Asset Price Booms, Debt Overhang and Debt Disorganization

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*Motivations

- To replicate empirical regularities of financial crises
  - **Credit-fueled asset price booms**
  - followed by **busts**, and deep and persistent **productivity declines**
- To provide new perspectives on inefficiency and policy
  - Inefficiency: Corporate **debt overhang** that discourages firms' activities
  - Why **debt overhang**?
    - Observe shortage of demand (for credit) in the aftermath of financial crisis
    - Literature: Credit supply frictions (borrowing constraint, credit crunch, . . .)
  - Policy: Ex-post **debt restructuring** can mitigate output externality
    - **Zombie firms can restore productivity if debt is (partially) forgiven**
    - Literature emphasizes congestion externality and elimination of Zombie firms
    - Time inconsistency may not be dominant

* What we do: Ingredients

- Two-period model
- **Risk-shifting asset boom** (Allen and Gale 2000; Allen, Barlevy and Gale 2022)
  - Firms buy the risky asset by borrowed money and can default on the debt
  - Borrowers bid up the ex-ante asset price by shifting the risk to banks
- **Debt overhang** (Sachs 1988; Krugman 1988; Kobayashi, Nakajima and Takahashi 2022)
  - Firms can produce output from the risky asset
  - When the lenders take all, borrowers do not expend efforts
    $\Rightarrow$ exit and stand-alone production (zombie)
- **Aggregate output externality** (Lamont 1995; Blanchard and Kiyotaki 1987)
  - Firms operate in monopolistic competition $\Rightarrow$ Spillover effect
  - Zombie firms exit due to debt overhang
  - Exit of one firm reduces productivity of all the other firms (love for variety)
    $\approx$ disorganization of supply network (**Debt Disorganization**)

*What we show: Results

- Ex-ante optimism

  ⇒ Asset-price boom is large (Risk-shifting boom) [Period 1]

  ⇒ Debt overhang if the revealed asset productivity is low [Period 2]

  ⇒ Firms' exit has negative **externality** and TFP declines (Debt disorganization)

- Larger asset boom ⇒ Larger debt overhang ⇒ More exits ⇒ Lower TFP

  ⇒ Fewer new entry ⇒ Persistent recession

- Knowing debt Laffer curve, lenders reduce debt voluntarily
- However, debt reduction is insufficient due to **externality**
  - For small debt, social optimum is achieved in equilibrium
  - For large debt, inefficiency prevails
- Ex-post subsidy for debt reduction improve social welfare by restoring aggregate productivity
- Ex-post subsidy may not induce ex-ante distortion

* Literature: Empirical regularities

- **Credit-fueled asset price booms** may lead to financial crises followed by deep and persistent recessions:
  - Jordà, Schularick, and Taylor (2015), Greenwood , Hanson, Shleifer and Sørensen (2022)
- Financial crisis followed by persistent **productivity slowdown**
  - Duval et al. (2020), Hayashi and Prescott (2002), Kehoe and Prescott (2002)
  - Zombie lending: Caballero, Hoshi and Kasyap (2008)
- **Corporate-credit booms** have a significant effect in persistent recessions
  - Greenwood et al. (2022), Jordà et al. (2022), Ivashina et al . (2024), Kornejew et al. (2024).

Our model: Integrated account for asset price, credit and productivity

Literature: Theoretical ingredients

- Risk shifting booms of asset prices
  - Allen and Gale (2000), Allen, Barlevy and Gale (2022)
- Debt overhang
  - Lack of lenders' commitment
    - Sachs (1988), Krugman (1988), Occhino and Pescatori (2015), Kobayashi, Nakajima Takahashi (2022)
    - Empirics: Honda, Ono, Uesugi and Yasuda (2024)
  - Lack of borrowers' commitment
    - Albuquerque and Hopenhayn (2004), Kovrijnykh and Szentes (2007), Aguiar, Amador and Gopinath (2009)
- Aggregate output externality ($\approx$ Debt disorganization)
  - Exit of one firm reduces revenues of the other firms in monopolistic competition
    - Related to the spillover effect in Lamont (1995) and Blanchard and Kiyotaki (1987)

Our model: New attempt to combine these theories

* Literature: Financial crisis

- Source of inefficiency $\Rightarrow$ **Our model: Debt overhang**
  - Pecuniary externality due to borrowing constraint: Aguiar and Amador (2011); Benigno et al. (2023); Bianchi (2011, 2016); Bianchi and Mendoza (2010); Farhi, Golosov, and Tsyvinski (2009); Gertler, Kiyotaki, and Queralto (2012); Lorenzoni (2008); Lorenzoni and Werning (2019)
  - Coordination failure: Diamond and Dybvig (1983); Gertler and Kiyotaki (2015); Keister (2016)
- Propagation $\Rightarrow$ **Our model: Lower productivity due to output externality (= debt disorganization)**
  - Inefficient consumption allocation: Bianchi (2011); Chari and Kehoe (2016); Farhi, Golosov, and Tsyvinski (2009); Jeanne and Korinek (2020); Keister (2016)
  - Lower output due to shortage of credit supply (i.e., credit crunch): Bianchi (2016); Bianchi and Mendoza (2010); Gertler, Kiyotaki, and Queralto (2012); Lorenzoni (2008).
- Time inconsistency in bailout policy
  - Bianchi (2016); Chari and Kehoe (2016); Green (2010); Keister (2016)

* Literature: Zombie lending

- Zombie lending: Bank loans with distorted incentives to non-viable firms
  - Japan: Peek and Rosengren (2005), Caballero, Hoshi and Kashyap (2008)
  - Acharya, Lenzu and Wang (2024) and references therein
- Zombie firms are intrinsically unproductive and exert negative congestion externalities. Should be eliminated.
- Becker and Ivashina (2022): Inefficient bankruptcy procedures amplify zombie lending

Our model: Debt reduction restores the efficiency of zombie firms

- Nakamura and Fukuda (2013): Zombie firms in the 1990s revived in the 2000s

* Model overview:

$A_H \uparrow \Rightarrow D \uparrow \Rightarrow n \downarrow \Rightarrow$ **TFP and** $Y \downarrow$ **(New entry** $\downarrow$**)**

- **Two-period model:** period 1, period 2.
  - Unit mass of firms (= borrowers) and households (= lenders)
  - Production and consumption take place only in period 2.
- In period 1, firms buy capital K on credit (promising to pay $D = QK$ in period 2). They produce $y = A_s K$ in period 2, where $A_s \in \{A_M, A_H\}$, $A_M < A_H$.
- Productivity A_s is **aggregate shock**: Not known in period 1, revealed in period 2
  - $A_s = A_H$ with p_H, $A_s = A_M$ with $1 - p_H$.
- Risk-shifting boom: Q (in period 1) is higher than the fundamental price.
- Then, if $A_s = A_M$ in period 2, debt D may not be repayable (Debt overhang)
- Lender i has three options about debt D under **agency friction** $\varepsilon_i \in [0, \varepsilon_{\max}]$
  - **Debt restructuring** (to reduce D to $\hat{D}$);
  - **Liquidation** (to operate K on her own);
  - **Zombie lending** (to keep D unchanged)
- n firms stay in **S-sector**, and $1 - n$ firms exit and go to **C-sector**

  $\Rightarrow$ Spillover (Aggregate output externality) $\Rightarrow$ **TFP** and $Y \downarrow$

Model setup

- Two-period closed economy: period 1, period 2
  - The productivity is uncertain in period 1, and is revealed in period 2
- Unit mass of households (HH) and firms: one HH owns one firm.
- Firm i has to buy k $(\leq K)$ units of capital at price Q in period 1 from other HH
- **Social welfare = Total consumption = Total output**

*Two production technologies

- **Specialized production sector (S-sector)**
  - Firms are in S-sector in period 1
  - S-production in period 2: $y = A_s k$, where $A_s \in \{A_M,\ A_H\}$
- **Common production sector (C-sector)**
  - Firms can move to C-sector any time
  - Stand-alone production
  - C-production in period 2: $y = A_L k$

- Productivity parameters: $0 < A_L < A_M < A_H$

- **Utility cost ε_i for S-production**: Firm i needs to expend (small) utility cost ε_i in period 2 to produce output in S-sector. (No need in C-sector) The cost ε_i is an idiosyncratic shock, where

$$\varepsilon_i \sim F(\varepsilon) \quad \text{and} \quad 0 \leq \varepsilon_i \leq \varepsilon_{\max}$$

*Production technology: Specialized production (S-sector)

- Firm i needs to install k_i in **Period 1**
- In **Period 2**, Firm i produces

$$y_i = A_s k_i,$$

  where $A_s \in \{A_M, A_H\}$ and $0 < A_M \ll A_H$.
- $A_s = A_H$ with prob p_H and $A_s = A_M$ with prob $p_M = 1 - p_H$.
- Symmetric equilibrium: The total output Y_S is given by

$$Y_S = \left(\int_0^n y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} = n^{\frac{\sigma}{\sigma-1}} A_s k$$

- $TFP = n^{\frac{\sigma}{\sigma-1}} A_s$: $\quad n \uparrow \Rightarrow TFP \uparrow$
- Suppose n firms produce output in S-sector
  - The S-firms choose price p to maximize earnings $\pi = py$:

$$p = p(y) \equiv Y_S^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}},$$

$$\pi = p(y)y = n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}}$$

Production technology: Common production (C-sector)

- No need to install capital in **Period 1**
- A firm with k can exit S-sector and go to C-sector anytime to produce $A_L k$ units of consumption good
- Total amount produced in C-sector Y_C is

$$Y_C = A_L(K - nk),$$

where $0 < A_L < A_M \ll A_H$.

*Debt overhang effect

- Suppose lenders reduce D to $\hat{D}$ under agency frictions (next page)
- Borrower's action: $\pi = p(y)y = n^{\frac{1}{\sigma-1}} A_M K$
  If $\hat{D} \leq \pi - \varepsilon_i \Rightarrow$ **no default** and repay $\hat{D}$
  - Borrower chooses to **produce $n^{\frac{1}{\sigma-1}} A_M K$ and repay $\hat{D}$ in S-sector**

  If $\hat{D} > \pi - \varepsilon_i \Rightarrow$ exit S-sector, and go to C-sector with **default**
  - Borrower's payoff in S-sector: $\max\{\pi - \hat{D},\ 0\} - \varepsilon_i < 0.$
  - Borrower's payoff in C-sector: $\max\{A_L K - \hat{D},\ 0\} = 0.$
  - Borrower chooses to **produce and repay $A_L K$ in C-sector**

- Debt overhang: **Larger debt makes output lower**
  - Produce $\pi = n^{\frac{1}{\sigma-1}} A_M K$ if $\hat{D}_i \leq \pi - \varepsilon_i$
  - Produce $A_L K$ if $\hat{D}_i > \pi - \varepsilon_i$

*Debt-restructuring technology (1/3)

- Lenders have incentive to reduce debt
- Lenders have three options about debt D:
  - **Liquidation**: To seize K and operate on her own
  - **Debt restructuring**: To reduce D to $\hat{D} = \pi - \varepsilon_i$ $(< D)$
  - **Zombie lending**: To keep D unchanged

- **Liquidation**
  - To reduce D to 0
  - seize K and operate it on her own in S-sector
  - spend utility cost ε_h in production, where $\varepsilon_h \sim F(\varepsilon)$
  - Liquidation decision is made before lender picks ε_h
  - **Expected value of liquidation (endogenous):** $R_L \equiv \pi - E[\varepsilon]$

*Debt-restructuring technology (2/3)

- In period 2, after A_s is revealed and before production, Lender i can choose
  1. **Liquidation**
  2. **Debt restructuring** to reduce debt from $D = Qk$ to $\hat{D}$
  3. **Zombie lending** to keep D unchanged
- They choose under **agency friction**:
  - Lender i consists of bank manager i (BM i) and unit mass of depositors (BM i is one of the depositors)
  - Depositors: **principal**
  - BM i: **agent** whose reward = $\phi \times$ [Depositors payoff (**subjective expectation**)]
  - **Information asymmetry**
    1. Liquidation or Debt restructuring makes depositors know the true value of payoff $\max\{R_L, R(\hat{D})\}$
    2. If D unchanged, **depositors believe the payoff is** D with prob. z where z is **probability of misperception**
- Zombie lending $\Rightarrow$ Expected value of depositors' belief: $zD + (1-z)A_L K$
- If D large, BM i earns higher rewards by misleading depositors

*Debt-restructuring technology (3/3)

- Given $(A_M,\ \varepsilon_i)$ revealed, BM i chooses to maximize the reward

$$\max\{\ \underbrace{\phi R_L,}_{\text{Liquidation}}\quad \underbrace{\max_{\hat{D}\le D}\phi R(\hat{D}),}_{\text{Debt restructuring}}\quad \underbrace{\phi[zD+(1-z)A_L K]}_{\text{Zombie lending}}\}$$

- **BM i choose Liquidation or Debt restructuring iff the utility cost ε_i is small**, such that

$$\max\{R_L,\quad \max_{\hat{D}\le D} R(\hat{D})\} \ge zD+(1-z)A_L K$$

- Larger $D \Rightarrow$ Lower threshold ε^e, where lenders choose liquidation or debt restructuring if $\min\{\varepsilon_i, E[\varepsilon]\} \le \varepsilon^e$

  $\Rightarrow$ Fewer firms operating in S-sector: $n^e = F(\varepsilon^e)$

* Equilibrium determination in period 1

- **Period 1**: Equilibrium variables are (Q, k)
  - **Borrower's profit max**: Revenue $\pi \equiv p(y)y = n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}}$, debt $D = Qk$

$$\max_k E[\max\{\pi - \varepsilon - D,\ 0\}],$$

    - Why D, not $\hat{D}$?: "$\pi - \varepsilon - D \leq 0$" $\leftrightarrow$ "$\pi - \varepsilon - \hat{D} = 0$" (page 24)
    - FOC wrt k decides

$$Q = E[n^{\frac{1}{\sigma-1}} A_s \mid \pi - \varepsilon - D \geq 0]\left(\frac{\sigma - 1}{\sigma}\right)$$

  - **Lender's participation decision:**

    Participation condition for HHs who sell k in exchange for risky debt in period 1

$$\rho Q > A_L \quad \Rightarrow \quad k = K,$$

    where ρ is the recovery rate of debt (endogenous).

* Equilibrium determination in period 2

- **Period 2**: Equilibrium variables are $(n, \hat{D})$
  - **Lender's debt restructuring decision** $\hat{D}$ (when $\pi - \varepsilon_i - D < 0$):

$$\max\{\underbrace{R_L}_{\text{Liquidation}}, \underbrace{\max_{\hat{D} \le D} R(\hat{D})}_{\text{Debt restructuring}}, \underbrace{zD + (1-z)A_L K}_{\text{Zombie lending}}\}, \quad \text{where} \quad R(\hat{D}) = \begin{cases} \hat{D} & \text{if } \pi - \varepsilon_i \ge \hat{D}, \\ A_L K & \text{if } \pi - \varepsilon_i < \hat{D}. \end{cases}$$

$\Rightarrow$ Restructure D to $\hat{D}_i = \pi - \varepsilon_i = \arg\max_{\hat{D}} R(\hat{D})$, if $\varepsilon_i \le \varepsilon^e$

  - **Borrower's exit decision:**

$$\max\{\underbrace{\max\{\pi - \hat{D},\ 0\} - \varepsilon_i}_{\text{Stay in S-sector}}, \underbrace{\max\{A_L K - \hat{D},\ 0\}}_{\text{Exit and go to C-sector}}\}$$

Free Entry Condition (FEC) for firms

- $\pi - \varepsilon_i - \hat{D} \ge 0$: Firms stay in S-sector
- $\pi - \varepsilon_i - \hat{D} < 0$: Firms exit to C-sector

Debt restructuring decision in period 2

- Lender chooses Zombie lending iff $\min\{\varepsilon_i,\ H(n)\} > G(n)$, where

$$G(n) = n^{\frac{1}{\sigma-1}} A_M K - zD - (1-z)A_L K,$$

$$H(n) = \int_0^{\varepsilon(n)} \varepsilon dF(\varepsilon) + (n^{\frac{1}{\sigma-1}} A_M K - A_L K) \int_{\varepsilon(n)}^{\varepsilon_{\max}} dF(\varepsilon),$$

$$\varepsilon(n) = \max\{0, n^{\frac{1}{\sigma-1}} A_M K - A_L K\}.$$

  - If $\varepsilon_i \leq H(n)$ and $\varepsilon_i \leq G(n)$, then **Debt restructuring**: $\hat{D} = \pi - \varepsilon_i$ and firm i stays in S-sector,
  - if $\varepsilon_i > H(n)$ and $H(n) \leq G(n)$, then **Liquidation**: $\hat{D} = 0$ and capital of firm i stays in S-sector,
  - if $\min\{\varepsilon_i, H(n)\} > G(n)$ then **Zombie lending**: $\hat{D} = D$ and firm i goes to C-sector.

- **Lender takes all for any** $\hat{D}$, if $\pi - \varepsilon_i - D \leq 0$
- **Borrower obtains nothing**, if $\pi - \varepsilon_i - D \leq 0$

*Equilibrium

- For smaller A_H, **No default**
  - No default in any state, A_M or A_H
  - $Q^N = \left(\frac{\sigma-1}{\sigma}\right)[p_H A_H + (1-p_H)A_M]$, and $D^N = Q^N K$
  - $n = 1$ (social optimum)

- For larger A_H, **Default → Debt Overhang**
  - No default if A_H, and **default if** A_M
  - $Q^B = \left(\frac{\sigma-1}{\sigma}\right)A_H$, and $D^B = Q^B K \Rightarrow$

$$\pi(n) = n^{\frac{1}{\sigma-1}} A_M K,$$

$$D = D^B = (1-\sigma^{-1})A_H K$$

  - A_H larger $\Rightarrow$ n smaller $\Rightarrow$ TFP and output smaller

1. For A_H relatively small, **liquidation** and **debt restructuring** → Social optimum
2. For A_H relatively large, **Zombie lending** → Inefficiency

Appendix: Equilibrium

- For smaller A_H, no default in any state, A_M or A_H
  - $k = K$, $Q^N = \left(\frac{\sigma-1}{\sigma}\right)\{p_H A_H + (1-p_H)A_M\} \approx \left(\frac{\sigma-1}{\sigma}\right)A_M$, and $D^N = Q^N k$
  - $n = 1$. Output is $Y = A_s K$, where $A_s = A_M$ or A_H.
  - Social welfare: $W^N = p_H A_H K + (1-p_H)A_M K - E[\varepsilon]$. (Socially optimal)

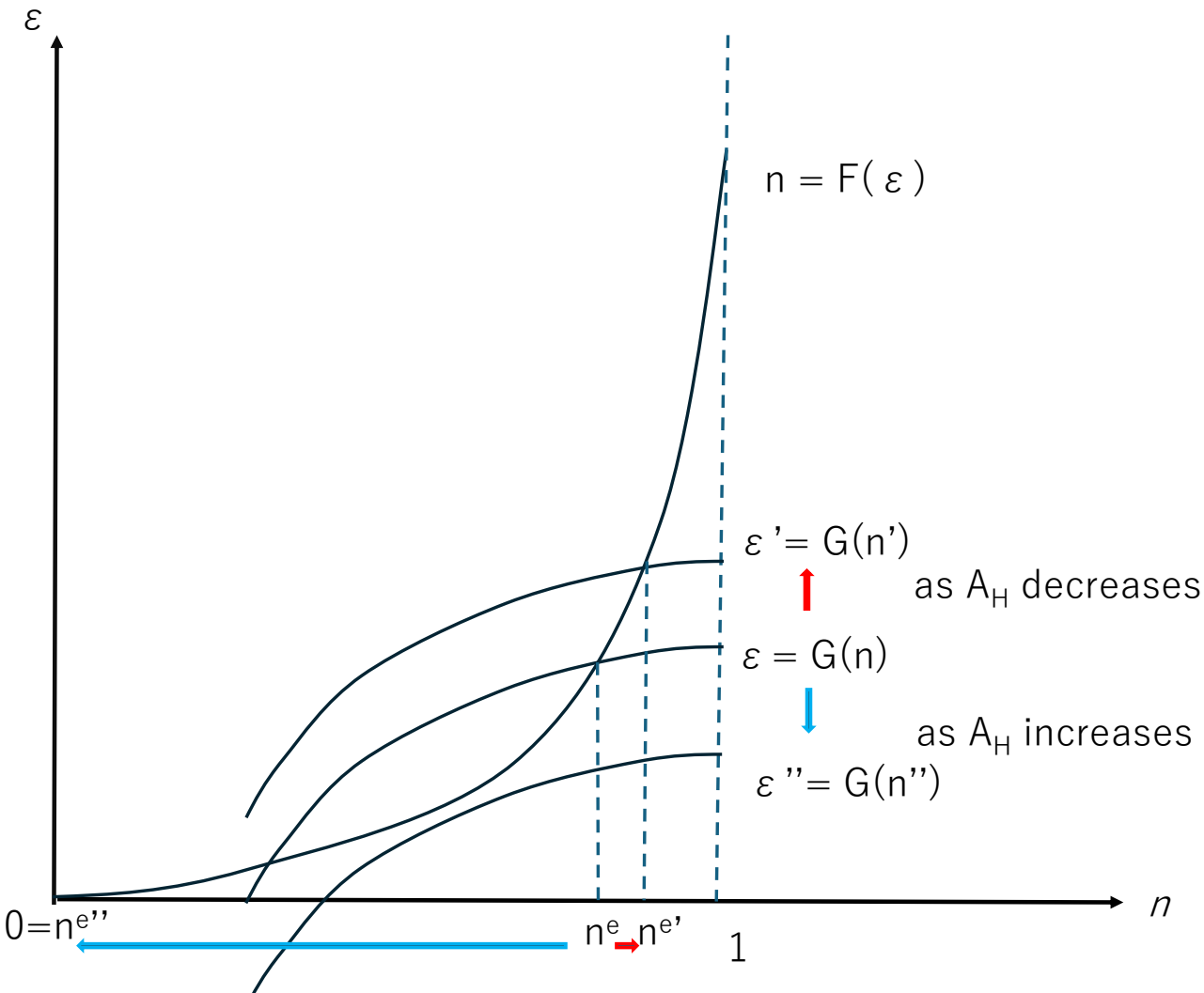
- For larger A_H, no default if A_H, and default if A_M
  - $k = K$, $Q^B = \left(\frac{\sigma-1}{\sigma}\right)A_H$, and $D^B = Q^B k$
  - If $A_s = A_H$, then $n = 1$ and $Y = A_H K$.
  - If $A_s = A_M$, then ε^e and n^e are given by $\varepsilon = G(n)$ and $n = F(\varepsilon)$.
    - n **is smaller for a larger boom (A_H)** $\Rightarrow$ See next slide
    - Firms with $\varepsilon_i \leq \varepsilon^e$ stay in S-sector, and those with $\varepsilon_i > \varepsilon^e$ default and go to C-sector. $Y(n) = Y_S(n) + (1-n)A_L K < A_M K$
  - Social welfare: $W^B = p_H A_H K + (1-p_H)Y(n) - n\varepsilon$, where $Y(n) < A_M K$.

*Equilibrium value of n

- Can show: when ex-ante optimism is larger (A_H larger), ex-post recession is deeper (n smaller)
- There exist thresholds A' and A'', where $A' < A''$.
- **Proposition 5**
  - For $A_H \leq A'$, all firms stay in S-sector ($n^e = 1$)
    - Lenders with $\varepsilon_i \in [0, E[\varepsilon]]$ choose **Debt Restructuring**
    - Lenders with $\varepsilon_i \in (E[\varepsilon], \varepsilon_{\max}]$ choose **Liquidation**
  - For $A_H \in (A', A'')$, n^e is given by $n = F(\varepsilon)$ and $\varepsilon = G(n)$.
    - Lenders with $\varepsilon_i \in [0, \varepsilon^e]$ choose **Debt restructuring**. n^e firms stay in S-sector.
    - Lenders with $\varepsilon_i \in (\varepsilon^e, \varepsilon_{\max}]$ choose **Zombie lending**. $1 - n^e$ firms go to C-sector.
  - For $A_H \in (A'', +\infty)$, all lenders choose **Zombie lending** and all firms go to C-sector ($n^e = 0$)

*Larger boom ⇒ Deeper recession: Graphical explanation

$$\varepsilon = G(n) = n^{\frac{1}{\sigma-1}} A_M K - z\left(\frac{\sigma-1}{\sigma}\right) A_H K - (1-z)A_L K, \qquad n = F(\varepsilon).$$



* Larger boom $\Rightarrow$ Deeper recessions: Intuition

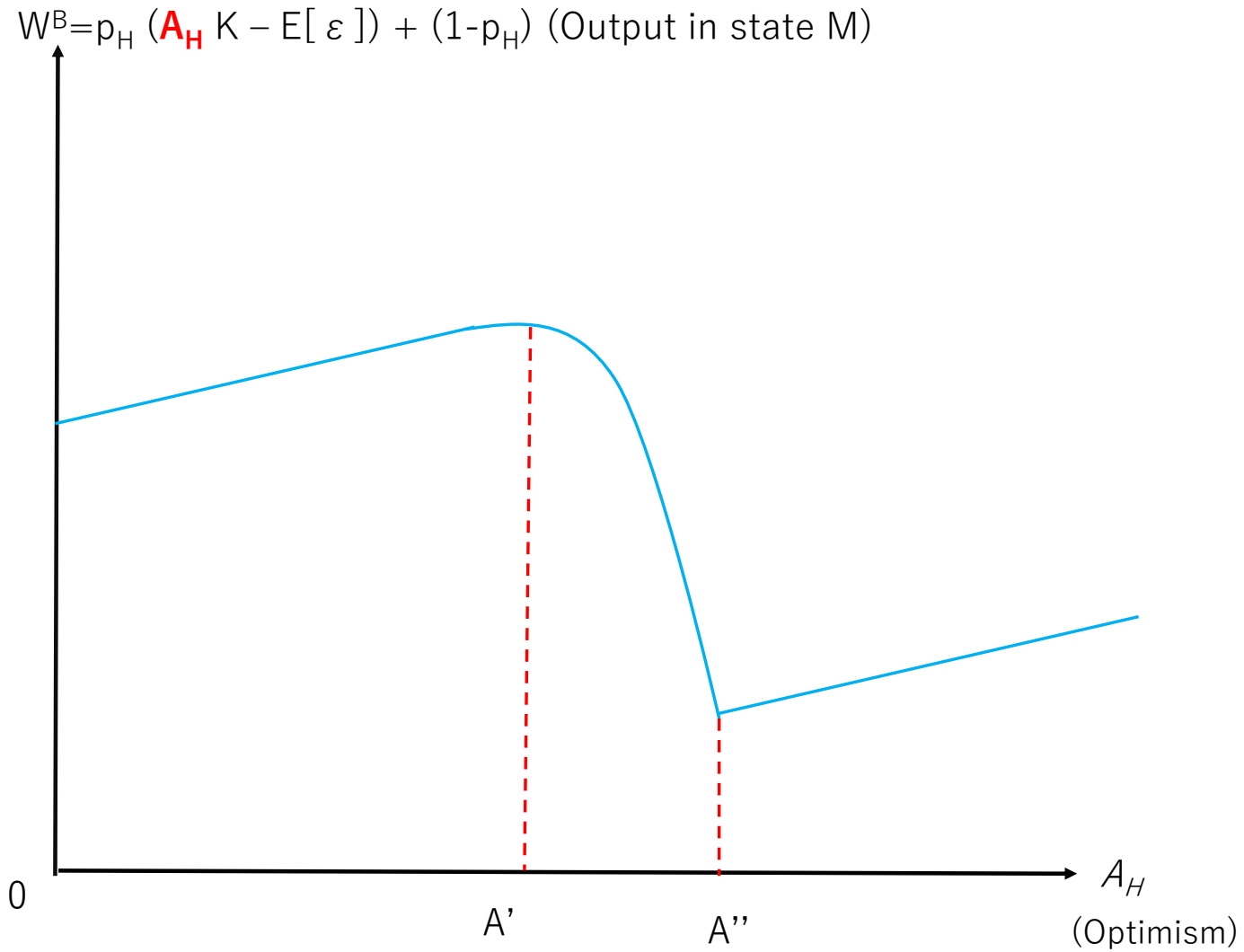
- State M in the equilibrium with default
- Larger $A_H \Rightarrow$ Larger $D \Rightarrow$ Larger agency friction $zD + (1-z)A_L K$
  $\Rightarrow$ Lower ε^e and n^e.

- A larger boom (larger A_H) leads to lower n

$$Y_S(n) = n^{\frac{\sigma}{\sigma-1}} A_M K$$

- Productivity $n^{\frac{\sigma}{\sigma-1}} A_M$ in the state M is lower, as the asset boom (A_H) is larger

*Larger boom ⇒ Deeper recession: Ex-ante social welfare



Benchmark: Borrower subsidy

- **Fixed amount of subsidy S to borrowers** in period 2 such that

$$\pi - \varepsilon - D + S > 0$$

where $\pi = A_s K$ for any $A_s \in \{A_M, A_H\}$. $\Rightarrow$ **No default in any state**

  - Risk-shifting asset boom disappears (Allen and Gale 2000, . . .)
  - All inefficiencies of this model disappear

- Borrower subsidy to avoid all default may be unrealistic
  - Serious moral hazard can arise (Time inconsistency)

* Ex-ante macroprudential policy

- Policy that imposes the borrowing constraint $\bar{D}$ s.t.

$$A_L K < \bar{D} \leq A_M K - \varepsilon_{\max}.$$

- As $Q = \frac{\bar{D}}{K} \leq A_M$ and $k = K$, there is no default when $A_s = A_M$

$$QK \leq A_M K - \varepsilon_{\max}$$

- Then, $n = 1$ and $k = K$ for any state (First best)

- Difficult to find optimal values of $\bar{D}$ for individual firms

* Ex-post subsidy to debt restructuring

- Policy intervention is welfare improving due to aggregate output externality

1. **Social planner** maximizes the total output

$$\max_{n} \; n^{\frac{\sigma}{\sigma-1}} A_M K - \int_0^{\bar{\varepsilon}(n)} \varepsilon dF(\varepsilon) + (1-n)A_L K, \qquad \text{s.t. } n = F(\varepsilon)$$

$$\Rightarrow \text{Social optimum: } (\varepsilon^o,\ n^o) = (\varepsilon_{\max},\ 1)$$

2. Social optimum can be attained by subsidy S to lenders who implement debt restructuring or liquidation:
   - For $A_H \leq A'$, no policy is necessary. $S = 0$ and $n^e = 1$.
   - For $A_H > A'$, the optimal policy to achieve $n^e = 1$ is $S = H(1) - G(1)$, where $H(1) = E[\varepsilon]$, and $G(1) = A_M K - z(1-\sigma^{-1})A_H K - (1-z)A_L K$.
   - Lender i with $\varepsilon_i \in [0, E[\varepsilon]]$ chooses **debt restructuring**, and lender i with $\varepsilon_i \in (E[\varepsilon], \varepsilon_{\max}]$ chooses **liquidation**.

* What if ex-post subsidy to debt restructuring is anticipated?

- Ex-post policy intervention is subsidy to banks, not firms.
- **Firms get nothing in the default state** ($\pi - \varepsilon - D < 0$), with or without subsidy to lenders.

$$\max\{\ \underbrace{\max\{\pi - \hat{D},\ 0\} - \varepsilon,}_{\text{S-sector}}\ \underbrace{\max\{A_L k - \hat{D},\ 0\}}_{\text{C-sector}}\ \} = 0.$$

- **Firms in period 1 decide the equilibrium allocation**

- Even if ex-post policy is anticipated, **equilibria do not change**, because **policy affects only the default state** and **firms do not care about the default state**
  - Equilibrium variables in period 1, $\{k, Q, D\}$, are not altered by anticipation
- Time inconsistency does not arise

* Monetary policy: Introducing nominal variables

- Period 1
  - Q': Asset price (nominal)
  - I: Nominal interest rate
  - Nominal debt $D' = Q'K$ grows to $(1+I)D' = (1+I)Q'K$ in period 2
- Period 2
  - P_H: Goods price in $s = H$ (nominal)
  - P_M: Goods price in $s = M$ (nominal)

- Debt Overhang Equilibrium

$$(1+I)D' = (1+I)Q'K = (1-\sigma^{-1})A_H K P_H,$$

$$D_H = \frac{(1+I)D'}{P_H} = (1-\sigma^{-1})A_H K,$$

$$D_M = \frac{(1+I)D'}{P_M} = (1-\sigma^{-1})A_H K \frac{P_H}{P_M}.$$

* Monetary policy

- **Ex-ante Monetary Policy**: raising nominal loan rate $I \uparrow$
  - No effect
  - A change in nominal loan rate is completely offset by the response of the asset price

$$(1+I)D' = (1+I)Q'K = (1-\sigma^{-1})A_H K P_H$$

- **Ex-post Monetary Policy**: raising price level in M $P_M \uparrow$
  - Effective
  - Ex-post inflation decreases the real burden of debt overhang and improves the social welfare

$$D_M = \frac{(1+I)D'}{P_M} = (1-\sigma^{-1})A_H K \frac{P_H}{P_M}$$

* Modified model for persistence

- Modified model to analyze persistence: Still **two-period model** . . .
  - λ new firms $(0 < \lambda < 1)$ born in period 2
- If the **new firms enter** S-sector and produce output, we say **recession is short-lived** (cyclical downturn)
- If the **new firms do not enter** S-sector, we say **recession is persistent** (secular stagnation)

* Larger booms lead to persistent recessions

- **Equilibrium**: $(n + e(n))$ firms operate in S-sector
  - n **incumbents** stay in S-sector
  - $e(n)$ **new firms** enter S-sector, where $e(n) \leq \lambda$

- **Proposition 13**: There exist $\bar{A}'$ and $\bar{A}''$ s.t.
  - For $A_H \leq \bar{A}'$,

    Short-term recession: $n^e = 1$ and $e(n) = \lambda$, i.e., **all new firms enter** S-sector.
  - For $A_H \geq \bar{A}''$,

    Persistent recession: $n^e = 0$ and $e(n) = 0$, i.e., **no new firms enter** S-sector.
  - For $A_H \in (\bar{A}', \bar{A}'')$, **Medium-term recession**: $n^e \in [0, 1]$ and $e(n) \in [0, \lambda]$, i.e., **some new firms enter** S-sector.

- **Intuition**:
  - New firms' revenue $((n + e)^{\frac{1}{\sigma-1}} A_H K)$ is increasing in n
  - New firms enter if payoff is larger than entry cost: $(n + e)^{\frac{1}{\sigma-1}} A_M K - \varepsilon - \gamma K \geq 0$.
  - If A_H is small: D small $\Rightarrow$ n large $\Rightarrow$ New firms enter
  - If A_H is large: D large $\Rightarrow$ n small $\Rightarrow$ New firms do not enter

* Conclusion

- Risk shifting boom, ex-post debt overhang, and aggregate output externality can replicate empirical regularities, i.e., boom, bust and productivity declines.
- Larger asset boom may lead to deeper and more persistent recession
- Lenders know their payoff will increase if they restructure debt. They voluntarily reduce debt.
- Their debt restructuring can achieve social optimum when the debt is small. It is insufficient due to externality when the debt is large.
- The ex-post subsidy to lenders that encourage debt restructuring can improve productivity and welfare. Time inconsistency may be minor.