Behavioral Sticky Prices*

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Abstract

We develop a model in which households make decisions using a dual-process framework. System 1 relies on fast, intuitive heuristics but is prone to error, while System 2 demands cognitive effort but yields more accurate decisions. Monopolistic firms can influence which system households engage through pricing. This strategic influence creates a novel source of price inertia. The model accounts for the "rockets and feathers" phenomenon (prices rise quickly but fall slowly), explains why firms with unexpectedly high demand often avoid price changes, and why hazard functions are downward sloping. Our model implies that price stability is not optimal.

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1 Introduction

We study a model in which households make decisions according to a dual-process framework widely used in the cognitive psychology literature to describe human decision making (see, e.g., Stanovich and West (2000)). In this framework, System 1 uses heuristics to make fast, low-effort decisions that are prone to errors. System 2 engages in slower, more deliberate reasoning that is cognitively costly but more accurate. Our analysis builds on the elegant formulation of dual-process reasoning proposed by Ilut and Valchev (2023).

In our model, households make purchase errors because optimizing their consumption bundle involves cognitive effort. Monopolistic producers, for whom these errors result in high levels of demand relative to the rational optimum, have an incentive to keep their prices constant to discourage households from activating System 2 and reconsidering their purchasing decisions. This strategic behavior gives rise to a novel form of price inertia.

Our model is consistent with three important empirical facts. The first is the puzzling empirical regularity documented by Karrenbrock (1991), Neumark and Sharpe (1992), Borenstein, Cameron, and Gilbert (1997), and Peltzman (2000) known as ”rockets and feathers”: prices rise rapidly when costs increase, but fall slowly when costs decrease. In our model, when costs rise substantially, all firms increase prices to avoid losses, leading costs and prices to rise in tandem. In contrast, when costs fall, firms enjoying strong demand have an incentive to keep prices fixed to avoid triggering re-optimization by consumers. As a result, prices decline, on average, more slowly than costs.

The second fact is the “sticky winners” phenomenon documented by Ilut, Valchev, and Vincent (2020), whereby firms experiencing unexpectedly high demand at prevailing prices are less likely to adjust them. This behavior is central to our model: firms with favorable demand realizations avoid changing prices to prevent households from engaging System 2 and reoptimizing, which could trigger a new, potentially less favorable, demand shock.

The third empirical regularity is the downward-sloping hazard functions observed within narrowly defined goods categories (see Nakamura and Steinsson (2008) and Campbell and Eden (2014)). This pattern arises naturally from demand heterogeneity across firms producing the same type of good: those facing weak demand are more likely to adjust prices early, while firms experiencing strong demand tend to keep prices fixed for longer.

In standard models of cashless economies with sticky prices, price stability is typically optimal because it eliminates relative price distortions caused by inflation (see Woodford (2003)). In contrast, our framework implies that price stability is not optimal due to the strategic interaction between monopolistic firms and boundedly rational households. When average inflation is zero, there is still dispersion in the consumption of varieties because of cognitive errors. Firms that benefit from high demand keep their prices unchanged, which locks in consumption errors and leads households to settle on inefficient consumption bundles. Deflation is optimal because it increases the relative price of goods produced by sticky-price firms, reducing demand for those goods and mitigating the effects of behavioral biases.

We now discuss three observations consistent with the importance of System 1 in consumer behavior. The first is "shrinkflation," a situation where manufacturers reduce product sizes while keeping prices constant. The UK Office for National Statistics (2019) identified 206 cases between September 2015 and June 2017 in which product size was reduced with prices remaining largely unchanged. Budianto (2024) reports that 35 percent of the products included in the U.K. consumer price index between 2012 and 2023 experienced changes in product size, with prices remaining constant in most instances.

This practice suggests that some manufacturers are prepared to incur considerable expenses to keep prices stable, presumably to avoid triggering a re-optimization of household purchasing decisions.[1]

The second phenomenon is the increasing adoption of subscription-based business models, such as streaming or software-as-a-service, and the tendency for subscription prices to remain stable over long periods. This stability can be interpreted as a tactic producers use to dissuade households from engaging System 2 and reassessing the value of their subscriptions.[2]

Amazon Prime subscription prices are remarkably sticky. Initially offered at an annual rate of $79 in 2011, the fee has only been adjusted a few times: to $99 in 2014, $119 in 2018, and $139 in 2022. These adjustments were often accompanied by enhancements in service offerings, including the introduction of Amazon Prime Day, which served to justify the higher fees.

Netflix provides a case study of both price stability and shrinkflation. The standard subscription price remained at $7.99 from November 2010 until May 2014. At that point, the price was increased to $8.99, but only for new subscribers. Existing subscribers were grandfathered in at the $7.99 rate for an additional two years. Concurrently, Netflix rolled out a new basic plan priced at $7.99, which offered only standard-definition video on a single screen, a downgrade from the two high-definition screens available under the regular plan. The price for this basic plan remained unchanged until 2019.

The third observation consistent with the elements of our model is that convenient prices that are slightly below a round number (e.g., $9.99 instead of $10) are widely used (Kashyap (1995) and Blinder, Canetti, Lebow, and Rudd (1998)), and less likely to change than other prices (Levy, Lee, Chen, Kauffman, and Bergen (2011) and Ater

[1]President Biden deemed shrinkflation important enough to merit discussion in a February 2024 Super Bowl video broadcast. The president noted that "sports drinks bottles are smaller, a bag of chips has fewer chips, but they're still charging us just as much [...] ice cream cartons have shrunk in size but not in price. [...] Some companies are trying to pull a fast one by shrinking the products little by little and hoping you won't notice."

[2]See Della Vigna and Malmendier (2006) for evidence that consumers often fail to rationally assess the value they derive from subscription services.

and Gerlitz (2017)). This practice can be interpreted as a way to exploit System 1 thinking, creating the perception that the price is lower than its actual value.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents a version of the model with fully rational households, and Section 4 introduces bounded rationality into household decision-making. Section 5 demonstrates that the model is consistent with the rockets and feathers phenomenon. Section 6 examines the welfare costs of bounded rationality and evaluates how large these costs must be for the model's rockets-and-feathers behavior to align with empirical estimates. Section 7 analyzes optimal fiscal and monetary policy. Section 8 develops a dynamic partial-equilibrium model of the firm and shows that it implies downward sloping hazard functions. Section 9 concludes.

2 Related literature

Our paper builds on the cognitive psychology literature (e.g., Evans and Stanovich (2013) and Stanovich and West (2000)), which distinguishes between two modes of decision-making: low-cost, heuristic thinking (System 1) and high-cost, analytical reasoning (System 2).

Ilut and Valchev (2023) develop a formulation of the dual-system framework and use it to study the household consumption-savings behavior in an incomplete markets environment. In familiar contexts, where beliefs about the policy function are precise, households rely on prior beliefs to make decisions. In unfamiliar situations, where beliefs are imprecise, households draw costly signals to update their beliefs about the policy function.

Building on Ilut and Valchev (2023), we model household decisions regarding the consumption of differentiated products. We show how strategic interactions between firms and boundedly rational consumers give rise to a new form of price inertia.

We extend Ilut and Valchev (2023)'s framework in two directions. First, we use a quadratic approximation to embed the tracking problem that determines signal precision within the utility maximization problem, rather than treating it separately. Second, we ensure that behavioral decisions satisfy the budget constraint directly, removing the need to specify a residual variable (the savings rate in their analysis) that adjusts so that the budget constraint holds.

The cognitive costs in our model are consistent with the findings of Afrouzi, Dietrich, Myrseth, Priftis, and Schoenle (2024). Using survey evidence, these authors show that households prefer inflation to be zero. Seen through the lens of our model, this preference reflects the fact that cognitive costs are minimized when inflation is zero.

Our paper is linked to the literature on limited attention, limited information, or costly control by firms, including Mankiw and Reis (2002), Woodford (2009), Maćkowiak and Wiederholt (2009), Costain, Nakov, and Petit (2019), and Ilut, Valchev, and Vincent (2020).

In addition, our work relates to prior research on the strategic interaction between firms and consumers. Matějka (2015) show that firms strategically adopt a limited set of reference prices in the presence of inattentive consumers. De Clippel, Eliaz, and Rozen (2014) explore how limited household attention impacts competition. Rotemberg (1982) proposes a framework where consumer anger over price changes incentivizes firms to limit price adjustments.

The mechanism in our model complements those that produce asymmetric price adjustments in menu cost models (see, e.g., Ellingsen, Friberg, and Hassler (2006) and Burstein and Hellwig (2007)). Using a New Keynesian model with menu costs, Cavallo, Lippi, and Miyahara (2023) show that prices tend to rise faster than they fall following significant cost shocks, such as the 2022 surge in energy prices. This phenomenon occurs because firms adjust prices more frequently when profit margins are under pressure. In order for these models to generate substantial price asymmetries, menu costs must be relatively large–around one percent of revenue (see Ellingsen, Friberg, and Hassler (2006)).[3]

An extended version of our model could potentially shed light on micro-level price rigidities that traditional models struggle to explain (see, for example, Alvarez, Le Bihan, and Lippi (2014)). These phenomena include the presence of small price changes (Klenow and Kryvtsov (2008) and Eichenbaum, Jaimovich, Rebelo, and Smith (2014)), the coexistence of high-frequency price changes with sticky reference prices (Eichenbaum, Jaimovich, and Rebelo (2011)), and the observation that price adjustments for new products are larger and more frequent (Argente and Yeh (2022)).

At the macro level, our mechanism offers insights into the non-neutrality of monetary policy. Unlike standard menu-cost models (Golosov and Lucas (2007)), where firms with large price gaps dominate adjustments, our framework allows for heterogeneous endogenous adjustment costs. Firms with small price gaps may still adjust prices. As a result, monetary policy might be more effective.

Finally, our analysis complements other explanations of the rockets and feathers phenomenon. For instance, Tappata (2009) proposes a model where persistent cost shocks interact with consumers' limited information about market prices and production costs.

3 Model with fully rational households

In this section, we present a version of the model in which households are fully rational. We describe the household problem, the problem of monopolistic producers, the government's fiscal and monetary policies, and the economy's equilibrium.

To streamline the presentation, we relegate the proofs of most lemmas and propositions in the remainder of the text to the Appendix.

[3] According to data compiled by Aswath Damodaran (see data on operating and net margins by industry sector for the U.S. at this link) in January 2025, the pre-tax operating margin for grocery and food retail–defined as operating income (revenue minus cost of goods sold minus operating expenses) as a fraction of revenue–is 3.3 percent. So, a seemingly modest one percent menu cost would represent an implausibly large fraction–roughly 1/3–of operating income.

3.1 Household problem

There is a representative household that maximizes its utility,

$$U = \frac{C^{1-\sigma} - 1}{1-\sigma} - \vartheta \frac{N^{1+\psi}}{1+\psi}, \;\; \sigma, \eta > 0, \tag{1}$$

which depends on aggregate consumption (C) and hours worked (N).

Aggregate consumption results from a composite of differentiated goods,

$$C = \left(\int_0^1 C_i^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \; \theta > 1, \tag{2}$$

where C_i denotes consumption of good i.

The household's budget constraint is given by,

$$\int_0^1 P_i C_i di \leq WN + \Pi - T, \tag{3}$$

where P_i is the nominal price of good i, W is the nominal wage, Π are total firm profits, and T are nominal taxes.

It is convenient to solve the household problem in two steps.

Step 1 For a given level of consumption expenditure, E, determine the purchases of differentiated goods, C_i, that maximize the utility derived from consumption. The Lagrangian for this problem is

$$\mathcal{L}_e = \frac{C^{1-\sigma} - 1}{1-\sigma} + \Lambda_e \left(E - \int_0^1 P_i C_i di \right), \tag{4}$$

where C is given by equation (2).

The solution to this problem is given by,

$$C_i = \left(\frac{P_i}{P} \right)^{-\theta} \frac{E}{P}, \tag{5}$$

$$\Lambda_e = \left(\frac{E}{P}\right)^{-\sigma} \frac{1}{P}, \tag{6}$$

where

$$P \equiv \left(\int_0^1 P_i^{1-\theta} di\right)^{\frac{1}{1-\theta}}. \tag{7}$$

Equations (2), (5), and (7) imply that $C = E/P$.

Step 2 Given the solutions, C_i, to the first problem, choose the optimal levels of total consumption expenditure and hours worked. The Lagrangian for this problem is,

$$\mathcal{L}_u = U\left(C, N\right) + \Lambda_u \left(WN + \Pi - T - PC\right). \tag{8}$$

The first-order conditions imply the familiar intratemporal condition for hours worked,

$$\vartheta C^{\sigma} N^{\psi} = \frac{W}{P}.$$

3.2 Firm's Problem

Differentiated goods producers are monopolistically competitive. Firm i produces Y_i units of good i using N_i hours according to a linear production function,

$$Y_i = AN_i. \tag{9}$$

The firm's nominal profits, Π_i, are given by

$$\Pi_i = \left[P_i - (1-\tau)\frac{W}{A}\right]\left(\frac{P_i}{P}\right)^{-\theta} C.$$

where τ is the rate at which the government subsidizes wages.

The profit-maximizing price takes the familiar form,

$$P_i = \left(\frac{\theta}{\theta-1}\right)(1-\tau)\frac{W}{A}, \tag{10}$$

which implies that all firms choose the same price.

3.3 Fiscal and monetary policy

For simplicity, we assume that the central bank uses monetary policy to target nominal expenditure,

$$M = \int_0^1 P_i \, C_i \, di. \tag{11}$$

The government finances the wage subsidies provided to firms at a rate τ through lump-sum taxes,

$$T = \tau W N. \tag{12}$$

3.4 Equilibrium

Suppose $A = \overline{A}$, $M = \overline{M}$, and $\tau = \overline{\tau}$, where

$$1 - \overline{\tau} = \frac{\theta - 1}{\theta}.$$

This value of the labor subsidy eliminates the monopoly distortion, so that the price equals marginal cost.

Let $\overline{C}$, $\overline{N}$, and $\overline{P}$ denote the equilibrium values of aggregate consumption, labor, and the price level associated with $\overline{A}$, $\overline{\tau}$, and $\overline{M}$.

The equations above imply that,

$$\overline{C} = \left(\frac{1}{\vartheta}\right)^{\frac{1}{\sigma+\psi}} \overline{A}^{\frac{1+\psi}{\sigma+\psi}},$$

$$\overline{N} = \left(\frac{1}{\vartheta}\right)^{\frac{1}{\sigma+\psi}} \overline{A}^{\frac{1-\sigma}{\sigma+\psi}},$$

and

$$\overline{P} = \frac{\overline{M}}{\overline{C}}.$$

Since each firm's price equals marginal cost, profits are equal to the labor subsidies received: $\overline{\Pi} = \overline{T}$, and $\overline{P}\,\overline{C} = \overline{W}\,\overline{N}$.

3.5 A Second-Order Approximation

To set the stage for the study of household decisions under bounded rationality, we consider a log-quadratic approximation to the household's problem around the rational baseline equilibrium associated with $\overline{A}$, $\overline{\tau}$, and $\overline{M}$.

Throughout, we use lowercase variables to denote the logarithmic deviation of a variable from the rational baseline equilibrium, i.e., for any X, $x \equiv \ln\left(X/\overline{X}\right)$. Given a function $f(X)$, we define $df \equiv f(X) - f(\overline{X})$.

The following lemma presents quadratic approximations to the utility function and the two Lagrangians for the household problem.

Lemma 1. *Let $\hat{\mathcal{L}}_e \equiv d\mathcal{L}_e/\overline{C}^{1-\sigma}$ and $\hat{\mathcal{L}}_u \equiv d\mathcal{L}_u/\overline{C}^{1-\sigma}$. Then*

$$\hat{\mathcal{L}}_e = -\frac{1}{2}\sigma c^2 - \frac{1}{2\theta}\mathrm{Var}_i[c_i] - \int_0^1 p_i c_i \, di + \lambda_e(e - p - c) + \Omega_e, \tag{13}$$

and

$$\begin{aligned}\hat{\mathcal{L}}_u = &-\frac{1}{2}\sigma c^2 - \frac{1}{2\theta}\mathrm{Var}_i[c_i] - \frac{1}{2}\psi n^2 + wn - \int_0^1 p_i c_i \, di \\ &+ \lambda_u\left[w + n + \tfrac{1}{\theta}\left(\ln\frac{\Pi}{\overline{\overline{\Pi}}} - \ln\frac{T}{\overline{\overline{T}}}\right) - p - c\right] + \Omega_u,\end{aligned} \tag{14}$$

where

$$p \equiv \int_0^1 p_i \, di, \tag{15}$$

$c \equiv \int_0^1 c_i di$, $\mathrm{Var}_i[c_i] \equiv \int_0^1 (c_i - c)^2 di$, and Ω_e and Ω_u are exogenous to the household problem.

Under rational expectations, the first-order conditions from Lagrangian (13) yield the standard demand function, which in logarithmic form is given by:

$$c_i^*(\mathbf{z}) = c - \theta(p_i - p),$$

where $c \equiv e - p$ and $\mathbf{z} \equiv \left(c, \{p_i\}_{i\in[0,1]}\right)$.

To set the stage for our discussion of bounded rationality, it is useful to restate the problem of optimally choosing consumption varieties as follows.

Lemma 2. *Let $\hat{\mathcal{L}}_e^*$ denote the optimized Lagrangian (13), and define $\Delta\hat{\mathcal{L}}_e \equiv \hat{\mathcal{L}}_e - \hat{\mathcal{L}}_e^*$ as the percentage deviation of the Lagrangian evaluated at arbitrary values c_i from its optimized value. Then*

$$\Delta\hat{\mathcal{L}}_e = -\frac{1}{2\theta}\left[\int_0^1 \left[c_i - c_i^*\left(\mathbf{z}\right)\right]^2 di + \left(\theta\sigma - 1\right)\left(\int_0^1 \left[c_i - c_i^*\left(\mathbf{z}\right)\right] di\right)^2\right] + \mu_e\left(c - \int_0^1 c_i di\right). \tag{16}$$

The proof of this lemma follows directly from the properties of quadratic forms. Under full rationality, the household chooses $\{c_i\}_{i\in[0,1]}$ and μ_e to maximize expression (16), which yields $c_i = c_i^*\left(\mathbf{z}\right)$, $i \in [0,1]$.

4 Model with boundedly-rational households

This section presents a version of the model in which households make decisions under bounded rationality along the lines of Ilut and Valchev (2023).

To isolate the effects of bounded rationality on the purchases of differentiated consumption goods, we assume that the household makes fully rational decisions regarding aggregate consumption expenditure and labor supply. Bounded rationality applies only to the choice of individual consumption varieties.

Households can compute the demand for the baseline equilibrium associated with $\overline{A}$, $\overline{\tau}$, and $\overline{M}$, but are uncertain about how to respond to shocks. The household solves its problem using the second-order approximations described in Lemma **??**.

When deciding the composition of the consumption basket, the household observes perfectly the state variables, $\mathbf{z}$, but is uncertain about $c_i^*\left(\mathbf{z}\right)$, $i \in [0,1]$. The household enters the period with a prior belief, $c_i^b(\mathbf{z})$, about $c_i^*(\mathbf{z})$ governed by a Gaussian Process ($\mathcal{GP}$),

$$c_i^b\left(\mathbf{z}\right) \sim \mathcal{GP}\left(\mu_i\left(\mathbf{z}\right), \gamma_i\left(\mathbf{z}, \widetilde{\mathbf{z}}\right)\right),$$

where $c_i^b(\mathbf{z})$ and $c_j^b(\mathbf{z})$ are orthogonal and

$$\mu_i(\mathbf{z}) = \mathbb{E}\left[c_i^b(\mathbf{z})\right], \;\; \gamma_i(\mathbf{z}, \widetilde{\mathbf{z}}) \equiv \text{Cov}\left[c_i^b(\mathbf{z}), c_i^b(\widetilde{\mathbf{z}})\right].$$

The household can obtain a noisy signal about the optimal consumption of variety i,

$$s_i(\mathbf{z}) = c_i^*(\mathbf{z}) + \gamma_\epsilon(\mathbf{z})\epsilon_i,$$

where $\epsilon_i \sim \mathcal{N}(0,1)$, and ϵ_i and ϵ_j are orthogonal for $i \neq j$.

This signal induces a posterior distribution for the optimal consumption of variety i, given by

$$c_i^b(\mathbf{z}) \mid s_i \sim \mathcal{GP}\left(\mu_{i|s}(\mathbf{z}), \gamma_{i|s}(\mathbf{z}, \widetilde{\mathbf{z}})\right),$$

where $\mu_{i|s}(\mathbf{z})$ and $\gamma_{i|s}(\mathbf{z}, \widetilde{\mathbf{z}})$ are computed using the standard expressions for the conditional mean and covariance of a Gaussian process.

To generate a signal for the optimal consumption of good i, the household incurs a cognitive cost that increases with the precision of the signal. The utility of the boundedly rational household is $\hat{U} - \mathcal{I}$, where $\mathcal{I}$ is the total cognitive cost of all the signals generated by the household.

We assume that cognitive costs are proportional to the reduction in uncertainty. Following Sims (2003), we measure this reduction as the decrease in entropy, or equivalently, as the Shannon mutual information,

$$\mathcal{I} = \frac{\kappa}{2}\int_0^1 \left[\ln \gamma_i^2(\mathbf{z}) - \ln \gamma_{i|s}^2(\mathbf{z})\right] di,$$

where

$$\gamma_i^2(\mathbf{z}) \equiv \text{Var}\left[c_i^b(\mathbf{z})\right], \;\; \gamma_{i|s}^2 \equiv \text{Var}\left[c_i^b(\mathbf{z}) \mid s_i\right].$$

The expression for the conditional distribution of a normal random variable implies that

$$\gamma_{i|s}^2(\mathbf{z}) = \frac{\gamma_i^2(\mathbf{z})\,\gamma_\epsilon^2(\mathbf{z})}{\gamma_i(\mathbf{z}) + \gamma_\epsilon^2(\mathbf{z})},$$

so we can model the selection of the signal variance as a choice over the posterior variance, $\gamma_{i|s}^2(\mathbf{z})$.

It is useful to define the expression analogous to (2) for the household making decisions under bounded rationality:

$$\Delta\hat{\mathcal{L}}_e^b = -\frac{1}{2\theta}\left[\int_0^1 \left[c_i - c_i^b(\mathbf{z})\right]^2 di + (\theta\sigma - 1)\left(\int_0^1 \left[c_i - c_i^b(\mathbf{z})\right] di\right)^2\right] + \mu_e\left(c - \int_0^1 c_i di\right). \tag{17}$$

Under bounded rationality, the problem of allocating spending across differentiated goods to maximize utility for a given total consumption expenditure can be written as

$$\max_{c_i, \gamma_{i|s}^2(\mathbf{z}), \mu_E} \mathbb{E}\left[\Delta\hat{\mathcal{L}}_e^b\right] - \mathcal{I} \text{ s.t. } \gamma_{i|s}^2(\mathbf{z}) \leq \gamma_i^2(\mathbf{z}) \quad i \in [0,1], \tag{18}$$

where the constraint guarantees that the solution is consistent with Bayes' rule.

We begin by solving for c_i. The first-order conditions yield:

$$c_i = \mu_{i|s}(\mathbf{z}) + c - \int_0^1 \mu_{i|s}(\mathbf{z})\, di. \tag{19}$$

The demand for each good equals its posterior mean, adjusted by a constant term ($c - \int_0^1 \mu_{i|s}(\mathbf{z})\, di$) to ensure that the constraint, $c = \int_0^1 c_i\, di$, is satisfied.

Having derived the demand functions given a set of signals, we now solve for the optimal posterior variance for each consumption variety, which is equivalent to selecting the optimal signal precision.

Lemma 3. *The optimal posterior variance for the optimal consumption of good i is the solution to the following problem,*

$$\max_{\gamma_{i|s}^2(\mathbf{z})} -\frac{1}{2\theta}\int_0^1 \gamma_{i|s}^2(\mathbf{z})\, di - \frac{\kappa}{2}\int_0^1 \left[\ln\gamma_i^2(\mathbf{z}) - \ln\gamma_{i|s}^2(\mathbf{z})\right] di \;\; s.t. \;\; \gamma_{i|s}^2(\mathbf{z}) \leq \gamma_i^2(\mathbf{z}), \;\; i \in [0,1]. \tag{20}$$

The first-order conditions for problem (20) are:

$$\gamma_{i|s}^{2}\left(\mathbf{z}\right)=\min\left\{\gamma_{i}^{2}\left(\mathbf{z}\right);\theta\kappa\right\}. \tag{21}$$

This condition implies that the household activates System 2 for good i whenever the value of p_i is unfamiliar, i.e., when the prior uncertainty about the optimal value for c_i corresponding to p_i is high $(\gamma_i^2(\mathbf{z}) > \theta\kappa)$.

The likelihood of activating System 2 declines with κ and θ. A higher κ increases cognitive costs, reducing the incentive to engage System 2. A higher θ implies greater substitutability across goods, reducing the utility contribution of each variety. Consequently, for a given level of aggregate consumption, c, the value of learning the optimal demand for each variety declines.

Learning in the pre-period There is a pre-period in which households choose their consumption for each variety, i.

To ensure that ex-ante biases do not drive our results, we assume, as in Ilut and Valchev (2023), that the pre-period prior distribution is centered on the rational demand,

$$\mu_{i,0}\left(\mathbf{z}\right)=c_{i}^{*}\left(\mathbf{z}\right). \tag{22}$$

The diagonal elements of the pre-period covariance function, $\gamma_{i,0}\left(\mathbf{z},\widetilde{\mathbf{z}}\right)$, are equal to γ_c^2 and the off-diagonal elements are equal to zero. We assume that $\gamma_c^2 > \theta\kappa$, so the initial level of uncertainty justifies activating System 2.

The form of the pre-period covariance function, $\gamma_{i,0}\left(\mathbf{z},\widetilde{\mathbf{z}}\right)$, incorporates two key assumptions. First, cognitive uncertainty about the demand for good i depends only on its nominal price. The household knows how to adjust the consumption of each variety i to changes in the aggregate price level, p, or aggregate consumption, c, but not in response to shifts in individual prices, p_i. Second, prior demands at different prices are uncorrelated. Knowing the demand at one price conveys no information about the optimal demand for a different price. This independence assumption preserves the computational simplicity that is the hallmark of System 1 reasoning.

Equation (21) implies that the household chooses to learn whenever its prior variance about optimal consumption of good i exceeds $\theta\kappa$. In the pre-period, all prior variances are above this threshold. As a result, the household generates a signal for the prices set by firms, $p_{i,0}$, and updates its beliefs about the corresponding optimal consumption levels.

The household does not update its beliefs for prices not posted by firms in the pre-period. For those prices, the posterior distribution about optimal consumption is equal to the prior (recall that we assume the priors are centered on the rational demand). Given these considerations, the resulting pre-period posterior means and variances are:

$$\mu_i(\mathbf{z}) = \begin{cases} c - \theta(p_i - p) + \alpha\gamma_\epsilon\epsilon_{i,0}, & \text{if } p_i = p_{i,0} \\ c - \theta(p_i - p), & \text{if } p_i \neq p_{i,0} \end{cases},$$

where

$$\alpha \equiv 1 - \frac{\theta\kappa}{\gamma_c^2}, \quad \gamma_\epsilon = \sqrt{\frac{\theta\kappa}{\alpha}}, \tag{23}$$

and

$$\gamma_i^2(\mathbf{z}) = \begin{cases} \theta\kappa, & \text{if } p_i = p_{i,0} \\ \gamma_c^2, & \text{if } p_i \neq p_{i,0} \end{cases}.$$

Since $\gamma_c^2 > \theta\kappa$, the household relies on System 2 only when $p_i \neq p_{i,0}$. When the price of good i is the same as in the pre-period, uncertainty about the optimal consumption of good i is sufficiently low that the household chooses to avoid cognitive costs and follows the rule inherited from the pre-period.

The period-one posterior means $\mu_{i|s}(\mathbf{z})$ are

$$\mu_{i|s}(\mathbf{z}) = c - \theta(p_i - p) + \alpha\gamma_\epsilon\widetilde{\epsilon}_i, \tag{24}$$

where $\widetilde{\epsilon}_i = \epsilon_{i,0}$ if $p_i = p_{i,0}$, and $\widetilde{\epsilon}_i = \epsilon_{i,1} \sim \mathcal{N}(0,1)$ otherwise.

When firm i sets its price, it knows the value of $\epsilon_{i,0}$ but not $\epsilon_{i,1}$. By combining equations (19) and (24) we obtain the following expression for the demand for good i:

$$c_i = c - \theta \left(p_i - p \right) + \alpha \gamma_\epsilon \left[\widetilde{\epsilon}_i - \mathbb{E}_i \left(\widetilde{\epsilon} \right) \right], \tag{25}$$

where the term $\mathbb{E}_i \left[\widetilde{\epsilon} \right] \equiv \int_0^1 \widetilde{\epsilon}_i di$ ensures that the constraint $c = \int_0^1 c_i \, di$ is satisfied.

Combining equations (14) and (25) and taking the first-order conditions with respect to c, n, and λ_u yields the standard intratemporal condition for labor choice, expressed in logarithmic form,

$$\sigma c + \psi n = w - p. \tag{26}$$

4.1 The firm's problem

We now revisit the firm's problem, taking into account the fact that households make decisions under bounded rationality. We write the problem in levels, using the relation $X = \overline{X} e^x$.

The ex-post nominal profits of firm i are given by,

$$\Pi_i = \left[P_i - (1 - \tau) \frac{W}{A} \right] C_i.$$

The firm makes two decisions: whether to adjust its price, and if so, by how much.

Suppose that $P_{i,0} = P_0$ for all $i \in [0,1]$. The following lemma characterizes the optimal pricing policy of firm i.

Lemma 4. *Define $\pi \equiv p - p_0$ and $\Theta \equiv \ln \left(\theta / (\theta - 1) \right)$. Firm i's optimal pricing policy is:*

$$p_i = \begin{cases} p_{adj}, & \text{if } \epsilon_{i,0} < \overline{\epsilon} \\ p_0, & \text{if } \epsilon_{i,0} \geq \overline{\epsilon} \end{cases}. \tag{27}$$

where

$$p_{adj} = w + \ln \left(\frac{1 - \tau}{1 - \overline{\tau}} \right) - a, \tag{28}$$

and

$$\bar{\epsilon} = \begin{cases} \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left\{(\theta-1)\left[\left(p_{adj}-p\right)+\pi\right] + \ln\left[\frac{1-e^{\left(p_{adj}-p\right)+\pi-\Theta}}{1-e^{-\Theta}}\right]\right\}, & \text{if } \left(p_{adj}-p\right)+\pi < \Theta \\ \infty, & \text{if } \left(p_{adj}-p\right)+\pi \geq \Theta \end{cases}. \tag{29}$$

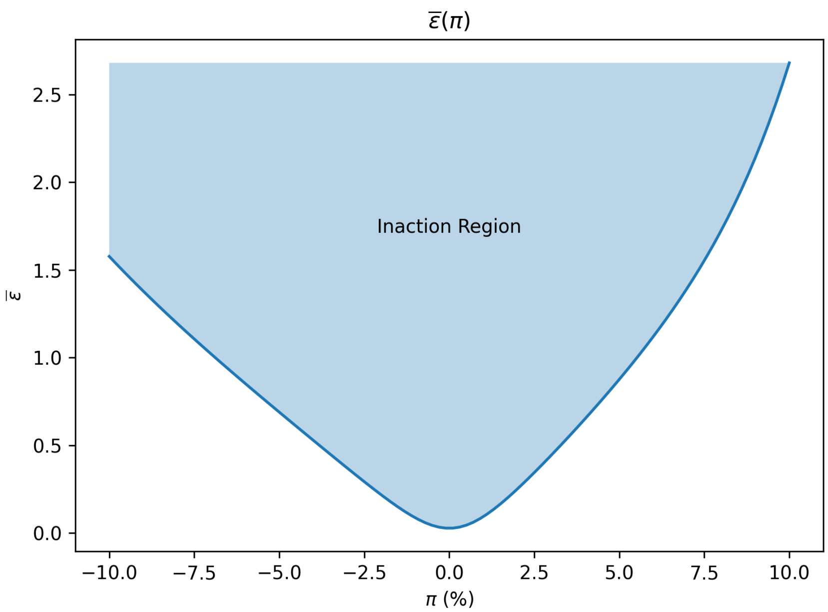

Figure 1: Inaction region for firms price setting

The optimal reset price, p_{adj}, coincides with the price in the model with fully rational households for two reasons. First, firms set prices before observing the cognitive errors made in the current period. Second, conditional on a price change, cognitive errors are uncorrelated with prices.

Equation (29) implies that when $\left(p_{\text{adj}}-p\right)+\pi \geq \Theta$, no realization of the past noise $\epsilon_{i,0}$ makes it optimal for the firm to keep its current price. In this case, the implied profit margin is non-positive, so the firm can always increase profits by adjusting its price. This asymmetry, which plays a central role in our "rockets and feathers" result, is illustrated in Figure 1.

High cost inflation erodes margins evaluated at fixed prices, prompting most firms to abandon their existing demand shocks and raise prices. In contrast, when costs decline, firms may choose to maintain their favorable demand shocks rather than reduce prices. Although a price cut could increase the quantity sold, some firms already sell more than they would if households were fully rational. Lowering prices would activate the household's System 2, prompting a reassessment of demand and generating a new demand shock. To avoid this reset and preserve their high demand levels, firms often choose to keep their prices constant.

The pricing policy described in Lemma 4 implies that a fraction χ of firms choose not to adjust their prices, where

$$\chi \equiv 1 - \Phi\left(\bar{\epsilon}\right), \tag{30}$$

and Φ is the cumulative distribution function of the standard normal distribution. Using the definition of the aggregate price level from equation (15), we obtain the following standard relationship between p_{adj} and π:

$$0 = -\chi\pi + (1-\chi)(p_{\text{adj}} - p). \tag{31}$$

4.2 Equilibrium

To define the equilibrium, we normalize the initial price level to one, which implies that the log price level at time zero is zero, $p_0 = 0$.

An equilibrium consists of allocations $\{c_i\}_{i\in[0,1]}, c, n$, prices $\{p_i\}_{i\in[0,1]}, w$, and information acquisition strategies $\{\gamma_{i|s}\}_{i\in[0,1]}$, such that, given a, m, τ, $p_0 = 0$, and $\{\mu_i, \gamma_i\}_{i\in[0,1]}$, the following conditions are satisfied:

1. Given c, the price vector $\{p_i\}_{i\in[0,1]}$, and the belief parameters $\{\mu_i, \gamma_i\}_{i\in[0,1]}$, the household chooses c_i and $\{\gamma_{i|s}\}_{i\in[0,1]}$ to solve the optimization problem (18);

2. Given consumption decisions for c_i, the household chooses c and n to maximize utility;

3. Each firm i chooses p_i to maximize profits;

4. The aggregate price level p satisfies equation (15);

5. Markets clear:

$$\pi + c = m, \tag{32}$$

$$\int_0^1 n_i \, di = n. \tag{33}$$

$$c_i = a + n_i \tag{34}$$

The government budget constraint is redundant.

Using equations (33) and (34) we obtain:

$$c = a + n, \tag{35}$$

which shows that, to a first-order approximation, there are no productive distortions.

The equilibrium conditions for the aggregate variables are given by equations (26), (28), (29), (30), (31), (32), and (35). Substituting equations (28) and (31) into this system of equations allows us to reduce the equilibrium conditions to equation (30), (32), and the condition for the cutoff $\overline{\epsilon}$:

$$\overline{\epsilon} = \begin{cases} \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[(\theta - 1)\frac{\pi}{1-\chi} + \ln\left(\frac{1-e^{\frac{\pi}{1-\chi}-\Theta}}{1-e^{-\Theta}}\right)\right], & \text{if } \frac{\pi}{1-\chi} < \Theta, \\ \infty, & \text{if } \frac{\pi}{1-\chi} \geq \Theta, \end{cases}$$

and

$$c = c^* + \frac{1}{\psi + \sigma}\left[\frac{\chi}{1-\chi}\pi - \ln\left(\frac{1-\tau}{1-\overline{\tau}}\right)\right],$$

where

$$c^* \equiv \left(\frac{1+\psi}{\psi+\sigma}\right) a,$$

denotes aggregate output in the equilibrium of the model with fully rational households.

The following proposition summarizes the existence and uniqueness properties of our model.

Proposition 1. *An equilibrium exists. Moreover, if $\psi + \sigma \geq 1$, the equilibrium is unique.*

4.3 The Phillips curve

For $\tau = \overline{\tau}$, the Phillips curve for this economy is given by

$$c - c^* = \frac{1}{\psi + \sigma}\left(\frac{\chi}{1-\chi}\right)\pi. \tag{36}$$

Figure 2 displays this Phillips curve. The output gap is defined as the current level of log output minus the level of log output in the economy with fully rational households. When inflation exceeds 12 percent, the Phillips curve becomes approximately vertical: firms face low or negative profit margins at current prices and choose to adjust prices regardless of their demand shocks. The Phillips curve has a conventional upward slope when the inflation rate is between -3.8 and 3 percent. In this range, higher inflation is associated with a more positive output gap. As inflation rises beyond 3 percent, price flexibility increases as more firms adjust their prices, causing the Phillips curve to slope backward and the output gap to approach zero. Because firms are more responsive to inflation than to deflation, the inflation rate at which the Phillips curve becomes approximately vertical is lower in absolute value when inflation is positive (12 percent) than negative (19 percent).

We now examine the properties of the equilibrium. The following proposition characterizes the relationship between the threshold $\overline{\epsilon}$ and inflation π, using the fact that $\chi = 0$ when $\overline{\epsilon} = \infty$.

Proposition 2. *The equilibrium relationship between $\overline{\epsilon}$ and π is given by:*

$$\overline{\epsilon}(\pi) = \begin{cases} \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[(\theta-1)\frac{\pi}{1-\chi} + \ln\left(\frac{1-e^{\frac{\pi}{1-\chi}-\Theta}}{1-e^{-\Theta}}\right)\right], & \text{if } \pi < \Theta, \\ \infty, & \text{if } \pi \geq \Theta. \end{cases} \tag{37}$$

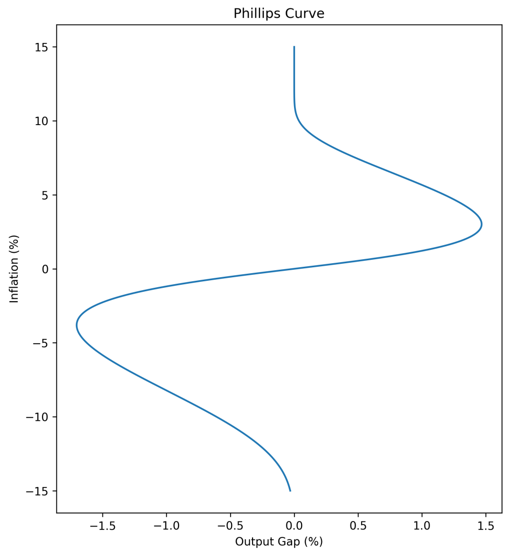

Figure 2: Phillips curve: inflation rate versus output gap.

Moreover, the function $\overline{\epsilon}(\pi)$ satisfies the following properties:

1. *$\overline{\epsilon}(\pi)$ attains its minimum at $\pi = 0$;*

2. *For any $a > 0$, we have $\overline{\epsilon}(a) > \overline{\epsilon}(-a)$.*

Proposition 2 implies that the function

$$\chi(\pi) \equiv 1 - \Phi\left[\overline{\epsilon}(\pi)\right],$$

attains its maximum at $\pi = 0$ and satisfies $\chi(a) < \chi(-a)$ for all $a > 0$. Hence, the model predicts an *asymmetric hazard function*, with the price-change probability rising faster with positive inflation than with deflation. This asymmetry plays a key role in explaining the rockets and feathers phenomenon discussed in Section 5.

5 Rockets and Feathers

We now study the impact of cost shocks and show that our model is consistent with the rockets and feathers phenomenon: prices rise quickly when costs increase but fall slowly when costs fall.

To do so, we examine the equilibrium response to symmetric shocks to potential output, $\nu > 0$ and $-\nu$, assuming $m = 0$. Define the deviation of consumption from its steady-state value as

$$\tilde{c}(\pi) \equiv c(\pi) - c^* = \frac{1}{\psi + \sigma} \left(\frac{\chi}{1 - \chi} \right) \pi.$$

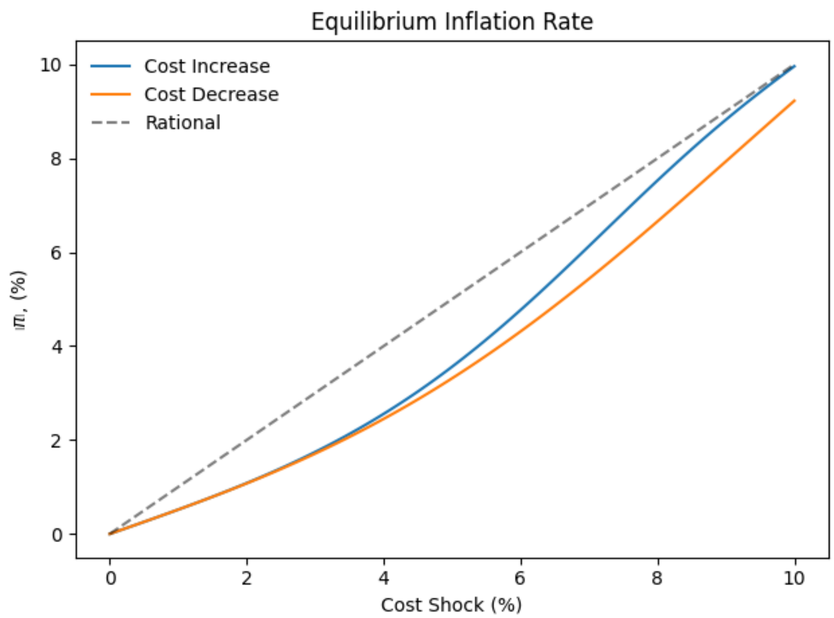

Figure 3: The impact of cost shocks on the absolute value of the logarithm of inflation

A cost increase ($\nu > 0$) leads to inflation, while a cost decrease ($\nu < 0$) results in deflation. To compare the price response to both types of shocks, Figure 3 plots the absolute value of the logarithm of gross inflation against the absolute value of the cost shock, $|\nu|$. The orange line represents cost decreases, and the blue line cost increases. In a fully rational model, these lines would overlap, as inflation and deflation would be symmetric in magnitude.

This symmetry holds in our model for infinitesimal shocks. However, for larger values of $|\nu|$, prices adjust more in response to cost increases than to equivalent decreases. When costs rise substantially, all firms adjust prices upward to avoid negative margins, causing prices to move closely with costs. By contrast, when costs fall, firms with strong demand prefer to maintain their prices to avoid triggering household reoptimization. As a result, average price declines are smaller than cost declines.

As $|\nu|$ increases, the orange and blue curves in Figure 3 converge. When positive cost shocks exceed roughly 12 percent, most firms adjust their prices. When costs fall by more than 19 percent, almost all firms choose to reduce prices.

The following proposition states our key result:

Proposition 3. *Let $\nu > 0$, and consider the equilibria with $c^* = \nu$ and $m = 0$. Then the corresponding inflation rates satisfy $\pi(\nu) < 0$, $\pi(-\nu) > 0$, and*

$$\pi(-\nu) > -\pi(\nu).$$

For sufficiently large cost shocks, inflation responds more strongly (in percentage terms) than deflation to shocks of equal absolute size.

6 The Welfare Cost of Dual-Process Decisions

In this section, we examine the welfare costs of dual-process reasoning in a model calibration that matches the inflation asymmetries documented in the rockets and feathers literature. We quantify these costs using a consumption-equivalent measure: the fraction of consumption the representative household would be willing to forgo to eliminate cognitive frictions and behave fully rationally.

To set ideas, suppose that the logarithm of productivity, a, follows a normal distribution with mean zero and variance γ_a^2 and let $\widetilde{u}\left(a; \kappa, \gamma_c^2\right)$ be the utility associated with the equilibrium in our economy. In the analogous economy with rational households, aggregate consumption and labor are

$$c^* = \frac{1+\psi}{\sigma+\psi}a, \;\; n^* = \frac{1-\sigma}{\sigma+\psi}a.$$

Lemma 1 implies that utility in the rational economy is:

$$u^*\left(c^*,n^*\right)=c^*+\frac{1-\sigma}{2}\left(c^*\right)^2-n^*-\frac{1+\psi}{2}\left(n^*\right)^2.$$

Now suppose that, for any level of log productivity, a, consumption is reduced by $100\times\lambda_w$ percent. In that case, the representative household would receive

$$u^*\left[c^*\left(a\right)-\lambda_w,n^*\left(a\right)\right].$$

The consumption-equivalent welfare cost of the friction, λ_w, is the solution to:

$$\mathbb{E}_a\left[\widetilde{u}\left(a;\kappa,\gamma_c^2\right)\right]=\mathbb{E}_a\left\{u^*\left[c^*\left(a\right)-\lambda_w,n^*\left(a\right)\right]\right\}.$$

In our calibration, we set $\theta=5$, $\sigma=1$, and $\psi=0$. We choose γ_a so that the standard deviation of log consumption in the efficient economy matches 0.032, the standard deviation of consumption originally used by Lucas (1987) in his study of the cost of business cycles,

$$\gamma_a=\frac{\sigma+\psi}{1+\psi}\times 0.032.$$

Figure 4 depicts our results. We consider two scenarios, where the variance of the prior is such that the welfare cost of bounded rationality is $\lambda=0.07$ and 0.5 percent, respectively. We measure asymmetry by the difference between the absolute values of the inflation and deflation responses, for a given prior variance and varying cost shocks. Each time the household engages in System 2 reasoning, it reduces the posterior variance of optimal consumption for good i to $\theta\kappa$ (see equation 21). Thus, the posterior variance is independent of the prior variance. However, the prior variance still affects α, which determines how much the household updates its prior mean using the signal (see equation (23)). When the prior variance is high–meaning the prior is relatively uninformative–the household places greater weight on the signal, resulting in more firms facing strong demand and choosing to keep their prices fixed.

The model with $\lambda_w=0.07$ generates inflation asymmetries of approximately 0.15×8 for cost shocks around 8 percent. This magnitude is consistent with the

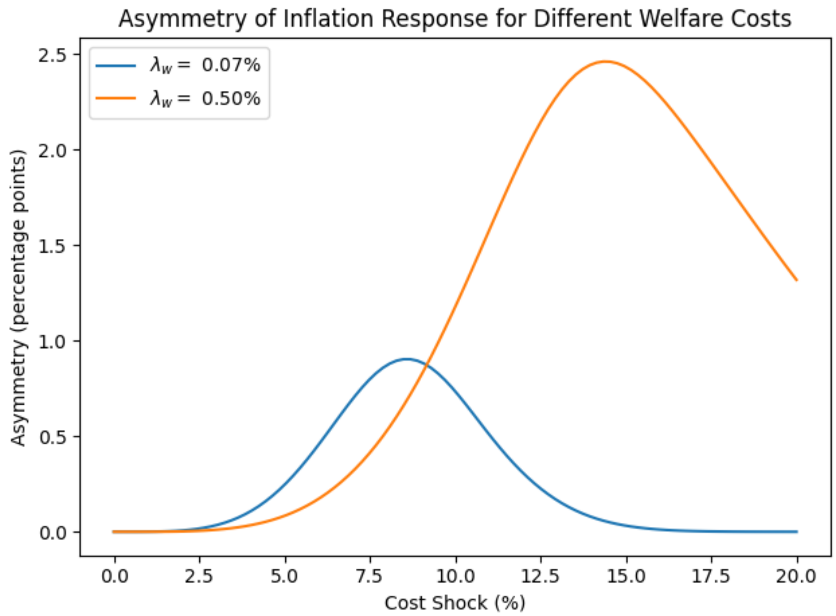

Figure 4: Asymmetry

estimates of Peltzman (2000), which imply an asymmetry of 0.15 for each one percent change in costs.

7 Optimal Policy

We now characterize the optimal values for the labor subsidy rate, τ, and the growth rate of money, m. We start by computing the indirect utility, net of cognitive costs, for a particular equilibrium.

The government can eliminate monopolistic distortions by setting,

$$\ln\left(\frac{1-\tau}{1-\overline{\tau}}\right) = p_{\text{adj}} - p = \frac{\chi}{1-\chi}\pi.$$

The next Lemma implies that the aggregate level of consumption and labor, c and n, can be chosen independently from π.

Lemma 5. *The Lagrangian associated with the planner's problem is*

$$\hat{\mathcal{L}}_p = -\frac{1}{2}\sigma c^2 - \frac{1}{2}\psi n^2 - \frac{1}{2\theta}\Delta\left(\pi\right) + a + \frac{1}{2}a^2 + an + \lambda_p\left(n + a - c\right).$$

where

$$\Delta\left(\pi\right) \equiv \mathrm{Var}_i\left[c_i\right] + 2\theta\mathcal{I},$$

$$\mathrm{Var}_i[\,c_i\,] = \theta^2\,\frac{1-\Phi\big(\overline{\epsilon}(\pi)\big)}{\Phi\big(\overline{\epsilon}(\pi)\big)}\,\pi^2 \;+\; \alpha^2\,\gamma_\epsilon^2\Big\{\,1 + \overline{\epsilon}\,\phi\big(\overline{\epsilon}(\pi)\big) \;-\; \phi^2\big(\overline{\epsilon}(\pi)\big)\Big\}$$
$$+\,2\,\theta\,\alpha\,\gamma_\epsilon\,\pi\,\phi\big(\overline{\epsilon}(\pi)\big)\,,$$

$$2\theta\mathcal{I} = \Phi\left[\overline{\epsilon}\left(\pi\right)\right]\alpha\gamma_\epsilon^2\ln\left(\frac{1}{1-\alpha}\right),$$

and λ_p denotes the Lagrange multiplier.

The solution to the planner's problem involves

$$c = c^* = \left(\frac{1+\psi}{\sigma+\psi}\right)a,$$

and

$$n = n^* = \left(\frac{1-\sigma}{\sigma+\psi}\right)a.$$

That is, the natural rate of output corresponds to the first-order response of aggregate output in the rational economy.

The previous Lemma implies that the optimal inflation rate solves

$$\min_{\pi}\Delta\left(\pi\right).$$

We first show that if prior uncertainty is sufficiently high, then price stability ($\pi = 0$) is preferable to incentivizing all firms to change prices ($\pi \geq \Theta$).

Lemma 6. *There is $\overline{\gamma}_c$ such that, if $\gamma_c \geq \overline{\gamma}_c$, then $\Delta\left(0\right) < \Delta\left(\Theta\right)$.*

We now show that even under parameter conditions that ensure that price stability is preferable to full flexibility, it is not optimal to set $\pi = 0$.

Lemma 7. *There is $\delta > 0$ such that for all $\pi \in (-\delta, 0)$, $\Delta(\pi) < \Delta(0)$.*

Proof. The result follows from the fact that

$$\Delta'(0) = 2\theta\alpha\gamma_{\epsilon}\phi\left[\bar{\epsilon}(0)\right] > 0.$$

□

The intuition for this result is as follows. When average inflation is zero, firms experiencing high demand due to household decision errors do not change their prices. Other firms slightly increase or decrease their prices to draw a new demand shock. As a result, sizeable behavioral mistakes become ingrained, leading households to select a highly suboptimal consumption basket. Moving from zero inflation to deflation mitigates this inefficiency by improving consumption choices.

Why is deflation locally better than inflation? The logic is as follows. Due to cognitive costs, households do not choose the fully-rational value of c_i. The planner would like to reduce the consumption of goods supplied by firms that have sticky prices, since these firms received positive demand shocks. When inflation is positive, the relative price of the goods produced by firms with sticky prices falls, inducing households to consume more of these goods and exacerbating the impact of behavioral biases. In contrast, when inflation is negative, the relative price of the goods produced by firms with sticky prices rises. As a result, the consumption of these goods falls, mitigating the impact of behavioral biases.

8 A Dynamic Model

In this section, we consider a dynamic partial equilibrium model of a firm that faces the dual-process demand discussed in Section 4. To simplify notation, we omit the subscript i, so, in this section, P_t denotes the price of the individual firm. We assume that the aggregate price level is constant and normalized to one.

Consider a firm with marginal cost Ξ_t and nominal price P_t. The firm's demand is given by:

$$C_t = P_t^{-\theta} \begin{cases} e^{\alpha\gamma\epsilon_{t-1}}, & \text{if } p_t = p_{t-1} \\ \mathbb{E}\left(e^{\alpha\gamma_\epsilon\epsilon_t}\right), & \text{if } p_t \neq p_{t-1} \end{cases},$$

where $\epsilon_t \sim \mathcal{N}(0,1)$. As before, lowercase variables denote logarithmic deviations, e.g. $\ln(P_t/1) = \ln(p_t)$.

This demand specification assumes that households have incomplete memory: when the price changes, they forget past prices and purchase decisions, and learn a noisy estimate of their optimal demand at the new price. So, prices observed before period $t-1$ are irrelevant for household choices.

Flow profits are given by:

$$\Pi_t = \left(P_t - \Xi_t\right) P_t^{-\theta} \begin{cases} e^{\alpha\gamma_\epsilon\epsilon_{t-1}}, & \text{if } p_t = p_{t-1} \\ \mathbb{E}\left(e^{\alpha\gamma_\epsilon\epsilon_t}\right), & \text{if } p_t \neq p_{t-1} \end{cases}.$$

To simplify, we use a second-order log-approximation to flow profits around the following solution to the firm's problem: $\overline{\Xi} \equiv 1$, $\overline{P} \equiv \theta/(\theta-1)$, and $\overline{\Pi} = (1/\theta)[\theta/(\theta-1)]^{1-\theta}$.

Lemma 8. *Let $r_t \equiv \ln\left(\Pi_t/\overline{\Pi}\right)$. The firm's per-period reward, computed using a second-order approximation, is*

$$r_t = -\left(\theta-1\right)\xi_t - \frac{\theta\left(\theta-1\right)}{2}\left(p_t - \xi_t\right)^2 + \begin{cases} \alpha\gamma_\epsilon\epsilon_{t-1}, & \textit{if } p_t = p_{t-1} \\ \frac{1}{2}\left(\alpha\gamma_\epsilon\right)^2, & \textit{if } p_t \neq p_{t-1} \end{cases}.$$

We assume that the logarithm of marginal cost, $\xi_t = log(\Xi_t)$, evolves according to a random walk,

$$\xi_t = \xi_{t-1} + v_t,$$

where the innovation v_t follows a jump-diffusion process:

$$v_t \begin{cases} 0, & \text{with probability } \rho \\ \sim \mathcal{N}\left(0, \gamma_v^2\right), & \text{with probability } 1-\rho \end{cases}.$$

Let $x_t = p_{t-1} - \xi_t$ denote the beginning-of-period price gap, and let $\widetilde{x}_t = p_t - \xi_t$ be the price gap chosen in period t. Under the random walk assumption for marginal costs, we have:

$$x_{t+1} = p_t - \xi_{t+1} = \widetilde{x}_t - v_{t+1}.$$

The firm's problem can be formulated recursively with two state variables: x_t and ϵ_{t-1}. Let $\beta < 1$ denote the firm's discount factor. The firm's value function is given by:

$$V(x,\epsilon) = \max\left\{V_{\text{No Adj}}(x,\epsilon)\,; V_{\text{Adj}}\right\}, \tag{38}$$

where

$$V_{\text{No Adj}}(x,\epsilon) = (1-\beta)\left[-\frac{\theta(\theta-1)}{2}x^2 + \alpha\gamma_\epsilon\epsilon\right] + \beta\mathbb{E}_v\left[V\left(x - v', \epsilon\right)\right], \tag{39}$$

and

$$V_{\text{Adj}} = \max_x\left\{(1-\beta)\left[-\frac{\theta(\theta-1)}{2}x^2 + \frac{1}{2}(\alpha\gamma_\epsilon)^2\right] + \beta\mathbb{E}_\epsilon\left[\mathbb{E}_v\left[V\left(x - v', \epsilon'\right)\right]\right]\right\}. \tag{40}$$

The following lemma describes some key properties of the firm's value function.

Lemma 9. *$V_{No\ Adj}(x,\epsilon)$ is strictly increasing in ϵ and $V(x,\epsilon)$ is nondecreasing in ϵ.*

Proof. Suppose $V(x,\epsilon)$ is nondecreasing in ϵ. From equation (39), $V_{\text{No Adj}}(x,\epsilon)$ is strictly increasing in ϵ. Since V_{Adj} is a constant, the operator implied by equation (38) maps into a nondecreasing function. Because the space of nondecreasing functions is closed, $V(x,\epsilon)$ is nondecreasing. □

Corollary 1. *The optimal policy involves a threshold $\overline{\epsilon}(x)$ such that if $\epsilon > \overline{\epsilon}(x)$, $V_{No\ Adj}(x,\epsilon) > V_{Adj}$.*

Figure 5 shows how the discount factor affects the inaction region–the set of conditions under which firms keep prices unchanged. The orange curve represents

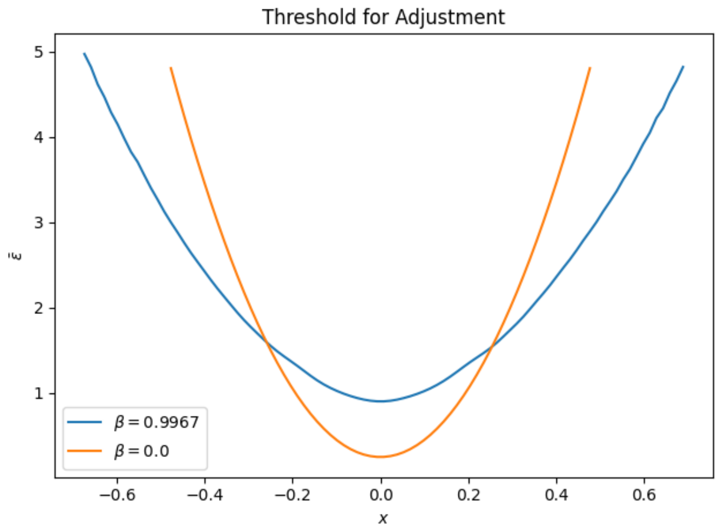

Figure 5: Inaction region

myopic firms ($\beta = 0$), while the blue curve corresponds to forward-looking firms that value future profits.[4]

Myopic firms place less value on favorable demand shocks than forward-looking firms. This property has two implications illustrated in Figure 5. First, when the price gap is small, myopic firms are less inclined to adjust prices to activate System 2 in hopes of eliciting a strong demand realization, because they fail to account for the future value of a high demand shock. Second, when the price gap is large, they require unusually strong demand shocks to justify leaving prices unchanged, again because they disregard the future benefits of high demand. In contrast, forward-looking firms are more likely to keep prices fixed even with large price gaps, because they recognize that positive demand shocks are valuable in the future.

We now highlight an important property of the dynamic model: it can account for a key empirical regularity emphasized by Nakamura and Steinsson (2008) and

[4]Figure 5 is analogous to Figure 1. Although Figure 1 displays inflation on the x-axis, inflation effectively determines the price gap in the static model. However, Figure 5 lacks the asymmetries seen in Figure 1 because the firm problem is solved using a quadratic approximation.

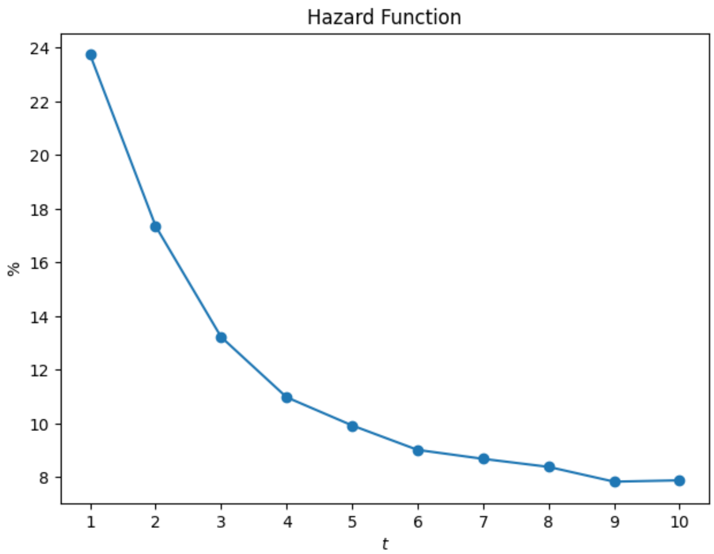

Figure 6: Hazard function: probability that a firm changes its price in period t, conditional on the price having remained unchanged for the previous $t-1$ periods

Campbell and Eden (2014): the hazard function for individual goods categories is downward sloping.[5] In contrast, standard menu cost models typically generate upward-sloping hazard functions.

Figure 6 plots the hazard function implied by our model. In our framework, firms facing unfavorable demand shocks are more likely to adjust prices early, while those experiencing favorable shocks tend to keep prices fixed for longer periods. This heterogeneity in demand conditions naturally gives rise to downward-sloping hazard rates.

[5]The aggregate hazard function across all CPI categories is sharply downward-sloping (see, e.g., Klenow and Kryvtsov (2008)). This property is primarily due to a composition effect across different categories. Prices of goods such as gasoline and fresh food products change frequently, while service prices are more stable. At short durations, all categories are represented, but at longer durations, services dominate. Our focus is not on these compositional effects across categories but on the fact that hazard functions tend to be downward-sloping even within narrowly defined categories.

9 Conclusion

This paper develops a model in which households make decisions according to a dual-process framework. This approach gives rise to a novel form of price rigidity that stems from the strategic interaction between consumers and monopolistic producers. There is a range of cost shocks for which some producers refrain from adjusting prices so that households do not reassess their purchasing decisions.

The model is consistent with three important empirical facts. First, it accounts for the well-known ”rockets and feathers” phenomenon: prices rise quickly in response to cost increases but fall slowly when costs decline. Second, it is consistent with the finding of Ilut et al. (2020) that firms experiencing strong demand realizations are less likely to adjust their prices. Third, it produces downward-sloping hazard functions, consistent with those estimated from micro data.

References

Afrouzi, H., A. Dietrich, K. Myrseth, R. Priftis, and R. Schoenle (2024): "Inflation preferences," NBER Working Paper 31990, National Bureau of Economic Research, Cambridge, MA.

Alvarez, F., H. Le Bihan, and F. Lippi (2014): "Small and large price changes and the propagation of monetary shocks," NBER Working Paper 20228, National Bureau of Economic Research, Cambridge, MA.

Argente, D. and C. Yeh (2022): "Product life cycle, learning, and nominal shocks," *The Review of Economic Studies*, 89, 2992–3054.

Ater, I. and O. Gerlitz (2017): "Round prices and price rigidity: Evidence from outlawing odd prices," *Journal of Economic Behavior & Organization*, 144, 188–203.

Blinder, A., E. R. Canetti, D. E. Lebow, and J. B. Rudd (1998): *Asking about prices: a new approach to understanding price stickiness*, Russell Sage Foundation.

Borenstein, S., A. C. Cameron, and R. Gilbert (1997): "Do Gasoline Prices Respond Asymmetrically to Crude Oil Price Changes?" *The Quarterly Journal of Economics*, 112, 305–339.

Budianto, F. (2024): "Shrinkflation," Working paper, TU Wien.

Burstein, A. and C. Hellwig (2007): "Prices and market shares in a menu cost model," NBER Working Paper 13459, National Bureau of Economic Research, Cambridge, MA.

Campbell, J. R. and B. Eden (2014): "Rigid prices: Evidence from us scanner data," *International Economic Review*, 55, 423–442.

Cavallo, A., F. Lippi, and K. Miyahara (2023): "Large Shocks Travel Fast," NBER Working Paper 31488, National Bureau of Economic Research.

Costain, J., A. Nakov, and B. Petit (2019): "Monetary Policy Implications of State-Dependent Prices and Wages," Working Paper 1916, Bank of Spain, Madrid.

De Clippel, G., K. Eliaz, and K. Rozen (2014): "Competing for consumer inattention," *Journal of Political Economy*, 122, 1203–1234.

Della Vigna, S. and U. Malmendier (2006): "Paying not to go to the gym," *American Economic Review*, 96, 694–719.

Eichenbaum, M., N. Jaimovich, and S. Rebelo (2011): "Reference prices, costs, and nominal rigidities," *American Economic Review*, 101, 234–262.

Eichenbaum, M., N. Jaimovich, S. Rebelo, and J. Smith (2014): "How frequent are small price changes?" *American Economic Journal: Macroeconomics*, 6, 137–155.

Ellingsen, T., R. Friberg, and J. Hassler (2006): "Menu costs and asymmetric price adjustment," .

Evans, J. S. B. T. and K. E. Stanovich (2013): "Dual-process theories of higher cognition: Advancing the debate," *Perspectives on Psychological Science*, 8, 223–241.

Golosov, M. and R. E. Lucas (2007): "Menu costs and Phillips curves," *Journal of Political Economy*, 115, 171–199.

Ilut, C. and R. Valchev (2023): "Economic agents as imperfect problem solvers," *The Quarterly Journal of Economics*, 138, 313–362.

Ilut, C., R. Valchev, and N. Vincent (2020): "Paralyzed by fear: Rigid and discrete pricing under demand uncertainty," *Econometrica*, 88, 1899–1938.

KARRENBROCK, J. D. (1991): "The behavior of retail gasoline prices: symmetric or not?" *Federal Reserve Bank of St. Louis Review*, 73, 19–29.

KASHYAP, A. K. (1995): "Sticky prices: New evidence from retail catalogs," *The Quarterly Journal of Economics*, 110, 245–274.

KLENOW, P. J. AND O. KRYVTSOV (2008): "State-dependent or time-dependent pricing: Does it matter for recent US inflation?" *The Quarterly Journal of Economics*, 123, 863–904.

LEVY, D., D. LEE, H. CHEN, R. J. KAUFFMAN, AND M. BERGEN (2011): "Price points and price rigidity," *Review of Economics and Statistics*, 93, 1417–1431.

LUCAS, R. E. J. (1987): *Models of Business Cycles*, Oxford: Basil Blackwell.

MAĆKOWIAK, B. AND M. WIEDERHOLT (2009): "Optimal sticky prices under rational inattention," *American Economic Review*, 99, 769–803.

MANKIW, N. G. AND R. REIS (2002): "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve," *The Quarterly Journal of Economics*, 117, 1295–1328.

MATĚJKA, F. (2015): "Rigid pricing and rationally inattentive consumer," *Journal of Economic Theory*, 158, 656–678.

NAKAMURA, E. AND J. STEINSSON (2008): "Five facts about prices: A reevaluation of menu cost models," *The Quarterly Journal of Economics*, 123, 1415–1464.

NEUMARK, D. AND S. A. SHARPE (1992): "Market structure and the nature of price rigidity: evidence from the market for consumer deposits," *The Quarterly Journal of Economics*, 107, 657–680.

PELTZMAN, S. (2000): "Prices rise faster than they fall," *Journal of Political Economy*, 108, 466–502.

ROTEMBERG, J. J. (1982): "Sticky prices in the United States," *Journal of Political Economy*, 90, 1187–1211.

SIMS, C. A. (2003): "Implications of rational inattention," *Journal of Monetary Economics*, 50, 665–690.

STANOVICH, K. E. AND R. F. WEST (2000): "Individual Differences in Reasoning: Implications for the Rationality Debate," *Behavioral and Brain Sciences*, 23, 645–665.

TAPPATA, M. (2009): "Rockets and feathers: Understanding asymmetric pricing," *The RAND Journal of Economics*, 40, 673–687.

UK OFFICE FOR NATIONAL STATISTICS, T. (2019): "Shrinkflation: How many of our products are getting smaller?" Tech. rep., U.K. Office for National Statistics.

WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, NJ: Princeton University Press, 1 ed.

——— (2009): "Information-constrained state-dependent pricing," *Journal of Monetary Economics*, 56, S100–S124.

10 Appendix

10.1 Proof of Lemma 1

Let

$$u(\mathbf{c}) \equiv \frac{\left(\frac{1}{n}\sum_{k=0}^{n} C_{\frac{k}{n}}^{\frac{\theta-1}{\theta}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}} - 1}{1-\sigma},$$

and let

$$C_{\frac{k}{n}} \equiv \overline{C}e^{c_{\frac{k}{n}}}.$$

Then

$$u(\mathbf{c}) = \frac{\overline{C}^{1-\sigma}\left(\frac{1}{n}\sum_{k=0}^{n} e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}} - 1}{1-\sigma}$$

Now

$$u_k(\mathbf{c}) = \overline{C}^{1-\sigma}\left(\frac{1}{n}\sum_{k=0}^{n} e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}-1} \frac{1}{n}e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}$$

$$u_{kj}(\mathbf{c}) = \overline{C}^{1-\sigma}\left[(1-\sigma)\left(\frac{\theta}{\theta-1}\right)-1\right]\left(\frac{1}{n}\sum_{k=0}^{n} e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}-2}\left(\frac{1}{n}e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}\right)\left(\frac{1}{n}\right)\left(\frac{\theta-1}{\theta}\right)e^{\left(\frac{\theta-1}{\theta}\right)c}$$

and

$$u_{kk}(\mathbf{c}) = \overline{C}^{1-\sigma}\left[\frac{1}{n}e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}\left[(1-\sigma)\frac{\theta}{\theta-1}-1\right]\left(\frac{1}{n}\sum_{k=0}^{n} e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}-2}\frac{1}{n}\left(\frac{\theta-1}{\theta}\right)e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}\right] +$$
$$+\overline{C}^{1-\sigma}\left(\frac{1}{n}\sum_{k=0}^{n} e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}-1}\left(\frac{\theta-1}{\theta}\right)\frac{1}{n}e^{\left(\frac{\theta-1}{\theta}\right)c_{\frac{k}{n}}}$$

Evaluated at $\mathbf{c} = \mathbf{0}$, we get

$$u(\mathbf{0}) = \frac{\overline{C}^{1-\sigma}-1}{1-\sigma},$$

$$u_k(\mathbf{0}) = \frac{1}{n}\overline{C}^{1-\sigma},$$

$$u_{kj}(0) = \overline{C}^{1-\sigma}\left[(1-\sigma) - \left(\frac{\theta-1}{\theta}\right)\right]\left(\frac{1}{n}\right)^2,$$

and

$$u_{kk}(\mathbf{0}) = \overline{C}^{1-\sigma}\left[(1-\sigma) - \left(\frac{\theta-1}{\theta}\right)\right]\left(\frac{1}{n}\right)^2 + \overline{C}^{1-\sigma}\left(\frac{\theta-1}{\theta}\right)\frac{1}{n}$$

Therefore

$$u(\mathbf{c}) \approx u(\mathbf{0}) + \overline{C}^{1-\sigma}\frac{1}{n}\sum_{k=0}^{n} c_{\frac{k}{n}} + \left(\frac{1}{2}\right)\overline{C}^{1-\sigma}\left(\frac{\theta-1}{\theta}\right)\frac{1}{n}\sum_{k=0}^{n} c_{\frac{k}{n}}^2 + \frac{1}{2}\overline{C}^{1-\sigma}\left[(1-\sigma) - \left(\frac{\theta-1}{\theta}\right)\right]\left(\frac{1}{n}\sum_{k=0}^{n} c_{\frac{k}{n}}\right)$$

so that taking $n \to \infty$,

$$\frac{u(\mathbf{c}) - u(\mathbf{0})}{\overline{C}^{1-\sigma}} \approx \int_0^1 c_i di + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\int_0^1 c_i^2 di + \frac{1}{2}\left[(1-\sigma) - \left(\frac{\theta-1}{\theta}\right)\right]\left(\int_0^1 c_i di\right)^2$$

Now consider the disutility of labor,

$$g(n) = \vartheta\frac{\overline{N}^{1+\psi}}{1+\psi}e^{(1+\psi)n}.$$

Note that at the equilibrium,

$$\vartheta\overline{N}^{\psi}\overline{C}^{\sigma} = \overline{A} = \frac{\overline{C}}{\overline{N}} \iff \vartheta\overline{N}^{1+\psi} = \overline{C}^{1-\sigma},$$

so that

$$\begin{aligned} g(n) &= \frac{\overline{C}^{1-\sigma}}{1+\psi}e^{(1+\psi)n} \\ &\approx \frac{\overline{C}^{1-\sigma}}{1+\psi}\left[1 + (1+\psi)n + \frac{1}{2}(1+\psi)^2 n^2\right], \end{aligned}$$

so that

$$\begin{aligned} \hat{U} &\equiv \frac{u(\mathbf{c}) - u(\mathbf{0})}{\overline{C}^{1-\sigma}} - \frac{g(n) - g(0)}{\overline{C}^{1-\sigma}} \\ &\approx \int_0^1 c_i di + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\int_0^1 c_i^2 di + \frac{1}{2}\left[(1-\sigma) - \left(\frac{\theta-1}{\theta}\right)\right]\left(\int_0^1 c_i di\right)^2 - n - \frac{1}{2}(1+\psi)n^2 \\ &= c - n - \frac{1}{2}\sigma c^2 - \frac{1}{2}\psi n^2 + \frac{1}{2}\int_0^1 c_i^2 di - \frac{1}{2}n^2 - \frac{1}{2\theta}\text{Var}_i[c_i], \end{aligned}$$

where

$$\text{Var}_i\left[c_i\right] = \int_0^1 c_i^2 di - \left(\int_0^1 c_i di\right)^2$$

Now consider the constraint terms associated with each of the problems.

$$G_e \equiv \Lambda_e \left(E - \int_0^1 P_i C_i di\right).$$

Let

$$\overline{\Lambda}_e \equiv \frac{\overline{C}^{-\sigma}}{\overline{P}},$$
$$\overline{E} \equiv \overline{P} \times \overline{C}.$$

Write

$$\begin{aligned} G_e &= \overline{\Lambda}_e e^{\lambda_e} \left(\overline{E} e^e - \overline{P} \times \overline{C} \int_0^1 e^{p_i + c_i} di\right) \\ &= \overline{C}^{1-\sigma} e^{\lambda_e} \left(e^e - \int_0^1 e^{p_i + c_i} di\right), \end{aligned}$$

so

$$\frac{G_e}{\overline{C}^{1-\sigma}} = e^{\lambda_e} \left(e^e - \int_0^1 e^{p_i + c_i} di\right).$$

Let

$$f_e\left(\mathbf{c}, \lambda_e, \mathbf{p}, e\right) = e^{\lambda_e + e} - \int_0^1 e^{\lambda_e + p_i + c_i} di.$$

Then

$$\begin{aligned} e^{\lambda_e + e} &\approx 1 + \lambda_e + e + \frac{1}{2}\left(\lambda_e + e\right)^2 \\ &= 1 + \lambda_e + e + \frac{1}{2}\lambda_e^2 + \lambda_e e + \frac{1}{2} e^2 \end{aligned}$$

and

$$\begin{aligned} e^{(\lambda_e + p_i + c_i)} &\approx 1 + \left(\lambda_e + p_i + c_i\right) + \frac{1}{2}\left(\lambda_e + p_i + c_i\right)^2 \\ &= 1 + \left(\lambda_e + p_i + c_i\right) + \frac{1}{2}\lambda_e^2 + \lambda_e\left(p_i + c_i\right) + \frac{1}{2}\left(p_i + c_i\right)^2 \end{aligned}$$

Therefore

$$
\begin{aligned}
f_e(\mathbf{c}, \lambda_e, \mathbf{p}, e) \approx\ & 1 + \lambda_e + e + \frac{1}{2}\lambda_e^2 + \lambda_e e + \frac{1}{2}e^2 \\
& - \int_0^1 \left[1 + (\lambda_e + p_i + c_i) + \frac{1}{2}\lambda_e^2 + \lambda_e (p_i + c_i) + \frac{1}{2}(p_i + c_i)^2\right] di \\
& = e + \lambda_e e + \frac{1}{2}e^2 - \int_0^1 \left[(p_i + c_i) + \lambda_e (p_i + c_i) + \frac{1}{2}(p_i + c_i)^2\right] di \\
& = e + \frac{1}{2}e^2 - \int_0^1 (p_i + c_i)\, di + \lambda_e \left[e - \int_0^1 (p_i + c_i)\, di\right] - \frac{1}{2}\int_0^1 (p_i + c_i)^2\, di \\
& = e + \frac{1}{2}e^2 - p - c + \lambda_e (e - p - c) - \frac{1}{2}\int_0^1 p_i^2 di - \int_0^1 p_i c_i di - \frac{1}{2}\int_0^1 c_i^2 di
\end{aligned}
$$

Now consider the constraint of Step 2,

$$
G_u = \Lambda_u \left(WN + \Pi - T - \int_0^1 P_i C_i di\right),
$$

Analogously, we can write

$$
G_u = \overline{\Lambda}_u e^{\lambda_u} \left(\overline{W} \times \overline{N} e^{w+n} + \overline{\Pi} e^{\ln\left(\frac{\Pi}{\overline{\Pi}}\right)} - \overline{T} e^{\ln\left(\frac{T}{\overline{T}}\right)} - \overline{P} \times \overline{C} \int_0^1 e^{p_i + c_i} di\right).
$$

Note that $\overline{W} \times \overline{N} = \overline{P} \times \overline{C}$, and $\overline{\Pi} = \overline{T}$. Moreover, $\overline{\Lambda}_u = \overline{C}^{-\sigma} / \overline{P}$. Therefore

$$
\frac{G_u}{\overline{C}^{1-\sigma}} = e^{\lambda_u + w + n} + \frac{\overline{\Pi}}{\overline{P}\,\overline{C}} \left[e^{\lambda_u + \ln\left(\frac{\Pi}{\overline{\Pi}}\right)} - e^{\lambda_u + \ln\left(\frac{T}{\overline{T}}\right)}\right] - \int_0^1 e^{\lambda_u + p_i + c_i} di
$$

Now

$$
\begin{aligned}
\overline{\Pi} &= \left[\overline{P} - \left(\frac{\theta - 1}{\theta}\right) \frac{\overline{W}}{\overline{A}}\right] \overline{C} \\
&= \frac{1}{\theta} \overline{PC}.
\end{aligned}
$$

Therefore

$$\frac{G_u}{\overline{C}^{1-\sigma}} = e^{\lambda_u+w+n} + \frac{1}{\theta}\left(e^{\lambda_u+\ln\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)} - e^{\lambda_u+T}\right) - \int_0^1 e^{\lambda_u+p_i+c_i}di$$

$$= w + \frac{1}{2}w^2 + n + \frac{1}{2}n^2 + wn + \lambda_u(w+n) +$$

$$+\frac{1}{\theta}\left[\ln\frac{\Pi}{\overline{\overline{\Pi}}} + \frac{1}{2}\ln^2\frac{\Pi}{\overline{\overline{\Pi}}} + \lambda_u\left(\ln\frac{\Pi}{\overline{\overline{\Pi}}} - \ln\frac{T}{\overline{\overline{T}}}\right) - \ln\frac{T}{\overline{\overline{T}}} - \frac{1}{2}\ln^2\frac{T}{\overline{\overline{T}}}\right]$$

$$-\int_0^1\left[p_i + \frac{1}{2}p_i^2 + c_i + \frac{1}{2}c_i^2 + \lambda_u(p_i+c_i) + p_ic_i\right]di$$

$$= n + \frac{1}{2}n^2 - c - \frac{1}{2}\int_0^1 c_i^2di + wn - \int_0^1 p_ic_idi + \lambda_u\left[w + n + \frac{1}{\theta}\left(\ln\frac{\Pi}{\overline{\overline{\Pi}}} - \ln\frac{T}{\overline{\overline{T}}}\right) - p - c\right]$$

$$+ w - p + \frac{1}{2}w^2 - \frac{1}{2}\int_0^1 p_i^2di + \frac{1}{\theta}\left(\ln\frac{\Pi}{\overline{\overline{\Pi}}} + \frac{1}{2}\ln^2\frac{\Pi}{\overline{\overline{\Pi}}} - \ln\frac{T}{\overline{\overline{T}}} - \frac{1}{2}\ln^2\frac{T}{\overline{\overline{T}}}\right)$$

Summing utility with the constraint terms yield the results.

10.2 Proof of Lemma 3

Combining equation (19) and $\mathbb{E}\left[\Delta\hat{\mathcal{L}}_e \mid \mathbf{s}\right]$,

$$\mathbb{E}\left[\Delta\hat{\mathcal{L}}_e \mid \mathbf{s}\right] = -\frac{1}{2\theta}\mathbb{E}\left[\int_0^1\left[c - \int_0^1 \mu_{i|s}(\mathbf{z})\,di + \mu_{i|s}(\mathbf{z}) - c_i^b(\mathbf{z})\right]^2 di \mid \mathbf{s}\right]$$

$$= +(\theta\sigma - 1)\,\mathbb{E}\left[\left(\int_0^1\left[c + \mu_{i|s}(\mathbf{z}) - \int_0^1 \mu_{i|s}(\mathbf{z})\,di - c_i^b(\mathbf{z})\right]di\right)^2 \mid \mathbf{s}\right].$$

Let $\int_0^1 \mu_{i|s}(\mathbf{z})\,di \equiv \overline{\mu}_{|s}$. The first expectation is

$$\mathbb{E}\left[\int_0^1\left[c - \overline{\mu}_{|s} + \mu_{i|s}(\mathbf{z}) - c_i^b(\mathbf{z})\right]^2 di \mid \mathbf{s}\right] = \left(c - \overline{\mu}_{|s}\right)^2 + \int_0^1 \gamma_{i|s}^2(\mathbf{z})\,di.$$

The second expectation is

$$\mathbb{E}\left[\left(\int_0^1 \left[c-\overline{\mu}_{|s}+\mu_{i|s}\left(\mathbf{z}\right)-c_i^b\left(\mathbf{z}\right)\right]di\right)^2 \mid \mathbf{s}\right] = \mathbb{E}\left[\left(\int_0^1 \left[c-\overline{\mu}_{|s}+\mu_{i|s}\left(\mathbf{z}\right)-c_i^b\left(\mathbf{z}\right)\right]di\right)^2 \mid \mathbf{s}\right]$$
$$= \mathbb{E}\left[\left(c-\overline{\mu}_{|s}+\int_0^1 \left[\mu_{i|s}\left(\mathbf{z}\right)-c_i^b\left(\mathbf{z}\right)\right]di\right)^2 \mid \mathbf{s}\right]$$
$$= \left(c-\overline{\mu}_{|s}\right)^2+\mathbb{E}\left[\left(\int_0^1 \left[\mu_{i|s}\left(\mathbf{z}\right)-c_i^b\left(\mathbf{z}\right)\right]di\right)^2 \mid \mathbf{s}\right]$$
$$= \left(c-\overline{\mu}_{|s}\right)^2.$$

The last equality results from the law of large numbers. Therefore

$$\mathbb{E}\left[\Delta\hat{\mathcal{L}}_e \mid \mathbf{s}\right] = -\frac{1}{2\theta}\int_0^1 \gamma_{i|s}^2\left(\mathbf{z}\right)di-\frac{1}{2}\sigma\left(c-\overline{\mu}_{|s}\right)^2.$$

Finally, letting $\overline{\mu}\equiv\int_0^1 \mu_i\left(\mathbf{z}\right)di$,

$$\mathbb{E}\left[\left(c-\overline{\mu}_{|s}\right)^2\right] = \mathbb{E}\left[\left(c-\overline{\mu}+\overline{\mu}-\overline{\mu}_{|s}\right)^2\right]$$
$$= \left(c-\overline{\mu}\right)^2+\mathbb{E}\left[\left(\overline{\mu}_{|s}-\overline{\mu}\right)^2\right]$$
$$= \left(c-\overline{\mu}\right)^2,$$

where the last equality follows again from the law of large numbers. Therefore

$$\mathbb{E}\left[\Delta\hat{\mathcal{L}}_e\right] = -\frac{1}{2\theta}\int_0^1 \gamma_{i|s}^2\left(\mathbf{z}\right)di-\frac{1}{2}\sigma\left(c-\overline{\mu}\right)^2,$$

which implies that only the first term depends on the distribution of the signal.

10.3 Proof of Lemma 4

Using the approximation $X=\overline{X}e^x$, we have

$$\Pi_i=\overline{PC}\left(e^{p_i-p}-e^{w-p-a+\tau-\Theta}\right)e^{c-\theta(p_i-p)+\alpha\gamma_\epsilon\{\tilde{\epsilon}_i-\mathbb{E}[\tilde{\epsilon}]\}}.$$

Conditional on a price change, expected profits are

$$\Pi_i = \overline{PC}\left(e^{p_i - p} - e^{w-p-a+\tau-\Theta}\right)e^{c-\theta(p_i-p)+\frac{1}{2}(\alpha\gamma_\epsilon)^2 - \alpha\gamma_\epsilon\mathbb{E}[\widetilde{\epsilon}]}.$$

Taking the first-order condition with respect to p_i yields

$$p_i = w - a + \ln\left(\frac{1-\tau}{1-\overline{\tau}}\right) \equiv p_{\text{adj}}.$$

Therefore optimized profits conditional on a price change are

$$\Pi_{\text{adj}} = \overline{PC}\left(1 - e^{-\Theta}\right)e^{-(\theta-1)\left(p_{\text{adj}}-p\right)+c-\alpha\gamma_\epsilon\mathbb{E}[\widetilde{\epsilon}]}e^{\frac{1}{2}(\alpha\gamma_\epsilon)^2}.$$

Conditional on keeping the price, profits are:

$$\Pi_{\text{no-adj}} = \overline{P}\,\overline{C}\left(1 - e^{(p_{\text{adj}}-p)+\pi-\Theta}\right)e^{c+(\theta-1)\pi+\alpha\gamma_\epsilon\left(\epsilon_{i,0}-\mathbb{E}[\widetilde{\epsilon}]\right)}.$$

Provided that

$$e^{(p_{\text{adj}}-p)+\pi-\Theta} < 1,$$

$\Pi_{\text{no-adj}}$ is strictly increasing in $\epsilon_{i,0}$, so a threshold rule is optimal. The threshold $\overline{\epsilon}$ is given by:

$$\Pi_{\text{no-adj}} = \Pi_{\text{adj}},$$

which implies:

$$\overline{\epsilon} = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left\{(\theta-1)\left[(p_{\text{adj}}-p)+\pi\right] + \ln\left[\frac{1-e^{(p_{\text{adj}}-p)+\pi-\Theta}}{1-e^{-\Theta}}\right]\right\}.$$

10.4 Proof of Proposition 2

We show the properties included in this proposition one at a time.

Uniqueness of $\overline{\epsilon}(\pi)$. We first show that when $\pi < \Theta$, (37) is a well-defined function. Let

$$f(\overline{\epsilon}, \pi) = \frac{1}{2}\alpha\gamma_{\epsilon} - \frac{1}{\alpha\gamma_{\epsilon}}\left[(\theta - 1)\frac{\pi}{\Phi(\overline{\epsilon})} + \ln\left(\frac{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}{1 - e^{-\Theta}}\right)\right] - \overline{\epsilon}.$$

First note that $f(\overline{\epsilon}, \pi)$ is only defined if $\pi < \Phi(\overline{\epsilon})\Theta$. Therefore, if $\pi < 0$, f is always well-defined. Otherwise, it is only defined for

$$\overline{\epsilon} > \Phi^{-1}\left(\frac{\pi}{\Theta}\right).$$

Hence, this function is only defined for $\pi < \Theta$. First suppose that $\pi < 0$. Then

$$\begin{aligned}
\lim_{\overline{\epsilon} \to -\infty} f(\overline{\epsilon}, \pi) &= \frac{1}{2}\alpha\gamma_{\epsilon} - \frac{1}{\alpha\gamma_{\epsilon}}\left[\ln\left(\frac{1}{1 - e^{-\Theta}}\right)\right] - \lim_{\overline{\epsilon} \to -\infty}\left[\frac{1}{\alpha\gamma_{\epsilon}}(\theta - 1)\frac{\pi}{\Phi(\overline{\epsilon})} - \overline{\epsilon}\right] \\
&= \frac{1}{2}\alpha\gamma_{\epsilon} - \frac{1}{\alpha\gamma_{\epsilon}}\left[\ln\left(\frac{1}{1 - e^{-\Theta}}\right)\right] - \lim_{\overline{\epsilon} \to -\infty}\left\{\frac{1}{\Phi(\overline{\epsilon})}\left[\frac{1}{\alpha\gamma_{\epsilon}}(\theta - 1)\pi - \overline{\epsilon}\Phi(\overline{\epsilon})\right]\right\} \\
&= \frac{1}{2}\alpha\gamma_{\epsilon} - \frac{1}{\alpha\gamma_{\epsilon}}\left[\ln\left(\frac{1}{1 - e^{-\Theta}}\right)\right] - \frac{1}{\alpha\gamma_{\epsilon}}(\theta - 1)\pi \lim_{\overline{\epsilon} \to -\infty}\left\{\frac{1}{\Phi(\overline{\epsilon})}\right\} = \infty,
\end{aligned}$$

and

$$\lim_{\overline{\epsilon} \to \infty} f(\overline{\epsilon}, \pi) = \lim_{\overline{\epsilon} \to \infty}\left\{\frac{1}{2}\alpha\gamma_{\epsilon} - \frac{1}{\alpha\gamma_{\epsilon}}\left[(\theta - 1)\pi + \ln\left(\frac{1 - e^{\pi - \Theta}}{1 - e^{-\Theta}}\right)\right] - \overline{\epsilon}\right\} = -\infty.$$

The case where $\pi = 0$ is trivial, since in that case

$$\overline{\epsilon} = \frac{1}{2}\alpha\gamma_{\epsilon}.$$

When $\pi \in (0, \Theta)$,

$$\lim_{\overline{\epsilon} \to \Phi^{-1}\left(\frac{\pi}{\Theta}\right)} f(\overline{\epsilon}, \pi) = \frac{1}{2}\alpha\gamma_{\epsilon} - \frac{1}{\alpha\gamma_{\epsilon}}\left[(\theta - 1)\frac{\pi}{\pi/\Theta} + \lim_{x \to 0}\ln(x)\right] - \Phi^{-1}\left(\frac{\pi}{\Theta}\right) = \infty.$$

Therefore, the equation $f(\overline{\epsilon}, \pi) = 0$ has at least one solution in $\overline{\epsilon}$. To show that it has only one solution, note that

$$\begin{aligned}
f_{\overline{\epsilon}}(\overline{\epsilon}, \pi) &= -\frac{1}{\alpha\gamma_{\epsilon}}(\theta - 1)\left[-\frac{\pi}{\Phi^2(\overline{\epsilon})}\phi(\overline{\epsilon})\right] - \frac{1}{\alpha\gamma_{\epsilon}} \times \left[\frac{-e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}\right] \times \left[-\frac{\pi}{\Phi^2(\overline{\epsilon})}\phi(\overline{\epsilon})\right] - 1 \\
&= \frac{1}{\alpha\gamma_{\epsilon}}\left[(\theta - 1) - \frac{e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}\right]\left[\frac{\pi}{\Phi^2(\overline{\epsilon})}\phi(\overline{\epsilon})\right] - 1.
\end{aligned}$$

Note that

$$\ln\left(\frac{\theta}{\theta-1}\right)=\Theta$$
$$\iff \theta-1=\frac{e^{-\Theta}}{1-e^{-\Theta}}.$$

Therefore

$$f_{\bar{\epsilon}}\left(\bar{\epsilon},\pi\right)=\frac{1}{\alpha\gamma_{\epsilon}}\left[\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}-e^{\frac{\pi}{\Phi(\bar{\epsilon})}}\right]\frac{e^{-\Theta}}{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}\frac{\pi}{\Phi\left(\bar{\epsilon}\right)}\left[\frac{\phi\left(\bar{\epsilon}\right)}{\Phi\left(\bar{\epsilon}\right)}\right]-1.$$

It is easy to show that the first term in square brackets is negative as long as

$$0<\frac{\pi}{\Phi\left(\bar{\epsilon}\right)}.$$

Therefore the first term in $f_{\bar{\epsilon}}\left(\bar{\epsilon},\pi\right)$ is negative, which implies that $f\left(\bar{\epsilon},\pi\right)$ is strictly decreasing in $\bar{\epsilon}$. Hence, there is a unique solution for $f\left(\bar{\epsilon},\pi\right)=0$, and the implicit function theorem globally defined $\bar{\epsilon}\left(\pi\right)$. Now

$$\begin{aligned}f_{\pi}\left(\bar{\epsilon},\pi\right)&=-\frac{1}{\alpha\gamma_{\epsilon}}\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{1}{\Phi\left(\bar{\epsilon}\right)}-\frac{1}{\alpha\gamma_{\epsilon}}\frac{-\frac{1}{\Phi(\bar{\epsilon})}e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}\\&=\frac{1}{\alpha\gamma_{\epsilon}}\frac{1}{\Phi\left(\bar{\epsilon}\right)}\frac{e^{-\Theta}}{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}\left[e^{\frac{\pi}{\Phi(\bar{\epsilon})}}-\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right].\end{aligned}$$

The implicit function theorem then yields

$$\begin{aligned}\bar{\epsilon}'\left(\pi\right)&=-\frac{f_{\pi}\left(\bar{\epsilon}\left(\pi\right),\pi\right)}{f_{\bar{\epsilon}}\left(\bar{\epsilon}\left(\pi\right),\pi\right)}\\&=\frac{\frac{1}{\alpha\gamma_{\epsilon}}\frac{1}{\Phi(\bar{\epsilon})}\frac{e^{-\Theta}}{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}\left[e^{\frac{\pi}{\Phi(\bar{\epsilon})}}-\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right]}{1+\frac{1}{\alpha\gamma_{\epsilon}}\left[e^{\frac{\pi}{\Phi(\bar{\epsilon})}}-\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right]\frac{e^{-\Theta}}{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}\frac{\pi}{\Phi(\bar{\epsilon})}\left[\frac{\phi(\bar{\epsilon})}{\Phi(\bar{\epsilon})}\right]},\end{aligned}$$

or

$$\bar{\epsilon}'\left(\pi\right)\pi=\frac{\Omega\left(\pi\right)}{1+\frac{\phi[\bar{\epsilon}(\pi)]}{\Phi[\bar{\epsilon}(\pi)]}\Omega\left(\pi\right)},$$

where

$$\Omega(\pi) \equiv \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{\varphi(\pi)-\Theta}}{1-e^{\varphi(\pi)-\Theta}} - \frac{e^{-\Theta}}{1-e^{-\Theta}}\right]\varphi(\pi),$$

and $\varphi(\pi) \equiv \pi/\Phi[\bar{\epsilon}(\pi)]$.

Minimum at $\pi = 0$. Note that $\bar{\epsilon}'(\pi) > 0 \iff \pi > 0$. Therefore $\bar{\epsilon}(\pi)$ has a minimum at $\pi = 0$. Therefore, $\chi \equiv 1 - \Phi(\bar{\epsilon})$, the fraction of sticky firms, has a maximum at $\pi = 0$.

Limits as $\pi \to \Theta$ or $\pi \to -\infty$. When $\pi \to \Theta$, we have

$$\lim_{\pi\to\Theta}\bar{\epsilon}(\pi) = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left\{\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{\Theta}{\lim_{\pi\to\Theta}\Phi(\bar{\epsilon}(\pi))} + \ln\left[\frac{1-e^{\frac{\Theta}{\lim_{\pi\to\Theta}\Phi(\bar{\epsilon}(\pi))}-\Theta}}{1-e^{-\Theta}}\right]\right\}.$$

The term in the logarithm is smaller than zero unless $\Phi[\bar{\epsilon}(\pi)] \to 1$, i.e., $\bar{\epsilon}(\pi) \to \infty$. Moreover, $\bar{\epsilon}(\pi) \to \infty$ is a fixed-point of the equation above and, for all $\pi < \Theta$, there is a unique $\bar{\epsilon}(\pi)$. Therefore $\lim_{\pi\to\Theta}\bar{\epsilon}(\pi) = \infty$. For $\pi \to -\infty$, note that $\Phi[\bar{\epsilon}(\pi)] \in [0,1]$ implies that

$$\ln\left[\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon}(\pi))}-\Theta}}{1-e^{-\Theta}}\right] \to \ln\left[\frac{1}{1-e^{-\Theta}}\right],$$

and

$$\frac{\pi}{\Phi[\bar{\epsilon}(\pi)]} \to -\infty.$$

These two facts imply that $\bar{\epsilon}(\pi) \to \infty$ (and that therefore $\Phi[\bar{\epsilon}(\pi)] \to 1$).

Asymmetry of $\bar{\epsilon}(\pi)$. To derive the asymmetry in $\bar{\epsilon}(\pi)$, it is convenient to write

$$\bar{\epsilon}(\pi) = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{\pi}{\Phi(\bar{\epsilon})} + \ln\left(\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right)\right].$$

Let a be some strictly positive scalar. Clearly, if $a \geq \Theta$, $\overline{\epsilon}(a) = \infty > \overline{\epsilon}(-a)$. Now consider $a \in (0, \Theta)$. Since $f(\overline{\epsilon}, \pi)$ is strictly decreasing in $\overline{\epsilon}$, to show that $\overline{\epsilon}(a) > \overline{\epsilon}(-a)$ it suffices to show that $f[\overline{\epsilon}(-a), a] > 0$. Now

$$\overline{\epsilon}(-a) = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{-a}{\Phi[\overline{\epsilon}(-a)]} + \ln\left(\frac{1-e^{\frac{-a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}{1-e^{-\Theta}}\right)\right],$$

and

$$f[\overline{\epsilon}(-a), a] = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{a}{\Phi[\overline{\epsilon}(-a)]} + \ln\left(\frac{1-e^{\frac{a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}{1-e^{-\Theta}}\right)\right] - \overline{\epsilon}(-a).$$

Therefore

$$\begin{aligned} f[\overline{\epsilon}(-a), a] &= \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{a}{\Phi[\overline{\epsilon}(-a)]} + \ln\left(\frac{1-e^{\frac{a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}{1-e^{-\Theta}}\right)\right] - \overline{\epsilon}(-a) \\ &\propto \ln\left(\frac{1-e^{\frac{-a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}{1-e^{\frac{a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}\right) - 2\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{a}{\Phi[\overline{\epsilon}(-a)]}. \end{aligned}$$

Let

$$\omega \equiv \frac{a}{\Phi[\overline{\epsilon}(-a)]},$$

and consider the function

$$g(\omega) \equiv \ln\left(\frac{1-e^{-\omega-\Theta}}{1-e^{\omega-\Theta}}\right) - 2\frac{e^{-\Theta}}{1-e^{-\Theta}}\omega.$$

Clearly $g(0) = 0$. Moreover,

$$g'(\omega) = \frac{1}{e^{\Theta+\omega}-1} + \frac{1}{e^{\Theta-\omega}-1} - \frac{2}{e^{\Theta}-1}.$$

Again, $g'(0) = 0$, and

$$g''(\omega) = \frac{e^{\Theta-\omega}}{(e^{\Theta-\omega}-1)^2} - \frac{e^{\Theta+\omega}}{(e^{\Theta+\omega}-1)^2}.$$

Now note that

$$\begin{aligned}
h\left(t\right) &= \frac{e^{t}}{\left(e^{t}-1\right)^{2}} \\
\Longrightarrow h'\left(t\right) &= \frac{e^{t}\left(e^{t}-1\right)-e^{t}\times 2\left(e^{t}-1\right)e^{t}}{\left(e^{t}-1\right)^{4}} \\
\Longrightarrow h'\left(t\right) &\propto 1-2e^{t}<0,
\end{aligned}$$

for $t>0$. Therefore, for $\omega>0$,

$$g''\left(\omega\right)>0 \Longrightarrow g'\left(\omega\right)>0 \Longrightarrow g\left(\omega\right)>0,$$

which implies that $f\left[\overline{\epsilon}\left(-a\right),a\right]>0$ for $a>0$.

10.5 Proof of Proposition 1 (existence of equilibrium)

Define $\mathcal{E}(\pi)=\pi+\widetilde{c}(\pi)$. Observe that $\widetilde{c}(\pi)\to 0$ as $\pi\to\pm\infty$. Therefore,

$$\lim_{\pi\to-\infty}\mathcal{E}(\pi)=-\infty, \quad \lim_{\pi\to\infty}\mathcal{E}(\pi)=\infty,$$

Implying that the equation $\mathcal{E}(\pi)=-c^{*}$ has at least one solution by the intermediate value theorem.

To show uniqueness, consider the derivative:

$$\begin{aligned}
\widetilde{c}'(\pi) &= \frac{1}{\psi+\sigma}\left\{\left[\frac{\chi'(\pi)}{[1-\chi(\pi)]^{2}}\right]\pi+\frac{\chi(\pi)}{1-\chi(\pi)}\right\} \\
&= \frac{1}{\psi+\sigma}\left[\frac{1}{1-\chi(\pi)}\right]\left\{\chi(\pi)-\frac{\frac{\phi[\overline{\epsilon}(\pi)]}{\Phi[\overline{\epsilon}(\pi)]}\Omega(\pi)}{1+\frac{\phi[\overline{\epsilon}(\pi)]}{\Phi[\overline{\epsilon}(\pi)]}\Omega(\pi)}\right\} \\
&> \frac{1}{\psi+\sigma}\left[\frac{1}{1-\chi(\pi)}\right]\left\{\chi(\pi)-1\right\} \\
&= -\frac{1}{\psi+\sigma}.
\end{aligned}$$

Hence, $\mathcal{E}'(\pi) = 1 + \tilde{c}'(\pi) > 1 - \frac{1}{\psi+\sigma} \geq 0$, which implies that $\mathcal{E}(\pi)$ is strictly increasing and ensures uniqueness when $\psi + \sigma \geq 1$. Therefore,

$$\mathcal{E}'(\pi) = 1 + \tilde{c}'(\pi) > 1 - \frac{1}{\psi + \sigma}.$$

If $\psi + \sigma \geq 1$, it follows that $\mathcal{E}'(\pi) > 0$ for all π. Hence, $\mathcal{E}(\pi)$ is strictly increasing, and the equilibrium is unique.

10.6 Proof of Proposition 3 ("rockets and feathers")

The equilibrium condition can be written as,

$$\mathcal{E}(\pi) \equiv \pi + \tilde{c}(\pi) = -c^*.$$

Assume $m = 0$. The equilibrium condition for inflation is

$$\pi + \tilde{c}(\pi) = -\nu.$$

To show that $\pi(\nu)\nu < 0$, consider the function

$$\mathcal{E}(\pi) = \pi + \tilde{c}(\pi) = \left\{1 + \frac{1}{\psi + \sigma}\left[\frac{\chi(\pi)}{1 - \chi(\pi)}\right]\right\}\pi.$$

The expression in curly brackets is strictly positive, so $\text{sign}(\pi(\nu)) = -\text{sign}(\nu)$.

Since $\mathcal{E}'(\pi) > 0$, it suffices to show that

$$\mathcal{E}[-\pi(\nu)] < \nu.$$

Note that

$$\mathcal{E}[-\pi(\nu)] = -\pi(\nu) + \tilde{c}[-\pi(\nu)],$$

and from the equilibrium condition,

$$\pi(\nu) + \tilde{c}[\pi(\nu)] = -\nu.$$

Therefore,

$$\begin{aligned}
\mathcal{E}[-\pi(\nu)] < \nu &\iff -\pi(\nu) + \widetilde{c}[-\pi(\nu)] < \nu \\
&\iff -\left[-\nu - \widetilde{c}[\pi(\nu)]\right] + \widetilde{c}[-\pi(\nu)] < \nu \\
&\iff \widetilde{c}[\pi(\nu)] + \widetilde{c}[-\pi(\nu)] < 0.
\end{aligned}$$

Substituting the expression for $\widetilde{c}(\pi)$, we obtain:

$$\begin{aligned}
&\frac{1}{\psi+\sigma}\left[\frac{\chi(\pi(\nu))}{1-\chi(\pi(\nu))}\right]\pi(\nu) + \frac{1}{\psi+\sigma}\left[\frac{\chi(-\pi(\nu))}{1-\chi(-\pi(\nu))}\right](-\pi(\nu)) < 0 \\
\iff &\left[\frac{\chi(\pi(\nu))}{1-\chi(\pi(\nu))} - \frac{\chi(-\pi(\nu))}{1-\chi(-\pi(\nu))}\right]\pi(\nu) < 0.
\end{aligned}$$

Since $\pi(\nu) < 0$, the inequality above holds if and only if

$$\frac{\chi(\pi(\nu))}{1-\chi(\pi(\nu))} > \frac{\chi(-\pi(\nu))}{1-\chi(-\pi(\nu))},$$

which is equivalent to

$$\chi(\pi(\nu)) > \chi(-\pi(\nu)).$$

This inequality follows directly from Proposition 2, completing the proof.

10.7 Proof of Lemma 5

We first derive the expression for $\Delta(\pi)$. From equation (25),

$$\mathrm{Var}_i[c_i] = \theta^2 \mathrm{Var}(p_i - p) + (\alpha\gamma_\epsilon)^2 \mathrm{Var}\left[\widetilde{\epsilon}_i - \mathbb{E}_i[\widetilde{\epsilon}_i]\right] - 2\theta\alpha\gamma_\epsilon \mathrm{Cov}\left[p_i - p, \widetilde{\epsilon}_i - \mathbb{E}_i[\widetilde{\epsilon}_i]\right].$$

First note that

$$\mathrm{Var}(p_i - p) = \frac{\chi}{1-\chi}\pi^2.$$

Second,

$$\mathbb{E}_i[\widetilde{\epsilon}_i] = \chi\mathbb{E}\left[\epsilon_{i,0} \mid \epsilon_{i,0} \geq \widetilde{\epsilon}\right] = \phi(\overline{\epsilon}).$$

Moreover, from the properties of the truncated normal distribution,

$$\mathbb{E}\left[\widetilde{\epsilon}_i^2\right] = \chi\left[1+\frac{\bar{\epsilon}\phi\left(\bar{\epsilon}\right)}{\chi}\right] + (1-\chi) = 1+\bar{\epsilon}\phi\left(\bar{\epsilon}\right),$$

from which it follows that

$$\text{Var}\left[\widetilde{\epsilon}_i - \mathbb{E}_i\left[\widetilde{\epsilon}_i\right]\right] = \mathbb{E}\left[\widetilde{\epsilon}_i^2\right] - \left(\mathbb{E}\left[\widetilde{\epsilon}_i\right]\right)^2 = 1+\bar{\epsilon}\phi\left(\bar{\epsilon}\right) - \left[\phi\left(\bar{\epsilon}\right)\right]^2.$$

As for the covariance, note that

$$\begin{aligned}
\text{Cov}[p_i - p, \widetilde{\epsilon}_i - \mathbb{E}_i\left[\widetilde{\epsilon}_i\right]] &= \mathbb{E}\left[\left(p_i - p\right)\left(\widetilde{\epsilon}_i - \mathbb{E}_i\left[\widetilde{\epsilon}_i\right]\right)\right] \\
&= \chi\left(-\pi\right)\left(\frac{\phi\left(\bar{\epsilon}\right)}{\chi} - \phi\left(\bar{\epsilon}\right)\right) + (1-\chi)\left(p_{\text{adj}} - p\right)\left(-\phi\left(\bar{\epsilon}\right)\right) \\
&= \chi\left(-\pi\right)\left(\frac{\phi\left(\bar{\epsilon}\right)}{\chi} - \phi\left(\bar{\epsilon}\right)\right) + (1-\chi)\frac{\chi\pi}{1-\chi}\left(-\phi\left(\bar{\epsilon}\right)\right) \\
&= -\chi\pi\left(\frac{\phi\left(\bar{\epsilon}\right)}{\chi} - \phi\left(\bar{\epsilon}\right)\right) - \chi\pi\phi\left(\bar{\epsilon}\right) \\
&= -\pi\phi\left(\bar{\epsilon}\right).
\end{aligned}$$

Therefore

$$\text{Var}_i\left[c_i\right] = \theta^2\left(\frac{\chi}{1-\chi}\right)\pi^2 + \alpha^2\gamma_\epsilon^2\left[1+\bar{\epsilon}\phi\left(\bar{\epsilon}\right) - \phi\left(\bar{\epsilon}\right)^2\right] + 2\theta\alpha\gamma_\epsilon\pi\phi\left(\bar{\epsilon}\right).$$

As for cognitive costs,

$$\mathcal{I} = \frac{\kappa}{2}\left[\chi \times 0 + (1-\chi)\left(\ln\gamma_c^2 - \ln\theta\kappa\right)\right] = \frac{1}{2\theta}\Phi\left[\bar{\epsilon}\left(\pi\right)\right]\theta\kappa\ln\left(\frac{\gamma_c^2}{\theta\kappa}\right).$$

Now,

$$\alpha = 1 - \frac{\theta\kappa}{\gamma_c^2},$$

and

$$\gamma_\epsilon = \sqrt{\frac{\theta\kappa}{\alpha}} \iff \theta\kappa = \alpha\gamma_\epsilon^2.$$

Therefore

$$\alpha = 1 - \frac{\theta\kappa}{\gamma_c^2} \iff \frac{\theta\kappa}{\gamma_c^2} = 1-\alpha,$$

from which it follows that

$$\mathcal{I} = \frac{1}{2\theta}\Phi\left[\bar{\epsilon}\left(\pi\right)\right]\alpha\gamma_{\epsilon}^{2}\ln\left(\frac{1}{1-\alpha}\right).$$

We start by showing that the implementable set of equilibria is characterized by equation (35).

Recall that the equilibrium conditions for c, n, π, and $\bar{\epsilon}$ can be summarized as

$$\sigma c + \psi n = \frac{1-\Phi\left(\bar{\epsilon}\right)}{\Phi\left(\bar{\epsilon}\right)}\pi - \tau + a, \tag{41}$$

$$c = a + n, \tag{42}$$

$$c + \pi = m, \tag{43}$$

$$\bar{\epsilon} = \begin{cases} \frac{1}{2}\alpha\gamma_{\epsilon} - \frac{1}{\alpha\gamma_{\epsilon}}\left[\frac{e^{-\vartheta}}{1-e^{-\vartheta}}\frac{\pi}{\Phi(\bar{\epsilon})} + \ln\left(\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\vartheta}}{1-e^{-\vartheta}}\right)\right], & \text{if } \pi < \vartheta \\ \infty, & \text{if } \pi \geq \vartheta \end{cases}. \tag{44}$$

Given π, equation (44) determines $\bar{\epsilon}$. Given c and n, equation (43) determines m, while equation (41) determines τ.

Since the set of implementable equilibria is characterized by equation (35), we can write the (non-linear) Lagrangian associated with the Ramsey problem as

$$\mathcal{L}_p = U + \Lambda_p\left(AN - \int_0^1 C_i di\right).$$

From Lemma 1,

$$\hat{U} = -c - \frac{1}{2}\sigma c^2 - \frac{1}{2}\int_0^1 c_i^2 di - n - \frac{1+\psi}{2}n^2 - \frac{1}{2\theta}\text{Var}_i\left[c_i\right]$$

To derive the second-order approximation of the constraint term, write

$$\begin{aligned} G_p &= \Lambda_p\left(AN - \int_0^1 C_i di\right) \\ &= \bar{\Lambda}_p e^{\lambda_p}\left(\bar{A}\times\bar{N}e^{a+n} - \bar{C}\int_0^1 e^{c_i}di\right). \end{aligned}$$

So, again, we can write

$$\begin{aligned}\frac{G_p}{\overline{C}^{1-\sigma}} &= e^{\lambda_p+a+n} - \int_0^1 e^{\lambda_p+c_i}di \\ &\approx 1+\lambda_p+a+n+\frac{1}{2}\left(\lambda_p+a+n\right)^2 - \\ &- \int_0^1 \left(1+\lambda_p+c_i\right)di - \frac{1}{2}\int_0^1 \left(\lambda_p+c_i\right)^2 di \\ &= a+n+\frac{1}{2}\left(\lambda_p+a+n\right)^2 - \int_0^1 c_i di - \frac{1}{2}\int_0^1 \left(\lambda_p+c_i\right)^2 di \\ &= a+n+\lambda_p\left(a+n\right)+\frac{1}{2}\left(a+n\right)^2 - \int_0^1 c_i di - \lambda c - \frac{1}{2}\int_0^1 c_i^2 di \\ &= a+\frac{1}{2}a^2+n+\frac{1}{2}n^2+\lambda_p\left(a+n-c\right)+an-c-\frac{1}{2}\int_0^1 c_i^2 di\end{aligned}$$

Letting

$$\hat{\mathcal{L}}_p = \hat{U} - \mathcal{I} + \frac{G_p}{\overline{C}^{1-\sigma}}$$

yields the result. The solution to c and n follows from taking first-order conditions with respect to c, n, and λ_p.

10.8 Proof of Lemma 6

It is easy to show that

$$\Delta\left(\Theta\right) = \alpha^2\gamma_\epsilon^2 + \alpha\gamma_\epsilon^2 \ln\left(\frac{1}{1-\alpha}\right),$$

and

$$\Delta\left(0\right) = \alpha^2\gamma_\epsilon^2\left\{1+\overline{\epsilon}\left(0\right)\phi\left[\overline{\epsilon}\left(0\right)\right] - \phi^2\left[\overline{\epsilon}\left(0\right)\right]\right\} + \Phi\left[\overline{\epsilon}\left(0\right)\right]\alpha\gamma_\epsilon^2\ln\left(\frac{1}{1-\alpha}\right).$$

Therefore

$$\Delta(\Theta) - \Delta(0) = \{1 - \Phi[\overline{\epsilon}(0)]\}\,\alpha\gamma_\epsilon^2 \ln\left(\frac{1}{1-\alpha}\right) - \alpha^2\gamma_\epsilon^2\left\{\overline{\epsilon}(0)\,\phi[\overline{\epsilon}(0)] - \phi^2[\overline{\epsilon}(0)]\right\}$$

$$\Longleftrightarrow \frac{\Delta(\Theta) - \Delta(0)}{\alpha\gamma_\epsilon^2\phi[\overline{\epsilon}(0)]} = \left[\frac{1 - \Phi\left(\frac{1}{2}\sqrt{\frac{\theta\kappa}{\alpha}}\right)}{\phi\left(\frac{1}{2}\sqrt{\frac{\theta\kappa}{\alpha}}\right)}\right]\ln\left(\frac{1}{1-\alpha}\right) - \alpha\left[\frac{1}{2}\sqrt{\frac{\theta\kappa}{\alpha}} - \phi\left(\frac{1}{2}\sqrt{\frac{\theta\kappa}{\alpha}}\right)\right].$$

As $\alpha \to 0$, the expression goes to zero (because the Mills ratio declines monotonically to zero). As $\alpha \to 1$, the expression above goes to ∞. Therefore, there must be a $\overline{\alpha}$ (potentially zero) such that if $\alpha \geq \overline{\alpha}$, then $\Delta(\Theta) - \Delta(0) > 0$. But

$$\alpha = 1 - \frac{\theta\kappa}{\gamma_c^2}.$$

Therefore

$$\alpha \geq \overline{\alpha} \iff \gamma_c^2 \geq \frac{\theta\kappa}{1-\overline{\alpha}} \equiv \overline{\gamma}_c^2.$$

10.9 Proof of Lemma 8

Using the logarithmic approximation,

$$\begin{aligned}
\overline{\Pi}e^{r_t} &= \left(\overline{P}e^{p_t} - \overline{\Xi}e^{\xi_t}\right)\overline{P}^{-\theta}e^{-\theta p_t}\mathbb{E}\left[e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right] \\
\Longleftrightarrow \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta} e^{r_t} &= \left(e^{p_t} - \frac{\theta-1}{\theta}e^{\xi_t}\right)\left(\frac{\theta}{\theta-1}\right)^{1-\theta} e^{-\theta p_t}\mathbb{E}\left[e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right] \\
\Longleftrightarrow e^{r_t} &= \left[\theta e^{p_t} - (\theta-1)\,e^{\xi_t}\right]e^{-\theta p_t}\mathbb{E}\left[e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right] \\
\Longleftrightarrow r_t &= \ln\left[\theta e^{p_t} - (\theta-1)\,e^{\xi_t}\right] - \theta p_t + \ln\left(\mathbb{E}\left[e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right]\right).
\end{aligned}$$

The standard second-order approximation of $\ln\left[\theta e^{p_t} - (\theta-1)\,e^{\xi_t}\right]$ around $p_t = 0$ and $\xi_t = 0$ yields

$$\ln\left[\theta e^{p_t} - (\theta-1)\,e^{\xi_t}\right] \approx \theta p_t - (\theta-1)\,\xi_t - \frac{\theta(\theta-1)}{2}(p_t - \xi_t)^2.$$

Moreover,

$$\ln\left(\mathbb{E}\left[e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right]\right) = \begin{cases} \frac{1}{2}\left(\alpha\gamma_\epsilon\right)^2, & \text{if } p_t \neq p_{t-1} \\ \alpha\gamma_\epsilon\epsilon_{t-1}, & \text{if } p_t = p_{t-1} \end{cases}.$$

Plugging into r_t yields the result.