solution to $p = H(p, S^F)$ is smaller than the solution to $p = H(p)$, implying that the firm subsidy hinders debt restructuring. We have shown the following lemma.

**Lemma 13.** *If $\phi$ is sufficiently small, the firm subsidy $S^F$ hinders debt restructuring.*

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