

Online Appendix for “Hidden Stagflation”

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September 27, 2023

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1 Detailed Explanation of the Methodology by Auer et al. (2019)

For the sake of completeness, we review the methodology developed by Auer et al. (2019) in this section.

1.1 Framework by Auer et al. (2019)

There are N countries, indexed by i and j , and S sectors indexed by s and k . The primitive of the analysis is the cost $W_{i,s,t}$ of sector s in country i at date t , which is given by

$$W_{i,s,t} = F_{i,s}(\chi_{i,s,t}, \mathbf{P}_{i,s,t}),$$

where $F_{i,s}$ is the cost function, $\chi_{i,s,t}$ is the domestic cost factor, and $\mathbf{P}_{i,s,t}$ is the input price vector, which is given by $\mathbf{P}_{i,s,t} = (P_{ji,ks,t})_{j \in N, k \in S}$ in US dollars. Here, production function $F_{i,s}$ is assumed to be time-invariant and to exhibit constant returns to scale.

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Log-linearizing the cost, $W_{i,s,t}$, at $(W_{i,s,t-1}, \chi_{i,s,t-1}, \mathbf{P}_{i,s,t-1})$, we obtain

$$\hat{W}_{i,s,t} = \gamma_{i,s,t}^C \hat{\chi}_{i,s,t} + \sum_{j,k} \gamma_{ji,ks,t-1} \hat{P}_{ji,ks,t}, \quad (1)$$

where

$$\gamma_{i,s,t-1}^C = \frac{\frac{\partial F_{i,s}}{\partial \chi_{i,s,t}} \chi_{i,s,t-1}}{W_{i,s,t-1}} \quad \gamma_{ji,ks,t-1} = \frac{\frac{\partial F_{i,s}}{\partial P_{ji,ks,t}} P_{ji,ks,t-1}}{W_{i,s,t-1}},$$

and the variable with a hat represents the first log difference of the original variable (e.g., $\hat{W}_{i,s,t} = \ln W_{i,s,t} / W_{i,s,t-1}$). It follows from the envelop theorem that $\gamma_{i,s,t-1}^C$ corresponds to the share of costs associated with the domestic factor in the total costs and $\gamma_{ji,ks,t-1}$ corresponds to the share of costs associated with intermediate input from sector k in country j in the total costs of sector s in country i . The constant-returns-to-scale property of $F_{i,s}$ implies that

$$\gamma_{i,s,t-1}^C + \sum_{j,k} \gamma_{ji,ks,t-1} = 1$$

for all i, s , and t . Equation (1) corresponds to equation (2) in [Auer et al. \(2019\)](#).

Note that we cannot directly observe the change in the marginal cost, $\hat{W}_{i,s,t}$. We assume that $\hat{W}_{i,s,t}$ is equal to the change in the output price of that good, which is observable:

$$\hat{W}_{i,s,t} = \hat{P}_{i,s,t}. \quad (2)$$

This assumption holds, for example, if the markup charged by the firm in country i in sector s is time-invariant.

Moreover, we assume that $\hat{P}_{ji,ks,t}$ is equal to the change in the cost of the destination:

$$\hat{P}_{ji,ks,t} = \hat{W}_{j,k,t}. \quad (3)$$

Substituting equations (2) and (3) into equation (1), we obtain the following linear equations with respect to $\hat{\chi}_{i,s,t}$:

$$\hat{P}_{i,s,t} = \gamma_{i,s,t-1}^C \hat{\chi}_{i,s,t} + \sum_{j,k} \gamma_{ji,ks,t-1} \hat{P}_{j,k,t}. \quad (4)$$

Solving the system of the linear equations, we can recover the domestic factors, $(\hat{\chi}_{i,s,t})_{i,s,t}$.

1.2 Implementation

To back out the domestic factors ($\hat{\chi}_{i,s,t}$) by solving equation (4), we need to specify the empirical counterparts of $\hat{P}_{i,s,t}$, $\hat{E}_{ji,t}$, $\gamma_{i,s,t-1}^C$, and $\gamma_{ji,ks,t-1}$. All the data we use are downloaded from the World Input-Output Database 2016 Version (Timmer et al. (2015)). This database provides (1) the world IO table (WIOT) in current prices denoted in US dollars. (2) the WIOT in US collars of the previous year (WIOT-PYP), and (3) the exchange rates used to convert national values into US dollars.

The first two datasets allow us to construct $\hat{P}_{i,s,t}$ in US collars (not in national currencies). We measure $\gamma_{ji,ks,t}$ as the expenditure for input imported from sector k in country j divided by the gross output of sector s in country i . Then, $\gamma_{i,s,t}^c$ is residually determined by

$$\gamma_{i,s,t}^c = 1 - \sum_{k,j} \gamma_{ji,ks,t}.$$

To solve the linear equation, we rewrite the linear system in the following vector format year-by-year.

$$\underbrace{\mathbf{p}_t}_{NS \times 1} = \underbrace{\text{diag}(\Gamma_t^C)}_{NS \times NS} \times \underbrace{\mathbf{c}_t}_{NS \times 1} + \underbrace{\Gamma_t^T}_{NS \times NS} \times \underbrace{\mathbf{p}_t}_{NS \times 1},$$

where

$$\begin{aligned} \underbrace{\mathbf{p}_{i,t}}_{1 \times S} &= (p_{i,1,t}, \dots, p_{i,S,t}) \quad , \quad \underbrace{\mathbf{c}_{i,t}}_{1 \times S} = (\hat{\chi}_{i,1,t}, \dots, \hat{\chi}_{i,S,t}), \\ \underbrace{\mathbf{p}_t}_{NS \times 1} &= (\mathbf{p}_{1,t}, \mathbf{p}_{2,t}, \dots, \mathbf{p}_{N,t})^T \quad , \quad \underbrace{\mathbf{c}_t}_{NS \times 1} = (\mathbf{c}_1, \dots, \mathbf{c}_N)^T, \\ \underbrace{\tilde{\Gamma}_t^{ij}}_{S \times S} &= (\gamma_{t,ij,k,s})_{k,s} \quad \underbrace{\Gamma_t}_{NS \times NS} = \left(\tilde{\Gamma}_t^{ij} \right)_{i,j \in N} \\ \underbrace{\Gamma_{i,t}^C}_{1 \times S} &= (\gamma_{i,1,t}^C, \dots, \gamma_{i,S,t}^C) \quad \underbrace{\Gamma_t^C}_{NS \times 1} = (\Gamma_{1,t}^C; \dots; \Gamma_{N,t}^C). \end{aligned}$$

With this formulation, it is easy to compute the domestic factors in the following manner:

$$\mathbf{c}_t = (\text{diag}(\Gamma_t^C))^{-1} (\mathbf{I}_{NS \times NS} - \Gamma_t^T) \mathbf{p}_t.$$

Now, we identify the effects of the imports to the Japanese computer sector (C26) on the relative

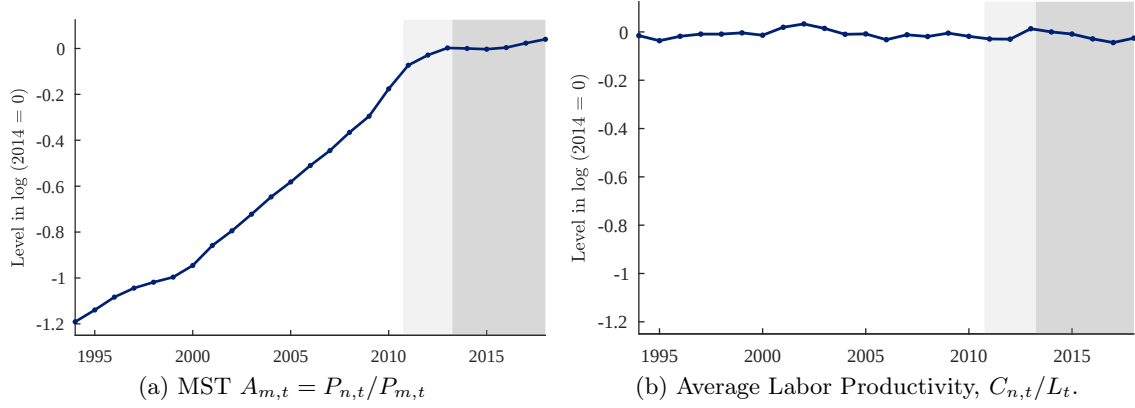
computer price as depicted below:

$$\hat{P}_{i,s,t} - \hat{e}_{i,s,t} - \pi_{s,t} = \underbrace{\gamma_{i,s,t-1}^C (\hat{X}_{i,s,t} - \hat{e}_{i,s,t} - \pi_{s,t})}_{\text{Domestic Factor Contribution}} + \underbrace{\sum_{j,k} \gamma_{ji,ks,t-1} (\hat{P}_{j,k,t} - \hat{e}_{i,s,t} - \pi_{s,t})}_{\text{International/Sectoral Trade Contribution}},$$

where $\hat{e}_{i,s,t}$ is the change in the nominal exchange rate between the Japanese yen and the US dollar.

2 Additional Figure

We list additional figures mentioned in the main text.



Notes: Strictly speaking, average labor productivity does not correspond to economy-wide TFP due to the presence of the pricing distortion. But this distortion does not impact the growth rate of ALP. This figure illustrates that the technology of the non-manufacturing firm has not improved over time.

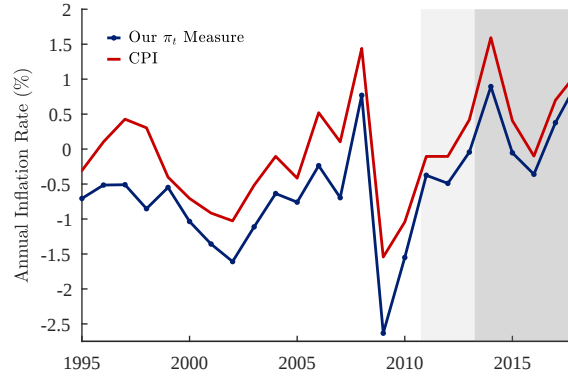


Figure 1: CPI VS Our Inflation Measure

Notes: This figure depicts the CPI inflation rate, excluding the impact of the VAT hikes and our inflation measure before adjusting the level. It is evident that these two inflation rates move similarly, and the only main difference between the two is the difference in level.

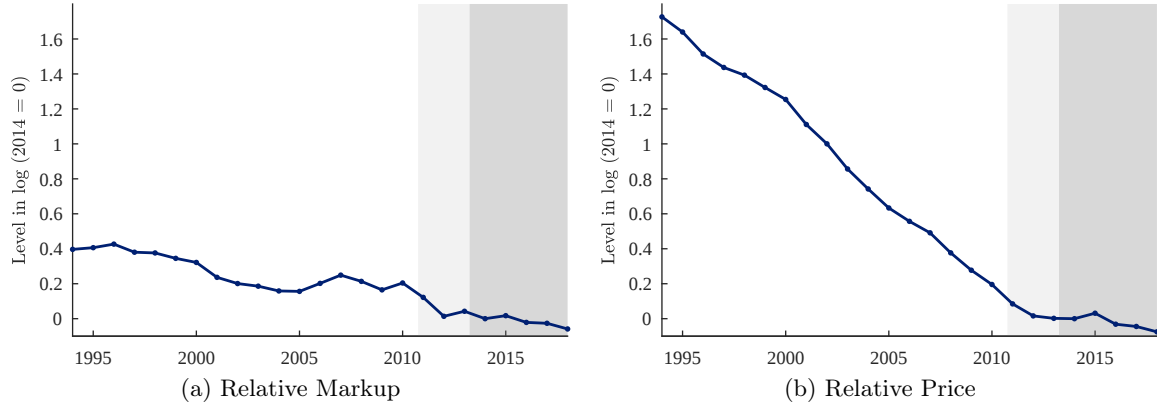


Figure 2: ICT Equipment

Notes: We depict the relative markup and the relative price of the ICT equipment in Figure 2. The growth rate of the relative markup has been weakly declining, and the magnitude of the decline in the relative markup is much smaller than that of the relative price of the ICT equipment.

3 New Keynesian Model

In the main text, we argue that the impulse response of inflation to a temporary TFP shock qualitatively differs from that to a permanent TFP shock. We formally establish this result in Section 3.1. Next, using the same model, we show that when the nominal interest rate is pegged, an almost permanent TFP shock has the unrealistically huge impacts on inflation and the output gap. We then demonstrate how to eliminate these implausible features.

3.1 Sensitivity of Expectation

Consider the canonical three-equation New Keynesian models. The so-called IS curve is

$$x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1} - r_t^*],$$

where x_t denotes output gap and r_t^* is the natural rate of interest. The Taylor rule is

$$r_t = \phi \pi_t,$$

where $\phi > 1$. The Phillips curve is

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

where $\kappa > 0$ and β is a discount factor. Output gap, interest rate, and inflation are endogenous variables, and the natural rate is exogenously given.

$$r_t^* = E_t a_{t+1} - a_t,$$

where a_t denotes a TFP shock.

We consider the following two cases. First, the growth rate of a_t follows a first-order autoregressive process:

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t \quad \Delta a_t \equiv a_t - a_{t-1}, \quad (5)$$

where $\rho \in [0, 1)$ and ε_t is an iid shock. Second, TFP shock a_t follows a first-order autoregressive process:

$$a_t = \rho a_{t-1} + \varepsilon_t. \quad (6)$$

Under the first case, (5), the natural rate is

$$r_t^* = \rho \Delta a_t.$$

It is easy to show that the locally unique rational expectation equilibrium is given by

$$\begin{aligned} x_t &= \rho(1 - \rho\beta) \psi \Delta a_t \quad \text{and} \\ \pi_t &= \rho\kappa\psi \Delta a_t, \end{aligned}$$

where $\psi = [(1 - \rho)(1 - \rho\beta) + \kappa(\phi - \rho)]^{-1}$. Thus, the inflation initially rises.

Now, consider the second case, (6). Then, the natural rate boils down to

$$r_t^* = (\rho - 1) a_t.$$

Again, we derive the locally unique rational expectation equilibrium in the following manner:

$$\begin{aligned} x_t &= -(1 - \rho)(1 - \beta\rho) \psi a_t \quad \text{and} \\ \pi_t &= -(1 - \rho) \kappa \psi a_t. \end{aligned}$$

Now, inflation initially declines.

This result arises for the following reasons. Consider a simplified version of the the first case: the TFP today increases, $a_0 > 0$, but tomorrow and onward it will be higher, $a_0 < a_1 = a_2 = \dots$. The economy reaches the steady state tomorrow. Then, from the household's perspective, the economy

will be better technologically tomorrow and the households want to consume more tomorrow. Thus, the natural rate of interest at date $t = 0$ increases.

$$r_0^* = \rho \Delta a_0 > 0.$$

If the interest rate does not increase sufficiently (i.e., $r_0 < r_0^*$), households consume more today and the labor demand becomes higher. Then, the wage today increases and, consequently, so does the price today. This is why we have high inflation today.

Now, consider a simpler version of the second case. Suppose that the TFP is temporarily higher today, $a_0 > a_1 = a_2 \cdots = 0$. Then, from the households' perspective, the economy will be worse tomorrow, and the natural rate declines, not increases. Again, if the interest rate does not decline sufficiently (i.e., $r_0 > r_0^*$), then the households consume less today and the labor demand shrinks. This weak demand lowers the nominal wage and, consequently, the price today. This is why inflation is lower today. Note that in both cases, as $\phi \rightarrow \infty$, $(x_t, \pi_t) \rightarrow \mathbf{0}$.

3.2 Almost Permanent Shock

We can use the same New Keynesian model to study how the economy responds to the following almost permanent technology shock. Consider the following almost permanent shock.

$$\Delta a_t = \begin{cases} \Delta a & 0 \leq t \leq T \\ 0 & t > T \end{cases},$$

where T is a large integer. We assume that from $T+1$, the economy goes back to the original steady state. In other words, for all $k = 1, 2, 3, \dots$,

$$x_{T+k} = \pi_{T+k} = 0.$$

Given these terminal conditions, we solve the model backward:

$$\begin{aligned} x_t &= x_{t+1} - \phi \pi_t + \pi_{t+1} + \Delta a \quad \text{and} \\ \pi_t &= \beta \pi_{t+1} + \kappa x_t. \end{aligned}$$

Then, the output gap and inflation at date $t = 0$ is

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = (I + A + A^2 + \cdots + A^T) B \begin{pmatrix} \Delta a \\ 0 \end{pmatrix}, \tag{7}$$

where

$$A = \begin{pmatrix} 1 & \phi \\ -\kappa & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 \\ 0 & \beta \end{pmatrix}, B = \begin{pmatrix} 1 & \phi \\ -\kappa & 1 \end{pmatrix}^{-1}.$$

Suppose that the nominal interest rate is pegged, $\phi = 0$. Then,

$$A = \begin{pmatrix} 1 & 0 \\ -\kappa & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 \\ 0 & \beta \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \kappa & \kappa + \beta \end{pmatrix}.$$

The characteristic function of A , $f(\cdot)$, is

$$f(\lambda) = (\lambda - 1)(\lambda - \kappa - \beta) - \kappa.$$

We know that $f(1) = -\kappa < 0$ and $f(0) = \beta > 0$. Thus, there are two distinct eigenvalues. One of them is greater than one and the other is positive but smaller than one. Therefore, the sum in equation (7) eventually diverges, as $T \rightarrow \infty$.¹

This instability can be easily understood. Suppose that $\Delta a < 0$. This decline in the real rate pushes down output gap at date T . This decline in output gap lowers inflation at date T through the Philips curve. Then, these declines further lower output gap and inflation at date $T - 1$ through the Euler equation and the Philips curve. These compounding effects force the sum in equation (7) to diverge as $T \rightarrow \infty$.

Now, we introduce cognitive discounting formalized by [Gabaix \(2020\)](#) and show that this modification eliminates the counterintuitive results mentioned above. We generalize the Euler equation and the Philips curve in the following manner:

$$\begin{aligned} x_t &= Mx_{t+1} - \phi\pi_t + \pi_{t+1} + \Delta a \quad \text{and} \\ \pi_t &= M^f\beta\pi_{t+1} + \kappa x_t, \end{aligned}$$

where M and M^f are the parameters that govern cognitive discounting.

We establish the following result:

Proposition 1. *Suppose that $(M, M^f) \in \mathcal{M}$, where*

$$\mathcal{M} = \left\{ (M, M^f) \in [0, 1); M^f < \frac{1 - \kappa - M}{(1 - M)\beta} \right\}.$$

Then, (x_0, π_0) is finite even in the limit that $T \rightarrow \infty$.

According to Proposition 1, if agents have sufficiently strong cognitive discounting, the implau-

¹This feature implies that the model is not solved linearly.

sible implication disappears. Thus, this behavioral New Keynesian model allows us to model the durability of the manufacturing good.

Proof. Let A denote

$$A = \begin{pmatrix} 1 & 0 \\ -\kappa & 1 \end{pmatrix}^{-1} \begin{pmatrix} M & 1 \\ 0 & M^f \beta \end{pmatrix} = \begin{pmatrix} M & 1 \\ M\kappa, & \kappa + M^f \beta \end{pmatrix}.$$

It suffices to show that the largest absolute eigenvalue of matrix A is less than one. The characteristic function of A , $f(\cdot)$, boils down to

$$f(\lambda) = (\lambda - M)(\lambda - \kappa - M^f \beta) - M\kappa.$$

We know that $f(M) = -M\kappa < 0$ and

$$f(-1) = 1 + \kappa + M^f \beta + M + MM^f \beta > 0.$$

Therefore, the absolute values of the eigenvalues are strictly less than one if and only if

$$f(1) > 0.$$

It is easy to show that

$$f(1) = 1 - \kappa - M + (M - 1)M^f \beta.$$

Then, $f(1) = 0$ if and only if

$$M^f = \frac{1 - \kappa - M}{(1 - M)\beta}.$$

Therefore, as long as M^f is less than $\frac{1 - \kappa - M}{(1 - M)\beta}$, $f(1) > 0$. Thus, we have established the claim. $\square$

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