Endogenous Ambiguity in Nonlinear Macro-Finance Models

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Introduction

Ambiguity refers to the *prior* uncertainty over parameterized economic models (*likelihoods*)

- decision makers do not know which parametric model is the correct specification

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How do decision makers in existing theories construct such an "*admissible*" set of models?

- ***exogenous*** admissible sets
  Gilboa and Schmeidler (1989); Chen and Epstein (2002); Hansen and Sargent (2022)
- learning from ***exogenous*** signals
  Epstein and Schneider (2008)

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This paper: construct the admissible set using ***endogenous*** signals

- signals are determined in general equilibriums (***endogenous ambiguity***)

Incorporate the endogenous ambiguity into a canonical nonlinear macro-finance model

Standard RBC model with a financial intermediary sector of He and Krishnamurthy (2019)

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- *real-time* aggregate intermediary capital is NOT observable

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Investors fear ***the low risk premium*** in the unconstrained scenario (RBC models)

- before crises, their adverse scenario is the ***high*** aggregate intermediary capital

Results: Nonlinear Belief Dynamics

During crises, *the financial accelerator* generates distinct implications for asset prices

- non-binding scenarios are NOT admissible
- adverse scenarios switch to the ***low*** aggregate intermediary capital

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Use **IBES financial analyst forecasts** to empirically support the endogenous belief dynamics

- endogenize and validate the ***neglected risk*** before crises
  Gennaioli, Shleifer, Vishny (2012)

Model Environment

Technology and Markets

The canonical continuous-time RBC model augmented with a financial intermediary sector

- builds on He and Krishnamurthy (2019) with slight modifications

Production capital K_t with price Q_t, producing nondurable goods $Y_t = A_t K_t$

Technology and Markets

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- TFP level A_t follows a geometric Brownian motion

$$\frac{dA_t}{A_t} = gdt + \sigma dZ_t$$

- capital investment follows the standard q-theory under investment adjustment costs

$$i_t = \delta + \frac{\overbrace{\frac{Q_t}{A_t K_t}}^{\text{macro-finance linkage}} - 1}{\kappa}.$$

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Housing capital with price P_t and a fixed unit supply $H \equiv 1$

Risk-free asset market with zero net supply and return r_t

Household Sector

Identical households indexed by i

- consume nondurable consumption goods ($C^y_{i,t}$) and housing services ($C^h_{i,t}$) with rent

$$D_t = \underbrace{\frac{\phi}{1-\phi}}_{\text{relative demand for housing}} \times \frac{C^y_{i,t}}{C^h_{i,t}}$$

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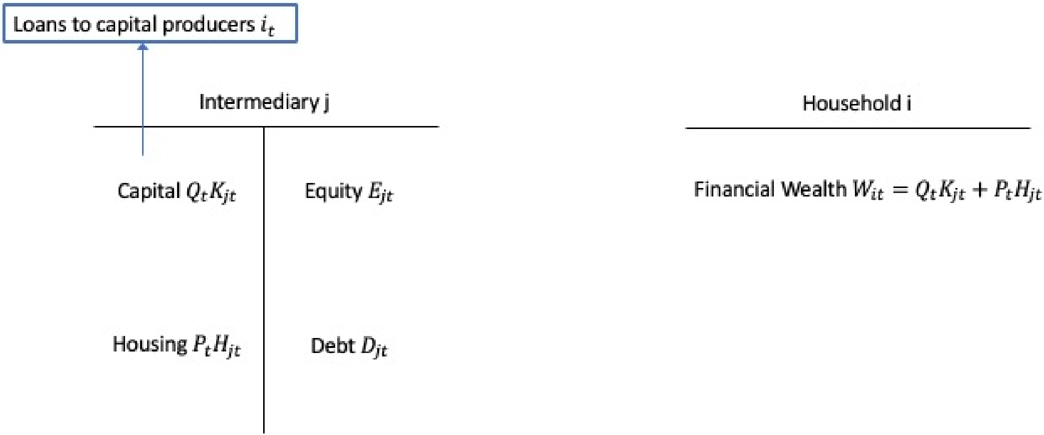
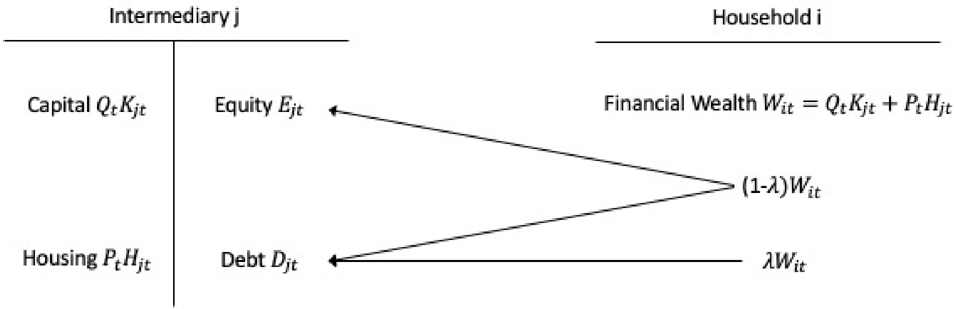
- save in equity and risk-free debt in the intermediary sector
  - subject to equity issuance constraints

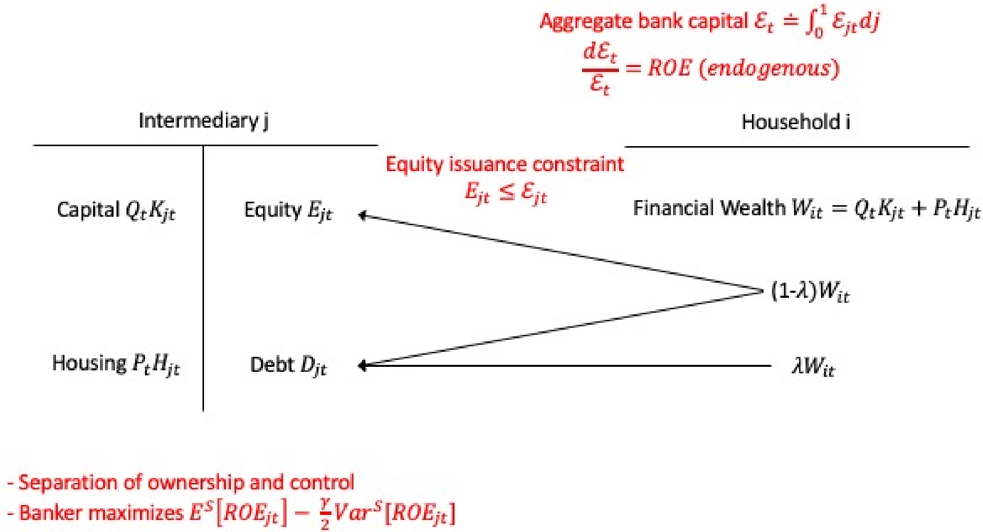
$$\underbrace{E_{j,t}}_{\text{Equity finance to intermediary } j} = \min \left\{ \underbrace{(1-\lambda)W_{i,t}}_{\text{max contribution}}, \underbrace{\mathcal{E}_{i,t}}_{\text{Intermediary capital}} \right\}$$

Market Structure





- Separation of ownership and control
- Banker maximizes $E^S[ROE_{jt}] - \frac{\gamma}{2} Var^S[ROE_{jt}]$



Aggregate bank capital $\mathcal{E}_t \doteq \int_0^1 \mathcal{E}_{jt} dj$
$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = ROE\ (endogenous)$
Intermediary j
Household i
Equity issuance constraint
$E_{jt} \leq \mathcal{E}_{jt}$
Capital $Q_t K_{jt}$
Equity E_{jt}
Financial Wealth $W_{it} = Q_t K_{jt} + P_t H_{jt}$
(1-λ)W_{it}
Housing $P_t H_{jt}$
Debt D_{jt}
λW_{it}
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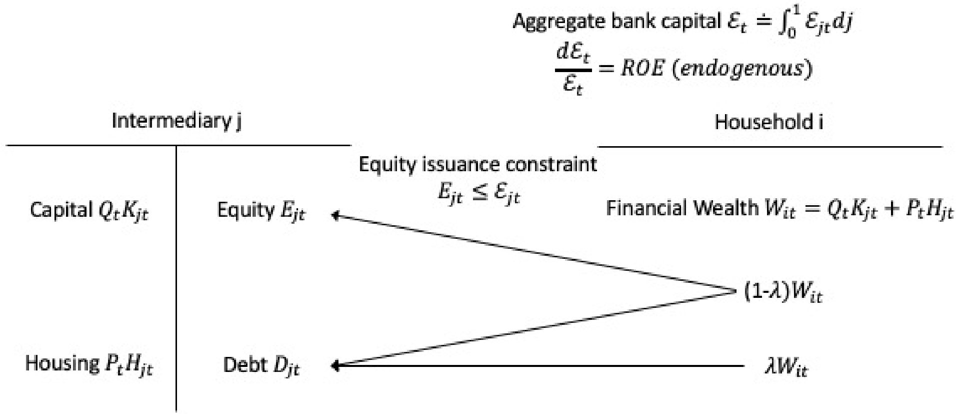

The (scaled) aggregate intermediary capital $e_t \doteq \frac{\mathcal{E}_t}{A_t K_t}$ becomes an endogenous state variable

- Separation of ownership and control
- Banker maximizes $E^S[ROE_{jt}] - \frac{\gamma}{2} Var^S[ROE_{jt}]$

Max Problem:

$$\max_{\alpha_k,\alpha_h} \underbrace{\alpha_k \pi_k + \alpha_h \pi_h}_{\text{expected excess portfolio return}} - \frac{\gamma}{2} \times \underbrace{(\alpha_k \sigma_k + \alpha_h \sigma_h)^2}_{\text{variance of excess portfolio return}},$$

Individual Intermediary Problem

Max-Min Problem:

$$\min_{\{\pi_a^S,\sigma_a^S\}_{a\in\{k,h\}}\in\Xi} \max_{\alpha_k,\alpha_h} \underbrace{\alpha_k\pi_k{}^S + \alpha_h\pi_h{}^S}_{\text{subjective expected excess portfolio return}} - \frac{\gamma}{2} \times \underbrace{(\alpha_k\sigma_k{}^S + \alpha_h\sigma_h{}^S)^2}_{\text{subjective variance of excess portfolio return}},$$

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Each intermediary constructs the admissible set of return distributions Ξ.

- understand **equilibrium mapping (likelihood)** parametrized by unobservable vector θ:

$$\{ \underbrace{\pi_a(\theta)}_{\text{expected excess return}}, \underbrace{\sigma_a(\theta)}_{\text{return volatility}} \}_{a\in\{h,k\}},$$

where $\theta \doteq (\underbrace{e}_{\text{scaled aggregate intermediary capital}}, \underbrace{g}_{\text{long-run TFP growth}}, \underbrace{\phi}_{\text{household housing demand}})$

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Construct the admissible set of $\theta \longrightarrow \Xi$

Endogenous Ambiguity: Admissible Set of Latent Objects

Current asset prices restrict the admissible set of θ's:

$$\underbrace{(P_t, Q_t, r_t)}_{\text{Currently observed price}} = \underbrace{(P(\theta_t), Q(\theta_t), r(\theta_t))}_{\text{Model-implied equilibrium price functions}}$$

- denote the set of admissible θ_t's by $\Theta(P_t, Q_t, r_t)$.

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The **admissible set of return distributions**:

$$\Xi(P_t, Q_t, r_t) \doteq \{(\pi_a(\theta_t), \sigma_a(\theta_t)), \forall a \in \{k, h\} : \theta_t \equiv (e_t, g_t, \phi_t) \in \Theta(P_t, Q_t, r_t)\}.$$

Related literature:

- abstract from learning
  Gilboa and Schmeidler (1989); Chen and Epstein (2002); Hansen and Sargent (2022)
- endogenous signal but with exogenously specified class of parametric models
  Molavi (2025)

Model Solution

Calibration

Terminology

- e_{crisis}: crisis threshold where the constraint binds
- e_{distress}: distress threshold as the 33rd percentile of e in the stationary distribution

Set the parameter values equal to He and Krishnamurthy (2019)

- match the conditional moments on being in normal and distress states of macroeconomic quantities and asset prices

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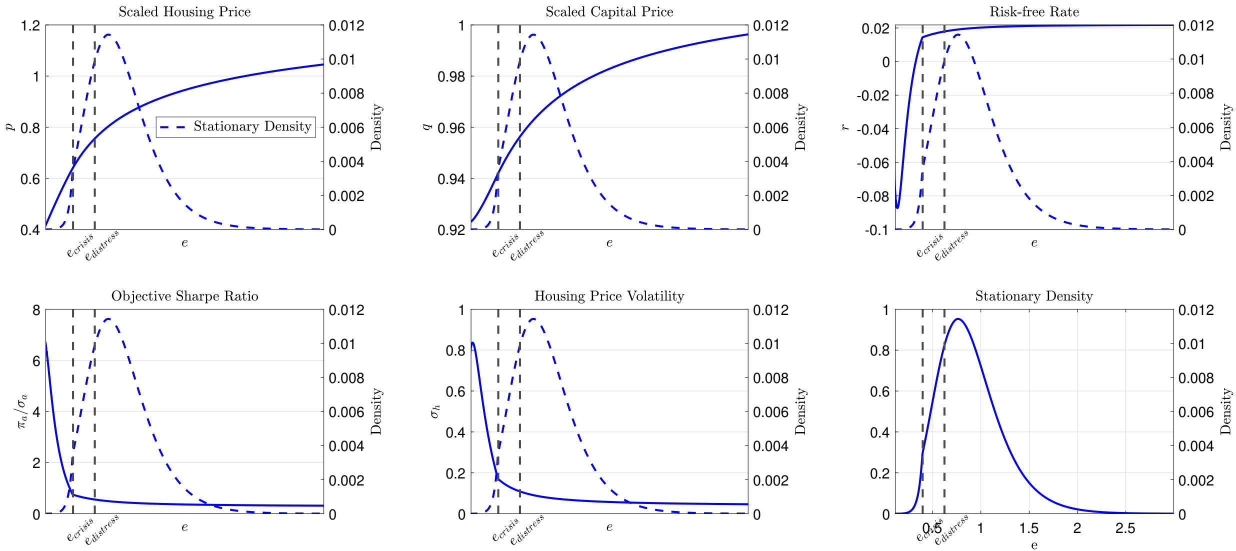
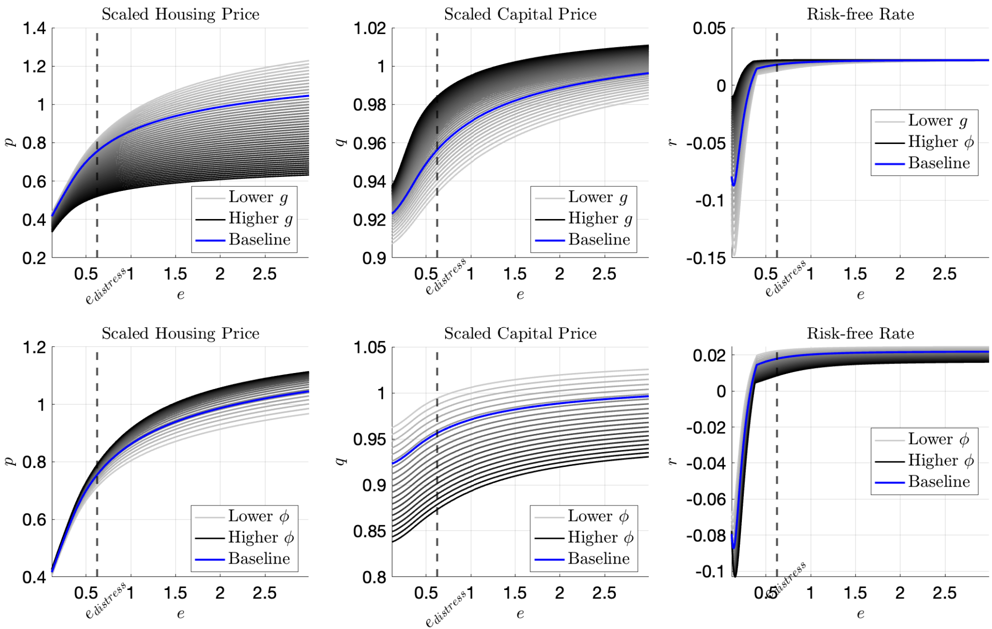
Unrestricted set of parameters (g, ϕ)

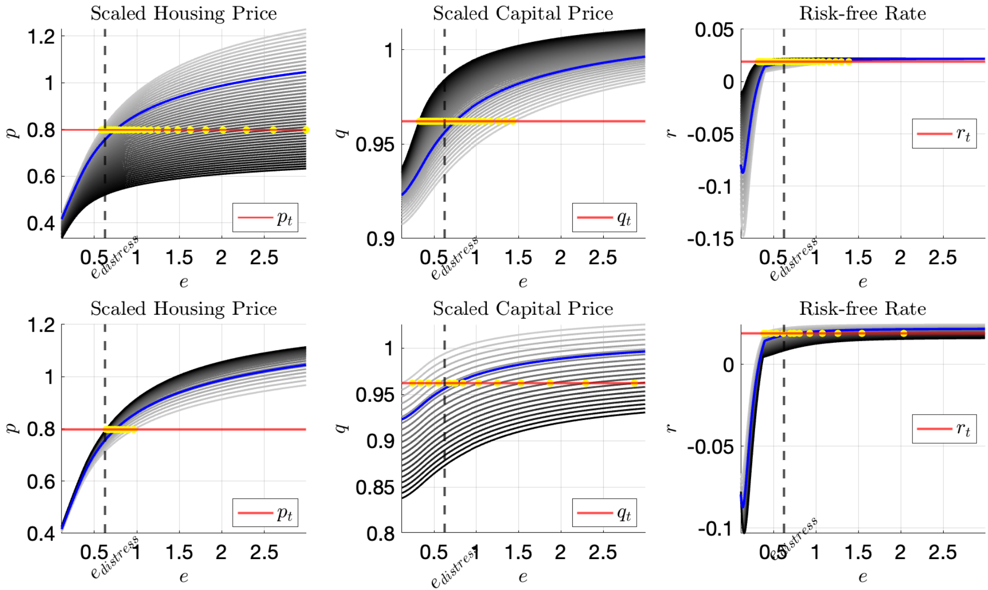
- baseline economy takes the same parameter values as in HK
- alternative economies take different values
  - the worst-case (g, ϕ) in baseline is never binding in unrestricted set

Nonlinear Equilibrium Prices from the Baseline Economy



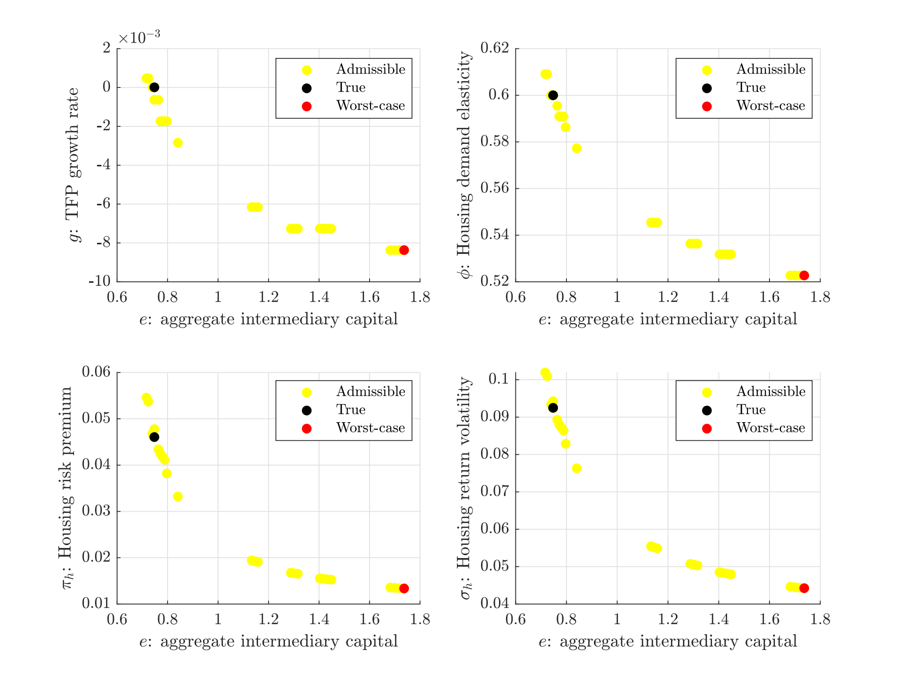
Observational Equivalence for Asset Prices



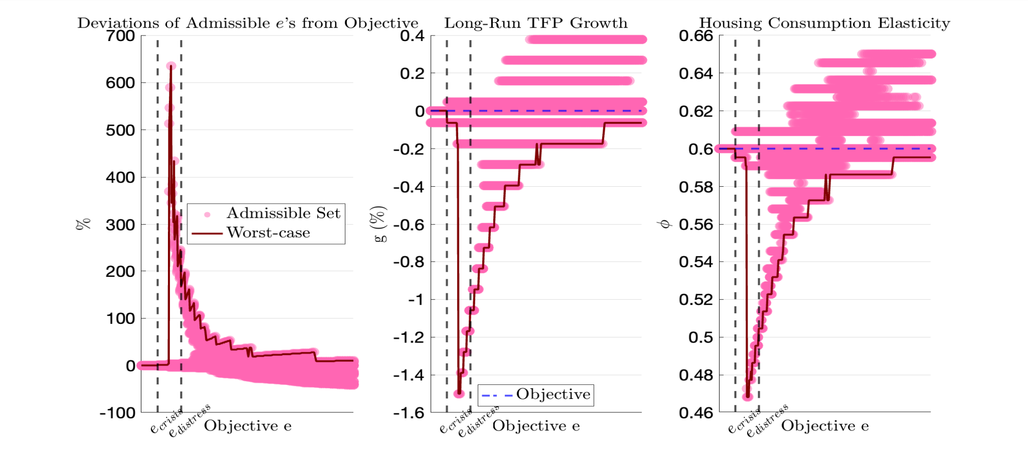


Scaled Housing Price
Scaled Capital Price
Risk-free Rate
p
q
r
e
$e_{distress}$
p_t
q_t
r_t
1.2
1
0.8
0.6
0.4
0.95
0.9
0.85
0.05
0
-0.05
-0.1
-0.15
0.5
1.5
2
2.5

Observational Equivalence: Cash Flow or Discount Rate?



Admissible Set of Latent Objects

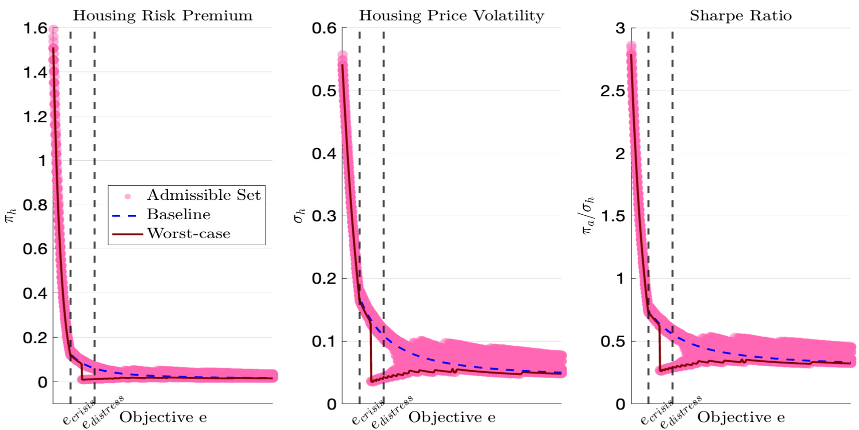


Asset prices fluctuate due to the revisions of future cash flows (g, ϕ), not risk premium (e)

- consistent with survey evidence of Delao and Myers (2021)
- contrast with the rational expectations equilibrium

Campbell-Shiller decomposition

Admissible Set of Return Distributions



Consistent with Nagel and Xu (2023)

- subjective risk premium is less predictable by P-D ratio than objective
- subjective risk premium covaries with subjective variance but not objective

NX regression | Risk-return tradeoff

Empirical Evidence

Survey Evidence for Subjective Beliefs

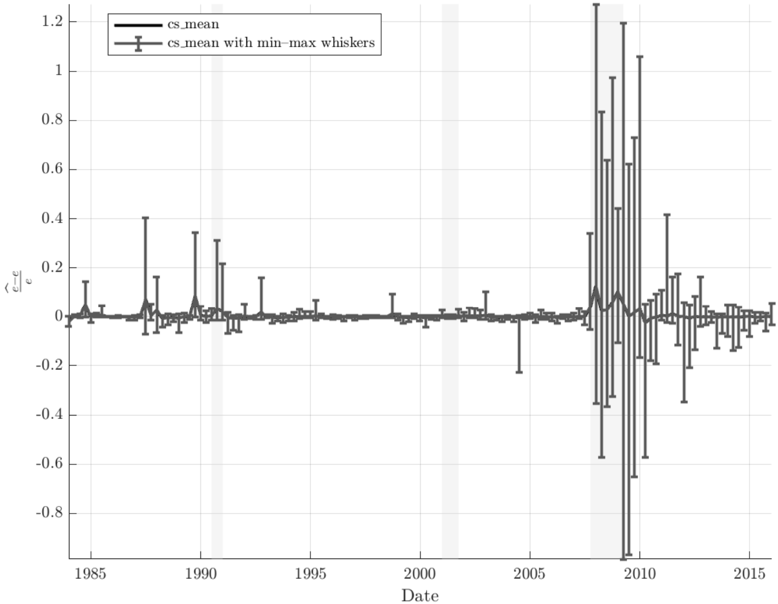
Survey data from IBES

- detailed files of financial analysts' contemporaneous estimates for earnings in the financial sector

$$\underbrace{\frac{\widehat{e}^{i}_{j,t} - e_t}{e_t}}_{\text{model-implied deviations of admissible } e} \approx \underbrace{\frac{\widehat{\text{Earning}}^{i}_{j,t} - \text{Earning}_{j,t}}{\mathcal{E}_{j,t}}}_{\text{data}}.$$

- forecasts for future cash flow growth and returns from the S&P 500 index (De la O and Myer (2021))

Estimate Errors (%) for Contemporaneous Earnings of the Financial Sector



Subjective Beliefs over e and g

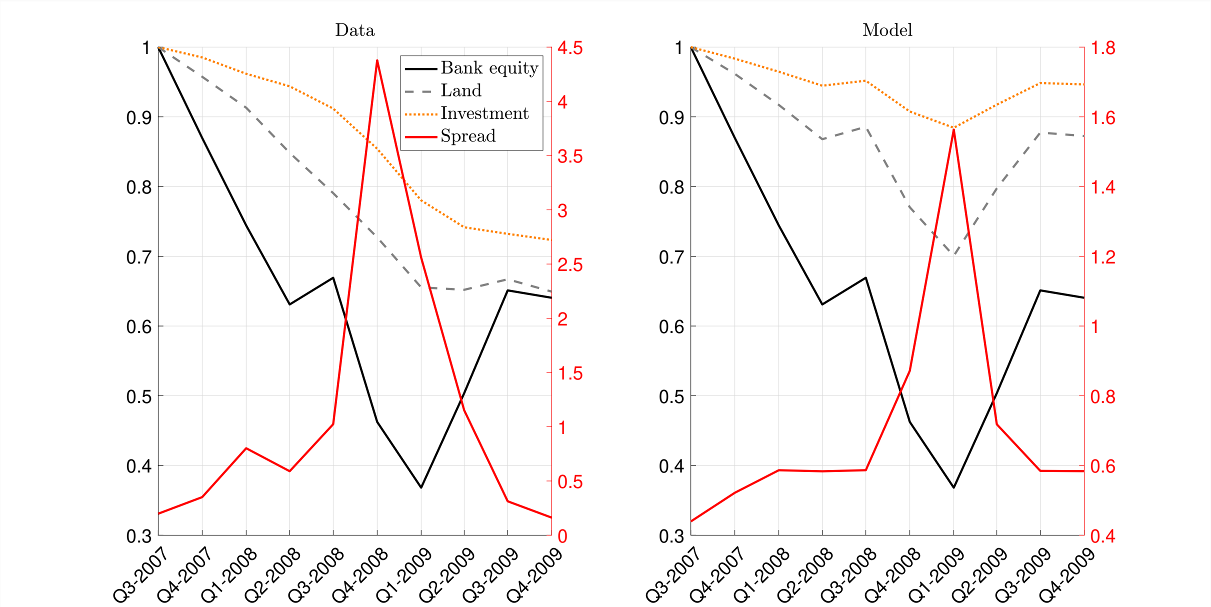
Moments	Model	Data	REE
Panel A: Predictability of cash flow forecast errors by intermediaries capital estimates			
Aggregate cashflow growth by the maximum forecast error of e $((\widehat{e}^{\max} - e)/e)$	0.69	0.18*	0
Aggregate cashflow growth by the minimum forecast error of e $((\widehat{e}^{\min} - e)/e)$	0.52	0.14	0
Panel B: Conditional moments of intermediaries capital estimates			
$\text{Mean}\left(\text{std}_t\left(\frac{\widehat{e}-e}{e}\right)\middle\vert \text{distress}\right)/\text{Mean}\left(\text{std}_t\left(\frac{\widehat{e}-e}{e}\right)\right)$	1.15	1.92	NA
$\text{Mean}\left(\frac{e^{\max}-e}{e}\vert\text{distress}\right)/\text{Mean}\left(\frac{e^{\max}-e}{e}\right)$	1.47	1.79	NA

Consistent with the theory:

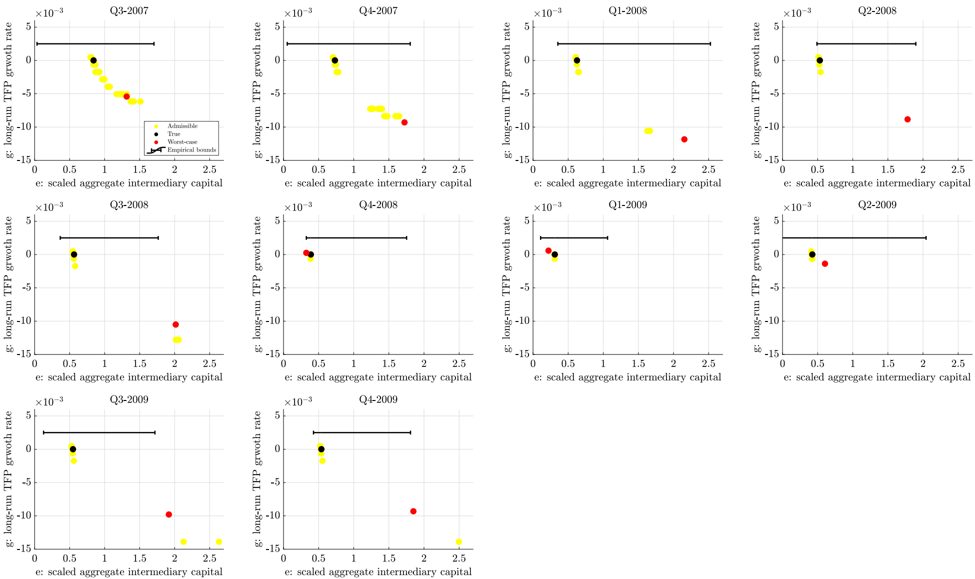
- aggregate cash flow growth are predicted to be lower when the largest e estimates are higher, not so much for the smallest e
- during financial distress, the dispersion of the e estimates is large, particularly upward

Crisis Dynamics

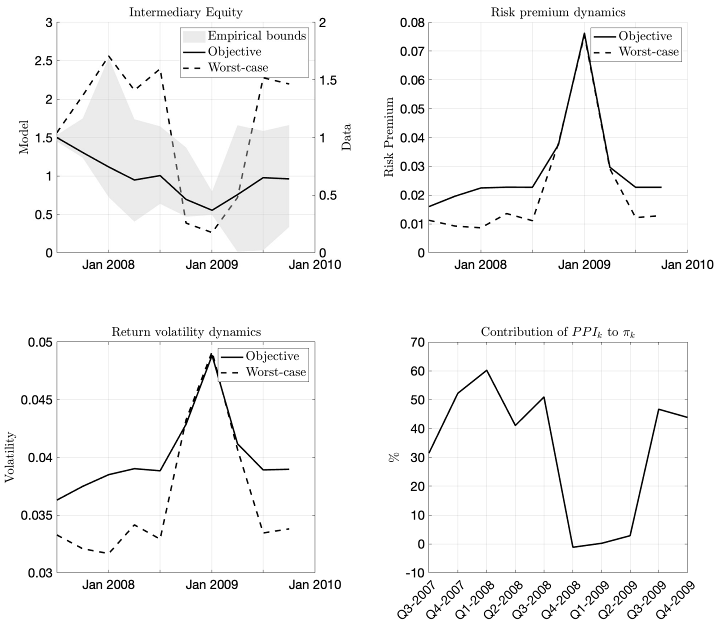
Macro-Financial Dynamics during the 2007-2009 Crisis



Subjective Belief Dynamics during the 2007-2009 Crisis



Subjective Return Distributions during the 2007-2009 Crisis



Concluding Remarks

Admissible set of models are dependent on the endogenous information in equilibrium

- asset price declines associated with Lehman collapse eliminated the optimistic view on the intermediary sector
- more room for policy promises to alter market beliefs, e.g. asset purchase announcement Haddad, Moreira, Muir (2025)

Follow-up theory

- dynamic extensions: dynamic consistency problem and learning Epstein and Schneider (2007)
- model misspecification concerns $\Rightarrow$ all parameterized models are misspecified Hansen and Sargent (2022)

Appendix

Objective Risk Premium Decomposition

The objective risk premium is decomposed as

$$\pi_a = \underbrace{\gamma(\alpha_k \sigma_k^w + \alpha_h \sigma_h^w)\sigma_a^w}_{\text{subjective risk exposure}=\pi_a^w} + \underbrace{PPI_a}_{\text{compensation for ambiguity}},$$

- subjective risk exposure explains 67% unconditionally
- compensation for endogenous ambiguity accounts for 33%

In REE, the risk exposure explains everything

What Drives Asset Pricing Fluctuations?

Moments	Model	Data	REE
Panel B: Campbell-Shiller price-dividend ratio decomposition (De la O and Meyer (2021))			
1-year ahead cash flow growth subjective expectations $(\text{Cov}(g^s, pd_t)/\text{Var}(pd_t))$	0.37	0.39	0
1-year ahead subjective discount rate $(\text{Cov}(ER^s, pd_t)/\text{Var}(pd_t))$	-0.22	-0.05	0.52

- many REE asset pricing models rationalize the volatile asset prices by *discount rates*
- call for models that drive asset prices through volatile *cash flow growth expectations*

Back

Are Return and Cash Flow Expectations consistent with FIRE?

Moments	Model	Data	REE
Panel C: Predictability of cash flow and return forecast errors (De la O and Meyer (2021))			
Forecast error predictability of aggregate cashflow growth by P-D ratio	-0.27	-0.30	0
Forecast error predictability of excess return by P-D ratio	-0.29	-0.25	0
Panel D: Cyclicality of risk premium for capital (Nagel and Xu (2023))			
Regression coefficient of 1-year subjective risk premium on dividend-price ratio	-0.09	-0.24	0.32
Regression coefficient of 1-year objective risk premium on dividend-price ratio	0.32	6.4	0.32
Ratio of subjective to objective coefficients	-0.3	-0.03	1

Predictability of forecast errors implies deviations from the FIRE

- the error declines when the asset prices are high and/or;
- the worst-case e is closer to the objective

Back

What Drives the Subjective Risk Premium?

Moments	Model	Data	REE
Panel E: Risk premium and return variance (Nagel and Xu (2023))			
Regression coefficient of subjective risk premium π_k^S on subjective variance $(\sigma_k^S)^2$	24	4	50
Regression coefficient of subjective risk premium π_k^S on subjective variance $(\sigma_k^S)^2$	-8.8	-0.01	50
Regression coefficient of objective risk premium π_k on objective variance $(\sigma_k)^2$	28.7	1.49	50

- the *subjective* risk premium is driven by the *subjective* risk perception, not *objective* one
- the *objective* risk premium is correlated with the *objective* return volatility

Back