

# Innovation and Pricing Frictions

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- **Standard view:** Nominal rigidities are short-lived and primarily relevant for business-cycle dynamics.
- Long-run outcomes—innovation, creative destruction, growth—are typically viewed as independent of pricing frictions.
- We revisit this question using rich product–firm longitudinal data for U.S. consumer goods.
- We develop an endogenous growth model with long-run pricing frictions to rationalize the empirical patterns and quantify their impact on innovation and growth.

## Key Question:

How do long-run pricing frictions shape innovation incentives and growth?

- **Entry Price Premia**
  - New products enter 10–20% above incumbents
  - Not explained by observable quality
- **Long-run Pricing Frictions**
  - Nominal prices show little growth over life cycle
  - Relative prices decline 2–3% per year
- **Adjustment via New Products**
  - Larger premia in rigid sectors
  - Cost shocks pass through via new products

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## Mechanism

- New products as a margin of price adjustment.
- Inflation erodes markups over time.

**Implication:** Pricing frictions may affect innovation decisions

- **Core friction:**

- Infrequent price adjustment with trend inflation
- $\Rightarrow$  Markups erode over the product life cycle

- **Environment:**

- Endogenous growth with multi-product firms
- Innovation through quality improvements (replacement and expansion)

- **Key feature:**

- New products reset markups
- $\Rightarrow$  Innovation provides a margin of price adjustment

- **Implications:**

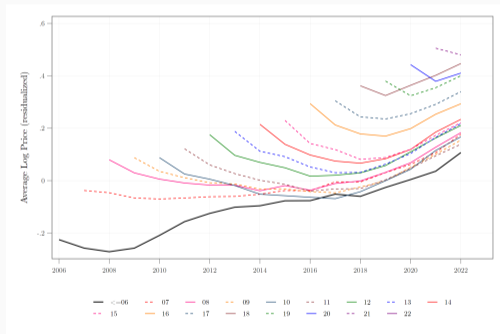
- Price overshooting at entry
- Pricing frictions shape product values and innovation incentives
- Monetary conditions affect growth through innovation

## Empirical Analysis

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## Fact 1: New Products Enter with Price Premia

- **Dataset:** Scanner-level data covering non-durable & semi-durable consumer goods sectors (14% U.S. consumption of goods). [▶ Details](#)
- Compute average log prices (residualized with sector FE) by product-cohort and follow over time.

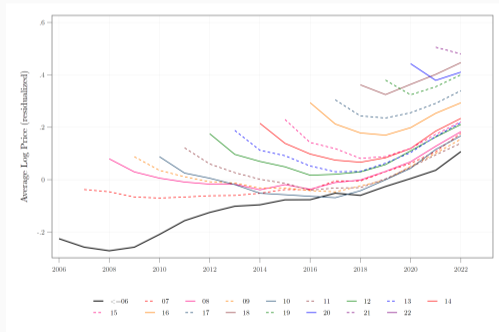


(a) Without Time Controls

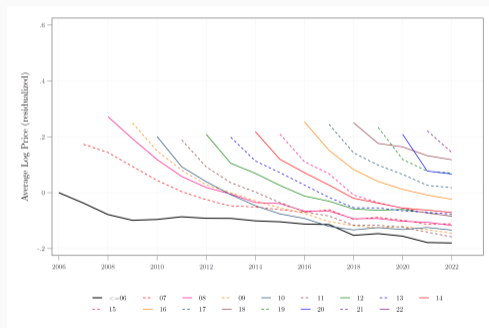
- Each successive cohort enters at  $\approx 10\text{--}20\%$  higher prices than existing products in the same sector.

## Fact 1: New Products Enter with Price Premia

- **Dataset:** Scanner-level data covering non-durable & semi-durable consumer goods sectors (14% U.S. consumption of goods). [▶ Details](#)
- Compute average log prices (residualized with sector sector x year FE) by product-cohort and follow over time.



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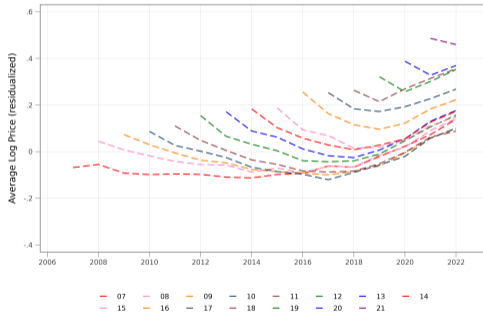


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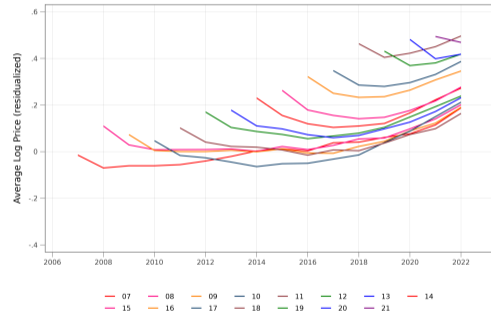
## Fact 1: Price Premia and Quality Upgrading

- Some newly introduced products replace existing products without materially expanding product attributes, while others expand the set of characteristics available in the market.

### Expansion Products



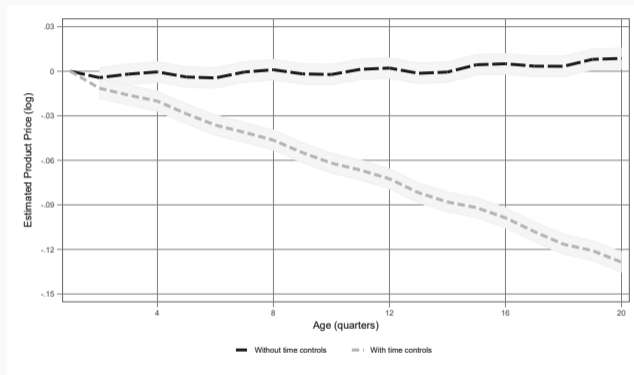
### Replacement Products



- Replacement products also show substantial price premia.
- Higher entry prices do not solely reflect observed quality upgrading.

## Fact 2: Incumbent Prices over the Product Life Cycle

- Estimate age profiles of log prices:  $\log P_{u(j,c)t} = \alpha + \sum_a \beta_a D_a D_c + \eta(1 - D_c) + \lambda_j (\text{or } \lambda_{jt}) + \theta_c + \varepsilon_{ut}$



- Nominal prices exhibit little systematic growth over the product life cycle, despite positive inflation.
- Relative to sector  $\times$  time, it declines  $\sim 2\text{--}3\%$  per year over first 5 years.

Does price rigidity affect the initial price premium of new products?

1. Use cross-sector variation in pricing rigidity.

- Sectors with **more frequent** price changes have **smaller** overshooting.
- Sectors with **flatter/declining** life-cycle prices have **larger** entry premia.

▶ Details

2. Explore firm's exposure to import tariffs as a proxy for cost shocks.

- Cost shocks primarily pass through via **new** products, not incumbents.
- **Stronger in sectors with higher price rigidity.**

▶ Details

- **Fact 1: Entry Price Premia.**

New products enter with sizable price premia relative to existing products.

- **Fact 2: Long-run Price Rigidity**

Nominal prices exhibit little systematic growth over the product life cycle, despite positive inflation.

- **Fact 3: Entry Premia Rise with Incumbent Price Rigidity.**

The relative price of new products is larger in sectors with more rigid pricing.

**Implication: Firms use new product introductions as a margin of long-run price adjustment.**

## Model

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- **Key ingredients:**

- Multi-product firms and creative destruction (Klette-Kortum; Akcigit-Kerr).
- **Standard:** firms maintain optimal markups.
- **Here:** firms cannot freely adjust prices → **markups erode over time.**

- **Key Mechanism**

- **Innovation = price reset**
- New products reset markups and avoid erosion.

- **Implications**

- Product values depend on expected markup dynamics.
- New product value reflects:
  - future erosion as it becomes incumbent
  - reset of markups at introduction
- Pricing frictions generate dispersion in markups across products.
- Pricing frictions distort innovation incentives and aggregate growth.

## Environment: Households and Money

Representative household:

$$\max \int_0^{\infty} e^{-\rho t} \left[ \ln C_t - L_t + \chi \ln \left( \frac{M_t}{\hat{P}_t} \right) \right] dt$$

$$\text{s.t. } \dot{A}_t + \dot{M}_t = W_t L_t + R_t A_t - \hat{P}_t C_t$$

First-order conditions:

$$\hat{P}_t C_t = W_t \quad (\text{Labor supply})$$

$$\hat{P}_t C_t = \chi^{-1} R_t M_t \quad (\text{Money demand})$$

$$\frac{\dot{C}_t}{C_t} = R_t - \frac{\dot{\hat{P}}_t}{\hat{P}_t} - \rho \quad (\text{Euler})$$

Monetary authority:

$$\frac{\dot{M}_t}{M_t} = \bar{\pi}$$

Implication (BGP):

$$R = \rho + \bar{\pi}$$

Money growth pins down inflation  $\Rightarrow$  drives markup erosion.

### Demand and production

$$Y_t = \left( \int (q_{it} y_{it})^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad y_{it} = \ell_{it}$$

### Product-level markup

$$\mu_{it} \equiv \frac{p_{it}}{W_t}$$

### With sticky nominal prices, inflation erodes markups

$$\frac{\dot{\mu}_{it}}{\mu_{it}} = -\bar{\pi}$$

### Implications

- Markups decline over the product life cycle.
- Products of different ages have different markups.
- Markup dispersion generates misallocation.

**Inflation creates a dynamic incentive to reset markups.**

- Firms set a price when a product is introduced, and afterwards prices are sticky.
- Three potential **channels to restore markups**:

**Calvo reset**  
Poisson rate  $\gamma$

same product  
reset to  $\mu_u$

**Menu-cost reset**  
when  $\mu = \mu_b$

same product  
pay menu-cost, reset markup  $\mu_u$

**Innovation**  
quality improvement

new product  
pay innov. cost, reset markup  $\mu_u$

**Innovation is an additional margin of price adjustment.**

Incumbents innovate along two margins:

### (1) Replacement innovation

- Improves own product lines
- Rate:  $x^I$
- Upgrade:  $q' = \lambda_I q$

Relaxes pricing frictions at the product line level

### (2) Expansion innovation

- Improves a competitor's product line (randomly)
- Rate:  $x^E$
- Upgrade:  $q' = \lambda_E q$

Relaxes pricing frictions at the portfolio level

**Innovation is both a growth margin and a price-adjustment margin.**

**State variables:** markup  $\mu$ , relative quality  $\hat{q}$

**Dynamic value of a product line:**  $V(\mu, \hat{q})$

## Firm chooses

- Replacement innovation  $x^I$
- Expansion innovation  $x^E$
- Whether to adjust price (menu cost)
- Adjusted new price ( $\mu^U$ )

## Sources of value

- Current profits
- Future quality improvements
- Markup resets

## Sources of value loss

- Markup erosion from inflation
- Creative destruction
- Innovation costs
- Menu costs

**Pricing frictions make product value depend on the markup state.**

# Product Value and Markup Dynamics

## Value function

$$V(\mu, \hat{q}) = \hat{q}^{\sigma-1} A(\mu) Y_t$$

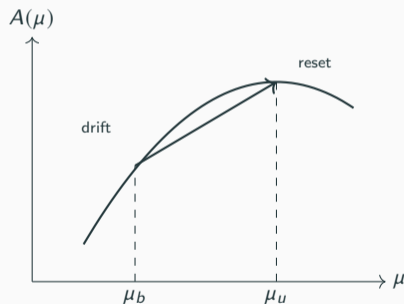
**Key object:**  $A(\mu)$  captures how future markup dynamics shape product value.

## Markup dynamics

- Inflation erodes markups
- Higher markups raise profits
- Resets restore markups

## Optimal policy

- quality-invariant
- **Inaction region:** no adjustment for  $\mu > \mu_b$ , unless Calvo reset or innovation reset.
- **Adjustment:** reset to  $\mu_u$  when  $\mu \leq \mu_b$



# Optimal Reset Markup and Price Overshooting

*Firms choose reset markup anticipating future erosion.*

## Proposition

*The optimal reset markup  $\mu_u$  satisfies*

$$(\varphi - g) A(\mu_u) = m^{\sigma-1} (\mu_u - 1) \mu_u^{-\sigma} + \Lambda_t^l(\mu_u) / Y_t$$

## Implications:

- **Price overshooting:**

$$\mu_u > \mu^* = \frac{\sigma}{\sigma - 1}$$

whenever inflation is positive.

- **Intuition:** after a reset, inflation gradually erodes markups. Firms therefore choose a markup above the static optimum so that the average markup between resets remains closer to  $\mu^*$ .
- **Frictionless benchmark:** if  $\bar{\pi} \rightarrow 0$  or  $\gamma \rightarrow \infty$ ,

$$\mu_u \rightarrow \mu^*.$$

Overshooting is optimal because firms care about the average markup between resets.

### Proposition

In equilibrium, the balanced growth rate is:

$$g = x^{I*} \frac{(1 + \lambda^I)^{\sigma-1} - 1}{\sigma - 1} + x^{E*} \frac{(1 + \lambda^E)^{\sigma-1} - 1}{\sigma - 1}$$

### Interpretation:

- Growth comes from **quality improvements**:

- replacement innovation  $x^{I*} = \left[ \frac{1-\alpha}{\xi^I} \left( A(\mu_u) (\lambda^I)^{\sigma-1} - A(\mu) \right) \right]^{\frac{1-\alpha}{\alpha}}$
- expansion innovation  $x^{E*} = \left[ \frac{1-\alpha}{\xi^E} A(\mu_u) (\lambda^E)^{\sigma-1} \right]^{\frac{1-\alpha}{\alpha}}$

### Role of pricing frictions:

- Inflation and adjustment frictions affect product value  $A(\mu)$
- This changes incentives to innovate:

$$A(\mu) \Rightarrow x^{I*}, x^{E*} \Rightarrow g$$

Monetary distortions affect growth **through innovation incentives**

- Inflation erodes markups over time
- Firms respond via:
  - price adjustment
  - **innovation (new margin)**
- Pricing frictions create:
  - markup dispersion
  - misallocation
  - distorted innovation incentives
- ⇒ Monetary policy affects growth through innovation

## Estimation and Calibration

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10 structural parameters  $\{\vartheta, \sigma, r, \alpha, \pi, \xi^I, \xi^E, \lambda^I, \lambda^E, \gamma\}$

- Externally set:  $r = 0.02$  (Interest rate),  $\alpha = 0.5$  (Akcigit-Kerr, 2018)
- Estimates of elasticity of substitution between products for the products in the consumer goods industry:  $\sigma = 6$  (Argente-Lee-Moreira 2024)
- $\pi \approx 2.65\%$  (average inflation rate over product categories in the consumer goods industry)
- Internally calibrated  $\{\xi^I, \xi^E, \lambda^I, \lambda^E, \psi, \gamma, \kappa\}$  to match:
  1. Incumbent replacement innovation rate,
  2. Incumbent expansion innovation rate,
  3. Aggregate growth rate,
  4. Share of expansion in incumbent growth,
  5. Expected life of a product
  6. Empirical price overshooting,
  7. Maximal magnitude of price adjustment.

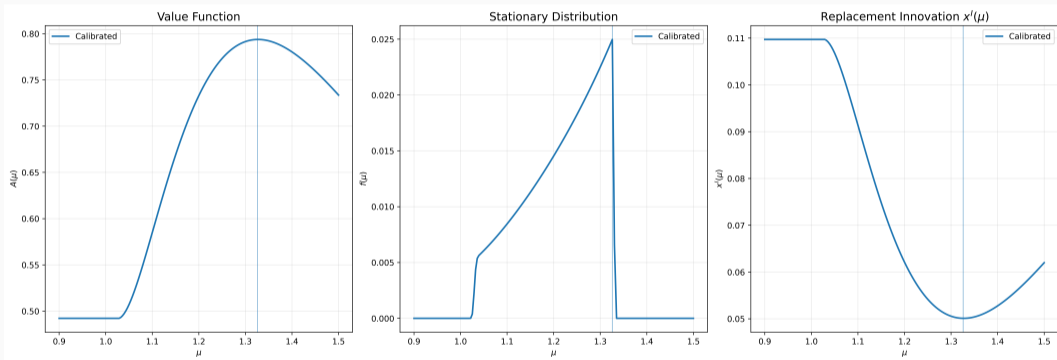
| Targeted Moments                 |        |        | Key Calibrated Parameters |                          |        |
|----------------------------------|--------|--------|---------------------------|--------------------------|--------|
| Moment                           | Model  | Data   | Parameter                 | Description              | Value  |
| Incumbent replacement innovation | 0.0657 | 0.0657 | $\xi_I$                   | Replacement R&D cost     | 2.728  |
| Incumbent expansion innovation   | 0.0488 | 0.0488 | $\xi_E$                   | Expansion R&D cost       | 21.079 |
| Aggregate growth rate            | 0.0203 | 0.0203 | $\lambda_I$               | Replacement quality step | 1.057  |
| Share of expansion growth        | 0.7930 | 0.7930 | $\lambda_E$               | Expansion quality step   | 1.215  |
| Expected life                    | 7.4978 | 7.5000 | $\gamma$                  | Calvo reset rate         | 0.0834 |
| Price overshooting               | 0.0948 | 0.0948 | $\psi$                    | Exogenous exit rate      | 0.0188 |
| P99 of price adj.                | 0.3316 | 0.3318 | $\kappa$                  | Menu cost                | 0.7409 |

- Consistent with life-cycle evidence: many products never fully reset their trend price.
  - $\gamma = 0.0834$  implies random price adjustment chance to trend price is small.
  - It takes 12.5 years to reach  $\mu_b$  while expected life is only 7.5 years.
- The majority of adjustments occur through the introduction of new products.
  - On avg, 55% would adjust prices using innovation; 40% Calvo reset; 5% pay menu cost.

| Baseline Equilibrium              |       |
|-----------------------------------|-------|
| Reset markup $\mu_0$              | 1.330 |
| Static markup $\mu^*$             | 1.20  |
| Exit markup $\mu_b$               | 0.954 |
| Aggregate markup $m$              | 1.189 |
| Efficiency $v$                    | 0.978 |
| Replacement innovation rate $x^I$ | 6.57% |
| Expansion innovation rate $x^E$   | 4.88% |
| Growth $g$                        | 2.03% |

- **Price overshooting:**  $\mu_u = 1.33$  exceeds static markup  $\mu^* = 1.20 \Rightarrow$  optimal reset markup overshoots frictionless level.
- **Price adjustment threshold:**  $\mu_b = 0.954$ . Products which survive until then and have not been improved show largest price adjustment.
- **Misallocation:** Markup dispersion implies TFP loss  $v = 0.978$  ( $\sim 2.2\%$  efficiency loss).

# Baseline Economy: Illustration



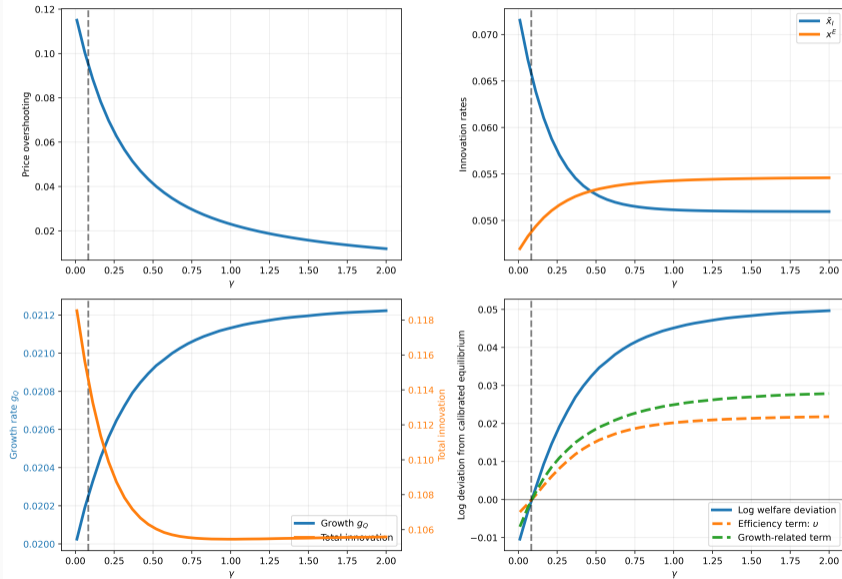
## Counterfactual 1: Frictionless Benchmark

|                                   | Baseline | Frictionless |
|-----------------------------------|----------|--------------|
| Reset markup $\mu^u$              | 1.330    | 1.201        |
| Static markup $\mu^*$             | 1.200    | 1.201        |
| Exit markup $\mu^b$               | 0.954    | –            |
| Aggregate markup $m$              | 1.189    | 1.200        |
| Efficiency $\nu$                  | 0.978%   | 1.00%        |
| Replacement innovation rate $x^I$ | 6.57%    | 5.09%        |
| Expansion innovation rate $x^E$   | 4.88%    | 5.47%        |
| Growth $g$                        | 2.03%    | 2.12%        |
| Welfare (CE)                      | 100.0    | 105.2        |

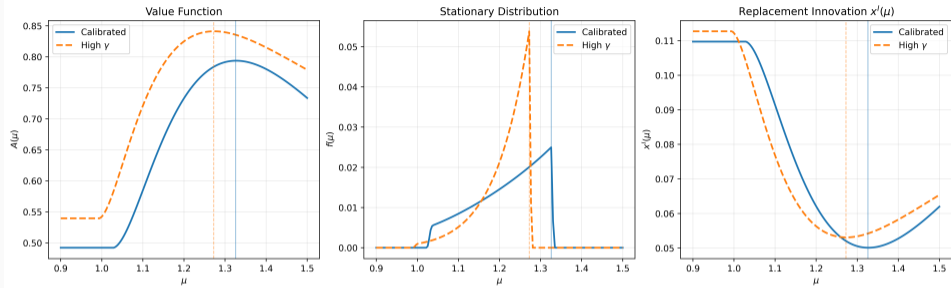
- **No pricing frictions:**  $\mu^u = \mu^*$ .
- **No markup dispersion:** Efficiency fully restored.
- Total **innovation** declines, with a shift in composition: replacement innovation decreases while expansion innovation increases.
- Growth increases. Welfare gain of 5.2 ppts, due to growth and misallocation.

Pricing frictions distort innovation decisions.

# Counterfactual 1: How Price Flexibility Shapes Innovation?

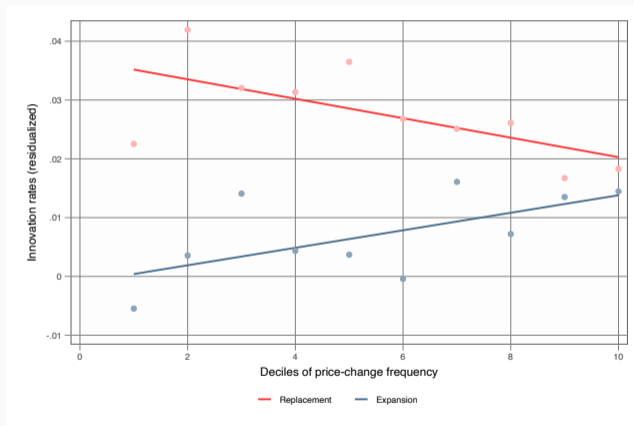


# Why does the nature of innovation matter?

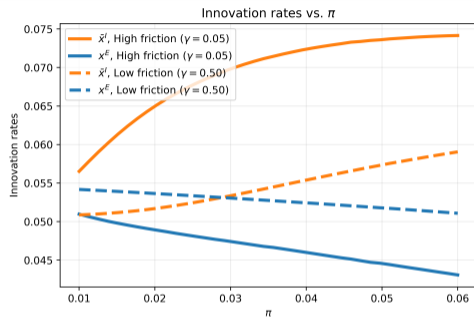
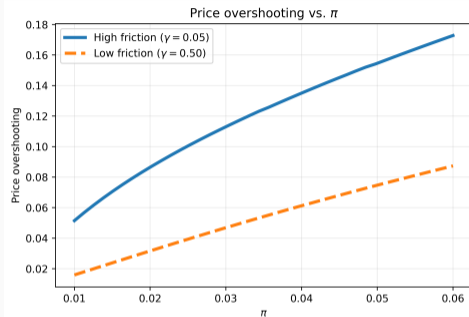


- Higher reset frequency ( $\gamma$ ) **lowers the average replacement innovation  $\bar{x}^I$ :**
  - **Value gap shrinks:**
    - Replacement innovation depends on  $A(\mu_u)(\lambda^I)^{\sigma-1} - A(\mu)$
    - More frequent resets  $\Rightarrow$  smaller gap  $\Rightarrow$  lower  $x^I(\mu)$
  - **Reallocation across firms:**
    - Distribution of  $\mu$  concentrates near  $\mu_u \Rightarrow$  more firms with low replacement incentives
- In contrast, higher reset frequency ( $\gamma$ ) **increases expansion innovation  $x^E$** 
  - Expansion innovation depends on  $A(\mu_u)$

We evaluate **empirically** this prediction using cross-sector variation in price rigidity:

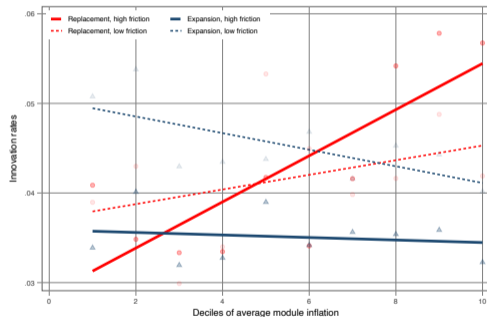
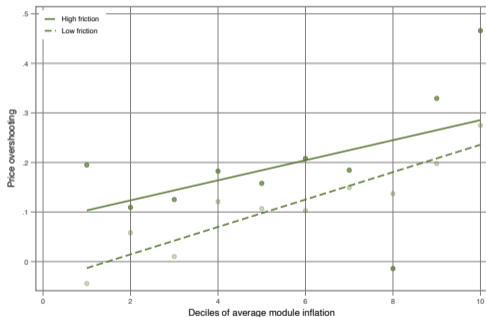


## Counterfactual 2: What happens in high inflation periods?



- Higher inflation erodes markups more quickly, so firms choose a higher reset markup to protect against future markup erosion.
  - stronger in high friction environments.
- Inflation also changes the composition of innovation, towards replacement innovation.
  - stronger in high friction environments.

We evaluate **empirically** this prediction using cross-sector variation in inflation and price rigidity:



## Empirical Findings

- New products enter with sizable price premia; incumbent prices are flat over the life cycle.
- Cost shocks pass through primarily via new products, especially in rigid sectors.

## Model and Mechanism

- Pricing frictions generate markup erosion over the product life cycle.
- Innovation introduces new products and resets markups.
- $\Rightarrow$  Innovation becomes a margin of price adjustment.

## Quantitative Implications

- Pricing frictions distort product values and innovation incentives.
- Affect the composition of innovation (replacement vs. expansion).
- Implications for aggregate growth and welfare.

Thank you!

Questions and comments welcome.

## **Appendix: Additional Details**

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## Data Overview

- NielsenIQ Retail Measurement Services (RMS) scanner data, 2006–2022.
  - Weekly, store-level sales and quantities.
  - ~1.7 million UPCs.
- Detailed product hierarchy:
  - 1,066 modules → 115 product groups → 10 departments.
- Link to manufacturing firms:
  - GS1 barcode prefixes ⇒ parent firms and product portfolios.

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- Detailed product hierarchy:
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- Link to manufacturing firms:
  - GS1 barcode prefixes ⇒ parent firms and product portfolios.
- For each product  $u$  in period  $t$  (quarter/year):

$$\text{Sales } Y_{ut} = \sum_{\text{stores, weeks}} \text{value}$$

$$\text{Quantity } y_{ut} = \sum_{\text{stores, weeks}} \text{units}$$

$$p_{ut} = \frac{Y_{ut}}{y_{ut}}$$

## A. Data: Robustness Samples

- Balanced store sample (constant set of stores, 2006–2022).
- Including private label products.
- Excluding incomplete spells (seasonal/promotional items).
- Life-cycle price patterns are robust across all these variants.

## A. Life-Cycle Regressions

- Baseline:

$$\log Y_{ut} = \alpha + \sum_a \beta_a D_a D_c + \eta(1 - D_c) + \lambda_j + \theta_c + \varepsilon_{ut}$$

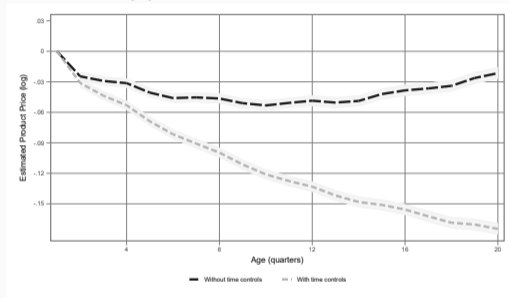
- With sector×time FE:

$$\log Y_{ut} = \alpha + \sum_a \beta_a D_a D_c + \eta(1 - D_c) + \lambda_{jt} + \theta_c + \varepsilon_{ut}$$

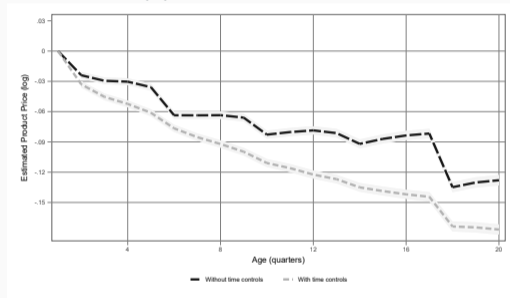
- Robust to alternative cohort normalizations and firm×sector×time FE.

# Estimated Life Cycle: Allowing for Selection

(a) Semi-balanced Sample

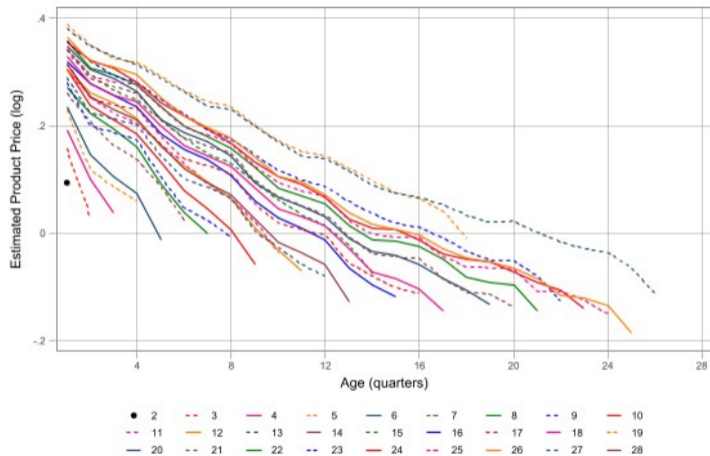


(b) Unbalanced Sample



Notes: The figure shows the estimated age fixed effects of (log) prices of products computed using equations (??) and (??). The estimation uses all products. In panel (a) the age fixed effects are computed for cohorts 2006q3-2016q3, including products that exit before they complete 20 quarters of sales. In panel (b) the age fixed effects includes products from cohorts 2006q3 to 2021q4. The gray area indicates the 95% confidence interval.

# Estimated Life Cycle by Survival



Notes: The figure shows the estimated age fixed effects of (log) prices of products (controlling for category x time FE), where age effects are interacted with duration, for products that lasted between 2 and 28 quarters in the market.

## B. Store-Level Life Cycle

- Construct product-store-year panel.
- Define age since product first appears in each store.
- Find:
  - store-level prices also show little upward drift,
  - relative prices fall with age, consistent with national aggregates.
- Confirms results are not driven by changing store composition.

### Fact 3: Entry Premia and Incumbent Rigidity - Cross Sectional Evidence

- Construct product category-level measures:
  - frequency of weekly price changes from product-store-week data.
  - slope of life-cycle price profile (age effects specific for each category).
- Regress price premia of new products on these measures.

|                             | Price Premia New Products $\tilde{P}_{m,t} = \left( P_{m,t}^{\text{new}} / P_{m,t}^{\text{inc}} \right) - 1$ |                      |                      |                      |
|-----------------------------|--|----------------------|----------------------|----------------------|
|                             | (1)  | (2)                  | (3)                  | (4)                  |
| Frequency Price Changes (m) | -0.591***<br>(0.036)   | -0.269***<br>(0.057) |                      |                      |
| Life Cycle Estimated (m)    |  |                      | -2.437***<br>(0.094) | -1.299***<br>(0.113) |
| Time FE                     | Y  | Y                    | Y                    | Y                    |
| Sector FE                   | N  | Y                    | N                    | Y                    |

- Sectors with **more frequent** price changes have **smaller** overshooting.
- Sectors with **flatter/declining** life-cycle prices have **larger** entry premia.

## Fact 3: Entry Premia and Incumbent Rigidity - Tariff Exposure Evidence

- Use firm-module-year **tariff exposure** as plausibly exogenous cost shocks.
  - Tariff changes are largely unexpected or not fully anticipated by firms (Boehm-Levchenko-Pandalai-Nayar 2023, Cavallo-Llamas-Vazquez 2025).
  - Tariff data: UNCTAD TRAINS / WITS, effectively applied tariffs at exporter  $\times$  HS8; Aggregate to HS6 using import-weighted averages; Map HS codes to Nielsen modules (Bai-Stumpner concordance).
  - Country-of-origin: LabellInsights package data with COO at the barcode level.
  - Firm-module-year tariff exposure:

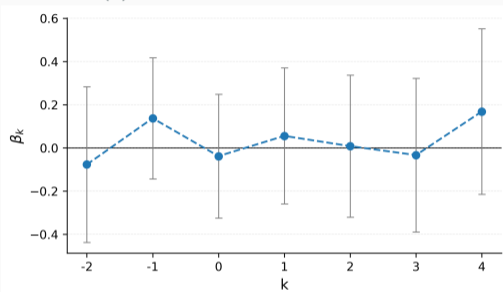
$$\bar{T}_{ijt} = \sum_{u \in j} w_{uijt} T_{uijt}, \quad w_{uijt} = \frac{q_{uijt}}{\sum_{u \in j} q_{uijt}}, \quad \bar{T}_{ijt} \in [0, 1]$$

- Local projection diff-in-diff (Jordà 2005).

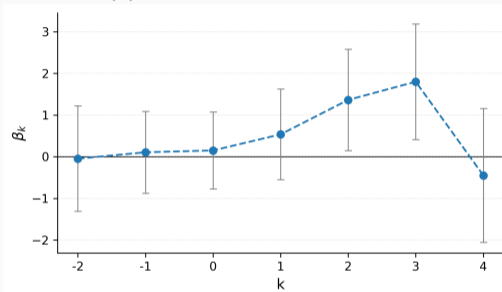
$$y_{ij,t+k} = \beta \bar{T}_{ij,t} + \omega_{j,t+k} + \phi_{ij} + \varepsilon_{ij,t+k}, \quad k = -2, \dots, 4$$

### Fact 3: Entry Premia and Incumbent Rigidity - Tariff Exposure Evidence

(a) Price of Incumbent Products

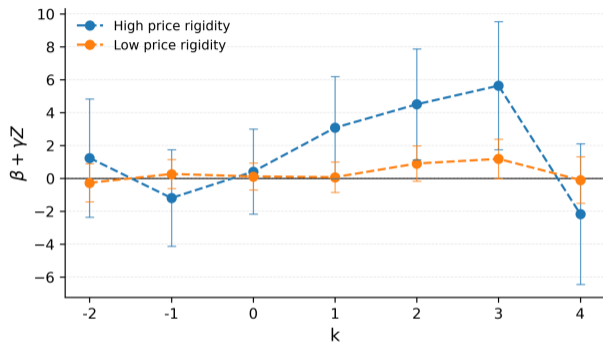


(b) Price Premia New Products



- No detectable effect on incumbent prices.
- Significant effect on new product prices.

### Fact 3: Entry Premia and Incumbent Rigidity - Tariff Exposure Evidence



- Cost shocks primarily pass through via **new** products, not incumbents.
- **Stronger in sectors with higher price rigidity.**

## C. Tariff Specification Details

- Regression:

$$y_{ij,t+k} = \beta T_{ij,t} + \gamma T_{ij,t} \times Z_j + \omega_{j,t+k} + \phi_{ij} + \text{controls}_{ij,t+k} + \varepsilon_{ij,t+k}$$

- $Z_j$ : standardized frequency of price changes (sector-level rigidity measure).
- Outcomes:
  - log price of incumbents,
  - relative price of new products.
- Stronger tariff pass-through to new products in high-rigidity sectors.

# Firm Problem with Pricing Frictions

**State variables:** markup  $\mu$ , relative quality  $\hat{q}$

**Firm chooses:** innovation  $(x^I, x^E)$ , whether to adjust price (menu cost), adjusted price  $(\mu^u)$

$$\begin{aligned} \max \left\{ \right. & \partial_t V_t(\mu, \hat{q}) - \bar{\pi} \mu V_\mu - g_Q \hat{q} V_{\hat{q}} + \pi(\mu, \hat{q}) - F_t && \text{Markup + quality drift} \\ & && \text{operating profits + fixed cost} \\ & + \max_{x^I} \left[ x^I (V(\mu_u, \lambda_I \hat{q}) - V(\mu, \hat{q})) - \xi^I (x^I)^{\frac{1}{1-\alpha}} \hat{q}^{\sigma-1} Y_t \right] && \text{Replacement innovation} \\ & + \max_{x^E} \mathbb{E} \left[ x^E V(\mu_u, \lambda_E \hat{q}) - \xi^E (x^E)^{\frac{1}{1-\alpha}} \hat{q}^{\sigma-1} Y_t \right] && \text{Expansion innovation} \\ & - \tau V(\mu, \hat{q}) && \text{Creative destruction} \\ & + \gamma \left[ V(\mu_u, \hat{q}) - V(\mu, \hat{q}) \right] && \text{Calvo reset} \\ & , V(\mu_u, \hat{q}) - V(\mu, \hat{q}) - \kappa \hat{q}^{\sigma-1} Y_t \left. \right\} = 0 && \text{Menu-cost adjustment} \end{aligned}$$

# Stationary Equilibrium Summary

- Value function:

$$0 = \max \left\{ -(\phi + \gamma - g^Y)A(\mu) - \bar{\pi}\mu A'(\mu) + \bar{\pi}(\mu) + \tilde{\lambda}^I(\mu) + \gamma A(\mu^u), A(\mu^u) - \kappa - A(\mu) \right\}. \quad (1)$$

- Kolmogorov forward equation:

$$0 = \underbrace{-\partial_{\mu}(\bar{\pi}\mu f(\mu))}_{\text{drift}} \underbrace{-\iota(\mu)f(\mu)}_{\text{random reset}} + \delta(\mu - \mu^u) \left[ \int_{\mu^b}^{\mu^u} \iota(\tilde{\mu})f(\tilde{\mu}) d\tilde{\mu} + \underbrace{\bar{\pi}\mu^b f(\mu^b)}_{\text{menu cost reset}} \right], \quad \mu \in [\mu^b, \mu^u].$$

where  $\iota(\mu) = \psi + \gamma + x^E + x^I(\mu)$ .

- Innovations:

$$x^I(\mu) = \left[ \frac{1-\alpha}{\xi^I} \left( A(\mu^u)(\lambda^I)^{\sigma-1} - A(\mu) \right) \right]^{\frac{1-\alpha}{\alpha}}; \quad x^E = \left[ \frac{1-\alpha}{\xi^E} A(\mu^u)(\lambda^I)^{\sigma-1} \right]^{\frac{1-\alpha}{\alpha}}. \quad (2)$$

## Appendix: Optimal Innovation Choices

Replacement innovation ( $x^I$ ):

$$\max_{x^I} \left[ x^I \Delta^I(\mu) - \xi^I (x^I)^{\frac{1}{1-\alpha}} \right]$$

where

$$\Delta^I(\mu) = \lambda_I^{\sigma-1} A(\mu_U) - \mathbf{A}(\mu) \mathbf{A}(\mu) \mathbf{A}(\mu)^{A(\mu)}$$

FOC:

$$\Delta^I(\mu) = \xi^I \frac{1}{1-\alpha} (x^I)^{\frac{\alpha}{1-\alpha}}$$

Solution:

$$x^I(\mu) = \left[ (1-\alpha) \frac{\Delta^I(\mu)}{\xi^I} \right]^{\frac{1-\alpha}{\alpha}}$$

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## Appendix: Expansion Innovation

Expansion innovation ( $x^E$ ):

$$\max_{x^E} \left[ x^E \Delta^E - \xi^E (x^E)^{\frac{1}{1-\alpha}} \right]$$

where

$$\Delta^E = \lambda_E^{\sigma-1} A(\mu_u)$$

FOC:

$$\Delta^E = \xi^E \frac{1}{1-\alpha} (x^E)^{\frac{\alpha}{1-\alpha}}$$

Solution:

$$x^E = \left[ (1-\alpha) \frac{\Delta^E}{\xi^E} \right]^{\frac{1-\alpha}{\alpha}}$$

*Key difference:*  $x^I(\mu)$  is state-dependent, while  $x^E$  is constant.

# Markup Dispersion and Inflation

- Inflation increases the speed of markup erosion
- Effects on distribution:
  - more firms at low markups
  - higher dispersion of markups
- Consequences:
  - lower aggregate markup  $m_t$
  - more misallocation ( $v_t \downarrow$ )
- **Key mechanism:**
  - inflation  $\Rightarrow$  markup dispersion  $\Rightarrow$  efficiency losses

# Aggregation and Markup Distribution

**Pricing policy implies a distribution of markups:**

- markups drift down over time (inflation)
- firms reset to  $\mu_u$  (Calvo, menu cost, innovation)

**Stationary distribution of  $(\mu, \hat{q})$ :**

- determined by:
  - inflation (speed of drift)
  - adjustment rates (Calvo, menu cost, innovation)

**Aggregate markup:**

$$m_t = \left( \int \mu_{it}^{1-\sigma} \hat{q}_{it}^{\sigma-1} di \right)^{\frac{1}{1-\sigma}}$$

**Misallocation:**

$$v_t = \frac{m_t^{-\sigma}}{E_{\hat{q}}[\mu_{it}^{-\sigma}]} \leq 1$$

**Aggregate output:**

$$Y_t = Z_t Q_t v_t L_t$$

# Counterfactual: Varying degrees of menu cost frictions

