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Fertility, Education, Inequality, and Economic Growth

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Introduction

This paper extends the Barro–Becker model of endogenous fertility and intergenerational transfer by incorporating

- parental education spending or human capital investment,
- idiosyncratic working ability shocks, wealth shocks, and human capital shocks.

This paper then calibrates the model to the U.S. economy, consistent with the empirical findings,

- a positive relationship between labor income or working ability and the number of children,
- a positive/negative relationship between the number of children married couple's working hours.

This paper finally evaluates child-related policies, including child allowances, education subsidies, childcare & paid leave subsidies.

Previous Literature

Dynasty-OLG model with intergenerational transfers

- Loury (ECMA 1981), Laitner (JET 1992), Nishiyama (RED 2002)

Dynasty model with endogenous fertility and intergen. transfers

- Becker & Barro (QJE 1988), Barro & Becker (ECMA 1989)

Bewley dynasty model with endogenous fertility and intergen. transfers

- Alvarez (RED 1999)

OLG model with endogenous fertility and human capital investment

- De la Croix and Doepke (AER 2003)

Dynasty-OLG model with child-related policies

- Stantcheva (NBER 2015), Guner et al. (RES 2020), and others

OLG/dynasty model with fertility, transfers, and human cap. investment

- Córdoba et al. (JME 2016), Daruich & Kozlowski (RED 2020), Zhou (2022), and others

Income/Hourly Wages on Number of Children

The models we estimate with WLS are

$$kids = \beta_0 + \beta_1 \ln income + \boldsymbol{\gamma}\mathbf{X} + u,$$

$$kids = \beta_0 + \beta_1 \ln wageinc + \boldsymbol{\gamma}\mathbf{X} + u,$$

$$kids = \beta_0 + \beta_1 \ln hwage + \boldsymbol{\gamma}\mathbf{X} + u,$$

$$kids = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \boldsymbol{\gamma}\mathbf{X} + u,$$

where

$$\gamma\,\mathbf{X} = \gamma_1\, age_f + \gamma_2\, age_f^2 + \gamma_3\, educ\_hs_m + \gamma_4\, educ\_hs_f + \gamma_5\, race\_wh + \gamma_6\, year_{2016}.$$

Summary Stats of SCF 2016 & 2019 (Ages 35–54)

	Mean	SD	Min	Max	N
Married Households with Wives' Ages between 35 *and* 54					
$kids$	1.5541	1.2449	0.	7.	2606
$\ln income$	11.7285	0.8479	7.7666	18.8776	2606
$\ln wageinc$	11.5352	0.9126	2.4683	16.1175	2606
$\ln hwage$	3.2668	0.6848	2.3026	8.1069	2606
$\ln hwage_m$	3.2376	0.7630	2.3026	8.4780	2606
$\ln hwage_f$	2.9109	0.6540	2.3026	7.5104	2606
age_f	44.1600	5.7351	35.	54.	2606
$educ\_hs_m$	0.8834	0.3210	0.	1.	2606
$educ\_hs_f$	0.9144	0.2798	0.	1.	2606
$race_wh$	0.6440	0.4789	0.	1.	2606

Income/Hourly Wages on Number of Children 1

	$kids$				$hours_m$	$hours_f$
	(1)	(2)	(3)	(4)	(6)	(7)
$\ln income$	0.1613					
	(0.0238)					
$\ln wageinc$		0.1279				
		(0.0220)				
$\ln hwage$			0.1947			
			(0.0287)			
$\ln hwage_m$				0.1628	356.94	-275.17
				(0.0260)	(19.48)	(19.73)
$\ln hwage_f$				-0.0145	-125.87	758.63
				(0.0335)	(19.86)	(26.95)
$kids$					34.27	-113.77
					(8.85)	(10.91)
$health_m$					-155.86	35.68
					(18.05)	(19.81)
$health_f$					56.36	-57.40
					(18.94)	(19.21)

Income/Hourly Wages on Number of Children 2

age_m					56.41	
					(8.24)	
age_m^2					-0.75	
					(0.10)	
age_f	0.2344	0.2355	0.2472	0.2426		62.99
	(0.0762)	(0.0759)	(0.0763)	(0.0767)		(14.77)
age_f^2	-0.0034	-0.0034	-0.0035	-0.0035		-0.81
	(0.0009)	(0.0009)	(0.0009)	(0.0009)		(0.18)
$educ\_hs_m$	-0.2941	-0.2758	-0.2782	-0.2834	172.14	92.06
	(0.0682)	(0.0669)	(0.0643)	(0.0648)	(42.59)	(44.61)
$educ\_hs_f$	-0.0910	-0.0703	-0.0553	-0.0157	-78.51	409.50
	(0.0769)	(0.0766)	(0.0756)	(0.0757)	(54.12)	(50.76)
$race\_wh$	-0.1518	-0.1361	-0.1383	-0.1372	109.70	-26.03
	(0.0462)	(0.0457)	(0.0453)	(0.0454)	(26.11)	(24.29)
$year_{2016}$	0.0135	0.0116	0.0300	0.0233	19.47	15.99
	(0.0426)	(0.0425)	(0.0430)	(0.0432)	(20.89)	(22.26)
$intercept$	-3.5009	-3.1816	-2.6097	-2.4116	385.62	-1244.96
	(1.6765)	(1.6558)	(1.6696)	(1.6754)	(187.71)	(282.36)
N	2606	2606	2606	2606	3559	3559
R^2	0.1145	0.1122	0.1149	0.1136	0.1518	0.2953
SD	0.0004	0.0003	0.0003	0.0002	0.0011	0.0015

Hourly Wages & No. of Children on Work Hours

The models we estimate with WLS are

$$hours_m = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \beta_3\, kids + \boldsymbol{\delta}\mathbf{X}_m + u,$$

$$hours_f = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \beta_3\, kids + \boldsymbol{\delta}\mathbf{X}_f + u,$$

where

$$\begin{aligned}\boldsymbol{\delta}\mathbf{X}_m &= \delta_1\, health_m + \delta_2\, health_f + \delta_3\, age_m + \delta_4\, age_m^2 \\ &\quad + \delta_5\, educ\_hs_m + \delta_6\, educ\_hs_f + \delta_7\, race\_wh + \delta_8\, year_{2016}, \\ \boldsymbol{\delta}\mathbf{X}_f &= \delta_1\, health_m + \delta_2\, health_f + \delta_3\, age_f + \delta_4\, age_f^2 \\ &\quad + \delta_5\, educ\_hs_m + \delta_6\, educ\_hs_f + \delta_7\, race\_wh + \delta_8\, year_{2016}.\end{aligned}$$

Summary Stats of SCF 2016 & 2019 (Ages 25–54)

	Mean	SD	Min	Max	N
Married Households with Wives' Ages between 25 and 54					
$hours_m$	2151.95	882.62	0.	6760.	3563
$hours_f$	1473.30	994.98	0.	5200.	3563
$kids$	1.5010	1.2598	0.	7.	3563
$\ln hwage_m$	3.1769	0.7125	2.3026	9.0628	3563
$\ln hwage_f$	2.8697	0.6178	2.3026	7.5104	3563
$health_m$	1.9564	0.7190	1.	4.	3563
$health_f$	1.8770	0.7230	1.	4.	3563
age_m	41.9362	9.8801	21.	86.	3563
age_f	39.3668	8.3633	25.	54.	3563
$educ\_hs_m$	0.8910	0.3116	0.	1.	3563
$educ\_hs_f$	0.9236	0.2658	0.	1.	3563
$race_w h$	0.6391	0.4803	0.	1.	3563

Hourly Wages & No. of Children on Work Hours 1

	$kids$				$hours_m$	$hours_f$
	(1)	(2)	(3)	(4)	(6)	(7)
$\ln income$	0.1613					
	(0.0238)					
$\ln wageinc$		0.1279				
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$health_f$					56.36	-57.40
					(18.94)	(19.21)

Hourly Wages & No. of Children on Work Hours 2

age_m					56.41	
					(8.24)	
age_m^2					-0.75	
					(0.10)	
age_f	0.2344	0.2355	0.2472	0.2426		62.99
	(0.0762)	(0.0759)	(0.0763)	(0.0767)		(14.77)
age_f^2	-0.0034	-0.0034	-0.0035	-0.0035		-0.81
	(0.0009)	(0.0009)	(0.0009)	(0.0009)		(0.18)
$educ\_hs_m$	-0.2941	-0.2758	-0.2782	-0.2834	172.14	92.06
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	(0.0426)	(0.0425)	(0.0430)	(0.0432)	(20.89)	(22.26)
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Gov't Policy Variables and Factor Prices

Let Ω_t be a time series of gov't policy variables and factor prices,

$$\Omega_t = \left\{\tau_{k,s}, \tau_{h,s}, \tau_{c,s}, \tau_{n,s}, \tau_{b,s}, \tau_{e,s}, \tau_{p,s}, tr_{p,s}, \vartheta_s, \psi_s, g_s, r_s, w_s\right\}_{s=t}^{\infty},$$

where

- $\tau_{h,t}$ is a labor income tax rate,
- $\tau_{c,t}$ is a consumption tax rate,
- $\tau_{n,t}$ is a child tax (allowance if negative) rate,
- $\tau_{b,t}$ is an estate (inter vivos transfer) tax rate,
- $\tau_{e,t}$ is an education tax (subsidy if negative) rate,
- ϑ_t is public education spending per child,
- ψ_t is childcare and paid leave subsidies (in hours) per child.

Household's Optimization Problem 1

The household's optimization problem is

$$v(a_t, h_t, \varepsilon_t; \Omega_t) = \max_{c_t, s_t, n_t, b_t, e_t} \Big\{ u(c_t) + \tilde{\beta} u(d_{t+1}) + \tilde{\gamma}\, \Phi(n_t) \mathbb{E}_t \big[\, v(a_{t+1}, h_{t+1}, \varepsilon_{t+1}; \Omega_{t+1}) \,\big] \Big\}$$

subject to the budget constraint,

$$(1+\tau_{c,t})(1+n_t)^{\xi} c_t + s_t + \tau_{n,t} n_t + (1+\tau_{b,t}) b_t n_t + (1+\tau_{e,t}) e_t n_t$$
$$= (1+(1-\tau_{k,t}) r_t) a_t + (1-\tau_{h,t}-\tau_{p,t}) w_t h_t \exp(\varepsilon_t)(1-(\phi-\psi_t) n_t),$$

$$(1+\tau_{c,t+1}) d_{t+1} = (1+(1-\tau_{k,t+1}) r_{t+1}) \frac{s_t}{1+\rho} + (1-\tau_{h,t+1}-\tau_{p,t+1}) w_{t+1} \frac{h_t \exp(\varepsilon_t)\, \zeta}{1+\rho} + tr_{p,t+1},$$

Household's Optimization Problem 2

and the laws of motion of the state variables,

$$a_{t+1} = \frac{1}{1+\rho} b_t \exp(\epsilon_{a,t+1}) \geq 0, \qquad \epsilon_{a,t+1} \sim N(-\sigma^2_{\epsilon_a}/2, \sigma^2_{\epsilon_a}),$$

$$h_{t+1} = \frac{1}{1+\rho} \Lambda(e_t, h_t, \vartheta_t; \bar{h}_t) \exp(\epsilon_{h,t+1}) > 0, \qquad \epsilon_{h,t+1} \sim N(-\sigma^2_{\epsilon_h}/2, \sigma^2_{\epsilon_h}),$$

$$\varepsilon_{t+1} \sim N(-\sigma^2_{\varepsilon}/2, \sigma^2_{\varepsilon}),$$

where the inter-generational law of motion of human capital is

$$\Lambda(e_t, h_t, \vartheta_t; \bar{h}_t) = B(\vartheta_t + e_t)^{\eta}\, h_t^{\tau}\, \bar{h}_t^{\kappa},$$

and non-negativity constraints are

$$c_t > 0, \qquad s_t \geq 0, \qquad n_t \geq 0, \qquad b_t \geq 0, \qquad e_t \geq 0.$$

Household's Optimization Problem 3

The period utility function is

$$u(c_t) = \frac{c_t^{1-\sigma} - c_{\min}^{1-\sigma}}{1-\sigma}, \qquad u'(c_t) > 0, \qquad u''(c_t) < 0,$$

where $c_{\min}$ is a minimum level of consumption so that $u(c_t) > 0$.

The scaling function of n_t children is

$$\Phi(n_t) = \frac{1 - \exp(-\mu n_t)}{1 - \exp(-\mu)}, \qquad \Phi(0) = 0, \qquad \Phi(1) = 1,$$

$$\lim_{n_t \to \infty} \Phi(n_t) = \frac{1}{1 - \exp(-\mu)}, \quad \lim_{\mu \to 0} \Phi(n_t) = n_t, \quad \lim_{\mu \to \infty} \Phi(n_t) = 1,$$

where μ is inversely related to the elasticity of $\Phi(n_t)$ with respect to n_t.

Joint Distribution of Households

Let $X_t(a_t, h_t, \varepsilon_t)$ be a cumulative distribution function, where

$$\int_{A\times H\times \mathbf{R}} dX_t(a_t, h_t, \varepsilon_t) = 1.$$

Let $\bar{n}_t$ be the total (and average) number of children,

$$\bar{n}_t = \int_{A\times H\times \mathbf{R}} n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t).$$

The law of motion of the density funcion, $x_t(a_t, h_t, \varepsilon_t)$, is

$$\begin{aligned} x_{t+1}(a_{t+1}, h_{t+1}, \varepsilon_{t+1}) = \frac{1}{\bar{n}_t} \int_{A\times H\times \mathbf{R}^3} \\ \mathbf{1}_{\{a_{t+1}=a_{t+1}(a_t,h_t,\varepsilon_t,\epsilon_{a,t+1};\Omega_t)\}} \mathbf{1}_{\{h_{t+1}=h_{t+1}(a_t,h_t,\varepsilon_t,\epsilon_{h,t+1};\Omega_t)\}} \\ \times\, n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dF_a(\epsilon_{a,t+1})\, dF_h(\epsilon_{h,t+1})\, dX_t(a_t, h_t, \varepsilon_t). \end{aligned}$$

Aggregation 1

Total private consumption in period t is

$$C_t = \int_{A\times H\times \mathbf{R}} (1+n_t(a_t,h_t,\varepsilon_t;\Omega_t))^{\xi} c_t(a_t,h_t,\varepsilon_t;\Omega_t)\, dX_t(a_t,h_t,\varepsilon_t)$$
$$+\frac{1}{\bar{n}_{t-1}}\int_{A\times H\times \mathbf{R}} d_t(a_{t-1},h_{t-1},\varepsilon_{t-1};\Omega_{t-1})\, dX_{t-1}(a_{t-1},h_{t-1},\varepsilon_{t-1}).$$

Total private wealth at the beginning of period t is

$$W_t = \int_{A\times H\times \mathbf{R}} a_t\, dX_t(a_t,h_t,\varepsilon_t)$$
$$+\frac{1}{\bar{n}_{t-1}}\int_{A\times H\times \mathbf{R}} \frac{s_{t-1}(a_{t-1},h_{t-1},\varepsilon_{t-1};\Omega_{t-1})}{1+\rho}\, dX_{t-1}(a_{t-1},h_{t-1},\varepsilon_{t-1}).$$

Aggregation 2

Total labor supply in efficiency units in period t is

$$L_t^s = \int_{A\times H\times \mathbf{R}} h_t \exp(\varepsilon_t)(1-(\phi-\psi_t)n_t(a_t,h_t,\varepsilon_t;\Omega_t))\, dX_t(a_t,h_t,\varepsilon_t) + \frac{1}{\bar{n}_{t-1}}\int_{A\times H\times \mathbf{R}} \frac{h_{t-1}\exp(\varepsilon_{t-1})\zeta}{1+\rho}\, dX_{t-1}(a_{t-1},h_{t-1},\varepsilon_{t-1}).$$

Total inter vivos transfers at the end of period t is

$$B_t = \int_{A\times H\times \mathbf{R}} b_t(a_t,h_t,\varepsilon_t;\Omega_t)n_t(a_t,h_t,\varepsilon_t;\Omega_t)\, dX_t(a_t,h_t,\varepsilon_t).$$

Total private education spending in period t is

$$E_t = \int_{A\times H\times \mathbf{R}} e_t(a_t,h_t,\varepsilon_t;\Omega_t)n_t(a_t,h_t,\varepsilon_t;\Omega_t)\, dX_t(a_t,h_t,\varepsilon_t).$$

Representative Firm's Problem

The production function is

$$Y_t = F(K_t, L_t) = A\,K_t^{\alpha} L_t^{1-\alpha},$$

the profit maximization problem is

$$\max_{K_t, L_t} A\,K_t^{\alpha} L_t^{1-\alpha} - (r_t + \delta)K_t - w_t L_t,$$

and the first order conditions are

$$r_t = \alpha A \left(\frac{K_t}{L_t}\right)^{\alpha-1} - \delta, \qquad w_t = (1-\alpha)A\left(\frac{K_t}{L_t}\right)^{\alpha}.$$

The market clearing conditions are

$$K_t = W_t, \qquad L_t = L_t^s.$$

Government's Budget 1

Total government consumption in period t is

$$G_t = \left(1 + \frac{1}{\bar{n}_{t-1}}\right) g_t,$$

total public education spending in period t is

$$\Theta_t = \int_{A \times H \times \mathbf{R}} \vartheta_t n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t) = \vartheta_t \bar{n}_t,$$

total cost of childcare and paid leave subsidies in period t is

$$\Psi_t = \int_{A \times H \times \mathbf{R}} w_t \bar{h}_t \psi_t n_t(a_t, h_t, \varepsilon_t; \Omega_t)\, dX_t(a_t, h_t, \varepsilon_t) = w_t \bar{h}_t \psi_t \bar{n}_t,$$

where $\bar{h}_t$ is the average human capital of the young households.

Government's Budget 2

Total public pension benefits to old households in period t is

$$P_t = \frac{1}{\bar{n}_{t-1}} \int_{A \times H \times \mathbf{R}} tr_{p,t}\, dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}) = \frac{tr_{p,t}}{\bar{n}_{t-1}}.$$

The government's budget constraint of the general account is

$$\tau_{c,t} C_t + \tau_{k,t} r_t K_t + \tau_{h,t} w_t L_t + \tau_{b,t} B_t + \tau_{e,t} E_t + \tau_{n,t} \bar{n}_t = G_t + \Theta_t + \Psi_t,$$

and the budget constraint of the public pension account is

$$\tau_{p,t} w_t L_t = P_t.$$

In the policy experiments, the general account and the pension account are combined, and the consumption tax finances policies.

Main Parameter Values and Baseline Policy Values 1

Main Parameters			
Share parameter of capital income	α	0.3600	
Discount factor	β	0.5277	$K/Y = 3.0/30$ (baseline)
Growth-adjusted discount factor	$\tilde{\beta}$	0.4545	$(1+\rho)^{1-\sigma}\beta$
Growth-adjusted discount and altruism factor	$\tilde{\gamma}$	0.3636	$0.8\,\tilde{\beta}$
Capital depreciation rate	δ	0.7854	$1-0.95^{30}$ (5% annual rate)
Share of working years in the old period	ζ	0.3333	$10/30$ (retire at age 65)
Education elasticity of human capital	η	0.5000	
Parents' human capital elasticity of human capital	τ	0.2000	De la Croix and Doepke (2003)
Average human capital elasticity of human capital	κ	0.0000	
Hazard rate of exponential child discount factor	μ	1.7397	Wage elasticity of $n_t = 0.125$
Adult equivalent scale for consumption	ξ	0.5000	
Long run productivity growth rate	ρ	0.3478	$1.01^{30}-1$ (1% annual rate)
Coefficient of relative risk aversion	σ	1.5000	
Time cost of childcare per unit of children	ϕ	0.1000	

Main Parameter Values and Baseline Policy Values 2

Scaling and Other Parameters			
Scaling parameter (TFP) of production function	A	3.0482	$w_t = 1.0$ (baseline)
Scaling parameter of human capital function Λ	B	3.2810	$\bar{h}_t = 1.0$ (baseline)
Shifting parameter of utility function u	$c_{\min}$	0.1014	$\bar{n}_t = 1.0$ (baseline)
SD of log labor income shocks ε_t	σ_ε	0.2000	
SD of log wealth shocks $\epsilon_{a,t}$	σ_a	0.1000	
SD of log human capital shocks $\epsilon_{h,t}$	σ_h	0.1000	
Baseline Government Policies			
Capital income tax rate	$\tau_{k,t}$	0.1000	
Labor income tax rate	$\tau_{h,t}$	0.1000	
Consumption tax rate	$\tau_{c,t}$	0.1000	
Child tax rate	$\tau_{n,t}$	0.0000	
Estate (gift inter vivos) tax rate	$\tau_{b,t}$	0.0000	
Education tax rate	$\tau_{e,t}$	0.0000	
PAYG public pension (payroll) tax rate	$\tau_{p,t}$	0.1000	
PAYG public pension benefit	$tr_{p,t}$	0.1146	
Public education spending per child	ϑ_t	0.0716	$\vartheta_t \bar{n}_t = 0.04\,Y_t$
Childcare and paid leave subsidies per child	ψ_t	0.0000	
Other government consumption	g_t	0.1069	

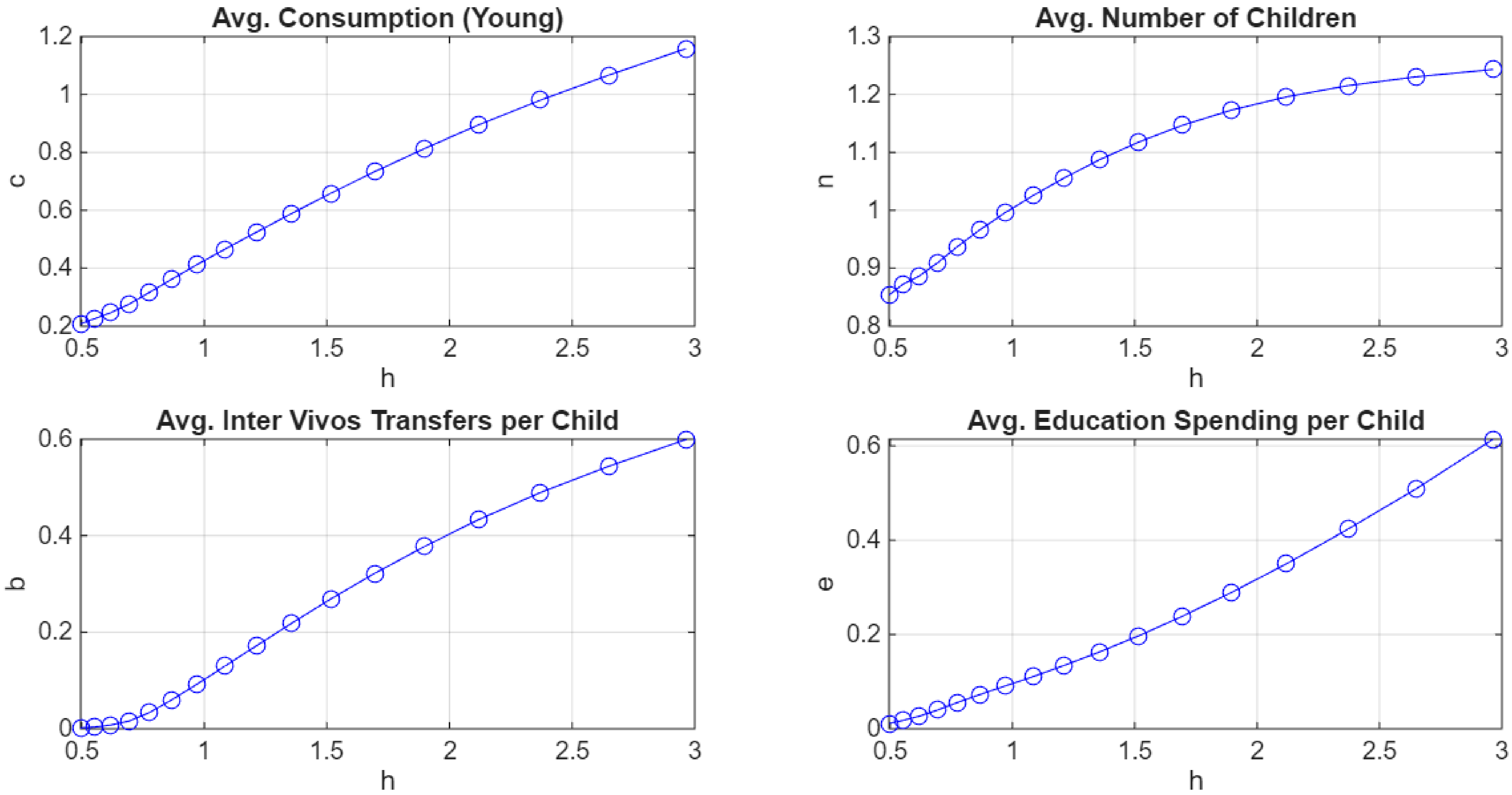
Main Variables and Statistics in the Baseline 1

Individual Variables (population weighted average)			
Initial wealth (young)	$\bar{a}_t$	0.0801	
Human capital	$\bar{h}_t$	1.0000	normalized to unity
Consumption (young)	$\bar{c}_t$	0.4244	
Saving	$\bar{s}_t$	0.1335	
Number of children	$\bar{n}_t$	1.0000	normalized to unity
Inter vivos transfer per unit of children	$\bar{b}_t$	0.1040	
Education expenditure per unit of children	$\bar{e}_t$	0.0964	
Consumption (old)	$\bar{d}_{t+1}$	0.6022	
Initial wealth of children	$\bar{a}_{t+1}$	0.0771	
Human capital of children	$\bar{h}_{t+1}$	0.9921	
Value (lifetime utility)	$\bar{v}_t$	7.4946	
Aggregate Variables (per young household)			
Consumption	C_t	1.2043	$\bar{c}_t + \bar{d}_t/\bar{n}_t$
Capital stock	K_t	0.1791	$\bar{a}_t + \bar{s}_t/(\bar{n}_t(1+\rho))$
Labor supply	L_t	1.1463	
Output	Y_t	1.7912	
Gross interest rate	$1 + r_t$	3.8146	4.56% annual rate
Wage rate	w_t	1.0000	normalized to unity

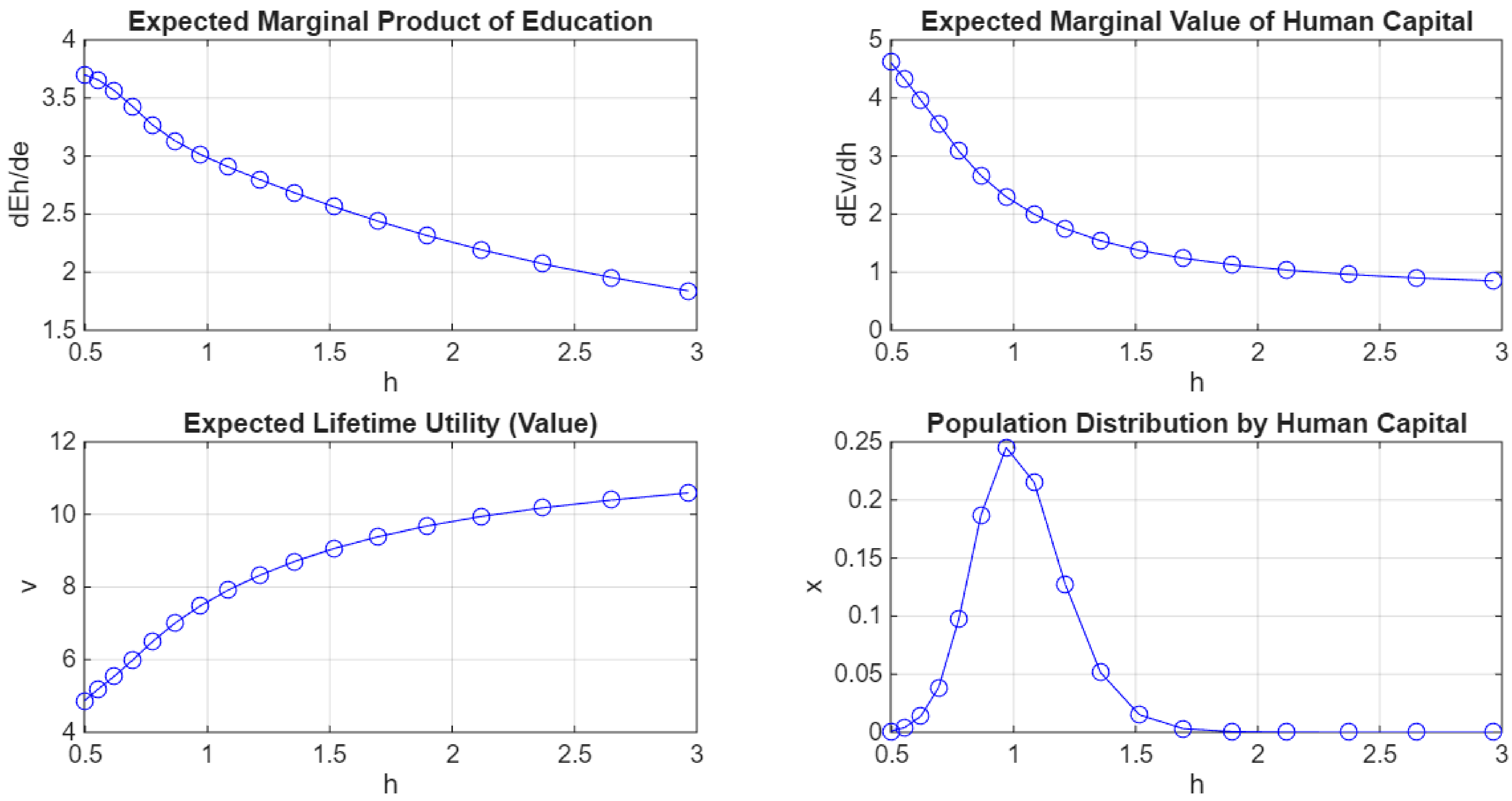
Main Variables and Statistics in the Baseline 2

Gini Coefficients		
Consumption	(c_t, d_t)	0.1763
Consumption and education	$(c_t + e_t n_t, d_t)$	0.1628
Labor income (young)	$(\tilde{h}_t l_t)$	0.1511
Labor income (young and old)	$(\tilde{h}_t l_t, \tilde{h}_{t-1}\zeta)$	0.3053
Wealth (young and old)	(a_t, s_{t-1})	0.3318
Wealth before inter vivos transfers	$(0, b_{t-1} + s_{t-1})$	0.6698
Intergenerational Correlations		
Human capital	(h_t, h_{t+1})	0.7707
Working ability	$(\tilde{h}_t, \tilde{h}_{t+1})$	0.4573
Labor income	$(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.4324
OLS Coefficients (Semi-elasticities)		
Income elasticity of n_t	$(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.1489
Labor income elasticity of n_t	$(n_t, \ln \tilde{h}_t l_t)$	0.1235
Working ability elasticity of n_t	$(n_t, \ln \tilde{h}_t)$	0.1250

Baseline Household Behaviors by Human Capital 1



Baseline Household Behaviors by Human Capital 2



Households' Problem Revisited

The child-related part of the household's optimization problem is

$$\max_{n_t, b_t, e_t} \tilde{\gamma}\Phi(n_t)v(a_{t+1}, h_{t+1})$$

subject to the budget constraint,

$$\begin{aligned}
&(1+\tau_{c,t})\big[\,(1+n_t)^{\xi}-1\,\big]c_t \\
&\qquad + \big[\,\tau_{n,t} + (1+\tau_{b,t})b_t + (1+\tau_{e,t})e_t + (1-\tau_{h,t}-\tau_{p,t})w_t h_t(\phi-\psi_t)\,\big]n_t \\
&\qquad = (1-\varsigma)\,\big[\,(1+(1-\tau_{k,t})r_t)a_t + (1-\tau_{h,t}-\tau_{p,t})w_t h_t\,\big] \equiv \mathrm{inc}_t,
\end{aligned}$$

and the laws of motion of the state variables,

$$a_{t+1} = b_t \geq 0, \qquad h_{t+1} = B(\vartheta_t + e_t)^{\eta}\, h_t^{\tau} > 0.$$

First Order Conditions

The first order conditions for the interior solution are

$$n_t: \quad \tilde{\gamma}\Phi'(n_t)v(a_{t+1}, h_{t+1}) = \lambda_t \Big[(1+\tau_{c,t})\xi(1+n_t)^{\xi-1}c_t + \tau_{n,t} + (1+\tau_{b,t})b_t$$
$$+ (1+\tau_{e,t})e_t + (1-\tau_{h,t}-\tau_{p,t})w_t h_t(\phi - \psi_t) \Big] \equiv \lambda_t \mathrm{mc}_{n,t},$$

$$b_t: \quad \tilde{\gamma}\Phi(n_t)v_a(a_{t+1}, h_{t+1}) = \lambda_t(1+\tau_{b,t})n_t \equiv \lambda_t \mathrm{mc}_{b,t},$$

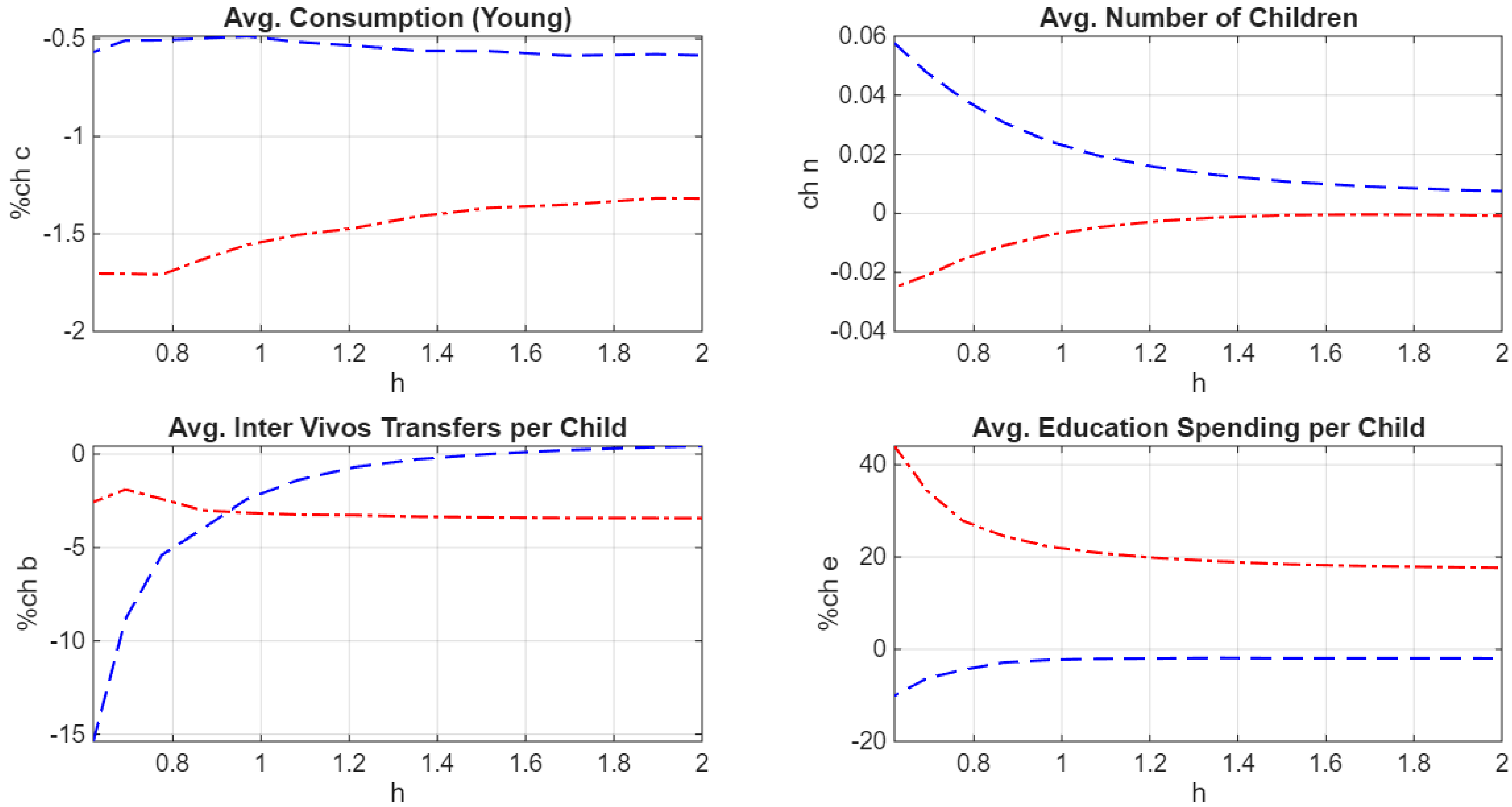
$$e_t: \quad \tilde{\gamma}\Phi(n_t)v_h(a_{t+1}, h_{t+1})B\eta(\vartheta_t + e_t)^{\eta-1} h_t^{\tau} = \lambda_t(1+\tau_{e,t})n_t \equiv \lambda_t \mathrm{mc}_{e,t},$$

where $\lambda_t > 0$ is the Lagrange multiplier and

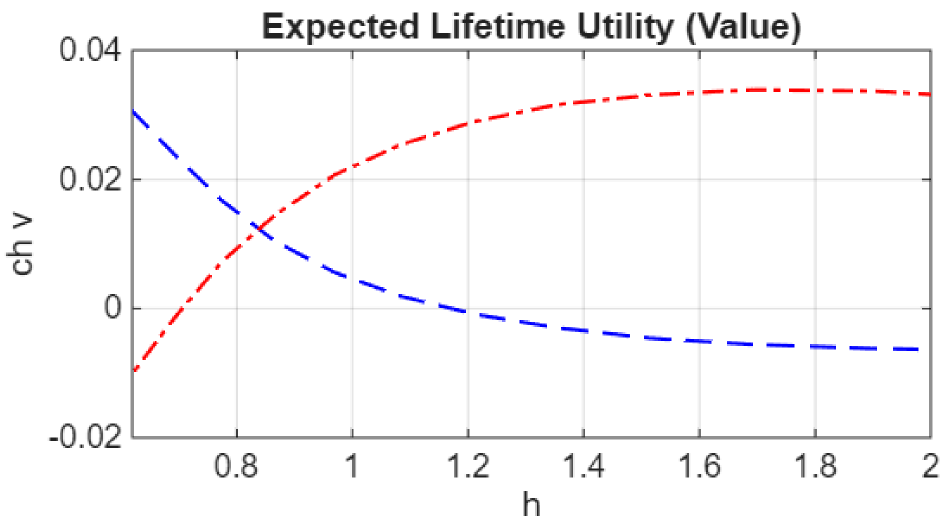
$$\lambda_t = \frac{u'(c_t)}{(1+\tau_{c,t})(1+n_t)^{\xi}}$$

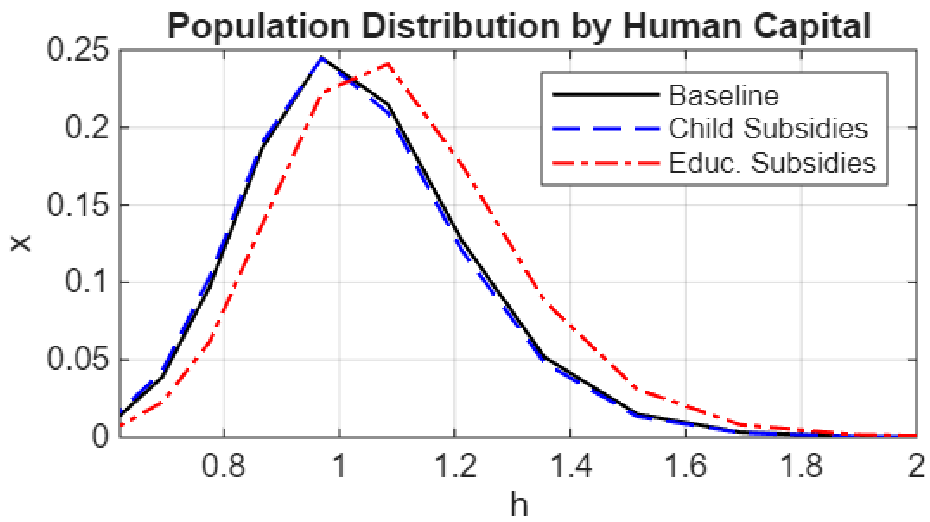
in the full model.

Child Allowances vs. Education Subsidies 1



Child Allowances vs. Education Subsidies 2





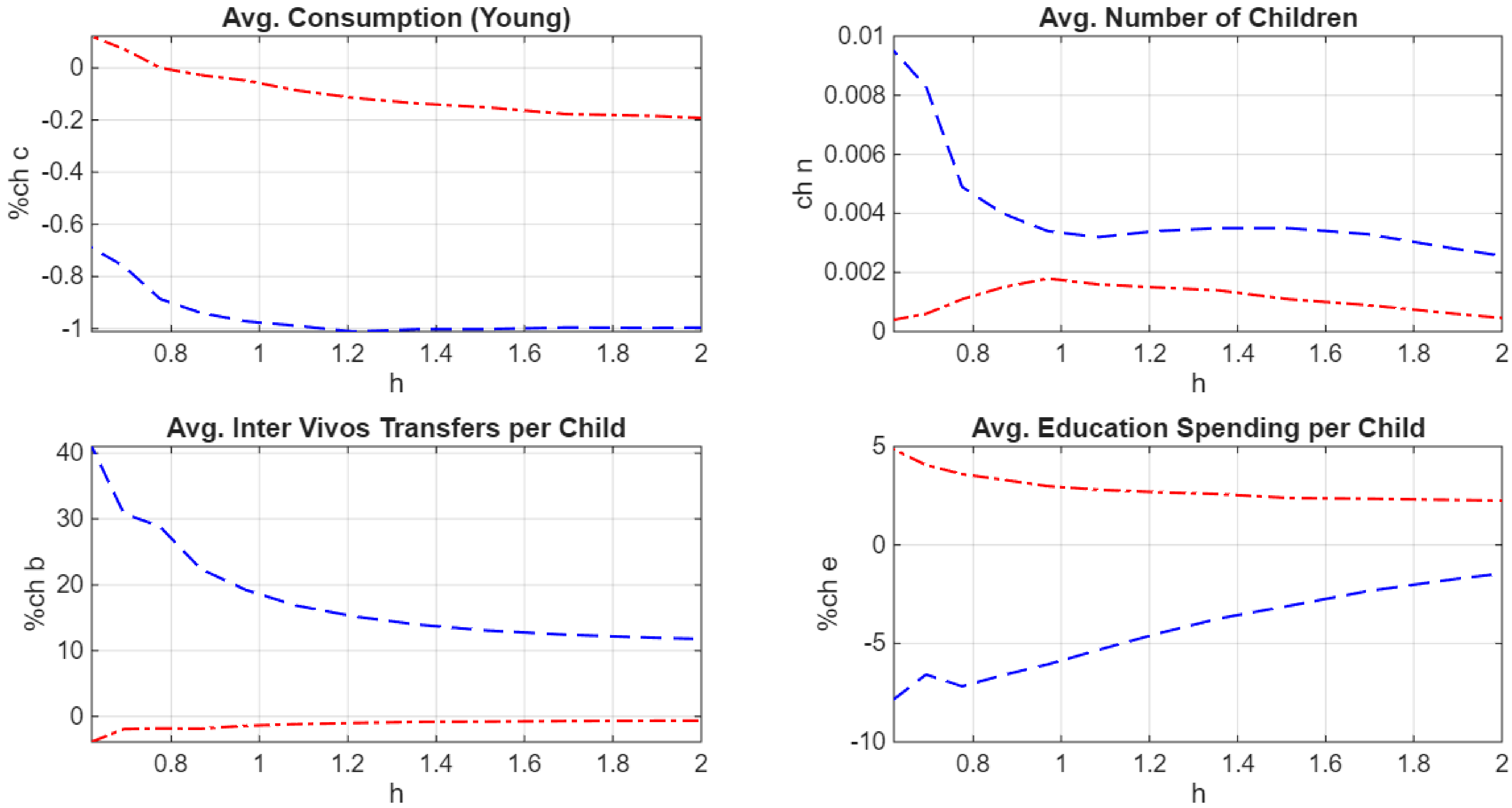
Child Allowances vs. Education Subsidies 1

	Baseline	Child Allowances				Education Subsidies			
Period t	0	1	2	3	∞	1	2	3	∞
Individual Variables (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	-2.5	-3.6	-6.9	-0.0	-2.7	1.5	15.6
$\bar{h}_t$	1.000	0.0	-0.9	-1.8	-4.3	0.0	6.2	9.0	19.9
$\bar{c}_t$	0.424	-0.5	-1.5	-2.5	-5.3	-1.5	1.4	4.4	15.7
$\bar{s}_t$	0.133	1.7	0.3	-0.5	-3.1	0.2	1.6	4.4	14.5
$\bar{n}_t$	1.000	2.6	2.2	1.8	0.9	-0.8	0.2	1.3	4.7
$\bar{b}_t$	0.104	-1.7	-2.9	-3.8	-6.4	-3.2	1.2	4.2	15.7
$\bar{e}_t$	0.096	-2.5	-4.8	-6.4	-11.2	22.0	27.6	33.8	58.9
$\bar{d}_{t+1}$	0.602	-0.0	-1.3	-2.2	-4.9	1.0	4.7	7.7	19.1
$\bar{a}_{t+1}$	0.077	-1.7	-2.9	-3.8	-6.4	-3.2	1.2	4.2	15.7
$\bar{h}_{t+1}$	0.992	-0.7	-1.6	-2.2	-4.1	6.1	8.9	11.1	19.7
$\bar{v}_t$	7.495	0.1	-0.5	-1.0	-2.4	0.3	1.8	3.2	7.7
$\bar{cv}_t$	0.000	0.2	-0.1	-0.8	-2.6	1.6	2.4	4.0	11.0
Initial Old Households (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-1.0			
$\bar{v}_{t-1}$	7.495	-0.0				0.0			
$\bar{cv}_{t-1}$	0.000	-0.9				0.5			

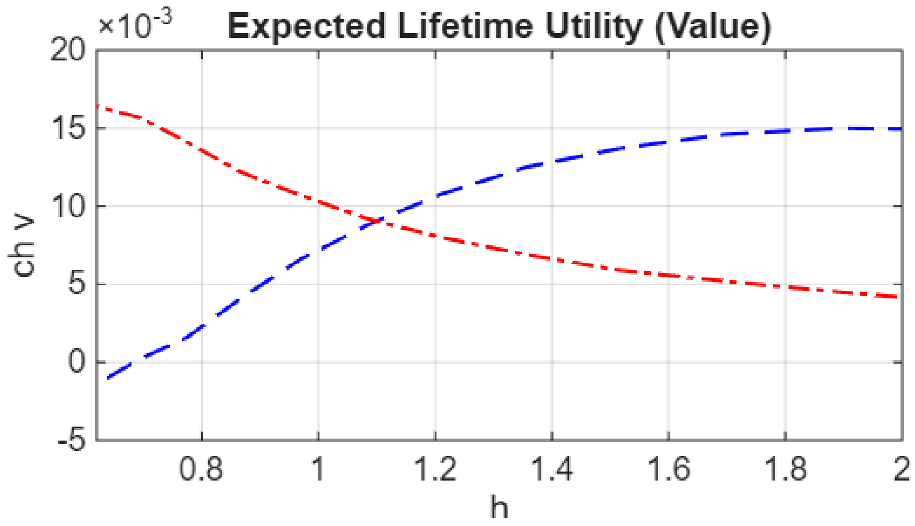
Child Allowances vs. Education Subsidies 2

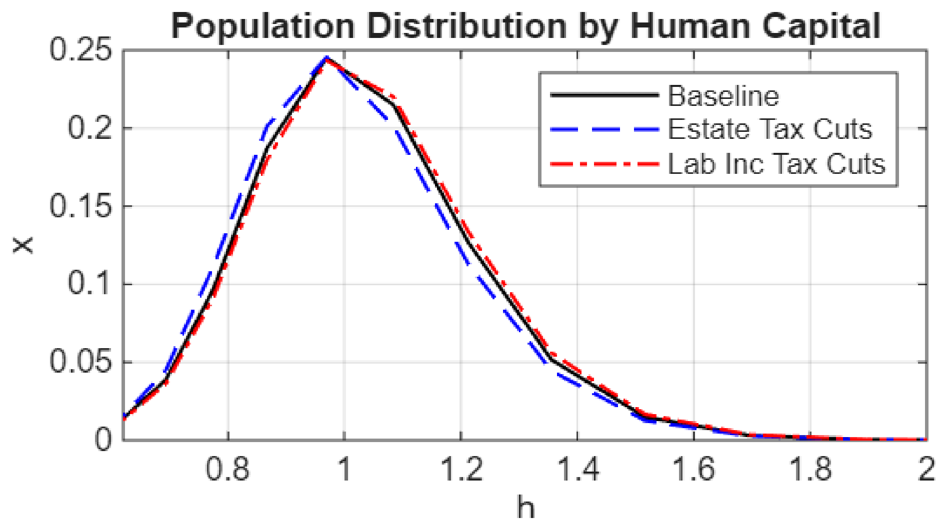
Aggregate Variables (per young household, % ch. from the baseline)									
C_t	1.204	-0.5	-1.8	-2.7	-5.5	-1.3	1.7	4.6	15.4
K_t	0.179	0.0	-1.6	-2.6	-5.3	0.0	-0.6	1.5	12.2
L_t	1.146	-0.2	-1.4	-2.2	-4.5	0.1	5.0	8.2	18.2
Y_t	1.791	-0.1	-1.5	-2.3	-4.8	0.0	2.9	5.7	16.0
$1+r_t$	3.815	-0.1	0.1	0.3	0.5	0.0	3.4	4.0	3.2
w_t	1.000	0.1	-0.1	-0.2	-0.3	-0.0	-2.0	-2.3	-1.9
Government Policy Variables (%, ch. in p.p. from the baseline)									
$\tau_{n,t}$	0.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00
$\tau_{e,t}$	0.00	0.00	0.00	0.00	0.00	-10.10	-10.10	-10.10	-10.10
$\tau_{c,t}$	0.00	**1.09**	1.06	1.41	2.52	**1.09**	0.32	-0.66	-3.88
Gini Coefficients									
$gini(c_t, d_t)$	0.176	0.177	0.179	0.180	0.181	0.179	0.176	0.177	0.183
$gini(\tilde{h}_t l_t)$	0.151	0.152	0.152	0.153	0.153	0.151	0.151	0.152	0.157
$gini(a_t, s_{t-1})$	0.332	0.332	0.338	0.342	0.353	0.332	0.330	0.324	0.315
$gini(0, b_{t-1}+s_{t-1})$	0.670	0.670	0.674	0.676	0.678	0.670	0.668	0.667	0.672
Intergenerational Correlations									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.466	0.468	0.469	0.469	0.461	0.467	0.475	0.531
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.443	0.445	0.446	0.445	0.435	0.442	0.449	0.508

Estate Taxes vs. Labor Income Taxes 1



Estate Taxes vs. Labor Income Taxes 2





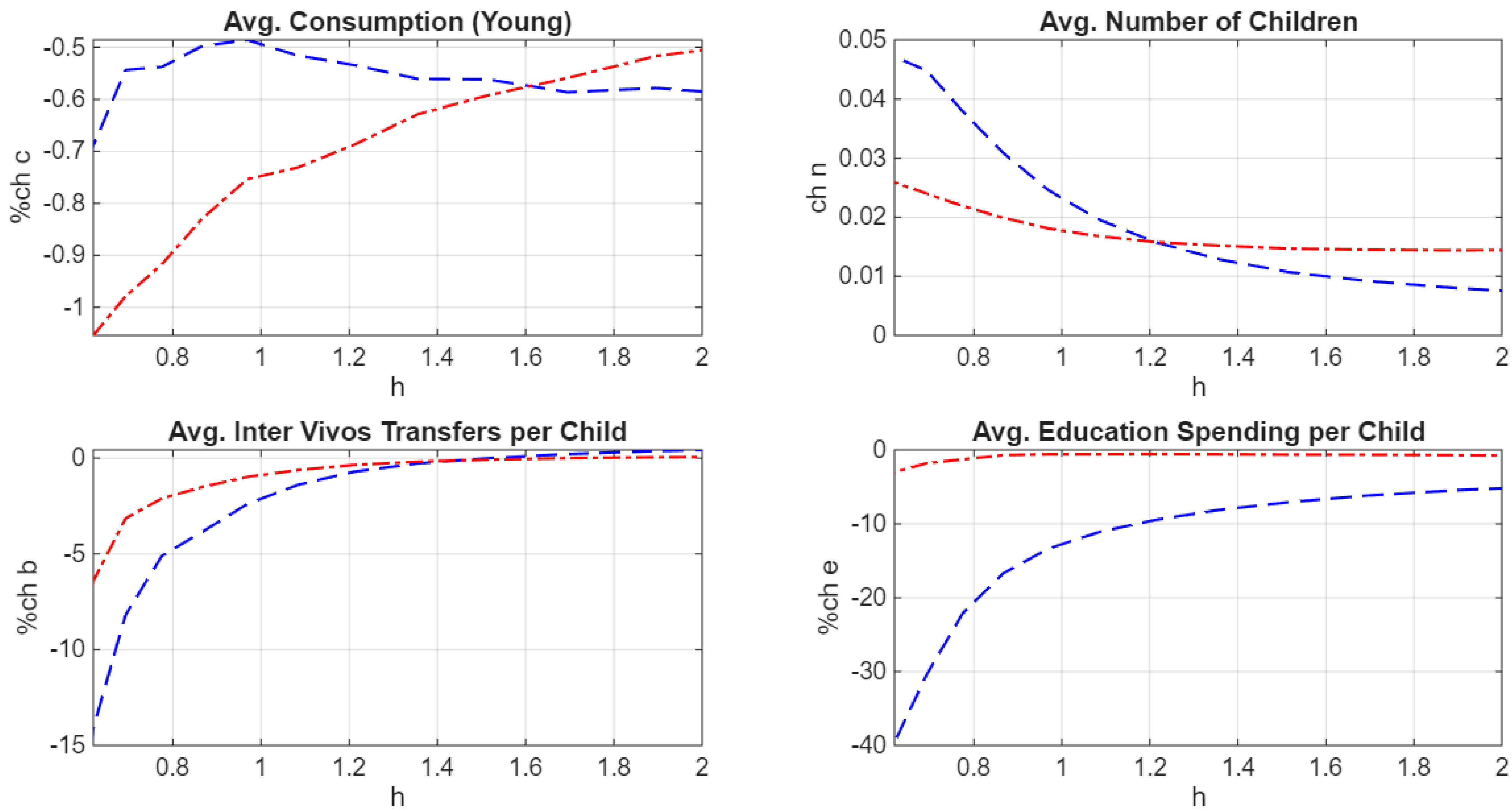
Estate Taxes vs. Labor Income Taxes 1

	Baseline	Estate Tax Cuts				Labor Income Tax Cuts			
Period t	0	1	2	3	∞	1	2	3	∞
Individual Variables (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	17.3	19.3	30.7	-0.0	-1.2	-0.7	0.6
$\bar{h}_t$	1.000	0.0	-1.6	0.2	7.3	0.0	0.8	1.1	2.0
$\bar{c}_t$	0.424	-1.0	2.1	4.1	12.1	-0.1	0.2	0.5	1.6
$\bar{s}_t$	0.133	-2.7	3.0	4.8	12.0	1.1	1.1	1.4	2.3
$\bar{n}_t$	1.000	0.4	1.8	2.4	4.8	0.1	0.2	0.3	0.7
$\bar{b}_t$	0.104	17.6	19.6	22.0	31.2	-1.2	-0.7	-0.4	0.5
$\bar{e}_t$	0.096	-5.5	2.1	5.9	21.0	2.9	3.2	3.8	5.7
$\bar{d}_{t+1}$	0.602	-3.6	-1.0	0.9	8.5	0.3	0.6	0.9	1.9
$\bar{a}_{t+1}$	0.077	17.6	19.6	22.0	31.2	-1.2	-0.7	-0.4	0.5
$\bar{h}_{t+1}$	0.992	-1.6	0.3	1.7	7.2	0.8	1.1	1.3	2.0
$\bar{v}_t$	7.495	0.1	1.4	2.3	5.6	0.1	0.3	0.4	0.9
$\bar{cv}_t$	0.000	0.6	2.2	4.0	9.4	0.7	0.9	1.0	1.6
Initial Old Households (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-0.6			
$\bar{v}_{t-1}$	7.495	-0.0				0.0			
$\bar{cv}_{t-1}$	0.000	-0.6				0.0			

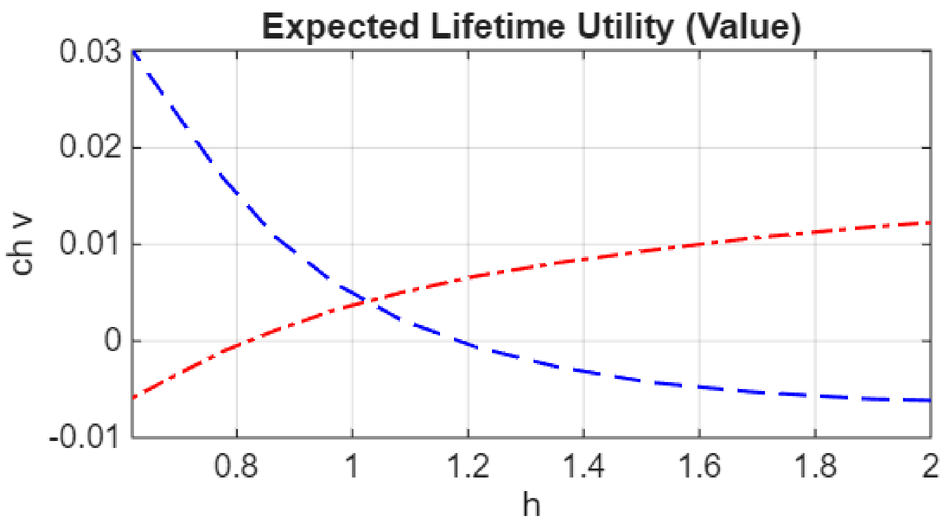
Estate Taxes vs. Labor Income Taxes 2

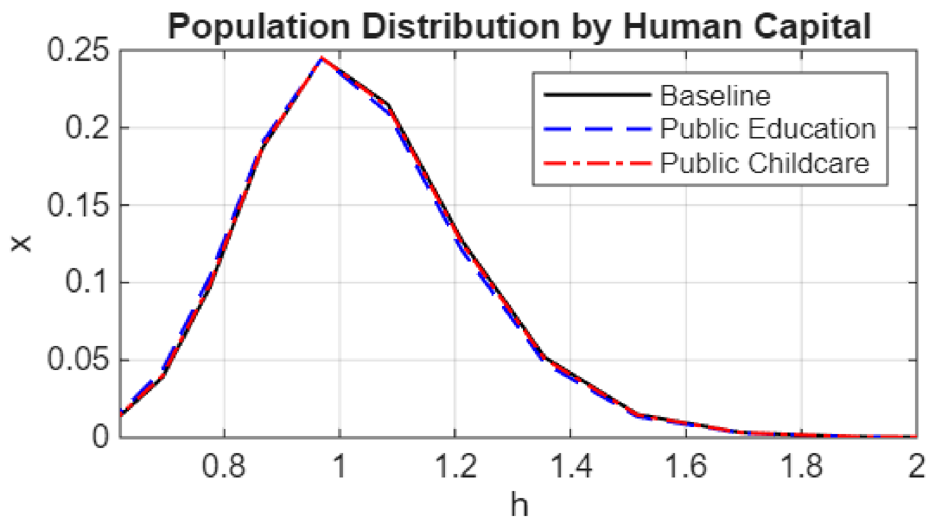
Aggregate Variables (per young household, % ch. from the baseline)									
C_t	1.204	-0.9	-0.7	1.0	8.5	-0.3	0.2	0.5	1.5
K_t	0.179	0.0	6.0	9.3	17.5	0.0	0.0	0.2	1.1
L_t	1.146	-0.0	-1.5	-0.7	5.8	-0.0	0.6	1.0	1.8
Y_t	1.791	-0.0	1.2	2.8	9.8	-0.0	0.4	0.7	1.6
$1+r_t$	3.815	-0.0	-4.3	-5.6	-6.1	-0.0	0.4	0.5	0.4
w_t	1.000	0.0	2.7	3.5	3.9	0.0	-0.2	-0.3	-0.2
Government Policy Variables (%, ch. in p.p. from the baseline)									
$\tau_{b,t}$	0.00	-9.10	-9.10	-9.10	-9.10	0.00	0.00	0.00	0.00
$\tau_{h,t}$	10.00	0.00	0.00	0.00	0.00	-1.10	-1.10	-1.10	-1.10
$\tau_{c,t}$	0.00	**1.09**	0.89	0.16	-2.29	**1.09**	0.92	0.82	0.46
Gini Coefficients									
$gini(c_t, d_t)$	0.176	0.178	0.168	0.169	0.174	0.176	0.176	0.176	0.177
$gini(\tilde{h}_t l_t)$	0.151	0.151	0.151	0.152	0.156	0.151	0.151	0.151	0.152
$gini(a_t, s_{t-1})$	0.332	0.332	0.328	0.327	0.322	0.332	0.333	0.332	0.333
$gini(0, b_{t-1}+s_{t-1})$	0.670	0.670	0.673	0.674	0.678	0.670	0.670	0.670	0.671
Intergenerational Correlations									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.460	0.465	0.474	0.519	0.459	0.460	0.462	0.467
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.435	0.441	0.449	0.496	0.434	0.436	0.437	0.442

Public Education vs. Childcare Paid Leave Subsidies 1



Public Education vs. Childcare Paid Leave Subsidies 2





Public Education vs. Childcare Paid Leave Subsidies 1

	Baseline	Public Education				Childcare Paid Leave Subsidies			
Period t	0	1	2	3	∞	1	2	3	∞
Individual Variables (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	-2.5	-3.5	-6.6	-0.0	-1.0	-1.3	-2.3
$\bar{h}_t$	1.000	0.0	-0.9	-1.7	-4.0	0.0	-0.3	-0.5	-1.3
$\bar{c}_t$	0.424	-0.5	-1.5	-2.4	-5.1	-0.8	-1.0	-1.3	-2.2
$\bar{s}_t$	0.133	1.6	0.4	-0.5	-2.9	1.3	0.9	0.6	-0.2
$\bar{n}_t$	1.000	2.5	2.1	1.8	0.9	1.8	1.7	1.6	1.3
$\bar{b}_t$	0.104	-1.7	-2.8	-3.7	-6.2	-0.7	-1.1	-1.4	-2.2
$\bar{e}_t$	0.096	-12.8	-15.0	-16.6	-21.1	-0.7	-1.4	-1.9	-3.4
$\bar{d}_{t+1}$	0.602	-0.0	-1.2	-2.1	-4.7	0.1	-0.3	-0.6	-1.5
$\bar{a}_{t+1}$	0.077	-1.7	-2.8	-3.7	-6.2	-0.7	-1.1	-1.4	-2.2
$\bar{h}_{t+1}$	0.992	-0.7	-1.5	-2.1	-3.9	-0.2	-0.5	-0.7	-1.2
$\bar{v}_t$	7.495	0.1	-0.5	-0.9	-2.3	0.0	-0.1	-0.3	-0.7
$\bar{cv}_t$	0.000	0.2	-0.1	-0.7	-2.4	0.3	0.3	0.1	-0.4
Initial Old Households (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-0.8			
$\bar{v}_{t-1}$	7.495	-0.0				-0.0			
$\bar{cv}_{t-1}$	0.000	-0.8				-0.6			

Public Education vs. Childcare Paid Leave Subsidies 2

Aggregate Variables (per young household, % ch. from the baseline)									
C_t	1.204	-0.5	-1.8	-2.7	-5.2	-0.6	-1.2	-1.5	-2.3
K_t	0.179	0.0	-1.6	-2.5	-5.1	0.0	-0.7	-1.0	-1.9
L_t	1.146	-0.2	-1.4	-2.1	-4.3	1.0	0.4	0.2	-0.5
Y_t	1.791	-0.1	-1.4	-2.2	-4.6	0.6	-0.0	-0.3	-1.0
$1+r_t$	3.815	-0.1	0.1	0.3	0.5	0.6	0.7	0.7	0.8
w_t	1.000	0.1	-0.1	-0.2	-0.3	-0.4	-0.4	-0.4	-0.5
Government Policy Variables (%, ch. in p.p. from the baseline)									
ϑ_t	7.16	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
ψ_t	0.00	0.00	0.00	0.00	0.00	1.27	1.27	1.27	1.27
$\tau_{c,t}$	0.00	**1.09**	1.05	1.39	2.44	**1.09**	0.96	1.06	1.38
Gini Coefficients									
$gini(c_t, d_t)$	0.176	0.177	0.179	0.180	0.180	0.178	0.179	0.179	0.179
$gini(\tilde{h}_t l_t)$	0.151	0.152	0.152	0.152	0.152	0.151	0.152	0.152	0.152
$gini(a_t, s_{t-1})$	0.332	0.332	0.337	0.342	0.351	0.332	0.334	0.336	0.338
$gini(0, b_{t-1}+s_{t-1})$	0.670	0.670	0.674	0.675	0.678	0.670	0.673	0.673	0.674
Intergenerational Correlations									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.464	0.464	0.464	0.461	0.460	0.461	0.461	0.460
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.441	0.441	0.441	0.437	0.439	0.439	0.439	0.439

Concluding Remarks

Children are normal goods. Therefore, child allowances would improve fertility rates at least in the short run.

There exists a quantity–quality tradeoff, however. So, child allowances would decrease human capital and physical capital investments.

The number of children, education spending, and inter vivos transfers are highly substitutable to each other.

The optimal child-related policy would be a combination of child allowances, education subsidies, and inter vivos transfer subsidies.

If the households are dynastically altruistic and the elasticity of intertemporal substitution is less than one, child-related policies would not likely increase income and wealth inequalities significantly.