Smooth Diagnostic Expectations

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Introduction: representativeness and time-series

Kahneman & Tversky (1972) representativeness heuristic

- Memory overweights events/types associated with (“representative” of) new info

This paper: extend the cognitive insight into time-series/macroeconomic environment

- Distinct role of news and uncertainty in shaping representativeness
- “Smooth” version of Diagnostic Expectations (DE) by Bordalo, Gennaioli, Shleifer
- Tight connection between uncertainty, over-reaction, over-confidence

Introduction: macroeconomic implications

Parsimonious account for stylized properties of surveys

- Stronger over-reaction for longer forecast horizons
  (Bordalo et al. 2019, D'Arienzo, 2019, Augenblick et al. 2021)
- Joint explanation for over-reaction and over-confidence
  (Altig et al. 2020, Barrero, 2022, Born et al. 2022)

Real Business Cycle model with Smooth DE

- Representativeness: propagation & amplification mechanism for uncertainty shocks
  - Asymmetry, countercyclical micro volatility, countercyclical macro volatility from state-dependent over-reaction w/o other frictions
  - (Perceived uncertainty increase) $>>$ (actual uncertainty increase) by more than $\times 3$
- Policy that reduces uncertainty has first-moment effects
- Tractable & portable solution method

News, uncertainty, and over-reaction in a static two-state setting

- Representativeness of a trait $\widehat{x}$ for a group G, given the reference group $-G$ (Gennaioli and Shleifer, 2010):

$$\frac{P(\widehat{x}|G)}{P(\widehat{x}|-G)}$$

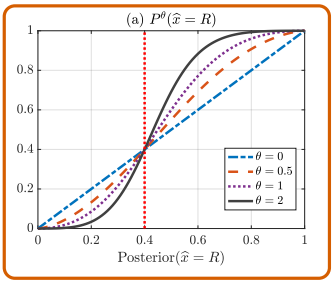
- Distorted probability of a person's hair color being red ($\widehat{x} = R$):

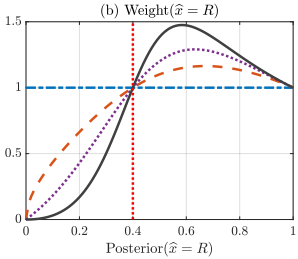
$$P^{\theta}(\widehat{x} = R) = \text{Posterior}(\widehat{x} = R)\,\text{Weight}\,(\widehat{x} = R),$$

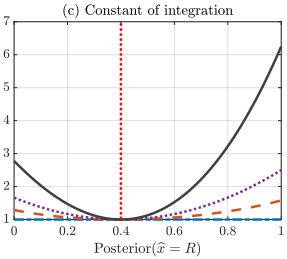
$$\text{Weight}\,(\widehat{x} = R) = \left(\frac{\text{Posterior}(\widehat{x} = R)}{\text{Prior}(\widehat{x} = R)}\right)^{\theta}\frac{1}{Z}$$

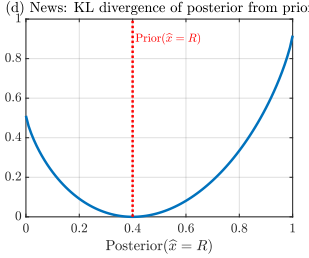
$$= \left(\frac{P(\widehat{x} = R|\mathit{Irish})}{P(\widehat{x} = R|\mathit{Unknown})}\right)^{\theta}\frac{1}{Z}$$

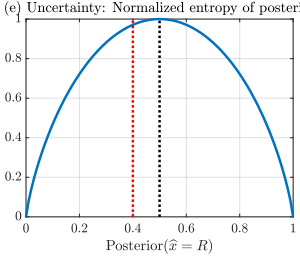
News, uncertainty, and over-reaction in a static two-state setting

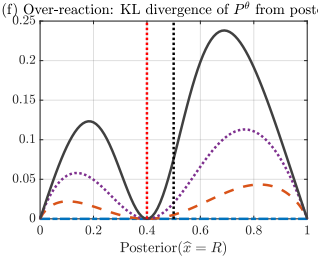




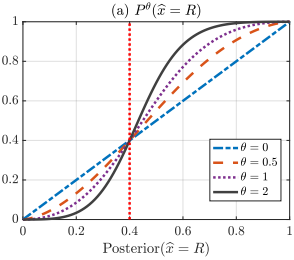


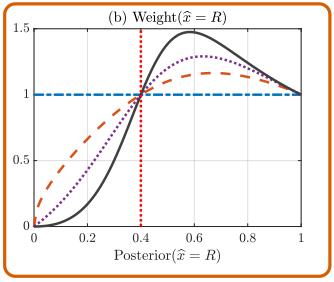


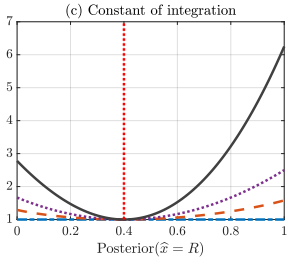


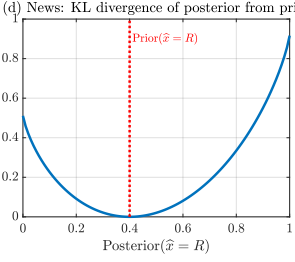


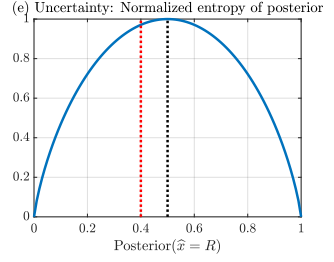
News, uncertainty, and over-reaction in a static two-state setting

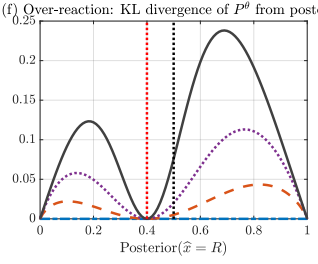












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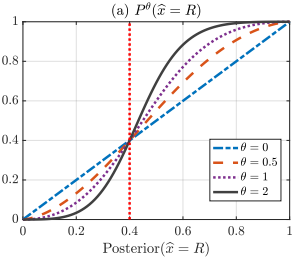
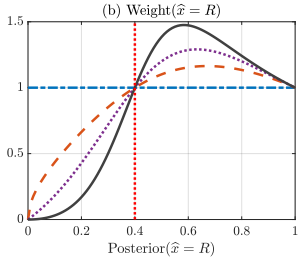
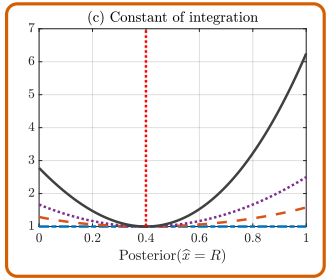
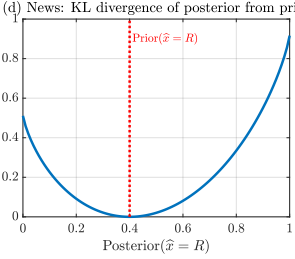
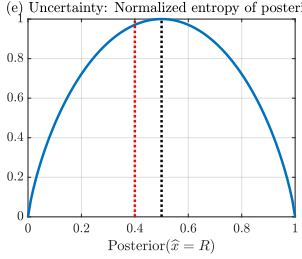
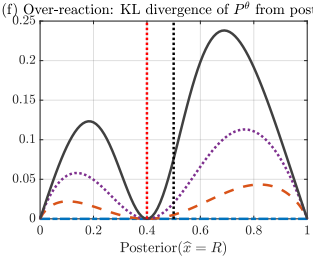




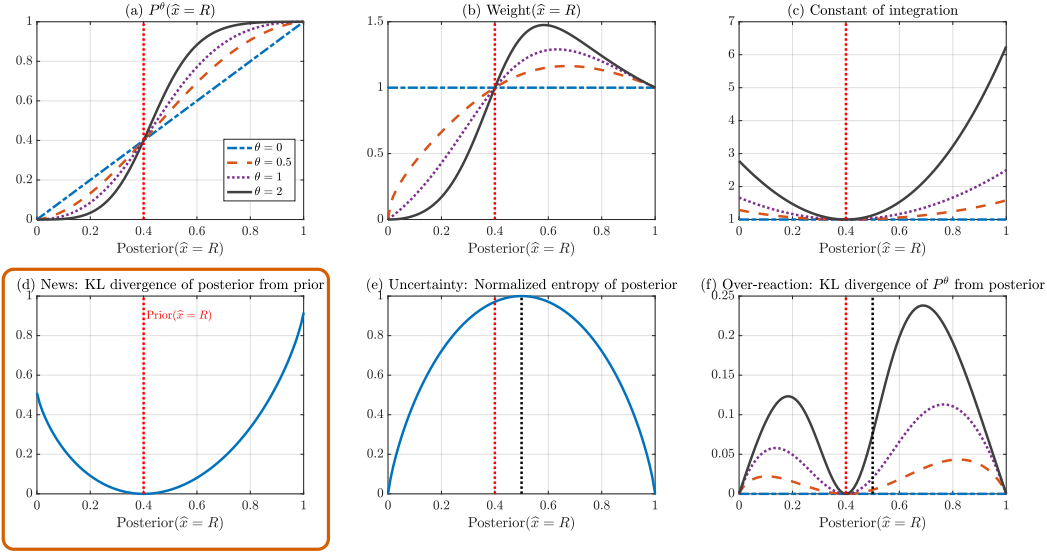








News, uncertainty, and over-reaction in a static two-state setting



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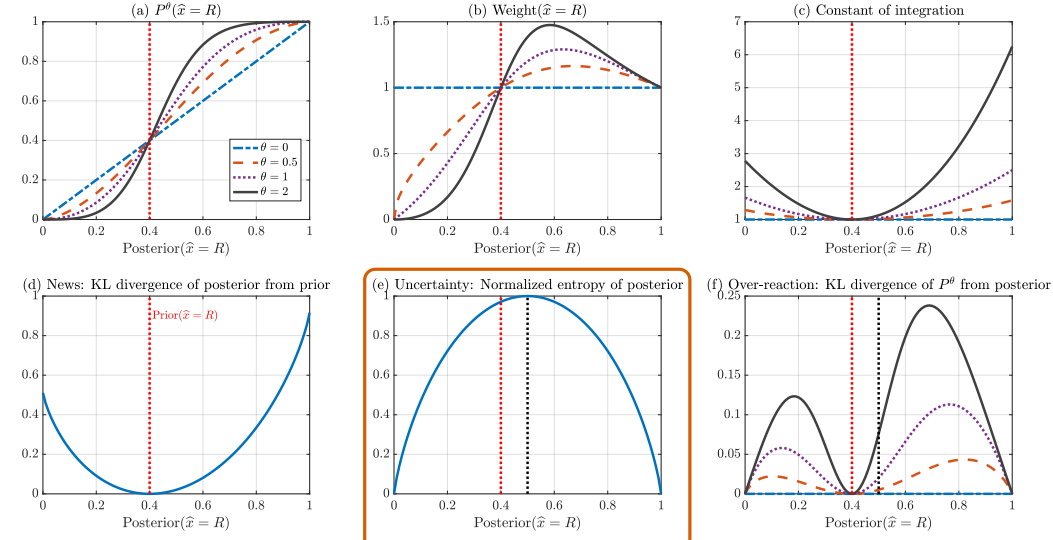

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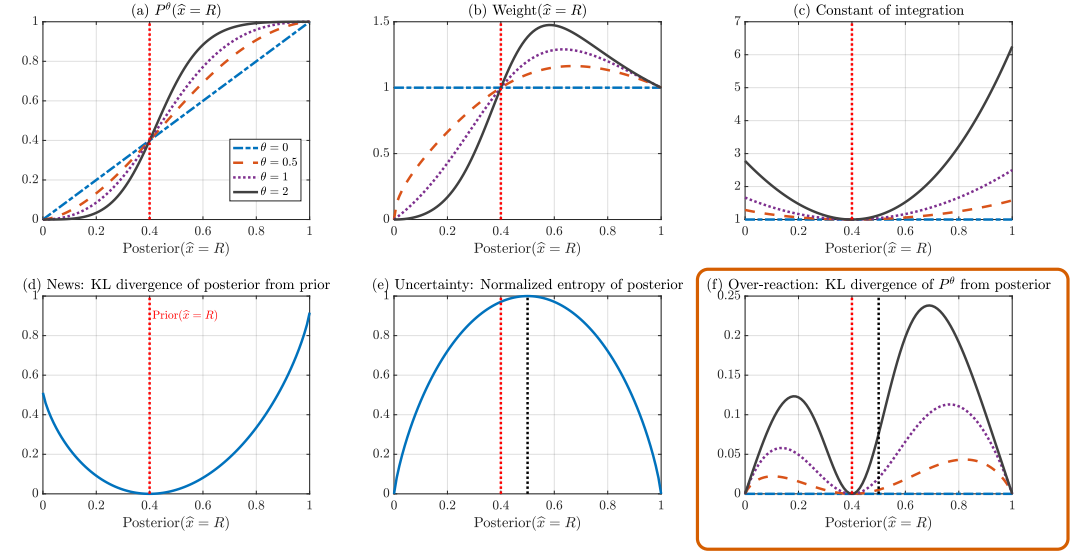


Representativeness in time series

- Diagnostic Expectations (DE): Bordalo, Gennaioli & Shleifer, 2018 (BGS)

$$f^{\theta}\left(\widehat{x}_{t+1}|\mathcal{I}_t\right) = f\left(\widehat{x}_{t+1}|\mathcal{I}_t\right)\left[\frac{f\left(\widehat{x}_{t+1}|\mathcal{I}_t\right)}{f\left(\widehat{x}_{t+1}|\mathcal{I}_t^{ref}\right)}\right]^{\theta}\frac{1}{Z}$$

- Representativeness induces distorted conditional density $f^{\theta}\left(\widehat{x}_{t+1}|\mathcal{I}_t\right)$ if $\theta > 0$
  - $f\left(\widehat{x}_{t+1}|\mathcal{I}\right)$: true density over future event $\widehat{x}_{t+1}$, given info set $\mathcal{I}$
  - Current group: current info set $\mathcal{I}_t$
  - Reference group: reference info set $\mathcal{I}_t^{ref} \subseteq \mathcal{I}_t$ to define current representativeness
- Our approach: reference = whole information set available $J \geq 1$ periods ago

$$\mathcal{I}_t^{ref} = \mathcal{I}_{t-J}$$

Prior information as the reference density

- Let true (RE) densities be (normals help with tractability)

$$\text{Posterior}: \quad f\left(\widehat{x}_{t+1}|\mathcal{I}_t\right) = \mathcal{N}\left(\widehat{x}_{t+1}; \mu_t, \sigma_t^2\right)$$

$$\text{Prior}: \quad f\left(\widehat{x}_{t+1}|\mathcal{I}_{t-J}\right) = \mathcal{N}\left(\widehat{x}_{t+1}; \mu_{t-J}, \sigma_{t-J}^2\right)$$

- Forming reference density $f\left(\widehat{x}_{t+1}|\mathcal{I}_t^{ref}\right)$:

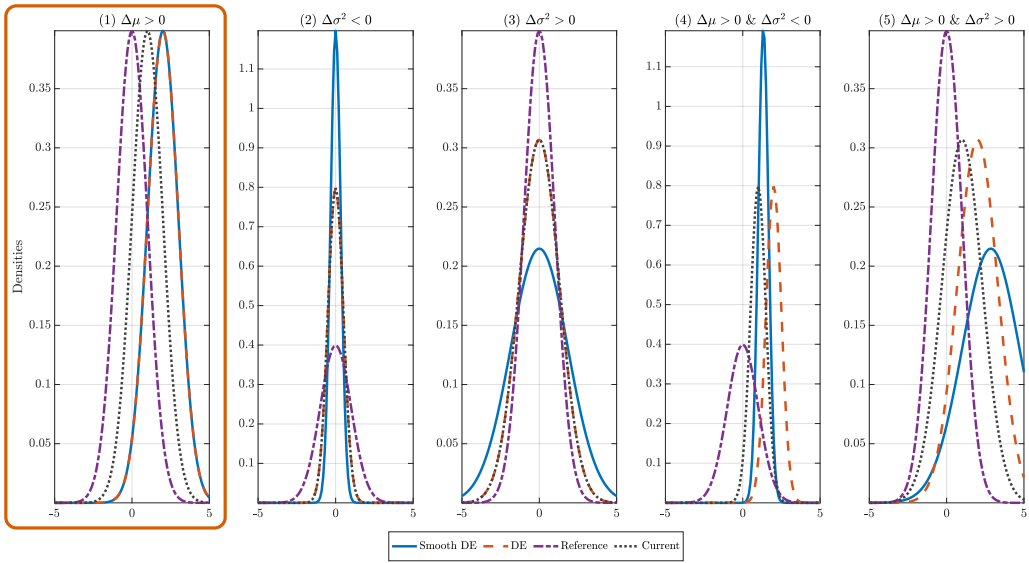
1. Smooth DE: we let $\mathcal{I}_t^{ref} = \mathcal{I}_{t-J}$ then

$$f\left(\widehat{x}_{t+1}|\mathcal{I}_t^{ref}\right) = \mathcal{N}\left(\widehat{x}_{t+1}; \mu_{t-J}, \sigma_{t-J}^2\right)$$

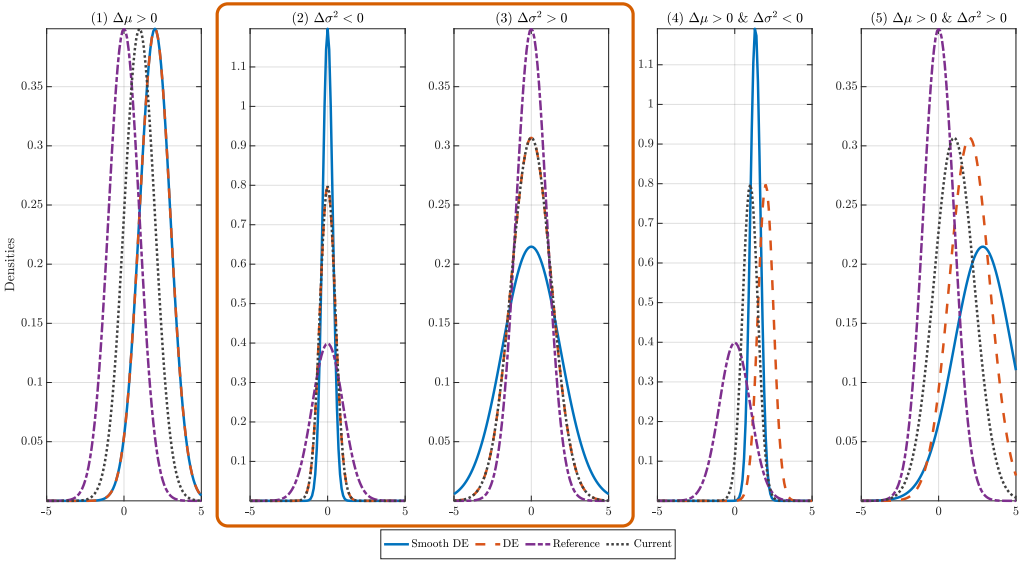
2. DE (BGS, Bianchi, Ilut and Saijo, 2024)

$$f\left(\widehat{x}_{t+1}|\mathcal{I}_t^{ref}\right) = \mathcal{N}\left(\widehat{x}_{t+1}; \mu_{t-J}, \sigma_t^2\right)$$

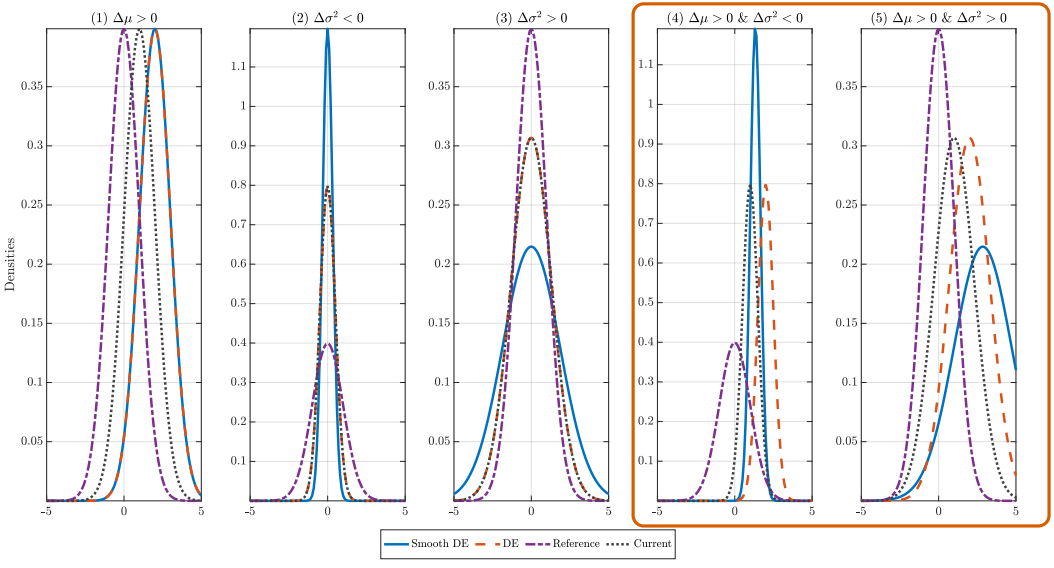
Distorted densities: Smooth DE and DE



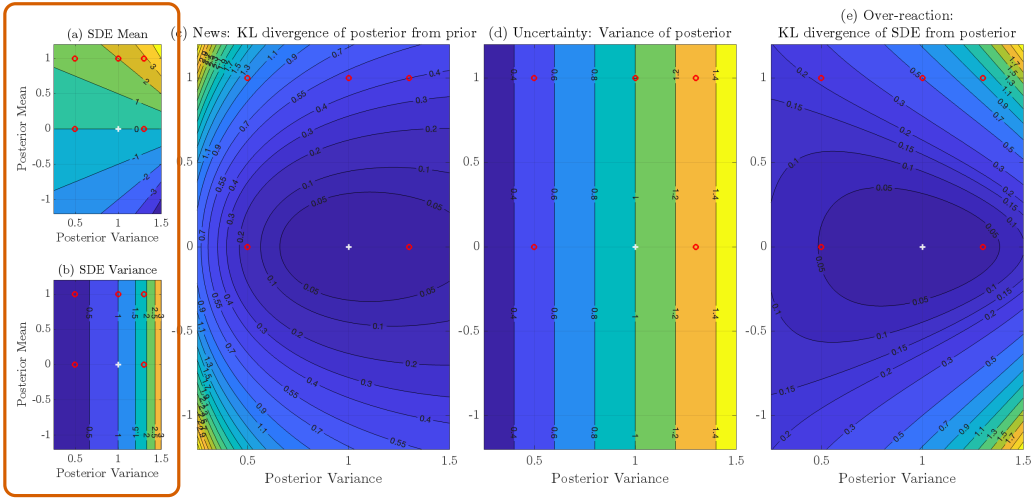
Distorted densities: Smooth DE and DE



Distorted densities: Smooth DE and DE



News, uncertainty, and over-reaction: Normal distribution



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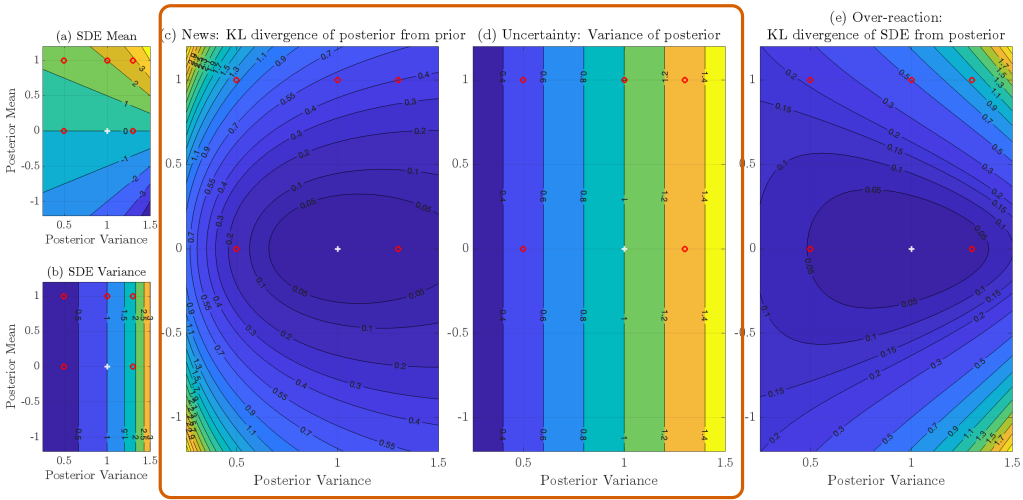

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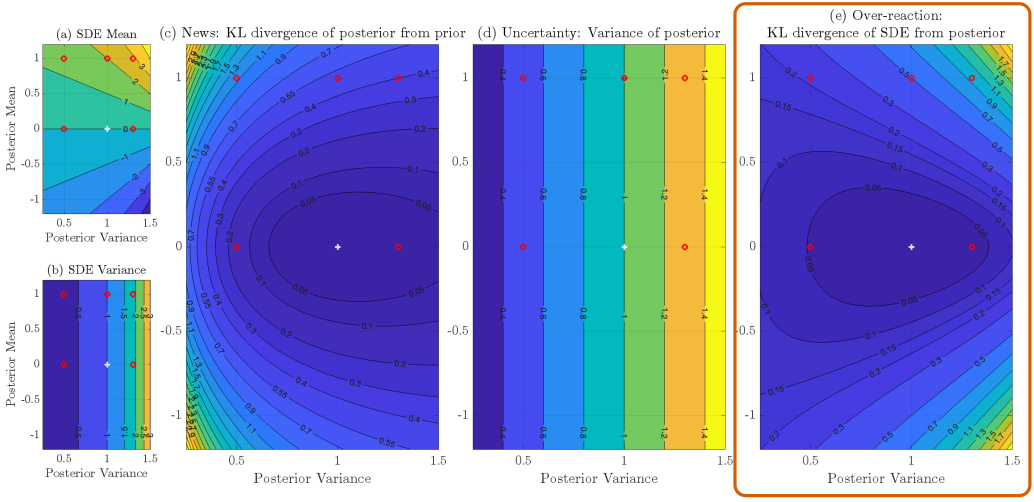

Smooth Diagnostic Expectations

Proposition

Denote the ratio of variances for the current and reference groups as

$$R_t \equiv \sigma_t^2 / \sigma_{t-J}^2$$

The Smooth DE density $f^\theta\left(\widehat{x}_{t+1}\right)$ *is Normal with conditional mean*

$$\mathbb{E}_t^\theta\left(x_{t+1}\right) = \mu_t + \underbrace{\theta \frac{R_t}{1+\theta\left(1-R_t\right)}}_{\equiv \widetilde{\theta}_t}\left(\mu_t - \mu_{t-J}\right)$$

and conditional variance

$$\mathbb{V}_t^\theta\left(x_{t+1}\right) = \frac{\sigma_t^2}{1+\theta\left(1-R_t\right)}$$

Novel properties

Corollary

Compared to the RE forecast $(\theta = 0)$*, the Smooth DE forecast exhibits*

1. *Over-reaction of the conditional mean to new information, since* $\widetilde{\theta}_t > 0$
2. ***Novel**: An effective over-reaction* $\widetilde{\theta}_t$ *of the conditional mean that is increasing in the ratio* R_t *between current and past uncertainty*
3. ***Novel**: Over-confidence when* $R_t < 1$*, since*

$$\mathbb{V}_t^{\theta}\left(x_{t+1}\right) < \sigma_t^2$$

or under-confidence when $R_t > 1$*, since*

$$\mathbb{V}_t^{\theta}\left(x_{t+1}\right) > \sigma_t^2$$

Parsimonious account for stylized properties of surveys

1. Stronger over-reaction for longer horizons $\mathbb{E}_t^\theta [x_{t+h}]$
   - Documented in
     - surveys: equity, interest rates (Bordalo et al. 2019, D'Arienzo, 2019)
     - field data: sports betting, equity index options (Augenblick et al. 2021)
   - Smooth DE: New info less informative for longer horizons h
2. Over-confidence in subjective uncertainty (over-precision)

$$\mathbb{V}_t^\theta < \sigma_t^2$$

   - Typical stationary process: new info reduces uncertainty $\sigma_t^2 < \sigma_{t-1}^2$
     - can be reverted under uncertainty shocks (e.g., volatility changes)
   - Survey evidence: Altig et al. 2020, Barrero, 2022, Born et al. 2022

Here representativeness $\rightarrow$ jointly over-reaction & (typically) over-confidence

RBC model with Smooth DE

- Continuum of islands $i \in [0, 1]$:

$$U(c_{i,t}, h_{i,t}) = \frac{c_{i,t}^{1-\gamma}}{1-\gamma} - \beta \frac{h_{i,t}^{1+\eta}}{1+\eta}$$

- Production/resource constraint (labor in advance)

$$c_{i,t} = y_{i,t} = \exp(z_{i,t}) h_{i,t-1}$$

- Technology: AR(1) aggregate + i.i.d. island-specific

$$z_{i,t+1} = A_{t+1} + a_{i,t+1}$$

- Predictable component (known in advance) $s_{i,t} \sim N(0, \sigma_s^2)$ + unpredictable component $u_{a,i,t+1} \sim N(0, \sigma_{a,t}^2)$

$$a_{i,t+1} = s_{i,t} + u_{a,i,t+1}$$

  Volatility $\sigma_{a,t}$ increases when A_t innovation negative (Bloom 2009)

Smooth DE solution

- Optimal labor supply (*naïveté* approach, Bianchi, Ilut and Saijo, 2024)

$$\eta \widehat{h}_{i,t} = \mathbb{E}^{\theta}_{i,t}\left(-\gamma \widehat{c}^{RE}_{i,t+1} + z_{i,t+1}\right)$$

- Equilibrium hours

$$\widehat{h}_{i,t} = \frac{1-\gamma}{\eta+\gamma}\mathbb{E}_{i,t}z_{i,t+1} + \widetilde{\theta}_t \frac{1-\gamma}{\eta+\gamma}\left(\mathbb{E}_{i,t}z_{i,t+1} - \mathbb{E}_{i,t-J}z_{i,t+1}\right)$$

- Hours increase with positive news if substitution effect is strong enough ($\gamma^{-1} > 1$)
- Higher uncertainty in recessions ($\sigma^2_{a,t} \uparrow$)

  $\rightarrow$ higher endogenous & exogenous conditional uncertainty $\mathbb{V}_{i,t}\left(-\gamma \widehat{c}^{RE}_{i,t+1} + z_{i,t+1}\right)$

  $\rightarrow$ larger over-reaction $\widetilde{\theta}_t$ to news

Business cycle implications

1. Asymmetry

   Negative economy-wide TFP shock $\rightarrow$ negative news and higher uncertainty

   $\rightarrow$ larger over-reaction $\rightarrow$ recessions sharper than expansions

2. Countercyclical micro volatility

   Larger over-reaction to idiosyncratic shocks in recessions

   $\rightarrow$ Cross-sectional variance of actions higher in recessions

3. Countercyclical macro volatility

   Larger over-reaction to economy-wide shocks in recessions

   $\rightarrow$ Time-series variance higher in recessions

4. Amplification of uncertainty shocks

   Tail events become more representative

   $\rightarrow$ (Perceived uncertainty increase) $>>$ (actual uncertainty increase)

Policy implications & calibration

Policy implication

- Policy that reduces micro uncertainty (eg. redistribution depending on $a_{i,t+1}$)
  $\rightarrow$ lower equilibrium micro uncertainty $\rightarrow$ lower $\widetilde{\theta}_t \rightarrow$ stabilize macroeconomy

Calibration

- Idiosyncratic uncertainty $\sigma_{a,t}$ rises by 10% in recessions
  (Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry 2018, Ilut, Kehrig and Schneider 2018)
- Set long-run diagnosticity $\widetilde{\theta}$ and idiosyncratic shocks σ_a, σ_s to match survey & micro/macro moments ▸ Details
  - Calibrated $\widetilde{\theta} = 1.54$: in line with other studies

Untargeted survey moments

Table: Over-reaction and over-confidence

	(1)	(2)	(3)	(4)
	$F_t(\Delta y_{i,t+4\|t}) - \Delta y_{i,t+4\|t}$	Absolute forecast error		
	on $\Delta y_{i,t\|t-1}$	Realized	Subjective	$\frac{\text{Subjective}}{\text{Realized}}$
Data (Barrero 2022)	0.173	0.143	0.023	0.16
Model	0.095	0.143	0.017	0.12

Business cycle implications

- Countercyclical micro and macro volatilities

	(1) Data	(2) Smooth DE	(3) DE	(4) RE
A. Cross-sectional standard deviation of labor growth				
(Recessions)/(Expansions)	1.16	1.12	1	1
B. Volatility of aggregate labor growth				
(Recessions)/(Expansions)	1.23	1.22	1	1

- (Perceived uncertainty increase) $>>$ (actual uncertainty increase): In recessions
  - Perceived uncertainty $\mathbb{V}^{\theta}_{i,t}\left(-\gamma \widehat{c}_{i,t+1} + z_{i,t+1}\right)$ rises by 69%
  - Realized uncertainty $\mathbb{V}_{i,t}\left(-\gamma \widehat{c}_{i,t+1} + z_{i,t+1}\right)$ rises by 19%

Conclusion

- Tractable application of the representativeness logic to time series
  - Distinct role of news and uncertainty in shaping over-reaction
  - Tight connection between uncertainty, over-reaction, over-confidence
- Parsimonious account for stylized properties of surveys
- RBC model with Smooth DE
  - Representativeness as a propagation mechanism for uncertainty shocks

Additional slides

Table: Internally calibrated parameters and targeted moments

Parameters		Targeted moments		
			Data	Model
σ_a	0.022	Realized absolute forecast error	0.143	0.143
σ_s	0.027	Residual uncertainty	0.41	0.41
$\widetilde{\theta}$	1.547	Skewness of aggregate hours	-0.21	-0.21

Return