The Origins and Propagation of Animal Spirits Shocks

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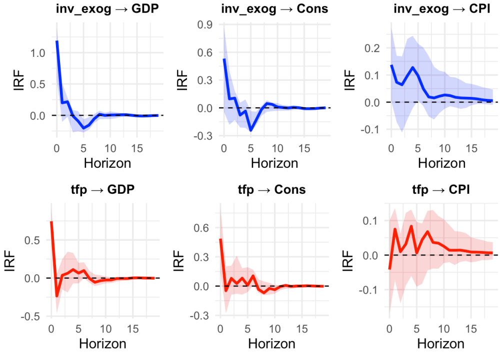
13th Annual CIGS conference on Macroeconomic Theory and Policy

June 17, 2025

What this paper does

- Develop a business cycle model with:
  - Comovement of investment, consumption, and inflation
  - No exogenous aggregate shocks such as:
    - Aggregate TFP shock
    - Monetary policy shock
- The model features "investment avalanches."
  - State-dependent synchronization of firms' investments
  - with underlying idiosyncratic productivity shocks
    - No common shocks assumed
- Motivation
  - Animal spirits
    - Investments fluctuate without apparent shocks.
    - We provide microfoundation.
  - Procyclical inflation
    - in contrast to productivity shocks

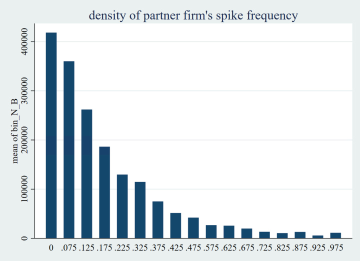
Comovement of investment, consumption, and inflation



- Structural VAR for US 1947Q2-2024Q4
- TFP (Fernald 2014; utilization-adjusted for non-equipment production)
- Investment shocks orthogonalized to lags and predicted TFP
- Cholesky identification in the order of (inv-exog, tfp, GDP, C, CPI)
- Quarterly, one st.d. shock (3.72% for inv), responses annualized %

Lumpy investments

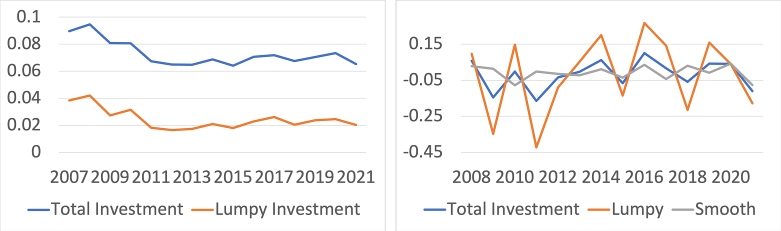
- Investment rate (gross investment/capital) $> \lambda - 1$
- Typical histogram:



(Annual data for Japanese firms, provided by Tokyo Shoko Research Ltd.; Nirei 2024)

Lumpy investments in business cycles

- Lumpy investments account for one-third of aggregate investments
- Lumpy investments account for most of the time-variation of aggregate investments
  - Cooper and Haltiwanger 1996; Doms and Dunne 1998; Gourio and Kashyap 2007



Left: ratio to GDP. Right: growth rate.
Source: Basic Survey of Japanese Business Structure and Activities; Nirei 2024

Literature

- Investment shocks
  - Fisher 2006; Justiniano, Primiceri, and Tambalotti 2010
  - Financial imperfections (Christiano, Motto, and Rostagno 2014)
  - Animal spirits and higher-order belief (Angeletos and La'O 2013)
  - We generate inflationary investment shocks without exogenous aggregate shocks.
- Sectoral business cycle models feature realistic technological shocks, but they aggregate too fast
- (S,s) models feature non-linear dynamics
  - Irrelevance in continuum-of-firms models (Caplin and Spulber 1987; Caballero and Engel 1991; Thomas 2002; Khan and Thomas 2008)
  - Reevaluations (Auclert, Rognlie, and Straub 2020; Koby and Wolf 2020; Winberry 2021)
- Sunspots / multiple equilibria
  - Galí 1994; Brock and Durlauf 2001; Wang and Wen 2008
  - Our model features a determinate, locally unique equilibrium.
- Fat tails
  - Granular effects (Gabaix 2011; Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi 2012)
  - Stochastic synchronization (Scheinkman and Woodford 1994)

Model

- Final goods wholesalers with sticky pricing
  - Aggregate output Y_t and price P_t
- Intermediate goods producers
  - Aggregate output Y_t^m and price P_t^m
  - Differentiated intermediate good y_{it}^m, $i = 1, \ldots, n$
  - Each firm invests and owns lumpy capitals
- Representative households
  - consume final goods, supply labor, hold bonds, and own firms
- Taylor-rule policy for nominal interest rate
- Markets clear
  - Final goods
  - Intermediate goods
  - Labor
  - Net zero supply of risk-free bonds

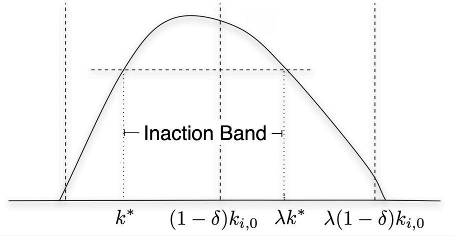
Intermediate producer $i = 1, 2, \ldots, n$

- Production function: $y^m_{it} = a_{it} k^{\alpha}_{it} l^{1-\alpha}_{it}$
- Demand function: $y^m_{it} = \left(\frac{p^m_{it}}{P^m_t}\right)^{-\eta} Y_t$
  - $P^m_t = \left(\sum_{i=1}^n (p^m_{it})^{1-\eta}/n\right)^{\frac{1}{1-\eta}}, \quad \eta > 1$
  - $m_t := P^m_t / P_t$: real intermediate price
- Firm i's real value: $\max \mathbb{E}\left[\sum_t \Lambda_t(\mu(a_{it}, k_{it}) - x_{it})\right]$ where
  - $\mu(a_{it}, k_{it}) := \max_{l_{it}} \frac{p^m_{it} y^m_{it}}{P_t} - w_t l_{it}$
  - $x_{it} := k_{i,t+1} - (1-\delta) k_{it}$
  - $\Lambda_t := \beta^t u'(c_t)$: stochastic discount factor
- Lumpy capital $k_{i,t+1} \in \{(1-\delta) k_{it} \lambda^s\}_{s=0,\pm 1}$
  - Lumpiness parameter $\lambda > 1/(1-\delta)$
  - Possible to endogenize λ by fixed adjustment costs

Lumpy investment of intermediate producers

- Operating surplus $\mu(a_{it}, k_{it}; w_t, m_t, K_t)$
- Lower threshold k^*_{it} for investment spikes
- Indifference between k^* and λk^*:

$$
\begin{aligned}
&\mathbb{E}_t\left[\Lambda_{t+1}(\mu_{t+1}(a, k^*) + (1-\delta)k^*) - \Lambda_t k^*\right] \\
=&\mathbb{E}_t\left[\Lambda_{t+1}(\mu_{t+1}(a, \lambda k^*) + (1-\delta)\lambda k^*) - \Lambda_t \lambda k^*\right].
\end{aligned}
$$



Threshold policy for lumpy investment

- Investment spikes in t if $(1-\delta)k_{i,t} < k^*_{i,t+1}$
- $K_t := \left(\sum_{i=1}^n (a_{it}^{1/\alpha} k_{it})^\rho / n\right)^{1/\rho}$
  - $\rho := \frac{(1-1/\eta)\alpha}{1-(1-1/\eta)(1-\alpha)} \in (0,1)$
- Threshold is $k^*_{i,t+1} = \tilde{a}_{i,t+1}\Phi_t K_{t+1}$, where $\tilde{a}_{i,t} := a_{i,t}^{\eta-1}$ and

$$\Phi_t := \left(\frac{\kappa(\lambda^\rho - 1)}{\lambda - 1} \frac{\mathbb{E}_t\left[\Lambda_{t+1} m_{t+1}^{1/\alpha} w_{t+1}^{1-1/\alpha}\right]}{\mathbb{E}_t[\Lambda_t - \Lambda_{t+1}(1-\delta)]}\right)^{\frac{1}{1-\rho}}$$

  - $\kappa := \left(\left(1 - \frac{\eta-1}{\eta}(1-\alpha)\right)^\alpha \left(\frac{\eta-1}{\eta}(1-\alpha)\right)^{1-\alpha}\right)^{1/\alpha}$
- Normalized capital: $s_{it} := \frac{\log k_{it} - \log k^*_{it}}{\log \lambda} \in [0,1)$
- State distribution: $F_t(a_{it}, s_{it})$

Dynamics of normalized capital profile $(s_{it})_i$

- Transition of $s_{it} = \frac{\log k_{it} - \log k_{it}^*}{\log \lambda} \in [0, 1)$
  - $s_{i,t+1} = \begin{cases} s', & \text{if } s' \geq 0 \\ s' + 1, & \text{otherwise} \end{cases}$
  - $s' = \frac{\log(1-\delta) + \log k_{it} - \log k_{i,t+1}^*}{\log \lambda} = \frac{\log(1-\delta) - \log k_{i,t+1}^* + \log k_{it}^*}{\log \lambda} + s_{it}$
- Assumption
  - (i) Productivity $a_{i,t}$ is i.i.d. with finite support $\mathcal{A}$ and $\max_{h,h'} |a(h) - a(h')| < \frac{-\log(1-\delta)}{\log \lambda}$
  - (ii) Initial normalized capital $s_{i,0}$ is uniformly distributed over $[0, 1)$ conditional on every value of $a_{i,0} \in \mathcal{A}$.
- s_{it} stays in a uniform distribution $\rightarrow F(a, s)$ stationary
- Aggregating $k_{i,t+1}^* = \tilde{a}_{i,t+1} \Phi_t K_{t+1}$ across i gives

$$\Phi_t = \Phi =: \mathbb{E}^F[\tilde{a}\lambda^{\rho s}]^{-1/\rho}$$

Rest of the model: Sticky prices

- Wholesaler $j \in [0,1]$ purchases intermediate goods y^m_{jit}, produces $y_{jt} = Y^m_{jt}$ where $Y^m_{jt} = (\int (y^m_{jit})^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$, and sells y_{jt} at price p_{jt}
  - Wholesalers incur price-adjustment costs $(\psi_P/2)\pi_t^2$
  - Cost-subsidy τ_t^Y financed by lump-sum tax t_t^Y corrects monopoly distortion
  - Aggregate final goods: $Y_t = \left(\int y_{jt}^{\frac{\epsilon_c-1}{\epsilon_c}} dj\right)^{\frac{\epsilon_c}{\epsilon_c-1}}$
  - CPI: $P_t = \left(\int p_{jt}^{1-\epsilon_c} dj\right)^{\frac{1}{1-\epsilon_c}}$
  - Inflation rate: $\pi_t := P_t/P_{t-1} - 1$
  - Wholesale profits: $\Omega_t = \left(1 - m_t - \frac{\psi_P}{2}\pi_t^2\right) Y_t$
- New Keynesian Phillips curve is derived (in the first order) as:

$$\pi_t = \frac{\epsilon_c - 1}{\psi_P}(m_t - 1) + \beta \mathbb{E}_t[\pi_{t+1}]$$

  - Intermediate cost m_t and $\mathbb{E}_t[\pi_{t+1}]$ determine π_t.

Representative households' choice

- Preference: $\sum_{t=0}^{\infty} \beta^t (u(C_t) - v(N_t))$
  - $u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ and $v(N) = \chi_n \frac{N^{1+1/\psi}}{1+1/\psi}$
    - Choose χ_n so that $N_{ss} = 1$
- Saving choice: $u'(C_t) = \beta \mathbb{E}_t \left[\frac{R_t^N}{1+\pi_{t+1}} u'(C_{t+1}) \right]$
- Budget constraints:
  $P_t C_t + A_{t+1} = R_t^N A_t + P_t (w_t N_t + \Omega_t + \sum_{i=1}^{n} (\mu(a_{it}, k_{it}) - X_t)/n)$
  - Households own monopolistic intermediate firms and instruct discount factors $\Lambda_{t+\tau}$.
  - Aggregate investment $X_t = \sum_{i=1}^{n} x_{it}$
    - $x_{it} = k_{i,t+1} - (1-\delta) k_{it}$
  - Firms choose dividends Ω_t and $\sum_{i=1}^{n} \mu(a_{it}, k_{it}) - X_t$
    - exogenous to households

Closing the model

- Monetary policy
  - sets risk-free nominal rate at: $R_t^N = (1 + r_{ss})(1 + \pi_t)^{\phi}$
  - Taylor principle $\phi > 1$
- Sticky real wage: for a $g \in [0, 1]$,
  - $w_t = (w_t^f)^g (w_{ss})^{1-g}$
  - $w_t^f = v'(N_t)/u'(C_t)$
- Markets clear
  - $N_t = \sum_{i=1}^n l_{it}/n$
  - $Y_t = C_t + X_t + \frac{\psi_P}{2}\pi_t^2 Y_t$
  - $A_t = 0$ (zero net supply of risk-free asset)

Recursive equilibrium when $n \to \infty$: Irrelevance

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \quad \text{(Production)}$$

$$L_t = ((1 - 1/\eta)(1-\alpha)m_t/w_t)^{\frac{1}{\alpha}} K_t \quad \text{(Labor demand)}$$

$$w_t = (v'(L_t)/u'(C_t))^{g} w_{ss}^{1-g} \quad \text{(Labor supply)}$$

$$C_t + X_t = \left(1 - \frac{\psi_P}{2}\pi_t^2\right) Y_t \quad \text{(Goods market clearing)}$$

$$K_{t+1} = (1-\delta)K_t + A_X X_t \quad \text{(Capital accumulation)}$$

$$1 = \mathbb{E}_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \frac{(1+r_{ss})(1+\pi_t)^{\phi}}{1+\pi_{t+1}}\right] \quad \text{(EE and Taylor)}$$

$$1 = \mathbb{E}_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \left(\tilde{\kappa} m_{t+1}^{1/\alpha} w_{t+1}^{1-1/\alpha} + 1 - \delta\right)\right] \quad \text{(Factor prices)}$$

$$\pi_t(1+\pi_t) = \frac{\epsilon_c - 1}{\psi_P}(m_t - 1) + \beta\mathbb{E}_t\left[\frac{u'(C_{t+1})}{u'(C_t)} \frac{Y_{t+1}}{Y_t} \pi_{t+1}(1+\pi_{t+1})\right] \quad \text{(Phillips curve)}$$

- These conditions determine an expected equiibrium path $K_{t+1}^e = \Xi(K_t)$

Investment avalanches

- Now, consider an economy with finite n firms.
- Rule-of-thumb: Agents expect $(a_{it}, s_{it})_{i=1}^{n}$ to follow F^n.
  $\Rightarrow k_{it}^* = \tilde{a}_{it}\Phi K_t$
- Investment in the finite economy is $X_t = (1 + \epsilon_t)X_t^e$,
  - Expected investment $X_t^e = (\Xi(K_t) - (1 - \delta)K_t)/A_X$.
  - Avalanche effect ϵ_t.
- Timing. $(K_t, X_t^e, (s_{it})_i)$ is predetermined.

$$(a_{it})_i \to \epsilon_t \to K_{t+1} \to C_t, L_t, w_t, m_t, \pi_t$$

- Investment avalanche
  - Idiosyncratic productivity shocks a_{it}
  - Normalized capital s_{it} is heterogeneous due to past productivity shocks
  - Aggregate variations caused by (a, s) is substantial (later).

Rule-of-thumb and the propagation of avalanche shocks

Expectation for $t \geq 2$ given K_2

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

$$L_t = ((1-1/\eta)(1-\alpha)m_t/w_t)^{\frac{1}{\alpha}} K_t$$

$$w_t = (v'(L_t)/u'(C_t))^g w_{ss}^{1-g}$$

$$Y_t = C_t + X_t^e$$

$$K_{t+1} = (1-\delta)K_t + A_X X_t^e$$

$$1 = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \frac{(1+r_{ss})(1+\pi_t)^{\phi}}{1+\pi_{t+1}} \right]$$

$$\pi_t = \frac{\epsilon_c - 1}{\psi_P}(m_t - 1) + \beta \mathbb{E}_t [\pi_{t+1}]$$

$$1 = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \left(\tilde{\kappa} m_{t+1}^{1/\alpha} w_{t+1}^{1-1/\alpha} + 1 - \delta \right) \right]$$

Realization after ϵ_1

$$Y_1 = K_1^{\alpha} L_1^{1-\alpha}$$

$$L_1 = ((1-1/\eta)(1-\alpha)m_1/w_1)^{\frac{1}{\alpha}} K_1$$

$$w_1 = (v'(L_1)/u'(C_1))^g w_{ss}^{1-g}$$

$$Y_1 = C_1 + X_1$$

$$K_2 = (1-\delta)K_1 + A_X X_1$$

$$1 = \mathbb{E}_t \left[\frac{\beta u'(C_2)}{u'(C_1)} \frac{(1+r_{ss})(1+\pi_1)^{\phi}}{1+\pi_2} \right]$$

$$\pi_1 = \frac{\epsilon_c - 1}{\psi_P}(m_1 - 1) + \beta \mathbb{E}_t [\pi_2]$$

$$X_1 = X_1^e + \epsilon_1$$

A four-equation model

- Special case of constant real wage ($g = 0$)
- Equilibrium conditions of a finite economy boil down to:

$$\frac{K_{ss}^{\alpha} L_{ss}^{1-\alpha}}{C_{ss}}(1-\alpha)\tilde{L}_t = \tilde{C}_t + \frac{X_{ss}}{C_{ss}}\epsilon_t$$

$$\tilde{m}_t = \alpha \tilde{L}_t$$

$$\pi_t = \frac{\epsilon - 1}{\psi_P}\tilde{m}_t + \beta \pi_{t+1}$$

$$\tilde{C}_{t+1} - \tilde{C}_t = \frac{1}{\sigma}\left(\phi \pi_t - \pi_{t+1}\right)$$

- Investment demand ϵ_t shifts out labor demand.
- An increase in labor reduces the marginal product of labor.
  - Capital is essential ($\alpha > 0$).
- Pushing up intermediate goods price m_t, given real wages

Impulse responses when $g > 0$

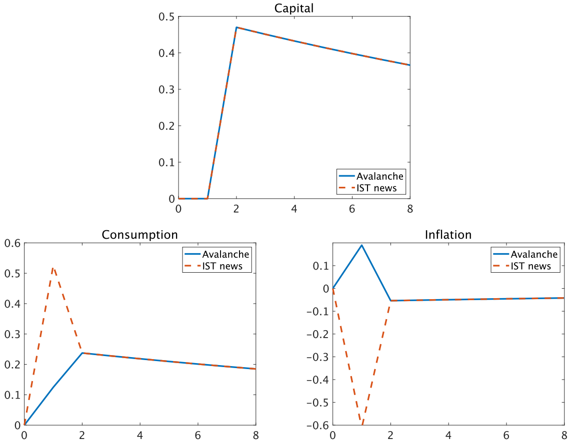


Figure: Annual IRFs for capital, consumption, and inflation rates.
X: years. Y: % deviations from steady states.

Time to build

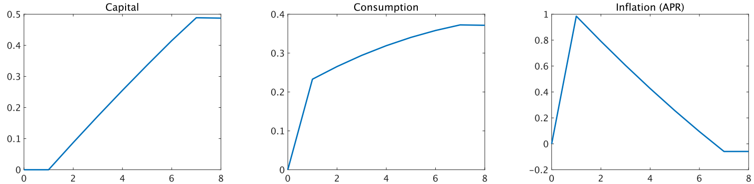


Figure: Quarterly IRFs for capital, consumption, and inflation rates (%) with six quarters time-to-build.

- Parameters calibrated at the quarterly frequency

α	β	δ	ψ	ϵ_c	η	σ	ψ_P	λ	ϕ
0.36	0.995	0.025	0.5	6	30	3	30	1.2	1.2

- g is set at 0.03 (0.12 at the annual frequency) so that the real wage volatility matches data (st.d. 0.73% in the model; 0.44% in NIPA weekly real earnings of wage and salary workers 1979Q1-2024Q3)

Procyclical inflation

- Negative inflation after a capital increase
  - Capital and consumption are above the steady-state at $t+1$ and decrease afterward.
  - Euler equation: $1 = \mathbb{E}_t \frac{\beta u'(C_{t+1})}{u'(C_t)} \frac{(1+r_{ss})(1+\pi_t)^{\phi}}{1+\pi_t}$
  - Taylor principle $\phi > 1 \Rightarrow$ future $\pi < 0$
- Positive inflation on the impact of an investment demand shock
  - Increased goods demand by investment shock.
  - Wealth effect of future capital raises consumption.
  - Sticky real wage raises marginal unit cost and intermediate price, leading to inflation.
- Policy analysis
  - If inflation is suppressed on impact, $C_1 < C_{ss}$ and $L_1 = L_{ss}$
  - Inflation helps intermediate production and facilitates higher consumption and hours worked

Contrasting with investment-specific technological (IST) shocks

- $K_{t+1} = (1-\delta)K_t + A_X X_t$
- A shock on A_X not known in $t \Rightarrow$ No change in t
- The case of "news shock": IST shock known in t
  $\Rightarrow$ leading to disinflation on impact

- News shock: $a_2^x > 0$ is known in $t = 1$

$$
\begin{aligned}
\tilde{C}_{t+1} - \tilde{C}_t &= \frac{1-\beta(1-\delta)}{\sigma}\Big((1-\alpha)(\tilde{L}_{t+1} - \tilde{K}_{t+1}) + \tilde{m}_{t+1}\Big) \\
\frac{K_{ss}^{\alpha} L_{ss}^{1-\alpha}}{C_{ss}}(1-\alpha)\tilde{L}_t &= \tilde{C}_t + \frac{X_{ss}}{C_{ss}}\tilde{X}_t \\
\delta(a_t^x + \tilde{X}_t) &= \tilde{K}_{t+1} \\
\tilde{m}_t &= \alpha \tilde{L}_t \\
\pi_t &= \frac{\epsilon - 1}{\psi_P}\tilde{m}_t + \beta\pi_{t+1} \\
\tilde{C}_{t+1} - \tilde{C}_t &= \frac{1}{\sigma}(\phi\pi_t - \pi_{t+1})
\end{aligned}
$$

- $\tilde{K}_2 > 0$, $\tilde{m}_2 < 0$, and $\tilde{L}_2 < 0$. Thus $\tilde{C}_2 < \tilde{C}_1$, leading to $\pi_1 < 0$.

Is ϵ_t large? Economy with n firms

- A finite number (n) of intermediate producers
  - Curse of dimensionality
  - Behavioral assumption: Agents expect $(a_{it}, s_{it})_{i=1}^{n}$ to follow F^n $\Rightarrow k_{it}^* = \tilde{a}_{it}\Phi K_t$
- Equilibrium of a model with n firms
  - Equilibrium condition:

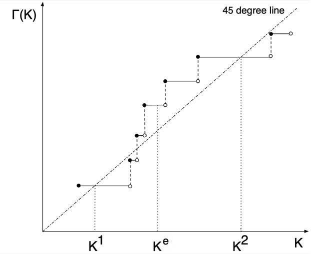
$$k_{it} \in [k_{it}^*, \lambda k_{it}^*), \quad \forall i = 1, 2, \ldots, n$$

  - Future expectation is given by $K^e = \Xi(K_{-1})$

Equilibrium selection

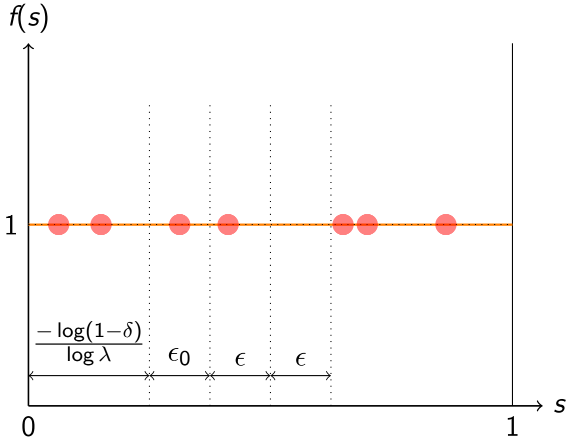
- Aggregate reaction function

$$\Gamma(K;\Phi) := \left(\sum_{i=1}^{n} \frac{(a_i^{1/\alpha} k_i)^{\rho}}{n}\right)^{1/\rho}, \quad k_i \in [k^*, \lambda k^*), \quad k^* = \tilde{a}_i \Phi K$$

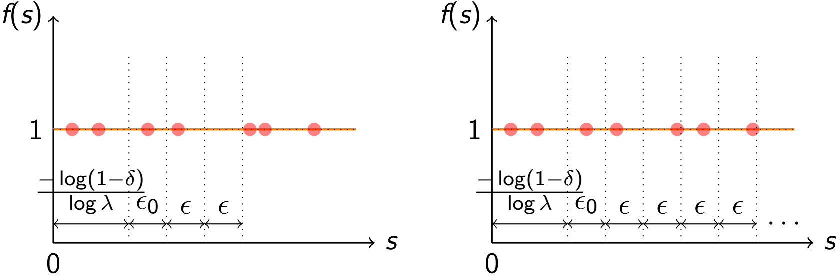


- Select K_t closest to $K^e = \Xi(K_{t-1})$
- A finite number of locally-unique equilibria K_t
  $\rightarrow$ Firms' decisions restrict output level.
  $\leftrightarrow$ K_t is indeterminat in a continum-of-firms economy.

Investment avalanche



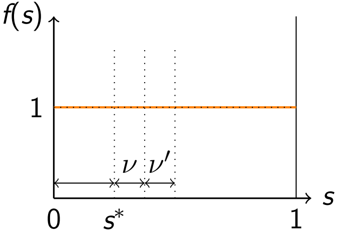
State-dependent multiplier effects



- State-dependent multiplier: Small changes in $(s_i)_i$ cause a large variation in the avalanche size.

Analysis: complementarity of investment spikes

- Perturbation experiment
  - Environment: a stationary economy with $n \to \infty$. Divide firms in H^2 groups where firms in group $h = (h_0, h_1)$ experience $a_{i,t} = a(h_0)$ and $a_{i,t+1} = a(h_1)$. Group-h firms have stationary measure $\omega(h)$.
  - Firms in h with $s_{i,t} \leq s^*(h) := \frac{-\log(1-\delta)+\Delta \log \tilde{a}(h)}{\log \lambda}$ invest.
  - Suppose firms in $[s^*(h), s^*(h) + \nu(h))$ additionally invest.
  - K_{t+1} increases, and $s_{i,t+1}$ shifts.
  - Firms of measure ν' hit s^*.
  - $\vartheta(h) := \lim_{\nu(h)\to 0} \frac{\nu'}{\omega(h)\nu(h)}$
  - $\vartheta := \sum_h \omega(h)\vartheta(h)$: Degree of complementarity



Complementarity of investment spikes

- Generalized model with $y_{jt}^{m} = a_{jt}(k_{jt}^{\alpha} l_{jt}^{1-\alpha})^{\theta}$, $\theta \leq 1$
- Proposition
  i. $\vartheta(h) = \frac{\tilde{a}(h_1)}{\mathbb{E}^F[\tilde{a}]}\tilde{\theta}$
     - $\tilde{\theta} = \frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}$

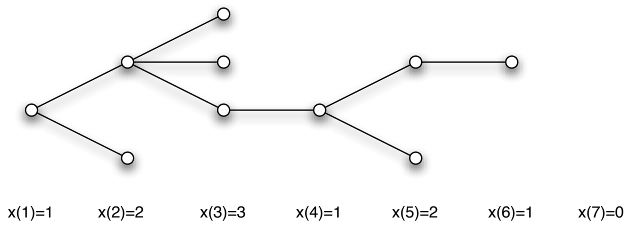
  ii. $\vartheta = \tilde{\theta}$

  iii. $\tilde{\theta} \nearrow 1$ as $\theta \nearrow 1$
- Implications
  - A firm's lumpy investment induces $\tilde{\theta}$ firms' lumpy investment on average.
  - $\tilde{\theta} = 1$ under constant returns to scale ($\theta = 1$).

Analytical results

- Investment avalanche: best response dynamics
  1. $s_i(0) \sim U[0,1)$
  2. $s_i(1) = s_i(0) + \frac{\log(1-\delta)-\Delta \log \tilde{a}_i}{\log \lambda}$
  3. $\mathcal{Z}_0 = \{i : s_i(1) < 0\}$. $s_i(1) \mapsto s_i(1) + 1$ for $i \in \mathcal{Z}_0$
  4. $K(0)$ is updated to $K(1)$. Stop if $K(1) = \Xi(K(0))$.
  5. For $u > 0$, $s_i(u+1) = s_i(u) - \frac{\tilde{\theta}(\log K(u) - \log K(u-1))}{\log \lambda}$
  6. $\mathcal{Z}_u = \{i : s_i(u+1) < 0\}$. $s_i(u+1) \mapsto s_i(u+1) + 1$ for $i \in \mathcal{Z}_u$
  7. $K(u)$ is updated to $K(u+1)$. Stop if $K(u+1) = K(u)$ and set $U = u$. Otherwise, $u \mapsto u+1$ and go to 5.

Branching process and avalanche distribution



- An avalanche size follows a Generalized Poisson distribution
  - Power-law tail with exponential truncation
  - Nirei 2015; Nirei and Scheinkman 2024
- The probability of the avalanche size being infinite is 0 if and only if the mean number of children (ϑ) is less than or equal to 1

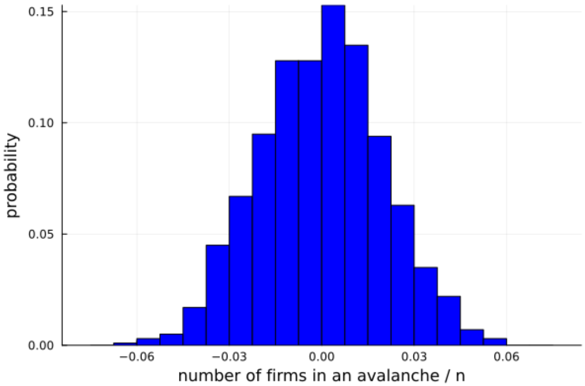
Fat tail of multiplier effects

- Analytical results
  - Let $z_u = |\mathcal{Z}_u|$ and $L = \sum_{u=1}^{U} z_u$: avalanche size
  - As $n \to \infty$, L converges in total variation to a sum of the Poisson branching process
  - $\Pr(L = \ell \mid z_1 = 1) \sim e^{-(\vartheta - 1 - \log\vartheta)\ell} \ell^{-1.5}$
  - $\mathbb{E}[(L/n)^2 \mid z_1 = 1] \propto \int^n (L/n)^2 L^{-1.5} dM \sim 1/\sqrt{n}$, for $\vartheta = 1$
  - $\mathbb{E}[z_1] \propto \sqrt{n}$
- Criticality at $\vartheta = 1$
  - Power-law tail of L if $\vartheta = 1$
  - Exponential tail if $\vartheta < 1$
  - Explosive if $\vartheta > 1$
- Constant returns to scale, indivisible capital, and business cycles

Numerical simulations

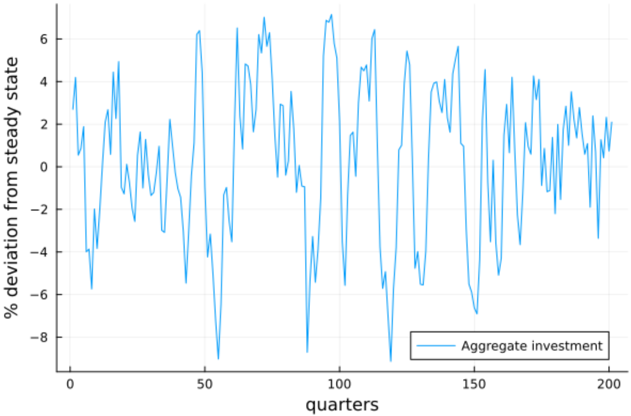
- Idiosyncratic productivity follows an AR(1):
  $\log a_{i,t} = 0.9 \log a_{i,t-1} + 0.03\epsilon_{i,t}$
- Number of firms $n = 30000$
- 6 quarters of time-to-build

Distribution of the avalanche size



- Negative values are "retracted" investment decisions.

Simulated time-series of aggregate investments



Cost of rule-of-thumb

- Optimal rule: $k^*_{i,t+1} = \tilde{a}_{i,t+1}\Phi_t K_{t+1}$ and

$$\Phi_t = \left(\frac{\kappa(\lambda^\rho - 1)}{\lambda - 1} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{m_{t+1}}{w_{t+1}^{1-\alpha}} \right)^{\frac{1}{\alpha}} \right] \mathbb{E}_t \left[1 - \frac{\Lambda_{t+1}}{\Lambda_t}(1-\delta) \right]^{-1} \right)^{\frac{1}{1-\rho}}$$

- Cost of not updating Φ_t is 0.12% of operating surplus for the firm at the extensive margin.
- In aggregation, households lose 0.84% of dividend revenues.
- Deviations from $F(a, s)$ are small
  - $\mathbb{E}[s^k_{it}]$ for $k = 1, 2, 3, 4$ and $t = 1, \ldots, 1000$
  - Average deviations are less than 0.02% of population moments
  - Maximum deviations are less than 2.5% of population moments
  - Correlation coefficient of (a_{it}, s_{it}) is 0.14%

Discussions

- Aggregate shocks can be incorporated. They will shift X_t^e
- Rule-of-thumb
  - Constant returns to scale with $n \to \infty$ means constant Φ_t
  - Timing of investment decisions and purchases
- Do firms internalize shifts in SDF caused by avalanche shocks?
  - Financial imperfections
  - Interest-elasticity of investment
  - Dampened general equilibrium effects
- Investment avalanches via production networks

Network propagation of lumpy investments

Linear-probability model: lumpy investments regressed on lumpy investments of their trading partners give positive coefficient

	(1)		(2)		(3)	
Customer	.016***	(.0044)	.015***	(.0047)	.020***	(.0047)
Customer (lag)			.006	(.0046)	.015***	(.0046)
Supplier	.011**	(.0046)	.013***	(.0049)	.029***	(.0050)
Supplier (lag)			.010**	(.0048)	.024***	(.0049)
Profit (lag)	.0003	(.00018)	.0003	(.0002)	.0008***	(.0002)
Liquidity (lag)	.0003***	(.00004)	.0002***	(.0000)	.0004***	(.0000)
Year FE	✓		✓		✓	
Firm FE	✓		✓			
Industry FE					✓	
N firms	23,224		22,326		22,326	
N obs	205,052		194,366		194,366	

Table 4: Estimates of the linear probability model of a firm's investment spike.

(BSJBSA; Nirei 2024)

Logit estimates with TSR sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Customer	1.190***	1.176***	1.190***	1.176***	1.191***	1.176***	1.194***
	(.0289)	(.0301)	(.0289)	(.0301)	(.0290)	(.0302)	(.0339)
Customer (lag)		1.074**		1.074**		1.074**	1.074*
		(.0278)		(.0278)		(.0278)	(.0308)
Supplier	1.162***	1.143***	1.162***	1.143***	1.161***	1.143***	1.148***
	(.0309)	(.0318)	(.0309)	(.0318)	(.0308)	(.0318)	(.0347)
Supplier (lag)		1.106***		1.106***		1.104***	1.100**
		(.0309)		(.0309)		(.0309)	(.0334)
Profit (lag)	1.000***	1.000***	1.000***	1.000***	1.000***	1.000***	1.000***
	(1.7e-5)	(1.7e-5)	(1.7e-5)	(1.7e-5)	(1.7e-5)	(1.7e-5)	(2.0e-5)
Asset (lag)	1.000***	1.000***	1.000***	1.000***	1.000***	1.000***	1.000***
	(1.3e-9)	(1.3e-9)	(1.3e-9)	(1.3e-9)	(1.4e-9)	(1.4e-9)	(2.0e-9)
Liquidity (lag)			1.000	1.000	1.000	1.000	
			(7.0e-5)	(7.3e-5)	(7.0e-5)	(7.3e-5)	
Liquidity (lag, censored)							2.364***
							(.0608)
N Suppliers					1.004***	1.004***	1.006***
					(6.1e-4)	(6.0e-4)	(8.5e-4)
N Customers					0.999	0.999	.999
					(4.9e-4)	(4.9e-4)	(5.9e-4)
Year FE	✓	✓	✓	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓	✓	✓	✓
N firms	23226	22440	23226	22440	23226	22440	20195
N obs	309477	294857	309477	294857	309477	294857	247155

Table 7: Logit estimates (odds ratio) of a firm's investment spike: Balanced TSR sample.

Conclusion

- Lumpy investments bring about investment avalanches under constant returns to scale production technology.
- This provides microfoundation for animal spirits, or aggregate investment demand shocks.
- Investment demand shocks account for procyclical inflation under sticky price and real wages.
- Time-to-build generates autocorrelation in investments and consumption.
- Future research on investment propagations over production networks.