Heterogeneity and Aggregate Fluctuations: Insights from TANK Models

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Background

- The representative household paradigm
- The HANK revolt: focus on household heterogeneity, in the form of *idiosyncratic income shocks*

  $\Rightarrow$ complexity: distribution of wealth as additional state variable
  $\Rightarrow$ numerical methods, no simple intuition

- Is heterogeneity important for understanding *aggregate* fluctuations?
- Can a simple tractable model approximate well the *aggregate* properties of a HANK model?

Present Paper

- Analysis of aggregate properties of HANK models
- Focus on distinction between *unconstrained* and *hand-to-mouth* households
- Three versions of HANK:

I: No binding borrowing constraints + illiquid stocks + profit allocation rule
II: Binding borrowing constraints + illiquid stocks + profit allocation rule
III: Binding borrowing constraints + optimal portfolio choice

- For each HANK model we propose a tractable counterpart drawn from the TANK family, and assess its merits as an approximation
- Focus on monetary policy and technology shocks.

Preview of Findings

- Is heterogeneity important for understanding *aggregate* fluctuations?

- Potentially yes, but possibly less than you may have thought (and not independently of the monetary policy rule in place)

- Can a simple tractable model approximate well the *aggregate* properties of a HANK model?

- Yes, but it must be properly designed and calibrated (an off-the-shelf TANK model won't generally do).

A Baseline HANK Model

Households

- Continuum of infinite-lived households, $j \in [0,1]$, with preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C(j)^{1-\sigma} - 1}{1-\sigma} - \frac{\mathcal{N}(j)^{1+\varphi}}{1+\varphi} \right)$$

- Period budget constraint

$$C_t(j) + B_t(j) = R_{t-1} B_{t-1}(j) + \Xi_t(j) W_t \mathcal{N}_t(j) + F_t(j)$$

where $\Xi_t(j) \equiv \exp\{\zeta_t(j)\}$ with $\zeta_t(j) = \rho_\zeta \zeta_{t-1}(j) + \varepsilon_t^\zeta(j)$ and $\int_0^1 \Xi_t(j) dj = 1$

- Borrowing constraint

$$R_t B_t(j) \geq -\Psi_t(j)$$

A Baseline HANK Model

Firms

- Monopolistic competition + quadratic price adjustment costs
- Implied New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa(y_t - y_t^n)$$

where $y_t^n \equiv \frac{1+\varphi}{\sigma+\varphi} a_t$

$\Rightarrow$ *supply block invariant to heterogeneity*

Monetary Policy

- Exogenous real interest rate

HANK-I

- Assumption #1: *natural* borrowing limit

  $\Rightarrow$ borrowing constraint *not* binding in equilibrium

- Assumption #2: illiquid stocks + profit allocation rule

$$F_t(j) \equiv \Theta_t(j) D_t$$

where $\Theta_t(j) = [\vartheta + (1-\vartheta)\Xi_t(j)]$

- For all t and $j \in [0,1]$:

$$1 = \beta R_t \mathbb{E}_t\{(C_{t+1}(j)/C_t(j))^{-\sigma}\}$$

- Aggregate consumption (Debortoli-Galí 2024):

$$\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t - \frac{\sigma+1}{2}\widehat{v}_t$$

where

$$v_t \equiv \int \frac{C_t(j)}{C_t} v_t(j) dj$$

and $v_t(j) \equiv var_t\{c_{t+1}(j)\}$ ("individual consumption risk")

RANK: A Tractable Counterpart to HANK-I

- Aggregate consumption (RANK)

$$\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t$$

- Aggregate consumption (HANK-I):

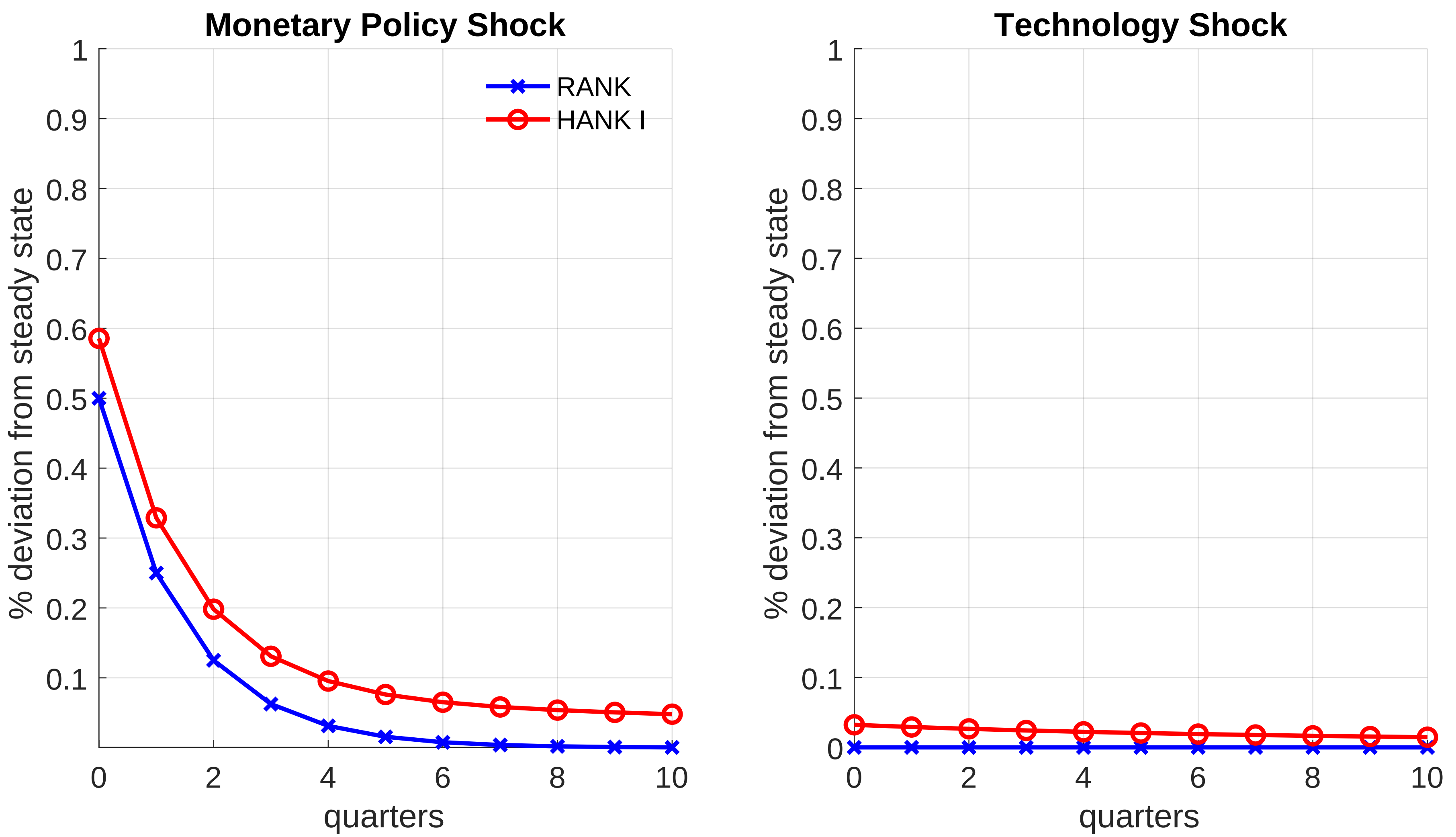
$$\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t - \frac{\sigma+1}{2}\widehat{v}_t$$

where

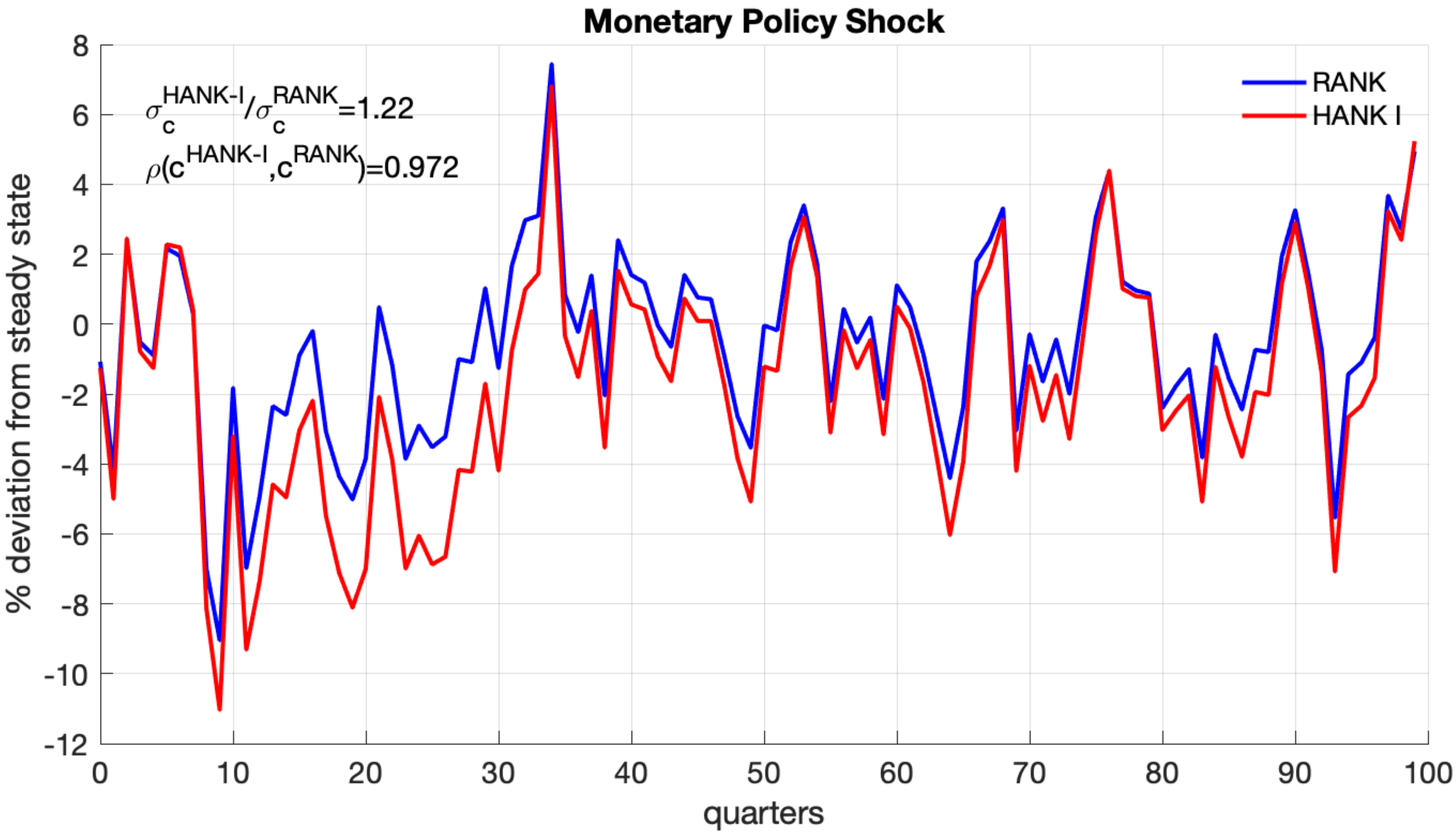
$$v_t \equiv \int \frac{C_t(j)}{C_t} v_t(j) dj$$

and $v_t(j) \equiv var_t\{c_{t+1}(j)\}$ ("individual consumption risk")

- Calibration and quantitative analysis



Monetary Policy Shock
Technology Shock
RANK
HANK I
% deviation from steady state
quarters
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
4
6
8
10

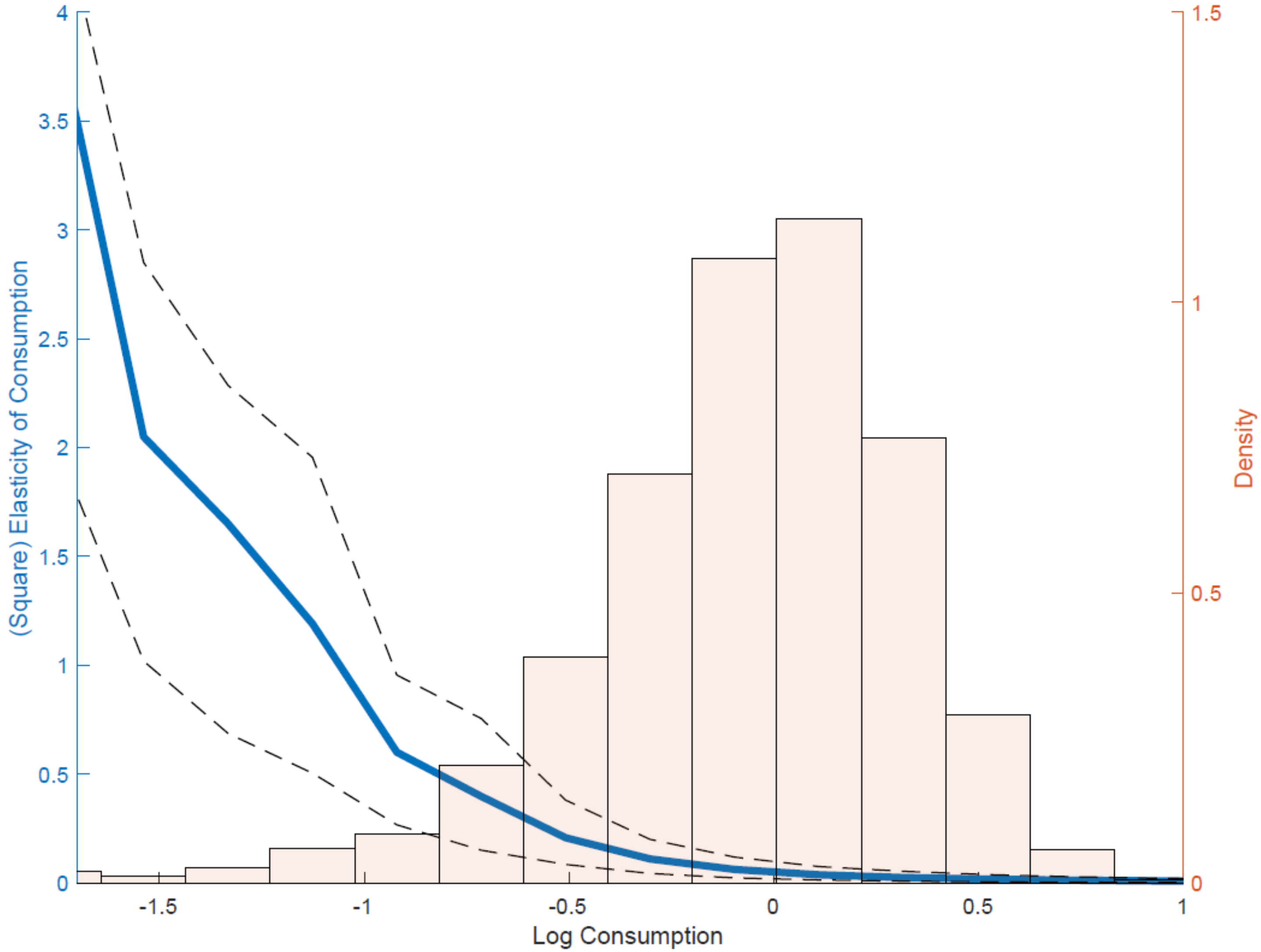


Monetary Policy Shock
$\sigma_c^{HANK-I}/\sigma_c^{RANK}$=1.22
$\rho(c^{HANK-I},c^{RANK})$=0.972
RANK
HANK I
% deviation from steady state
quarters
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0
10
20
30
40
50
60
70
80
90
100

HANK-I vs RANK

- Quantitative analysis: small impact of heterogeneity
- Changes in consumption risk are concentrated in low consumption households

  $\Rightarrow$ small aggregate impact



(Square) Elasticity of Consumption
Density
Log Consumption
4
3.5
3
2.5
2
1.5
1
0.5
0
1.5
1
0.5
0
-1.5
-1
-0.5
0
0.5
1

HANK-I vs RANK

- Quantitative analysis: small impact of heterogeneity
- Changes in consumption risk are concentrated in low consumption households

  $\Rightarrow$ small aggregate impact

- Implication: small role of idiosyncratic income risk *by itself.*

Does it have a larger role in the presence of *occasionally binding borrowing constraints?*

HANK-II

- Borrowing limit

$$\Psi_t(j) = \psi Y$$

- In any given period, two subsets of households: unconstrained and hand-to-mouth
- Unconstrained households (measure $1 - \lambda_t^H$):

$$1 = \beta R_t \mathbb{E}_t\{(C_{t+1}(j)/C_t(j))^{-\sigma}\}$$

Average consumption:

$$\widehat{c}_t^U = \mathbb{E}_t\{\widehat{c}_{t+1}^U\} - \frac{1}{\sigma}\widehat{r}_t - \frac{\sigma+1}{2}\widehat{v}_t^U - \widehat{h}_t^U$$

HANK-II

- Hand-to-mouth households (measure λ_t^H):

$$C_t(j) = \Xi_t(j) W_t N_t + \Theta_t(j) D_t + R_{t-1} B_{t-1}(j) + \frac{\psi Y}{R_t}$$

where $\Theta_t(j) \equiv \vartheta + (1-\vartheta)\Xi_t(j)$

Average consumption:

$$C_t^H = \left[\Xi_t^H \frac{\mathcal{M}}{\mathcal{M}_t} + \Theta_t^H \left(1 - \frac{\mathcal{M}}{\mathcal{M}_t} \right) \right] Y_t - \psi Y (\widehat{R}_t + \Omega_{t-1}^H - 1)$$

where $\widehat{R}_t \equiv \frac{R_t - 1}{R_t}$ and $\Omega_{t-1}^H \equiv -R_{t-1} B_{t-1|t}^H / \psi Y$

- Two channels absent from RANK:
  - "interest rate exposure": $\frac{\partial C_t^H}{\partial R_t} < 0$
  - "income distribution": $\frac{\partial C_t^H}{\partial \mathcal{M}_t} > 0$, given that in equilibrium $\Xi_t^H < \Theta_t^H < 1$

TANK-II: A Tractable Counterpart to HANK-II

- Unconstrained households: identical, constant measure $1 - \lambda^H$

$$\widehat{c}_t^U = \mathbb{E}_t\{\widehat{c}_{t+1}^U\} - \frac{1}{\sigma}\widehat{r}_t$$

- Hand-to-mouth households: identical, constant measure λ^H

$$C_t^H = \Xi^H W_t N_t + \Theta^H D_t - \psi Y \ \widehat{R}_t$$

where $\Theta^H \equiv \vartheta + (1-\vartheta)\Xi^H$

- Differences with standard TANK: $\Xi^H < 1$, $\Theta^H > 0$, $\psi > 0$
- Aggregate consumption (TANK-II):

$$\begin{aligned} C_t &= \lambda^H C_t^H + (1-\lambda^H) C_t^U \\ &= \lambda^H \left[\Xi^H \frac{\mathcal{M}}{\mathcal{M}_t} + \Theta^H \left(1 - \frac{\mathcal{M}}{\mathcal{M}_t}\right)\right] Y_t - \lambda^H \psi Y \ \widehat{R}_t + (1-\lambda^H) C_t^U \end{aligned}$$

TANK-II: A Tractable Counterpart to HANK-II

- Aggregate consumption (TANK-II):

$$C_t = \lambda^H \left[\Xi^H \frac{\mathcal{M}}{\mathcal{M}_t} + \Theta^H \left(1 - \frac{\mathcal{M}}{\mathcal{M}_t} \right) \right] Y_t - \lambda^H \psi Y \ \widehat{R}_t + (1 - \lambda^H) C_t^U$$

- Aggregate consumption (HANK-II):

$$C_t = \lambda_t^H \left[\Xi_t^H \frac{\mathcal{M}}{\mathcal{M}_t} + \Theta_t^H \left(1 - \frac{\mathcal{M}}{\mathcal{M}_t} \right) \right] Y_t - \lambda_t^H \psi Y (\widehat{R}_t + \Omega_{t-1}^H - 1) + (1 - \lambda_t^H) C_t^U$$

- Good approximation if $\lambda_t^H \simeq \lambda^H$, $\Omega_{t-1}^H \simeq 1$, $\Xi_t^H \simeq \Xi^H$, $\Theta_t^H \simeq \Theta^H$, $\widehat{v}_t^U \simeq 0$

TANK-II: A Tractable Counterpart to HANK-II

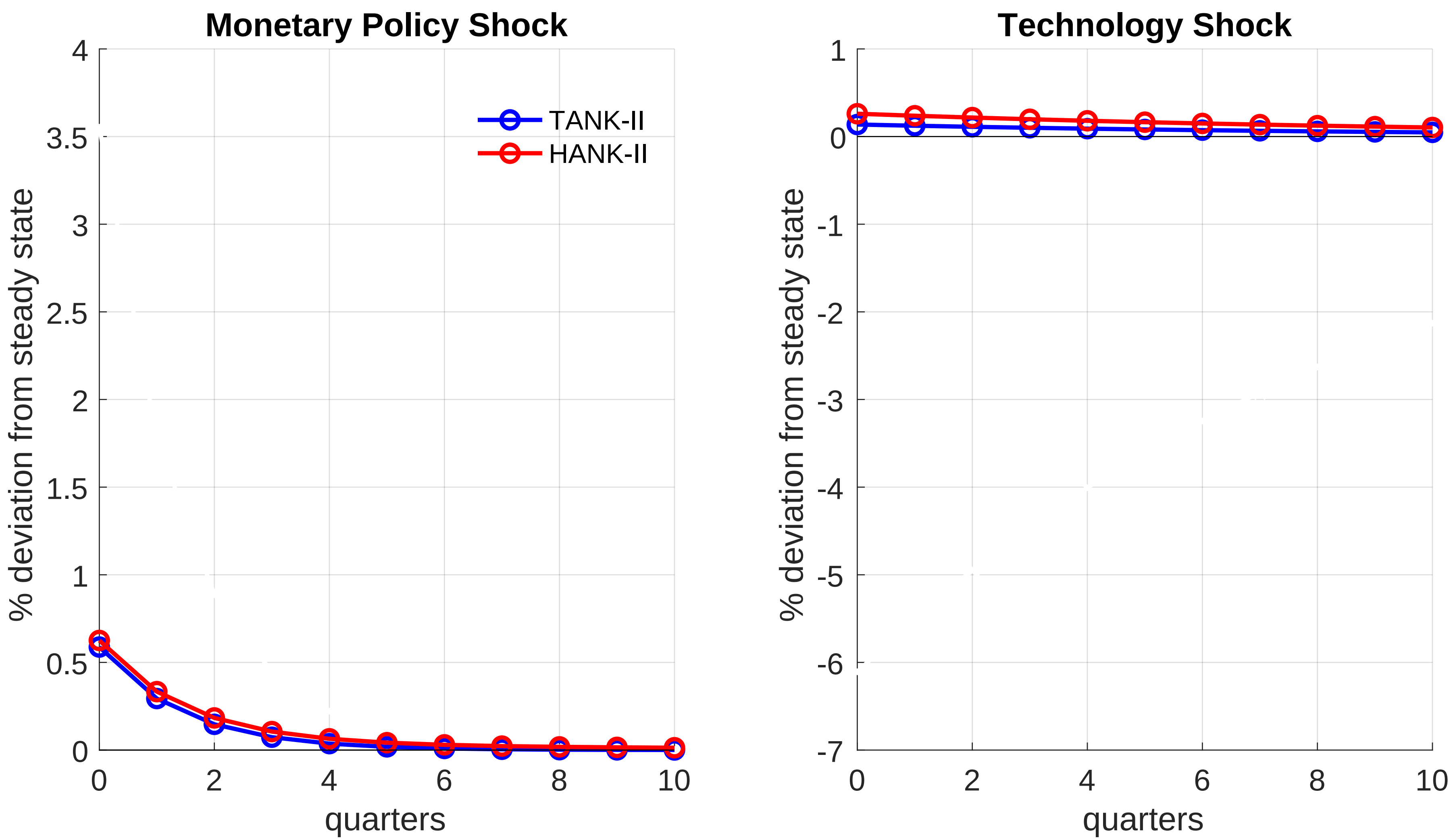
- Aggregate consumption (TANK-II):

$$C_t = \lambda^H \left[\Xi^H \frac{\mathcal{M}}{\mathcal{M}_t} + \Theta^H \left(1 - \frac{\mathcal{M}}{\mathcal{M}_t} \right) \right] Y_t - \lambda^H \psi Y \widehat{R}_t + (1 - \lambda^H) C_t^U$$

- Aggregate consumption (HANK-II):

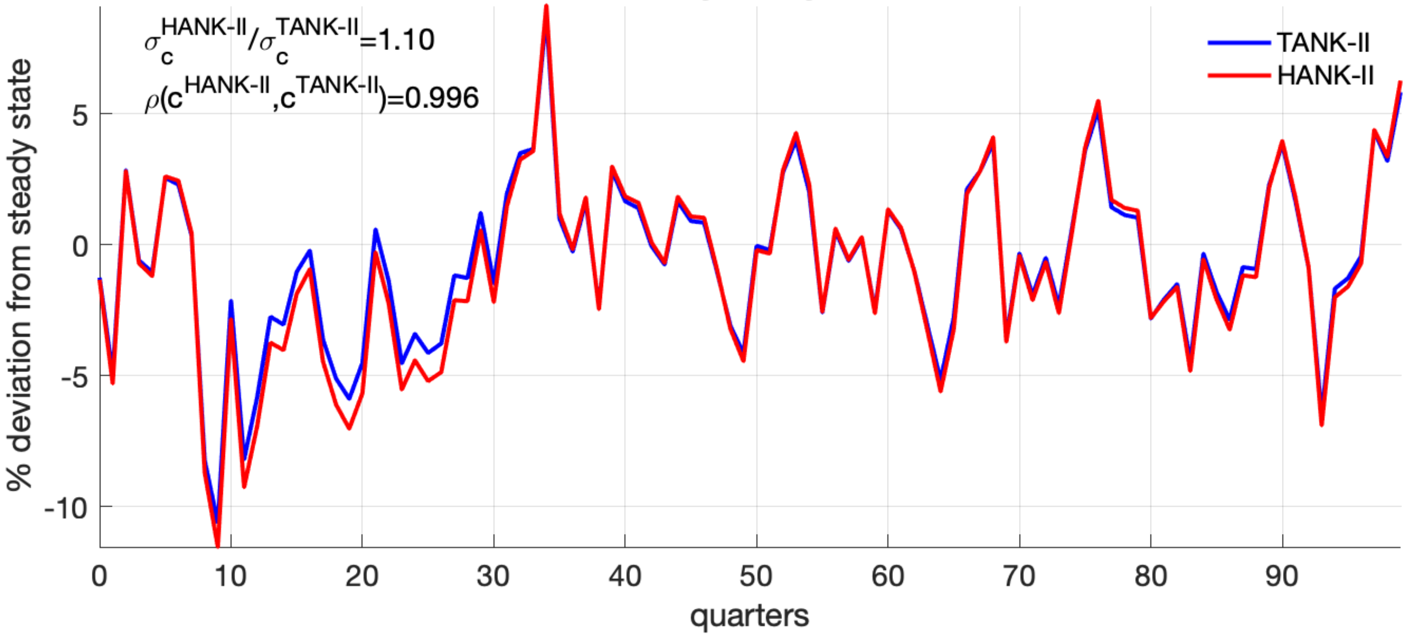
$$C_t = \lambda_t^H \left[\Xi_t^H \frac{\mathcal{M}}{\mathcal{M}_t} + \Theta_t^H \left(1 - \frac{\mathcal{M}}{\mathcal{M}_t} \right) \right] Y_t - \lambda_t^H \psi Y (\widehat{R}_t + \Omega_{t-1}^H - 1) + (1 - \lambda_t^H) C_t^U$$

- Good approximation if $\lambda_t^H \simeq \lambda^H$, $\Omega_{t-1|t} \simeq 1$, $\Xi_t^H \simeq \Xi^H$, $\Theta_t^H \simeq \Theta^H$, $\widehat{v}_t^U \simeq 0$
- Calibration: ψ so that $\mathbb{E}\{\lambda_t^H\} = 0.30$, $\lambda^H = \mathbb{E}\{\lambda_t^H\}$, $\Xi^H = \mathbb{E}\{\Xi_t^H\}$, $\Theta^H = \mathbb{E}\{\Theta_t^H\}$
- Quantitative analysis

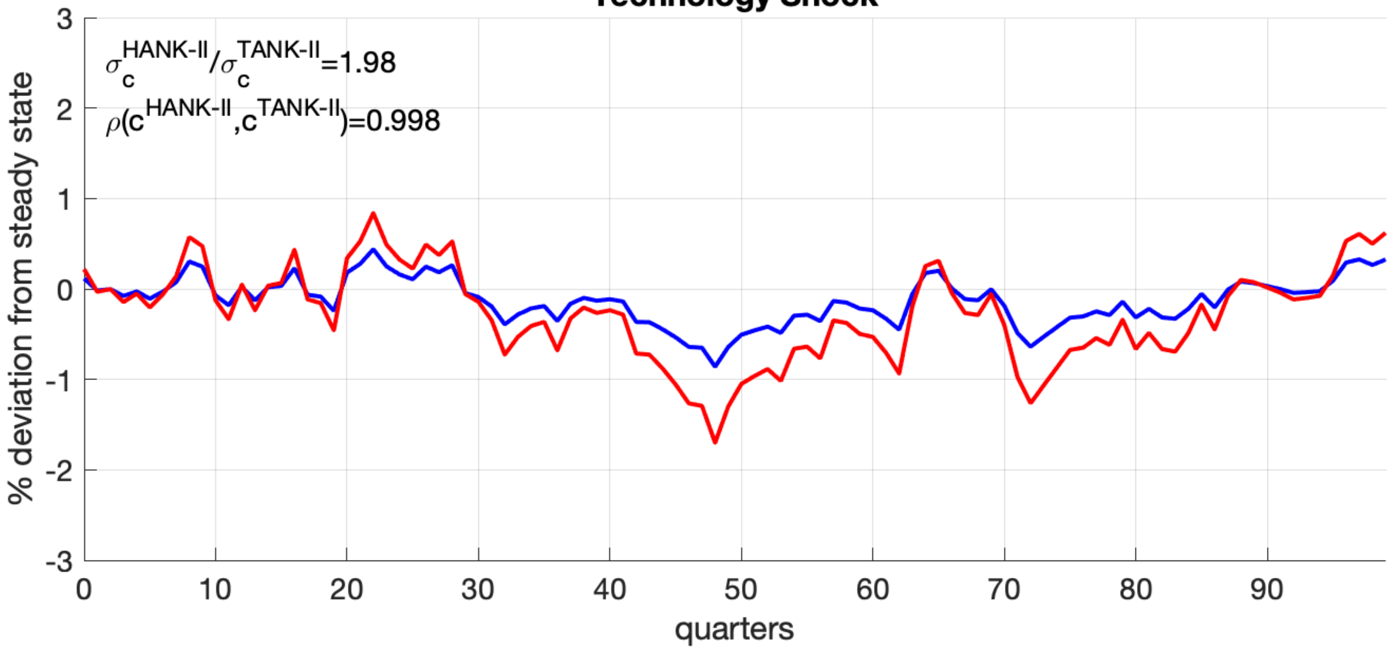


Monetary Policy Shock
Technology Shock
TANK-II
HANK-II
% deviation from steady state
quarters
4
3.5
3
2.5
2
1.5
1
0.5
0
1
-1
-2
-3
-4
-5
-6
-7
0
2
4
6
8
10

Monetary Policy Shock



Technology Shock



TANK-II: A Tractable Counterpart to HANK-II

- By contrast, in the standard TANK model (TANK-I)

$$C_t^H = W_t N_t$$

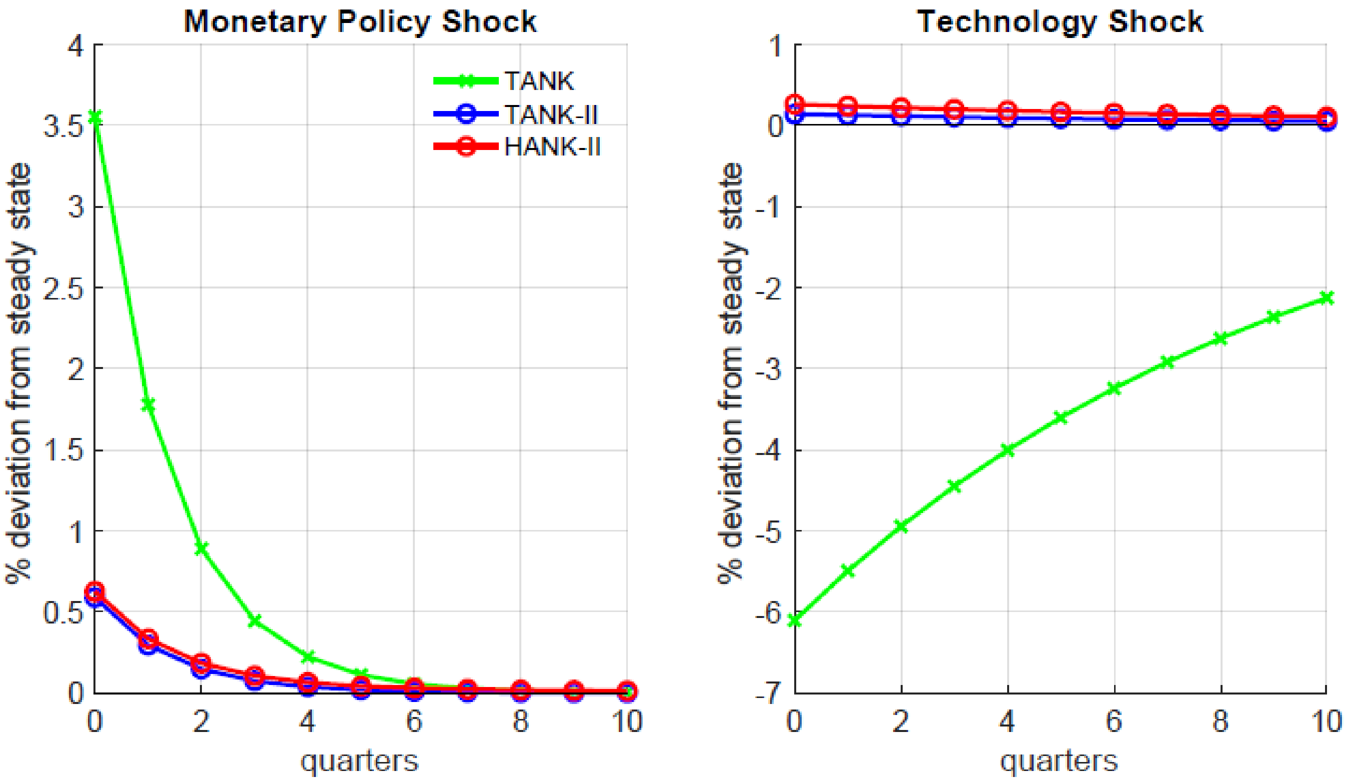
$$C_t = \lambda^H \frac{\mathcal{M}}{\mathcal{M}_t} Y_t + (1 - \lambda^H) C_t^U$$

$\Rightarrow$ no interest rate exposure
$\Rightarrow$ income distribution channel: wrong sign

- Quantitative analysis

Figure 5: Simple Alternatives to HANK-II

Panel (a): Impulse Responses



HANK-III

- Borrowing limit:

$$\Psi_t(j) = \psi Y$$

- Optimal portfolio choice: additions/withdrawals to/from equity account subject to adjustment costs and short-sale constraint

$$E_t(j) \geq 0$$

- Two types of hand-to-mouth households:

(i) poor hand-to mouth (measure λ_t^P): $B_t(j) = -\psi Y$, $E_t(j) = 0$

$$C_t^P = \Xi_t^P W_t N_t - \psi Y \left(\widehat{R}_t + \Omega_{t-1}^H - 1\right)$$

(ii) wealthy hand-to-mouth (measure λ_t^W): $B_t(j) = -\psi Y$, $E_t(j) > 0$

$$C_t^W = \Xi_t^W W_t N_t + F_t^W - \psi Y \left(\widehat{R}_t + \Omega_{t-1}^H - 1\right)$$

where $F_t^W \equiv R_t^e E_{t-1}^W - E_t^W - \chi_t^W$

TANK-III: A Tractable Counterpart to HANK-III

- Hand-to-mouth agents: identical, constant measure λ^H

$$C_t^H = \Xi^H W_t N_t + \Theta^H D_t - \psi Y \ \widehat{R}_t$$

where Θ^H is now decoupled from Ξ^H

- Aggregate consumption (TANK-III)

$$C_t = \lambda^H \Xi^H W_t N_t + \lambda^H \Theta^H D_t - \lambda^H \psi Y \ \widehat{R}_t + (1 - \lambda^H) C_t^U$$

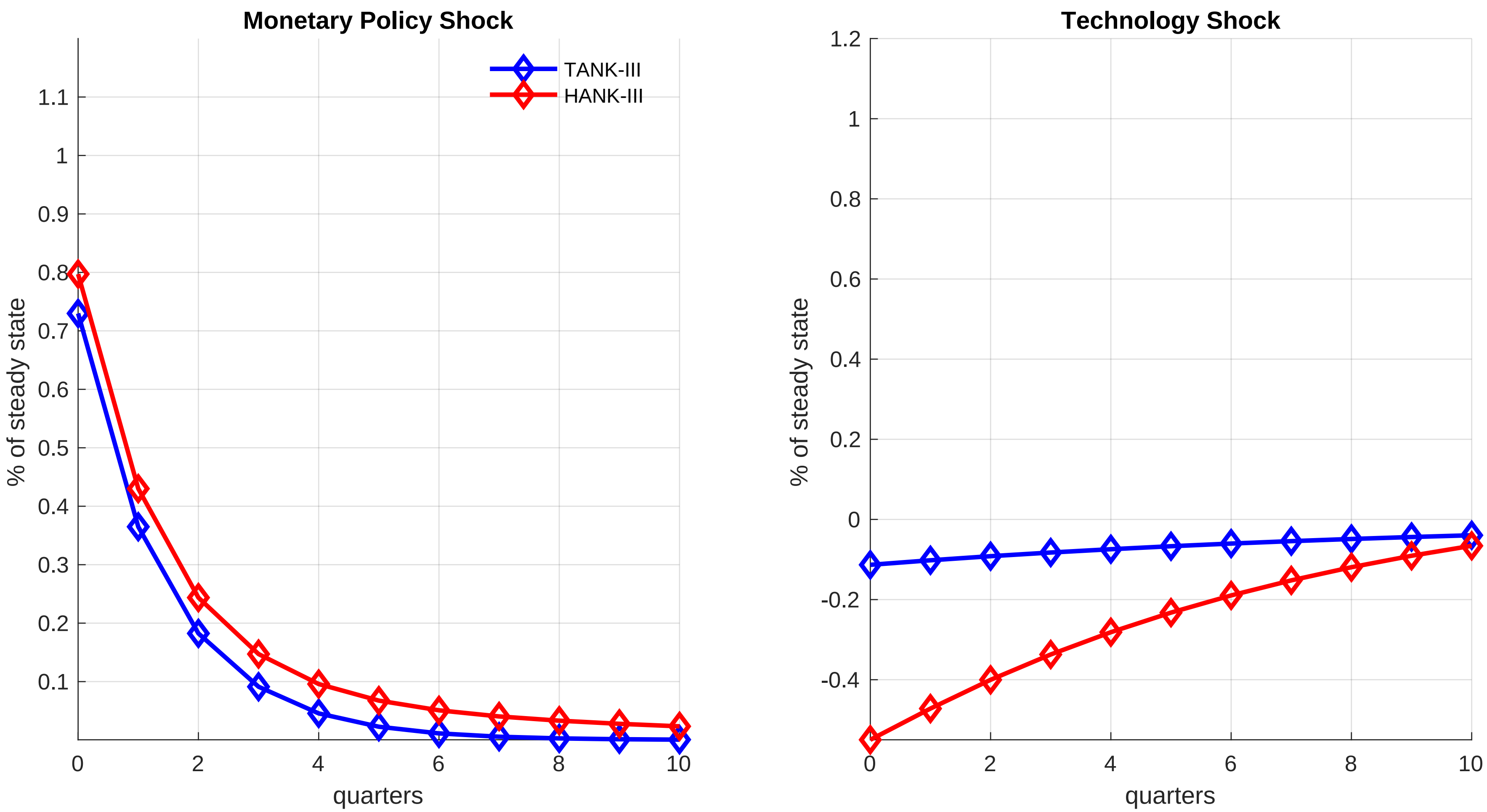
- Aggregate consumption (HANK-III)

$$C_t = \lambda_t^H \Xi_t^H W_t N_t + \lambda_t^W F_t^W - \lambda_t^H \psi Y \ [\widehat{R}_t + \Omega_{t-1}^H - 1] + (1 - \lambda_t^H) C_t^U$$

- Quantitative analysis

Endogenous Monetary Policy

- A Taylor-type rule
$$i_t = r + 1.5\pi_t + 0.125\widehat{y}_t + v_t$$
where $v_t = \rho_v v_{t-1} + \varepsilon_t^v$.
- Quantitative analysis



Monetary Policy Shock
TANK-III
HANK-III
% of steady state
quarters
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
0
2
4
6
8
10
Technology Shock
% of steady state
quarters
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
0
2
4
6
8
10

Endogenous Monetary Policy

- A Taylor-type rule

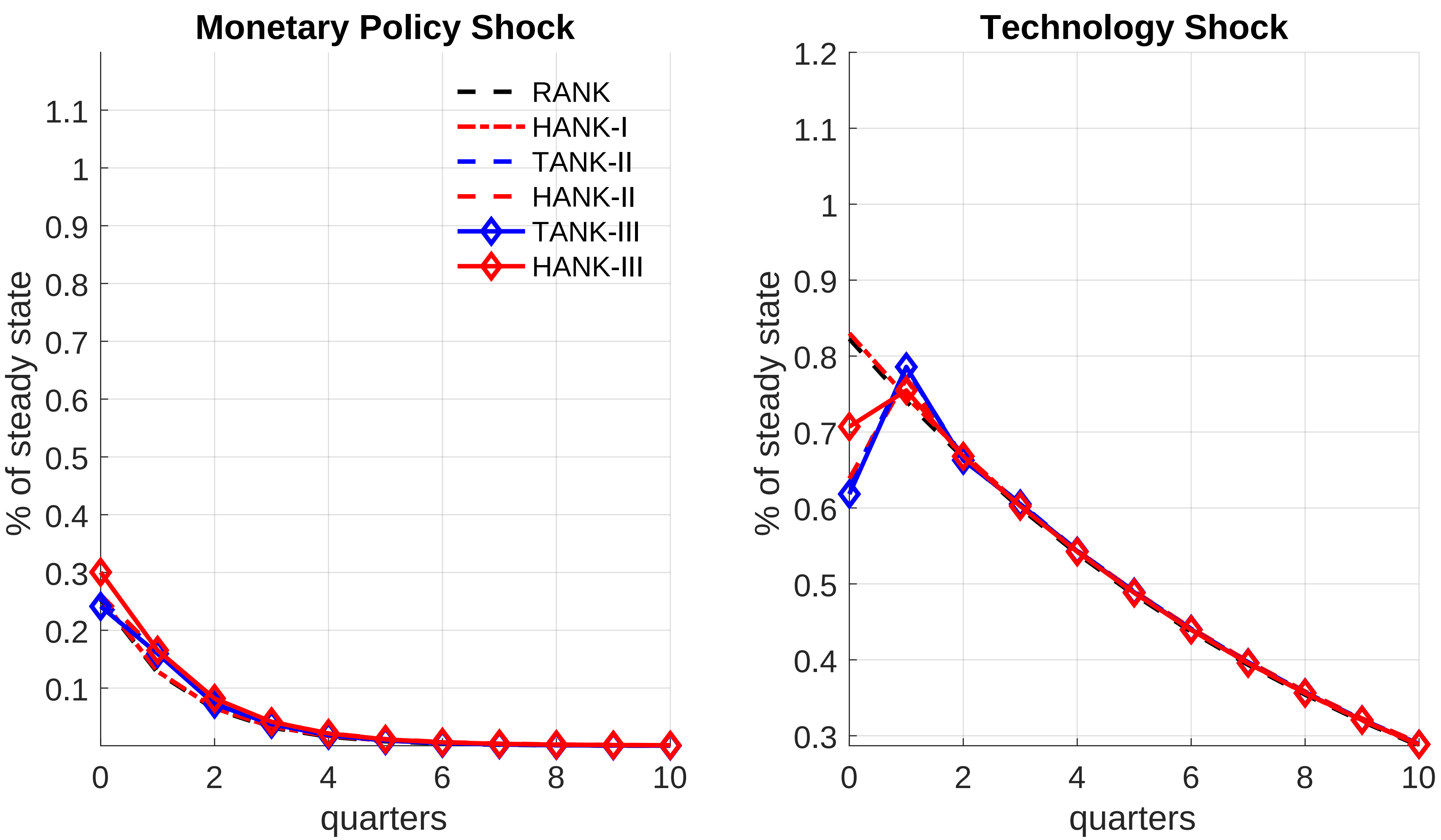
$$i_t = r + 1.5\pi_t + 0.125\widehat{y}_t + v_t$$

where $v_t = \rho_v v_{t-1} + \varepsilon_t^v$.

- Quantitative analysis.

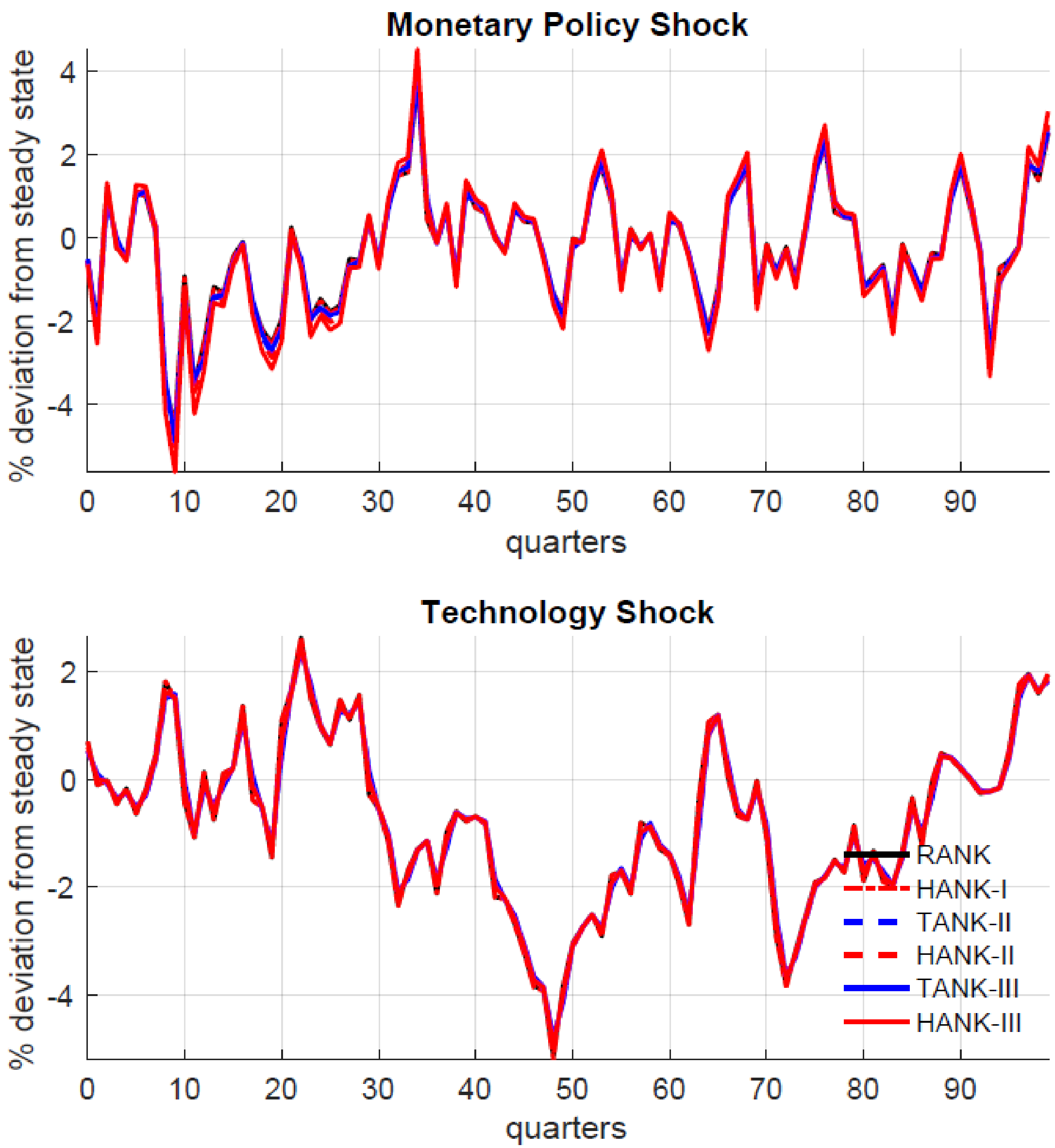
- **Proposition:** under a strict inflation targeting policy (i.e. $\pi_t = 0$ for all t) equilibrium output is invariant to the presence of heterogeneity.

Proof: follows from "divine coincidence" property of NKPC



Monetary Policy Shock
Technology Shock
RANK
HANK-I
TANK-II
HANK-II
TANK-III
HANK-III
% of steady state
quarters
0
2
4
6
8
10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2

Panel (b): Simulations



Concluding Remarks

- Idiosyncratic income risk: key source of complexity in HANK, but small role in *aggregate fluctuations*
- Aggregate predictions of HANK models can be approximated by a suitably calibrated TANK model (qualitatively, and often quantitatively as well).
- More so if monetary policy is focused on inflation stabilization.
- HANK models potentially useful for analysis of:

  - distributional impact of aggregate shocks
  - role of inequality in optimal monetary policy design

- Future research: Can TANK models approximate HANK models' predictions on those fronts as well?

Balanced Budget
Deficit Financed
RANK
HANK II
TANK II
% deviation from steady state
quarters
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9

Balanced Budget
Deficit Financed
RANK
HANK II
TANK I
TANK II
% deviation from steady state
quarters
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5
6
7
8
9


Balanced Budget
Deficit Financed
RANK
HANK II
TANK I
TANK II
% deviation from steady state
quarters
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9