Does Market Incompleteness Matter for Monetary Policy? Theory and Evidence

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Introduction

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    - Renewed interest in this question since the Great Recession.
    - Research frontier in mon. econ: Heterogeneous Agent NK model.

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- How? Develop a general methodology to quantify the differences between incomplete markets (IM) and complete markets (CM) models.

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- Our objective: quantify the role of market incompleteness in shaping the response to monetary policy.
- How? Develop a general methodology to quantify the differences between incomplete markets (IM) and complete markets (CM) models.
- Apply methodology to measure the role of market incompleteness on the effects of monetary policy directly from micro data.

Challenges to Assessing the Role of IM

- Current frontier of HANK calibrated to match limited set of cross-sectional moments (e.g., MPCs, wealth Gini, ...)
- Can be difficult to assess how those moments identify differences between RANK and HANK
- For example, McKay et al (2016) claimed HANK can solve "Forward Guidance Puzzle" but a small change in their tax function leads to the opposite conclusion (while leaving steady states roughly unchanged)
- Potentially important redistributions across households in response to shocks that are not informed by the data too much

New Method to Quantify the Role of IM

- Given those challenges, instead of making assumptions we move directly to the *budget constraints* of households in the data
- We add a large amount of empirical information to the limited moments typically used
- To the extent possible, use the data to discipline the important channels in the model
- Our methodology shows how that can be used to quantify the difference between CM and IM in response to shocks
- Can't put everything into the model. Will show how we use the data and be explicit about what our methodology does and what assumptions are needed

Theoretical Objective

- Quantify the differences between CM and IM in the data.
- Therefore: Construct a transfer scheme $\Delta_{i,t}$ which renders the IM identical to CM (in terms of aggregates).
- Can measure $\Delta_{i,t}$ in the data.
- Properties of $\Delta_{i,t}$ informative on how close IM and CM are.
  - If MP implements the same real rate in RANK and HANK can use $\Delta_{i,t}$ to recover HANK GE response
  - If MP runs a Taylor rule results depend on how much the real rate changes because of the Taylor rule $\Rightarrow$ will depend on the extent of price rigidities

Model

Model: Households

Continuum of ex-ante identical households with preferences:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) - g(h_t)\}$$

where:

$$\begin{aligned} u(c) &= \frac{c^{1-\sigma}-1}{1-\sigma} \\ g(h) &= \psi \frac{h^{1+1/\varphi}}{1+1/\varphi} \end{aligned}$$

and $\beta \in (0,1)$ is the discount factor.

- Households' labor productivity $\{s_t\}_{t=0}^{\infty}$ is stochastic
- $s_t \in \mathscr{S} = \{s^1, \cdots, s^N\}$ with transition probability characterized by $p(s_{t+1}|s_t)$

MODEL: RECRUITING FIRMS

A representative, competitive recruiting firm aggregates a continuum of differentiated households labor services indexed by $j \in [0,1]$ and nominal wages per efficiency unit W_{jt}:

$$H_t = \left(\int_0^1 s_{jt} (h_{jt})^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}.$$

Given a level of aggregate labor demand H, demand for the labor services of household j is given by:

$$h_{jt} = h(W_{jt}; W_t, H_t) = \left(\frac{W_{jt}}{W_t} \right)^{-\varepsilon_w} H_t.$$

where W_t is the (equilibrium) nominal wage,

$$W_t = \left(\int_0^1 s_{jt} W_{jt}^{1-\varepsilon_w} dj \right)^{\frac{1}{1-\varepsilon_w}}.$$

Model: Wage Setting

- A union sets a nominal wage $W_{jt} = \hat{W}_t$ for an effective unit of labor to maximize profits.
- Quadratic wage adjustment as in Rotemberg (1982):

$$s_{jt}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w\right)^2 H_t.$$

- Union's wage setting problem is to maximize

$$\begin{aligned} & V_t^w\left(\hat{W}_{t-1}\right) \\ \equiv \quad & \max_{\hat{W}_t} \int \left(\frac{s_{jt}(1-\tau_t^w)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{g(h(\hat{W}_t; W_t, H_t))}{u'(C_t)}\right) dj \\ - \quad & \int s_{jt}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w\right)^2 H_t dj + \frac{1}{1+r_t} V_{t+1}^w\left(\hat{W}_t\right) \end{aligned}$$

- Symmetry: $h_{jt} = H_t$ and $\hat{W}_t = W_t$. Real wage $w_t = \frac{W_t}{P_t}$. $C_t =$ aggregate consumption.

Model: Worker Households

Can write their problem recursively:

$$V(a,s;\Omega) = \max_{c\geq 0, a'\geq 0} u(c,h) + \beta \sum_{s\in\mathscr{S}} p(s'|s)V(a',s';\Omega')$$

subject to

$$c + a' = (1+(1-\tau^a)r^a)a + (1-\tau^w)whs + \lambda\Gamma + \lambda^{MF}D^{MF}$$

$$\Omega' = \Upsilon(\Omega)$$

- Γ is a real transfer and D^{MF} are dividends from the mutual fund
- λ and λ^{MF} are how those are allocated across HHs
- $\Omega(a,s) \in \mathscr{M}$ is the distribution on the space $X = A \times S$.
- Υ equilibrium object determines evolution of Ω.

Model: Final Goods Production

A final good producer aggregates a continuum of intermediate goods indexed by $j \in [0,1]$ and with prices p_j:

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

Given a level of aggregate demand Y, cost minimization for the final goods producer implies that the demand for the intermediate good j is given by

$$y_{jt} = y(P_{jt}; P_t, Y_t) = \left(\frac{P_{jt}}{P_t} \right)^{-\varepsilon} Y_t,$$

where P_t is the (equilibrium) price of the final good and can be expressed as

$$P_t = \left(\int_0^1 P_{jt}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} .$$

Model: Intermediate Goods Production

- Production technology takes capital and labor:

$$Y_{jt} = \begin{cases} Z_t K_{jt}^{\alpha} H_{jt}^{1-\alpha} - Z_t F & \text{if} \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

  where Z_t is aggregate productivity and F is fixed cost of production.

- Marginal costs given by

$$mc_t = \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \frac{(r_t^k)^{\alpha}(w_t)^{1-\alpha}}{Z_t}$$

- Price adjustment costs a la Rotemberg (1982):

$$\frac{\theta}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \bar{\Pi}\right)^2 Y_t.$$

- Make profits D_t^{IG}

Model: Mutual Fund

- Collects HH savings A_{t+1}, promises real return $\tilde{r}_t^a$, invests in real bonds B_{t+1} and capital K_{t+1} subject to adjustment costs $\Phi(K_{t+1}, K_t)$.
- Owns intermediate goods firms and collects profits D_t^{IG}
- In equilibrium: ▸ Mutual Fund's Problem

$$
\begin{aligned}
& r_{t+1} = \tilde{r}_{t+1}^a \\
& 1 + r_{t+1}^k - \delta = (1 + \tilde{r}_{t+1}^a)(1 + \Phi_1(K_{t+1}, K_t)) + \Phi_2(K_{t+2}, K_{t+1}) \\
& A_{t+1} = K_{t+1} + B_{t+1} + \Phi(K_{t+1}, K_t).
\end{aligned}
$$

- The total profits of the fund are

$$
D_{t+1}^{MF} = (1 + r_{t+1}^k - \delta)K_{t+1} + (1 + r_{t+1})B_{t+1} - (1 + \tilde{r}_{t+1}^a)A_{t+1},
$$

- MF Dividends distributed according to λ_{it}^{MF}
- IG Dividends distributed proportional to assets.
- So that households receive:

$$
r_t^a = \tilde{r}_t^a + D_t^{IG}/A_{t+1}
$$

Model: Government

Government taxes real labor & capital income and provides transfers:

$$T(wsh, r^a a, \lambda) = \tau^w wsh + \tau^a r^a a - \lambda \Gamma.$$

- Government issues real bonds B_t
- Exogenous unvalued real expenditures G_t
- Government budget constraint given by:

$$B_{t+1} = (1 + r_t) B_t + G_t - \int T_t(w_t s_{it} h_t, r_t^a a_{it}, \lambda_i) d\Omega.$$

- Monetary policy sets nominal interest rate i_t
- Inflation π_t and real rate relationship $1 + r_t = \frac{1+i_t}{1+\pi_t}$

Equilibrium

Definition: A monetary competitive equilibrium is a sequence of tax rates τ_t^w, τ_t^a, real transfers Γ_t, real government spending G_t, bonds B_t, value functions v_t, policy functions a_{t+1} and c_t, H_t, pricing functions $r_t, r_t^k, r_t^a, \tilde{r}_t^a$ and w_t, and law of motion Υ, such that:

1. v_t satisfies the Bellman equation with corresponding policy functions a_t, c_t given price sequences r_t^a, w_t.
2. Prices are set optimally by firms.
3. Wages are set optimally by unions.
4. For all $\Omega \in \mathscr{M}$: Markets clear
5. Aggregate law of motion Υ generated by a_{t+1} and $p(s_{t+1}|s_t)$.

Focus on steady state equilibria where all real variables are constant, and constant rate of inflation.

Monetary Policy in Complete Markets

The complete markets economy arises as a special case when there is no idiosyncratic risk:

$$
\begin{aligned}
Y_t^{CM} = Z_t (K_t^{CM})^{\alpha} (H_t^{CM})^{1-\alpha} \quad &= \quad C_t^{CM} + G_t + Z_t F + K_{t+1}^{CM} - (1-\delta) K_t^{CM} + \Phi(K_{t+1}^{CM}, K_t^{CM}) \\
u_c(C_t^{CM}) = (C_t^{CM})^{-\sigma} \quad &= \quad \beta (1 + r_{t+1}^{a,CM}) (C_{t+1}^{CM})^{-\sigma} \\
(1-\varepsilon) + \varepsilon mc_t^{CM} \quad &= \quad \theta \left(\pi_t^{CM} - \overline{\Pi} \right) \pi_t^{CM} - \frac{1}{1 + r_t^{CM}} \theta \left(\pi_{t+1}^{CM} - \overline{\Pi} \right) \pi_{t+1} \frac{Y_{t+1}^{CM}}{Y_t^{CM}} \\
mc_t^{CM} \quad &= \quad \left(\frac{1}{\alpha} \right)^{\alpha} \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \frac{(r_t^{k,CM})^{\alpha} (w_t^{CM})^{1-\alpha}}{Z_t} \\
\frac{K_t^{CM}}{H_t^{CM}} \quad &= \quad \frac{\alpha w_t^{CM}}{(1-\alpha) r_t^{k,CM}} \\
\vdots \quad & \quad \quad \vdots
\end{aligned}
$$

Measurement: Differences IM $\leftrightarrow$ CM

- Complete Markets:
  - Steady state in CM: $C_{ss}^{CM}, A_{ss}^{CM}, H_{ss}^{CM}, Y_{ss}^{CM}, w_{ss}^{CM}$.
  - β chosen to match the same capital to output ratio in the data
  - Monetary Policy shock: $i_0 = i^*, i_1, i_2, \ldots, i_t, \ldots i^*$
  - Consumption/Assets/Hours/Output/Wages Responses:

$$\begin{aligned}
\text{Consumption, Assets:} \quad & \gamma_t^C = \frac{C_t^{CM}}{C_{ss}^{CM}}, \quad \gamma_t^A = \frac{A_t^{CM}}{A_{ss}^{CM}} \\
\text{Hours, Output:} \quad & \gamma_t^H = \frac{H_t^{CM}}{H_{ss}^{CM}}, \quad \gamma_t^Y = \frac{Y_t^{CM}}{Y_{ss}^{CM}} \\
\text{Wages, Transfers:} \quad & \gamma_t^w = \frac{w_t^{CM}}{w_{ss}^{CM}}, \quad \gamma_t^\Gamma = \frac{\Gamma_t^{CM}}{\Gamma_{ss}^{CM}}
\end{aligned}$$

- Incomplete Markets:
  - Distributional Impact of MP $\rightarrow$ Different Responses
  - Compute transfers $\Delta_{i,t}$ to undo $\rightarrow$ Same aggregate response

IM Transfers

Household dynamic program in response to MIT MP shock:

$$V_t(a^{IM}_{i,t}, s_{i,t}) = \max_{c^{IM}_{i,t}, a^{IM}_{i,t+1} \geq 0} u(c^{IM}_{i,t}, h^{IM}_{i,t}) + \beta \mathbb{E}_{s_{t+1}} V_{t+1}(a^{IM}_{i,t+1}, s_{i,t+1})$$

$$\begin{aligned} \text{subj. to} \quad c^{IM}_{i,t} + a^{IM}_{i,t+1} &= (1 + (1-\tau^a) r^{a,IM}_t) a^{IM}_{i,t} + (1-\tau^w) w^{IM}_t h^{IM}_{i,t} s_{i,t} \\ &\quad + \lambda_{it} \Gamma^{IM}_t + \lambda^{MF}_{it} D^{MF,IM}_t + \Delta_{i,t} \end{aligned}$$

Note: $\Delta_{i,t} = \Delta(a_{i,0}; s_{i,0}, \ldots, s_{i,t})$ does not depend on any choices.

Construct $\Delta_{i,t}$ such that

$$\begin{aligned} \frac{c^{IM}_{i,t}}{c^{IM,ss}_{i,t}} &= \frac{C^{CM}_t}{C^{CM}_{ss}} \\ \frac{h^{IM}_{i,t}}{h^{IM,ss}_{i,t}} &= \frac{H^{CM}_t}{H^{CM}_{ss}} \\ \frac{a^{IM}_{i,t+1}}{a^{IM,ss}_{i,t+1}} &= \frac{A^{CM}_{t+1}}{A^{CM}_{ss}} \end{aligned}$$

IM Transfers

Related to Werning (2015):

- No savings in that environment, $C = Y$
- Individual income proportional to output, $s_{i,t} Y_t$
- That implies equally proportional income changes with Y, $\forall i,j$:

$$\frac{s_{i,t} Y_t}{s_{i,t} Y_{ss}} = \frac{s_{j,t} Y_t}{s_{j,t} Y_{ss}}$$

- Typically, does not hold, e.g. with transfers Ξ, $\forall i,j$:

$$\frac{s_{i,t} Y_t + \Xi_t}{s_{i,t} Y_{ss} + \Xi_{ss}} \neq \frac{s_{j,t} Y_t + \Xi_t}{s_{j,t} Y_{ss} + \Xi_{ss}}$$

- We construct Δ_{it}, Δ_{jt} so that $\forall i,j$:

$$\frac{s_{i,t} Y_t + \Xi_t + \Delta_{it}}{s_{i,t} Y_{ss} + \Xi_{ss}} = \frac{s_{j,t} Y_t + \Xi_t + \Delta_{jt}}{s_{j,t} Y_{ss} + \Xi_{ss}}$$

- Preserves heterogeneity: Δ_{it} not related to Arrow securities.

CONSTRUCT TRANSFERS

- 1. Take away labor income, transfer and dividend change

$$
\begin{aligned}
&(w_{ss}^{IM}(1-\tau^{w})h_{i,t}^{IM,ss}s_{i,t}+\lambda_{it}\Gamma_{ss}^{IM}+\lambda_{it}^{MF}D_{ss}^{MF,IM}) \\
&-(w_{t}^{IM}(1-\tau^{w})h_{i,t}^{IM}s_{i,t}+\lambda_{it}\Gamma_{t}^{IM}+\lambda_{it}^{MF}D_{t}^{MF,IM})
\end{aligned}
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- 2. Take away asset income changes

$$a_{i,t}^{IM,ss}[(1+(1-\tau^a)r_{ss}^{a,IM})-(1+(1-\tau^a)r_t^{a,IM})]$$

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$$(a_{i,t}^{IM,ss}-a_{i,t}^{IM})(1+(1-\tau^a)r_{t}^{a,IM})$$

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- 4. Add resources for consumption

$$c_{i,t}^{IM}-c_{i,t}^{IM,ss}$$

Construct transfers

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$$-(w_t^{IM}(1-\tau^w)h_{i,t}^{IM}s_{i,t}+\lambda_{it}\Gamma_t^{IM}+\lambda_{it}^{MF}D_t^{MF,IM})$$

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- 4. Add resources for consumption

$$c_{i,t}^{IM}-c_{i,t}^{IM,ss}$$

- 5. Add resources for asset accumulation

$$a_{i,t+1}^{IM}-a_{i,t+1}^{IM,ss}$$

Construct transfers

Taking into account that in the desired equilibrium:

$$\begin{aligned}
w_t^{IM} - w_{ss}^{IM} &= \Big(\frac{w_t^{CM}}{w_{ss}^{CM}} - 1\Big) w_{ss}^{IM} = (\gamma_t^{w} - 1) w_{ss}^{IM} \\
h_{i,t}^{IM} - h_{i,t}^{IM,ss} &= \Big(\frac{H_t^{CM}}{H_{ss}^{CM}} - 1\Big) h_{i,t}^{IM,ss} = (\gamma_t^{H} - 1) h_{i,t}^{IM,ss} \\
c_{i,t}^{IM} - c_{i,t}^{IM,ss} &= \Big(\frac{C_t^{CM}}{C_{ss}^{CM}} - 1\Big) c_{i,t}^{IM,ss} = (\gamma_t^{C} - 1) c_{i,t}^{IM,ss} \\
\ldots & \qquad\qquad \ldots
\end{aligned}$$

We get:

$$\begin{aligned}
\Delta_{i,t} &= (\gamma_t^{C} - 1) c_{i,t}^{IM,ss} - \left(\gamma_t^{H}\gamma_t^{w} - 1\right) w_{ss}^{IM} (1 - \tau^{w}) s_{it} h_{i,t}^{IM,ss} \\
&- \lambda_{i,t} (\gamma_t^{\Gamma} - 1) \Gamma_{ss}^{IM} - \lambda_{i,t}^{MF} (\gamma_t^{MF} - 1) D_{ss}^{MF,IM} \\
&+ a_{i,t}^{IM,ss} [(1 + (1 - \tau^{a}) r_{ss}^{a,IM}) - (1 + (1 - \tau^{a}) r_t^{a,IM})] \\
&- a_{i,t}^{IM,ss} (\gamma_t^{A} - 1)(1 + (1 - \tau^{a}) r_t^{a,IM}) + a_{i,t+1}^{IM,ss} (\gamma_{t+1}^{A} - 1).
\end{aligned}$$

PROPERTIES OF Δ_{it}

- Δ_{it} captures redistributive impact:

$$\int \Delta_{it} di = 0 \quad \forall t$$

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- Knife-edge case (also Werning)

$$\Delta_{i,t} \equiv 0 : \text{CM} = \text{IM}.$$

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$$\Delta_{i,t} \equiv 0 : \text{CM} = \text{IM}.$$

- Δ_{it} depends on the steady-state joint cross-sectional distribution of $c, a, swh, \ldots \Longleftarrow$ DATA

Equivalence Between IM and CM

Theorem

Consider the CM economy $\{C_t^{CM}, K_t^{CM}, H_t^{CM}, w_t^{CM}, \pi_t^{CM}, 1+i_t\}$. The IM economy with transfers $\Delta_{i,t}$ as above and the same policies has the same aggregate consumption, capital, hours, wages and inflation rates as the corresponding complete markets model. Furthermore, individual consumption, hours, and savings satisfy

$$
\begin{aligned}
c_{i,t}^{IM} &= \gamma_t^C c_{i,t}^{IM,ss}, \\
h_{i,t}^{IM} &= \gamma_t^H h_{i,t}^{IM,ss}, \\
a_{i,t+1}^{IM} &= \gamma_{t+1}^A a_{i,t+1}^{IM,ss}.
\end{aligned}
$$

IDEA OF PROOF

1. Budget constraints are satisfied (by construction)
2. Proposed allocations satisfy Euler equation for all HHs:

$$(c_{i,t}^{IM})^{-\sigma} \quad = \quad \beta(1 + r_{t+1}^{a,IM}(1-\tau^a))E_t(c_{i,t+1}^{IM})^{-\sigma}$$

Idea of proof

1. Budget constraints are satisfied (by construction)
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$$
\begin{aligned}
\left(\frac{C_t^{CM}}{C_{ss}^{CM}} c_{i,t}^{IM,ss}\right)^{-\sigma} &= \beta(1+r_{t+1}^{a,IM}(1-\tau^a))E_t\left(\frac{C_{t+1}^{CM}}{C_{ss}^{CM}} c_{i,t+1}^{IM,ss}\right)^{-\sigma} \\
c_{i,t}^{IM} &= \frac{C_t^{CM}}{C_{ss}^{CM}} c_{i,t}^{IM,ss} \\
c_{i,t+1}^{IM} &= \frac{C_{t+1}^{CM}}{C_{ss}^{CM}} c_{i,t+1}^{IM,ss}
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\left(\frac{C_t^{CM}}{C_{ss}^{CM}}\right)^{-\sigma} &= \left(\frac{C_{t+1}^{CM}}{C_{ss}^{CM}}\right)^{-\sigma} \frac{\left(1+r_{t+1}^{a,IM}(1-\tau^a)\right)}{\left(1+r_{ss}^{a,IM}(1-\tau^a)\right)}
\end{aligned}
$$

Idea of proof

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2. Proposed allocations satisfy Euler equation for all HHs:

$$
\begin{aligned}
(c_{i,t}^{IM,ss})^{-\sigma} &= \beta(1+r_{ss}^{a,IM}(1-\tau^a))E_t(c_{i,t+1}^{IM,ss})^{-\sigma} \\
(\frac{C_t^{CM}}{C_{ss}^{CM}})^{-\sigma} &= (\frac{C_{t+1}^{CM}}{C_{ss}^{CM}})^{-\sigma}\frac{(1+r_{t+1}^{a,IM}(1-\tau^a))}{(1+r_{ss}^{a,IM}(1-\tau^a))}
\end{aligned}
$$

3. Given that consumption and savings paths identical and only the consumption block is different, all other optimality conditions and constraints satisfied and all markets clear

Return heterogeneity

- Evidence of heterogenous returns across households
- Incorporate return heterogeneity $\lambda^r_{i,t}(1+(1-\tau^a)r^a_t)$ on their assets a into the model
- This gives the following definition of $\Delta_{i,t}$

$$\begin{aligned}
\Delta_{i,t} = {} & (\gamma^C_t - 1)c^{IM,ss}_{i,t} && \text{Consumption} \\
& - (\gamma^H_t \gamma^W_t - 1) w^{IM}_{ss}(1-\tau^w) s_{i,t} h^{IM,ss}_{i,t} && \text{Labor} \\
& - \lambda_{i,t}(\gamma^\Gamma_t - 1)\Gamma^{CM}_{ss} && \text{Transfers} \\
& + a^{IM,ss}_{i,t} \lambda^r_{i,t}(1 - \gamma^A_t + r_{ss}(1-\tau^a)(1-\gamma^A_t \gamma^r_t)) && \text{Assets} \\
& + a^{IM,ss}_{i,t+1}(\gamma^A_{t+1} - 1) && \text{Savings}
\end{aligned}$$

Mapping to Data

Use comprehensive data from the PSID to construct the HH budget constraint:

$$c_t + a_{t+1} = \lambda^r (1 + (1 - \tau^a) r^a_{ss}) a_t + (1 - \tau^w) w h_t s_t + \lambda_t \Gamma$$

- c_t, total household expenditures
- $\lambda^r (1 + (1 - \tau^a) r^a_{ss}) a_t$, household wealth + asset income less the capital tax bill $T^k_{i,t}$, where λ^r allows for return heterogeneity.
- $(1 - \tau^w) w h_t s_t$, household labor income less the labor portion of the tax bill $T^l_{i,t}$.
- $\lambda \Gamma$, total government and private transfers (social security, UI benefits, TANF, SSI, other welfare, VA pensions, disability, alimony, child support.)
- a_{t+1}, calculated as the residual of the previous items such that the budget constraint holds.

PSID Data, Aggregates

Consumption	1,952,278,226
Saving, A_{t+1}	18,521,166,788
Wealth, A_t	19,063,437,809
Asset Income	217,406,381
Labor Income	2,516,248,282
Transfers	197,007,137
Dividends	124,371,449
Taxes	-734,704,632

PSID Data, Mapping to Model

- We want data/model consistency: PSID $\leftrightarrow$ model aggregates

$$Y = G + C + \delta K \tag{1}$$

$$Y = D^{IG} + wH + r^k K \tag{2}$$

$$\varepsilon = D^{IG}/Y \tag{3}$$

$$\Gamma = T^{tax} - rB - G \tag{4}$$

$$A = K + B \tag{5}$$

$$r^k = \tilde{r} + \delta = r + \delta \tag{6}$$

$$r^a = \tilde{r}^a + d^{IG} = r + d^{IG} \tag{7}$$

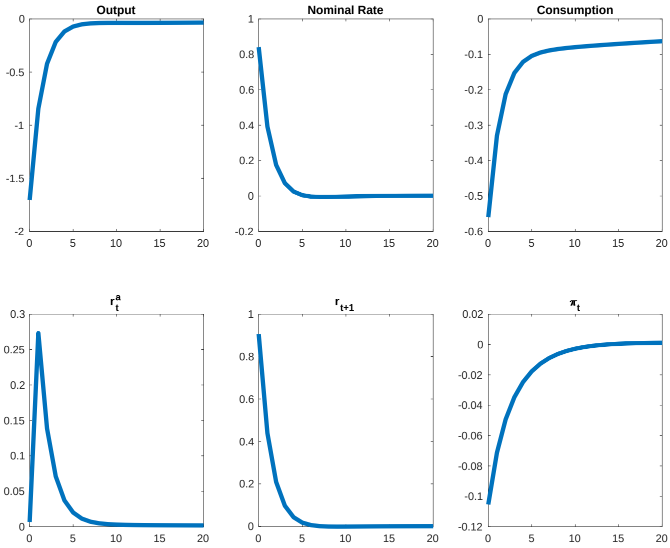
PSID Data, Mapping to Model

- We want data/model consistency
- PSID ‘misses’ wealthy households and certain consumption categories (e.g. health expenditures)
- We add representative top 1% to match aggregate $A/wH \to r^a$
- Assume labor share of 0.64 $\to Y$
- Assume firm profits are distributed as dividends $\to \varepsilon$
- Fix K/Y to the data counterpart $\to r, \delta, r^k$
- Remaining items $G/Y, C/Y, B/Y$ balance accounting identities.
- Rescale consumption based on $C/Y = 0.655$, close to NIPA.

PSID Data, Mapping to Model

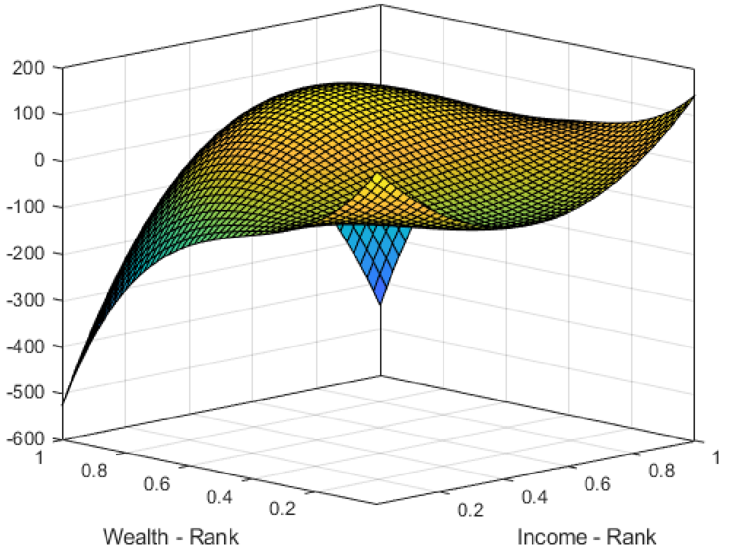
	Description	Quarterly Values
	Fixed to match	
$w_{ss}H_{ss}/Y_{ss}$	Labor share	0.640
K_{ss}/Y_{ss}	Capital to output	13.392
A_{ss}/Y_{ss}	Asset to output	18.8432
	Measured	
r^a	Return on assets	0.0080
r	Interest rate	0.0064
Γ_{ss}/Y_{ss}	Transfers to output	0.0501
T_{ss}/Y_{ss}	Taxes to output	0.1868
$\varepsilon = D_{ss}/Y_{ss}$	Dividend to output	0.0316
α	Output elasticity	0.3391
T^{labor}/wH	Labor income tax rate	0.2565
	Residuals	
C_{ss}/Y_{ss}	Consumption to output	0.6548
B_{ss}/Y_{ss}	Debt to output	5.4515
r^k_{ss}	Rental rate	0.0245
δ	Depreciation rate	0.0182
G_{ss}/Y_{ss}	Gov't expenditure to output	0.1021

Monetary Policy Shock in RANK

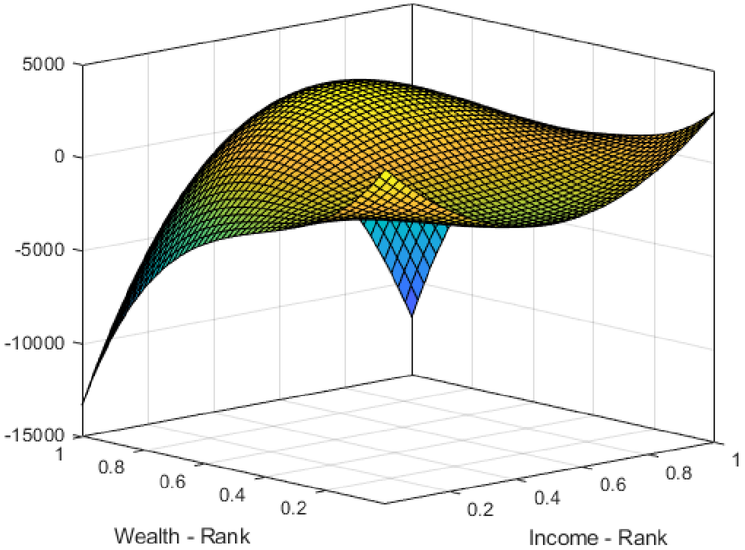


25 BP innovation to the Taylor Rule in RANK

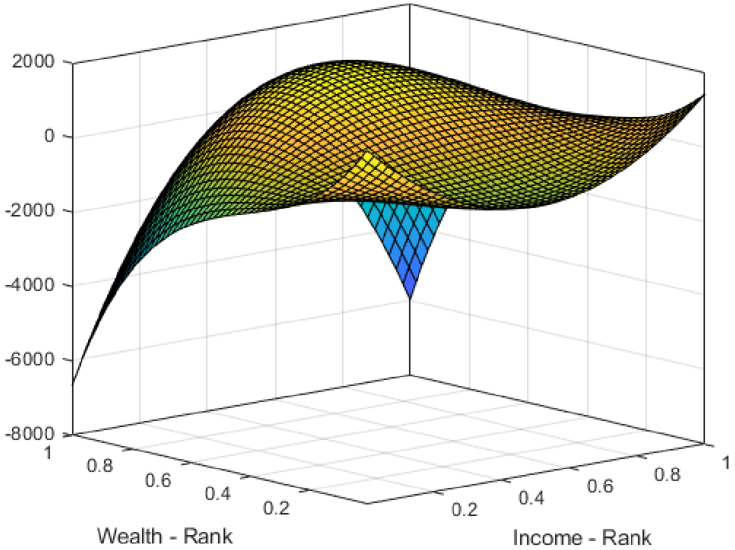
MODEL: DISTRIBUTION OF Δ_i AT $t = 0$



MODEL: DISTRIBUTION OF Δ_i AT $t = 1$



MODEL: DISTRIBUTION OF Δ_i AT $t = 2$

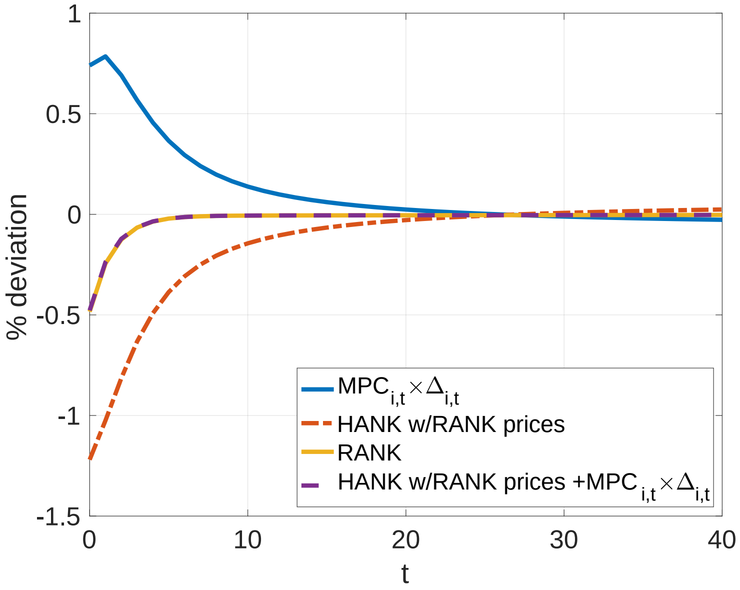


Going from Δs to aggregates

- We've now constructed $\Delta_k(a, y)$ in the data
- To construct impact on aggregate consumption ΔC_t we need the MPCs to those transfers
- Want $MPC_{t,k}(a, y)$, the *aggregate consumption* response in time t to a transfer in time k to a household with (a, y) in period k
- Can't directly measure in the data, so compute $MPC_{t,k}(a, y)$ in HA model

$$\Delta C_t = \sum_{k=0}^{\infty} \int_{a \in \mathscr{A}, y \in \mathscr{Y}} \Delta_k(a, y) MPC_{t,k}(a, y) da dy$$

Validation using Model Generated Data



Can We Recover the GE HANK IRF?

- Theory tells us difference between RANK and HANK for same *price sequence*
- Ideally, want to recover the GE HANK response to MP
- We show (next slides) our methodology:
  - Delivers HANK GE response if MP implements the same real rate in HANK as RANK
  - If MP runs a Taylor rule, results depend on how much real rate moves:
  - If prices are very sticky, $r^{RANK} \approx r^{HANK} \Rightarrow$

    $\Rightarrow GE^{HANK} \approx GE^{RANK} - \Delta C$

General Equilibrium

- To first order we, our theorem tells us we can express

$$dY^{RANK} = M^T\left(-Br^{RANK}\right) + \Delta C + M^r r^{RANK} + M^Y dY^{RANK}$$

  where ΔC is as defined before, an M^T, M^r and M^Y are matrices which represent the consumption response in the HANK economy to shocks to transfers, real rate, and output, respectively
- For HANK we have (r^{HANK} subs. r^{RANK})

$$dY^{HANK} = M^T\left(-Br^{HANK}\right) + M^r r^{HANK} + M^Y dY^{HANK}$$

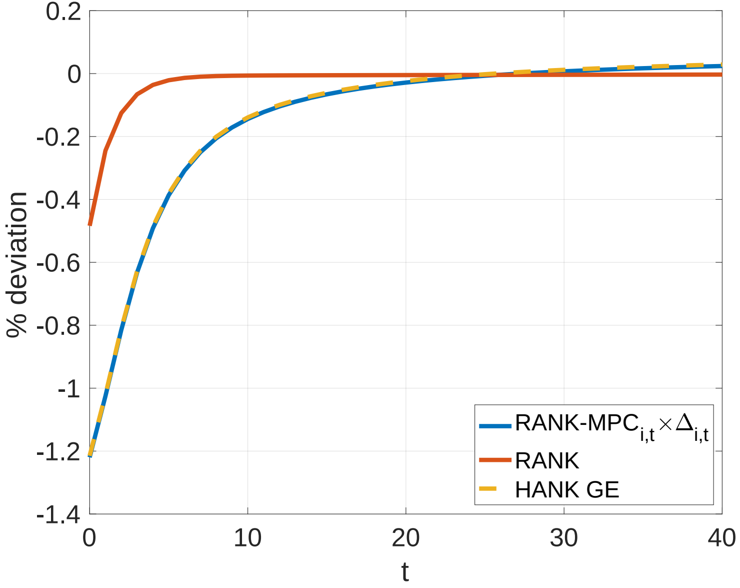
- Differencing between that and the HANK GE response:

$$\begin{aligned} dY^{RANK} - dY^{HANK} &= M^T\left(-B(r^{RANK} - r^{HANK}\right) \\ &+ \Delta C + M^r\left(r^{RANK} - r^{HANK}\right) \\ &+ M^Y(dY^{RANK} - dY^{HANK}) \end{aligned}$$

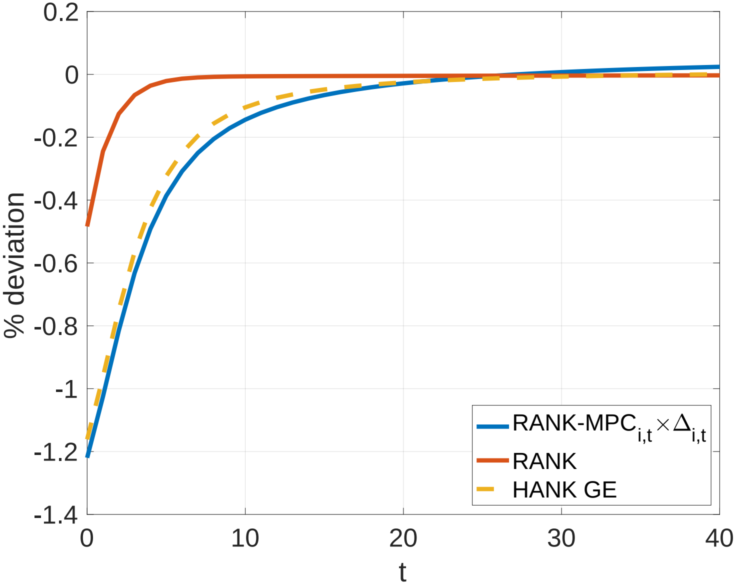
General Equilibrium

- If MP policy implements the same real rate, then all of the difference terms cancel, and $dY^{RANK} - dY^{HANK} \approx \Delta C$
- If prices are very sticky, then for the same nominal shock $r^{RANK} \approx r^{HANK}$, so again $dY^{RANK} - dY^{HANK} \approx \Delta C$
- If prices are (counterfactually) flexible, then the inflation responses will be different, leading to different real rates from the Taylor rule.
- Working on a way to recover the HANK general equilibrium, but would require more model assumptions

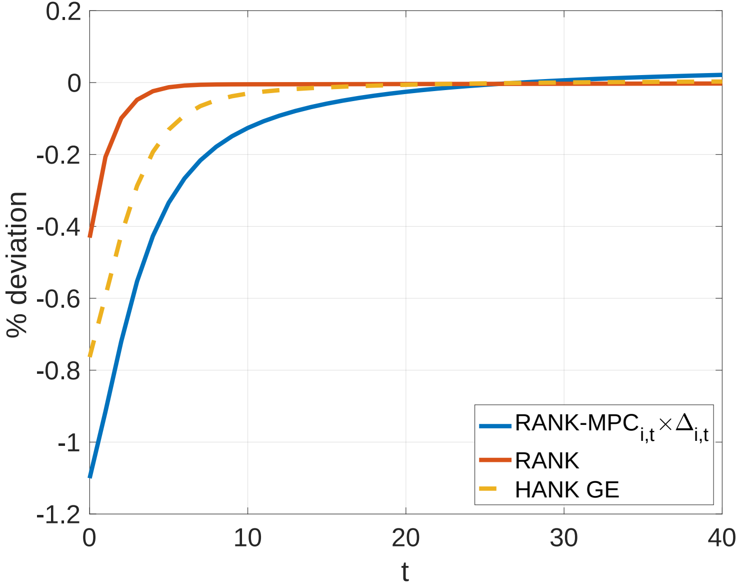
With MP implementing same real rate



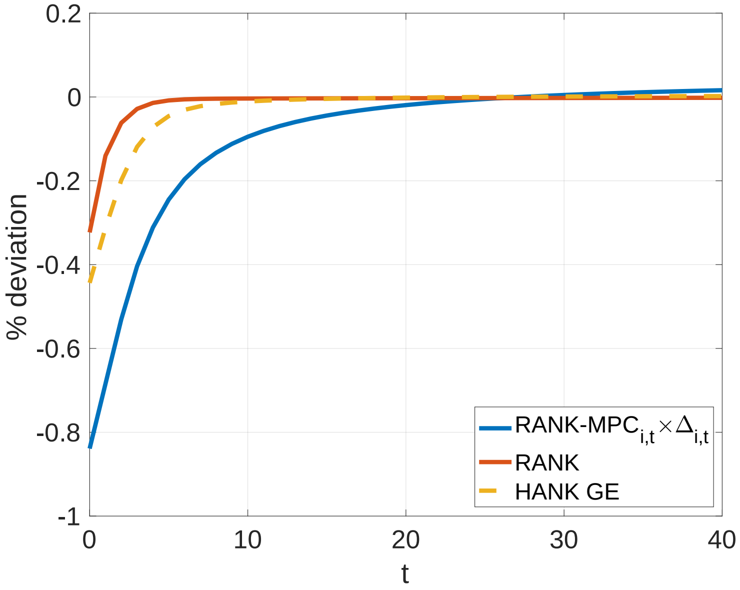
BENCHMARK, SLOPE OF NKPC = 0.03



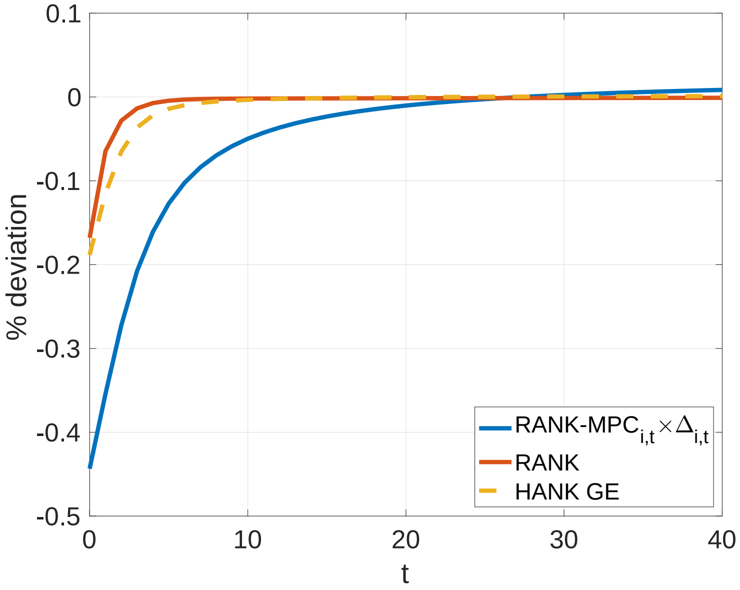
Slope of NKPC = 0.1



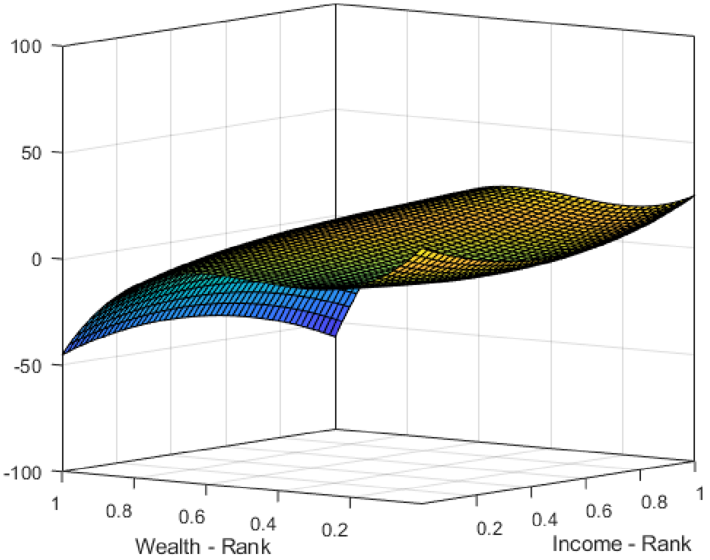
Slope of NKPC = 0.3



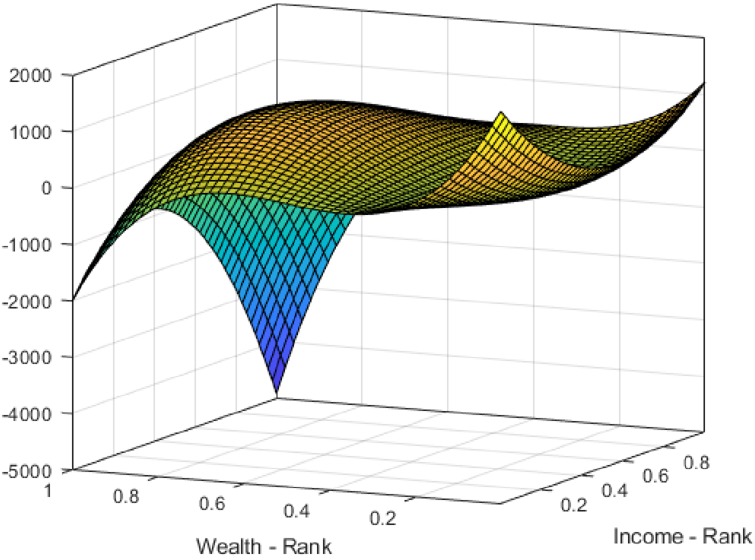
Slope of NKPC = 1



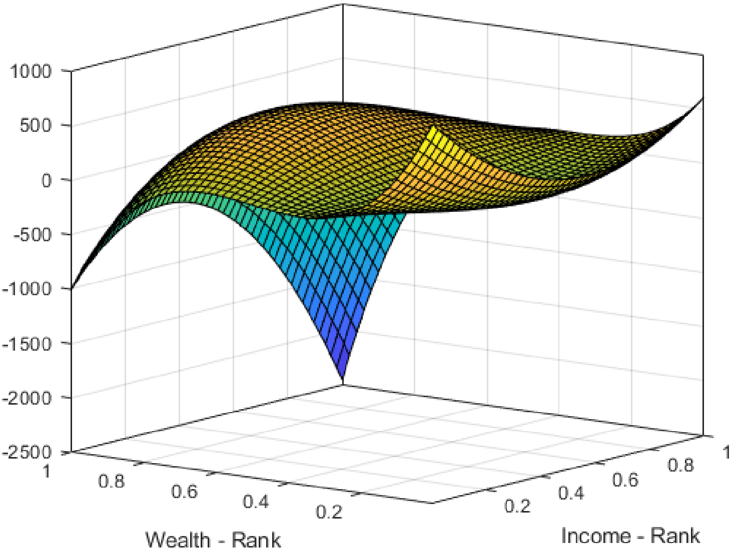
DATA: DISTRIBUTION OF Δ_i AT $t = 0$



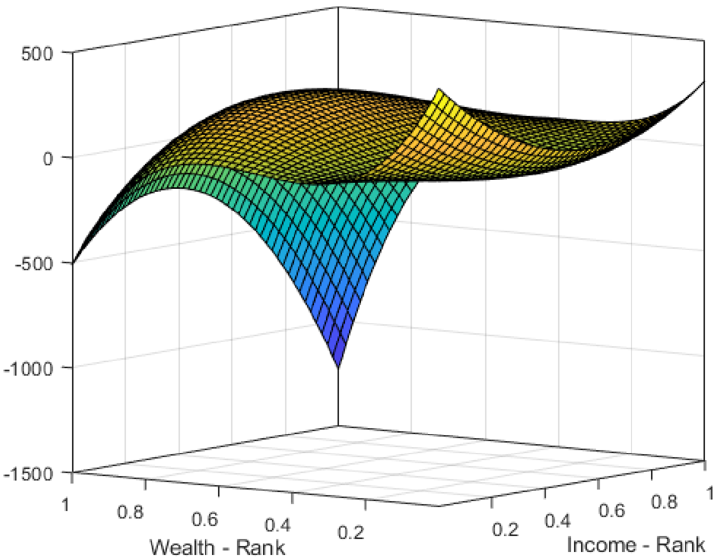
DATA: DISTRIBUTION OF Δ_i AT $t = 1$



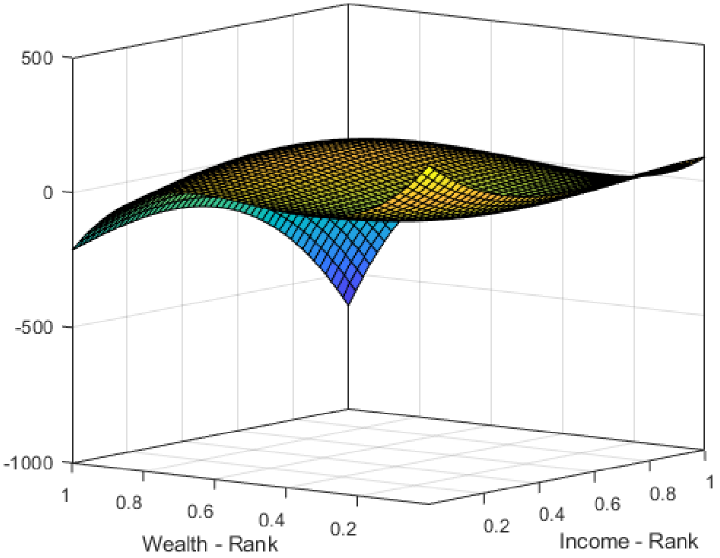
DATA: DISTRIBUTION OF Δ_i AT $t = 2$



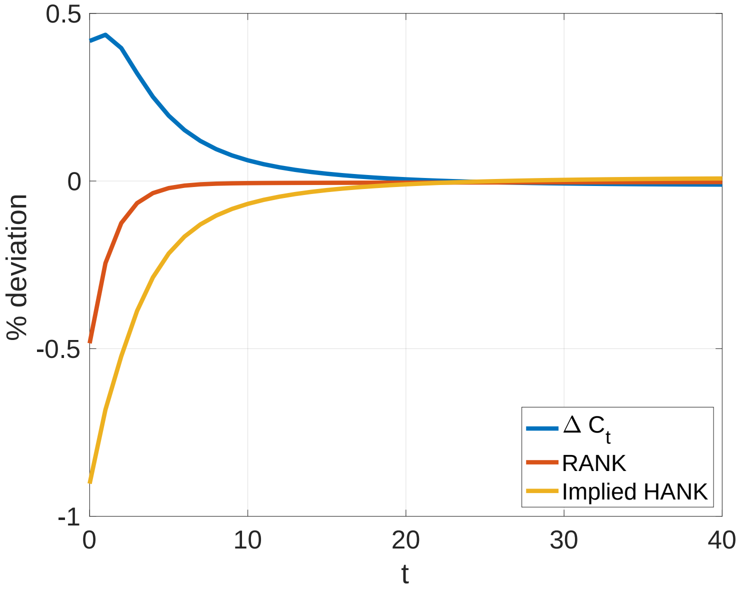
DATA: DISTRIBUTION OF Δ_i AT $t = 3$



DATA: DISTRIBUTION OF Δ_i AT $t = 4$



Consumption response to Δ in PSID



Conclusions

- HANK model predictions are very sensitive to assumptions on the distributional effects of shocks and policies.
- Developed a new methodology to theoretically analyze the the consequences of market incompleteness using micro data.
- Find significant differences between RANK and HANK
- Next steps:
  - Measures of MPCs directly from data.
  - Better understand the importance of nominal vs real fiscal policies.

Thanks!