A Macroeconomic Model with Rational Exuberance: Temporarily Explosive Land Price Dynamics

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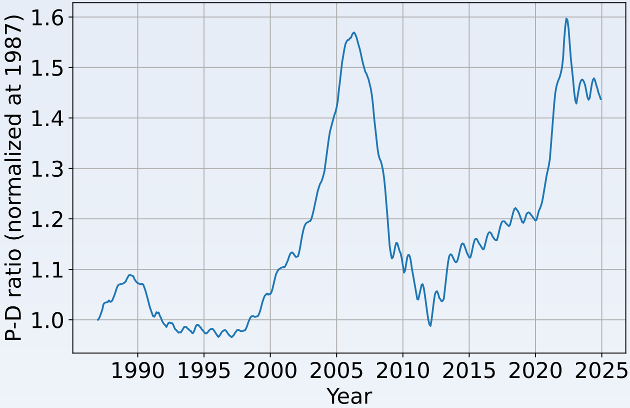
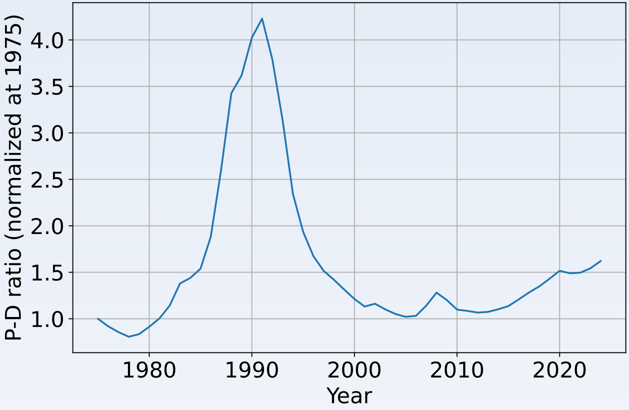
Motivation

- P-D ratio in the US housing sector
  - Mostly stationary, but explosive dynamics in the 2000s



Motivation

- P-D ratio in Japan land sector
  - Mostly stationary, but explosive dynamics in the 1980s



This Paper (1)

1. Develop a macro-finance model with rational exuberance
   - rational exuberance: temporarily explosive land price followed by a large reduction
   - probability of the large reduction is correctly taken into account

2. Explosive dynamics arise only in a certain environment
   - Strong spillover of capital + high leverages
     - so that the model generates growth
   - When this condition is met, the model generates qualitatively different dynamics

This Paper (2)

3. Analyze condition for the existence of bubbles
   - Bubbles arise during the temporarily explosive land price if followed by a larger land price reduction when explosive dynamics last longer
   - Connects macro-finance theory of bubbles and literature on bubble detection
4. Make a case for the cost of rational exuberance
   - Add downward wage rigidity
   - Show “higher you climb, harder you fall”

A Model of Rational Exuberance

Model Overview

- Agents: Entrepreneurs or savers with i.i.d. probability
  - Entrepreneurs: invest in capital and land, and borrow from savers
  - Savers: lend to entrepreneurs
- Production: normally decreasing in capital, but spillover of capital makes linear in capital
- Standard macro-finance model except
  - Stochastic spillover of capital that occasionally makes production linear in capital
  - Positive dividend from land

Agents

- Life time utility of agent i

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t \log(c_{i,t})\right]$$

- Budget constraint (given states $k_{i,t}$, $x_{i,t}$, and $b_{i,t}$)
  - For entrepreneurs (with probability π)

$$c_{i,t}+k_{i,t+1}+P_{X,t}x_{i,t+1}-b_{i,t+1}=R_t^K k_{i,t}+(P_{X,t}+D)\,x_{i,t}-(1+r_t)\,b_{i,t}$$

  - For savers (with probability $1-\pi$)

$$c_{i,t}-b_{i,t+1}=R_t^K k_{i,t}+(P_{X,t}+D)\,x_{i,t}-(1+r_t)\,b_{i,t}$$

Borrowing Constraint

- Borrowing constraint

$$b_{i,t+1} \leq \theta k_{i,t+1} + \theta^X P_{X,t} x_{i,t+1}$$

- θ and θ^X represent leverages
  - e.g.: to purchase $k_{i,t+1}$, only $(1-\theta)k_{i,t+1}$ needs to be self financed

Optimality condition

- Consumption function

$$c_{i,t} = (1-\beta)\left[R_t^K k_{i,t} + (P_{X,t} + D)\,x_{i,t} - (1+r_t)\,b_{i,t}\right],$$

- Non-arbitrage condition (under the timing assumption we will make in the coming slides)

$$\frac{\frac{P_{X,t+1}+D}{P_{X,t}} - (1+r_{t+1})\,\theta^X}{1-\theta^X} = \frac{R_{t+1}^K - (1+r_{t+1})\,\theta}{1-\theta}$$

  - Leveraged return must be equated

Production

- Perfect competition
- Production function for firm j

$$Y_{j,t} = AK_{j,t}^{\alpha}\left(K_t^{\phi_t}\right)^{1-\alpha}$$

  - K_t: aggregate capital
  - ϕ_t: degree of spillover
- If $\phi_t = 1$,
  - $\phi_{t+1} = 1$ with probability λ (H-state)
  - $\phi_{t+1} = \underline{\phi}$ with probability $1-\lambda$ (L-state)
- If $\phi_t = \underline{\phi}$, $\phi_{t+1} = \underline{\phi}$ with probability 1 (absorbing state)

Return on Capital

- Rate of return on capital

$$R_t^K = \alpha A K_t^{\alpha+\phi_t(1-\alpha)-1} = \begin{cases} \alpha A & \text{when } \phi_t = 1 \;\; (H\text{-state}) \\ \alpha A K_t^{\left(1-\underline{\phi}\right)(\alpha-1)} & \text{when } \phi_t = \underline{\phi} \;\; (L\text{-state}) \end{cases}.$$

- Timing assumtion: ϕ_{t+1} is realized at the beginning of period t

Equilibrium

- Agents and firms optimally make their decisions
- Following the equilibrium conditions are satisfied

$$\int_0^1 x_{i,t} di = \overline{X}$$

$$\int_0^1 k_{i,t} di = K_t$$

$$\int_0^1 b_{i,t} di = 0$$

- Focus on an equilibrium where the borrowing constraint binds and the linear consumption function holds

System of Equations

- Investment-Saving equation

$$K_{t+1} + P_{X,t}\overline{X} = \beta\left[R_t^K K_t + (P_{X,t} + D)\,\overline{X}\right]$$

- Capital accumulation

$$K_{t+1} = \frac{1}{1-\theta}\left[\beta\pi\left(R_t^K K_t + (P_{X,t} + D)\,\overline{X}\right) - \left(1-\theta^X\right)P_{X,t}\overline{X}\right]$$

- Non-arbitrage condition

$$\frac{\frac{P_{X,t+1}+D}{P_{X,t}} - (1 + r_{t+1})\,\theta^X}{1-\theta^X} = \frac{R_{t+1}^K - (1 + r_{t+1})\,\theta}{1-\theta}$$

Recursive Formulation

- Land price (for $t \geq 1$)

$$P_{X,t+1} = \mathcal{C}_K R^K_{t+1} P_{X,t} + \mathcal{C}_P D$$

- Capital

$$K_{t+1} = \mathcal{C}_K R^K_t K_t + \mathcal{C}_K D \overline{X}$$

  - with $\mathcal{C}_K := \frac{\theta^X - (1-\pi)}{\frac{1}{\beta}(\theta^X - \theta) - (1-\pi-\theta)}$ and $\mathcal{C}_P := \frac{1-\pi-\theta}{\frac{1}{\beta}(\theta^X - \theta) - (1-\pi-\theta)}$

About the Parameters

- Focus on $\theta < 1 - \pi < \theta^X$ and $0 < \frac{1}{\beta}\left(\theta^X - \theta\right) - (1 - \pi - \theta)$

  $\Rightarrow$ then, $\mathcal{C}_K > 0$ and $\mathcal{C}_P > 0$

  - In reality, θ^X tends to be high, θ tends to be low

- $\mathcal{C}_P$ is decreasing in θ^X and θ

- $\mathcal{C}_K$ is increasing in θ^X and θ

Macroeconomic Dynamics

Uniqueness of Equilibrium

Proposition

In this economy, the equilibrium we focus on is unique.

- "the equilibrium we focus on": the borrowing constraint binds and the linear consumption function holds

Explosive Dynamics

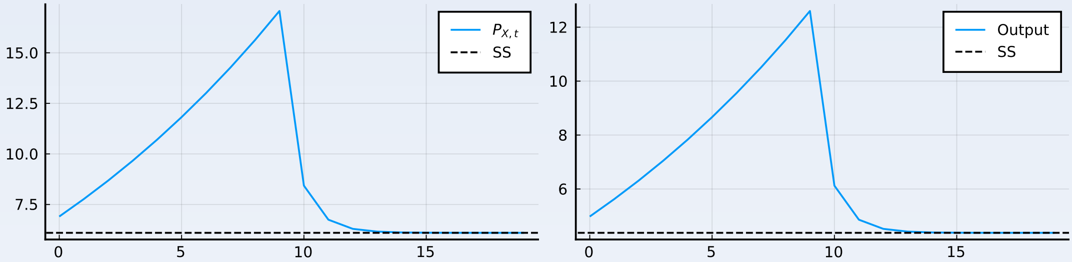
Proposition

The land price and the P-D ratio shows explosive dynamics in the *H*-state if $\mathcal{C}_K \alpha A > 1$. If $\mathcal{C}_K \alpha A < 1$, they are stationary in the *H*-state. In the *L*-state, they are stationary too.

- Growth is required for explosive dynamics
  - *H*-state ($\phi_t = 1$) corresponds to very high spillover
- $\mathcal{C}_K$ and A need to be high enough to generate explosive dynamics
  - Recall: $\mathcal{C}_K$ is increasing in θ^X and θ
- High leverages and high spillover → rational exuberance

Sample paths

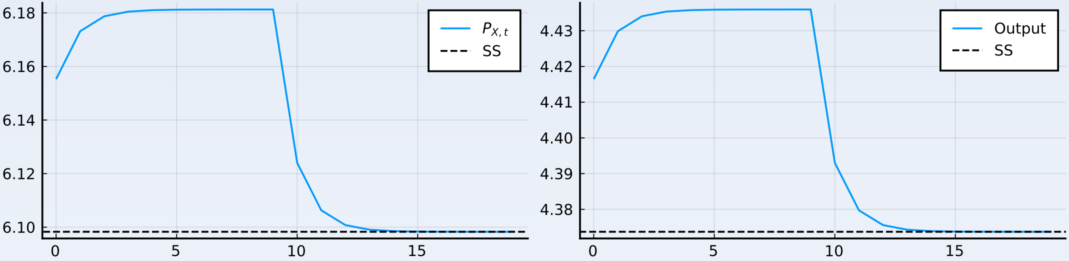
– IRF of $\phi = 1$ for 10 periods



– A rise in ϕ_t leads to an explosive boom

Sample paths

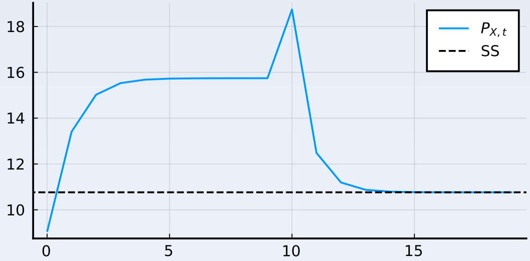
– IRF of a rise in A in the L-state for 10 periods

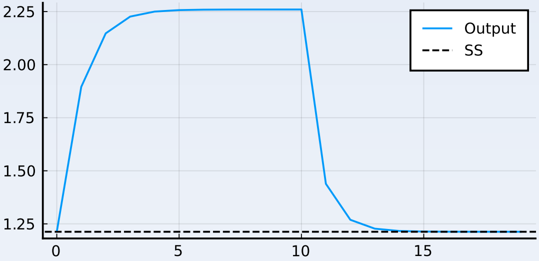


– Qualitatively different dynamics from ϕ_t shock

Sample paths

– IRF of a rise in θ^X in the L-state for 10 periods

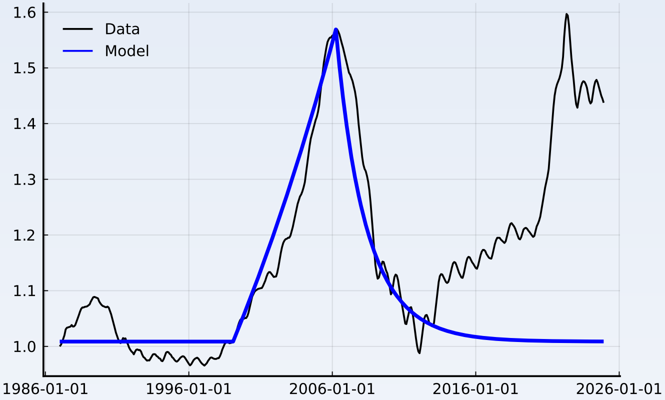




– Qualitatively different dynamics from ϕ_t shock

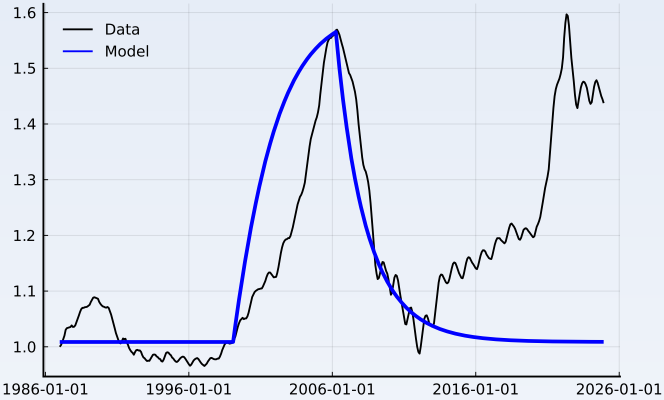
Matching with Data

- Calibrate $A = 3.198$ to match the growth with $\phi_t = 1$ from 1999/3 to 2006/5 and $\underline{\phi} = 0.95$ for other periods



Example of Failed Matching with Data

- Shock to A (from 3.198 to 3.248) with always $\underline{\phi} = 0.95$ cannot generate the explosive dynamics



Bubbles

Defining Bubbles

- From the non-arbitrage condition, can derive

$$P_{(H),X,0} = \underbrace{\sum_{T=1}^{\infty} \lambda^{T-1} \frac{(1-\lambda) P_{(L,T-1),X,T} + D}{\prod_{t=1}^{T} \mathcal{R}_{(H),t-1,t}}}_{\text{Fundamental term } (F_t)} + \underbrace{\lim_{T\to\infty} \frac{\lambda^T P_{(H),X,T}}{\prod_{t=1}^{T} \mathcal{R}_{(H),t-1,t}}}_{\text{Bubble term } (B_t)}.$$

  where $\mathcal{R}_{(H),t-1,t}$ is the expected rate of return on land

- When the bubble term> 0, the land price contains a bubble

Condition for the Existence of Bubbles

$$\text{Bubble term} = \lim_{T \to \infty} \frac{\lambda^T P_{(H),X,T}}{\prod_{t=1}^{T} \mathcal{R}_{(H),t-1,t}}$$

$$\mathcal{R}_{(H),t-1,t} = \lambda \frac{P_{(H),X,t}}{P_{(H),X,t-1}} + (1-\lambda) \frac{P_{(L,t-1),X,t}}{P_{(H),X,t-1}} + \frac{D}{P_{(H),X,t-1}}$$

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- 1st term: growth rate of the numerator
  - the rest of the terms must converge to zero for the bubble term to be positive

Condition for the Existence of Bubbles

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- 1st term: growth rate of the numerator
  - the rest of the terms must converge to zero for the bubble term to be positive
- 2nd term excl. $1-\lambda$: goes to zero if the growth rate of the land price at the timing of the state change goes to zero

Condition for the Existence of Bubbles

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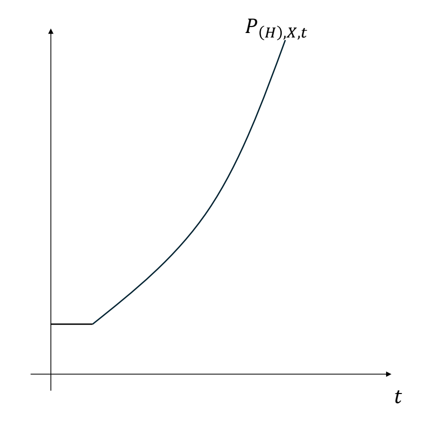
$$\mathcal{R}_{(H),t-1,t} = \lambda \frac{P_{(H),X,t}}{P_{(H),X,t-1}} + (1-\lambda)\frac{P_{(L,t-1),X,t}}{P_{(H),X,t-1}} + \frac{D}{P_{(H),X,t-1}}$$

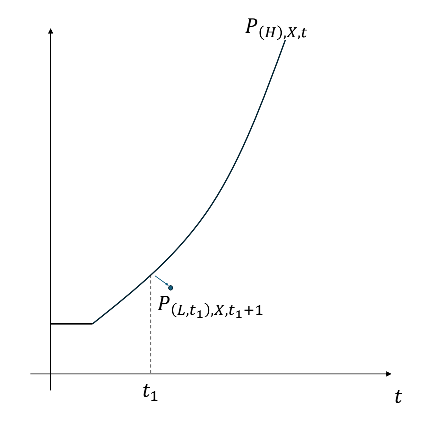
- 1st term: growth rate of the numerator
  - the rest of the terms must converge to zero for the bubble term to be positive
- 2nd term excl. $1-\lambda$: goes to zero if the growth rate of the land price at the timing of the state change goes to zero
- 3rd term: goes to zero if the land price diverges

Graphical Representation



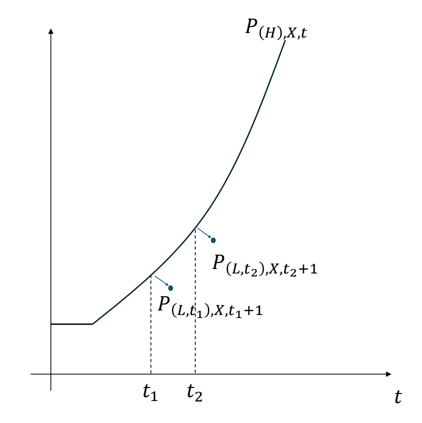
– $P_{(H),X,t}$: The path of the land price when the *H*-state continues

Graphical Representation



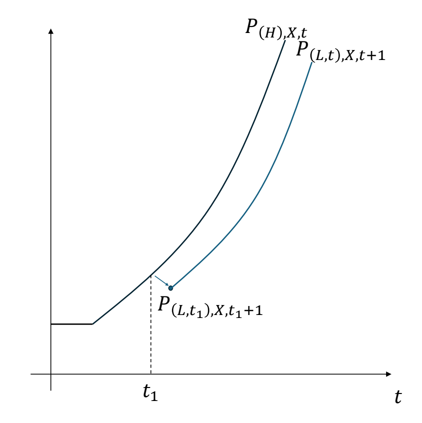
- $P_{(L,t_1),X,t_1+1}$: The land price when the state changed from H to the L between t_1 and t_1+1

Graphical Representation



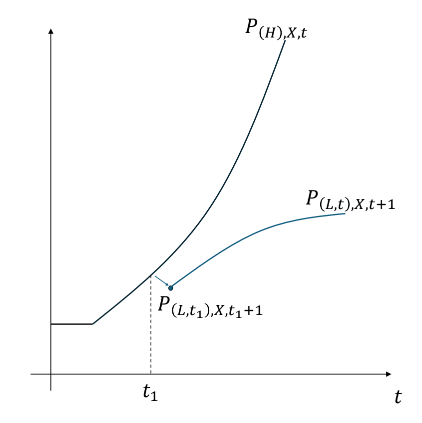
- $P_{(L,t_2),X,t_2+1}$: The land price when the state changed from H to the L between t_2 and $t_2 + 1$

Graphical Representation



- In this figure, the state change doesn't reduce the land price so much even when the *H*-state lasts long
  - Corresponds to the explosive fundamental = No bubble

Graphical Representation



- In this figure, the state change reduces the land price more when the *H*-state lasts longer
  - Corresponds to the non-explosive fundamental = Bubble

Existence of Bubbles

Proposition

Suppose $\phi_0 = \phi_1 = 1$. When $\mathcal{C}_K \alpha A > 1$, the equilibrium land price contains a bubble. When $\mathcal{C}_K \alpha A < 1$, the equilibrium land price does not contain a bubble.

- For bubbles to exist, large $\mathcal{C}_K$, A, and $\phi_t = 1$ are required

Connection to the Literature on Bubble Detection

- Literature on bubble detections
  - focus on the case where fundamental term is not explosive
    - e.g. Phillips et al. (2015) assume F_t is at most $I(1)$
  - check the explosiveness of the P-D ratio to detect bubbles

$$P_t = F_t + B_t$$

    - If P_t is explosive but F_t is not, then B_t is explosive, implying B_t is not zero

- Our theory: bubbles arise when
  - explosive land price
  - larger collapse for longer bubbles
    - = fundamental is not explosive

$\Rightarrow$ Consistent with the empirical literature

Signal of Bubbles

Proposition

When the P-D ratio behaves explosively, we may be warned that the land price contains a bubble.

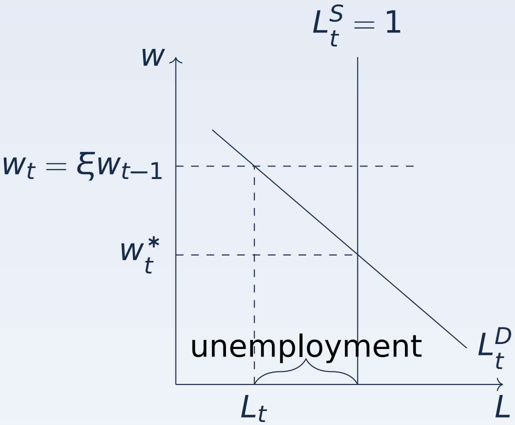
- Looking at the P-D ratio can be a signal of bubbles
  - Ideally, want to check if fundamental is explosive

Extension: Cost of Rational Exuberance

Downward Real Wage Rigidity (DRWR)

- Explicitly consider the hand-to-mouth workers
- Labor supply: $L_t^S = 1$
- Labor demand: $L_t^D = \left(\frac{AK_t^{\alpha + \phi_t(1-\alpha)}}{w_t} \right)^{\frac{1}{\alpha}}$
  - w: the real wage
- DRWR: $w_t = \max\{\xi w_{t-1}, w_t^*\}$
  - w_t^*: the flexible wage determined by $L_t^S = L_t^D$
  - ξ: the degree of downward rigidity
  - (Evidence: Holden and Wulfsberg (2009); **?**)
- When $w_t = \xi w_{t-1}$, equilibirum labor is less than 1
  - Unemployment: $1 - L_t^D$

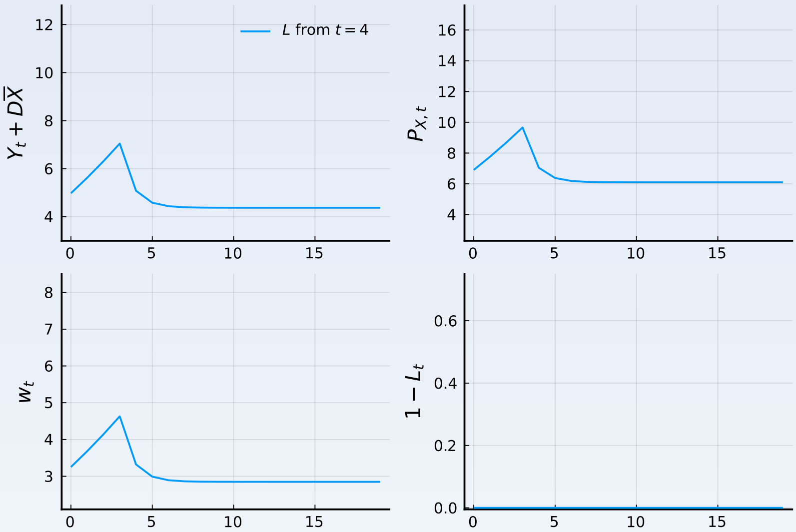
Graphical Exposition of DRWR



– When labor demand is low, rigid real wage cannot clear the market ($\xi w_{t-1} < w_t^*$)

→ Unemployment: $1 - L_t > 0$

The Higher You Climb, the Harder You Fall



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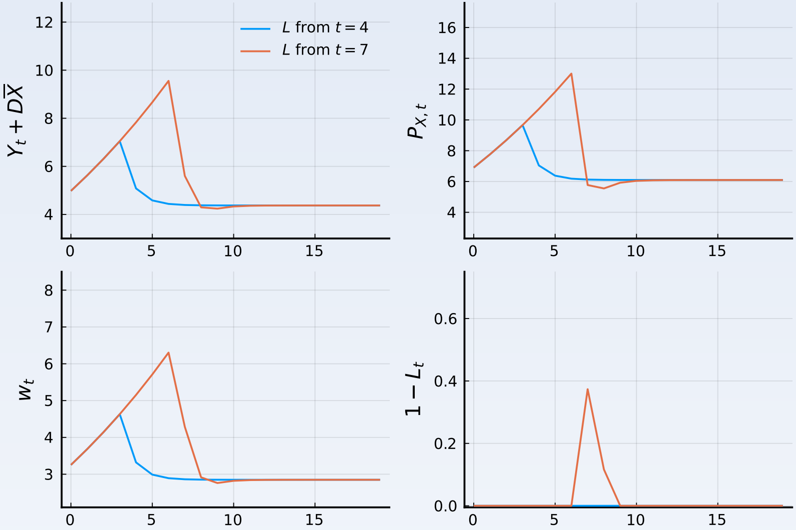

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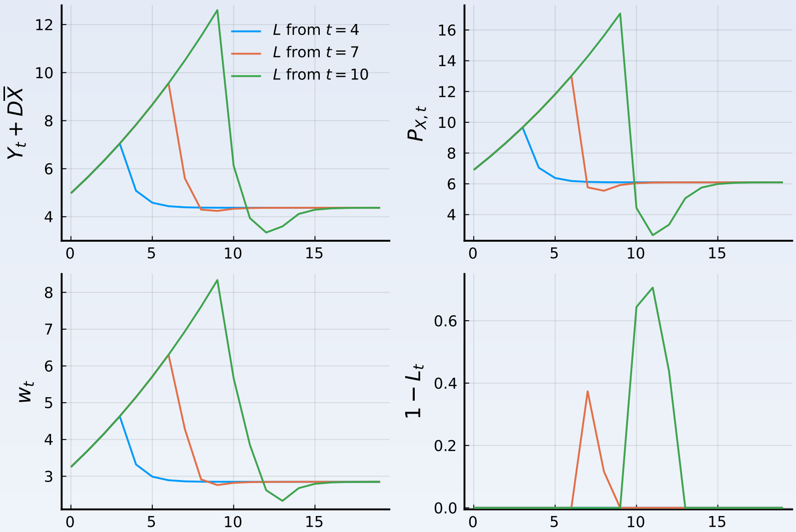
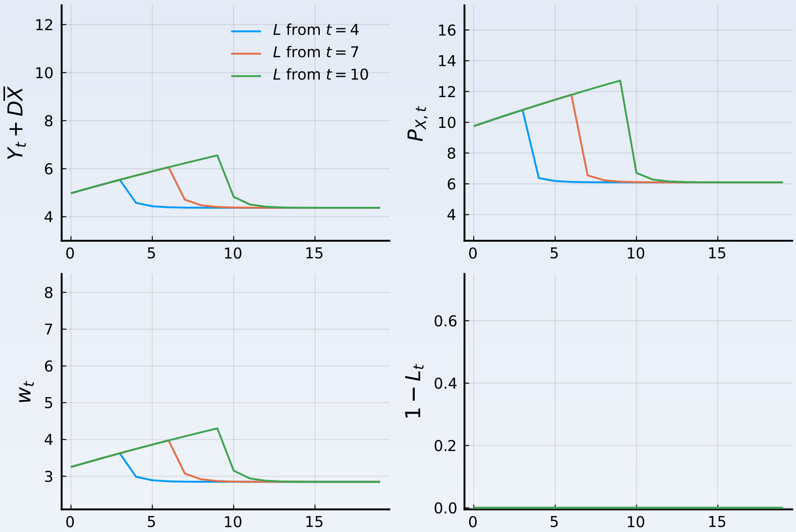


Financial Regulation Can Prevent Unemployment

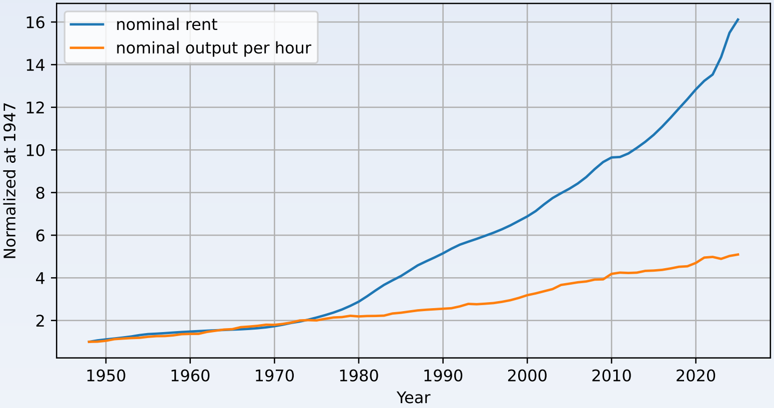


Conclusion

- Construct a model of rational exuberance: temporarily explosive land price dynamics
- Explosive dynamics only when the spillover is strong and leverages are high, stationary otherwise
- Analyze the condition for the existence of bubbles and connects the theory of bubbles to empirical literature on bubble detection
- Make a case for the cost of rational exuberance

Appendix

Growth rate of output productivity and rent



Discounting

- Non-arbitrage condition that holds every period

$$\mathcal{R}_{(HH),t,t+1} = \frac{P_{(H),X,t+1} + D}{P_{(H),X,t}} \text{ with probability } \lambda$$

or

$$\mathcal{R}_{(HL),t,t+1} = \frac{P_{(L,t)X,t+1} + D}{P_{(H),X,t}} \text{ with probability } 1 - \lambda$$

- Can obtain two relations

Discounting

- First one is on slide

$$P_{(H),X,t} = \frac{\mathbb{E}\left[P_{X,t+1} + D | \phi_t = 1\right]}{\mathcal{R}_{(H),t,t+1}} = \frac{\lambda P_{(H),X,t+1} + (1-\lambda) P_{(L,t),X,t+1} + D}{\mathcal{R}_{(H),t,t+1}}$$

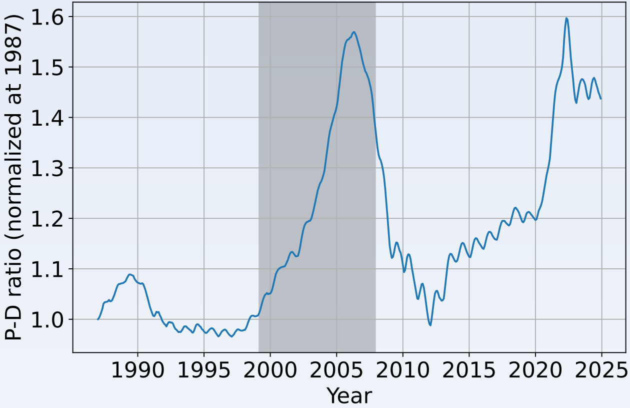
- Second one is

$$P_{(H),X,t} = \lambda \frac{P_{(H),X,t+1} + D}{\mathcal{R}_{(HH),t,t+1}} + (1-\lambda) \frac{P_{(L,t),X,t+1} + D}{\mathcal{R}_{(HL),t,t+1}}$$

  - This second one means "discount by the realized return"
  - $\mathcal{R}_{(HH),t,t+1} =$ growth rate of $P_{(H),X,t+1}$ in the long-lasting bubble, so bubble never exists

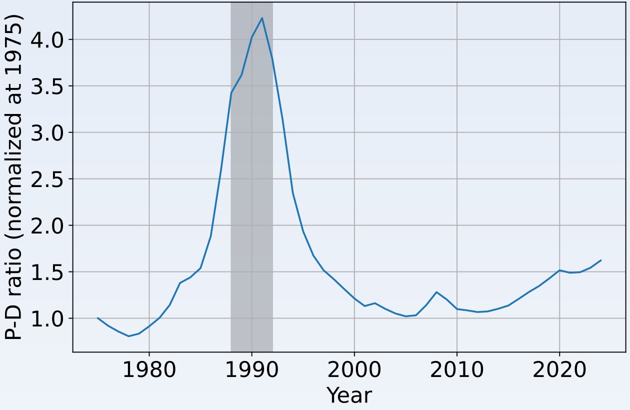
Bubble Detection Result: US

- Apply the bubble detection method of Phillips et al. (2015) to the US data

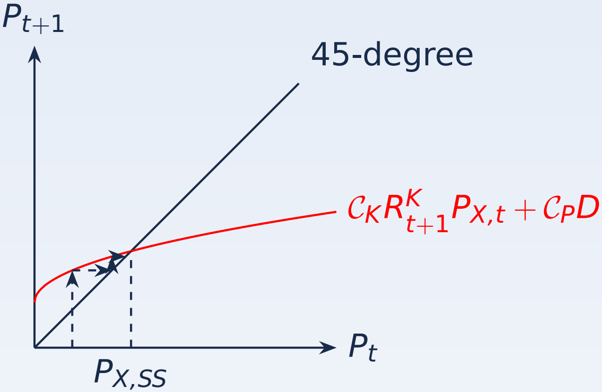


Bubble Detection Result: Japan

– Apply the bubble detection method of Phillips et al. (2015) to Japanese data

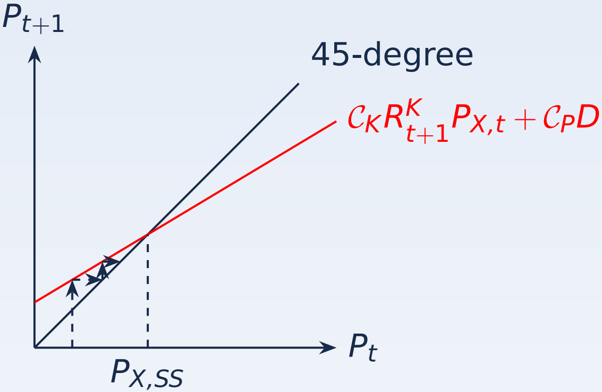


Changing Phase: L-state



- In L-state, the economy always has steady state
- In H-state, the economy has steady state if $\mathcal{C}_K \alpha A < 1$
- In H-state, the economy has explosive dynamics if $\mathcal{C}_K \alpha A > 1$

Changing Phase: H-state with $\mathcal{C}_K \alpha A < 1$



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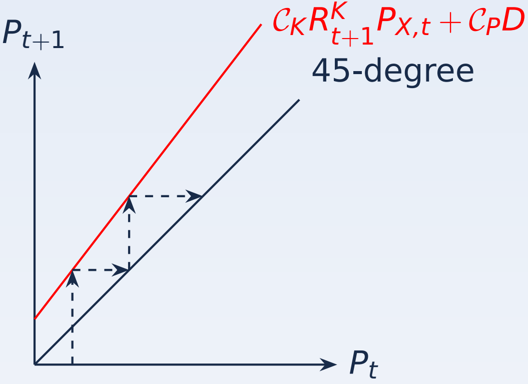
Changing Phase: H-state with $\mathcal{C}_K \alpha A > 1$



- In L-state, the economy always has steady state
- In H-state, the economy has steady state if $\mathcal{C}_K \alpha A < 1$
- In H-state, the economy has explosive dynamics if $\mathcal{C}_K \alpha A > 1$

Discussion: Alternative Specification

- Our model: ϕ_t is stochastic
  - Reduction in ϕ_t in the L-state creates larger reduction for longer bubble

- Alternative: always $\phi_t = 1$, but stochastic θ and θ^X
  - also generate explosive dynamics when θ and θ^X are sufficiently high, and stationary dynamics otherwise

$\Rightarrow$ However; land price does not contain a bubble
  - explosive boom is not followed by a large reduction even after a long-lasting bubble
  - recall: $P_{(L,t),X,t+1} = \mathcal{C}_K R^K_{(L,t),t+1} P_{(H),X,t} + \mathcal{C}_P D$

- (Alternative for a bubble: θ^X in L-state $\to 1 - \pi$ as $t \to \infty$)

Alternative Model

- Consider a model where savers can also invest in land
- Focus on an equilibrium where the return from capital is higher than the return from land

⇒ Entrepreneurs invest in capital and borrow, savers invest in land and lend

Equilibrium Conditions

- Investment-Saving equation

$$K_{t+1} + P_{X,t}\overline{X} = \beta\left[R_t^K K_t + (P_{X,t} + D)\,\overline{X}\right],$$

- Capital accumulation

$$K_{t+1} = \frac{1}{1-\theta}\beta\pi\left[R_t^K K_t + (P_{X,t} + D)\,\overline{X}\right],$$

- Non-arbitrage condition (of entreprenurs)

$$\frac{P_{X,t+1} + D}{P_{X,t}} = 1 + r_{t+1}.$$

Recursive Formulation

- Recursively,

$$P_{X,t+1} = \hat{\mathcal{C}}_K R^K_{t+1} P_{X,t} + \hat{\mathcal{C}}_P D.$$

$$K_{t+1} = \hat{\mathcal{C}}_K \left[R^K_t K_t + D\overline{X} \right],$$

  - where $\hat{\mathcal{C}}_K := \frac{\beta\pi}{(1-\beta)(1-\theta)+\beta\pi}$ and $\hat{\mathcal{C}}_P := \frac{\beta(1-\theta-\pi)}{(1-\beta)(1-\theta)+\beta\pi}$.

- Can show the existence of land price bubbles in this model too

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