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Asset Price Booms, Debt Overhang and Debt Disorganization*

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Abstract

We propose a tractable model of financial crises that replicates the empirical regularities: A credit-fueled asset-price boom tends to collapse, followed by a deep and persistent recession with productivity declines. Risk-shifting firms amplify the boom and bust of asset prices by purchasing assets with borrowed money. The resulting debt overhang reduces productivity by discouraging borrowing firms from spending additional effort. This inefficiency causes shrinkage of the production network through demand externality, which we call debt disorganization. The larger asset-price boom is followed by a deeper and more persistent recession. Lenders know that debt reduction can increase lenders' payoff, and when the debt burden is small, they restructure the debt on their own and attain social optimum. When debt is large, government subsidies to encourage lenders to implement debt restructuring are necessary to reduce externality and restore aggregate productivity.

Key words: Zombie lending, bank recapitalization, time inconsistency, the debt Laffer curve.

JEL Classification: E02, G01, G33

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1 Introduction

Recent studies show the following empirical regularities about booms and busts of asset prices: when the asset price boom is fueled with an increase in credit, the asset boom tends to end up with a bust, followed by a deep and persistent recession with lower observed total factor productivity (TFP). See, for example, Jordà, Schularick and Taylor (2015). We propose a parsimonious model to replicate these empirical regularities, which is also tractable in analyzing policy interventions. Our model could be considered a simple and consistent explanation for the movements of asset prices, credit, and productivity throughout the entire cycle of a financial crisis. In particular, a unique feature of our model is to relate productivity declines after the collapse of the asset price to *debt overhang*. The debt overhang in this study indicates the situation in which the stipulated amount of debt is higher than the repayable amount and the repayment is yet to be settled.

Why focus on debt overhang? As we shall see, our debt overhang theory of financial crisis provides a new perspective consistent with the observation that persistent stagnation after the crisis tends to be accompanied by the contraction of the aggregate demand for credit, rather than the tightening of the credit supply. A typical example is the three decades of stagnation in Japan since the 1990s. After the bursting of asset price bubbles at the beginning of the 1990s, Japan has suffered from persistently weak demand for credit in the corporate sector. Another reason to raise a debt overhang theory is that it provides a new perspective on policy interventions. The standard prescription in the literature is to close and liquidate zombie firms because they are regarded intrinsically unproductive. Our theory implies that they may not be intrinsically inefficient and that the reduction of debt overhang could restore the productivity of borrowers and also increase lenders' payoffs and social welfare.

What we do in this paper is the following. We construct a simple two-period model, in which we unify the model of risk-shifting booms of asset prices (Allen and Gale 2000; Allen, Barlevy and Gale 2022) and the model of macroeconomic debt overhang due to spillover effect through aggregate output (Lamont 1995). Our theory differs from these two models in that it accounts for productivity declines in the aftermath of a financial crisis. One key ingredient is our assumption that the risky asset, whose price can be driven up by risk-shifting, is also used as an input for production by borrowers. In the first period, the agents have the expectation that the productivity of the asset will be either A_H or A_M in the second period, where $A_H \geq A_M$. The parameter A_H represents the degree of optimistic expectations in period 1, concerning the productivity of the asset in period 2. When the ex-ante optimism is large (i.e., A_H is large), the price of the asset is high and debt is large in period 1. It then becomes a debt overhang if the productivity of the asset turns out to be A_M in period 2. The debt overhang is accompanied by a recession with lower aggregate productivity.

In equilibrium, the price of the asset is driven by firms that buy the asset with borrowed money. The borrowers bid up the asset price because they can push the loss on the lenders by defaulting on the debt, when the productivity of the asset turns out to be low (A_M). This is the *risk-shifting boom* of asset prices (Allen and Gale 2000; Allen, Barlevy and Gale 2022). In period 2, the borrower cannot repay the full amount of debt when the productivity of the asset turns out to be low. The TFP declines disproportionately because the debt overhang discourages borrowing firms from spending effort in production activity. They are discouraged because the lenders cannot commit to reward their efforts as the lenders have the legitimate right to take all as long as debt is larger than the borrowers revenue. This is called the lack of lender's commitment (Kobayashi, Nakajima and Takahashi 2023). The borrowers choose not to expend additional effort because they know they will get nothing, as the lenders take all.

The inefficiency of debt overhang would be internalized by lenders, who would optimally reduce debt on their own and achieve social optimum if there were no additional frictions. In reality, however, larger debt is usually difficult to restructure. Thus, we assume the agency problem on the lender side as follows. We suppose that a lender is a bank and that the bank manager can receive a high executive salary, which is proportional to the stipulated amount of debt if she can successfully hide the impairment of the value of debt from depositors (and regulators). We also assume that if the bank manager does not undertake debt restructuring or liquidation of the firm, the depositors (and regulators) tend to be misled to believe that the value of debt is not impaired, while if she undertakes debt restructuring or liquidation of the firm, depositors recognize the impairment of the value of the debt. Given this agency problem, it is shown that

- when debt is small, lenders optimally choose debt restructuring or liquidation of firms so that the social optimum is achieved in the competitive equilibrium without policy intervention, and
- when debt is larger, a larger number of lenders tends to keep the stipulated amount of debt unchanged and the inefficiency of debt overhang arises.

In addition to the inefficiency of debt overhang, the specialization among borrowers' products in the Dixit-Stiglitz production is lowered by the shrinkage of aggregate demand. We call this disruption of specialization the *debt disorganization*. We define debt disorganization as an external effect of an exit of one firm from the network of specialized production that decreases the other firms' revenues by reducing the aggregate demand. This externality discourages a firm from continuing participation in the specialization when some other firms exit due to debt overhang. Thus, debt disorganization lowers aggregate productivity due to the "love-for-variety" structure.[1]

It is shown that a larger asset-price boom may lead to a deeper recession: When the asset-price boom is larger in the first period, the resulting debt overhang becomes larger, leading to a larger number of exiting firms (varieties), which implies a lower aggregate productivity due to the love-for-variety. In the extended model in which new-born firms can enter specialized production in period 2, it is shown that a larger boom-and-bust leads to a more persistent stagnation in which fewer or no new firms enter the economy. The number of new entrants is smaller because the expected profits of new entrants are negatively affected by the exits of incumbent firms due to the externality of debt disorganization. Thus, the number of new entrants decreases in the deeper recession after the larger asset boom. This mechanism simply explains the persistence of the post-crisis stagnation in our two-period setting.

Using this model, we compare ex-ante and ex-post policy interventions, including monetary policy. In particular, we emphasize the effectiveness of the ex-post policy to facilitate debt restructuring or debt relief. A subsidy to lenders for the restructuring of debt overhang can increase the recovery of debt for lenders and also improve productivity and social welfare. The result that the lenders are better off by reducing the face value of debt is the same as the classical argument of the debt Laffer curve (Sachs 1988; Krugman 1988), which is about the sovereign debt, while our focus in this paper is on corporate debt similar to Kobayashi et al. (2023). As argued in these studies, lenders may know that restructuring of debt overhang increases their payoff, and they can optimally reduce debt. In other words, the debt Laffer curve does not imply the necessity of policy intervention because the lenders themselves can choose the efficient amount of debt restructuring, unless there exist exogenous frictions. However, because there exists the externality of debt disorganization in our model, the amount of debt reduction without policy intervention is smaller than the socially optimal level. Thus, a policy intervention to encourage debt restructuring can be welfare improving in our model. This policy implication is one novelty of our study. To facilitate debt restructuring, the government can subsidize the lenders to partially compensate the loss of debt write-off so that the optimal amount of debt reduction is realized. Our result that debt relief improves productivity of the borrowers can be seen as complementary to that of Caballero, Hoshi and Kashyap (2008). They stress that zombie firms with debt overhang are intrinsically inefficient and should be liquidated. Our result points to the possibility that zombie firms may become productive if their debts are

[1] Lamont (1995) argue that the investment is reduced by the macroeconomic debt overhang due to the spillover effect, that is similar to the debt disorganization in our model. The difference is as follows. In the Lamont model, the spillover effect decreases investment and does not change productivity because the exits of firms from the specialization are not allowed in his model, while the debt disorganization causes endogenous productivity declines in our model because firms may exit the specialization.

forgiven. We also show that ex-post policy to encourage debt restructuring does not necessarily distort ex-ante incentives, that is, the time-inconsistency problem may not arise when the ex-post policy is subsidy to the lenders while the ex-ante allocation is decided by the borrowers.

The paper is organized as follows. The next section reviews the related literature. In Section 3, we describe the setting of the baseline model. Section 4 specifies the equilibrium and shows that a larger asset boom causes a deeper recession. Section 5 discusses the policy implications. In Section 6, we analyze the extended model, where new-born firms can enter specialized production after the asset-price collapse, and show that a larger asset boom leads to a more persistent stagnation. Section 7 concludes.

2 Literature

2.1 Empirical regularities

There is a large empirical literature that reports empirical regularities concerning the asset-price and credit booms and their effects on subsequent economic growth. Our model is an attempt to give an integrated account for the empirical regularities of the crisis cycle reported by the following literature. Jordà, Schularick, and Taylor (2015) analyze data of 17 countries for the past 140 years and show that the asset-price booms fueled by credit booms tend to end up with financial crisis, followed by deep and persistent recession. Greenwood, Hanson, Shleifer and Sørensen (2022) also report that a rapid growth in private credit and asset prices predicts a financial crisis.

Putting asset prices aside, some studies show that credit booms alone can predict the crises.[2] Schularick and Taylor (2012) analyze data on 14 countries for 140 years and report that credit booms tend to lead to financial crises. Verner (2019) reports based on the data of 143 countries for 60 years that credit booms in the short-run usually lead to financial crises. Giroud and Mueller (2021) find that a firm leverage boom predicts a boom-bust cycle of employment. Krishnamurthy and Muir (2024) report that credit grows high with too low credit spread in pre-crisis periods. They also report that a severe and protracted recession tends to follow the crisis.

Greenwood, Hanson, Shleifer and Sørensen (2022) emphasize that corporate debt buildups have adverse effects on the economy, as well as household debt. As our model emphasizes that corporate debt may be a significant driver of a financial crisis, this point is noteworthy. The dominant view in the literature since the Global Financial Crisis (GFC) has been that the household debt is the driver of financial crisis, and less attention has been paid to the corporate debt. Recently, however, there have emerged empirical studies

[2] Credit deepening in the long-run leads to higher long-term economic growth (King and Levine 1993), while the credit booms in the short-run tend to end up with crises.

that emphasize the importance of the corporate debt (Greenwood et al. 2022; Jordà et al. 2022; Sever 2023; Ivashina et al. 2024). Jordà et al. (2022) argue that corporate debt has a little power in predicting crises and it predicts slow recovery only in the countries with inefficient bankruptcy procedures. Ivashina et al. (2024) with data of bank loans in 115 countries over the period 1940–2014 show that the corporate debt accounts for the vast majority of nonperforming loans after the crisis and predicts slower recovery in average countries. They argue that the difference from Jordà et al. (2022) is due to coverage of data and definitions of corporate debt and crisis events. In particular, Ivashina et al. (2024) focus on the bank loans to firms, while Jordà et al. (2022) include corporate bond, which seems uncorrelated with crises. Kornejew, Lian, Ma, Ottonello, and Perez (2024) document that business credit booms are often followed by severe declines in output in environments with poorly functioning business bankruptcy. They also construct a model that accounts for declines in output in the aftermath of non-fundamental credit booms, where efficient bankruptcy systems can mitigate the declines. The difference from our model is that Kornejew et al. (2024) do not have asset prices in their model and they do not analyze policies to enhance debt restructuring explicitly.

There are studies that point to distinction between good credit booms with high economic growth and bad credit booms with low growth (Gorton and Ordoñez 2020). Müller and Verner (2023) report, based on the data of 116 countries for 80 years, that bad credit booms are mostly debt booms in non-tradable sector.[3]

It is also well known that financial crises tend to be followed by persistent productivity slowdown. Duval, Hong and Timmer (2020) argue that financial frictions might have caused the productivity slowdown during the Great Recession. Adler et al. (2017) report that productivity growth fell sharply after the GFC in 2008.[4] Related literature is on the great depressions, a decade-long deep recessions observed in the 20th century. It is said that deep and persistent productivity declines are the major cause of the great depressions (Hayashi and Prescott 2002, Kehoe and Prescott 2002). Our paper is also related to the literature on the Secular Stagnation, e.g., Rachel and Summers (2019) and Eggertsson, Mehrotra and Robbins (2019). In this literature, changes in the aggregate productivity

[3] Although our focus in this paper is not on whether the asset booms are caused by credit-supply or credit-demand shocks, we note that Verner (2019) reports that the short-run credit booms are usually driven by credit-supply expansion. Justiniano, Primiceri and Tambalotti (2019) also argue that the empirical facts about the housing boom preceding the Great Recession are consistent with the explanation that the boom was caused by an increase in credit supply, not in credit demand. Adverse effect of credit supply shock is also reported by Mian, Sufi and Verner (2017). They show that a credit supply shock induces a decrease in the interest rate and an increase in household debt with consumption boom, followed by persistently lower GDP growth.

[4] There is an opposite view that labor productivity increased in GFC. See Lazear, Shaw and Stanton (2013) who argue that people tend to work harder during recessions.

is taken as given exogenously, while our model provides an endogenous mechanism for persistent stagnation of productivity (see Section 6).

2.2 Theoretical ingredients

Our theory is a new attempt to integrate the following theories of risk-shifting asset booms, debt overhang, and aggregate output externalities or debt disorganization.

Risk-shifting effect on asset prices: This study is related to the literature on risk-shifting booms of asset prices, which are theoretically analyzed by Allen and Gale (2000) and Allen, Barlevy and Gale (2022). They demonstrate that asset-price booms can be driven by risk shifting by investors who buy the asset with borrowed money. In their models, the cost of default is exogenous and no policy response is possible ex-post, whereas in our model the ex-post debt reduction can reduce the inefficiency. The risk shifting from the firms to the lenders (households) in our model is possible due to the technological constraint that only firms can produce output, and the households cannot produce anything from capital.

Debt overhang: Our study is related to the broad literature of debt overhang. As Kobayashi, Nakajima and Takahashi (2023) argue, debt overhang can be categorized into two types. The first type of debt overhang is due to the lack of borrowers' commitment, and the second type is due to the lack of lenders' commitment. The debt overhang in this paper is the second type. We choose the second type because it seems consistent with our experience in the lost decades in Japan, associated with a persistent shrinkage of credit demand. The first type of debt overhang is analyzed by, e.g., Albuquerque and Hopenhayn (2004), Kovrijnykh and Szentes (2007), and Aguiar, Amador, and Gopinath (2009). In these models, the inefficiency is generated from the lenders' offer of back-loading payoff schedule to the borrowers in order to prevent the borrowers' default at the early stage. The second type of debt overhang is argued in macroeconomics by Sachs (1988), Krugman (1988), Occhino and Pescatori (2015), and Kobayashi, Nakajima, and Takahashi (2023). In the second type, the inefficiency arises because borrowers choose not to expend effort as the lenders cannot commit to reward their effort. The lack of lenders' commitment is caused by the fact that the lenders have legitimate right to take all when the amount of debt is larger than the borrowers' revenues. In this case, the lenders cannot credibly commit to give positive amounts to the borrowers to reward their efforts. Anticipating that the lenders will take all, the borrowers refrain from expending effort and make their production inefficient. Empirical evidence for inefficiency of (the second type of) debt overhang is provided by Honda, Ono, Uesugi and Yasuda (2024), who study turnaround of small and medium-sized enterprises in Japan.

Aggregate output externality or debt disorganization: In our model, the inefficiency of debt overhang is aggravated by the aggregate output externality, which is a spillover in the monopolistic competition. This spillover effect is argued by Lamont (1995) in the context of debt overhang (see also, e.g., Blanchard and Kiyotaki 1987). A difference between our model and Lamont's model is that there are no exits of firms and the aggregate productivity is invariant in his model, whereas exits of firms from the specialized production endogenously lower the productivity in our model due to the love-for-variety structure.[5] As we emphasize the negative effect of disruption of specialization among firms or disorganization of supply chains, we call this externality *debt disorganization*.[6] Debt disorganization or the aggregate output externality in our model works through the shortage of the aggregate demand, which is similar to Illing, Ono and Schlegl (2018). The difference is that the demand shortage in their model is due to unlimited liquidity preference, which is exogenously assumed, while the demand shortage in our model is caused by debt overhang.

2.3 Theoretical studies on financial crises and policy responses

This paper is related to the vast literature on financial crises and the policy responses. We can clarify the difference of our model from the existing studies in three aspects: The source of inefficiencies, the nature of inefficiencies, and the policy interventions. First, concerning the source of inefficiency, the literature primarily focus on pecuniary externality due to borrowing constraints (Aguiar and Amador 2011; Benigno et al. 2023; Bianchi 2011, 2016; Bianchi and Mendoza 2010; Farhi, Golosov, and Tsyvinski 2009; Gertler, Kiyotaki, and Queralto 2012; Lorenzoni 2008; Lorenzoni and Werning 2019) or coordination failure such as bank runs (Diamond and Dybvig 1983; Gertler and Kiyotaki 2015; Keister 2016; Keister and Narasiman 2016). On the other hand, the source of inefficiency in our model is debt overhang, which can emerge from various reasons such as news shocks, asset bubbles and overconfidence, even if pecuniary externality or coordination failure are nonexistent. Second, concerning the nature of propagation of inefficiencies, many existing models feature consumption misallocations (Bianchi 2011; Chari and Kehoe 2016; Farhi, Golosov, and Tsyvinski 2009; Jeanne and Korinek 2020; Keister 2016) or inefficient production due to increases in the cost of credit, that is, the credit crunch (Bianchi 2016; Bianchi and Mendoza 2010; Gertler, Kiyotaki, and Queralto 2012; Lorenzoni 2008). In these models, the aggregate productivity does not decline, and the inefficiencies arise from allocative frictions. In contrast to them, our model features the declines in aggregate productivity due to shortage of the aggregate demand. Third, concerning policy interven-

[5] See Philippon (2010) for multiple equilibria due to the similar spillover of debt overhang for households and banks. See also Occhino (2017) for a simplified model of multiple equilibria due to debt overhang.

[6] See Kobayashi and Inaba (2005) for the first usage of the term *debt disorganization*.

tions, the existing literature primarily focus on time inconsistency, that is, the trade-off between ex-ante incentive and ex-post efficiency that the bailout policy induces (Bianchi 2016; Chari and Kehoe 2016; Green 2010; Keister 2016: Keister and Narasiman 2016). Chari and Kehoe (2016) argue that bailouts can be welfare reducing because of the time inconsistency, while Bianchi (2016), Green (2010), Keister (2016), and Keister and Narasiman (2016) make the case that welfare improving effects of bailout policies overwhelm the adverse effects of time inconsistency. It is shown in our model that the time inconsistency of ex-post policy disappears and only welfare-improving effects survive under some circumstances where ex-post policy is subsidy to lenders and ex-ante allocation is decided by borrowers.

2.4 Zombie lending

Our theory is very closely related to the growing literature on the zombie lending or evergreening in the wake of financial crises. Zombie lending is the bank lending to non-viable firms due to distorted bank incentives. The pioneering works by Peek and Rosengren (2005) and Caballero, Hoshi and Kashyap (2008) report the proliferation of zombie lending in Japan during the 1990s. See also Sekine, Kobayashi and Saita (2003) for an early report of zombie lending or "forbearance lending" in Japan. Acharya, Lenzu and Wang (2021) and the references therein analyze models of zombie lending and report related empirical findings.[7] Acharya et al. (2021) emphasize that the accommodative government policy can distort bank incentive and induce zombie lending, leading to persistent stagnation. Our model complements to the existing literature of zombie lending in the following three respects. First, even without distortionary policy, large debt overhang can induce persistent stagnation in our model. Second, Acharya et al.(2021) implies that removal of distortionary policy is welfare improving, while introduction of active policy intervention is necessary to mitigate the aggregate output externality in our model. Third, most of the literature assumes that zombie firms are intrinsically inefficient and that their exits improve productivity and welfare through mitigating congestion, while our model implies that zombie firms may be able to restore efficiency by reducing their debt burden. Nakamura and Fukuda (2013) could provide support for our theory. They report that a significant portion of zombie firms in the non-tradeable sector in Japan that had difficulties in repaying debt in the 1990s have recovered and become productive in the 2000s, imply-

[7]The literature usually defines the zombie firms as firms that are kept afloat with subsidized loans from the lenders. Recently Rocheteau (2024) raises the possibility that the equity shares of firms with negative net present values (NPV) can be positively priced in the market without any subsidy. He calls these firms zombies and shows that they can exist if the equity shares of zombie firms provide liquidity under the environment with high liquidity demand. Since Rocheteau (2024) assumes that the negative NPV of zombie firms are exogenously given, they are not in our interest in this paper.

ing that debt-ridden zombie firms may not have been intrinsically unproductive. Becker and Ivashina (2022) empirically show that inefficient bankruptcy procedures amplify the inefficiency of zombie lending. Kornejew et al. (2024) support their arguments empirically and theoretically. These results may also support our argument on the welfare improving effect of debt reduction.

3 Model

The model is a two-period closed economy, in which households and firms are inhabited. In period 1, firms buy capital from households on credit, that is, they promise to pay consumer goods to households in period 2 in exchange for getting capital in period 1. In period 1, firms then install capital for specialization though its productivity, which is an aggregate shock, has not been revealed yet. In period 2, the productivity of capital is revealed. After the productivity is revealed, the lending households have a chance to restructure the firms' debt. The lenders can also liquidate the firms if the debt is not repayable. With the debt restructured, the borrowing firms decide whether or not to exit the market of specialized goods and default on the debt. Production and consumption take place only in period 2. Social welfare is maximized when the total output in period 2 is maximized. In Section 6, we extend the model by introducing new entries of new-born firms in period 2. For the convenience of the readers, we display the timeline of events in this economy.

- Period 1
  - (1) Firms purchase capital from households on credit without knowing the productivity of capital. They issue debt $D = Qk$, where Q is the price of capital, which may be higher than the fundamental price.
  - (2) Firms install capital for production.
- Period 2
  - (1) Productivity of capital (A_H or A_M) is revealed. Utility cost of S-production (ε_i) is also revealed for each firm $i \in [0, 1]$.
  - (2) Lenders (households) can restructure debt D to some level $\hat{D}$ ($< D$). Lenders can also choose to liquidate the borrowers if the debt is not repayable. Even if it is not repayable, the lender can also choose to keep D unchanged (i.e., Zombie lending).
  - (3) Borrowers (firms) can choose whether to produce specialized goods in S-sector or to produce consumer goods in C-sector (with very low productivity).

(4) Firms produce and sell the goods. Firms spend utility costs ε_i when they produce output in S-sector. They pay debts fully or partially. Firms also pay dividends, if any, to owners (households). Specialized goods in S-sector are aggregated to the consumer goods competitively. Households consume them.

3.1 Setup

There are two periods, period 1 and period 2, in the economy. There inhabits a unit mass of identical households and each household owns a firm. Thus, the measure of the firms is also unity. There are two production sectors, specialized production sector (S-sector) and common production sector (C-sector). The firms are initially in S-sector. The firms in S-sector can produce the intermediate goods from capital only in period 2, and the intermediate goods are aggregated into the consumer goods through the Dixit-Stiglitz production technology. The firms in C-sector can produce consumer goods directly with low productivity. The households can consume the consumer goods only in period 2. Each household is endowed with K units of capital at the beginning of period 1. The total amount of capital in the economy is thus K. Firms can produce goods from capital, whereas households cannot produce anything. In period 1, the firms choose the amount of capital, k, where $k \leq K$, to use for production. Each firm has to buy k from (another) household and install k in period 1 to prepare for production in period 2. As firms have nothing to pay for k in period 1, they issue debt D to buy k. That is, a firm i, where $i \in [0, 1]$, purchases k_i units of capital from household i in exchange for a promise to pay $D_i \equiv Qk_i$ units of period-2 consumer goods to the household, where Q is the price of capital in terms of period-2 consumer good. We use the same subscript for a lender (household) and her borrower (firm). We simply posit that the debt contract is the optimal financial contract in this economy.[8]

Production technologies: Initially in period 1, all firms are in S-sector, which stands for "Specialized production." They install capital in period 1 for production in period 2. In period 2, the productivity of capital in S-sector is revealed and the possibility of repayment of D becomes clear. When it is revealed that D is not fully repayable, lending households can choose debt restructuring or liquidation of the borrowers. Debt restructuring is to reduce debt D to $\hat{D}$ ($\leq D$) subject to certain agency frictions (see the next paragraph titled "Debt-restructuring technology"). Liquidation of a firm is that the lender seizes capital k from the firm and makes another manager operate k. For more details, see the paragraph titled "Debt-restructuring technology." Lender can also choose to continue Zombie lending, which is to keep the stipulated amount D unchanged. After the lender's

[8]We implicitly assume that the debt contract is optimal because there exist asymmetric information and agency problems a lá Townsend (1979) or Gale and Helwig (1985).

choice is made, the firms can choose whether to produce output in S-sector or to exit S-sector. The exited firms move to C-sector, which stands for "Common production," and produce output with lower productivity. After producing output in S- or C-sector, the firms repay $\hat{D}$ if revenues are greater than $\hat{D}$. If revenues are less than $\hat{D}$, they repay all revenues to the lenders and default on the remaining debt.

- **S-sector:** In S-sector, each firm produces specialized intermediate goods in the monopolistically competitive market. Productivity of capital in S-sector, A_s, is common for all firms. A_s is stochastic and revealed at the beginning of period 2. There are two states $s \in \{M, H\}$ in period 2. The state s becomes $s = H$, where $A_s = A_H$, with probability p_H, and becomes $s = M$, where $A_s = A_M$, with probability $p_M = 1 - p_H$. We consider the case where $A_M < A_H$.[9] The state M is the medium or "normal" state, whereas H is the high or "good" state. Given the realization of A_s in period 2, firm i, where $i \in [0, 1]$, produces the intermediate goods

$$y_i = A_s k_i,$$

where k_i is the amount of capital that firm i installed in period 1. To use k_i for production in S-sector, firm i must install k_i in period 1, and no more capital can be added in period 2. The consumption goods Y_S is produced from the intermediate goods y_i by the Dixit-Stiglitz aggregator:

$$Y_S = \left(\int_0^n y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where σ is the elasticity of substitution with $\sigma > 1$ and $n \in [0, 1]$ is the number of remaining firms in S-sector, which is endogenously decided as a result of firms' choice of exit at the beginning of period 2. The firms who exit S-sector goes to C-sector. The demand function for firm i's good is given as the solution to $\max_{y_i} Y_S - \int_0^n p_i y_i di$, where p_i is the price of the intermediate good i. The first order condition (FOC) implies

$$p = Y_S^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}}.$$

In this paper we focus on a symmetric equilibrium where each firm uses the identical amount of capital $k_i = \bar{k}$. In the symmetric equilibrium, the aggregate output in S-sector is given by

$$Y_S = n^{\frac{\sigma}{\sigma-1}} A_s \bar{k},$$

[9] For convenience, we show the examples of parameter regions to have default in equilibrium. Suppose $\sigma = 4$. Then, default occurs at state M in equilibrium when $A_H > 9.34 A_M$ for $p_H = 0.1$; $A_H > 1.67 A_M$ for $p_H = 0.5$; and $A_H > 1.45 A_M$ for $p_H = 0.75$.

where $n^{\frac{\sigma}{\sigma-1}}A_s$ is the total factor productivity (TFP) in S-sector, which is increasing in n. Revenue of a firm in S-sector is

$$py = Y_S^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}} = n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}} \equiv \pi(n, A_s, k),$$

where k is the firm's capital and $\bar{k}$ is the social level of capital per firm in S-sector. In what follows we use π as the abbreviation of $\pi(n, A_s, k)$ flexibly. In the symmetric equilibrium where $k = \bar{k}$, the revenue is $\pi = n^{\frac{1}{\sigma-1}} A_s k$.

- **C-sector:** In C-sector, a firm can produce $A_L k$ units of consumer goods from k units of capital in period 2. The firms need not install capital in period 1 for production in C-sector. Households can sell capital in period 2 for the use of C-sector, or firms in S-sector can move to C-sector and can use their capital k for production in C-sector in period 2, although they were installed in period 1 for production in S-sector. Productivity parameter in C-sector, A_L, is deterministic and satisfies

$$0 < A_L < A_M < A_H.$$

C-sector is a perfectly competitive market and firms do not have monopoly power there. In the symmetric equilibrium where $k_i = k$ for all i, the total output in C-sector, Y_C, is given by

$$Y_C = A_L(K - nk),$$

where n is the number of S-sector firms, k is the amount of capital per one S-sector firm, and thus nk is the total amount of capital used in S-sector.

- **Utility cost for S-production:** We assume a utility cost is necessary to undertake S-production, which is to produce the specialized intermediate goods in S-sector in period 2. The firm wants compensation for the utility cost when it produces output in S-sector. We make the following assumption:

Assumption 1. Firm i needs to expend an idiosyncratic utility cost ε_i in period 2 when it produces output in S-sector, where ε_i is a random variable that follows the cumulative distribution function $F(\varepsilon)$ and the density function $f(\varepsilon) = F'(\varepsilon)$, over $\varepsilon \in [0, \varepsilon_{\max}]$. For later convenience, we define $F(\varepsilon)$ over all $\varepsilon \in \mathbb{R}$ such that $F(\varepsilon) = 0$ for $\varepsilon \leq 0$ and $F(\varepsilon) = 1$ for $\varepsilon \geq \varepsilon_{\max}$. No utility cost is necessary to produce output in C-sector. The value ε_i is the consumption equivalence of the utility cost in terms of period-2 consumer goods, where

$$\varepsilon_{\max} < A_M K - A_L K. \tag{1}$$

The parameter ε_i is revealed at the beginning of period 2, and all agents have the identical expectations $\Pr[\varepsilon_i \leq \varepsilon] = F(\varepsilon)$ in period 1 about a firm's utility cost.

Debt-restructuring technology: In period 2, after the state s and the productivity of capital, A_s, are revealed and before production takes place, a lending household has the opportunity to forgive the debt or liquidate the borrower if the debt is not repayable. Even when the debt turns out not to be repayable, the lender can choose to continue Zombie lending, which is to keep D unchanged. Liquidation is described in the following assumption.

Assumption 2. Liquidation of firm i is that lender i seizes the capital k_i from firm i. Then, lender i operates the capital k_i by herself and produces output $A_s k_i$. We assume that a household (lender) cannot install the capital in period 1 but if the households seize the capital which has been installed by a firm, she can operate it and produce output. When the lender produce output in S-sector, she need to spend utility cost ε_h, which is picked up from the distribution $F(\varepsilon)$. We assume the value of ε_h is revealed only after the lender i seized the capital k_i.

The value that lender i can recover by liquidation of firm i can be denoted by R_L for now. In general, when a lender, such as a bank, forgives the debt or liquidates the borrower, significant coordination cost for the lender emerges. In this paper, we introduce a simple agency cost as a model of the various costs of debt restructuring or liquidation in reality. The agency problem is described in the following Assumption 3.

Assumption 3. A lender i ($\in [0, 1]$), which is a household, consists of one bank manager i (BM i) and a unit mass of depositors. BM i may be one of the depositors in household i. The depositors are the principal and BM i is their agent. BM i is supposed to maximize the gain of the depositors in lender i, and the depositors decide the reward to BM i as

$$\phi \times [\text{the depositors' expectation on their payoff}],$$

where ϕ is an infinitesimally small parameter. But the reward for BM i is not proportional to the true payoff of the depositors, because the depositors' expectation on their payoff is not the true payoff but a subjective expectation under information asymmetry. If BM i reduces debt D to $\hat{D}$, then the depositors notice that the value of debt has been impaired and that the true value is $R(\hat{D})$, where $R(\hat{D})$ is the actual repayment from the borrowing firm i when the stipulated amount of debt is $\hat{D}$. If BM i liquidates firm i, the depositors notice that the value of debt becomes R_L. Thus, the subjective expectation of the payoff for depositors becomes $\hat{R}(\hat{D}) = \max\{R(\hat{D}),\ R_L\}$ if BM i chooses debt restructuring or liquidation. In this case, BM i receives $\phi\hat{R}(\hat{D})$ as a reward. If BM i chooses to continue Zombie lending and keeps the contractual amount D unchanged, then the depositors in lender i mistakenly perceive with probability z that their payoff will be D, and with probability $1 - z$ that their payoff will be $R(D)$. Thus, in this case, the expected value of BM i's reward is $\phi[zD + (1 - z)R(D)]$. The depositors equally divide

$R(D) - \phi[zD + (1-z)R(D)] \approx R(D)$, because ϕ is infinitesimally small. The parameter z, which we assume for simplicity is common for all lenders, is the probability that the depositors in lender i mistakenly perceive that the value of debt is D, while the true value of the debt is in fact $R(D)$.

This assumption says that the bank manager i (BM i), who is supposed to work on behalf of the depositors in lender i, can make money by misleading them, i.e., by continuing Zombie lending, because Zombie lending gives BM i the chance to make depositors believe mistakenly that their payoff is D. On the one hand, if BM i implements a debt restructuring that reduces D to $\hat{D}$ or liquidation, the impairment of the value of debt becomes public information and all depositors know that they receive $\hat{R}(\hat{D})$. Therefore, the lender i (or bank manager i) will choose Zombie lending iff the following condition is satisfied.

$$\hat{R}(\hat{D}) < zD + (1-z)R(D). \tag{2}$$

We will show later by analyzing this condition that the agency cost in lenders makes Zombie lending more prevalent when the stipulated amount of debt D is larger.

In what follows, we describe the decision-making by households and firms backward.

3.2 Decision making in period 2

In the previous period (period 1), capital stock of each firm k and the debt for each firm $D = Qk$ were already determined. At the beginning of period 2, the aggregate productivity of S-sector ($A_s \in \{A_M, A_H\}$) is revealed, and the idiosyncratic utility cost ε_i to firm i for all $i \in [0,1]$ is also revealed. The number of firms n in S-sector is determined in equilibrium by lenders' and borrowers' decision-making in period 2, whereas their decisions are made, taking the expectation, n, as given. At this point, there are following two cases.

- **No default**: $\pi - D - \varepsilon_i \geq 0$.
  Note that π depends on n. In this case, firm i choose to stay in S-sector and produce specialized goods and pay D to the lender.

- **Default:** $\pi - D - \varepsilon_i < 0$.
  In this case, the lender i and borrower i sequentially solve the following problems:

  - **Lender's decision on debt restructuring.**
    Lender i chooses either debt restructuring, liquidation of the borrower, or Zombie lending.

  - **Borrower's decision on production.**
    After the lender's choice is made, the borrower (the lender in the liquidation case) chooses whether to produce output in S- or C-sector.

We will examine backwardly the borrower's decision first, and then the lender's decision.

Firms' decision on production: We suppose that the lender i has made decision to reduce the debt D to $\hat{D}_i$. Note that if the lender chose debt restructuring, it should be $\hat{D}_i < D$; and that if the lender chose Zombie lending, it should be $\hat{D}_i = D$. In the next paragraph,we will consider the case where the lender chose liquidation. The Free Entry Condition (FEC) for firm i to choose to operate in S-sector is written as

$$\pi(n, A_s, k) - \hat{D}_i \geq \varepsilon_i. \tag{3}$$

The firm continues to operate in S-sector and repay $\hat{D}_i$ if (3) is satisfied. The firm with $\pi - \hat{D}_i < \varepsilon_i$ has two options, i.e., either to earn $\pi(n, A_s, k)$ in S-sector and repay $\min\{\pi(n, A_s, k),\ \hat{D}_i\}$ to the lender, or to move to C-sector to produce $A_L k$ units of consumer good and repay all of them to the lender.[10] Since $\pi - \hat{D}_i < \varepsilon_i$, the firm obtains $\max\{0, \pi - \hat{D}_i\} < \varepsilon_i$ if it operates in S-sector. Since the firm pays the utility cost ε_i to produce output in S-sector (Assumption 1), the payoff for the firm becomes $\max\{0, \pi - \hat{D}_i\} - \varepsilon_i < 0$ if it operates in S-sector, whereas the payoff becomes $0 = \max\{0, A_L k - \hat{D}_i\}$ if it operates in C-sector. Therefore, the firm with $\pi - \hat{D} < \varepsilon_i$ will optimally choose to exit S-sector and go to C-sector. In sum, the revenue of firm i can be given by $y(\hat{D}_i)$, where

$$y_i(\hat{D}_i) = \begin{cases} \pi(n, A_s, k) - \varepsilon_i, & \text{if} \quad \pi(n, A_s, k) \geq \hat{D}_i + \varepsilon_i, \\ A_L k, & \text{if} \quad \pi(n, A_s, k) < \hat{D}_i + \varepsilon_i. \end{cases} \tag{4}$$

Equation (4) says that firm i goes to S-sector if $\pi \geq \hat{D}_i + \varepsilon_i$ and to C-sector otherwise. The borrower (firm i) will pay $\min\{\hat{D}_i,\ y_i(\hat{D}_i)\}$ to the lender as debt repayment.

Case of liquidation: As we see in Assumption 2, when the lender i chooses liquidation of firm i, she takes over the operation of the capital k and all revenue y_{Lh} accrues to the lender i, where

$$y_{Lh} = \max\{\pi - \varepsilon_h, A_L k\},$$

where ε_h is the utility cost to the lender i. The value ε_h is revealed after lender i liquidates firm i. At the time when the lender decides whether to liquidate or not, the productivity A_s has been revealed, but ε_h has not been revealed yet. The expected value of liquidation for the lender is

$$E_\varepsilon[y_{Lh}] = n^{\frac{1}{\sigma-1}} A_s k - H(n, k),$$

where $E_\varepsilon[\cdot]$ is the expectation operator over the distribution $F(\varepsilon)$ and

$$H(n, k) \equiv \int_0^{\varepsilon(n,k)} \varepsilon dF(\varepsilon) + (n^{\frac{1}{\sigma-1}} A_s k - A_L k) \int_{\varepsilon(n,k)}^{\varepsilon_{\max}} dF(\varepsilon), \tag{5}$$

where

$$\varepsilon(n, k) = \max\{0,\ n^{\frac{1}{\sigma-1}} A_s k - A_L k\}.$$

[10] Without loss of generality, we can focus on the case where $A_L k \leq \hat{D}_i$.

Lenders' decision on debt restructuring: Taking n as given and anticipating firms' decision (4), the bank manager of lender i (BM i) makes decision on debt restructuring or liquidation or Zombie lending. Each lender i consists of a unit mass of depositors and BM i who is supposed to work on behalf of the depositors. If there is no agency problem, the lender (depositors) would choose

$$\max\{\max_{\hat{D}} \min\{\hat{D}, y(\hat{D})\},\ E_{\varepsilon}[y_{Lh}]\}$$

subject to (4) and $\hat{D} \leq D$. However, there is the agency problem between BM i and depositors in lender i, as described in the previous subsection. Thus, BM i solves the following:

$$\max\{\max_{\hat{D}} \min\{\hat{D}, y(\hat{D})\},\ E_{\varepsilon}[y_{Lh}],\ zD + (1-z)A_L k\} \tag{6}$$

The first term inside the max operator is the payoff for BM i of debt restructuring, the second term is the payoff of liquidation, and the third term is the payoff of Zombie lending, i.e., not reducing the stipulated amount D (we omit the parameter ϕ). The third term means that without reducing the stipulated amount D, BM i receives ϕD with probability z, and receives $\phi A_L k$ with probability $1-z$, because the true value of repayment is $A_L k$ when debt D is larger than $\pi - \varepsilon_i$, as shown in (4). The solution is given explicitly, as follows. If $D \leq \pi(n, A_s, k) - \varepsilon_i$, then the lender chooses $\hat{D} = D$, and the firm earns $\pi(n, A_s, k)$ and repay D. In the case where $D > \pi(n, A_s, k) - \varepsilon_i$, it is obvious from (4) that

$$\arg\max_{\hat{D}} \min\{\hat{D}, y(\hat{D})\} = \pi - \varepsilon_i.$$

The condition (2) for BM i to choose Zombie lending can be rewritten as

$$\varepsilon_{Xi}(n,k) > G(n,k,D) \equiv n^{\frac{1}{\sigma-1}} A_s k - zD - (1-z)A_L k, \tag{7}$$

where $\varepsilon_{Xi}(n,k) \equiv \min\{\varepsilon_i,\ H(n,k)\}$ because $\hat{D}_i = R(\hat{D}_i) = \pi - \varepsilon_i$ and $R(D) = A_L k$. The lender i with (7) satisfied chooses Zombie lending and keep D unchanged, and the borrower goes to C-sector to earn and repay $A_L k$ to the lender, whereas the lender with (7) violated chooses either debt restructuring or liquidation so that the firm or the lender earns π in S-sector. In sum, we have proven the following lemma.

Lemma 1. *For given n, A_s and k, the revenue of the firm is $\pi = n^{\frac{1}{\sigma-1}} A_s k$ if it produces output in S-sector. When $D > \pi - \varepsilon_i$, the lenders choose*

- *debt restructuring such that $\hat{D}_i = \pi - \varepsilon_i$, if (7) is not satisfied and $\varepsilon_{Xi}(n,k) = \varepsilon_i$,*
- *liquidation such that $\hat{D} = 0$ and seize k, if (7) is not satisfied and $\varepsilon_{Xi}(n,k) = H(n,k)$, and*

- *Zombie lending such that $\hat{D} = D$, if (7) is satisfied.*

Once $D > \pi - \varepsilon_i$, the borrowing firms do not obtain anything (except the compensation of the utility cost ε_i).

Equilibrium selection: Let $N(n)$ be the measure of the pairs of lender i and firm i that satisfy $\varepsilon_{Xi}(n,k) \leq G(n,k,D)$, where n is given in $\pi(n, A_s, k)$ and $\varepsilon_{Xi}(n,k)$. In equilibrium (where k is fixed), the rational expectations, i.e., $N(n) = n$ must hold. Since $N(n) = n$ may have multiple solutions, the equilibrium values of n can be multiple. For example, $n = 0$ is always an equilibrium value, as $N(0) = 0$ because $\varepsilon_{Xi}(0,k) = -A_L k > G(0,k,D) = -zD - (1-z)A_L k$ for any $\varepsilon_i \geq 0$. We make the following assumption that agents are optimistic to eliminate the possibility of multiple equilibria due to pure coordination failure of expectations.

Assumption 4. When there exist multiple values of n, which satisfies $N(n) = n$, the expectations of households and firms are coordinated such that the largest value of n prevails as the commonly-held expectation in equilibrium.

This assumption says that the macroeconomic expectations are coordinated such that the most optimistic one among all feasible expectations prevails.

Debt overhang effect: Condition (4) implies that, when debt $\hat{D}$ is large, firms tend to exit S-sector and go to C-sector. The exit from S-sector is inefficient, because the exiting firm's capital cannot be used efficiently in S-sector with productivity A_H or A_M, but is used inefficiently in C-sector with the lowest productivity A_L. This individual inefficiency for an exiting firm can be called debt overhang effect, which is the inefficiency caused by the lack of lender's commitment in the following sense (Kobayashi, Nakajima and Takahashi 2022): When $\pi - \hat{D} < \varepsilon_i$, the firm would have chosen to continue operations in S-sector if the lender could promised to give ε_i to the firm to compensate the utility cost (Assumption 1); but, the lender cannot credibly commit to give ε_i because the lender has the legitimate right to take $\hat{D}$ and leave $\pi - \hat{D}$ $(< \varepsilon_i)$ to the borrower. The borrower precisely anticipates that the lender will leave less than ε_i, and chooses to exit S-sector to save the utility cost ε_i. This inefficiency is caused by the lack of commitment of the lender, that the lender cannot credibly commit to make the repayment strictly less than the contractual amount of debt $\hat{D}$.

Debt disorganization effect: In addition to the inefficient use of capital for the exiting firm itself, the exit of the firm has a negative externality on the other firms. The exit of one firm reduces the other firms' expected revenues of operating in S-sector by reducing the aggregate output Y_S, because the revenue of a firm π depends on Y_S: $\pi = py = Y_S^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}}$.

Since $Y_S = n^{\frac{\sigma}{\sigma-1}} A_s \bar{k}$, we can also rephrase this result as debt overhang decreases the TFP of S-sector, $n^{\frac{\sigma}{\sigma-1}} A_s$, by decreasing the equilibrium value of n. The number of firms in S-sector, n, represents the degree of specialization or the degree of organization of the supplier network. As this negative externality works through reducing the degree of organization, we call it debt disorganization in this paper. It is similar to the spillover effect in Lamont (1995), whereas the external effect that the aggregate output affects the value of n is present in our model, while it is not in Lamont's model.

3.3 Decision making in period 1

Firms promise to pay $D(k) = Qk$ units of consumer goods in period 2 in exchange for receiving k in period 1. The firms install k in period 1 for the S-production in period 2. There are two variables in period 1, Q and k, which are to be determined as solutions to the borrower and lender problems.

Borrower's problem: Firms know that the lenders' decision making in period 2 implies that a firm obtains zero if $\pi(n, A_s, k) - D(k) < \varepsilon_i$ in period 2, as shown in Lemma 1. Knowing this and taking n as given, the firms in period 1 solve

$$\max_k E[\max\{\pi(n, A_s, k) - \varepsilon_i - D(k),\ 0\}], \tag{8}$$

where $E[\cdot]$ is the unconditional expectation over A_s and ε_i. The FOC with respect to k is

$$E\left[\left(\frac{\sigma-1}{\sigma}\right) n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{-\frac{1}{\sigma}} - Q \;\middle|\; \text{ND}\right] = 0, \tag{9}$$

where $E[\cdot|\ \text{ND}]$ is the expectation conditional on no default, i.e., $\pi(n, A_s, k) - D(k) \geq \varepsilon_i$. The FOC must hold with equality since otherwise k goes to 0 or $+\infty$. In equilibrium where $k = \bar{k}$, this condition implies

$$Q = \left(\frac{\sigma-1}{\sigma}\right) E[n_s^{\frac{1}{\sigma-1}} A_s \mid \text{ND}]. \tag{10}$$

When the price Q is given by (10), the quantity of capital k is determined as $k = \bar{k}$ by (9), while $\bar{k}$ is determined by the supply, i.e., $\bar{k} = K$, in the symmetric equilibrium where the strict inequality holds in the lender's participation condition (see the next paragraph).

Lender's problem: As we argued in the paragraph of debt-restructuring technology, the bank manager of household i (BM i) maximizes the expected value of her reward in period 2, given that their choice is either to sell capital K to the firms in exchange for the risky debt or to hold the capital and sell it in the next period for the use in C-sector. The households' choice is limited to the two options because they are subject to the technological constraint that they cannot produce output in S-sector nor C-sector. (Note that only when lenders liquidate the borrowing firms in period 2, they seize and operate the capital in S- or C-sector.) Thus, BM i's decision-making in period 1 is degenerated such that they sell the capital to the firms if the following participation condition (PC) is satisfied, and they hold the capital until period 2 if the PC is not satisfied. The PC for households is given as follows. On the one hand, BM i's reward would be $\phi \times \rho Q$ units of period-2 consumer good by selling one unit of capital in period 1 in exchange for the debt that matures in period 2, where ρ is BM i's expectation about the depositors' subjective expectation on the value of recovery rate of debt, which is determined in equilibrium endogenously (see the next paragraph). On the other hand, when the lender i does not sell one unit of capital in period 1, BM i can obtain the reward A_L units of period-2 consumer good by selling it in period 2 as an input to C-sector, because the capital is used in S-sector only if it is sold and installed for specialization in period 1. Thus the PC for selling capital in period 1 is

$$\rho Q \geq A_L. \tag{11}$$

If the inequality in PC is strict ($>$), then all capital K is sold to the firms in period 1:

$$k = K.$$

If the PC holds with equality ($=$), then $k \leq K$. If the PC does not hold ($\rho Q < A_L$), then $k = 0$, and all capital is used in C-sector.

Recovery rate of debt: In the case of no default, the recovery rate of debt is 1. In the case of default, i.e., $\pi - D < \varepsilon_i$, the recovery rate is lower than one. Note that ρ is BM i's expectation about the depositors' expectation on the recovery rate of the debt, which depends on BM i's action: debt restructuring, liquidation or Zombie lending. From (6), ρ can be written as

$$\rho = \frac{1}{D} \times \left[\max\{\max_{\hat{D}} \min\{\hat{D}, y(\hat{D})\},\ E_\varepsilon[y_{Lh}],\ zD + (1-z)A_L k\} \right] \geq z + (1-z)\frac{A_L}{Q} > \frac{A_L}{Q},$$

where we used $D = Qk$ and $Q > A_L$ in equilibrium. Thus, (11) holds in equilibrium. We have proven the following lemma.

Lemma 2. *The participation constraint for lenders holds with strict inequality ($\rho Q > A_L$) in any equilibrium where $Q > A_L$.*

From this lemma, we can consider that $k = K$ holds in any relevant equilibrium.

3.4 Social optimum

We can consider the problem for the social planner who chooses k, the amount of capital installed in period 1 for S-sector, and n, the number of remaining firms in period 2 in

S-sector facing the realization of $A_s \in \{A_M, A_H\}$. We measure the social welfare by

$$E[n^{\frac{\sigma}{\sigma-1}} A_s k - \int_0^{\varepsilon_e} \varepsilon dF(\varepsilon) + A_L(K - nk)],$$

where ε_e is given by $F(\varepsilon_e) = n$. Since $A_L < A_M < A_H$ and $A_M K - \varepsilon_{\max} > A_L K$ by assumption (see equation (1)), production in S-sector is always more efficient than production in C-sector if $n = 1$. Thus, the socially optimal allocation is obviously $k = K$ and $n = 1$.

4 Equilibrium

Lemma 2 implies $k = K$ in equilibrium because the PC holds with strict inequality: $\rho Q > A_L$. The productivity parameter A_H represents the optimistic expectations that prevail in period 1. We will describe in this section how the equilibrium changes as the value A_H changes. The summary of the results is as follows: a larger A_H induces a higher price of asset and a larger debt in period 1; and when the productivity of capital turns out to be A_M ($< A_H$) in period 2, the lenders tend to continue more Zombie lending and the recession is deeper.

4.1 Equilibrium with a small A_H

When the parameter of optimism A_H is small, that is, A_H is larger than but close to A_M, there is no default in equilibrium. We clarify the conditions for debt overhang not occurring in any state, $A_s = A_H$ or $A_s = A_M$. Define $\xi = p_H(A_H/A_M) + 1 - p_H$. In the equilibrium where no default occurs, it should be that $k = K$ and $n = 1$. Condition (10) implies

$$Q^N = \left(\frac{\sigma - 1}{\sigma}\right) \xi A_M,$$

and $D^N = Q^N K$, where the superscript N denotes "no default." For simplicity of exposition, we assume only in this subsection that the point distribution, $\varepsilon_i = \varepsilon$ for all $i \in [0, 1]$, and ε is infinitesimally small. We also define an infinitesimally small number $\bar{\varepsilon}$ by $\varepsilon = \bar{\varepsilon} A_M K \sigma^{-1}$. Then, we have the following proposition.

Proposition 3. *We assume $\varepsilon_i = \varepsilon$ for all $i \in [0, 1]$ and ε is infinitesimally small. Suppose that A_L sufficiently small and A_H not too large, so that the following three conditions are* *satisfied:*

$$A_H < \left(\frac{1}{(\sigma-1)p_H} + 1\right) A_M - \left(\frac{\sigma}{\sigma-1}\right)\frac{\varepsilon}{p_H K}, \tag{12}$$

$$A_L < \left(\frac{\sigma-1}{\sigma}\right) A_M, \tag{13}$$

$$A_H < \frac{(1-p_H)}{\left((1-(1-p_H)\bar{\varepsilon})^{-\frac{1}{\sigma}} - p_H^{\frac{\sigma-1}{\sigma}}\right) p_H^{\frac{1}{\sigma}}} A_M. \tag{14}$$

Then, there exists an equilibrium without default, where $n = 1$ and $k = K$ and the debt is always fully paid back. The asset price is $Q^N = \left(\frac{\sigma-1}{\sigma}\right)\xi A_M$ and the debt is $D^N = Q^N K$.

See Appendix A for the proof. Note that these conditions (12), (13), (14) are sufficient conditions for the existence of the equilibrium without default. Note also that in the limit $\varepsilon \to 0$, (14) can be rewritten as $A_H < \frac{1-p_H}{(1-p_H^{\frac{\sigma-1}{\sigma}})p_H^{\frac{1}{\sigma}}} A_M$. With A_L sufficiently small and $\varepsilon \to 0$, the conditions in Proposition 3 are satisfied if, for example, $A_H < 2A_M$ for $p_H = 0.1$ and $\sigma = 4$, or $A_M < 1.4A_H$ for $p_H = 2/3$ and $\sigma = 4$. The intuition of this proposition is as follows: If A_H is not too large, the asset price is not too high and the debt is not too large, leading to no default in the state M. TFP is either A_M or A_H, which is strictly bigger than A_L. As $k = K$ and $n = 1$ in all states, the equilibrium without default is socially optimal (See Section 3.4). The social welfare W^N is given by

$$W^N = [p_H A_H + (1-p_H)A_M]K - E[\varepsilon].$$

4.2 Equilibrium with a large A_H

First, in Section 4.2.1, we specify the nature of the equilibrium with a large A_H, where debt overhang occurs when the productivity turns out to be A_M in period 2, and does not occur when it turns out A_H, on the premise that the equilibrium exists. Second, in Section 4.2.2, we then clarify the (sufficient) condition for its existence.

4.2.1 Nature of the equilibrium with default

We assume that the parameters satisfy

$$\left(\frac{\sigma-1}{\sigma}\right) A_H > A_M, \tag{15}$$

$$A_M K - A_L K > \varepsilon_{\max}, \tag{16}$$

$$\sigma^{-1} A_H K > \varepsilon_{\max}. \tag{17}$$

Now, suppose that the equilibrium with default exists. We can show by the guess-and-verify the equilibrium price of capital is

$$Q^B = \left(\frac{\sigma-1}{\sigma}\right) A_H, \tag{18}$$

where the superscript B denotes the "Boom" of asset prices. This price is justified as follows: Given Q^B, the debt becomes $D^B = Q^B K = (1-\sigma^{-1})A_H K$; as we assume (15), the firms default on D^B when $A_s = A_M$; as we assume (17), the firms do not default on D^B when $A_s = A_H$; and the FOC (10) is satisfied with Q^B. Since the expected value of the productivity of capital is ξA_M and $Q^N = \left(\frac{\sigma-1}{\sigma}\right)\xi A_M$, the equilibrium price Q^B is higher than the "fundamental" price Q^N. This is risk shifting, in which the firms bid up the price to Q^B because they are willing to buy capital at a higher price as they only care about the state of no default, i.e., $s = H$, and they do not care about the lenders' loss from their default at $s = M$. The number of firms in S-sector is $n = 1$ for $s = H$, and n is endogenously determined for $s = M$ by the lenders' decisions on debt restructuring in period 2. In the next paragraph, we analyze how n is determined in period 2 when A_s turns out to be A_M.

Equilibrium value of n when productivity is A_M: When the productivity turns out to be A_M, all firms default on the debt due to (15). Condition (7) for Zombie lending can be rewritten as follows in equilibrium where $k = K$, $D = Q^B K$, and $A_s = A_M$:

$$\min\{\varepsilon_i, H(n)\} > G(n), \tag{19}$$

where

$$\begin{aligned}
G(n) &\equiv G(n, K, (1-\sigma^{-1})A_H K) = n^{\frac{1}{\sigma-1}} A_M K - z(1-\sigma^{-1})A_H K - (1-z)A_L K, \\
H(n) &\equiv H(n, K) = \int_0^{\varepsilon(n)} \varepsilon dF(\varepsilon) + (n^{\frac{1}{\sigma-1}} A_M K - A_L K) \int_{\varepsilon(n)}^{\varepsilon_{max}} dF(\varepsilon), \\
\varepsilon(n) &\equiv \varepsilon(n, K) = \max\{0, n^{\frac{1}{\sigma-1}} A_M K - A_L K\},
\end{aligned}$$

with $\int_{\varepsilon(n)}^{\varepsilon_{\max}} dF(\varepsilon) = 0$ for $\varepsilon(n) > \varepsilon_{\max}$. Condition (19) implies that, in equilibrium, BM i (lender i) chooses

- debt restructuring if $\varepsilon_i \leq H(n)$ and $\varepsilon_i \leq G(n)$,
- liquidation if $\varepsilon_i > H(n)$ and $H(n) \leq G(n)$,
- Zombie lending if $\min\{\varepsilon_i, H(n)\} > G(n)$.

We have the following lemma about $G(n)$ and $H(n)$.

Lemma 4. *Define $A' = \{A_M - E[\varepsilon]K^{-1} - (1-z)A_L\}/\{z(1-\sigma^{-1})\}$, which is the value of A_H that satisfies $G(1) = H(1)$. For $A_H \leq A'$, there exists $\bar{n} \in [0,1]$ such that $G(\bar{n}) = H(\bar{n})$, $G(n) < H(n)$ for $n < \bar{n}$ and $G(n) > H(n)$ for $n > \bar{n}$. For $A_H > A'$, $G(n) < H(n)$ for all $n \in [0,1]$.*

See Appendix A for the proof. Figure 1 shows the graphs of $n = F(\varepsilon)$ and $\varepsilon = G(n)$ in $n - \varepsilon$ space. Obviously, an increase (or a decrease) in parameter A_H shifts down (or up) the graph $\varepsilon = G(n)$. So we can define a unique value $A'' \in (-\infty, +\infty)$ such that for $A_H \leq A''$, the two graphs ($n = F(\varepsilon)$ and $\varepsilon = G(n)$) have intersections and that for $A_H > A''$, the two graphs have no intersection. Now we can specify n^e, the equilibrium value of n.

Proposition 5. *When optimism is small such that $A_H \leq A'$, $n^e = 1$ in equilibrium, and BM i with $\varepsilon_i \in [0, E[\varepsilon]]$ chooses debt restructuring to reduce D to $\hat{D} = A_M K - \varepsilon_i$, while BM i with $\varepsilon_i \in (E[\varepsilon], \varepsilon_{\max}]$ chooses liquidation and operates the capital by herself. When optimism is large such that $A_H > A'$, we have $n^e < 1$ in equilibrium. There are the following two cases.*

- *Case with $A' \leq A''$. If $A_H \in (A', A'']$, the value of n^e (< 1) is given by $n^e = F(\varepsilon^e)$ and $\varepsilon^e = G(n^e)$. The value n^e is the largest one that satisfies the above simultaneous equations, due to Assumption 4. BM i with $\varepsilon_i \in [0, \varepsilon^e]$ chooses debt restructuring, and BM i with $\varepsilon_i \in (\varepsilon^e, \varepsilon_{\max}]$ chooses Zombie lending in which firm i goes to C-sector. If $A_H \in (A'', \infty)$, the value of n^e is given by $n^e = 0$ and all BMs choose Zombie lending and all firms go to C-sector.*

- *Case with $A' > A''$. For all $A_H > A'$, the value of n^e is given by $n^e = 0$ and all BM i's choose Zombie lending and all firm i's go to C-sector.*

See Appendix A for the proof.

Larger boom leads to deeper recession: Here we confirm that greater optimism (i.e., a larger A_H) induces a larger boom (i.e., a larger $Q^B = (1 - \sigma^{-1})A_H$) and a larger debt (i.e., a larger $D^B = Q^B K$). If the productivity of capital turns out to be A_M in period 2, the debt overhang is larger, and the number of firms in S-sector (n^e) and total output become lower.

Proposition 5 implies the following:

- In the case where $A' \geq A''$. If $A_H \leq A'$, all lenders choose debt restructuring or liquidation, and all firms operate in S-sector. Thus, $n^e = 1$ and $Y = A_M K - E[\varepsilon]$. If $A_H > A'$, all lenders choose Zombie lending and all firms operate in C-sector. Thus, $n^e = 0$ and $Y = A_L K$.

- In the case where $A' < A''$. For $A_H \leq A'$, $n^e = 1$ and $Y = A_M K - E[\varepsilon]$. For $A_H > A''$, $n^e = 0$ and $Y = A_L K$. For $A_H \in (A', A'']$, the equilibrium values of n^e

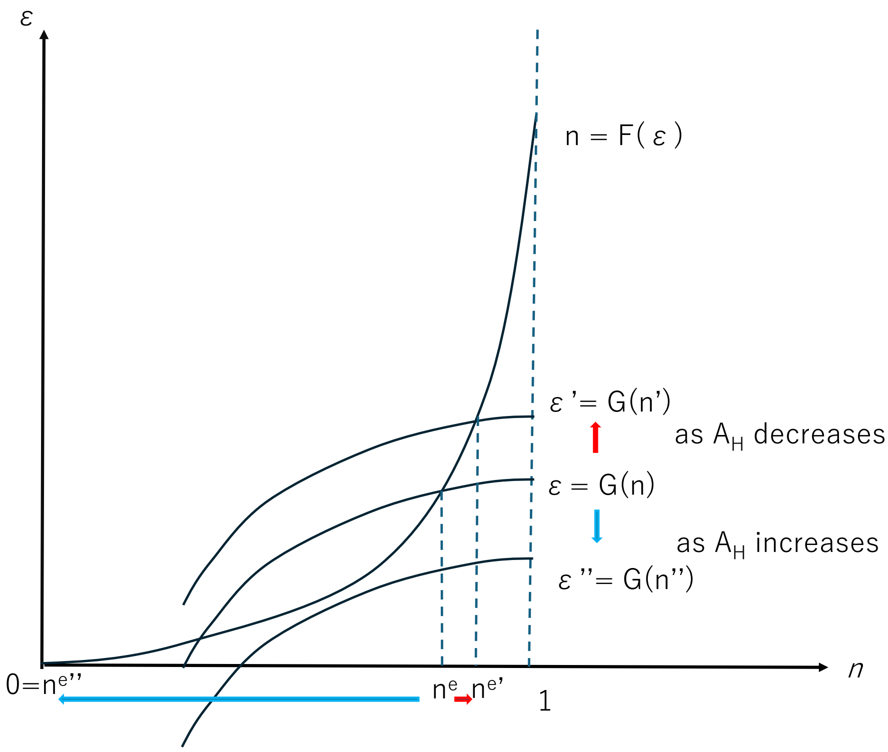

Figure 1: Larger boom (A_H) leads to smaller n

and Y are given by

$$n^e = F(G(n^e)),$$
$$Y = (n^e)^{\frac{\sigma}{\sigma-1}} A_M K - \int_0^{\varepsilon(n^e)} \varepsilon dF(\varepsilon) + (1-n^e)A_L K,$$

where $\varepsilon(n) = F^{-1}(n)$. It is shown that $\frac{\partial n^e}{\partial A_H} = -F'(G(n^e))z(1-\sigma^{-1})K < 0$. Thus, n^e is lower for a larger A_H. We have $\frac{dY}{dn} = \left(\frac{\sigma}{\sigma-1}\right) n^{\frac{1}{\sigma-1}} A_M K - \varepsilon(n) - A_L K$ with $n = n^e$. Substituting $\varepsilon(n^e) = G(n^e) = (n^e)^{\frac{1}{\sigma-1}} A_M K - z(1-\sigma^{-1})A_H K - (1-z)A_L K$, we have

$$\left.\frac{dY}{dn}\right|_{n=n^e} = \frac{1}{\sigma-1}(n^e)^{\frac{1}{\sigma-1}} A_M K + z((1-\sigma^{-1})A_H K - A_L K) > 0.$$

Thus, the total output Y is smaller for a larger A_H, because n^e is smaller for a larger A_H and Y is smaller for a smaller n^e.

On the aggregate productivity: The result that output in the ex-post recession is lower for a larger ex-ante asset boom is also shown by Allen, Barlevy and Gale (2022). Comparing our result with theirs makes clear the difference. Their result is derived from the assumption that an exogenous dead weight cost of default is increasing in the amount of defaulted debt. In our model, we endogenously derive the agency cost of debt restructuring, which is increasing in the amount of debt. In addition to this, it is shown in our model that the total production ($n^{\frac{\sigma}{\sigma-1}} A_M K$) and the total factor productivity in S-sector ($n^{\frac{\sigma}{\sigma-1}} A_M$) are increasing in n. A larger ex-ante boom leads to a lower n in our model, implying that the lower total factor productivity. On the other hand, in Allen, Barlevy and Gale (2022), there is no mechanism that an ex-ante boom leads to a lower productivity. The decreases in aggregate productivity in our model are due to the output externality or debt disorganization.

4.2.2 Existence of the equilibrium with default

In the following proposition, we specify the sufficient condition for the existence of the equilibrium.

Proposition 6. *The equilibrium with default exists if A_H is sufficiently large and satisfy*

$$A_H > \left(\frac{1}{(\sigma - 1)p_H} + 1 \right) A_M. \tag{20}$$

In this equilibrium, $k = K$, $Q^B = (\frac{\sigma-1}{\sigma})A_H$, and $D^B = Q^B K$. The number of firms in S-sector is $n = 1$ if $A_s = A_H$, and it is n^e, which is given in Proposition 5, if $A_s = A_M$.

Proof is given in Appendix A. Note that condition (20) is not compatible with condition (12), and it is equivalent to $A_M < Q^N$, meaning that the equilibrium without default cannot exist. This is because debt overhang is inevitable if the asset price is Q^N and A_s turns out to be A_M, as the revenue ($A_M K - \varepsilon$) would be strictly smaller than debt $D = Q^N K$. We focus in what follows on the case where condition (20) is satisfied.[11]

Asset boom may impair the ex-ante welfare: The ex-ante welfare is

$$W^B = p_H(A_H K - E[\varepsilon]) + (1 - p_H)(Y_M - \text{utility cost}),$$

where Y_M is the total output in the state M. Proposition 5 implies that $W^B = p_H A_H K + (1 - p_H) A_M K - E[\varepsilon]$ for $A_H \leq A'$, and $W^B = p_H(A_H K - E[\varepsilon]) + (1 - p_H) A_L K$ for $A_H > \max\{A', A''\}$. Figure 2 shows the ex-ante welfare as a function of A_H in the case where $A' < A''$. This figure shows that an increase in A_H or an increase in optimism has two opposing effects: a positive effect to increase W^B and a negative effect of debt overhang and debt disorganization to decrease W^B. The negative effect outweighs the positive effect at $A_H \in [A', A'']$ or at $A_H = A'$ if $A'' < A'$. The positive effect outweighs the negative effect otherwise.

[11] It may be possible that both equilibrium without default and with default coexist for A_H that satisfies (12). It may be possible to specify the conditions for multiple equilibria, but it is cumbersome and may not provide useful insights.

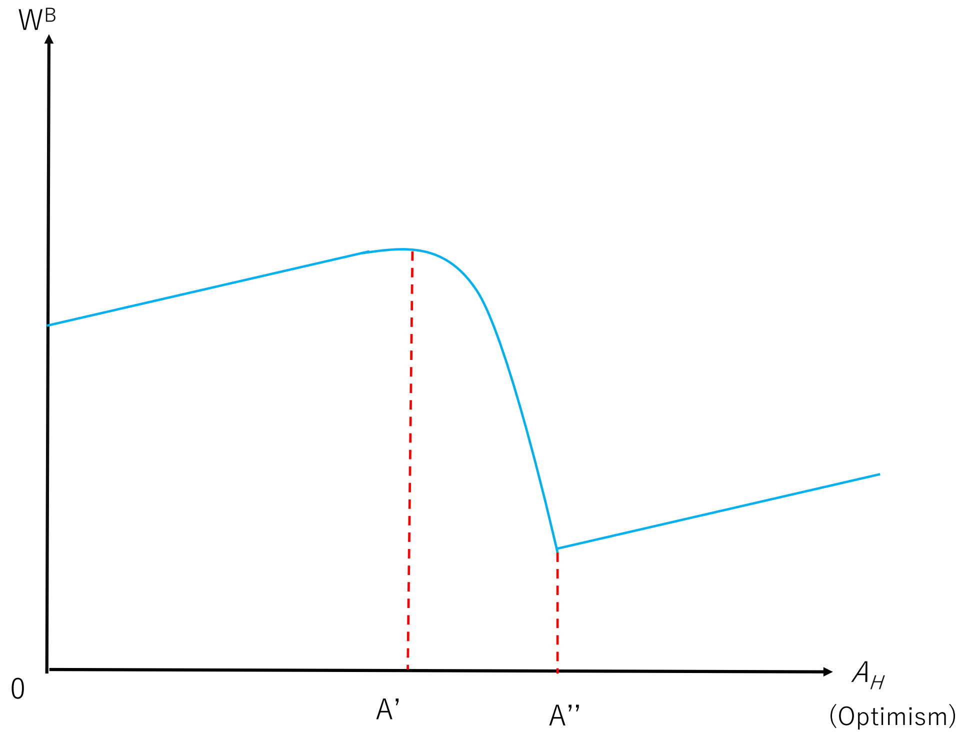

Figure 2: Ex-ante welfare and optimism

5 Policy

Our model enables us to assess ex-ante and ex-post policy interventions to the asset-price boom and subsequent debt overhang. In this section, we consider the case where (20) is satisfied, so that firms default on the debt when the productivity turns out to be A_M. In other words, we consider the case where there arrives a news shock in period 1 that the productivity of capital A_H can be extremely high in period 2. In Section 5.1, we first establish a benchmark policy that completely suppress the risk-shifting asset price boom. Then, we analyze ex-post subsidy to lenders for debt restructuring in Section 5.2, and ex-ante macroprudential policy in Section 5.3. Finally, in Section 5.4, we will argue about monetary policy in a modified model, in which nominal money is introduced as a unit of account.

The analysis in this section can be summarized in the following four points. First, the benchmark policy, that is the subsidy to borrowers to prevent default from occurring, can completely suppress the risk-shifting booms and attain the social optimum, though such a generous subsidy is difficult to implement in reality. Second, ex-post subsidy to encourage lenders to implement debt restructuring is welfare improving. In contrast to the existing literature, the ex-post policy does not cause time inconsistency in our model.

This is because the subsidy is to lenders, not to debt-ridden borrowers and the borrowers' actions in period 1 determine the equilibrium allocation. Third, ex-ante imposition of borrowing limit may be welfare improving, while finding the optimal borrowing limits for individual firms seems not possible in reality. Fourth, an ex-post monetary easing can be welfare improving if it can make inflation higher, as the inflation reduces the burden of debt overhang.

Parameter of debt restructuring cost: Before going on to the policy analysis, we confirm how z, which is a cost parameter of debt restructuring, affects the efficiency of the outcome. Obviously, reduction of z encourages the lenders to restructure the debt overhang, leading to an increase in n^e, that improves social welfare. This is similar to Kornejew et al. (2024) in that efficient bankruptcy systems increase output of defaulted firms. A key difference is that aggregate output externality (or debt disorganization) is present in our model, and not in theirs. The externality plays a crucial role in the policy implications in this section.

5.1 Benchmark: complete suppression of risk shifting booms

This subsection is based on discussion by Watanabe (2024). The asset price Q^B is higher than the fundamental price (Q^N) because the firms can default on their debt and they maximize $E[\max\{\pi - \varepsilon - D,\ 0\}]$ to solve (8). If the government can make the firms maximize $E[\pi - \varepsilon - D]$, instead of $E[\max\{\pi - \varepsilon - D,\ 0\}]$, then the FOC of (8) with respect to k would imply that the asset price equals the fundamental value (Q^N), and the risk-shifting boom of asset price is completely suppressed. It is straightforward from this argument to have the following lemma.

Lemma 7. *Suppose the government credibly announces in period 1 that it will give any borrower a sufficient amount of subsidy in period 2 to enable the borrower to pay the stipulated debt entirely. In this case, the asset price becomes the fundamental price (Q^N).*

This claim holds for any subsidy in general, as long as it can prevent the default, while it may be contingent on the realization of A_s but should be independent of the choice variable k. When the parameters satisfy (20), the government actually pays a positive amount of subsidy ex-post in state $s = M$. This is because (20) implies that the debt is larger than the revenue in the state $s = M$, that is, $D = Q^N K > A_M K = \pi$.

The above reasoning concerning Lemma 7 is basically given by Watanabe (2024) and it is so strong that any risk-shifting asset booms can be completely suppressed by the same policy intervention in the existing models such as Allen and Gale (2000) and Allen, Barlevy and Gale (2022).[12] However, the subsidy to make any borrower never default on

[12]The borrower subsidy that is financed by a lump-sum tax leads to the equilibrium where the funda-

their debt may not be realistic as a policy recommendation. Ex-ante moral hazard by the borrowers and the resultant amount of subsidy required would be unthinkably huge. So I describe the borrower subsidy here as a theoretical possibility, and move on to other policy tools in what follows.

5.2 Ex-post subsidy to debt restructuring

In this subsection, we focus on the case where the productivity of capital turns out to be A_M in period 2 and default occurs.

Assumption 5. There is a chance of government intervention at the beginning of period 2 after the aggregate shock $A_s = A_M$ is revealed and before lenders restructure the debt and borrowers produce outputs. A policy intervention at this stage is ex-post optimal if it maximizes the total output.

Ex-post problem for social planner: Given the debt overhang $D = Q^B K$, the social planner would maximize the total output Y by solving the following optimization problem:

$$\max_{n} \; n^{\frac{\sigma}{\sigma-1}} A_M K - \int_0^{\varepsilon(n)} \varepsilon dF(\varepsilon) + (1-n)A_L K, \tag{21}$$

where $\varepsilon(n) = F^{-1}(n)$. The threshold $\varepsilon(n)$ is imposed as the social planner internalizes the externality or debt disorganization. The social planner is also free from the agency problem. The optimal value ε^o is given by the FOC of the above problem:

$$(1-\sigma^{-1})^{-1} n^{\frac{1}{\sigma-1}} A_M K - \varepsilon(n) \geq A_L K. \tag{22}$$

The solution is $(\varepsilon^o, n^o) = (\varepsilon_{\max}, 1)$ as (16) implies that the inequality of the FOC is strict ($>$) at $(\varepsilon, n) = (\varepsilon_{\max}, 1)$.

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mental price of the risky asset is realized in Allen and Gale (2002):

$$r = f'(B - \bar{P}),$$
$$\bar{P} = \frac{1}{r}\left[\int_0^{R_{\max}} Rh(R)dR - c'(1)\right],$$

where $\bar{P}$ is the fundamental asset price and r is the loan rate, which is equal to the safe rate in the no-default equilibrium. The same is true for Allen, Barlevy and Gale (2022), in which the safe rate R and the fundamental asset price $\bar{p}^D$ are realized in the no-default equilibrium with borrower subsidy, where

$$R = \rho(\bar{p}^D),$$
$$1 + R = \frac{(1-\pi)(D + \bar{p}^D) + \pi(d + p^d)}{\bar{p}^D}.$$

See respective papers for notations.

Ex-post optimal policy: Consider that the government wants to attain $n^e = 1$ by giving a subsidy to the lenders.

Assumption 6. The government can give a subsidy to lender i, when she starts to implement debt restructuring or liquidation. The government cannot distinguish whether lender i is restructuring the debt or liquidating firm i, when the government gives the subsidy to the lender. Also, the government cannot observe the utility cost ε_i of firm i, when it grants a subsidy to lender i. The subsidy is financed by a lump sum tax on the households in period 2.

Suppose that the amount of subsidy is S. The payoff for the bank manager of lender i (BM i) is proportional to

- $n^{\frac{1}{\sigma-1}} A_M K - H(n) + S$, if she liquidates firm i,
- $n^{\frac{1}{\sigma-1}} A_M K - \varepsilon_i + S$, if she restructure the debt of firm i, and
- $zD + (1-z)A_L K$, if she continues Zombie lending.

This payoff implies that BM i chooses Zombie lending iff

$$\min\{\varepsilon_i,\ H(n)\} > G(n) + S,$$

which is a similar condition to (7). We can show the following proposition.

Proposition 8. *The optimal policy to achieve $n^e = 1$ is to give the following subsidy S to lenders who undertake debt restructuring or liquidation of the borrowers:*

$$S \equiv \max\{0,\ H(1) - G(1)\},$$

where $G(1) = A_M K - z(1-\sigma^{-1})A_H K - (1-z)A_L K$, and $H(1) = E[\varepsilon]$. For $A_H \leq A'$, the optimal policy is $S = 0$, that is, the policy intervention is not necessary to achieve $n^e = 1$. For $A_H > A'$, the optimal policy is $S = H(1) - G(1) > 0$. With this policy, lender i with $\varepsilon_i \in [0, E[\varepsilon]]$ chooses debt restructuring, and lender i with $\varepsilon_i \in (E[\varepsilon], \varepsilon_{\max}]$ chooses liquidation.

Proof. When $A_H \leq A'$, we have $G(1) \geq H(1)$. In this case, similar arguments as those in the proof of Proposition 5 imply that $n^e = 1$. For $A_H > A'$, we have $G(1) < H(1)$. In this case, if the government sets S such that $G(1) + S \geq H(1)$, then similar arguments to those above imply that $n^e = 1$. Thus, $S = H(1) - G(1)$ is the optimal policy.[13] □

[13] It can be easily shown that, if the government can distinguish whether the lender is undertaking debt restructuring or liquidation, then the optimal policy would be to give a subsidy $S = H(1) - G(1)$ only to the lenders who undertake liquidation. We do not take it as the main result because in reality the government cannot distinguish debt restructuring or liquidation at the onset.

This ex-post subsidy for debt restructuring can improve social welfare by mitigating the externality of debt disorganization. Debt disorganization can be seen as one example of externalities caused by the financial crisis, which can be resolved by debt restructuring, such as the counterparty risk among borrowing firms or the free-rider problem among lenders who have claims on the same borrower and want to free ride on the other lenders' debt restructuring. Our result demonstrates that an ex-post government intervention to enhance debt reduction can improve welfare by mitigating these externalities of financial crises.

Equilibrium with anticipated ex-post interventions: What happens if the lenders and borrowers anticipate in period 1 that government subsidy S will be given in period 2 when debt overhang occurs? The answer is that nothing changes except that n becomes $n^o = 1$ when debt overhang occurs. Given that the subsidy is for lenders, not borrowers, the firms obtain nothing when $\pi - \varepsilon_i < D$. While a change caused by the subsidy S is that the debt restructuring is implemented for some more firms, their payoffs are unchanged by debt restructuring. Their payoffs are kept at zero, as Lemma 1 shows. Note also that the restructured debt in period 2 changes by the policy intervention: $\hat{D} = \pi - \varepsilon$, as $\pi = n^{\frac{1}{\sigma-1}} A_M K$ changes because n changes from n^e to $n^o = 1$ with the subsidy to the lenders. We can show as follows that the equilibrium in period 1 does not change with anticipation of ex-post policy intervention. First, the ex-post debt restructuring policy affects the allocation only in the state where debt overhang occurs. Second, as long as the participation condition for lenders, $\rho Q \geq A_L$, continues to hold with strict inequality as shown in Lemma 2, the decision making by firms in period 1 is irrelevant to the anticipation about what happens in the state $s = M$ in period 2 where debt overhang occurs because the firms do not care about the debt-overhang state, where they obtain nothing in any case with or without subsidy. The conditions for the existence of the equilibrium without default are not affected by the anticipation of the government intervention, and thus Proposition 3 still holds. Concerning the equilibrium with default, we have the following proposition that shows the equilibrium is identical in period 1 no matter whether the ex-post policy interventions are anticipated.

Proposition 9. *We assume parameters satisfy* (20). *Suppose that all agents expect the government to give a subsidy S to lender i, conditional on implementing debt restructuring, if $D = D^B$ and $A_s = A_M$. Then, there exists the equilibrium with default, where $k = K$, $Q^B = \left(\frac{\sigma-1}{\sigma}\right) A_H$, and $D^B = Q^B K$. These values are the same as those in Proposition 6. If $A_s = A_H$, D^B is fully repaid and $n = 1$, while if $A_s = A_M$, the default occurs and the social optimum $(\varepsilon^o, n^o) = (\varepsilon_{\max}, 1)$ is realized by the subsidy S.*

Proof. The expectations of government intervention affects only ρ, which changes the participation condition (PC) for households' selling capital: $\rho Q > A_L$. It is obvious from

Lemma 2 that the PC holds with strict inequality, even when the government subsidy is anticipated. Therefore, nothing changes in conditions for equilibrium. □

5.3 Ex-ante macroprudential policy: Borrowing limit

It is easily shown that an imposition of the appropriately designed borrowing limit can modify the equilibrium in such a way that no default occurs when $A_s = A_M$. Suppose that the financial regulator imposes the borrowing constraint in period 1 that each firm's debt D cannot exceed $\bar{D}$, where

$$A_L K < \bar{D} \leq A_M K - \varepsilon_{\max}.$$

In this case, the asset price in equilibrium becomes $Q = \bar{D}/K$, and the PC is satisfied: $\rho Q = Q > A_L$. Each firm buys K units of capital in period 1, and when A_s turns out to be A_M in period 2, the firms can pay the debt $\bar{D}$, because their earnings are $A_M K$, given $n = 1$. There is no default and no exit from S-sector. The allocation, $k = K$ and $n = 1$, is socially optimal. In reality, it may be practically difficult to find the appropriate level of $\bar{D}$ for individual firms. Moreover, the optimality of the borrowing limit is crucially based on the assumption that A_s is a binary variable, i.e., $A_s \in \{A_M, A_H\}$. If A_s has a continuous value, the borrowing limit cannot prevent debt overhang from occurring with a positive probability, although it may be able to improve social welfare to some extent.

5.4 Monetary policy in a model with nominal variables

In this subsection, we analyze monetary policy. When the inefficiency is caused by debt overhang and its spillover, conventional monetary easing may not have a direct effect on restoring efficiency. However, we can show that an ex-post inflation may reduce debt overhang and improve efficiency.

Here, we modify our model by adding money. Money is just a unit of account used both in period 1 and period 2, and we assume that the quantity of money supplied is zero. Debt contract is made in terms of money. In period 1, a firm purchases k units of capital in exchange for debt $Q'k$, where Q' is the asset price in terms of money in period 1. Here the debt evolves at the loan rate $1 + I$ and the firm is obliged to repay $D' = (1 + I)Q'k$ in terms of money in period 2 to the lender household. We can define P_s as the price of period-2 consumer goods in terms of money in the state s, where $s \in \{M, H\}$. Then, the real burden of debt is $D_s = (1 + I)Q'/P_s$ in terms of period-2 consumer goods.

We assume that the central bank can set the nominal rate I and the nominal price levels P_s. Setting the nominal rate I in period 1 is ex-ante monetary policy, whereas setting P_s for $s \in \{M, H\}$ is ex-post monetary policy. We assume that the values of P_s

is anticipated by firms and households in period 1.[14] We will assess ex-ante and ex-post policies respectively.

Given I and P_s, a firm in period 1 maximizes the expected profit:

$$\max_k E[\max\{\pi - \varepsilon - D,\ 0\}],$$

where $\pi \equiv p(y)y = n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}}$ and $D_s = (1+I)Q'k/P_s$. FOC wrt k at $k = \bar{k}$ decides $(1+I)Q'$ by

$$E\left[P_s^{-1}|\text{ND}\right]\ (1+I)Q' = E[n^{\frac{1}{\sigma-1}} A_s\ |\text{ND}] \left(\frac{\sigma-1}{\sigma}\right)$$

The real burden of debt overhang D_s at the state $s \in \{M, H\}$ is

$$D_s = \frac{(1+I)Q'k}{P_s} = \frac{E[n^{\frac{1}{\sigma-1}} A_s\ |\text{ND}]}{E\left[P_s^{-1}|\text{ND}\right]} \left(\frac{\sigma-1}{\sigma}\right) P_s^{-1} k.$$

In this modified model, we focus on the equilibrium where the debt overhang ($\pi-\varepsilon-D < 0$) does not occur in the state H and occurs in the state M. Thus, since $n_H = 1$ and $E[P^{-1}|\text{ND}] = P_H^{-1}$, we have

$$(1+I)Q' = \left(\frac{\sigma-1}{\sigma}\right) A_H P_H,$$

$$D_H = \left(\frac{\sigma-1}{\sigma}\right) A_H K, \tag{23}$$

$$D_M = \left(\frac{\sigma-1}{\sigma}\right) A_H \frac{P_H}{P_M} K. \tag{24}$$

Ex-ante monetary policy: We assume period-2 prices (P_H and P_M) are fixed. Since $(1+I)Q' = \left(\frac{\sigma-1}{\sigma}\right) A_H P_H$ in equilibrium, a change in I is exactly offset by the corresponding change in Q' so that $(1+I)Q'$ is unchanged. (23) and (24) indicate that the nominal rate I is irrelevant to the real debt burden D_s and to the decision-makings by lenders and firms in both period 1 and period 2. It is obvious from this that ex-ante monetary policy, i.e., a change in I, has no effect on equilibrium allocation.

Ex-post monetary policy: Central bank decides period-2 prices, P_s for $s \in \{M, H\}$. We do not specify how central bank implement P_s, and just assume that it can decide P_s. This assumption is a shortcut for the description of monetary policy. We focus on the debt-overhang state $s = M$ in period 2, where lenders restructure debt to choose ε^e and n^e. As (24) indicates, a higher P_M reduces real burden of debt $D_M = \frac{(1+I)Q'K}{P_M} = \left(\frac{\sigma-1}{\sigma}\right) A_H P_H K P_M^{-1}$, and shifts $G(n) = n^{\frac{1}{\sigma-1}} A_M K - zD_M - (1-z)A_L K$ upward in Figure 1, increasing ε^e and n^e in equilibrium. A higher P_M is interpreted as ex-post monetary easing in state M. Therefore, the ex-post monetary easing, whether anticipated or unanticipated,

[14] Our results in this subsection hold qualitatively unchanged, even if the central bank can set the totally unexpected values of P_s.

can reduce the real debt burden D_M and increase efficiency and output. This policy implication holds on the premise that the central bank can control the price level.

There may also be other policy interventions such as tax/subsidy in C-sector.[15]

6 Sclerosis or Secular Stagnation

One of the empirical regularities of financial crises that we wanted to explain is that a persistent and decade-long recession often follows huge declines in asset prices. In this section, we demonstrate that an extended version of our model can explain the basic mechanism of this persistence, though it is still a two-period model. Before going on to the details, we summarize the intuition in advance: we consider period 2 when the productivity of capital turns out to be A_M and debt overhang occurs. Suppose that there exist new-born firms in period 2 who can potentially enter S-sector. They can enter S-sector by paying a fixed entry cost, and produce output in S-sector in period 2. If many new firms enter S-sector, the output will increase. In this case, we say, the recession is short-lived. If no one or very few new firms enter S-sector, we say, the recession is persistent. We can easily see that many firms enter when the debt overhang is small and no firms enter when the debt overhang is large. This is due to output externality or debt disorganization. Suppose that the mass e of new firms enter; the expected revenue for a new entrant, $\pi = (n+e)^{\frac{1}{\sigma-1}} A_M K$, is increasing in n, the number of remaining incumbent firms; when debt overhang is small, n is large and the expected revenue for a potential entrant exceeds the entry cost; When debt overhang is large, n is small and the expected revenue for a potential entrant is less than the entry cost; Then, the new firm chooses not to enter when debt overhang is large. In sum, we can explain the mechanism of persistence as follows: a large asset-price boom is often followed by a bust and huge debt overhang, which in turn depresses the new entry and leads the economy into a persistent recession. The persistent recession after the asset price collapse can be called “sclerosis” (Acharya, Lenzu and Wang 2021) or secular stagnation. On the other hand, the recession that follows a small asset boom is shallow and short-lived as there are many new entrants.

The policy implication of the extended model is basically the same as Section 5. In particular, our result implies that the policy intervention to subsidize debt restructuring

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[15] I thank Tack Yun for pointing to the policy issues of monetary policy and the tax/subsidy in C-sector. Consider a business income tax on firms in C-sector: $\tau A_L k$ for producing $A_L k$. With this policy, the effective productivity in C-sector becomes $(1-\tau)A_L$. An increase in τ increases n^e by shifting $G(n)$ upward in Figure 1, where

$$\varepsilon = G(n) = n^{\frac{1}{\sigma-1}} A_M K - z(1-\sigma^{-1})A_H K - (1-z)(1-\tau)A_L K.$$

The tax on C-sector, τ, can be welfare improving, given that tax revenue is transferred back to the households in a lump sum fashion.

may be able to attain the fast economic recovery without going through a deep recession.

6.1 Extended model – Larger boom leads to more persistent recession

We extend our model by adding the following assumption. Assumption 7 is a common knowledge for all agents in both period 1 and period 2. To simplify the model, we assume that the new-born firms are endowed with capital stock.

Assumption 7. In period 2, the mass λ of new firms are created, where $0 < \lambda < 1$. The new firms are owned by randomly selected λ households. A new firm is endowed with capital K when it is born in period 2. A new firm can enter S-sector by paying the entry cost, γK, which is a dead-weight loss, where (γ, λ) satisfy

$$A_M K - \gamma K - \varepsilon_{\max} > 0, \tag{25}$$

$$\lambda^{\frac{1}{\sigma-1}} A_M K - \gamma K < 0. \tag{26}$$

The new born firm makes the entry decision after its utility cost ε_i is revealed, where ε_i is a random variable with the distribution $F(\varepsilon)$. After the entry, the new firm produces and sells output to obtain the revenue

$$\pi = \{n + e(n)\}^{\frac{1}{\sigma-1}} A_M K,$$

where $e(n)$ is the measure of new entrants in period 2 that satisfies $0 \leq e(n) \leq \lambda$. The value of $e(n)$ is an equilibrium outcome. To produce output in S-sector, a new born firm needs to spend the utility cost ε_i.

In this extended model, we analyze how the entry decisions of new firms are affected by the size of A_H.

Entry decision of new firms: The entry decision of new-born firms is simple. As the payoff for a new firm to enter the S-sector is $\{n + e(n)\}^{\frac{1}{\sigma-1}} A_M K - \varepsilon_i - \gamma K$, it decides to enter iff the payoff is non-negative. This decision-making rule implies the following lemma.

Lemma 10. *For a given value of n, $e(n)$ is determined by*

$$e(n) = \lambda \int_0^{\bar{\varepsilon}(n)} dF(\varepsilon) = \lambda \Pr[\varepsilon \leq \bar{\varepsilon}(n)], \tag{27}$$

where

$$\{n + e(n)\}^{\frac{1}{\sigma-1}} A_M K - \bar{\varepsilon}(n) - \gamma K = 0, \tag{28}$$

and $\Pr[\varepsilon \leq \bar{\varepsilon}(n)] = 1$ *for* $\bar{\varepsilon}(n) > \varepsilon_{\max}$ *and* $\Pr[\varepsilon \leq \bar{\varepsilon}(n)] = 0$ *for* $\bar{\varepsilon}(n) < 0$.

Then we can show the following lemma.

Lemma 11. *$e(1) = \lambda$ and $e(0) = 0$.*

Proof. By definition, we have $(1+e(1))^{\frac{1}{\sigma-1}} A_M K - \gamma K = \bar{\varepsilon}(1)$. This equation and $e(1) \geq 0$ imply that $\bar{\varepsilon}(1) \geq A_M K - \gamma K > \varepsilon_{\max}$, where the last inequality comes from (25). Thus, we have $\Pr[\varepsilon \leq \bar{\varepsilon}(1)] = 1$, which implies $e(1) = \lambda$. Similarly, by definition, we have $(0+e(0))^{\frac{1}{\sigma-1}} A_M K - \gamma K = \bar{\varepsilon}(0)$. As $e(0) \leq \lambda$, we have $\bar{\varepsilon}(0) \leq \lambda^{\frac{1}{\sigma-1}} A_M K - \gamma K < 0$, where the last inequality comes from (26). Thus, $\Pr[\varepsilon \leq \bar{\varepsilon}(0)] = 0$ and $e(0) = 0$. $\square$

Determination of n: Simiar arguments as Section 4 determine the equilibrium value (n^e) of n. We define

$$
\begin{aligned}
G(n, e) &\equiv (n+e)^{\frac{1}{\sigma-1}} A_M K - z(1-\sigma^{-1}) A_H K - (1-z) A_L K, \\
H(n, e) &\equiv \int_0^{\varepsilon(n,e)} \varepsilon dF(\varepsilon) + ((n+e)^{\frac{1}{\sigma-1}} A_M K - A_L K) \int_{\varepsilon(n,e)}^{\varepsilon_{max}} dF(\varepsilon), \\
\varepsilon(n, e) &\equiv \max\{0, (n+e)^{\frac{1}{\sigma-1}} A_M K - A_L K\},
\end{aligned}
$$

We have the following lemma, which is a parallel to Lemma 4, about $G(n, e)$ and $H(n, e)$.

Lemma 12. *Define $\bar{A}' = \{(1+\lambda)^{\frac{1}{\sigma-1}} A_M - E[\varepsilon]K^{-1} - (1-z)A_L\}/\{z(1-\sigma^{-1})\}$, which is the value of A_H that satisfies $G(1, \lambda) = H(1, \lambda)$. For $A_H \leq \bar{A}'$, there exists $\bar{n} \in [0, 1]$ such that $G(\bar{n}, \lambda) = H(\bar{n}, \lambda)$, $G(n, \lambda) < H(n, \lambda)$ for $n < \bar{n}$ and $G(n, \lambda) > H(n, \lambda)$ for $n > \bar{n}$. For $A_H > \bar{A}'$, $G(n, \lambda) < H(n, \lambda)$ for all $n \in [0, 1]$.*

We omit the proof, which is the same as the proof for Lemma 4. Obviously, an increase (or a decrease) in parameter A_H shifts down (or up) the graph $\varepsilon = G(n, \lambda)$. So we can define a unique value $\bar{A}'' \in (-\infty, +\infty)$ such that for $A_H \leq \bar{A}''$, the two graphs ($n = F(\varepsilon, \lambda)$ and $\varepsilon = G(n, \lambda)$) in the $n - \varepsilon$ space have intersections and that for $A_H > \bar{A}''$, the two graphs have no intersection. Now we can show the following proposition, which is parallel to Proposition 5, about the equilibrium value of n.

Proposition 13. *Suppose that condition (20) is satisfied so that the equilibrium is the one with default, in which the equilibrium values of variables in period 1 are $k = K$, $Q^B = (\frac{\sigma-1}{\sigma})A_H$, and $D^B = Q^B K$. The number of firms in S-sector is $n = 1$ if $A_s = A_H$ in period 2. In the debt-overhang state where $A_s = A_M$ in period 2, the equilibrium becomes one of the following cases according to the value of A_H.*

- *When optimism is small such that $A_H \leq \bar{A}'$, $n^e = 1$ in equilibrium, and BM i with $\varepsilon_i \in [0, E[\varepsilon]]$ chooses debt restructuring to reduce D to $\hat{D} = (1+\lambda)^{\frac{1}{\sigma-1}} A_M K - \varepsilon_i$, while BM i with $\varepsilon_i \in (E[\varepsilon], \varepsilon_{\max}]$ chooses liquidation and operates the capital by herself. In this case, all the new born firms enter, that is, $e(1) = \lambda$.*

- *When optimism is large such that $A_H > \bar{A}'$, we have $n^e \leq 1$ in equilibrium. There are the following two cases.*

– *Case with $\bar{A}' \leq \bar{A}''$. If $A_H \in (\bar{A}', \bar{A}'']$, the value of n^e is given by $n^e = F(\varepsilon^e)$ and $\varepsilon^e = G(n^e, e(n^e))$. The value n^e is the largest one that satisfies the above simultaneous equations, due to Assumption 4. BM i with $\varepsilon_i \in [0, \varepsilon^e]$ chooses debt restructuring, and BM i with $\varepsilon_i \in (\varepsilon^e, \varepsilon_{\max}]$ chooses Zombie lending in which firm i goes to C-sector. In this case, $0 \leq e(n^e) \leq 1$. If $A_H \in (\bar{A}'', \infty)$, the value of n^e is given by $n^e = 0$ and all BMs choose Zombie lending and all firms go to C-sector. All the new born firms refrain from entering, that is, $e(0) = 0$.*

– *Case with $\bar{A}' > \bar{A}''$. For all $A_H > \bar{A}'$, the value of n^e is given by $n^e = 0$ and all BM i's choose Zombie lending and all firm i's go to C-sector. All the new born firms refrain from entering, that is, $e(0) = 0$.*

We omit the proof, as it is parallel to the proof of Proposition 5. The results about $e(n^e)$ is obvious from Lemma 11. This proposition says that one of the following three types of recession occurs in state M of the equilibrium with default, according to the degree of the optimism, i.e., A_H:

- **Short-term recession** for $A_H < \bar{A}'$: $n^e = 1$ and all new firms enter, i.e., $e(n^e) = \lambda$. The total number of firms in S-sector is $1 + \lambda$.

- **Persistent recession** for $A_H > \max\{\bar{A}', \bar{A}''\}$: $n = 0$ and $e(n) = 0$. The total number of firms in S-sector is 0.

- **Medium-term recession** for $A_H \in (\bar{A}', \bar{A}'')$: The medium-term recession occurs only if $\bar{A}' < \bar{A}''$. In this case, n^e and $e(n^e)$ are given by the equations $n^e = F(\varepsilon^e)$ and $\varepsilon^e = G(n^e, e(n^e))$. The equilibrium entry $e(n^e)$ is $0 \leq e(n^e) \leq 1$.

Broadly speaking, this proposition says that short-term recession with fast recovery occurs if debt overhang is small (i.e., A_H is small) and that persistent recession occurs if debt overhang is large (i.e., A_H is large).

6.2 Policy implications from the extended model

The previous subsection demonstrated that a large debt overhang subsequent to a large asset-price boom makes the stagnation persistent by discouraging entries of new firms. Policy implication of this extended model is that the ex-post optimal policy is to give sufficient subsidy that incentivize the lenders to implement debt restructuring and increase n, so that the new firms become willing to enter, i.e. $(n + \lambda)^{\frac{1}{\sigma-1}} A_M K - \varepsilon - \gamma K \geq 0$. Therefore, policy intervention to encourage the lender to restructure the debt is welfare improving in this model. This view could be interpreted as complementary to that in Acharya et al. (2021). Acharya et al. (2021) view that the persistent stagnation can result from the distortionary policy that facilitate zombie lending, which is a subsidy to the banks that extend and rollover the loans to the nonviable firms. Acharya et al. (2021) argues that the government policy that rewards the lenders for continuing to lend debt overhang makes the stagnation persistent, while we argue that the government policy that rewards the lenders for reducing debt overhang can stop the persistent stagnation. An implicit policy implication of Acharya et al. (2021) is that stopping the inefficient policy intervention may be sufficient to improve welfare. A value added of our argument to theirs is to indicate that stopping the policy to reward the lenders for continuing zombie lending may not be enough. As we show in the previous subsection, persistent stagnation can occur because of the aggregate output externality of debt disorganization, even without inefficient government policy. What is emphasized in our model is that it may be necessary for economic recovery to implement an active policy intervention that rewards the lenders for debt restructuring.

Our theory implies that policy intervention that encourages debt restructuring by the lenders attains the fast economic recovery without going through a deep (and short-lived) recession, because the zombie firms in our model can become productive once their burdens of debt are lifted. This result may be noteworthy as policymakers usually argue on the premise that the trade-off between the V-shaped (temporary but deep) and the L-shaped (shallow but persistent) recessions is inevitable. Our result implies that it may not be.

7 Conclusion

We demonstrated that the model of risk-shifting booms of asset prices and ex-post debt overhang can replicate empirical regularities of financial crises, i.e., credit-fueled asset boom tends to end up with the bust, followed by a deep and persistent recession with productivity declines. We focus on debt overhang as the main driver of inefficiencies in the aftermath of a financial crisis. The inefficiency of debt overhang simply accounts for the observation that a larger asset-price boom leads to a deeper and more persistent recession ex-post. Knowing that debt reduction increases lenders' payoff (the debt Laffer curve), lenders voluntarily reduce debt. However, as the inefficiency of debt overhang is aggravated by the externality of debt disorganization (or aggregate output externality), the lenders implement insufficient amount of debt relief on their own, and ex-post policy intervention that incentivizes debt restructuring can improve welfare by increasing the aggregate productivity and output. An example of such a policy intervention is an ex-post subsidy to the lenders for restructuring the debt overhang, which represents bank recapitalization in reality. We also showed that the inefficiency of time inconsistency may not emerge even when the ex-post subsidy is anticipated, as the subsidy is to the lenders while the equilibrium allocation is determined by the borrowers' actions. The tradeoff between the V-shaped (temporary but deep) recession and the L-shaped (shallow but persistent) stagnation may not be inevitable because timely and appropriate debt relief encouraged by policy intervention can achieve fast economic recovery without going through a deep recession. These results may shed some light on the relevance of the various policy responses to financial crises that may be worth analyzing further in a future study.

Appendix A: Proofs

A.1 Proof of Proposition 3

The condition for no debt overhang, $\pi - D - \varepsilon > 0$, in period 2 at $n = 1$ and $A_s = A_M$ is

$$\left[1 - \left(\frac{\sigma - 1}{\sigma}\right)\xi\right] A_M K > \varepsilon,$$

which is rewritten as (12). The PC for selling capital is satisfied with strict inequality if $\rho Q^N = Q^N > A_L$, where $\rho = 1$ because no default occurs in the equilibrium. This condition is satisfied if $\left(\frac{\sigma-1}{\sigma}\right)\xi A_M > A_L$. Since $\xi > 1$ the sufficient condition for $Q^N > A_L$ is (13). We focus on the parameter region where (13) is satisfied.

To complete the proof of existence of the equilibrium without default, we need to confirm there is no deviation. In the equilibrium, a firm could deviate in a way that it increases k to a certain value, k_d, such that it cannot repay $D_d = Q^N k_d$, when $A_s = A_M$, and it repays D_d only when $A_s = A_H$.[16] For the existence of the equilibrium, it is necessary to confirm that this deviation is not profitable. The expected profits for a firm when it does not deviate is $E[\pi^N - \varepsilon - D^N] = \xi A_M K/\sigma - \varepsilon$. The expected profits for a deviating firm is $E[\pi_d - \varepsilon - D_d \mid \text{ND}] = p_H\{A_H K^{\frac{1}{\sigma}} k_d^{\frac{\sigma-1}{\sigma}} - \varepsilon - Q^N k_d\}$. It is maximized by $k_d = \left(\frac{A_H}{\xi A_M}\right)^{\sigma} K$ and the maximized value of profits from deviation is

$$E[\pi_d - \varepsilon - D_d \mid \text{ND}] = p_H \frac{(\xi A_M)^{1-\sigma} A_H^{\sigma}}{\sigma} K - p_H \varepsilon$$

The condition for no deviation is $E[\pi^N - \varepsilon - D^N] > E[\pi_d - \varepsilon - D_d \mid \text{ND}]$. This condition can be rewritten as

$$(\xi A_M)^{\sigma} > p_H A_H^{\sigma} + \frac{\sigma}{K}\frac{(1 - p_H)\varepsilon}{(\xi A_M)^{1-\sigma}}.$$

By definition, we have

$$\varepsilon < \bar{\varepsilon}\xi A_M K \sigma^{-1}.$$

[16] Note that choosing k_d, which is larger than K, is feasible for an individual firm. This is because firms can choose any quantity under the market price Q^N. Although the optimal choice of quantity is K under the price Q^N in the symmetric equilibrium, the firms can choose a larger amount at will.

These two condition implies the sufficient condition for $E[\pi^N - \varepsilon - D^N] > E[\pi_d - \varepsilon - D_d \mid \mathrm{ND}]$ is

$$\{1 - (1 - p_H)\bar{\varepsilon}\}(\xi A_M)^\sigma > p_H A_H^\sigma,$$

which is rewritten as (14). This condition is satisfied if A_H is not so large.[17]

A.2 Proof of Lemma 4

Define $n_0 \equiv (A_L/A_M)^{\sigma-1}$. It satisfies $H(n_0) = 0$. For $n \in [0, n_0]$, it is easily shown that $H(n) = n^{\frac{1}{\sigma-1}} A_M K - A_L K$. Since $G(n) = n^{\frac{1}{\sigma-1}} A_M K - z(1 - \sigma^{-1})A_H K - (1 - z)A_L K$ and $(1 - \sigma^{-1})A_H > A_L$, it is obvious that $G(n) < H(n) \leq 0 = H(n_0)$ for $n \in [0, n_0]$. For $n > n_0$, it is shown that

$$G'(n) = (\sigma - 1)^{-1} n^{\frac{2-\sigma}{\sigma-1}} A_M K > (\sigma - 1)^{-1} n^{\frac{2-\sigma}{\sigma-1}} A_M K \int_{n^{\frac{1}{\sigma-1}} A_M K - A_L K}^{\varepsilon_{\max}} dF(\varepsilon) = H'(n).$$

This inequality, $G'(n) > H'(n)$ for $n > n_0$, implies that equation $G(n) = H(n)$ has at most one solution for $n \in [n_0, 1]$. Since an increase in A_H lowers $G(n)$ and $G(1) = H(1)$ for $A_H = A'$, it is straightforward that $G(n) = H(n)$ has only one solution for $A_H \leq A'$ and no solution for $A_H > A'$. This completes the proof.

A.3 Proof of Proposition 5

First, consider the case where $A_H \leq A'$. We will show $n^e = 1$ using the guess-and-verify method. By definition of A', we have $H(1) = E[\varepsilon] \leq G(1)$. For BM i with $\varepsilon_i \in [0, E[\varepsilon]]$, we have $\varepsilon_i \leq H(1) \leq G(1)$, which means that debt restructuring is preferable to liquidation or Zombie lending, on the premise that $n^e = 1$. For BM i with $\varepsilon_i \in (E[\varepsilon], \varepsilon_{\max}]$, we have $H(1) = E[\varepsilon] \leq \min\{\varepsilon_i, G(1)\}$, which means that liquidation is preferable to debt restructuring or Zombie lending, on the premise that $n^e = 1$. Therefore, given that $n^e = 1$, all lenders choose either debt restructuring or liquidation, resulting in all firms operating in S-sector. This result and Assumption 4 justifies the premise that $n^e = 1$.

Next, we consider the case where $A_H > A'$. This case is divided into the following two cases: $A' \leq A''$ and $A' > A''$.

- Case with $A' \leq A''$. There are two sub-cases.
  - Case where $A_H \in [A', A'']$. There exists at least one solution to $n = F(G(n))$. Since $G(n) < H(n)$ for all n, lenders never choose liquidation. Suppose that

[17] We could be interested in whether the deviated firm actually default on D_d when $A_s = A_M$, that is, whether $\pi(1, A_M, k_d) - Qk_d < \varepsilon$. But this inequality is not necessary for the existence of the equilibrium. Suppose $\pi(1, A_M, k_d) - Qk_d < \varepsilon$ is satisfied. In this case, the deviation is feasible and is not profitable as long as (14) is satisfied. Suppose $\pi(1, A_M, k_d) - Qk_d \geq \varepsilon$. In this case, the optimal deviation with default is not feasible and therefore the equilibrium can exist stably.

the equilibrium number of firms, n^e (> 0), is given as the largest solution to $n = F(G(n))$. Then, BM i with $\varepsilon_i \leq G(n^e)$ chooses debt restructuring and BM i with $\varepsilon_i > G(n^e)$ chooses Zombie lending. Thus the equilibrium condition is that $n^e = F(G(n^e))$ is satisfied. This is satisfied by definition of n^e.

– Case where $A_H \in (A'', +\infty)$. In this case, $n = F(G(n))$ has no solution. Suppose that agents have the expectations that the number of firms in S-sector is a positive number n^e (> 0). Then, BM i with $\varepsilon_i \leq G(n^e)$ chooses debt restructuring and BM i with $\varepsilon_i > G(n^e)$ chooses Zombie lending. Therefore, $n^e = F(G(n^e))$ must be satisfied in equilibrium, while this equation has no solution. Thus, $n^e = 0$, since $\varepsilon_i > G(0)$ for all i and all lenders choose Zombie lending when $n^e = 0$.

- Case with $A' > A''$. In this case, $A_H > A'$ means that $A_H > A''$, which means that $n = F(G(n))$ has no solution. Similar argument as above implies that $n^e = 0$ in equilibrium and all lenders choose Zombie lending. All firms go to C-sector.

A.4 Proof of Proposition 6

For the existence of the equilibrium with default, conditions (15) and (17) are necessary: (15) says firms default when the productivity is A_M, while (17) says firms can fully pay the debt when the productivity is A_H. Another necessary condition for existence of the equilibrium is that the firms have no incentive to deviate from the equilibrium. Now, we specify the condition for no deviation. The expected profit in period 1 for a firm in the equilibrium with default is

$$p_H(A_H K - E[\varepsilon] - D^B) = \frac{p_H A_H}{\sigma} K - p_H E[\varepsilon].$$

We used the fact that a firm with any value of ε_i operates in S-sector if productivity is A_H. Suppose that a firm considers to deviate from the equilibrium by reducing k to k_d so that it does not default on $D_d = Q^B k_d$ when $A_s = A_M$. The optimization problem for a deviating firm is

$$\max_{k_d} \; [p_H A_H + (1 - p_H) n^{\frac{1}{\sigma-1}} A_M] K^{\frac{1}{\sigma}} k_d^{\frac{\sigma}{\sigma-1}} - \left(\frac{\sigma - 1}{\sigma}\right) A_H k_d - E[\varepsilon], \tag{29}$$

$$\text{s.t.} \quad n^{\frac{1}{\sigma-1}} A_M K^{\frac{1}{\sigma}} k_d^{\frac{\sigma-1}{\sigma}} - \varepsilon_{\max} - \left(\frac{\sigma - 1}{\sigma}\right) A_H k_d \geq 0. \tag{30}$$

The condition (30) says that k_d is chosen such that the firm does not default on the debt when $A_s = A_M$. The solution to (29) on the premise that (30) is nonbinding is

$$k_d = \left[p_H + (1 - p_H) n^{\frac{1}{\sigma-1}} \frac{A_M}{A_H}\right]^{\sigma} K. \tag{31}$$

Substituting (31) into (30), it is shown that (30) is equivalent to

$$\left[1 - \left(\frac{\sigma - 1}{\sigma}\right)(1 - p_H)\right] n^{\frac{1}{\sigma-1}} A_M - \varepsilon'' \geq \left(\frac{\sigma - 1}{\sigma}\right) p_H A_H, \tag{32}$$

at the solution (31), where $\varepsilon'' = \varepsilon_{\max} K^{-1}[p_H + (1 - p_H)n^{\frac{1}{\sigma-1}}(A_M/A_H)]^{1-\sigma}$. If (32) is violated, the profit of the firm at A_M is negative, implying that (29) at k_d that satisfies (31) is smaller than the profit with default on the debt at A_M. With any k, the profit with default at A_M is weakly smaller than the profit of no deviation, i.e., $p_H A_H K/\sigma - p_H \varepsilon$, which is the maximized expected profit with default at A_M. Therefore, the deviation is not more profitable than no deviation, if (32) is violated. Thus, the sufficient condition for no deviation is

$$\left[1 - \left(\frac{\sigma - 1}{\sigma}\right)(1 - p_H)\right] n^{\frac{1}{\sigma-1}} A_M - \varepsilon'' < \left(\frac{\sigma - 1}{\sigma}\right) p_H A_H.$$

Since $\varepsilon'' > 0$ and $n \leq 1$, the sufficient condition for the above condition is $[1 - (\sigma - 1)\sigma^{-1}(1 - p_H)]A_M < (\sigma - 1)\sigma^{-1} p_H A_H$, which is equivalent to $A_M < Q^N$, and can be rewritten as (20).

Finally, the participation constraint for lenders is satisfied, as Lemma 2 implies that $\rho Q^B > A_L$ for $Q^B > A_L$.

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