Slow Learning

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based on joint work with Martin Eichenbaum and Ben Johannsen

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  - Harder to justify in unprecedented situations.

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- Ask: What features of the economy determine speed of convergence to REE?
  - Use a reduced form example which suggests a simple *learning principle*:
    - When expectations of a variable are partially self-fulfilling, then learning converges *slowly* to REE, if at all.

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  - Relate this result to the learning principle.

Outline

- Simple example:
  - Learning principle.
- New Keynesian analysis of shocks and policies in the ZLB using nonlinear version Eggertsson-Woodford (2003) model.
  - Government spending
  - Forward Guidance
  - Interpret results using learning principle.

Simple Example: REE

- Model analyzed in Bray and Savin (ECMA1986):

$$x_t = a + b\mathbb{E}_{t-1}x_t + \varepsilon_t,\ \varepsilon_t \sim iiN\left(0, \sigma^2\right), \sigma^2 < \infty$$

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  - When $b < 0$: Muth's (1961) version of Cobweb model,
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- Rational expectations equilibrium:

$$\mathbb{E}_{t-1}x_t = E_{t-1}x_t,\ x_t = \overbrace{\frac{a}{1-b}}^{\mu} + \varepsilon_t.$$

- In REE, $x_t \sim iiN\left(\mu, \sigma^2\right)$.

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$$\mu_t = \mu_{t-1} + \frac{1}{\lambda_0 + t}(x_t - \mu_{t-1})$$
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- Repeated substitution:

$$\mu_t - \frac{a}{1-b} = \sum_{j=1}^{t} \left\{ \frac{z_t}{z_j} \frac{\varepsilon_j}{\lambda_0 + j} \right\} + z_t \left(\mu_0 - \frac{a}{1-b} \right)$$

where

$$z_t = \prod_{j=1}^{t} (1 - b_j), \; b_j = \frac{1-b}{\lambda_0 + j}.$$

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  - Yes for $b < 1$.
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- how fast does convergence occur?
  - potentially, very slowly.

A Feedback Loop and Speed of Convergence

- To understand convergence rate, recall data-generating process under learning:

$$x_t = a + b\mu_{t-1} + \varepsilon_t$$
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- Suggests speed of convergence may be a *decreasing* function of b.

Simple Example: Learning Might be Very Slow (or, Fast)

- Consider expected gap relative to REE, as fraction of initial gap:

$$z_t = \frac{E\left(\mu_t - \frac{a}{1-b}\right)}{\mu_0 - \frac{a}{1-b}} = f\left(t, \lambda_0, b\right).$$

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T	2	10	120	2500	4 billion

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- Christopeit and Massmann (2018) derive various asymptotic properties, μ_t, as $t \to \infty$.
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- Learning principle:
  - positive feedback loop ($b > 0$): slow learning.
  - negative feedback loop ($b < 0$): relatively fast learning. constant gain

Turning to New Keynesian Model

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- Key findings:
  - When the ZLB model is binding, NK model corresponds to a *high-b economy*.
    - Model in ZLB implies a strong positive feedback loop in inflation expectations.
  - Convergence to a REE is very slow.
  - REE analysis very misleading for government spending multiplier and forward guidance.

NK Model with Learning

- Simple closed economy, NK model without capital, flexible wages, Rotemberg-sticky prices.
  - Up to period 0, economy is in unique steady state REE with
    - $\beta = 1/(1 + r_{ss})$, ss ˜ ‘steady state’
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  - Up to period 0, economy is in unique steady state REE with
    - $\beta = 1/(1 + r_{ss})$, ss ˜ 'steady state'
    - gross nominal interest rate, $R > 1$.
- In period 0, everyone discovers unexpectedly that r drops to $r_\ell < r_{ss}$ (Eggertsson-Woodford, 2003).
  - People know the law of motion of r, $r \in (r_\ell, r_{ss})$, r_{ss} is an absorbing state and $P[r_{t+1} = r_\ell | r_t = r_\ell] = p$.
  - When economy reverts to absorbing state, $r = r_{ss}$, everyone understands we're back to unique steady state REE with $R > 1$.

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- Learning:
  - When data arrive, they update beliefs using Bayes' rule.
  - In thinking about the future they internalize that their beliefs will continue evolving as new data arrive.

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    - ⋆ Density of x degenerate when $r = r_{ss}$, non-trivial with $r = r_\ell$.
- The h^{th} household forms plans for $C_h, N_h, b_h^{'}$ contingent on the not-yet-realized current value of x.

Household x-Contingent Plan

- For a range of values of $x = [C, \pi]$ the h^{th} household chooses $C_h, N_h, b_h^{'}$ to solve:

$$\max_{C_h, N_h, b_h^{'}} \{\log\left(C_h\right) - \frac{\chi}{2}\left(N_h\right)^2 + \frac{1}{1+r_\ell}\left[\left(1-p\right) V_h^{ss}\left(b_h^{'}\right) + p\mathbb{E}V_h\left(b_h', \Theta', x'\right)\right]\},$$

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subject to the budget constraint:

$$C_h + \frac{b_h'}{R(x)} \leq \frac{b_h}{\pi(x)} + w(x) N_h + T(x),$$

where V_h and V_h^{ss} denote the value functions in case $r = r^\ell$ or $r = r^{ss}$ in the next period, respectively. EquilibriumFunction

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$$C_h + \frac{b_h^{\prime}}{R\left(x\right)} \leq \frac{b_h}{\pi\left(x\right)} + w\left(x\right)N_h + T\left(x\right),$$

where V_h and V_h^{ss} denote the value functions in case $r = r^\ell$ or $r = r^{ss}$ in the next period, respectively. ▸ EquilibriumFunction

- Here,
  - $\mathbb{E}$ denotes the expectation operator over marginal data density of $x^{\prime}$, conditional on $r^{\prime} = r_\ell$, Θ, x.

Household x-Contingent Plan

- For a range of values of $x = [C, \pi]$ the h^{th} household chooses $C_h, N_h, b_h^{'}$ to solve:

$$\max_{C_h, N_h, b_h'} \{\log(C_h) - \frac{\chi}{2}(N_h)^2 + \frac{1}{1+r_\ell}\left[(1-p)\, V_h^{ss}\left(b_h^{'}\right) + p\mathbb{E}V_h\left(b_h', \Theta', x'\right)\right]\},$$

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- Here,
  - $\mathbb{E}$ denotes the expectation operator over marginal data density of x', conditional on $r' = r_\ell$, Θ, x.
  - Θ', next period's belief parameters constructed by combining Θ, x.

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- The vector Θ denotes the parameters that characterize these prior distributions.

Evolution of Beliefs over Time: Internalized Learning

- In making their $x-$contingent decisions, people internalize that Θ' is a function of Θ and the observed value of x:

$$\Theta' = f(\Theta, x).$$

Here, f has an analytic representation.

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- The people in our model are 'internally rational' in the sense of Adam and Marcet 2011.

- In period 0, Θ_0 are free parameters.

Household Value Function

- Value function satisfies the following fixed point property:

$$V_h\left(b_h, \Theta, x\right) = \max_{C_h, N_h, b_h'} \left\{ \log\left(C_h\right) - \frac{\chi}{2}\left(N_h\right)^2 + \frac{1}{1+r_\ell}\left[\left(1-p\right) V_h^{ss}\left(b_h^{'}\right) + p\mathbb{E} V_h\left(b_h', \Theta', x'\right)\right] \right\},$$

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- That households can map from x into the aggregate variables required for their budget constraints corresponds to our assumption that they are good at *static* general equilibrium reasoning.
  - However, they are not good at *dynamic* general equilibrium reasoning.
  - Their beliefs about the future are distorted.

Production and Firms

- Dixit-Stiglitz formalization standard in NK model.
  - Final good created by aggregating intermediate goods produced by monopolists.

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- Intermediate firms' problem expressed in recursive form.
- Have same beliefs as households.

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  - Baseline: $G = G_{ss} >$ fixed for all r.

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  - Government uses lump sum taxes to balance budget in each period.
- Monetary policy:

$$R = \max\left\{1, \frac{1}{\beta} + \alpha\left(\pi - 1\right)\right\}, \alpha > 1$$

- We also consider perturbations on this policy, including forward guidance.

Market Clearing in a *Period Learning Equilibrium*

- Given $r = r_\ell$ and Θ,
- The vector, $x = [C, \pi]$, is adjusted to ensure goods, bonds and labor markets clear in a way that is consistent with private sector optimization and government policy.
  - The approach is inspired by Eusepi, Gibbs and Preston, 2022.

Learning Equilibrium

- As long as $r = r_\ell$, economy is a sequence of period learning equilibria.
- When $r = r_{ss}$ economy jumps to $R > 1$ REE steady state.

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- Second,
  - How quickly does convergence occur?
- Third,
  - are predictions of REE about macro stabilization policies robust to learning?
- Related issue: there are also multiple *steady state* REE's in the NK model (BSGU).
  - Based on our experiments and the literature, we will focus on the zero inflation steady state.

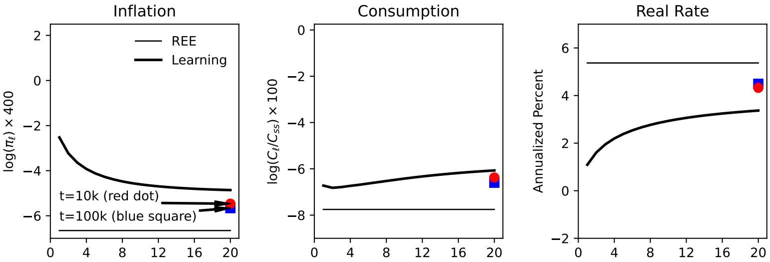
Parameter Values

$$p = 0.80,\ r_\ell = -0.0015\,(-0.6APR),\ \ G_{ss} = 0.20,\ \ r_{ss} = 0.005\,(2.0APR),$$

$$Y_{ss} = N_{ss} = 1,\ \varepsilon = 7,\ \phi = 110,\ \chi = 1.25,\ \alpha = 1.5$$

Experiment #1: Slow Learning in the ZLB

- r drops and G remains unchanged.



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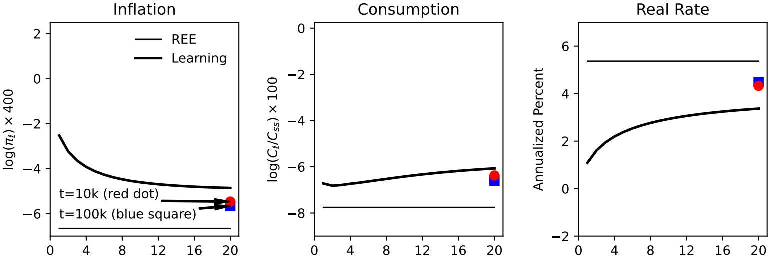

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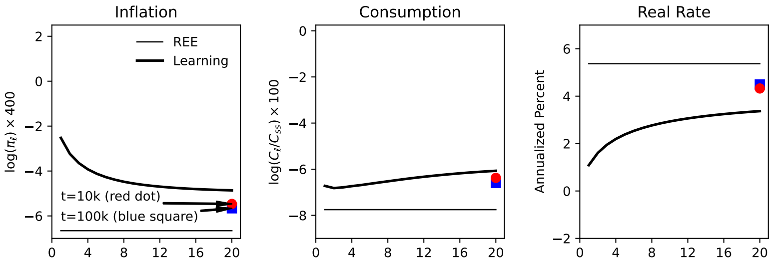

- Key results:
  - Economic impact of the shock under learning is small compared with REE.
    - ★ Learning is **extremely** slow.
  - Learning moves the model in the 'right' empirical direction:
    - ★ addresses 'missing deflation puzzle'. role of internalized learning

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  - With lower actual inflation now, expected inflation in future reduced.
    - ⋆ Actual inflation in the future reduced.
- In sum: Households and firms complement each other in creating a positive feedback loop that makes the NK model behave like a 'high-b' economy.

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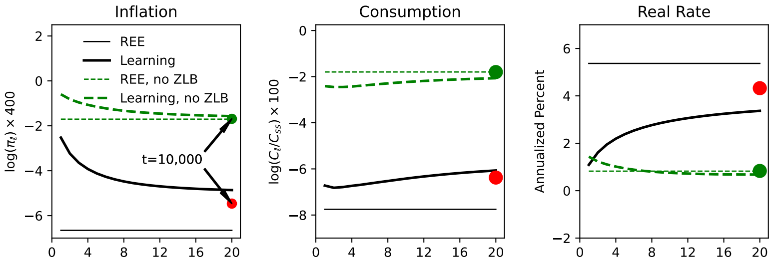

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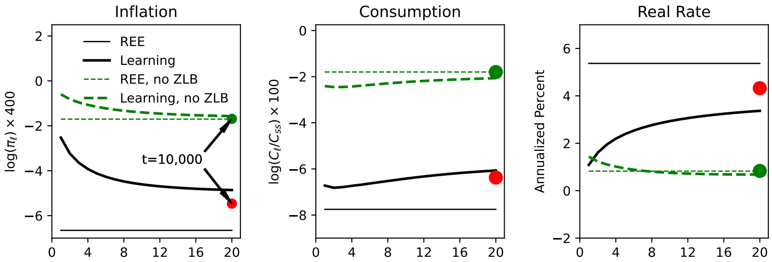

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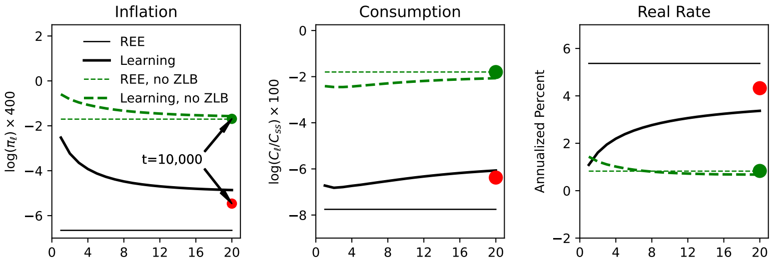

- Learning equilibrium converges relatively quickly to REE with Taylor Principle (ZLB ignored).
- Reflects learning principle:
  - Central Bank "Does what it Takes" to keep inflation on target, independent of prior expectations of the public: 'low b economy' with Taylor Principle.

Increase in G During ZLB

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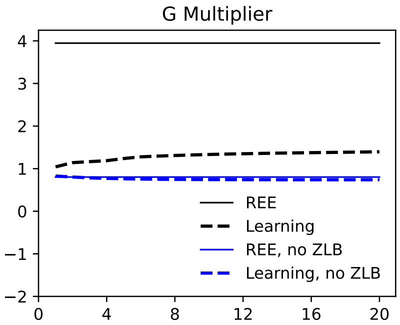
- Standard result in rational expectations (REE) literature:
  - multiplier on government spending can be very large in the ZLB.
  - But, large multiplier in REE happens chiefly by raising expected inflation.
    - If learning is backward-looking, then this inflation expectation channel broken.
- Our finding:
  - We find that the multiplier under learning is very small, compared to REE.
  - Rational expectations generates *very* misleading prediction about the effects of government spending.

The G Multiplier In ZLB

- Multiplier, $\frac{dY}{dG}$.



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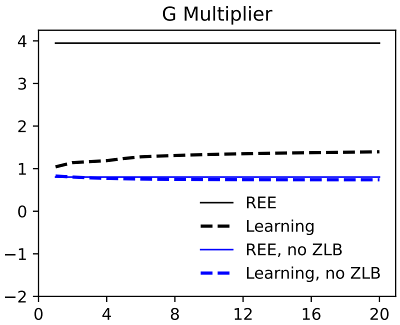

- A huge difference between REE and learning when Taylor Principle is out of the picture.

We Identify the Analog of b in the Linearized solution to the NK Model in a Neighborhood of a Stable REE Equilibrium

- Let $\hat{\mu}_t = \left[\hat{\pi}_t, \hat{C}_t\right]$ represent the log deviation of people's time t posterior of $\mathbb{E}_t x_{t+1}$ and the REE value of Ex_{t+1}.
- We show that for sufficiently large t, approximately

$$\hat{x}_t = B\hat{\mu}_{t-1}.$$

  - The structure maps beliefs about $\hat{x}_t$ (i.e., $\hat{\mu}_{t-1}$) into realized values of $\hat{x}_t$.
  - The Analog of b in Bray-Savin is the largest real part of the eigenvalue of B.
- The system is completed by Kalman-updating equations that map $\hat{x}_t$ into $\hat{\mu}_t$.
- Asymptotically $\hat{\mu}_t$ behaves like κt^{b-1} for $\kappa > 0$.

Findings

Table: Eigenvalues of B

	Eigenvalue 1	Eigenvalue 2	T_1	T
ZLB	0.92	-0.48	920,482	944,710
No ZLB, $\alpha = 1.5$	0.054+0.44i	0.054-0.44i	2	3
No ZLB, $\alpha = 3$	-0.135+0.84i	-0.135-0.84i	2	1

Note: The matrix, B, is defined on previous slide. The scalar, b, is the largest real part of the eigenvalues of B. The reported values of T are based on simulations of the linearized solution to the model.

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- Asymptotic formulas implies $T_1 = 920,482$ periods to close 2/3 of gap, nonlinear simulations imply $T = 944,710$ periods to converge.
- When ZLB is ignored, convergence is very fast, and asymptotic formulas reliable far away from REE.

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- REE analysis of government spending in ZLB can be very misleading.
  - Could induce government to rely excessively on fiscal policy.
- We generalize previous results for asymptotic rates of convergence to vector case and identify analog of Bray and Savin's b.
- Analysis confirms the wisdom of exploring the implication of replacing REE by alternative micro-founded mechanisms by which people form beliefs.

Appendix Materials

Period Price and Profit Functions

- Households (and firms) observe $x = \left[\begin{array}{cc} C, & \pi \end{array} \right]$
  - from x (as well as $r, G(r)$) they are able to deduce the variables needed to define their current-period budget constraint.

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  - from x (as well as $r, G(r)$) they are able to deduce the variables needed to define their current-period budget constraint.
- GDP (Y), aggregate employment (N), real wage (w), marginal firm cost (s), profits, taxes net of profits (T):

$$N = Y = (C + G(r))\left(1 + \frac{\phi}{2}(\pi - 1)^2\right)$$

$$w = \chi N C,\ s = (1 - \nu)\, w,\ R = \max\left\{1, 1 + r^h + \alpha(\pi - 1)\right\}.$$

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$$w = \chi NC, \; s = (1 - \nu) w, \; R = \max \left\{1, 1 + r^h + \alpha (\pi - 1)\right\}.$$

We assume the government issues no debt and finances its expenditures, with lump sum taxes:

$$G(r) + \nu wN,$$

where νwN represents the subsidy paid to intermediate good firms.

Period Price and Profit Functions, cnt'd

- Finally, profits net of taxes implied by x and r are:

$$T = \overbrace{(1-s)\,Y - \frac{\phi}{2}(\pi-1)^2\,(C+G(r))}^{\text{profits for intermediate good producers}} - \overbrace{(G(r)+\nu wY)}^{\text{lump sum taxes}}.$$

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- Note: none of these mappings use bond market clearing or the household's intertemporal Euler equation. Go Back

Cobweb Model

- Model of competitive market and a time lag in production.
  - John Muth, ‘Rational Expectations and the Theory of Price Movements’, ECMA, July 1961.
  - Coase and Fowler, ‘Bacon Production and the Pig-Cycle in Great Britain’, Economica, May, 1935.
- Demand:

$$d_t = m_I - m_p p_t + \upsilon_{1t}$$

- Supply decided in period t before υ_{1t} is observed:

$$s_t = r_I + r_p \mathbb{E}_{t-1} p_t + \upsilon_{2t}$$

- Equilibrium, $d_t = s_t$:

$$\overbrace{p_t}^{x_t} = \overbrace{\frac{m_I - r_I}{m_p}}^{a} \overbrace{- \frac{r_p}{m_p}}^{+b} \mathbb{E}_{t-1} p_t + \overbrace{\frac{\upsilon_{1t} - \upsilon_{2t}}{m_p}}^{\varepsilon_t}$$

Lucas Model

- Aggregate output:

$$q_t = \overline{q} + \pi \left(p_t - \mathbb{E}_{t-1} p_t \right) + \zeta_t$$

- Velocity equation:

$$m_t + \upsilon_t = p_t + q_t$$

- Monetary policy:

$$m_t = \bar{m} + u_t.$$

- Substitute second two equations into first, to obtain equilibrium condition:

$$\overbrace{p_t}^{x_t} = \overbrace{\frac{\bar{m} - \overline{q}}{1+\pi}}^{a} + \overbrace{\frac{\pi}{1+\pi}}^{b} \mathbb{E}_{t-1} p_t + \overbrace{\frac{u_t + \upsilon_t - \zeta_t}{1+\pi}}^{\varepsilon_t}$$

Rational Expectations Equilibrium

- Reduced form model:

$$x_t = a + b\mathbb{E}_{t-1}x_t + \varepsilon_t,\ \varepsilon_t \sim E\varepsilon_t = 0, E\varepsilon_t^2, E\varepsilon_t\varepsilon_{t-j} = 0, j \neq 0.$$

- In rational expectations equilibrium, $\mathbb{E}_{t-1}x_t = E_{t-1}x_t$, so

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- To verify this, note:

$$\begin{aligned} x_t &= a + bE_{t-1}x_t + \varepsilon_t \overbrace{=}^{REE} a + b\overbrace{\frac{a}{1-b}}^{E_{t-1}x_t} + \varepsilon_t \\ &= \frac{a}{1-b} + \varepsilon_t. \end{aligned}$$

Go Back

Constant-gain learning

- Assume people update their view of $\mu_{t-1}$by constant-gain learning:

$$\mu_t = \mu_{t-1} + \gamma \left(x_t - \mu_{t-1} \right), \tag{1}$$

for $0 < \gamma < 1$.

- Now

$$\mu_t - \frac{a}{1-b} = \sum_{j=0}^{t-1} (1-\gamma_b)^j \left(\frac{\varepsilon_{t-j}}{1-b} \right) \gamma_b + (1-\gamma_b)^t \left(\mu_0 - \frac{a}{1-b} \right),$$

where $\gamma_b = (1-b)\,\gamma$,

$$z_t = E \left(\frac{\mu_t - \frac{a}{1-b}}{\mu_0 - \frac{a}{1-b}} \right) = (1-\gamma_b)^t .$$

Learning principle again

- Again calculate how long it takes to close $2/3$ of the initial gap, i.e., calculate, T, the value of t such that $z_T \simeq 1/3$.

- Suppose $\gamma = 0.5$ and $b = 0, 0.5, 0.75, 0.85, .95$.

b	0	0.5	0.75	0.85	0.95
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b	0	0.5	0.75	0.85	0.95
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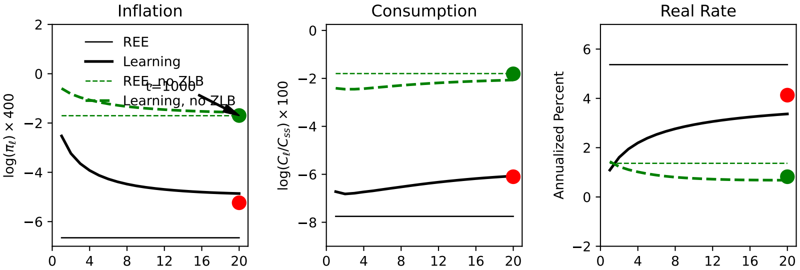
- Note: speed of convergence is quicker for 'small' values of b than under Bayesian learning.

- But again speed of convergence increases nonlinearly with b. Go Back

Dropping ZLB in Experiment #1

Figure: Effect of Ignoring ZLB



- Ignoring ZLB leads to smaller eventual drop in GDP, as Taylor principle enters picture.

Dropping ZLB in Experiment #1

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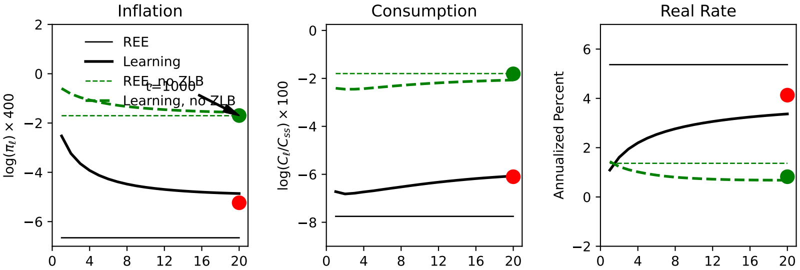

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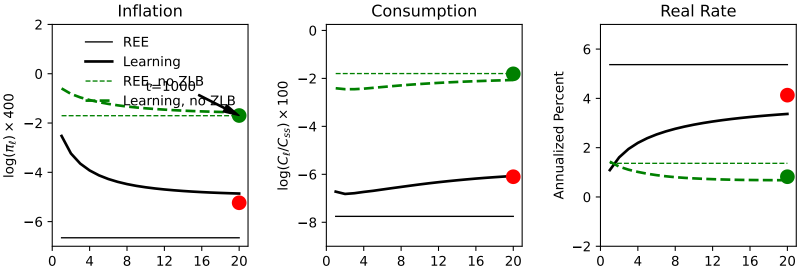
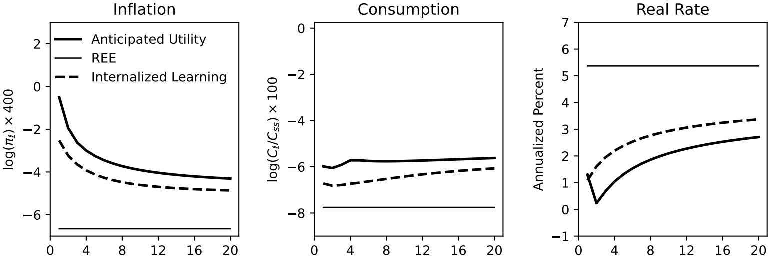


- Ignoring ZLB leads to smaller eventual drop in GDP, as Taylor principle enters picture.
- Seems to move economy in direction of 'low-*b* economy'.
- Still studying this result. ▸ Go Back

Role of Internalized Learning in Experiment #1

- r drops and G remains unchanged.



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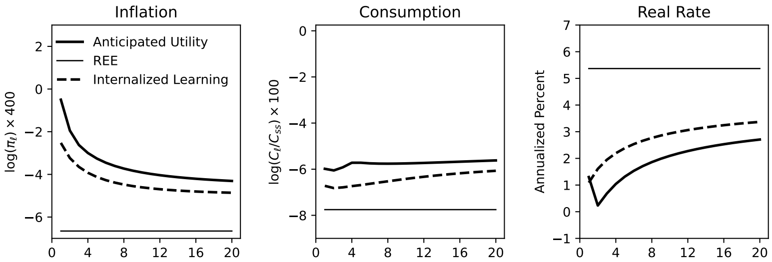

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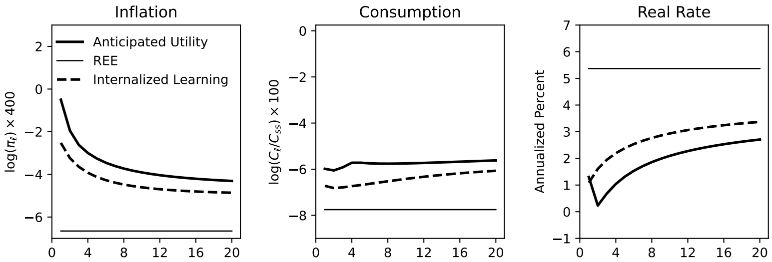

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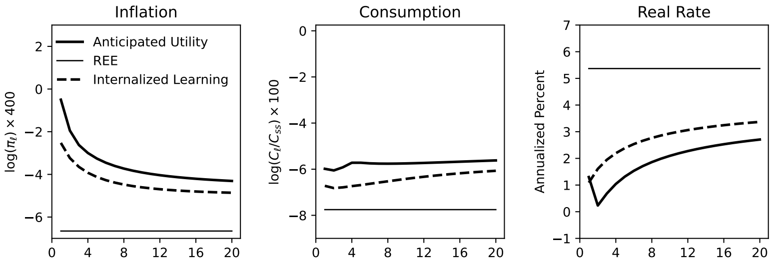

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  - Precautionary motive stronger under greater uncertainty of internalized learning.

Go Back

Multiple REE in ZLB

- Scenario:
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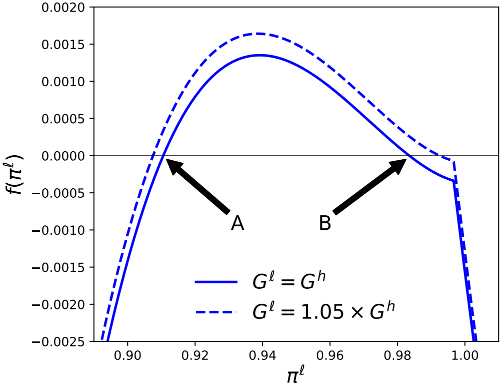
- Parameter values

$$p = 0.80,\ r_\ell = -0.0015\,(-0.6APR),\ G_{ss} = 0.20,\ r_{ss} = 0.005\,(2.0APR),$$

$$Y_{ss} = N_{ss} = 1,\ \varepsilon = 7,\ \phi = 110,\ \chi = 1.25,\ \alpha = 1.5$$

REEE Equilibria in ZLB

- Two ZLB equilibria
  - Bad-ZLB (A) equilibrium: substantial deflation, very high real rate, very low consumption.
  - Good-ZLB (B) equilibrium: more modest deflation, reduced consumption and high in real rate.



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