Endogenous Ambiguity in Nonlinear Macro-Finance Models*

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Abstract

This paper introduces a belief-formation mechanism under ambiguity embedded in a nonlinear macro-finance model with a financial intermediary sector, and shows that it rationalizes several survey-based deviations from rational expectations. Individual intermediaries use the equilibrium price functions to endogenously construct observationally equivalent models that match asset prices but imply different distributions of future returns. The resulting worst-case subjective beliefs overestimate aggregate capital in the sector until financial frictions trigger crises through nonlinear drops in risky asset prices; once crises unfold, this overoptimism disappears and subjective risk perceptions and premia spike. The model also generates an acyclical subjective risk premium and attributes asset-price fluctuations to countercyclical subjective cash-flow expectations, consistent with recent survey evidence. Compensation for endogenous ambiguity accounts for roughly half of the increase in risk premia during the 2007–2009 financial crisis.

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1 Introduction

Ambiguity arises when decision makers face uncertainty about the prior over future outcomes and consider a set of models, rather than a single model as assumed under standard risk. I develop a novel belief formation mechanism under ambiguity within general equilibrium models, in which the set of models agents consider is endogenously disciplined. Each model in this set is observationally equivalent with respect to contemporaneous endogenous observable information and the structure of the economy—the mapping from parameters and states to observable information, but implies distinct distributions over future payoff-relevant outcomes. Cautious agents select the worst-case model—i.e., the one that minimizes their payoffs—and adopt it as their subjective belief. They then choose actions to maximize payoffs given these subjective beliefs.

I incorporate this belief formation mechanism into macroeconomic models with a financial intermediary sector. Financial frictions in the models generate highly nonlinear dynamics in asset prices and macroeconomic fluctuations, driven by declines in aggregate intermediary capital and the resulting disruptions in intermediation. Under rational expectations, the model counterfactually predicts a countercyclical subjective risk premium and accurate forecasts for contemporaneous aggregate capital in the sector. In contrast, the belief formation mechanism under ambiguity aligns with novel survey evidence: the subjective beliefs arising from the worst-case model overestimate aggregate capital until financial frictions trigger crises through nonlinear declines in risky asset prices. Once crises unfold, overoptimism disappears, and subjective risk perceptions and premia spike. The model also produces an acyclical subjective risk premium and attributes asset price fluctuations to countercyclical subjective cash-flow expectations, consistent with survey data. Moreover, the novel survey evidence indicates the negative association between cash flow growth expectations and contemporaneous estimates of the capital in the intermediary sector, consistent with the theory.

The macro-financial environment I study builds on He and Krishnamurthy (2019). The He and Krishnamurthy (2019) model is a continuous-time Real Business Cycle (RBC) framework augmented with a financial intermediary sector subject to an occasionally binding equity issuance constraint. When intermediary constraints are slack, the model behaves similarly to a frictionless RBC model, making it difficult to distinguish between the two using observable information. However, a sequence of adverse aggregate shocks pushes intermediaries closer to their constraints, resulting in financial distress. During these periods, the nonlinearities introduced by financial frictions become quantitatively significant, producing observable outcomes that markedly differ from those of the frictionless model. Financial crises occur in tail states where the constraint binds, causing risk premia to spike and asset prices to fall sharply. In this environment, aggregate capital in the intermediary sector becomes a key endogenous state variable.

Building on this rich macro-financial framework, I depart from rational expectations by incorporating the aforementioned endogenous ambiguity into the model of financial intermediary frictions. This paper develops a method to generalize rational expectations equilibrium models to include endogenous ambiguity. Individual intermediaries face uncertainty over the distribution of future risky asset returns. Each distribution (model) corresponds to an equilibrium object in the same model but is parameterized by alternative parameter values—such as long-run TFP growth[1] or households' intratemporal substitution between consumption and housing services[2]—and/or the current state (e.g., the aggregate capital in the intermediary sector)[3], which are assumed to be directly unobservable.

The set of parameters and states that individuals view as admissible is disciplined by observable information from contemporaneous asset prices (capital, housing, and risk-free rates) and the knowledge of the structure of the economy—that is, the mapping from current states and parameters to asset prices. The resulting models, parameterized by admissible states and parameters, are observationally equivalent but imply distinct admissible distributions over future returns. Cautious intermediaries adopt the worst-case admissible model from this set as their subjective belief and make investment decisions to maximize utility under it. Importantly, these novel expectations do not introduce exogenous shocks; instead, they arise endogenously from the state of the economy through observable information and the underlying economic structure.

The interaction between the endogenous construction of admissible models and the nonlinear-

[1] The long-run risk literature initiated by Bansal and Yaron (2004) has long struggled to accurately identify and estimate the persistent component of consumption growth without relying on asset-pricing implications for guidance. Despite important recent contributions by Bansal, Kiku, and Yaron (2016) and Schorfeide, Song, and Yaron (2018), empirical evidence for the existence of such a persistent component remains controversial and plausibly uncertain.

[2] Gerardi, Foote, and Willen (2010) document substantial disagreements among economists regarding the causes of the rise in housing prices prior to the 2007–2009 financial crisis. Some attributed the increase to stronger fundamentals driven by optimistic expectations about future rental revenues, while others viewed it as evidence of housing bubbles. This illustrates the uncertainty surrounding the housing demand.

[3] Prior to the global financial crisis, little attention was paid to the resilience of the intermediary sector, and many failed to recognize that the assets on intermediaries' balance sheets were substantially exposed to losses on risky subprime morgages.

ities arising from financial intermediation frictions generates a novel state-dependent endogenous ambiguity. Starting from a nonbinding state with a high aggregate capital in the intermediary sector, consider a sequence of negative shocks to current TFP, which agents do not directly observe, causing risky asset prices to decline. All individual intermediaries incur losses on their leveraged positions, reducing the aggregate capital in the sector, which in turn lowers risky asset demand and further depresses asset prices.

However, once one acknowledges that intermediaries might differ in their balance-sheet compositions, an individual intermediary cannot confidently infer the underlying cause of an asset-price decline without directly observing aggregate capital in the sector. For example, in the spirit of standard RBC models, a lower long-run TFP growth rate or reduced housing demand could also rationalize observed price drops, even if aggregate capital remained unchanged. Critically, this alternative admissible scenario would imply a lower Sharpe ratio for risky assets because it features higher aggregate intermediary capital, thereby offering less advantageous investment opportunities from the perspective of each intermediary. In contrast, in the true scenario a decline in aggregate net worth raises the Sharpe ratio. As a result, intermediaries adopt the alternative scenario as their worst-case and form their subjective beliefs accordingly. Consistent with survey evidence in Nagel and Xu (2023), under this worst-case scenario the subjective risk premium is acyclical, whereas long-run TFP growth expectations become countercyclical. Hence, subjective beliefs attribute asset-price fluctuations to countercyclical cash-flow growth expectations rather than countercyclical discount-rate movements, in line with the findings of De la O and Myers (2021).

This belief distortion—reflecting an "optimistic" assessment of aggregate intermediary capital—cannot be sustained once a further sequence of negative shocks pushes intermediaries up against the equity-issuance constraint. At that point, the nonlinear declines in both risky asset prices and the risk-free rate can only be reconciled with a deterioration in aggregate capital, in sharp contrast to frictionless RBC environments. For the alternative scenario to match these highly depressed asset prices, the counterfactually higher capital would need to offset the price declines driven by lower long-run TFP growth and weaker housing demand. Yet once aggregate capital becomes sufficiently large, it ceases to influence asset prices because financial frictions no longer operate. As a result, the alternative scenario becomes incompatible with the observed market data and with the structure of the economy.

Hence, subjective beliefs in the model display optimism about aggregate intermediary capital during periods of financial distress, followed by an abrupt shift toward the distressed–capital scenario once a crisis unfolds. During distress, agents also underestimate expected TFP growth and housing demand, but this pessimism is partially reversed after the crisis reveals the true state of balance sheets. I present new empirical evidence from the 2007–2009 financial crisis showing that financial analysts systematically overestimated intermediaries' contemporaneous capital relative to real-time book values prior to Lehman's collapse. This belief distortion vanished once the crisis exposed the severe deterioration in capital across major financial institutions, coinciding with sharp declines in asset prices.

The existence of observationally equivalent models makes it difficult to infer future crisis probabilities from asset prices alone. The model indicates that roughly half of the increase in the risk premium during the 2007–2009 crisis reflects compensation for endogenous ambiguity stemming from uncertainty about aggregate intermediary capital. By contrast, in the rational-expectations equilibrium the rise in the risk premium is explained entirely by intermediaries' increased exposure to aggregate shocks through high leverage in risky assets.

The analysis in this paper builds on two methodological contributions. First, it develops a framework for endogenizing the set of admissible models under ambiguity, thereby capturing the feedback between endogenous observable information and subjective beliefs. I apply this framework to generalize the He and Krishnamurthy (2019) model to allow for non-rational expectations, enabling the study of belief formation without disturbing the equilibrium dynamics with which those beliefs coevolve. A central implication is that both endogenous ambiguity and financial frictions are necessary to understand the evolution of subjective beliefs around financial crises, as well as the associated dynamics of asset prices and the macroeconomy.

Second, this paper demonstrates the benefits of using global solution methods to fully characterize the equilibrium impact of economic dynamics on belief formation. The optimism during financial distress and the sudden reversal of beliefs during crises require the nonlinear dynamics of asset prices, which deviate from frictionless predictions. By employing global solution methods, the paper captures the complete interactions between belief formation and the global nonlinear macro-financial dynamics.

Related Literature

This paper contributes to two strands of the literature. First, the belief-formation mechanism developed here relates to the model-uncertainty literature. Hansen and Sargent (2022) classify this literature into structured ambiguity and model misspecification. Structured ambiguity dates back to the axiomatic foundations of Gilboa and Schmeidler (1989), in which decision makers entertain a parameterized set of models and assume that the true data-generating process lies within this set.

In contrast, the decision makers in this paper endogenously construct the set of admissible models, generating a feedback loop between belief formation and observed endogenous information. The analysis abstracts from dynamic extensions of endogenous ambiguity and from issues of dynamic consistency.[4]

Model misspecification concerns arise when decision makers do not fully trust any parameterized (structured) model and instead entertain a set of unstructured models, which may not admit a parametric representation.[5] Hansen and Sargent (2021) combine structured ambiguity with model-misspecification concerns to study their joint quantitative implications for risk premia. The belief-formation framework developed in this paper abstracts from model-misspecification considerations and focuses solely on endogenous structured ambiguity.

Second, this paper contributes theoretically and quantitatively to the macro-finance literature that incorporates financial frictions into general-equilibrium models. Seminal work includes Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). More recent research frequently employs continuous-time methods to study global dynamics in nonlinear environments, as in Adrian and Boyachenko (2012), He and Krishnamurthy (2013), He and Krishnamurthy (2019), Brunnermeier and Sannikov (2014), and Di Tella (2017). A growing strand of this literature introduces deviations from rational expectations into nonlinear macro-financial frameworks to explain the optimism observed before crises and the pessimism embedded in asset prices during crises; notable examples include models with diagnostic expectations such as Krishnamurthy and Li (2021) and Maxed (2023). In addition, Gertler, Kiyotaki, and Prestipino (2020) examines the role of

[4]See Chen and Epstein (2002), Epstein and Schneider (2003), and Hansen and Sargent (2022) for the rectangularity conditions required for dynamic consistency under exogenously specified sets of structured models.

[5]See Hansen and Sargent (2001) for the foundations of robustness. Applications to macroeconomics and asset pricing include Pouzo and Presno (2016) and Bhandari, Borovicka, and Ho (2023).

beliefs in generating booms and busts in financial markets.

In contrast, this paper demonstrates that the nonlinearity generated by financial frictions—operative only in part of the state space—makes it difficult to distinguish the model from its frictionless benchmark using only observable public information, such as asset prices. This feature helps explain why many observers failed to detect the likelihood of a severe financial crisis before 2007, even as some intermediaries had already begun to incur losses on housing-related investments. Survey data from financial analysts likewise reveal optimistic forecasts of contemporaneous earnings for these troubled intermediaries. Such assessments of balance-sheet resilience are difficult to reconcile with models that abstract from endogenous ambiguity. The framework developed here can account for these patterns and also replicates several key stylized facts about subjective beliefs over returns and cash-flow growth documented in Nagel and Xu (2023) and De la O and Myers (2021), which are inconsistent with traditional rational-expectations models.

The rest of this paper is structured as follows. Section 2 describes the model environment, the equilibrium definitions with endogenous ambiguity, and the analytical descriptions of the equilibrium mechanism of the subjective beliefs. In Section 3, I calibrate the model and shows the quantitative fits of the numerical solution of the model to both macro-finance moments and survey expectations evidence. Finally, Section 4 concludes. I relegate the detailed derivations of the equilibrium conditions, the description of the numerical algorithms, and the data sources in Online Appendices.

2 Model

This paper embeds a novel belief-formation mechanism under endogenous ambiguity into the macro-finance model of He and Krishnamurthy (2019) (hereafter HK). The HK model is one of the first fully quantitative continuous-time macro-finance frameworks and successfully captures the downside risks that arise from disruptions in financial intermediation. However, a growing body of survey data documents volatile, procyclical cash-flow growth expectations and acyclical return expectations—patterns that challenge the countercyclical discount rates and acyclical cash-flow growth implied by rational-expectations equilibria. Furthermore, as shown later, the HK model cannot account for the optimistic forecasts of intermediary-sector capital observed during periods of financial distress.

I extend the HK framework to address these shortcomings in two steps. First and most importantly, I generalize the model to incorporate endogenous ambiguity, enabling it to match the empirically observed dynamics of expectations—not only the dynamics of macro-financial quantities. Second, I replace the capital-quality shock in HK with a long-run TFP growth shock and allow for nonzero average long-run cash-flow growth rates, thereby introducing the possibility of sustained economic growth. Wherever possible, I preserve HK's original notation to facilitate comparison.

2.1 Model set-up

$(\Omega, \mathcal{F}, P)$ represents the objective probability space. Time is modeled as continuous, with t denoting the current period. The purpose of using continuous time is not to capture high-frequency dynamics, but rather to exploit its analytical and computational advantages. Continuous-time methods enable the local behavior of the model to be characterized analytically while allowing for a global solution. These features are critical for the analysis in this paper. First, the analytical tractability provides clear economic characterizations of future payoff distributions, such as infinitesimal return distributions. Second, the global solution allows for the characterization of nonlinear financial crises, alongside near-linear dynamics outside of crises. This, in turn, enables an examination of how these equilibrium dynamics endogenously shape ambiguity and subjective beliefs differently during and outside crisis episodes.

The economy consists of two sectors: households and financial intermediaries. There are two types of assets in positive supply: productive capital K_t and housing H. The supply of housing is fixed and normalized to $H \equiv 1$. The price of capital is denoted by Q_t, and the price of housing by P_t. Asset prices are endogenous and determined in equilibrium.

Only intermediaries can directly hold K_t and H. Consequently, this is an intermediary asset pricing model, as financial intermediaries act as the marginal investors in these asset markets in equilibrium. Their beliefs and risk pricing therefore determine the risk premium.

Intermediaries finance their asset purchases by issuing debt and equity to households. However, each intermediary faces an "equity capital constraint" that limits its ability to raise equity. This constraint is the key financial friction: when it binds, intermediaries must rely more heavily on debt financing.

I begin the rest of this section by describing the standard RBC component of the model, and then turn to the financial frictions and endogenous ambiguity in the intermediary sector.

2.1.1 Production capital

Output flow Y_t is produced according to an "AK" production function:

$$Y_t = \overline{A} A_t K_t,$$

where $\overline{A} A_t$ governs the productivity of capital. The time-varying component follows a geometric Brownian motion:

$$\frac{dA_t}{A_t} = g dt + \sigma dZ_t,$$

where g is the average long-run growth rate and Z_t is a standard Brownian motion. Capital accumulates according to

$$\frac{dK_t}{K_t} = (i_t - \delta) dt.$$

The effective capital stock then evolves as

$$\frac{d(A_t K_t)}{A_t K_t} = (i_t - \delta + g) dt + \sigma dZ_t.$$

Investment in capital is subject to quadratic adjustment costs. For a gross installation of $i_t K_t$, the cost is given by $\Phi(i_t, A_t K_t) \doteq i_t A_t K_t + \frac{\kappa}{2}(i_t - \delta)^2 A_t K_t$. Define the unit price of capital scaled by the effective capital stock as $q_t = \frac{Q_t K_t}{A_t K_t}$. Capital producers then solve the standard q-theory investment problem $\max_{i_t} Q_t i_t - \Phi(i_t, A_t K_t)$, and all profits are distributed to households. This yields the familiar investment rule

$$i_t = \delta + \frac{q_t - 1}{\kappa}. \tag{1}$$

This equation illustrates an important macro-financial linkage: the economy's growth rate depends on the investment rate i_t, which in turn depends on the endogenous capital price q_t. To the extent that financial frictions and ambiguity influence q_t, they propagate from financial markets into the broader economy's growth dynamics.

2.1.2 Households, housing rent, and risk-free rate

There are identical households with mass $i \in [0,1]$. Households consume output goods $c^y_{i,t}$ and housing services $c^h_{i,t}$. Output goods are the numeraire. Because households do not hold housing directly, they rent housing services at price D_t.

Households maximize

$$E_t \left[\int_t^\infty e^{-\rho(s-t)} \frac{c_{i,t}}{1-\gamma_h} ds \right],$$

where $c_{i,t}$ is a Cobb-Douglas consumption aggregator $c_{i,t} = (c^y_{i,t})^{1-\phi}(c^h_{i,t})^{\phi}$. Since households are identical, I drop the individual subscript for notational simplicity. Intratemporal maximization implies

$$\frac{c^y_t}{c^h_t} = \frac{1-\phi}{\phi} D_t. \tag{2}$$

Let W_t denote aggregate household wealth. Each household invests its individual wealth $w_{i,t} \equiv W_t$ in two assets: the debt and the equity issued by a randomly matched intermediary. Debt yields a risk-free return r_t, and equity yields a stochastic return $d\widetilde{R}_t$. I now impose reduced-form assumptions to guarantee that households purchase at least λW_t of intermediary debt. Because households are not the focal point of the analysis, these simplifying assumptions allow the leverage of the intermediary sector to be governed by the exogenous parameter λ.

Each household is divided into a "debt member" and an "equity member." The debt member can invest only in the intermediaries' risk-free debt. The equity member can purchase intermediary equity but cannot take levered positions. At the start of each period, the debt member receives a fraction λ of wealth, and the equity member receives the remaining fraction $1-\lambda$. Investments pay off at time $t+dt$, after which returns are pooled and the process repeats.

Collectively, equity members invest their assigned wealth in intermediary equity, subject to the restriction that, given the capital stock of their matched intermediary $\epsilon_{i,t}$, they cannot purchase more than $\epsilon_{i,t}$ of equity. If the constraint binds, equity members allocate their remaining wealth to debt. In aggregate, the total equity raised by the intermediary sector at time t is therefore

$$E_t \equiv \min\{\mathcal{E}_t, (1-\lambda)W_t\}, \tag{3}$$

where $\mathcal{E}_t$ denotes the aggregate capital held by intermediaries.

Households also determine the risk-free rate r_t through their intertemporal optimization:

$$r_t dt = \rho dt + E_t\left[\frac{dc_t^y}{c_t^y}\right] - \frac{\xi(\xi+1)}{2} Var_t\left[\frac{dc_t^y}{c_t^y}\right]. \tag{4}$$

The parameter $\xi = 1-(1-\phi)(1-\gamma_h)$ can be interpreted as the inverse of the elasticity of intertemporal substitution (EIS).

2.1.3 Aggregate intermediary sector

The aggregate capital of the entire financial sector evolves according to

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} \equiv d\tilde{R}_t - \eta dt + d\psi_t, \tag{5}$$

where η is an exogenous exit rate that prevents intermediaries from fully escaping the financial friction by accumulating unlimited net worth, thereby ensuring a nondegenerate stationary distribution. The final term $d\psi_t$ will be fully characterized when I describe the boundary conditions of the economy.

2.1.4 Endogenous ambiguity and intermediary portfolio choice

A decision maker working for each intermediary lives for an infinitesimal interval of length dt. He maximizes mean–variance preferences over the portfolio return generated by housing, capital, and risk-free assets. From this point onward, I refer to the decision maker and his intermediary interchangeably.

Importantly, he does not directly observe three key objects: the current aggregate capital in the financial sector $\mathcal{E}_t$, the underlying TFP growth rate g, and the intratemporal elasticity of substitution ϕ. Moreover, he acknowledges the possibility that these latent objects may evolve over time as i.i.d. random variables.[6]

He entertains ambiguity in the sense of uncertainty about the prior distribution of risky returns. More specifically, he understands the structure of the economy—that is, the mapping from the

[6] Chen and Epstein (2002), Hansen and Sargent (2021), and Hansen and Sargent (2022) consider the same extreme possibility, as the existing literature has not yet developed a theory that incorporates learning into the structured-ambiguity framework.

current values of $(\mathcal{E}_t, g_t, \phi_t)$ to the observed asset prices (P_t, Q_t, r_t), as well as the mapping from the latent objects to the distributions of future infinitesimal returns for capital and housing.

Given this prior uncertainty, the decision maker constructs the admissible set of latent objects that are consistent with both the structure of the economy and the information contained in observed asset prices. Letting $\theta_t \doteq (\mathcal{E}_t, g_t, \phi_t)$ denote the vector of latent objects, a consistent θ must satisfy the following conditions:

$$\begin{aligned} \underbrace{P_t}_{\text{Currently observed price}} &= \underbrace{\widetilde{P}(\theta_t)}_{\text{Model-implied price}} \quad ; \\ \underbrace{Q_t}_{\text{Currently observed price}} &= \underbrace{\widetilde{Q}(\theta_t)}_{\text{Model-implied price}} \quad ; \\ \underbrace{r_t}_{\text{Currently observed price}} &= \underbrace{\widetilde{r}(\theta_t)}_{\text{Model-implied price}} \quad . \end{aligned} \tag{6}$$

The model-implied price functions above are taken as given at this stage.[7] I will later impose cross-equilibrium restrictions on these price functions: for any latent parameter values (g, ϕ), the entertained price functions $\{\widetilde{P}(\mathcal{E}_t; g, \phi), \widetilde{Q}(\mathcal{E}_t; g, \phi), \widetilde{r}(\mathcal{E}_t; g, \phi)\}$ must coincide with the equilibrium price functions of the economy parameterized by (g, ϕ).

Clearly, the true (objective) latent object, denoted $\hat{\theta}_t$, satisfies these consistency conditions. I denote the set of admissible θ's by $\Theta(P_t, Q_t, r_t)$ since it depends on the observed asset prices. Because these prices depend on the unobservable aggregate capital, this set is itself an endogenous object. The elements in this admissible set all have observationally equivalent implications for the current asset prices given the prespecified price functions.

The infinitesimal return processes for a unit of capital k and housing h are represented by Itô processes: for an asset $a \in \{k, h\}$,

$$dR_{a,t} = (\pi_{a,t} + r_t)dt + \sigma_{a,t} dZ_t,$$

where $\pi_{a,t}$ is the infinitesimal objective risk premium and $\sigma_{a,t}$ is the objective return volatility.

Since each intermediary also entertains the prespecified mapping from θ to the risk premium

[7] I suppress the dependence of these price functions on the effective capital stocks $A_t K_t$ since decision makers are assumed to observe the current aggregate TFP level and capital stock.

and return volatility, he constructs the set of admissible models (i.e., distributions) over future risky returns using the admissible latent objects in $\Theta(P_t, Q_t, r_t)$:

$$\widetilde{\Xi}(P_t, Q_t, r_t) \doteq \{(\widetilde{\pi}_a(\theta), \widetilde{\sigma_a}(\theta)), \forall a \in \{k, h\} : \theta \equiv (\mathcal{E}_t, g_t, \phi_t) \in \Theta(P_t, Q_t, r_t)\}.$$

At this stage, I simply postulate functional forms for $\widetilde{\pi}_a(\theta_t)$ and $\widetilde{\sigma}_a(\theta_t)$ for $a \in k, h$. As with the price functions, I will later impose the same *cross-equilibrium* restriction on these return functions to discipline the set of admissible latent objects. Under this restriction, the entertained return distributions coincide with the equilibrium objective risk premium and volatility for asset a in the alternative economy characterized by (g, ϕ) and current aggregate capital $\mathcal{E}_t$. Each intermediary also understands that the uncertainty over the latent objects should depress the risky asset prices and deliver additional compensation in equilibrium. Consequently, it postulates that the return distribution parameterized by θ is specified as:

$$\pi_a^S(\theta) \doteq \widetilde{\pi}_a(\theta) + PPI_a^S;$$

$$\sigma_a^S(\theta) = \widetilde{\sigma}_a(\theta),$$

where denoting $\hat{\theta}$ be the baseline (true) state and parameter values,

$$PPI_a^S \doteq \pi_a(\hat{\theta}) - \pi_a^{REE}(\hat{\theta}).$$

PPI_a^S is the subjective compensation for endogenous ambiguity, termed as *the price of partial identification*[8], which intermediary takes as given. π_a^{REE} is the risk premium under the rational expectations equilibrium. If there is no ambiguity, meaning that $\Xi(p, q, r)$ is singleton, then the objective risk premium in this model coincides with the risk premium under the rational expectations equilibrium. In such a special case, there is no compensation for ambiguity: $PPI_a^S \equiv 0$.

After all, given the observed information of current asset prices, each intermediary views the following set of models as admissible:

$$\Xi(P_t, Q_t, r_t) = \left\{(\pi_a^S, \sigma_a^S)_{a\in\{k,h\}} : \exists \theta \in \Theta(P_t, Q_t, r_t) \quad \text{s.t} \quad \pi_a^S = \pi_a^S(\theta) \quad \text{and} \quad \sigma_a^S = \sigma_a^S(\theta)\right\}. \quad (7)$$

[8]This terminology is credited with Tom Sargent.

Given this set of models, each intermediary adopts a min–max decision rule by solving

$$\min_{\{\pi_k^S,\pi_h^S,\sigma_k^S,\sigma_h^S\}\in\Xi(P_t,Q_t,r_t)} \max_{\alpha_k,\alpha_h} \underbrace{\alpha_k\pi_k^S+\alpha_h^S\pi_h^S}_{\text{Subjective expected excess portfolio return}} + \frac{\gamma}{2}\times \underbrace{(\alpha_k\sigma_k^S+\alpha_h\sigma_h^S)^2}_{\text{Subjective variance of excess portfolio return}}, \tag{8}$$

where α_a denotes the portfolio share invested in asset $a \in k, h$. Given the portfolio choice, the "alter ego" of the cautious intermediary solves the minimization problem to guard against ambiguity concerns. I denote the solution to this minimization problem as, for $a \in k, h$,

$$\pi_{a,t}^w \doteq \pi_a^S(\theta_t^w);$$

$$\sigma_{a,t}^w \doteq \sigma_a^S(\theta_t^w),$$

where θ_t^w is the minimand, referred to as the worst-case scenario.

Given the worst-case return distribution, the intermediary then solves the maximization problem, and the solution satisfies

$$\underbrace{\frac{\pi_{k,t}^w}{\sigma_{k,t}^w}=\frac{\pi_{h,t}^w}{\sigma_{h,t}^w}}_{\text{Subjective Sharpe ratio}} = \gamma(\alpha_{k,t}\sigma_{k,t}^w+\alpha_{h,t}\sigma_{h,t}^w). \tag{9}$$

In words, the intermediary's subjective Sharpe ratios must increase to compensate for the subjective exposure to the aggregate shock.

Discussion on unobservability of housing demand elasticity ϕ: *The equation (2) shows that the demand elasticity ϕ governs the rental price of housing, which can be interpreted as the fundamental cash flow from holding housing capital. Gerardi, Foote, and Willen (2010) document substantial disagreement among economists about the causes of the rise in housing prices prior to the 2007–2009 financial crisis. Some attributed the increase to stronger fundamentals driven by optimistic expectations about future rental revenues, while others viewed the rapid appreciation as evidence of a bubble fueled by aggressive risk-taking and low interest rates associated with accommodative monetary policy. These conflicting interpretations highlight the uncertainty surrounding the true cash flows and demand conditions for housing capital by inferring from the observed housing prices.*

Discussion on unobservability of the long-run TFP growth rate g: *The long-run risk literature initiated by Bansal and Yaron (2004) identifies a persistent component of consumption growth to help explain the equity premium puzzle. Despite recent contributions by Bansal, Kiku, and Yaron (2016) and Schorfeide, Song, and Yaron (2018), accurately identifying and estimating these components using only aggregate consumption data remains difficult without relying on asset-pricing implications. This underscores the uncertainty surrounding long-run economic growth.*

Discussion on unobservability of aggregate intermediary capital $\mathcal{E}$: *It is now widely recognized that the intermediary sector was operating with much higher leverage than previously understood. This resulted from intermediaries' (banks') indirect exposure to subprime mortgages and mortgage-backed securities through guarantees on the liabilities of the shadow banking system, which held these assets on behalf of those banks. Given the complex network of financial linkages, there was likely substantial uncertainty about the true asset and liability positions of banks and, consequently, about their internal capital levels.*

2.1.5 Financial crisis

Following HK, financial crises are defined as states in which the equity issuance constraint binds for all intermediaries: $\mathcal{E}_t < (1-\lambda)W_t$. When this constraint binds, the aggregate share of risky asset holdings by intermediaries increases, while they borrow more in the risk-free asset market. The individuals' optimality conditions (9) indicate that the worst-case Sharpe ratio must rise to induce intermediaries to take on larger levered positions in risky assets.

The higher worst-case risk premium must be generated either through a higher worst-case Sharpe ratio $\pi_a^S(\theta^w)/\sigma_a^S(\theta^S)$ induced by changes in the admissible set of models (7), through an increase in PPI_a^S driven by a higher objective risk premium $\pi_a(\hat{\theta})$, or through both mechanisms. If the objective risk premium increases, asset prices must fall.

The binding equity issuance constraint makes asset-pricing dynamics nonlinear, in contrast to frictionless RBC models: negative aggregate shocks lead to sharper asset-price declines because intermediaries must take larger levered positions in risky assets and require greater compensation for bearing such exposures.

2.2 Equilibrium

This section presents the market-clearing conditions, the definition of equilibrium, and the boundary conditions. I then describe the set of admissible latent objects and models, drawing on comparative statics results.

2.2.1 Market clearing conditions

In the goods market, total output must equal consumption plus real investment. Using capital letters to denote aggregate variables,

$$Y_t = C_t^y + \Phi(i_t, A_t K_t). \tag{10}$$

Note that bankers do not consume and therefore do not appear in this market-clearing condition. Households receive all returns from investment.

The housing rental market clears according to

$$C_t^h = H \equiv 1. \tag{11}$$

The intermediary sector holds the entire stocks of capital and housing. In aggregate, the sector raises total equity financing of $E_t = \min \mathcal{E}_t, (1-\lambda)W_t$. The portfolio shares in capital and housing are identical across intermediaries and are denoted by α_t^k and α_t^h, respectively[9]. The total value of productive capital is $Q_t K_t$, and the total value of housing is P_t. Market clearing for capital and housing therefore requires

$$\alpha_{k,t} E_t = K_t Q_t \quad \text{and} \quad \alpha_{h,t} E_t = P_t. \tag{12}$$

These conditions determine the equilibrium values of the portfolio shares.

The aggregate financial wealth of the household sector equals the total value of the productive capital and housing stocks:

$$W_t = K_t q_t + P_t. \tag{13}$$

[9] In equilibrium, these shares exceed one because intermediaries borrow from households.

2.2.2 Markov equilibria

I consider Markov equilibria in the state variables A_tK_t and $\mathcal{E}_t$. The term A_tK_t scales the size of the economy, and $\mathcal{E}_t$ represents the intermediary sector's aggregate capital. HK use K_t and $\mathcal{E}_t$ as state variables, but the solution can be further simplified by scaling the economy by K_t. Define

$$e_t \doteq \frac{\mathcal{E}_t}{A_tK_t}.$$

The variable e_t captures the capital of the intermediary sector relative to the size of the overall economy. I then look for price functions of the form $P_t = p(e_t)A_tK_t$ and $Q_t = q(e_t)A_t$. The model is solved numerically as a function of e_t. Consequently, the equilibrium housing rental price takes the form $D_t = d(e_t)A_tK_t$, where the full expression for $d(e_t)$ is provided in Appendix A.

Given these scaling assumptions, the consistency conditions used in constructing the admissible set of models (6) can be rewritten as:

$$\begin{aligned} \underbrace{p_t}_{\text{Currently observed scaled price}} &= \underbrace{\widetilde{p}(\theta_t)}_{\text{Model-implied price}} \quad ; \\ \underbrace{q_t}_{\text{Currently observed scaled price}} &= \underbrace{\widetilde{q}(\theta_t)}_{\text{Model-implied price}} \quad ; \\ \underbrace{r_t}_{\text{Currently observed price}} &= \underbrace{\widetilde{r}(\theta_t)}_{\text{Model-implied price}} \quad , \end{aligned} \tag{14}$$

where the model-implied price functions are now scaled by the aggregate effective capital stock observed by intermediaries: $\widetilde{p}(\theta_t) = \widetilde{P}(\theta_t)/(A_tK_t)$ and $\widetilde{q}(\theta_t) = \widetilde{Q}(\theta_t)K_t/(A_tK_t)$. With a slight abuse of notation, I denote the admissible set of latent objects in period t by $\Theta(p_t, q_t, r_t)$, and the corresponding set of admissible return distributions by

$$\Xi(p_t, q_t, r_t) = \left\{(\pi_a^S, \sigma_a^S)_{a\in\{k,h\}} : \exists\theta \in \Theta(p_t, q_t, r_t) \quad \text{s.t} \quad \pi_a^S = \pi_a^S(\theta) \quad \text{and} \quad \sigma_a^S = \sigma_a^S(\theta)\right\}.$$

First, I define the *temporary* Markov equilibrium in a model with a particular baseline parameter values of $(\hat{g}, \hat{\phi})$, given the set of price functions $\{\tilde{p}(e; g, \phi), \tilde{q}(e; g, \phi), \tilde{r}(e; g, \phi)\}$ that individual financial intermediaries entertain in their set constructions $\widetilde{\Xi}(p, q, r)$ (see equation (6)) and the admissible set of risky asset return distributions $\{\widetilde{\pi}_a, \widetilde{\sigma}_a\}_{a\in\{k,h\}}$.

Definition 1. *(Temporary Markov Equilbrium) The temporary Markov equilibrium in the model with baseline parameter values $(\hat{g}, \hat{\phi})$, a prespecified set of price functions $(\widetilde{p}, \widetilde{q}, \widetilde{r})$ that individual intermediaries entertain in their set construction, and the set of risky asset return distributions $\{\widetilde{\pi_a}, \widetilde{\sigma}_a\}_{a \in \{k,h\}}$ that they use to form beliefs over risky asset returns is:*

1. *a set of price functions $\{q(e; \hat{g}, \hat{\phi}), p(e; \hat{g}, \hat{\phi}), r(e; \hat{g}, \hat{\phi}), d(e; \hat{g}, \hat{\phi})\}$;*
2. *household decisions $\{c^y(e_t), c^h(e_t), i(e_t)\}$;*
3. *intermediary decisions $\{\alpha_k(e_t), \alpha_h(e_t)\}$;*
4. *intermediaries' beliefs*
   $\{\Theta(p_t, q_t, r_t), \Xi(p_t, q_t, r_t), \theta^w(p_t, q_t, r_t), \pi_k^w(p_t, q_t, r_t), \sigma_k^w(p_t, q_t, r_t), \pi_h^w(p_t, q_t, r_t), \sigma_h^w(p_t, q_t, r_t)\}$;

such that:

1. *prices are given by $Q_t \equiv q(e_t; \hat{g}, \hat{\phi})K_t$, $P_t \equiv p(e_t; \hat{g}, \hat{\phi})A_t K_t$, $r_t \equiv r(e_t; \hat{g}, \hat{\phi})$, and $D_t \equiv d(e_t)A_t K_t$;*
2. *given prices, household decisions expressed by $C_t^y \equiv c^y(e_t)A_t K_t$, $C_t^h \equiv c^h(e_t)$, and $i_t \equiv i(e_t)$ satisfy their optimality conditions (1), (2), and (4);*
3. *given prices and the prespecified price functions, intermediaries' beliefs satisfy the consistency conditions (14), and solve the minimization problem in (8);*
4. *given prices and intermediaries' beliefs, intermediary decisions satisfy the optimality conditions in (9);*
5. *households' and intermediaries' decisions satisfy the market-clearing conditions (10), (11), and (12);*
6. *the equity issuance constraint (3) is satisfied;*
7. *the resource constraint (13) holds.*

The rational expectations equilibrium corresponds to the special case of a temporary Markov equilibrium in which the set of models consists solely of the true return distributions under rational expectations equilibrim and is therefore a singleton.

Proposition 1. *If the set of models consists only of the true risk premium and return volatility functions under rational expectations, then the equilibrium becomes a rational expectations equilibrium.*

Proof. By assumption, the alter ego of each intermediary selects the true return distribution as the worst-case model: $\pi_a^w \equiv \pi_a^{REE} + PPI_a^S$ and $\sigma_a^w \equiv \sigma_a^{REE}$ for $a \in \{k, h\}$. Consequently, the worst-case return distribution coincides with that under the rational expectations equilibrium: $\pi_a^w = \pi_a^{REE}$. Since the price functions from the rational expectations equilibrium satisfy the remaining equilibrium conditions, the equilibrium in this special case reduces to the rational expectations equilibrium. □

The Markov full equilibria defined below discipline the prespecified price functions across alternative equilibria.

Definition 2. *(Markov Full Equilbria) The set of Markov full equilibria in models parameterized by alternative values of (g, ϕ) satisfies the following conditions:*

1. *Each equilibrium is a temporary Markov equilibrium, and let $\{p(e; g, \phi), q(e; g, \phi), r(e; g, \phi)\}$ be the corresponding price functions in the equilibrium parameterized by (g, ϕ);*
2. *The price functions and the distributions of risky asset returns that intermediaries entertain when constructing the admissible set of models must coincide with the equilibrium price functions of the alternative economies:*

$$(\widetilde{p}, \widetilde{q}, \widetilde{r}) = (p, q, r);$$

$$\{\widetilde{\pi_a}, \widetilde{\sigma}_a\}_{a \in \{k,h\}} = \{\pi_a, \sigma_a\}.$$

The notion of Markov full equilibria is similar in spirit to the rational expectations equilibrium. In the rational expectations framework, decision makers' beliefs must coincide with the true law of motion in the economy. In contrast, Markov full equilibria impose a looser form of consistency: the perceived price functions need not match the actual price functions in the baseline economy, but must instead be consistent with those of alternative economies parameterized by different values of (g, ϕ).

2.2.3 Boundary conditions

The Markov full equilibria conditions reduce to a system of second-order ordinary differential equations for (p, q). I adopt the boundary conditions for the price functions (p, q) in each equilibrium within the Markov full equilibria, following He and Krishnamurthy (2019). A full discussion is provided in Appendix A.

At the upper boundary, I impose $p' = q' = 0$ as $e \to \infty$. These boundary conditions follow from the assumption that when the scaled net worth of the financial sector becomes sufficiently large, the equity issuance constraint will not bind in the near future, implying that asset prices become independent of e.

2.2.4 Observationally equivalent latent objects

In this section, I describe how alternative state e and parameter values (g, ϕ) generates the observationally equivalent implications for the currency asset prices (p, q, r), relying on analytical characterizations of the equilibrium.

First of all, the Campbell-Shiller decomposition approximately implies that the risky asset prices are the expected discounted sum of future cash flows. Since the risk-free rate is observable by each intermediary, the combinations of (e, g, ϕ) in the admissible set $\Theta(p, q, r)$ preserve the observationally equivalent implications for (p, q) by providing the offsetting implications for the cash flows and risk premia.

To illustrate this, capital and housing deliver cash flows through the dividend $\overline{A}A_t$ and the housing rental price D_t, respectively. The dividend grows at rate g and is exposed to the aggregate shock dZ_t. Since the capital price is given by $Q_t \equiv A_t q_t$ and A_t is observable, q_t increases with the growth rate g. Appendix A shows that the housing rental price can be expressed using the households' intratemporal optimality condition (2) combined with market-clearing conditions as:

$$D_t = \frac{\phi}{1-\phi} A_t K_t \left[\overline{A} - i_t - \frac{\kappa}{2}(i_t - \delta)^2 \right],$$

indicating that the housing rental price is increasing in the intratemporal elasticity of substitution ϕ. Since the housing price is given by $P_t \equiv p_t A_t K_t$ and $A_t K_t$ is observable, p_t is increasing in housing demand as represented by ϕ.

On the other hand, risky asset prices are decreasing in the scaled aggregate intermediary capital e because the risk premium declines as e increases. To illustrate this, Appendix A derives the following expression for the objective risk premium from the first-order conditions (9): for an asset $a \in \{k, h\}$,

$$\pi_a = \underbrace{\gamma(\alpha_k \sigma_k^w + \alpha_h \sigma_h^w)\sigma_a^w}_{\text{Subjective risk exposure}=\pi_a^w} + \underbrace{PPI_a}_{\text{Compensation for ambiguity}}, \tag{15}$$

where PPI_a is the objective compensation for ambiguity:

$$PPI_a \doteq \pi_a^{REE}(\hat{\theta}) - \pi_a(\theta^w).$$

When e is small, the equity issuance constraint (3), reexpressed as $\frac{E_t}{A_t K_t} = \min\{e_t, (1-\lambda)(p_t + q_t)\}$, binds. Consequently, by the market-clearing conditions (12), the portfolio exposure of the intermediary sector to risky assets becomes larger:

$$\alpha_t^k = \frac{A_t K_t}{E_t} q_t \quad \& \quad \alpha_t^h = \frac{A_t K_t}{E_t} p_t.$$

Hence, the subjective exposure to aggregate shocks increases, and the objective risk premium must rise accordingly.

Finally, it can be shown that the risk-free rate r_t is increasing in both the growth rate g and the aggregate intermediary capital e_t. Appendix A derives the following expression for the risk-free rate from the household Euler equation; suppressing the time subscript,

$$r = \rho + \xi \overbrace{\left[\underbrace{i - \delta + g}_{\text{Frictionless}} - \underbrace{\frac{q^2}{\kappa c}\left(\mu_q + \frac{\sigma_q^2}{2} + \sigma_q \sigma\right)}_{\text{Financial friction}} \right]}^{\text{Household expected consumption growth}} - \frac{\xi(\xi+1)}{2} \overbrace{\left[\underbrace{\sigma}_{\text{Frictionless}} - \underbrace{\frac{q^2}{\kappa c}\sigma_q}_{\text{Financial friction}} \right]^2}^{\text{Household consumption variance}},$$

where (μ_q, σ_q) are the drift and volatility terms of dq/q that must be solved for in equilibrium. These terms vanish as the financial friction disappears ($e \to \infty$).

The risk-free rate is increasing in the TFP growth rate g, which raises expected consumption growth. Aggregate intermediary net worth e raises the risk-free rate through its effect on the investment rate i in the frictionless component of household expected consumption growth. As shown earlier, the capital price is increasing in e because a higher e lowers the risk premium and therefore the discount rate, which in turn raises q. The higher capital price increases the investment rate in (1). Higher investment increases future output and thus raises the expected consumption growth rate.

These comparative statics show that combinations of lower cash flows (g, ϕ) and higher aggregate intermediary net worth e, or vice versa, can generate observationally equivalent implications for the contemporaneous asset prices (p, q, r). From the perspective of individual intermediaries, such uncertainty arises naturally because it is difficult to observe or estimate the latent objects involved, such as real-time granular balance sheet information for the entire intermediary sector.

As the numerical solutions later demonstrate, the worst-case scenario for individual intermediaries corresponds to a situation with lower cash flows and higher aggregate intermediary capital. The latter suppresses the risk premium (15) and, consequently, the expected excess returns on risky assets.

In the remainder of the paper, I present numerically solved global solutions for the calibrated model, which allow me to study the joint dynamics of beliefs, asset prices, and the macroeconomy during and outside of crisis episodes. The calibrated model also delivers novel testable implications for survey expectations evidence that I test in the data while preserving the success of matching the moments of macroeconomic and financial variables.

3 Global solution

This section presents the calibration of the model, the numerical solution strategy for solving the Markov full equilibria, and the resulting model solution. I then compare the model's predictions to unconditional and conditional moments during financial distress (defined later) as well as to the dynamics observed during crisis episodes.

3.1 Calibration

The HK model is a standard RBC framework augmented with a financial intermediary sector. The economy behaves like a standard RBC model when e_t is far from the constraint, but intermediary frictions become quantitatively important near the crisis region. Following HK, I define e_{distress} as the 33rd percentile of e_t in the model's stationary distribution. The value e_{distress} separates "normal" periods from periods of financial distress.

3.1.1 RBC parameters

Discount rate ρ, depreciation rate δ, and adjustment cost κ are standard RBC parameters.

The parameter σ governs the volatility of capital quality shocks. As in HK, I set $\sigma = 3\%$. HK report that from 1975 to 2015, the volatility of investment growth in non-distress periods was 6.9%, and the volatility of consumption growth was 1.47%. In the model, $\sigma = 3\%$ generates investment growth volatility of 4.72% and consumption growth volatility of 2.22% in nondistress periods. Thus, the model overpredicts consumption volatility and underpredicts investment volatility.

The parameter $\overline{A}$ is calibrated to match the average investment–output ratio. In the model, $\overline{A}$ generates an 8.3% ratio, which is typical of values used in the literature. I set the long-run TFP growth rate $g = 0$, consistent with HK, who assume no long-run TFP growth.

3.1.2 Intermediation parameters

Parameter γ represents bankers' risk aversion. As in HK, I set $\gamma = 2$, which generates an average realized Sharpe ratio of 46%. This is consistent with He, Kelly, and Manela (2017), who estimate an average Sharpe ratio of 48% for assets intermediated by the financial sector.

Intermediary leverage is governed by λ. Since intermediaries hold assets of $P_t + Q_t K_t = W_t$ and outside equity of E_t, equation (3) implies an aggregate leverage value of $\frac{W_t}{E_t} = \frac{1}{1-\lambda}$ in non-crisis states. Following HK, I set $\lambda = 0.75$, which generates a leverage ratio of 4 when the constraint does not bind.

3.1.3 Crisis parameters

I set the intermediaries' exit rate η equal to 15% to match the 3% historical incidence of financial crises in the United States over the past 100 years. The choice of η shifts the mean of the stationary distribution of e. With this value, the model generates a crisis probability of 3.4%.

Parameters $\underline{e}$ and β determine the lower boundary condition, represented by $d\psi_t$ in equation (5). The parameter $\underline{e}$ is the minimum level of capital capacity at which government intervention occurs, and β is the cost of the intervention required to restore intermediary capital. Following HK, I set $\underline{e}$ such that the objective Sharpe ratio at $\underline{e}$ is 6.5. The parameter β determines the slope of the house price P_t at $\underline{e}$, which in turn affects the volatility of P_t in distress states. As in HK, I set $\beta = 2.8$. HK report that the empirical volatility of land price growth from 1975 to 2015 was 11.9%. In the model, $\beta = 2.8$ generates a volatility of 11.5%.

3.1.4 Household parameters

The parameter ϕ affects housing demand and thus the aggregate value of land relative to the value of capital. This determines the rental rate D_t. Since P_t is the discounted value of these rental payments, ϕ directly influences the housing price. I set $\phi = 0.6$ to target the measured housing-to-wealth ratio reported in HK (45.5%). The model then generates a ratio of 46.4%.

The relative risk aversion parameter ξ, or the inverse of the EIS, governs the responsiveness of households' savings in risk-free assets to changes in expected future consumption growth. As in HK, I target the empirical volatility of risk-free rates (1%). Setting $\xi = 0.13$ generates a volatility of 0.5% in the model.

3.1.5 Unrestricted set of latent parameters

I set the minimum and maximum values of the parameters (g, ϕ) in the alternative economies such that these choices do not affect the equilibrium prices in the baseline economy, where (g, ϕ) are fixed at the values reported in Table 1. All other parameter values are held constant across economies. I set the minimum and maximum values of g to -1.5% and 0.6%, respectively, and the minimum and maximum values of ϕ to 0.45 and 0.65, respectively.

3.2 Numerical solution strategy

The equilibrium concept of Markov full equilibria in Definition 2 requires jointly solving all of the alternative equilibria parameterized by different values of (g, ϕ). Each equilibrium in this set affects the others, since the resulting price functions alter the objects that intermediaries use to discipline the admissible sets of latent objects and risky return distributions used to form expectations. Implementation details of the numerical algorithms are provided in Appendix B.

Parameter		Choice	Target
Panel A: Intermediation Parameters			
γ	Banker risk aversion	2^*	Mean Sharpe ratio
λ	Debt ratio	0.75^*	Intermediary leverage
η	Bank exit rate	0.15^*	Probability of crisis
$\underline{e}$	Lower entry barrier	0.080	Max Sharpe ratio
β	Entry cost	2.8^*	Land price volatility
g	Long-run TFP growth rate	0%	HK economy
Panel B: Technology parameters			
σ	Capital stock volatility	$3\%^*$	C and I volatility
δ	Depreciation rate	$10\%^*$	Literature
κ	Adjustment cost	3^*	Literature
$\overline{A}$	Scaling constant of output	0.133^*	Consumption-output ratio
Panel C: Household preference parameters			
ρ	Time discount rate	$2\%^*$	Literature
ξ	1/EIS	0.13^*	Risk-free rate volatility
ϕ	Housing expenditure share	0.6^*	Housing-wealth ratio
Panel D: Unrestricted set of latent parameters			
$g_{\min}$	Minimum TFP growth rate	-1.5%	Θ never binds
$g_{\max}$	Maximum TFP growth rate	0.6%	Θ never binds
$\phi_{\min}$	Minimum housing expenditure share	0.45	Θ never binds
$\phi_{\max}$	Maximum housing expenditure share	0.65	Θ never binds
Panel E: Unconditional simulated moments			
Mean $\left(\frac{\text{investment}}{\text{capital}}\right)$			8.3%
Mean (Realized Sharpe ratio)			46%
Probability of crisis			3.4%
Volatility(land price growth)			11.5%
Volatility(risk-free interest rate)			0.5%
Panel F: Non-distress simulated moments			
Volatility(investment growth)			4.72%
Volatility(consumption growth)			2.22%
Volatility(output growth)			3%
Mean $\left(\frac{\text{Housing wealth}}{\text{Total wealth}}\right)$			46.4%

Table 1: Baseline model calibration

Notes: The superscript * represents the same parameter values as in He and Krishnamurthy (2019).

I take the price functions and risky return distributions from the alternative rational expectations equilibria as the initial guess for the Markov full equilibria. Each intermediary is then assigned these equilibrium objects and constructs the admissible set of future return distributions for each model parameterized by a specific combination of (g, ϕ). Given these beliefs, I solve for a new equilibrium and obtain updated price functions and return distributions. I then take this new set of equilibrium functions as the next guess for the Markov full equilibria and repeat the procedure to find the fixed point until the new equilibrium price functions become numerically indistinguishable from the previous ones.

3.3 Model solution

This section describes the equilibrium in the baseline economy and then shows how alternative parameter values affect the equilibrium price functions in the alternative economies. Building on these insights, I present how the admissible sets and the worst-case beliefs vary across the state space in the baseline economy.

3.3.1 Nonlinear price functions in the baseline economy

Figures 1 and 2 plot the price functions for the baseline model. The horizontal axis in all panels is the scaled aggregate intermediary capital $e \equiv \mathcal{E}/(AK)$. I also plot the stationary distributions in the panels[10]. Figure 1 highlights the strong nonlinearity of the price functions: the slopes become steeper as aggregate intermediary capital approaches the distressed threshold e_{distress}. The upper boundary condition implies that the slopes approach zero as e becomes large, indicating that equilibrium prices no longer vary with e once intermediary capital is abundant and the economy behaves similarly to a standard RBC model.

Figure 2 zooms in on the region of the state space where the stationary distribution has positive mass. Once aggregate intermediary capital e reaches the constrained threshold e_{crisis}, risky asset prices decline nonlinearly because all intermediaries become more highly leveraged due to the binding equity issuance constraint. This increases their required subjective Sharpe ratio, which in turn raises

As documented in HK, asset prices begin to decline nonlinearly even before the equity issuance constraint actually binds. This occurs because the probability of hitting the constraint threshold rises, increasing future discount rates.

Figure 3 shows the consumption rate scaled by the effective capital stock AK and the investment rate. The nonlinearity of the equilibrium capital price q generates corresponding nonlinearities in these equilibrium quantities, as in HK. The investment rate declines as aggregate intermediary capital approaches distress states because the rate is increasing in the capital price. Households then substitute away from investment toward consumption as e falls, producing the downward-sloping curve for the consumption rate c^y.

[10]The stationary distribution is computed as the limiting solution of the Kolmogorov Forward equation. See Appendix A for details.

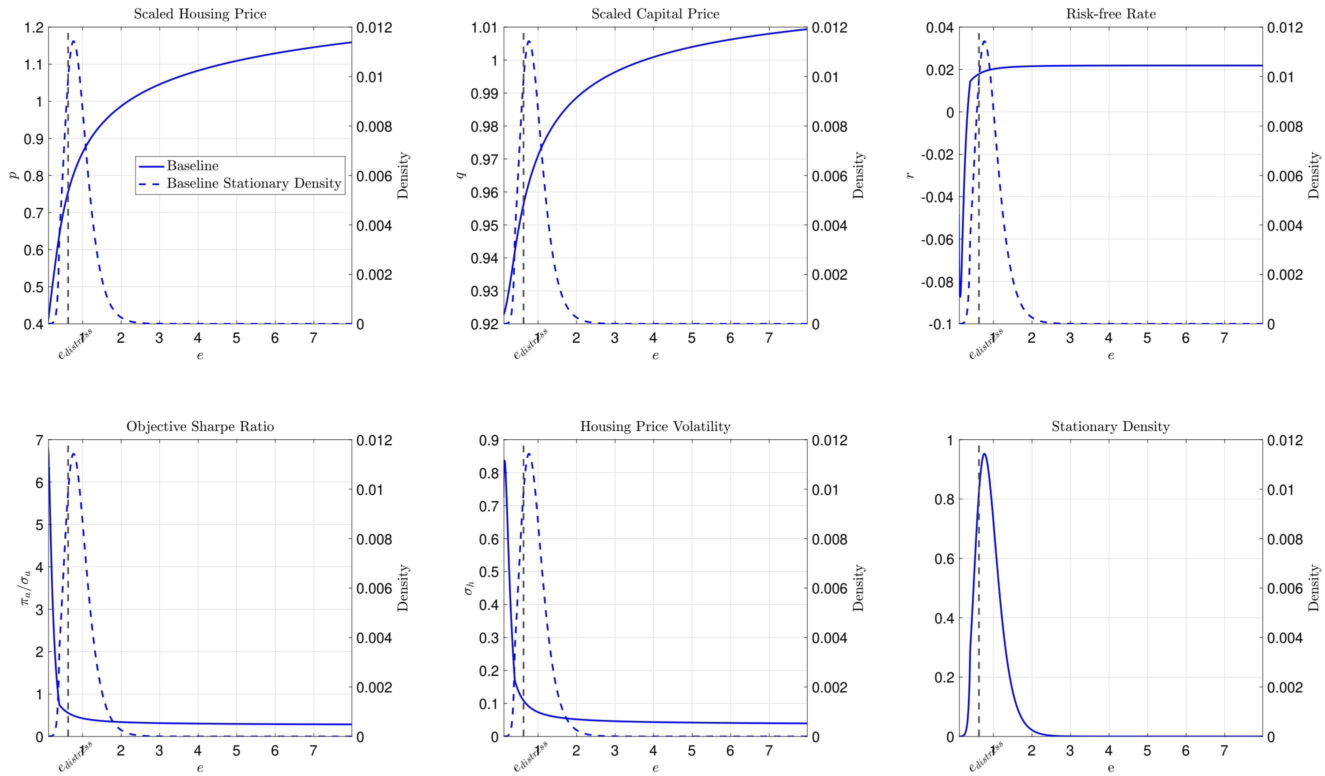

Figure 1: Baseline model solutions: prices

3.3.2 Price functions in alternative economies

I revisit the analytical comparative statics in Section 2.2.4 using the numerical solution to the Markov full equilibrium. Figure 4 plots the price functions (p, q, r) from various alternative economies that intermediaries use to construct the admissible sets of parameters and states. The upper panels plot the price functions parameterized by the same value of $\phi = 0.6$ as in the baseline economy but with alternative values of long-run TFP growth g. Functions with higher g are plotted in darker gray. Similarly, the lower panels plot price functions parameterized by the same value of g as in the baseline economy but with alternative values of the housing demand elasticity ϕ.

The upper panels show that the scaled housing price functions p shift downward as long-run TFP growth increases. Higher TFP growth raises future consumption growth and thus increases the risk-free interest rate; the resulting higher discount rate lowers the present value of housing. By contrast, the capital price rises despite the higher discount rate, because the dividend growth

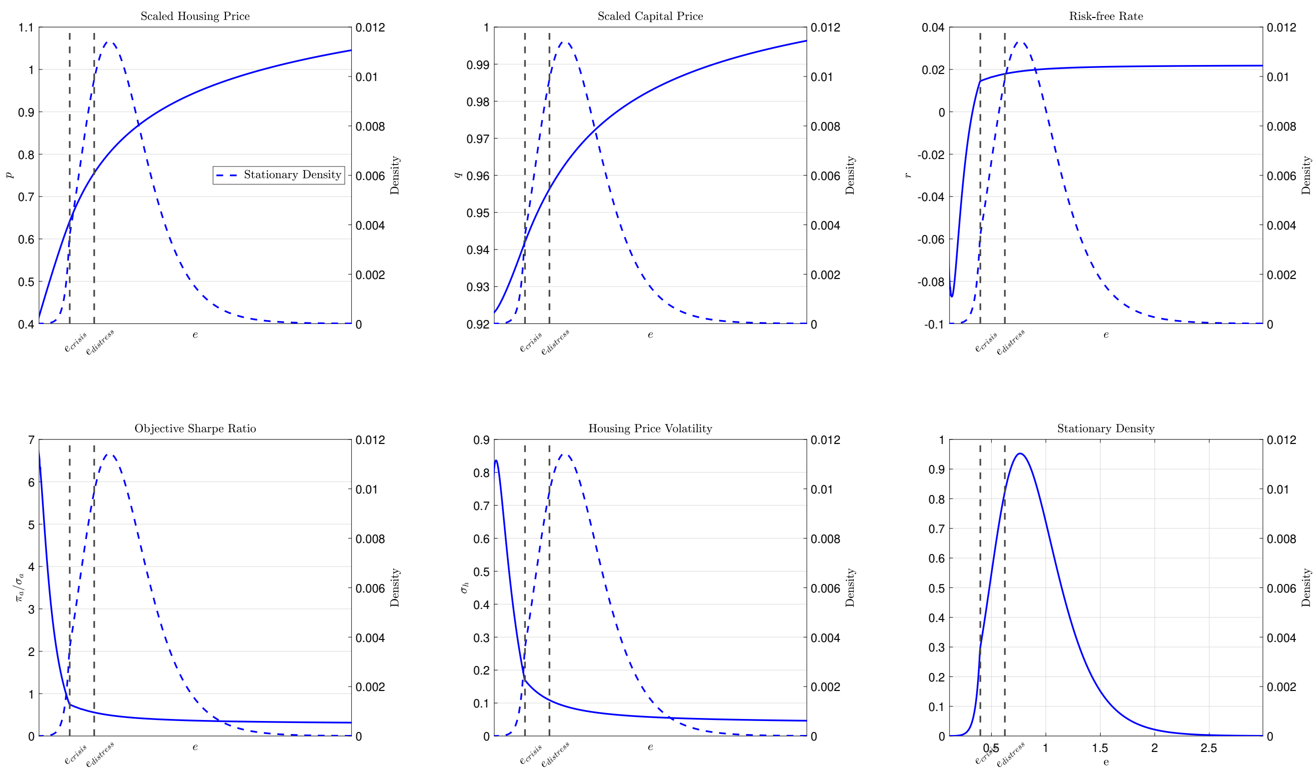

Figure 2: Baseline model solutions around distress threshold: prices

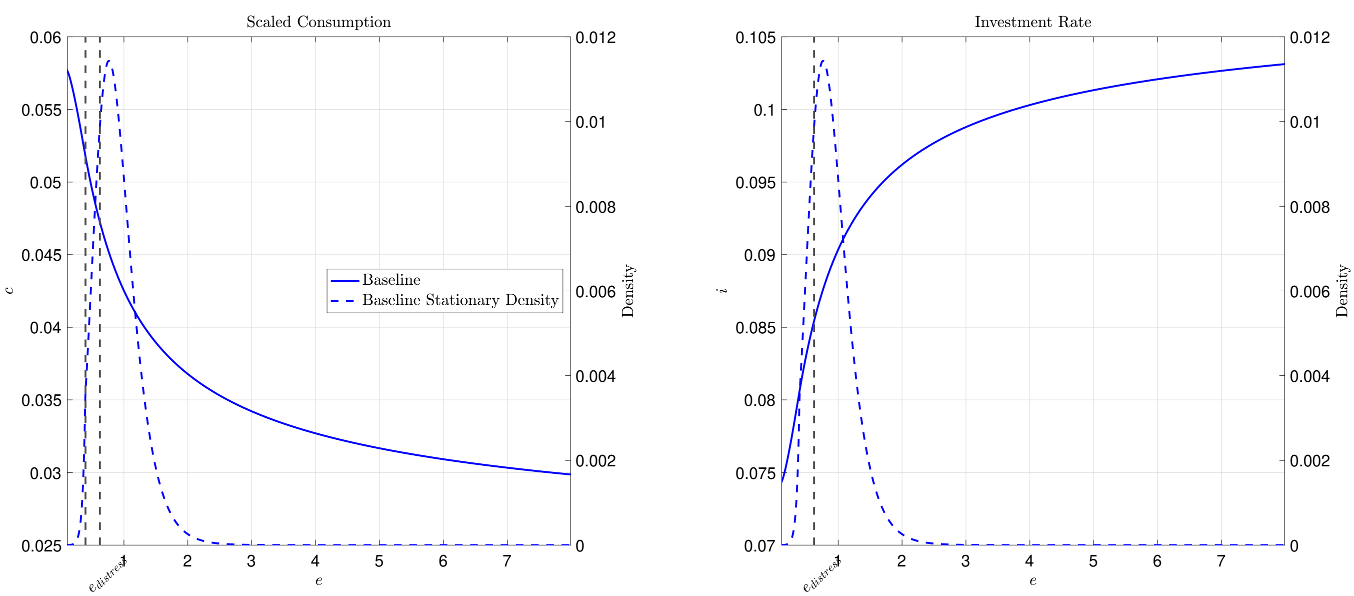

Figure 3: Baseline model solutions: quantities

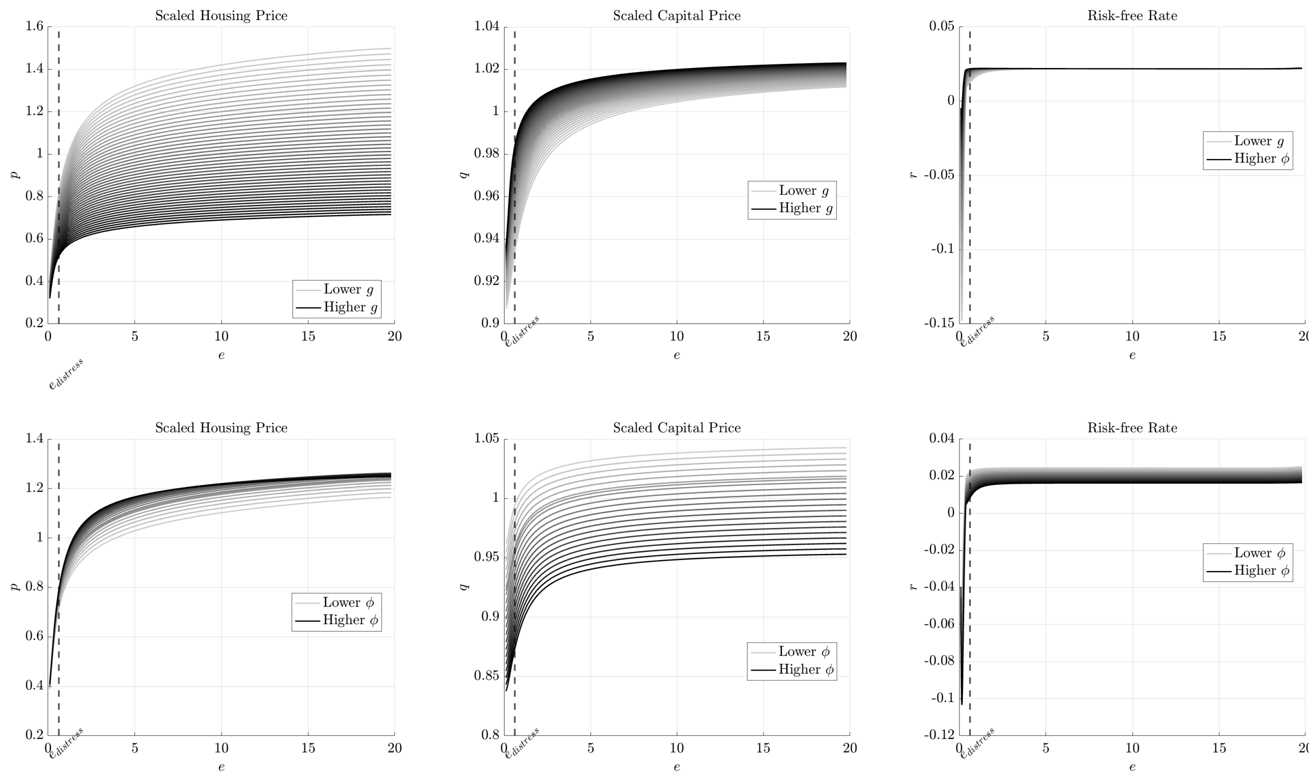

Figure 4: Alternative price functions

rate is higher under stronger TFP growth.

The lower panels show that the housing price increases with the housing demand elasticity ϕ due to higher rental prices. In contrast, the capital price declines because higher housing demand reduces future demand for nondurable goods, leading to lower dividends. The lower capital price also induces a lower investment rate and weaker consumption growth, depressing the risk-free rate.

These results illustrate that intermediaries face identification challenges when attempting to distinguish between alternative scenarios based solely on the observed asset prices (p, q, r), especially as actual aggregate intermediary capital approaches distress states. In the first scenario, low aggregate intermediary capital leads to low risky asset prices (p, q) and a low risk-free rate r. In an alternative scenario, similarly low asset prices arise because long-run TFP growth g and housing demand ϕ are weaker, even though aggregate intermediary capital e remains far from distress.

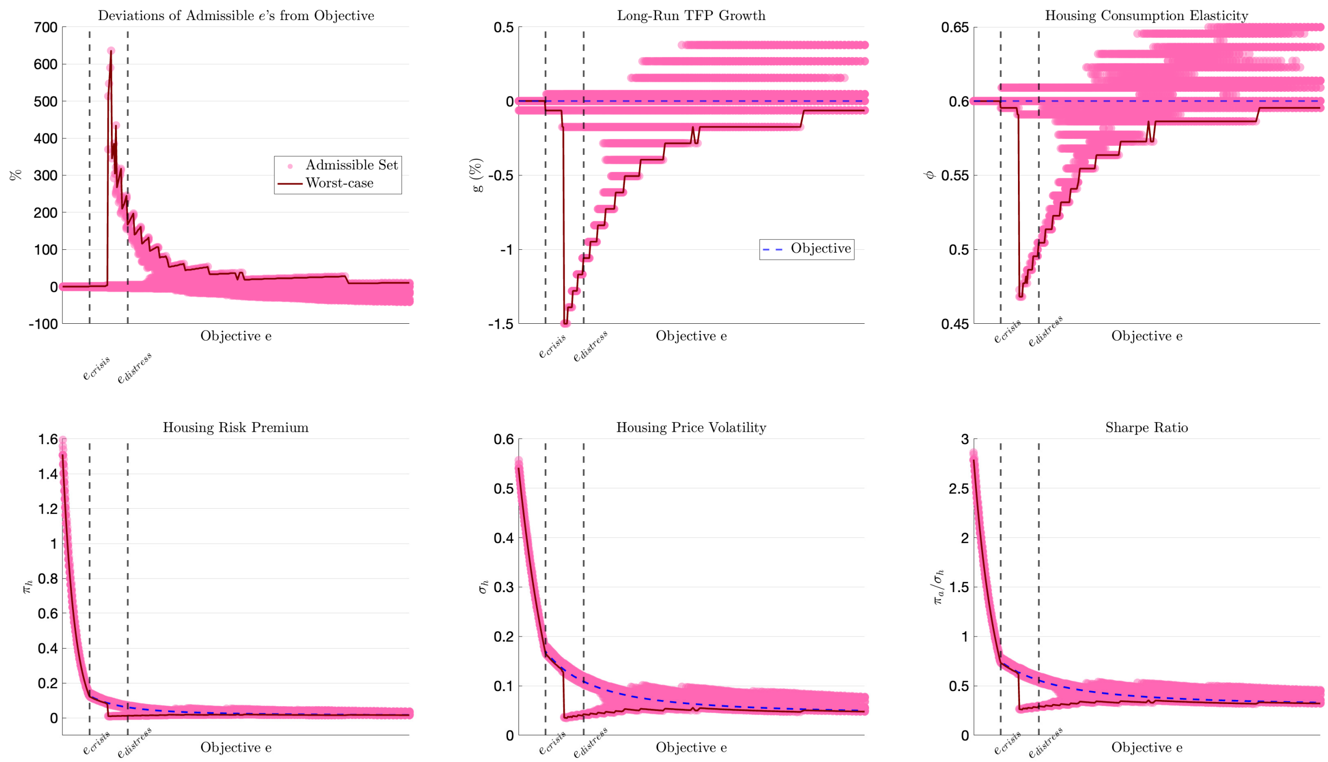

Figure 5: Set of admissible models and worst-case belief

Notes: The darker red lines represent the worst-case latent objects and the moments of return distributions. The dashed lines in blue show the objective counter parts. The shaded pink areas are the admissible sets.

3.3.3 Admissible set of models and worst-case beliefs

Figure 5 plots the admissible set Θ of latent objects (e, g, ϕ) in the upper panels and the implied set of moments of the infinitesimal housing return distributions in the lower panels, shown along the state space of the actual (objective) e in shaded pink areas. Note that individual intermediaries cannot directly observe the objective e.

When the objective aggregate intermediary capital e is far from distress states, intermediaries cannot distinguish among three alternative types of scenarios: (i) the objective scenario, with a higher e and baseline values of (g, ϕ); (ii) an alternative scenario, with a lower e but higher values of (g, ϕ); and (iii) another alternative scenario, with a much higher e and lower values of (g, ϕ). In other words, intermediaries cannot confidently infer why the observed risky asset prices and the risk-free rate are high. These observations may arise from strong economic growth and housing demand (ii), from strong aggregate intermediary risk-bearing capacity (iii), or from a combination of both factors (i).

The bottom panels visualize the set of admissible distributions of housing returns. Alternative scenario (ii) implies a higher risk premium and higher return volatility than the actual values shown by the dashed blue lines, because aggregate intermediaries could be constrained and asset prices should be underpriced relative to the frictionless benchmark to which the economy eventually converges. Moreover, return volatility should be high because risky asset prices would decline nonlinearly in the future once an adverse shock occurs. Consequently, this alternative scenario may generate a higher Sharpe ratio, which intermediaries might interpret as offering more attractive investment opportunities.

In contrast, alternative scenario (iii) implies a lower risk premium and lower return volatility, leading to a quantitatively lower Sharpe ratio. Under this scenario, intermediaries view current asset prices as overpriced and consider the scenario to offer less attractive investment opportunities.

As the objective e approaches the distress threshold e_{distress}, scenario (ii) becomes inconsistent, even from the perspective of individual intermediaries who cannot directly observe the objective e, with the lower observed asset prices and with the economic structure they understand. Alternative values of e that are lower than the objective level would imply counterfactually low risky asset prices and a low risk-free rate because of the nonlinearities in the price functions (see Figure 4). These nonlinear declines cannot be offset by high alternative values of (g, ϕ) in a way that would keep the model-implied prices consistent with the observed asset price levels.

By contrast, alternative scenario (iii) can remain admissible, featuring higher aggregate intermediary capital e and lower TFP growth g and housing demand ϕ. In this case, the nonlinearities are less severe when the alternative e exceeds the objective e, making the resulting asset prices more consistent with observed values. Accordingly, the lower panels show that the alternative return distributions in scenario (iii) feature a lower risk premium, lower return volatility, and a lower Sharpe ratio than their objective counterparts.

Given the set of return distributions in those states, the worst-case scenario corresponds to the less advantageous investment opportunities embodied in (iii), as intermediaries guard against the adverse possibility of overpricing. The moments of the worst-case distributions, shown in darker red, imply a lower risk premium and lower return volatility.

Finally, even alternative scenario (iii) becomes inconsistent as aggregate intermediary capital e approaches the constrained threshold e_{crisis}. Extremely high values of e cannot raise asset prices or the risk-free rate in the alternative economies, because such economies are far from the constraint threshold and behave similarly to a standard RBC model (see Figure 4). In this region, the effects of lower alternative values of (g, ϕ) on model-implied risky asset prices and the risk-free rate cannot be offset by higher values of e.

At this point, the overoptimism regarding aggregate intermediary capital vanishes, and intermediaries infer that the entire intermediary sector must be constrained and that asset prices are underpriced relative to the frictionless benchmark. Consequently, the discrepancy between the worst-case and objective return distributions disappears as the economy approaches the crisis region.

3.3.4 Risk premium decompositions

As shown in equation (15) in Section 2.2.4, the model features two distinct sources of the objective risk premium in equilibrium. The first source is compensation for exposure to aggregate shocks through risky asset holdings. The second source is compensation for exposure to uncertainty arising from endogenous ambiguity. In the rational expectations equilibrium, as in HK, the first source accounts for the entire objective (and subjective) risk premium.

In contrast, the model with endogenous ambiguity shows that this second, novel source contributes approximately 33.2% of the total objective risk premium unconditionally. Conditional on nondistress (distress) states, it accounts for 32.2% (35.4%), respectively. In distress states, the source of the risk premium switches around a threshold value of e, where the subjective risk premium jumps to the level of the objective risk premium. When the objective state is above this threshold, the main source of the objective risk premium is compensation for endogenous ambiguity, because subjective exposure to aggregate shocks is perceived to be small. When the objective state e falls below the threshold, compensation for the larger exposure to aggregate shocks becomes the primary source of the risk premium, since subjective exposures are correctly perceived to be large. Balancing these forces, the contribution of endogenous ambiguity is moderately larger in distress states than in nondistress states.

In Section 3.5, I will examine the sources of heightened risk premium dynamics during the

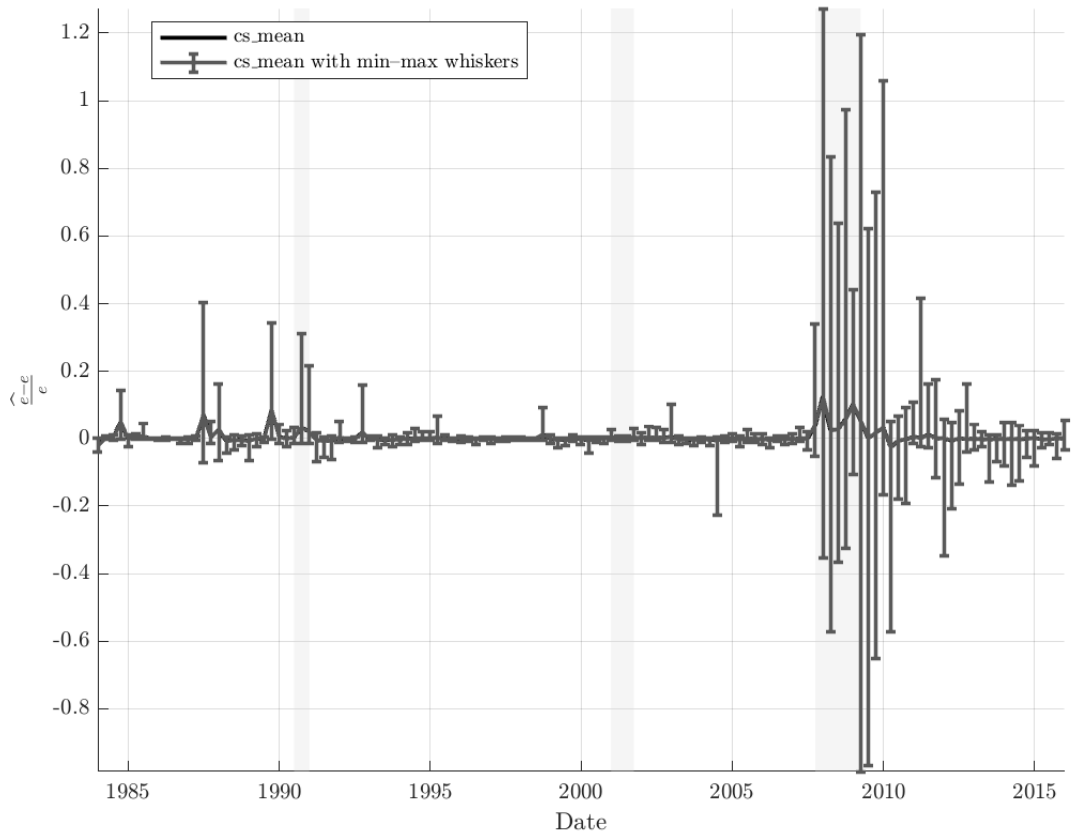

Figure 6: Time series of estimation errors as proxy for $(\widehat{e}_t - e_t)/e_t$

Notes: The black line shows the time-series of cross-sectional means of $(\widehat{e}_t - e_t)/e_t$, while the whiskers plot the minimum and maximum values of cross-sectional values. The shaded areas correspond to the NBER recession dates.

2007–2009 financial crisis through the lens of this decomposition and show that compensation for ambiguity regarding capital inside the intermediary sector accounts for the bulk of the elevated risk premium, in contrast to the predictions from the rational expectations equilibrium.

3.4 Empirical implications of the model

The model produces distinct implications for subjective belief dynamics relative to the rational expectations equilibrium, while preserving the nonlinear behavior of macroeconomic quantities and asset prices generated by the rational expectations equilibrium in HK. In this section, I relate the theoretical predictions for belief dynamics—including the admissible set of latent objects, the admissible return distributions, and the worst-case beliefs—to survey expectations evidence. In particular, I examine how well the model can quantitatively match the novel empirical properties of contemporaneous estimates by financial analysts of the capital positions of large intermediaries, as well as key anomalous empirical facts, including the sources of asset price fluctuations documented in De la O and Myers (2021) and the acyclicality of the subjective risk premium with respect to objective return volatility documented by Nagel and Xu (2023).

3.4.1 Data

I compute an empirical proxy for the set of admissible values of aggregate intermediary capital e by using the detailed files of the Thomson Reuters I/B/E/S Estimates Database and collecting individual financial analysts' forecasts of earnings per share (EPS) for major financial intermediaries. In this paper, I focus on the primary dealers of the Federal Reserve Bank of New York, following the intermediary asset pricing literature such as He, Kelly, and Manela (2017). I/B/E/S is a comprehensive forecast database containing analyst estimates for more than 20 forecast measures of U.S. publicly traded firms, including EPS, with coverage beginning in 1976[11].

The detailed file contains individual analysts' i forecast data ($\widehat{\text{Earning}}^{i}_{j,t}$ per share) as well as the realized values of the forecasted variables ($\text{Earning}_{j,t}$ per share) for each company j. I use these values to compute contemporaneous within-quarter estimation errors for primary dealers and convert them into a measure of earnings forecast errors per equity, $(\widehat{\text{Earning}}^{i}_{j,t} - \text{Earning}_{j,t})/\mathcal{E}_{j,t}$, by dividing the EPS values by each firm's equity $\mathcal{E}_{j,t}$ from Compustat.

I then use these measures to construct a proxy for the deviations of contemporaneous forecasts of e from their objective (realized) values. I postulate that current intermediary capital $\mathcal{E}_{j,t}$ can be proxied by $\mathcal{E}_{j,t} = \mathcal{E}_{j,t-1} + \text{Earning}_{j,t}$. Notice that contemporaneous earnings cannot be observed within the same quarter and are released only in the next quarter $t+1$. Hence, current-quarter earnings are unobservable. Using this relation, I construct the percentage deviations of analysts'

[11] Importantly, the forecasts compiled by Thomson Reuters come from a large number of brokerage and independent analysts who follow firms as part of their research activities. Each forecast is attributed to a specific analyst or brokerage, so the estimates are not anonymous. This transparency gives analysts strong incentives to report their expectations accurately. Prior work shows that forecast accuracy plays a significant role in analysts' career outcomes, including tenure and compensation (Mikhail, Walther, and Willis (1999); Cooper and Lewis (2001)). In addition, studies using the I/B/E/S Estimates Database document that financial institutions' trading behavior aligns with their own analysts' forecasts and recommendations, further indicating that the reported forecasts truly reflect the beliefs of the analysts and their firms (Bradshaw (2004); Chan, Chang, and Wang (2009)).

estimates $\widehat{e}_t$ from the realized value $e_{j,t}$ as

$$\frac{\widehat{e}^{i}_{j,t} - e_t}{e_t} = \frac{A_{t+1}K_{t+1}}{A_t K_t} \frac{\widehat{\text{Earning}}^{i}_{j,t} - \text{Earning}_{j,t}}{\mathcal{E}_{j,t}} \approx \frac{\widehat{\text{Earning}}^{i}_{j,t} - \text{Earning}_{j,t}}{\mathcal{E}_{j,t}}. \tag{16}$$

I take the set $\left\{(\widehat{e}^{i}_{j,t} - e_t)/e_t\right\}_{i,j}$ as the empirical proxy for $\{(\widehat{e}_t - e_t)/e_t : \widehat{e}_t \in \Theta_t \equiv \Theta(p_t, q_t, r_t)\}$—that is, the set of deviations of admissible latent values of e from the objective value in the model.

Figure 6 plots the time series of the cross-sectional means of the variable, together with its maximum and minimum bounds. The shaded area corresponds to the NBER recession dates. The figure highlights several episodes in which intermediaries' capital was overestimated, including the S&L crisis from 1985 to 1995, the global financial crisis from 2007 to 2009, and the onset of the European debt crisis in 2011. All of these episodes are associated with severe stress in the intermediary sector. Remarkably, financial analysts frequently overestimated intermediaries' capital, particularly at the beginning of these episodes.

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I also use the dataset of consensus forecasts for aggregate dividend and earnings growth rates, as well as the price–dividend ratio for firms in the S&P 500 index, from De la O and Myers (2021). These forecasts are likewise constructed from the I/B/E/S dataset. In addition, the data on asset prices and macroeconomic quantities come from He and Krishnamurthy (2019). This dataset includes aggregate consumption, investment, land prices (used as a proxy for housing prices in the model), bank equity, and the excess bond premium (ebp) from Gilchrist and Zakrajsek (2012).

Gilchrist and Zakrajsek (2012) show that the excess bond premium reflects the risk-absorbing capital capacity of primary dealers and influences the risk premium that compensates investors for exposure to default risk in corporate bond markets. Hence, ebp is a plausible empirical measure to match the objective risk premium (Sharpe ratio) in the model.

Moments	Model	Data
Panel A: Distress periods		
vol(Eq)	30.75	25.73
vol(I)	5.47	7.71
vol(C)	3.61	1.72
vol(PL)	11.4	15.44
vol(EB)	33.9	65.66
cov(Eq,I)	1.50	1.02
cov(Eq,C)	-0.01	0.20
cov(Eq,PL)	3.44	2.38
cov(Eq,EB)	-9.02	-8.50
Panel B: Nondistress periods		
vol(Eq)	23.13	20.54
vol(I)	4.73	5.79
vol(C)	2.26	1.24
vol(PL)	7.62	9.45
vol(EB)	6.09	16.56
cov(Eq,I)	0.95	-0.07
cov(Eq,C)	0.32	-0.01
cov(Eq,PL)	1.68	-0.43
cov(Eq,EB)	0.04	0.60

Table 2: Macro-finance moments in model and data

Notes: This table reports the standard deviations and covariances of aggregate intermediary capital growth (Eq), investment growth (I), consumption growth (C), housing-price growth (PL), and the Sharpe ratio (EB). Growth rates are measured as year-over-year log changes from t to $t+1$, while the Sharpe ratio is taken at time $t+1$. The "Model" column reports statistics conditional on distress and non-distress states. The "Data" column reports the corresponding moments for the sample period 1975Q1–2015Q4. The Sharpe ratio is constructed using the excess bond premium, and the remaining variables follow the standard definitions provided in the text.

The joint availability of these data spans from 1984Q1 to 2015Q4, except for aggregate dividend growth expectations, which are available only from 2003Q1. Consistent with the definition of distress states in the model and following HK, I classify distress periods as those quarters in which ebp lies in the top one-third of its empirical distribution.

3.4.2 Macro-finance moments

Table 2 reports the conditional moments of macroeconomic quantities and asset prices in both the model and the data. The model with endogenous ambiguity preserves the key successes of the rational expectations equilibrium in HK: it reproduces the heightened volatility of macroeconomic variables, intermediary capital, and the Sharpe ratio during distress periods relative to nondistress periods, reflecting the underlying nonlinear dynamics.

3.4.3 Unconditional and conditional belief moments

In this section, I evaluate the extent to which endogenous ambiguity in the model can account for the empirical unconditional and conditional moments of survey expectations—moments that are difficult to reconcile under the rational expectations equilibrium. Table 3 reports the model-implied moments (Model column), the corresponding empirical moments (Data column), and the moments generated by the model under the rational expectations equilibrium (REE column).

Sources of asset price fluctuations (Panel A): First, I examine the sources of asset price fluctuations in the model, connecting the results to the survey evidence in De la O and Myers (2021). Their study uses mean I/B/E/S forecasts for cash-flow growth—such as dividend and earnings growth for the S&P 500 index—along with expected returns for the aggregate stock market. They decompose the variance of the price–earnings (or price–dividend) ratio into contributions from expected cash-flow growth, expected returns, and expectations of the long-run price–earnings (dividend) ratio. I reproduce their results in the Data column of Panel C. Their findings show that fluctuations in the price–earnings (dividend) ratio are primarily driven by variation in expected cash-flow growth rather than by expected returns; if anything, expected returns dampen price fluctuations. De la O and Myers (2021) argue that many leading rational-expectations asset-pricing models struggle to match these patterns because they generate price movements mainly through discount-rate variation. The rational expectations equilibrium in HK is no exception—it produces asset-price fluctuations entirely through changes in expected returns.

In contrast, the model with endogenous ambiguity aligns more closely with the survey evidence, as shown in the Model column. I take the worst-case dividend growth rate g^w as the model-implied counterpart to subjective cash-flow growth in the data, postulating that empirical mean forecasts are heavily influenced by worst-case beliefs—those reflected in the maximum or minimum values of the cross-sectional distribution. As illustrated in Figure 5, the worst-case long-run TFP growth rate g^w is positively correlated with the objective intermediary capital e across most of the stationary distribution, and therefore positively correlated with the scaled capital price (price–dividend ratio) q. By contrast, the subjective discount rate—captured by the worst-case Sharpe ratio—declines with e and is thus negatively correlated with q[12].

[12]The subjective (worst-case) noninfinitesimal finite return expectations are computed by relying on the Feynman-

Facing uncertainty about the latent objects, intermediaries attribute declines in asset prices to deteriorating cash-flow growth rather than to increases in discount rates under the worst-case scenario. This is because higher discount rates would imply higher expected returns, whereas the worst-case belief should correspond to less advantageous investment opportunities.

Predictability of cash flow growth and return forecast errors (Panel B and C): In Panel B of Table 3, the Data column reports the predictability of aggregate dividend growth and return forecast errors by the price-dividend ratio of the S&P500 index, as documented in De la O and Myers (2021). The negative coefficients indicate that forecasters systematically predict higher future dividend growth and returns when the price-dividend ratio increases, relative to subsequent realizations. This evidence challenges rational expectations equilibrium models in which asset price fluctuations arise through time-varying rational forecasts of future dividend growth, as would be required to match the patterns in Panel A. Instead, De la O and Myers (2021) argue that these facts call for equilibrium models in which asset price volatility is driven by nonrational forecasts of future cash-flow growth, thereby generating forecast error predictability with respect to the price-dividend ratio.

Notably, this forecast-error predictability is consistent with the worst-case forecasts of future dividend growth and returns implied by endogenous ambiguity, as reported in the Model column of the same panel. As shown in Figure 5, the worst-case forecasts become more pessimistic—and their deviations from the objective forecasts grow larger—as the objective e declines and, correspondingly, the price-dividend ratio q falls over most of the stationary distribution. In contrast, the REE column confirms that the rational expectations version of the model cannot generate any forecast-error predictability.

The model further predicts that forecast errors in future cash-flow growth are larger when worst-case beliefs about e deviate more from its objective value, and similarly when the maximum value of e in the admissible set is farther from the truth. This arises because, under the worst-case scenario, investors attribute asset price declines to both lower dividend growth and higher aggregate intermediary capital, generating a positive correlation between future dividend-growth forecast errors and the maximum deviations of e from its objective value.

Kac formula. See Appendix A for details.

To evaluate this mechanism quantitatively, I regress the forecast errors of worst-case dividend growth on deviations of the maximum and minimum admissible e's, as shown in the Model column of Panel C. The forecast errors are more strongly related to the maximum e and only weakly related to the minimum, consistent with the fact that the worst-case cash-flow growth g^w is chosen endogenously based on the maximum e in the admissible set. I then run analogous regressions in the data using mean forecasts of earnings growth for the S&P500 index from De la O and Myers (2021) together with the cross-sectional maximum and minimum estimation errors of e's. The maximum estimation error significantly predicts the forecast error at the 90% level, while the minimum does not—mirroring the model's predictions. These empirical findings suggest that part of the observed forecast errors may stem from investors' difficulty in disentangling the resilience of the financial sector from long-run TFP growth based solely on real-time asset price fluctuations.

Predictability of risk premium by dividend-price ratio (Panel D): The Data column in Panel D reports regression coefficients of 1-year excess stock return forecasts and realized stock excess returns on the dividend-price ratio, measuring the cyclicality of the subjective and objective risk premium with respect to the dividend-price ratio in the U.S., as reported in Nagel and Xu (2023). The main takeaway from those values is that the forecasts for excess stock return are more acyclical than the objective risk premium and that the existing rational expectations equilibrium models are inconsistent with such acyclicality since those models generate volatile asset prices through the volatile objective and subjective countercyclical risk premium.

The model in this paper can rationalize these empirical regularities: the worst-case risk premium is much more acyclical (somewhat even procyclical), and the objective risk premium is countercyclical. As shown in Figure 5, the worst-case Sharpe ratio tends to be invariant or somewhat decreasing in the objective e and hence, decreasing in the dividend-price ratio $1/q$ on the bulk of the distribution except in the crisis states. On the other hand, the objective Sharpe ratio is monotonically decreasing in the objective e and consequently, increasing in the dividend-price ratio.

Risk premium and return variance (Panel E): I examine the model predictions for the risk–return tradeoff in terms of both objective and worst-case beliefs. Using the perceived variance data collected from the Graham-Harvey CFO survey, Nagel and Xu (2023) empirically document that the subjective risk premium does not comove with the objective return variance such as the

VIX^2 but instead comoves with the perceived variance measure, while the realized excess return is predicted by the objective return variance measures. This empirical regularity calls for an explanation of why subjective and objective return variances differ.

The worst-case risk premium and return volatility are consistent with this pattern. First, as Figure 5 shows, the worst-case return volatility σ_a^w is acyclical or even procyclical with respect to the objective e and, consequently, the price-dividend ratio q, while the objective return volatility is highly countercyclical. Under the worst-case beliefs, individual investors attribute asset price declines to lower TFP growth rather than lower e, inferring that the amplification of negative shocks will be weaker and that return volatility will be lower because they perceive lower leverage in the aggregate intermediary sector. By contrast, the objective return volatility increases as the objective e falls, since individual intermediaries become more levered after negative shocks and the amplification becomes stronger. Because the worst-case return volatility is acyclical, the first-order condition (9) implies that the worst-case risk premium is also acyclical but remains correlated with the worst-case return variances.

Conditional moments of capital of the intermediary sector capital (Panel F): Figure 6 shows that the cross-sectional dispersions and, in particular, the maximum estimation errors tend to be larger during episodes in which the financial sector incurs severe losses. The Data column in Panel F confirms this empirical regularity. These empirical patterns are also consistent with the admissible set in the model. As the objective e approaches the distress states, the dispersions and especially the maximum value of admissible e's become larger, as seen in Figure 5 and reported in the Model column in Panel F. This result implies that assessing the vulnerability of the financial sector becomes more challenging without directly observing its balance sheet composition as more negative shocks hit the economy and asset prices decline.

Moments	Model	Data	REE
Panel A: Campbell-Shiller price-dividend ratio decomposition (De la O and Myers (2021))			
Contribution of 1-year ahead cash flow growth subjective expectations $(\text{Cov}(g^s, pd_t)/\text{Var}(pd_t))$	0.37	0.39	0
Contribution of 1-year ahead subjective discount rate $(\text{Cov}(\text{ER}^s, pd_t)/\text{Var}(pd_t))$	-0.22	-0.05	0.52
Panel B: Predictability of cash flow and return forecast errors (De la O and Myers (2021))			
Forecast error predictability of aggregate cashflow growth by P-D ratio	-0.27	-0.30	0
Forecast error predictability of excess return by P-D ratio	-0.29	-0.25	0
Panel C: Predictability of cash flow forecast errors by intermediaries capital estimates			
Aggregate cashflow growth by the maximum forecast error of e $((\widehat{e}^{\max} - e)/e)$	0.69	0.18*	0
Aggregate cashflow growth by the minimum forecast error of e $((\widehat{e}^{\min} - e)/e)$	0.52	0.14	0
Panel D: Cyclicality of risk premium for capital (Nagel and Xu (2023))			
Regression coefficient of 1-year subjective risk premium on dividend-price ratio	-0.09	-0.24	0.32
Regression coefficient of 1-year objective risk premium on dividend-price ratio	0.32	6.4	0.32
Ratio of subjective to objective coefficients	-0.3	-0.03	1
Panel E: Risk premium and return variance (Nagel and Xu (2023))			
Regression coefficient of subjective risk premium π_k^S on subjective variance $(\sigma_k^S)^2$	24	4	50
Regression coefficient of subjective risk premium π_k^S on subjective variance $(\sigma_k^S)^2$	-8.8	-0.01	50
Regression coefficient of objective risk premium π_k on objective variance $(\sigma_k)^2$	28.7	1.49	50
Panel F: Conditional moments of intermediaries capital estimates			
$\text{Mean}\left(\text{std}_t\left(\frac{\widehat{e}-e}{e}\right)\middle\vert\text{distress}\right)/\text{Mean}\left(\text{std}_t\left(\frac{\widehat{e}-e}{e}\right)\right)$	1.15	1.92	NA
$\text{Mean}\left(\frac{e^{\max}-e}{e}\vert\text{distress}\right)/\text{Mean}\left(\frac{e^{\max}-e}{e}\right)$	1.47	1.79	NA

Table 3: Belief moments in model and data

Notes: Panel A (Model) reports the model-implied covariances of the 1-year ahead worst-case growth rate g^w and worst-case expected return for capital with the log price-dividend (P-D) ratio, scaled by the variance of the log P-D ratio. The Data column shows the corresponding covariances for the mean 1-year ahead forecasts and expected returns for the S&P 500, as in Table IV of De la O and Myers (2021). In Panel B, Model reports the correlations between the 1-year ahead worst-case forecast errors for aggregate dividend growth or excess returns on capital and the log P-D ratio; the Data column reports the matching values in De la O and Myers (2021) (Table III Panel B and p. 1357). Panel C reports the correlations of maximum (minimum) estimate errors of $e^{\max}$ ($e^{\min}$) with aggregate dividends in both the model and data; * denotes 90% significance. In Panel D, the first two rows show the regression coefficients of the 1-year ahead worst-case and objective risk premium on the dividend-price ratio, followed by their ratio (Model). The Data column reports the corresponding coefficients for mean forecasts of expected and realized excess returns on the S&P 500 over Treasury bills, as in Tables 3–4 of Nagel and Xu (2023). Panel E reports regression coefficients of worst-case and objective risk premia on worst-case and objective return variances (Model), while the Data column reports coefficients of mean CRSP excess-return forecasts on perceived variances (Table 10 in Nagel and Xu (2023)). Panel F presents the mean of the admissible set and the cross-sectional standard deviations of e in the data, along with the mean maximum admissible e in the model and in the data forecasts. The REE column reports the values under the rational expectations equilibrium.

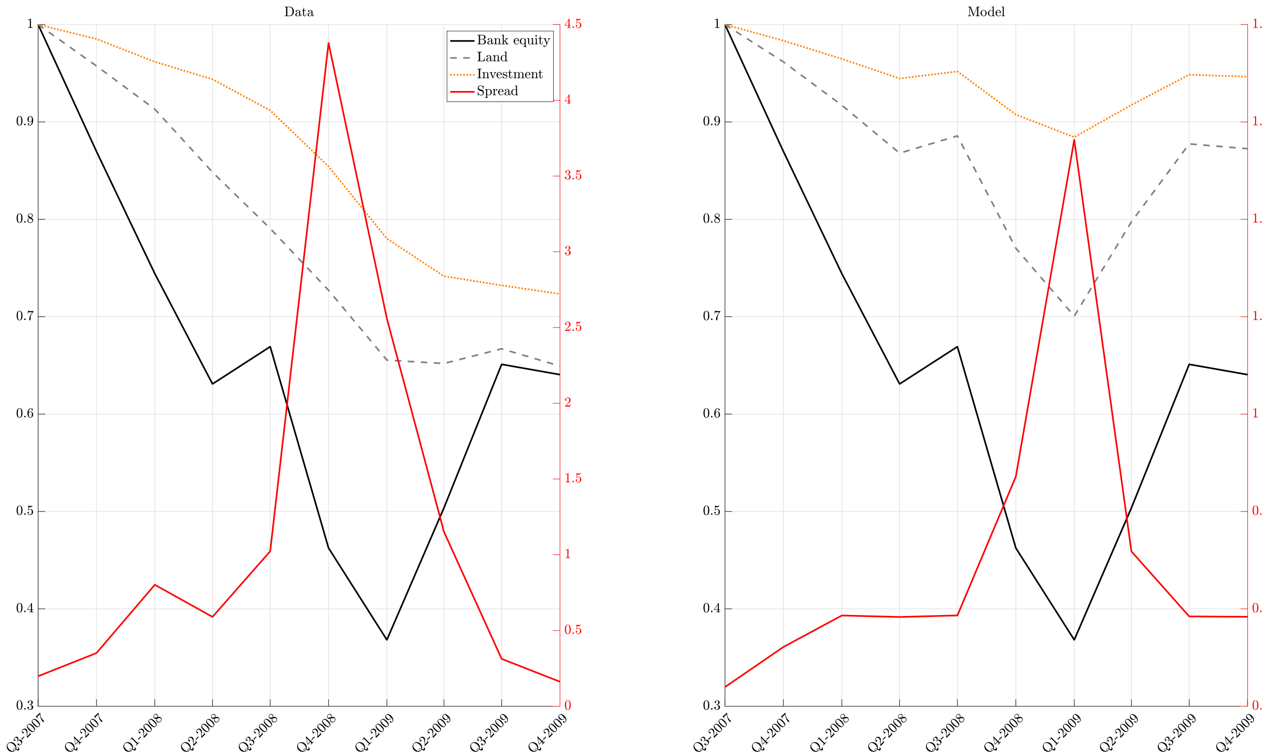

Figure 7: Asset price and macroeconomic dynamics during the 2007-2009 crisis

3.5 Crisis dynamics

I now turn to the crisis dynamics of asset prices, macroeconomic quantities, and subjective beliefs during the 2007–2009 crisis episode. Building on the success of the HK model under rational expectations in replicating the dynamics of asset prices and macroeconomic variables, I show that the model with endogenous ambiguity can additionally account for the belief dynamics associated with uncertainty over the intermediary sector's capital.

The model in this paper replicates the dynamics of asset prices and macroeconomic quantities just as well as the HK model under the rational expectations equilibrium. Figure 7 plots the dynamics of bank equity, land (housing) prices, and investment (left axis), together with the Sharpe ratio (right axis), in the data (left panel) and in the model (right panel). All prices and quantities are normalized to 1 in the first quarter of 2007. Following HK, I choose the sequence of shocks dZ_t in the model so that the model-implied dynamics exactly match those observed in the data.

The figure shows that beginning in the first quarter of 2007, aggregate intermediary capital

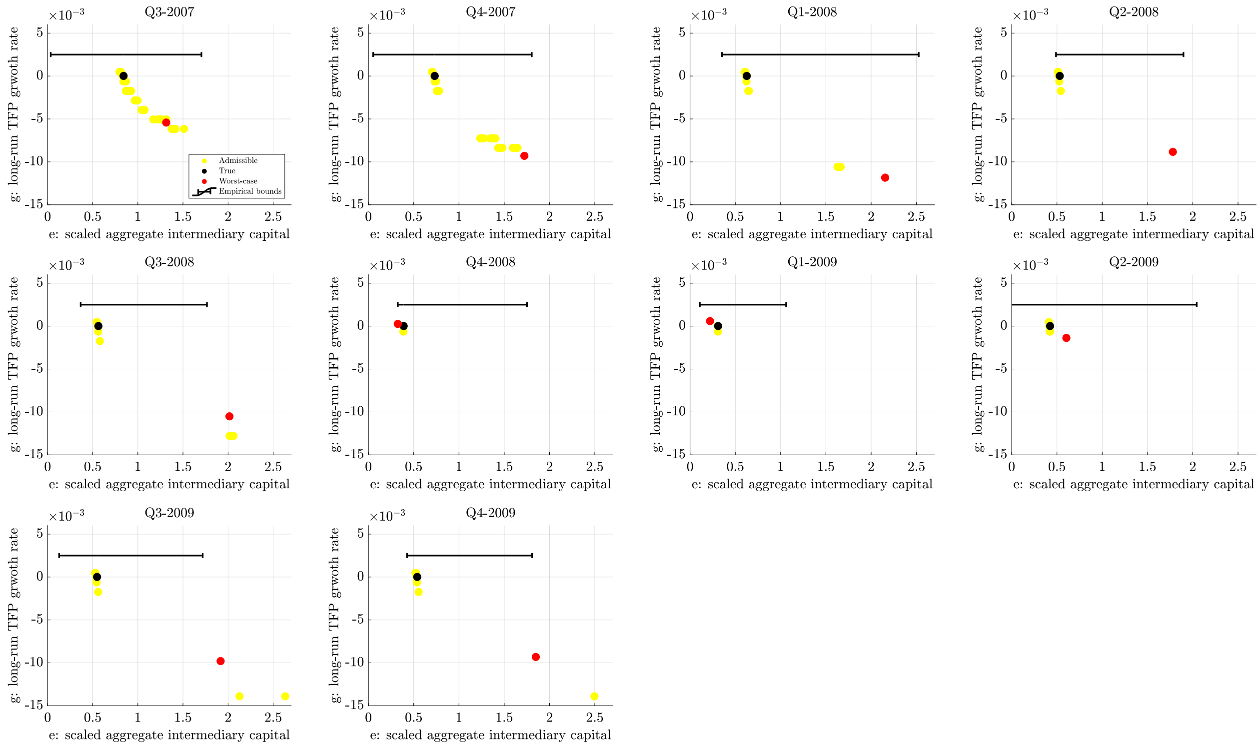

Figure 8: Subjective belief dynamics over e and g during the 2007-2009 crisis

started to decline, accompanied by a gradual increase in the objective Sharpe ratio. After the collapse of Lehman Brothers in the third quarter of 2008, conditions deteriorated sharply, leading to a sudden drop in capital and a pronounced amplification of the Sharpe ratio. In hindsight, the dynamics of the Sharpe ratio can be closely tracked by the movements in realized aggregate intermediary capital.

However, there was substantial uncertainty about the real-time level of aggregate intermediary capital during the crisis. Figure 8 plots the observationally equivalent combinations of (e, g) in yellow, the worst-case combination in red, and the objective trajectory in black, together with the empirical bounds derived from real-time estimates in the I/B/E/S dataset during the crisis episode. The empirical bounds show that real-time estimates of e remained highly dispersed through the end of 2008, even though the objective level of capital was steadily declining. Following the collapse of Lehman Brothers, the empirical bounds contracted sharply in the first quarter of 2009, plausibly reflecting reduced uncertainty due to the negative information revealed by the malfunctioning

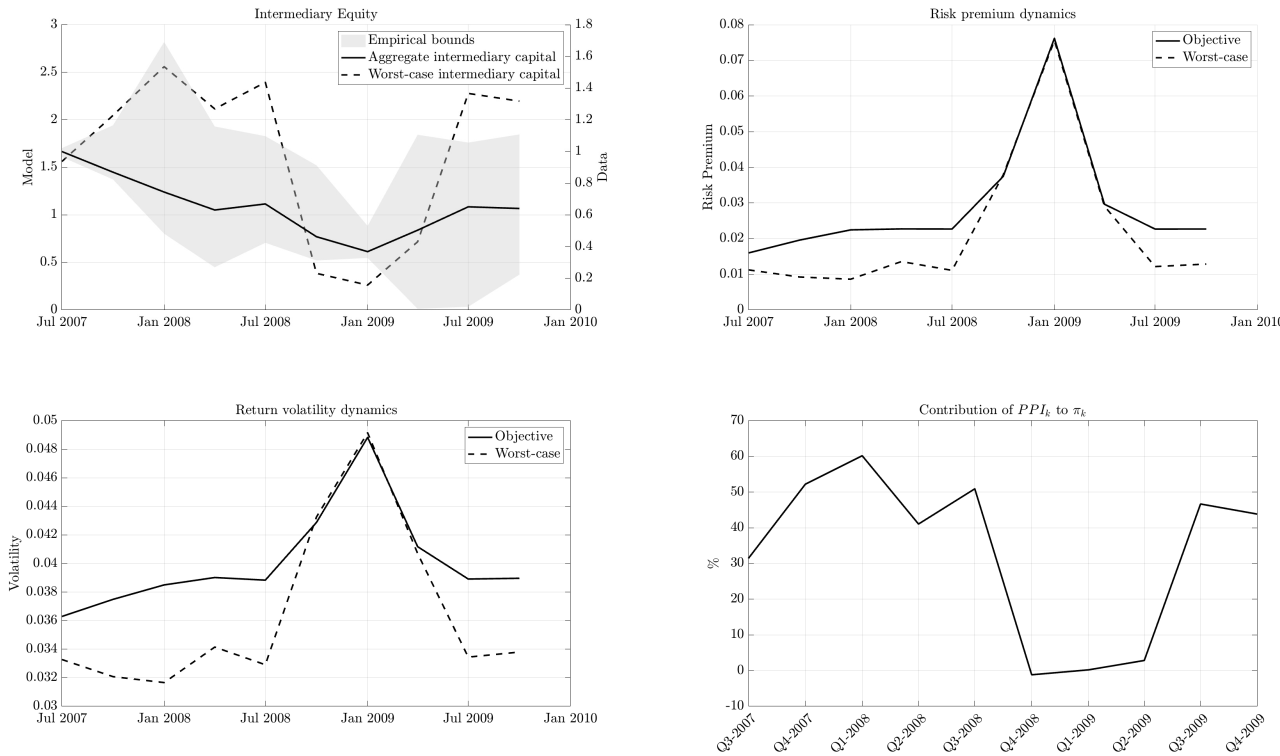

Figure 9: Return dynamics over e and g during the 2007-2009 crisis

intermediary sector. After government intervention in financial markets and the intermediary sector, the actual aggregate capital began to recover, and the empirical bounds once again expanded.

The observationally equivalent values of e shown in yellow lie largely within the empirical bounds, supporting the plausibility of the subjective uncertainty over e in the model. Moreover, the worst-case e values in red closely track the upper edge of the empirical bounds, suggesting that worst-case beliefs may lie at the boundary of the set of plausible estimates. Up to the third quarter of 2008, the worst-case e reflects intermediaries' belief that aggregate sectoral capital might not be the main problem and that stagnating cash-flow growth (lower g) could instead be driving asset-price declines. This "optimistic" view of the sector disappears between late 2008 and the second quarter of 2009 as asset prices fall nonlinearly, in a manner inconsistent with the frictionless benchmark. As asset prices subsequently recovered, intermediaries once again began to entertain the possibility that sectoral capital could be relatively abundant.

Figure 9 also shows the expected excess returns and volatilities under the objective and worst-case distributions. While the objective risk premium rises gradually—together with return volatility—before reaching its peak in the third quarter of 2008, the worst-case counterparts display no such warning signal and instead spike abruptly in that quarter, rapidly converging to the objective measures. This pattern arises because, prior to the crisis peak, the worst-case beliefs attribute asset-price declines to slowing TFP growth rather than to scarce intermediary capital. Once asset prices begin to fall nonlinearly, this counterfactual interpretation becomes inconsistent, and the worst-case beliefs correctly infer that deteriorating intermediary-sector capital is responsible, bringing them into alignment with the objective beliefs.

The model also provides an alternative explanation for the elevated objective risk premium during the crisis. As shown in the lower-left panel of Figure 9, with the exception of the crisis peak between 2008Q4 and 2009Q2, the compensation for endogenous ambiguity PPI_a accounts for roughly half of the total objective risk premium. This indicates that, in addition to the subjective exposure to aggregate shocks through risky-asset holdings—as in HK—the real-time uncertainty surrounding the resilience of the intermediary sector also contributed to higher discount rates and lower asset prices.

The results also highlight the challenges policymakers face when attempting to mitigate heightened risk premia during crisis episodes. Even if policymakers were able to influence the set of admissible e's perceived by investors—such as by disclosing information about the deteriorating capital position of the intermediary sector—such disclosure could instead lead investors to conclude that the sector is in even worse shape than expected, implying greater exposure of risky asset returns to aggregate shocks. Market participants would then demand even higher compensation for bearing this exposure, and information disclosure alone would fail to restore asset prices.

Thus, effective policy intervention may require reducing both the actual exposure of the intermediary sector to aggregate shocks and the uncertainty surrounding the state of the economy in order to restore market confidence and contain risk premia during crises. A detailed policy analysis is left for future work.

4 Conclusion

This paper develops a novel belief-formation framework under ambiguity, in which the state-dependent admissible set of models is endogenously disciplined by observable endogenous information and by agents' knowledge of the structure of the economy—that is, the equilibrium mapping from latent parameters and states to observable variables. Deviations from the rational expectations equilibrium of this form can generate empirically consistent uncertainty about the capital of the financial intermediary sector and the cash-flow growth of risky assets, both during and outside crisis episodes.

Cautious investors confronted with this endogenous ambiguity treat higher intermediary capital and lower cash-flow growth—rather than lower capital—as the worst-case scenario behind observed asset-price declines, because such a scenario implies a lower risk premium and therefore less advantageous investment opportunities. Systematic policy interventions that directly reshape the admissible set of observationally equivalent latent objects, or indirectly affect it through changes in observed endogenous information, will consequently influence worst-case beliefs and decisions, which feeds back to the endogenous observables.

The endogenous ambiguity considered in this paper abstracts from learning dynamics (Hansen and Sargent (2010)), dynamic consistency (Chen and Epstein (2002)), and interactions with model-misspecification concerns (Hansen and Sargent (2022)). The focus is on applying the endogenous ambiguity mechanism to the specific environment of He and Krishnamurthy (2012). Nevertheless, the underlying nonlinearities and the associated identification challenges should arise broadly in many macro-finance models featuring occasionally binding constraints. I leave further development of the endogenous-ambiguity framework and its application to other environments for future work.

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Online Appendix Not For Publications

A Appendix A. Theoretical Appendix

A.1 Exogenous processes and equilibrium laws of motion

The TFP process is given by a geometric Brownian motion:

$$\frac{dA_t}{A_t} = gdt + \sigma dZ_t,$$

where Z_t is a one-dimensional standard Brownian motion. The capital accumulation is governed by the law of motion:

$$\frac{dK_t}{K_t} = (i_t - \delta)dt,$$

where i_t is the investment rate and δ is the depreciation rate. Then the law of motion of the effective capital stock A_tK_t follows from Itô's Lemma:

$$\frac{dA_tK_t}{A_tK_t} = (i_t - \delta + g)dt + \sigma dZ_t.$$

Let P_t and Q_t be the unit price of housing and capital. Define the scaled housing and capital price by the effective capital stock:

$$p_t \doteq \frac{P_t}{A_tK_t};$$

$$q_t \doteq \frac{Q_tK_t}{A_tK_t}.$$

Consequently, the unit price of capital is given by $Q_t = A_tq_t$. Let $\mathcal{E}_t$ denote the aggregate net worth of the intermediaries. I denote the scaled aggregate net worth as $e_t \doteq \mathcal{E}_t/(A_tK_t)$. I solve for q_t, p_t, and r_t as the functions of the Markovian state e_t, where r_t is the risk-free rate.

I postulate the scaled aggregate intermediary capital e follows a diffusion process with drift and volatility characterized as functions of the state e_t:

$$\frac{de_t}{e_t} = \mu_e dt + \sigma_e dZ_t.$$

From Itô's Lemma, the equilibrium dynamics of (p, q) are also Itô processes:

$$\frac{dq}{q} = \mu_q dt + \sigma_q dZ;$$

$$\frac{dp}{p} = \mu_p dt + \sigma_p dZ_t,$$

where the drift and volatility terms are computed by Itô's Lemma as:

$$\mu_q = \frac{q'}{q} e\mu_e + \frac{1}{2}\frac{q''}{q}(e\sigma_e)^2; \tag{A1}$$

$$\mu_p = \frac{p'}{p} e\mu_e + \frac{1}{2}\frac{p''}{p}(e\sigma_e)^2; \tag{A2}$$

$$\sigma_q = \frac{q'}{q} e\sigma_e;$$

$$\sigma_p = \frac{p'}{p} e\sigma_e.$$

The goal is to derive the drift terms of asset prices (μ_p, μ_q, r) and (μ_e, σ_e) as the implicit functions of (p, q, r) and to jointly solve the resulting system of nonlinear partial differential equations for (p, q, r) (see (A9) and (A10) as well as the equation for r derived later).

A.2 Housing rental price

The household's intratemporal optimality condition between housing service and nondurable consumption is given by:

$$D_t = \frac{\phi}{1-\phi}\frac{C_t^y}{C_t^h} = \frac{\phi}{1-\phi} C_t^y,$$

where D_t is the rental price of housing, ϕ is the elasticity of substitution between housing and consumption, C_t^y is the nondurable consumption, C_t^h is the housing consumption, and the last equality comes from the housing rental market clearing condition $C_t^h \equiv 1$. Invoking the goods market clearing condition,

$$C_t^y + \Phi(i_t, A_t K_t) = \underbrace{\overline{A} A_t K_t}_{\text{Output}},$$

where the capital investment cost is specified as $\Phi(i_t, A_tK_t) \doteq i_tA_tK_t + \frac{\kappa}{2}(i_t - \delta)^2A_tK_t$ and $\overline{A}$ is the constant TFP scaling. Substituting this expression, The rental price of housing is then rewritten as:

$$D_t = \frac{\phi}{1-\phi}A_tK_t\left[\overline{A} - i_t - \frac{\kappa}{2}(i_t - \delta)^2\right].$$

A.3 Capital producers

The capital producer's problem is formulated as:

$$\max_{i_t} i_tQ_tK_t - \Phi(i_t, A_tK_t) = A_tK_t\left[i_tq_t - i_t - \frac{\kappa}{2}(i_t - \delta)^2\right]. \tag{A3}$$

The first-order condition gives the standard Tobin's q-theory:

$$i_t = \delta + \frac{q_t - 1}{\kappa}. \tag{A4}$$

A.4 Return for housing and capital

The infinitesimal return for a unit of housing is defined and computed by Itô's Lemma:

$$\begin{aligned}
dR_h &\doteq \frac{dP + Ddt}{P} = \frac{d(p\cdot AK)}{p\cdot AK} + \frac{D}{p\cdot AK} = \\
&= \frac{dp}{p} + \frac{d(AK)}{AK} + \left[\frac{dp}{p}, \frac{d(AK)}{AK}\right] + \frac{\phi}{1-\phi}\frac{\overline{A} - i - \frac{\kappa}{2}(i-\delta)^2}{p}dt = \\
&= \left(\mu_p + i - \delta + g + \sigma_p\sigma + \frac{\phi}{1-\phi}\frac{\overline{A} - i - \frac{\kappa}{2}(i-\delta)^2}{p}\right)dt + (\sigma_p + \sigma)dZ \doteq \\
&\doteq (\pi_h + r)dt + \sigma_h dZ.
\end{aligned} \tag{A5}$$

Notice that the financial frictions affect the drift of the infinitesimal return through μ_p and volatility through σ_p. The latter is the amplification term through the change in the aggregate net worth in the financial sector.

Similarly, the infinitesimal return for a unit of capital is given from the Itô's lemma by

$$
\begin{aligned}
dR_k &\doteq \frac{dA}{A}\frac{dq}{q} + \frac{dA}{A} + \frac{dq}{q} - \delta dt + \frac{\overline{A}}{q}dt \\
&= \left(\mu_q + g - \delta + \sigma\sigma_q + \frac{\overline{A}}{q}\right) dt + (\sigma_q + \sigma)dZ \\
&\doteq (\pi_k + r)dt + \sigma_k dZ.
\end{aligned}
$$

A.5 Household consumption and Euler equation

From the consumption goods market clearing condition, the household consumption is expressed as:

$$
\begin{aligned}
C_t^y &= Y_t - \Phi(i_t, A_t K_t) = \left[\overline{A} - i_t - \frac{\kappa}{2}(i_t - \delta)^2\right] A_t K_t \\
&= \underbrace{\left[\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{\kappa}{2}\left(\frac{q-1}{2}\right)^2\right]}_{\text{Consumption rate} \geq 0} AK,
\end{aligned}
$$

where the last equality follows from equation (A4).

The Itô's lemma, the household consumption process is written as:

$$
\begin{aligned}
dC_t^y =& AK \cdot d\left[\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{\kappa}{2}\left(\frac{q-1}{\kappa}\right)^2\right] + \left[\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{\kappa}{2}\left(\frac{q-1}{\kappa}\right)^2\right] d(AK) \\
&+ \left[d\left(\left[\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{\kappa}{2}\left(\frac{q-1}{\kappa}\right)^2\right]\right), d(AK)\right].
\end{aligned}
$$

The Itô's lemma further implies the law of motion of the consumption rate:

$$
d\left[\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{\kappa}{2}\left(\frac{q-1}{\kappa}\right)^2\right] = -\frac{dq}{\kappa} - \frac{d(q-1)^2}{2\kappa} = -\frac{q^2}{\kappa}\left[\left(\mu_q + \frac{\sigma_q^2}{2}\right) dt + \sigma_q dZ\right].
$$

Substituting this equation into (A21) and dividing both sides by C_t^y:

$$
\frac{dC_t^y}{C_t^y} = \left[i - \delta + g - \frac{\frac{q^2}{\kappa}\left(\mu_q + \frac{\sigma_q^2}{2} + \sigma_q\sigma\right)}{\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{(q-1)^2}{2\kappa}}\right] dt + \left[\sigma - \frac{\frac{q^2}{\kappa}\sigma_q}{\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{\kappa}{2}\left(\frac{q-1}{\kappa}\right)^2}\right] dZ.
$$

The Euler equation for households is expressed as:

$$r = \rho + \xi E_t\left[\frac{dC_t^y}{C_t^y}\right] - \frac{\xi(\xi+1)}{2}Var_t\left[\frac{dC_t^y}{C_t^y}\right]$$
$$= \rho + \xi\left[i - \delta + g - \frac{\frac{q^2}{\kappa}\left(\mu_q + \frac{\sigma_q^2}{2} + \sigma_q\sigma\right)}{\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{(q-1)^2}{2\kappa}}\right] - \frac{\xi(\xi+1)}{2}\left[\sigma - \frac{\frac{q^2}{\kappa}\sigma_q}{\overline{A} - \delta - \frac{q-1}{\kappa} - \frac{\kappa}{2}\left(\frac{q-1}{\kappa}\right)^2}\right]^2. \quad \text{(A6)}$$

A.6 Intermediaries Problem

Let $j \in [0,1]$ denote the intermediary index. The preferences of intermediary j is represented by the max-min preferences over the distribution of risky asset returns $\{\pi_k^S, \pi_h^S, \sigma_k^S, \sigma_h^S\}$ and the infinitesimal portfolio excess return over the risk-free rate, respectively:

$$\min_{\{\pi_k^S,\pi_h^S,\sigma_k^S,\sigma_h^S\}\in\Xi(p,q,r)} \max_{\alpha_k,\alpha_h} \underbrace{\alpha_k\pi_k^S + \alpha_h^S\pi_h^S}_{\text{Subjective expected excess portfolio return}} + \frac{\gamma}{2}\times \underbrace{(\alpha_k\sigma_k^S + \alpha_h\sigma_h^S)^2}_{\text{Subjective variance of excess portfolio return}}. \quad \text{(A7)}$$

I describe how the set of distributions (models) $\Xi(p,q,r)$ is constructed after I derive the standard optimal portfolio choice given the worst-case model as the solution to the minimization problem: $\{\pi_k^w, \pi_h^w, \sigma_k^w, \sigma_h^w\}$.

The first-order condition w.r.t. $\{\alpha_k, \alpha_h\}$ is given by

$$\underbrace{\frac{\pi_k^w}{\sigma_k^w} = \frac{\pi_h^w}{\sigma_h^w}}_{\text{Subjective Sharpe ratio}} = \gamma(\alpha_k\sigma_k^w + \alpha_h\sigma_h^w). \quad \text{(A8)}$$

Turning to the minimization problem, the individual intermediaries neither directly observe the underlying state nor parameter values, $\theta \equiv (e, g, \phi)$. Each intermediary understands the structure of the economy in the sense of the mapping from the unknown parameters and state θ to the asset prices (p, q, r). I denote this mapping as $(p(\theta), q(\theta), r(\theta))$. This mapping comes from the price functions of the possibly alternative equilibria characterized with the state e and parameter value (g, ϕ). Given this mapping, each intermediary restricts the set of unknown values of (e, g, ϕ) consistent with observable asset prices (p, q, r) and the structure of the economy, and constructs the set of consistent parameters and state denoted by $\Theta(p, q, r)$ that satisfies the following consistency conditions:

$$\underbrace{p}_{\text{Currently observed price}} = \underbrace{p(\theta)}_{\text{Model-implied price}} ;$$
$$\underbrace{q}_{\text{Currently observed price}} = \underbrace{q(\theta)}_{\text{Model-implied price}} ; \tag{A9}$$
$$\underbrace{r}_{\text{Currently observed price}} = \underbrace{r(\theta)}_{\text{Model-implied price}} .$$

Each intermediary does not learn from the past information of prices to learn about the unknowns, since they entertain the possibility that the parameters and state could evolve as i.i.d. processes in the spirit of Chen and Epstein (2002).

Let $\{\pi_k(\theta), \pi_h(\theta), \sigma_k(\theta), \sigma_h(\theta)\}$ be the objective expected excess return and volatility in the current state e in an alternative equilibrium parameterized by (g, ϕ). The individual intermediary postulates the following distribution of the infinitesimal risky asset returns: for $a \in \{k, h\}$,

$$\pi_a^S(\theta) \doteq \pi_a(\theta) + PPI_a^S; \tag{A10}$$

$$\sigma_a^S(\theta) = \sigma_a(\theta), \tag{A11}$$

where denoting $\hat{\theta}$ be the baseline (true) state and parameter values,

$$PPI_a^S \doteq \pi_a(\hat{\theta}) - \pi_a^{REE}(\hat{\theta}).$$

Each intermediary understands the underlying economic structure in the sense of how parameters and state alter the risk premium captured in the first term of $\pi_a(\theta)$ in equation (A10) and return volatility $\sigma_a(\theta)$ in equation (A11). Moreover, it knows that all other intermediaries are confronted with the uncertainty over the set of models and hence, correctly understands that the risk premium π_a^S should also entail the compensation for this uncertainty captured by PPI_a^S[13]. The latter component of the risk premium is represented by the difference between the objective risk premium $\pi_a(\hat{\theta})$ under the true data-generating process in this model parameterized by $\hat{\theta}$ and the objective risk premium under the rational expectations equilibrium parameterized by the same $\hat{\theta}$. I postulate

[13]This uncertainty comes from the partial-identification problem in econometricians' terminology. The notation for the compensation for the partial identification problem is inspired by "the price of partial identification problem", which is credited to Tom Sargent.

that each intermediary fully trusts the specification of PPI_a^S and takes it as given.

Given these specifications for return distributions, the set of models (distributions) is defined as

$$\Xi(p,q,r) = \left\{ (\pi_a^S, \sigma_a^S)_{a\in\{k,h\}} : \exists \theta \in \Theta(p,q,r) \quad \text{s.t} \quad \pi_a^S = \pi_a^S(\theta) \quad \text{and} \quad \sigma_a^S = \sigma_a^S(\theta) \right\}.$$

$\Xi(p,q,r)$ represents the set of return distributions consistent with the underlying economic structure and observable information. Since the latter is state-dependent, so is the set of models $\Xi(p,q,r)$. Unlike the existing literature of ambiguity and model misspecification concerns, this state-dependency is endogenous in this framework.

Due to the uncertainty aversion of each intermediary, its alter ego chooses the worst-case model in this set $\Xi(p,q,r)$ that minimizes the mean-variance utility given the choice of portfolio weights (α_k, α_h). For each risky asset $a \in \{k,h\}$, let

$$\pi_a^w \doteq \pi_a(\theta^w);$$

$$\sigma_a^w \doteq \sigma_a(\theta^w)$$

be the worst-case expected excess return and volatility, where θ^w is the corresponding worst-case parameters and state. The worst-case return distributions, parameters, and state are obviously state-dependent and endogenous as well.

Notice that the construction of $\Xi(p,q,r)$ depends not just on the equilibrium objects in the model parameterized by the baseline (true) $\hat{\theta}$, but also the set of equilibria in the models parameterized by alternative θ. Consequently, the baseline model parameterized by $\hat{\theta}$ must be jointly solved by those alternative equilibria. This point will be elaborated on in Appendix B, where I discuss the numerical method to solve the model.

Moreover, the solution to the minimization problem also shapes the subjective evolution of the latent state e conditional on the current information (p,q,r):

$$\frac{de^S}{e^S} = \mu_e(e^S; g^w, \phi^w)dt + \sigma_e(e^S; g^w, \phi^w)dZ^w,$$

where e^s starts with e^w, the drift and volatility are taken from the alternative economy in state

e^w, parameterized by (g^w, ϕ^w), and dZ^w is the subjective perception of the latent TFP shock. This conditional evolution of the subjective latent state e^w characterizes the conditional subjective beliefs over the evolution of the economy.

A.7 Objective expected excess returns and drifts of risky asset prices

For a risky asset $a \in \{k, h\}$, the first-order condition (A8) can be solve for the objective expected excess return:

$$\begin{aligned}\pi_a &= \underbrace{\gamma(\alpha_k \sigma_k^w + \alpha_h \sigma_h^w)\sigma_a^w}_{\text{Subjective risk exposure}=\pi_a^w} + \underbrace{PPI_a}_{\text{Adjustment for misspecified expected excess return}} \\ &= \pi_a^w + PPI_a,\end{aligned}$$

where PPI_a is the objective compensation for model uncertainty:

$$PPI_a \doteq \pi_a^{REE}(\hat{\theta}) - \pi_a^w.$$

The risky asset market clearing conditions are defined as:

$$\alpha_h E_t = P_t \quad \& \quad \alpha_k E_t = K_t Q_t,$$

where E_t is the equity raised from households that intermediaries can invest in risky asset markets. Combining these two conditions, the subjective risk exposure of the intermediary's portfolio is given by

$$\alpha_h \sigma_h^w + \alpha_k \sigma_k^w = \frac{AK}{E}(p\sigma_h^w + q\sigma_k^w).$$

Notice that the risk exposure is given by the leverage times the exposure of the risky asset return to the fundamental shock σ_a^w, $a \in \{h, k\}$. Then the objective expected excess return is characterized by

$$\pi_a = \gamma \frac{AK}{E}(p\sigma_h^w + q\sigma_k^w)\sigma_a^w + PPI_a,$$

for $a \in \{h, k\}$.

Combined with the expected return from capital (A5), the drift of a unit price of effective capital is expressed as

$$\mu_q = \underbrace{r+\delta-g}_{\text{Deterministic discounting}} - \underbrace{\frac{\overline{A}}{q}}_{\text{Dividend payout}} - \sigma\sigma_q + \underbrace{\gamma\frac{AK}{E}(p\sigma_h^w + q\sigma_k^w)\sigma_k^w + PPI_k}_{\text{Risk premium}}.$$

Similarly, the drift of a unit price of scaled housing is characterized as

$$\mu_p = \underbrace{\delta+r-i-g}_{\text{Deterministic discounting}} - \sigma_p\sigma - \underbrace{\frac{\phi}{1-\phi}\frac{\overline{A}-i-\frac{\kappa}{2}(i-\delta)^2}{p}}_{\text{Rent payout}} + \underbrace{\gamma\frac{AK}{E}(p\sigma_h^w + q\sigma_k^w)\sigma_h^w + PPI_h}_{\text{Risk premium}}.$$

A.8 State dynamics

The evolution of the aggregate intermediary capital is defined as

$$\frac{d\mathcal{E}}{\mathcal{E}} \equiv \underbrace{d\tilde{R}}_{\text{Aggregate intermediary portfolio return}} - \underbrace{\eta}_{\text{exit rate}}\,dt + \underbrace{d\psi_t}_{\text{government intervention at the left boundary as } e\downarrow 0}. \tag{A12}$$

Then using the expression for the objective risk premium (A36), the evolution in the interior of the state space is characterized as

$$\frac{d\mathcal{E}}{\mathcal{E}} = \big(\gamma(\alpha_k\sigma_k^w + \alpha_h\sigma_h^w)^2 + (\alpha_k PPI_k + \alpha_h PPI_h) + r - \eta\big)\,dt + (\alpha_k\sigma_k + \alpha_h\sigma_h)dZ.$$

Then the risky asset market clearing conditions are

$$\alpha_h E = P \quad \& \quad \alpha_k E = QK.$$

Multiplying both sides by the objective volatility terms σ_h and σ_k, respectively and combining those two conditions, we obtain

$$\alpha_h\sigma_h + \alpha_k\sigma_k = \frac{AK}{E}(\sigma_h p + \sigma_k q). \tag{A13}$$

Substituting this expression into (A12),

$$\frac{d\mathcal{E}}{\mathcal{E}} = \left(\gamma\left(\frac{AK}{E}\right)^2(\sigma_h^w p + \sigma_k^w q)^2 + \frac{AK}{E}(p\cdot PPI_h + q\cdot PPI_k) + (r-\eta)\right)dt + \frac{AK}{E}(\sigma_h p + \sigma_k q)dZ. \tag{A14}$$

Applying the Itô's lemma to $d\mathcal{E}/\mathcal{E}$ to express the drift and volatility terms in (μ_e, σ_e),

$$\begin{aligned}\frac{d\mathcal{E}}{\mathcal{E}} &= \frac{d(e\cdot AK)}{e\cdot AK} = \frac{d(AK)}{AK} + \frac{de}{e} + \left[\frac{d(AK)}{AK}, \frac{de}{e}\right] \\ &= \Bigg(\underbrace{(i-\delta+g)}_{\text{frictionless drift}} + \underbrace{\mu_e + \sigma\sigma_e}_{\text{friction drift}}\Bigg)dt + (\sigma+\sigma_e)dZ.\end{aligned}$$

Matching the drift and volatility terms in (A13) and (A14),

$$\sigma_e = \frac{AK}{E}(\sigma_h p + \sigma_k q) - \sigma;$$

$$\mu_e = \gamma\left(\frac{AK}{E}\right)^2(\sigma_h^w p + \sigma_k^w q)^2 + \frac{AK}{E}(p\cdot PPI_h + q\cdot PPI_k) + r - \eta - (i-\delta+g) - \sigma\sigma_e.$$

I express $(AK)/E$ in terms of (p,q) by rewriting the equity raising constraint:

$$E = \min\{\mathcal{E}, (1-\lambda)W\}.$$

Since $W_t = P_t Q_t K_t$,

$$E = \{\mathcal{E}, (1-\lambda)(P+QK)\} = \{\mathcal{E}, (1-\lambda)AK(p+q)\}.$$

Equivalently,

$$\frac{AK}{E} = \frac{1}{\min\{e, (1-\lambda)(p+q)\}}.$$

A.9 Equilibrium asset pricing dynamics

Given the expression for the state dynamics (μ_e, σ_e), I can now characterize equilibrium asset pricing dynamics in terms of (p,q) and the worst-case beliefs $\{\sigma_a^w, PPI_a\}_{a\in\{k,h\}}$. Recall the consumption rate is given by

$$c \doteq \overline{A} - \delta - \frac{q-1}{\kappa} - \frac{\kappa}{2}\left(\frac{q-1}{\kappa}\right)^2.$$

Then the risk-free rate (A6) can be written as

$$r = \rho + \xi \left[i - \delta + g - \frac{q^2}{\kappa c} \left(\mu_q + \frac{\sigma_q^2}{2} + \sigma_q \sigma \right) \right] - \frac{\xi(\xi+1)}{2} \left[\sigma - \frac{q^2}{\kappa c} \sigma_q \right]^2 .$$

I decompose the drift of dq as $\mu_q = \tilde{\mu}_q + r$, where

$$\tilde{\mu}_q \doteq \delta - g - \frac{\overline{A}}{q} - \sigma\sigma_q + \gamma \frac{AK}{E} (p\sigma_h^w + q\sigma_k^w)\sigma_k^w + PPI_k.$$

Then the risk-free rate is characterized in terms of (p, q) and $(PPI_a, \sigma_a)_{a \in \{k,h\}}$

$$r = \left[1 + \frac{\xi q^2}{\kappa c} \right]^{-1} \left[\rho + \xi \left\{ i - \delta + g - \frac{q^2}{\kappa c} \left(\tilde{\mu}_q + \frac{\sigma_q^2}{2} + \sigma_q \sigma \right) \right\} - \frac{\xi(\xi+1)}{2} \left(\sigma - \frac{q^2}{\kappa c} \sigma_q \right)^2 \right].$$

Given the risk-free rate, we obtain (μ_q, μ_e, μ_p) in terms of (p, q) and $(PPI_a, \sigma_a)_{a \in \{k,h\}}$.

Finally, we characterized the system of ordinary differential equations (A1) and (A2) coupled with the minimization problem (A7) that yields the worst-case distribution and PPI's, for all alternative combinations of parameter values for (g, ϕ). The set of those systems must be simultaneously solved.

A.10 Boundary Conditions

A.10.1 lower boundary

I adopt a similar lower boundary condition to He and Krishnamurthy (2019). When the scaled aggregate net worth e in the financial sector approaches to $\underline{e} \approx 0$, the government is assumed to intervene to sustain the asset prices. More specifically, it can convert βx units of capital (AK) to Ax units of net worth $\mathcal{E}$. Consequently, when a negative shock with magnitude ϵ sends $\mathcal{E}$ to $\underline{e}AK - \epsilon$ below $\underline{e}AK$,

$$\underbrace{\widehat{\mathcal{E}}}_{\text{net worth ex-post intervention}} = \underline{e}AK - A\epsilon + Ax = \overbrace{\underline{e}}^{\text{lower bound on } e} A \underbrace{\widehat{K}}_{\text{effective capital ex-post intervention}} = \underline{e}A(K - \beta x).$$

Solving for x,

$$x = \frac{\epsilon}{1 + \beta \underline{e}} > 0.$$

To preserve the no-arbitrage condition, the price of capital and housing must be the same ex-ante and ex-post intervention:

$$\underbrace{q\left(\underline{e} - \frac{\epsilon}{K}\right)A}_{\text{capital price before intervention}} = \underbrace{q(\underline{e})A}_{\text{after intervention}};$$

$$\underbrace{p\left(\underline{e} - \frac{\epsilon}{K}\right)AK}_{\text{housing price before intervention}} = \underbrace{p(\underline{e})A(K - \beta x)}_{\text{after intervention}}.$$

After rearranging these expressions, I obtain:

$$\frac{q(\underline{e}) - q\left(\underline{e} - \frac{\epsilon}{K}\right)}{\frac{\epsilon}{K}} = 0$$

and taking the limit $\epsilon/K \to 0$, the lower boundary condition for the capital price is given by

$$q'(\underline{e}) = 0.$$

Similarly,

$$\frac{p(\underline{e} - p\left(\underline{e} - \frac{\epsilon}{K}\right))}{\frac{\epsilon}{K}} = \frac{\beta p(\underline{e})}{1 + \underline{e}\beta}.$$

Taking the limit $\epsilon/K \to 0$, the lower boundary condition for the housing price is given by

$$p'(\underline{e}) = \frac{\beta p(\underline{e})}{1 + \underline{e}\beta}.$$

A.10.2 upper boundary

As $e \to \infty$, I impose that (p, q) are constant and the set of models $\Xi(p, q, r)$ converges to the singleton, and hence, there is no model uncertainty on the upper boundary. Then the lower boundary condition is given by $p'(e) = 0$ and $q'(e) = 0$ as $e \to \infty$.

These restrictions come from the following reasoning: as $e \to \infty$, the probability of the equity raising constraint binding becomes close to zero, so that the economy should behave similarly to the frictionless economy. This implication is similar to He and Krishnamurthy (2019). Then asset prices (p, q, r) should solely depend on (g, ϕ). Since there are three observables, each intermediary should be able to correctly back out the underlying two unknown parameter values (g, ϕ). Consequently, the model uncertainty should vanish on the upper boundary. Indeed, I confirm in the numerical solutions that as e becomes larger, the worst-case beliefs become indistinguishable from the actual values of (e, g, ϕ) before reaching the upper boundary. This illustrates the economic validity of this boundary condition.

A.11 Transition dynamics under objective and subjective beliefs

The Kolmogorov Forward equations characterize the distributions of transition dynamics of the underlying state variable e.

A.11.1 Objective distributions of the transition dynamics

Suppose the current state is denoted by e. Let $p_t(e, e')$ denote the density of the t-period ahead state e', conditional on the current state e. Then the evolution of this density is characterized by the solution to the following Kolmogorov forward equation:

$$\begin{aligned} \frac{\partial}{\partial t} p_t(e, e') &= -\frac{\partial}{\partial e'} \left[\mu_e(e') e' p_t(e, e')\right] + \frac{\partial^2}{\partial (e')^2} \left[\frac{1}{2} \sigma_e^2(e')(e')^2 p_t(e, e')\right] \\ &\doteq \mathcal{A}^* p_t(e, e'), \end{aligned} \tag{A15}$$

where $\mathcal{A}^*$ is the adjoint operator of the infinitesimal generater $\mathcal{A}$, which is defined for a function f,

$$\mathcal{A} f(e) \doteq \mu_e(e) e \frac{\partial}{\partial e} f(e) + \frac{1}{2} \sigma_e^2(e) e^2 \frac{\partial^2}{\partial e^2} f(e).$$

The long-run density $p(e)$ can be obtained by letting $t \to \infty$.

A.11.2 Subjective distributions of the transition dynamics

The subjective (worst-case) current latent state and parameters are denoted by $e^w(e)$. Then the subjective density over the $t-$period ahead future latent state, $p_t^w(e^w(e), e')$ is characterized as the solution to the following Kolmogorov Forward equation:

$$\frac{\partial}{\partial t}p_t^w(e^w(e),e') = -\frac{\partial}{\partial e'}\left[\mu_e^w(e')e'p_t^w(e^w(e),e')\right] + \frac{\partial^2}{\partial (e')^2}\left[\frac{1}{2}\sigma_e^2(e')(e')^2 p_t(e^w(e),e')\right]$$
$$\doteq \mathcal{A}^{w,*}p_t(e^w(e),e'),$$

where the drift and volatility terms are taken from the worst-case model in the state e:

$$\mu_e^w(e') \doteq \mu_e(e'; g^w, \phi^w);$$

$$\sigma_e^w(e') \doteq \sigma_e(e'; g^w, \phi^w).$$

Consequently, the functional form of the drift and volatility terms differs across true latent states e. $\mathcal{A}^{w,*}$ is the adjoint operator of the infinitesimal generater $\mathcal{A}^w$, which is defined for a function f, as

$$\mathcal{A}^w f(e) \doteq \mu_e^w(e)e\frac{\partial}{\partial e}f(e) + \frac{1}{2}(\sigma_e^w)^2(e)e^2\frac{\partial^2}{\partial e^2}f(e). \quad \text{(A16)}$$

The long-run subjective density $p^w(e';e)$ conditional on the current latent state e can be obtained by letting $t \to \infty$. The unconditional subjective long-run density is defined as

$$p_t^S(e') \doteq \int_{-\infty}^{\infty} p^w(e';e)p(e)de.$$

A.12 Formulas for computing conditional and unconditional moments

The conditional expectation of a random variable $X_{t+\tau} \doteq X(e_{t+\tau})$ under the objective probability is given by

$$E_t[X_{t+\tau}] = \int_{-\infty}^{\infty} X(e_{t+\tau})p(e_t, e_{t+\tau})de_{t+\tau}.$$

The unconditional counterpart is computed by

$$E[X] = \int_{-\infty}^{\infty} X(e)p(e)de.$$

Similarly, the conditional expectation of a random variable $X_{t+\tau} \doteq X(e_{t+\tau})$ under the subjective probability is given by

$$E_t^S[X_{t+\tau}] = \int_{-\infty}^{\infty} X(e_{t+\tau}) p^w(e^w(e_t), e_{t+\tau}) de_{t+\tau}.$$

The unconditional counterpart is computed by

$$E^S[X] = \int_{-\infty}^{\infty} X(e) p^S(e) de.$$

The $T-t$-horizon conditional risk premium for a risky asset $a \in \{k, h\}$ and risk-free asset return under the objective measure is defined as

$$g(e,t) \doteq E_t \left[\int_t^T h_s ds \right],$$

where h_t is the infinitesimal counterpart. Then the Feynman-Kac formula states that $g(e_t, t)$ is the solution to the following ordinary differential equation:

$$h(e,t) + g_e(e,t)\mu_e(e)e + \frac{1}{2} g_{ee}(e,t)\sigma_e(e)^2 e^2 + g_t(e,t) = 0.$$

with a terminal condition $g(e,T) \equiv 0$, where h^w is the infinitesimal counterpart under the subjective measure. Similarly, under the subjective measure

$$g^w(e^w(e),t) \doteq E_t^w \left[\int_t^T h_s^w ds \right],$$

The Feynman-Kac formula states that $g^S(e^w(e_t), t)$ is the solution to the following ordinary differential equation:

$$h^w(e,t) + g_e^w(e,t)\mu_e^w(e)e + \frac{1}{2} g_{ee}^w(e,t)\sigma_e^w(e)^2 e^2 + g_t^w(e,t) = 0.$$

B Appendix B. Numerical Appendix

The equilibrium concept in this paper requires the alternative equilibrium price functions $\{p(e; g, \phi), q(e; g, \phi)\}$ parameterized by different values of (g, ϕ) to be solved for jointly. This requirement comes from the minimization problem in (A25) that the alter egos of the uncertain-averse intermediaries solve to guard their portfolio choice against the ambiguity. They understand the structure of all the alternative economies in the sense of the equilibrium mapping from the unknown parameters and state (e, g, ϕ) onto the asset prices (p, q, r). Since the alter ego constructs the set of models given the set of alternative equilibrium price functions and the intermediary affects the asset prices under the worst-case beliefs through the portfolio choice, all the alternative equilibria affect each other jointly and hence, need to be solved for simultaneously.

B.1 Solution method for asset prices

I solve the set of Markov full equilibria by following the two step. First, I solve the set of rational expectations equilbria of the economies parameterized by alternative values of (g, ϕ). I Start from the solution for the frictionless economy as the initial guess and sequentially solve the set of ordinary differential equations given by (A9) and (A10) for (p, q) by using the quasi-implicit method (Achdou, Han, Lastry, Lions, and Moll (2022); Maxed (2023)).

Then I use the set of price functions (p, q, r) under those rational expectations equilibria as the initial guess for the set of Markov full equilibria. More specifically, intermediaries use those functions as the prespecified price functions to form the admissible sets. Given their worst-case beliefs, I obtain the set of ordinary differential equations (A9) and (A10) and can solve them for the set of Markov temporary equilibria using the quasi-implicit method. Then I use the new set of Markov temporary equilibria as the next guess as the prespecified price functions to solve the new set of Markov temporary equilibria. I repeat this procedure until the new solutions are numerically indistinguishable from the old solution, finding the Markov full equilibria.

B.2 Objective and subjective long-run density

Given the solution for the Markov full equilibria, I can solve the Kolmogorov forward equations in the long-run limit for the alternative stationary distributions. In the baseline economy, intermediaries have the state-dependent worst-case models parameterized by a worst-case (g, ϕ). In each state, their worst-case (subjective) long-run distribution corresponds to the stationary distribution of the Markov full equilibrium parameterized by the worst-case (g, ϕ).

B.3 Long-run objective and subjective expectations

Similarly to the long-run density, the worst-case (subjective) expectations operators are state-dependent. In each state, the subjective expectations use the transition dynamics under the Markov full equilibrium parameterized by the current worst-case (g, ϕ).

C Appendix C. Empirical Appendix

C.1 List of primary dealers included in I/B/E/S earning estimates

Intermediary	Start date	End date	Intermediary	Start date	End date
Bank of America	1997Q3	2024Q3	Mizuho Financial	2008Q2	2024Q3
Barclays	2012Q3	2024Q3	Morgan Stanley	1990Q1	2024Q3
Bank of Boston Corp	1985Q4	1999Q2	National Bank CP	1992Q1	1998Q2
Bank of Montreal	1994Q3	2024Q3	NCNB CP NC	1983Q4	1991Q3
BNP Paribas	2004Q4	2024Q3	Nomura Holdings	2004Q4	2023Q3
Bank of Nova Scotia	2002Q1	2024Q3	Northern Trust CP IL	1983Q4	2024Q3
Bear Sterns	1986Q3	2007Q4	Natwest Group	2020Q3	2024Q3
Bankers Trust	1984Q1	1999Q1	Bankc One CP OH	1986Q1	2004Q1
COML Credit	1987Q2	1988Q3	Prudential Financial	2002Q1	2024Q3
Citi Group	1998Q3	2024Q3	Royal Bank of Scotland	2012Q3	2020Q1
Countrywide Financial	2002Q4	2008Q1	Royal Bank of Canada	2004Q4	2024Q3
Chemical New York	1983Q4	1995Q4	Toronto-Dominion Bank	2004Q4	2024Q3
Credit Suisse	2005Q2	2023Q2	UBS	2005Q2	2024Q3
Deutsche Bank	2009Q1	2024Q3	Zions	1984Q1	2024Q4
Daiwa Securities	2004Q3	2023Q3	Goldman Sachs	1993Q2	2024Q3
First Interstate	2010Q3	2024Q3	HSBC Holdings	2012Q3	2024Q3
First Chicago	1983Q1	1995Q3	Jefferies Group	1999Q3	2024Q3
FNB Corporation	2004Q1	2024Q3	JP Morgan Chase	1996Q2	2024Q3
Societe Generale	2005Q2	2024Q3	MF Global	2007Q3	2011Q3

Table 4: List of primary dealers in I/B/E/S detailed files