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Transparency vs Privacy in Credit Markets*

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Abstract

We compare Transparency and Privacy in credit markets. A long-lived borrower, who has a risky investment opportunity, seeks loans from a sequence of short-lived lenders. Under Transparency, all the information about the past investment outcomes is shared among the future lenders, which helps the lenders learn the borrower's type. In contrast, no information is shared under Privacy. We first show that under both Transparency and Privacy, the iterated elimination of dominated strategies leaves unique outcomes. We then show that trade stops earlier under Transparency than under Privacy. A higher social welfare is achieved under Privacy than under Transparency.

Keywords: credit market; transparency; privacy; strategic experimentation

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1 Introduction

Should lenders be allowed to have access to the credit history of borrowers? Not many credit bureaus allow full access to it. Indeed, Elul and Gottardi (2015) document that "[o]f the 113 countries with credit bureaus as of January 2007, over 90 percent of them had provisions for restricting the reporting of adverse information after a certain period of time (page 295)." But why?

In this paper, we compare the performances of two alternative information structures. One is *Transparency*, where the credit history of a borrower is observed by the lenders. The other is *Privacy*, where it is the borrower's private information.

For this purpose, we propose a model in which a long-lived borrower (or an entrepreneur), who has a risky investment opportunity every period, seeks loans from a sequence of short-lived lenders. The borrower is either of a good type, who has a chance to succeed, or of a bad type, who has no chance to succeed. No agent knows the type of the borrower ex-ante, and the lenders must make lending decisions based on their beliefs about whether or not the borrower is of a good type.

Under Transparency, all the information about the past investment outcomes is shared among the future lenders, whereas no information is shared under Privacy. Under Transparency, the lenders can update their beliefs by continuing lending. Hence, the lenders experiment via lending. This channel is shut down under Privacy. That is, the lender has no access to information about the past investment outcomes of the borrower.

Our model is rather standard, and is built on strategic experimentation models, where agents learn about a state by experimenting (see Keller, Rady, and Cripps, 2005 among many others). We depart from them by assuming short-lived lenders. When lenders are short-lived, each lender cannot directly observe the past investment outcomes of the borrower by herself, and so, whether or not the lenders have access to that information can, and indeed, does make a difference.

In this setup, we first show that under both Transparency and Privacy, the *iterated elimination of dominated strategies* (IEDS) leaves *unique* outcomes.[1] In the unique outcome under Transparency, a lender terminates experimentation when she wants to. In contrast, under Privacy, it is the borrower who terminates trade, because the lenders do not learn about the history and cannot update their beliefs about the borrower's type.

Then, we compare the two outcomes and obtain the following theorem as the main result of the paper.

Theorem. *Under Privacy, trade stops later than under Transparency. A higher social welfare is always achieved under Privacy than under Transparency.*

Under Transparency, trade stops too early relative to the first best, because short-lived lenders do not internalize a positive externality that information about past outcomes benefits future lenders. Under Privacy, on the other hand, the lenders cannot learn from the credit history, and so, never become too pessimistic. This makes trade long-lasting—always longer than under Transparency, and for a wide range of parameters, even longer than in the first best.

Why does Privacy achieve a higher social welfare than Transparency does? To see this, consider the timing at which under Transparency, a lender is indifferent between lending and not lending. At the moment, there are still *flow* gains from trade because the borrower wants to proceed with his project at the expense of the lenders. This flow

[1] In our setup, there are multiple Nash equilibria. Under Privacy, information asymmetry develops endogenously, and hence, there are multiple sequential equilibria supported by certain off-equilibrium beliefs as well.

total surplus is exactly the same as the discounted present social value from lending forever thereafter. Because lending forever is no better than the allocation attained under Privacy, social welfare must be higher under Privacy than under Transparency.

Related Literature

Our modeling approach is akin to the one developed in the literature of strategic experimentation, nicely surveyed by Hörner and Skrzypacz (2017). Like Keller, Rady, and Cripps (2005), ours is a *good news* model, where a success arrives only when the borrower is of a good type. In this literature, Bergemann and Hege (1998, 2005) and Hörner and Samuelson (2013) consider environments where an entrepreneur interacts with a long-lived lender. Because the lender is long-lived, perfect recall implies that the lender also knows the past history , and hence, privacy does not play any role.

In a broad sense, this paper relates to two literature—one on privacy and the other on credit.[2] More specifically, our paper is closely related to the following papers that examine the role of credit history. Elul and Gottardi (2015) and Bhaskar and Thomas (2019) show that coarser information can improve welfare in such markets. In both papers, the driving forces are moral hazard of borrowers, which is not present in our model.[3] Thus, unlike ours, trade never occurs under perfect Privacy in which the lenders get no information at all, and hence, perfect Privacy never dominates Transparency. Using the framework of strategic experimentation, Kovbasyuk and Spagnolo (2023) also show that finer information about past history can be detrimental. They show that Transparency leads to market shutdown, and show that the optimal information structure is to disclose information at the beginning and shut it down at some point of time. They consider an environment in which borrowers' types vary over time. Our

[2]See, for example, Acquisti, Taylor, and Wagman (2016) for a survey on privacy, and Lagos, Rocheteau, and Wright (2017) on credit.

[3]In a more abstract setting of a two-period repeated game, Kandori (1991) shows that finer information about past history need not facilitate cooperation among players.

simpler assumption of good news with a fixed type of a borrower allows us to obtain a *unique* outcome of the IEDS procedure.

Organization of the Paper

The rest of the paper is organized as follows. Section 2 presents the basic setup. Section 3 characterizes the *unique* outcome of the IEDS procedure for each information structure. Section 4 presents the main result of the paper, a comparison of Transparency and Privacy. The omitted proofs can be found in Appendix A. The formal argument of the IEDS procedure is delegated to Appendix B. In Appendix C, we discuss the trading mechanism.

2 Preliminaries

We study a discrete-time infinite-horizon model. *One period*, in which one trade can potentially occur, is denoted as $\mathrm{d}t$. We study the limit as $\mathrm{d}t \to 0$ in the subsequent analysis. This structure is standard in so-called strategic experimentation models (see, for example, Keller, Rady, and Cripps, 2005).

There is an infinitely-lived borrower ("he") and a sequence of countably many one-period-lived lenders ("she").[4] All the agents are risk-neutral. Every period, the borrower meets a lender, and they can trade if they both agree.[5]

Every period, the borrower has a risky investment opportunity, whose outcome is either success or failure. The borrower yields 1 unit of a perishable consumption good

[4]Our assumption of one-period-lived lenders can also be interpreted as a turnpike structure (Townsend, 1980, among others) or a bilateral-meeting structure (Kiyotaki and Wright, 1989, among others), where a borrower meets a different (possibly infinitely-lived) lender every period. Some papers, including Sanches (2011) and He, Wright, and Zhu (2015) as well as Bhaskar and Thomas (2019) and Kovbasyuk and Spagnolo (2023), use bilateral-meeting setups to examine the role of credit. Elul and Gottardi (2015), like ours, assume short-lived lenders. Some of them explicitly mention the equivalence.

[5]A formal description of the game form is presented in Appendix B.

if he succeeds, and 0 unit if he fails. The outcome in the current period is verifiable to the current lender.

The borrower is either of a good type or of a bad type. The borrower's type is drawn at the beginning of the economy and does not change over time. If the borrower is of a good type, outcomes can vary across periods. He will succeed with a Poisson arrival rate $\lambda > 0$ if he pursues an investment opportunity. If the borrower is of a bad type, he will never succeed at any moment of time. Neither the borrower nor the lenders know the type of the borrower. We assume that all the agents share a common prior belief $\rho_0 \in [0, 1]$ that the borrower is of a good type.

In order to pursue an investment opportunity in a period, the borrower needs an amount $k\mathrm{d}t$ of a "production good," where $k > 0$. The borrower does not have the production good, whereas the lenders do. The borrower chooses whether to borrow or not. A lender chooses whether to lend or not.

We assume that the production good is used for an investment opportunity in an irreversible way. That is, once the production good is used for an opportunity, it cannot be used again for another opportunity. The net return from not lending is normalized to 0.

Throughout the paper, we assume that the borrower repays $x \in (0, 1)$ of the consumption good when he succeeds. While we treat x as exogenously given here, this can be derived as an outcome of ex-post Nash bargaining, where the borrower and a lender split 1 unit of the consumption good. The details can be found in Appendix C.

When the borrower fails, he pays a failure cost at a rate of c to the lender. We assume that the penalty c is simply a transfer from the borrower to the lender. That is, the presence of c per se does not generate any intrinsic social benefit nor cost, although it affects equilibrium behavior of agents. We assume that $k > c \geq 0$. In Table 1, we summarize the payoff structure in a period.

Let ρ be a belief of an agent. The expected flow payoff to the borrower is $(\lambda \rho \mathrm{d}t)(1 -$

$x) + (1 - \lambda\rho\mathrm{d}t)(-c)\mathrm{d}t$. The expected flow payoff to a lender is $(\lambda\rho\mathrm{d}t)(x - k\mathrm{d}t) + (1 - \lambda\rho\mathrm{d}t)(c - k)\mathrm{d}t$. Ignoring the terms of order $(\mathrm{d}t)^2$, we get Table 2.

	The borrower succeeds	The borrower fails
The net surplus to the borrower	$1 - x$	$-c\mathrm{d}t$
The net surplus to a lender	$x - k\mathrm{d}t$	$c\mathrm{d}t - k\mathrm{d}t$
The net surplus to the society	$1 - k\mathrm{d}t$	$-k\mathrm{d}t$

Table 1: This table summarizes the flow net surpluses within a period.

The flow payoff to the borrower	$\{\lambda\rho(1 - x) - c\}\,\mathrm{d}t$
The flow payoff to a lender	$\{\lambda\rho x + c - k\}\,\mathrm{d}t$
The flow payoff to the society	$\{\lambda\rho - k\}\,\mathrm{d}t$

Table 2: This table summarizes the expected flow payoffs.

For the problem to be non-trivial, we must have $\lambda > k$. Otherwise, even if everyone knows that the borrower is of a good type, there is no gains from trade, and so, trade never occurs. In fact, throughout the paper, we make the following stronger assumption that guarantees that the lenders have incentives to lend given the prior ρ_0.

Assumption 1. *For any $c \geq 0$, the initial lender has an incentive to lend, that is,*

$$\lambda\rho_0 x - k > 0.$$

We also make the following assumption.

Assumption 2. *The borrower incurs a sufficiently small cost from failing, that is,*

$$c < \bar{c} = \frac{k + r + \lambda(1 - x) - \sqrt{\{k + r + \lambda(1 - x)\}^2 - 4k(1 - x)(r + \lambda)}}{2}.$$

If Assumption 2 does not hold, the borrower always stops borrowing *before* a lender stops lending. In this case, the information structure that the lenders face, which is the main subject of this paper, does not play any role. See footnotes 8 and 9 for the details when the assumption is violated. Assumption 1 implies that $\lambda > k$, and this guarantees that $\bar{c}$ is a positive real number. An example of parameters that satisfy both assumptions is that $c = 0$, $k = 0.15$, $r = 0.02$, $\lambda = 0.4$, $x = 0.8$, and $\rho_0 = 0.7$.

The belief of an outside observer who sees the entire history is updated by the Bayes rule. Provided that the lenders keep lending and yet no success has arrived, the belief at period t is, in the limit as $\mathrm{d}t \to 0$, given by

$$\rho_t = \frac{\rho_0 e^{-\lambda t}}{\rho_0 e^{-\lambda t} + 1 - \rho_0}. \tag{1}$$

Thus, upon no success, the outside observer becomes increasingly pessimistic about the borrower's type. If experiments continue forever and yet no success arrives, the observer is almost certain that the borrower is of bad type, that is, $\lim_{t\to\infty} \rho_t = 0$. If a single success arrives, on the other hand, the belief jumps up to 1.

Information Structures

We compare two information structures, namely, *Transparency* and *Privacy*. Under either information structure, the borrower observes and remembers all the past outcomes, and upon no success, his belief follows (1). The difference between the two information structures arises when it comes to the lenders' knowledge about the past outcomes.

Under Transparency, the lenders observe all the past outcomes—whether trade has occurred, and if so, whether the borrower has succeeded or failed in the past. The lenders can update their beliefs about the borrower's type in the same manner as the borrower does. Thus, upon no success, their beliefs follow (1). In other words, there is no information asymmetry between the borrower and the lenders.

Under Privacy, the lenders do not observe any outcome from the past. We assume lenders know the calendar time, but it is easy to see the same conclusion holds when they do not. Under this information structure, ex-ante, there is no information asymmetry between the borrower and the lenders, but it endogenously arises during plays.

First Best

To define the first best in this economy, consider a social planner, who observes all the past outcomes, but does not know the true type of the borrower. The planner shares the prior ρ_0 with all the other agents. The information available to the planner is the same as that to the borrower, and under Transparency, the lenders as well. Thus, the belief of the planner follows (1). Given the information, the planner dictates whether trade occurs or not in each period. We assume that the planner shares the discount rate r with the borrower.

It is obvious that the planner lets trade occur forever once a single success arrives. Thus, the planner's problem is to determine the optimal stopping time, that is, when to stop lending if no success has arrived so far. Let T be a stopping time. The stock value of social welfare is denoted as $W(T)$, and we have

$$\begin{aligned} W(T) &= \left(1-\rho_0+\rho_0 e^{-\lambda T}\right)\int_0^T e^{-rt}(-k)\mathrm{d}t \\ &\quad +\rho_0\int_0^T\left(e^{-rs}+\int_s^\infty e^{-rt}\lambda\mathrm{d}t+\int_0^\infty e^{-rt}(-k)\mathrm{d}t\right)\lambda e^{-\lambda s}\mathrm{d}s. \end{aligned} \tag{2}$$

The first term is the case where no success has arrived until T. In this case, the planner stops lending at T. The second term is the case where the first success arrives at some time $s<T$. In this case, the planner realizes at s that the borrower is of a good type, and continues to lend forever thereafter.

A standard argument (for example, equation (4) of Keller, Rady, and Cripps, 2005) shows that

$$rW(T)=z(\rho_0)-z(\rho_T)\left(\frac{1-\rho_0}{1-\rho_T}\right)^{1+r/\lambda}\left(\frac{\rho_T}{\rho_0}\right)^{r/\lambda}, \tag{3}$$

where ρ_T is the belief of the planner at period T given by (1) and $z(\rho)=\lambda\rho-k$ is the flow payoff to the planner given a belief ρ.

The planner chooses T to maximize $W(T)$. We can show that W is single-peaked, and denote its unique maximizer as T^*. The proof is standard and reproduced in Appendix A.1.

3 Outcomes

3.1 Iterated Elimination of Dominated Strategies

Now, we characterize the outcomes of the two information structures in this section, and then we compare them in terms of social welfare in Section 4. Note first that there is always a Nash equilibrium in which trade does not occur. That is, an agent rejects trade because the other side of the agent rejects too. Moreover, under Privacy, there are many no-trade sequential equilibria supported by certain off-equilibrium beliefs. For example, the borrower does not borrow and a lender refuses to lend, believing that the borrower who wishes to borrow (which occurs only off the equilibrium path) is the one who has never succeeded.

Thus, we focus on the outcome of iterated elimination of dominated strategies. In the iterated process, in some rounds we eliminate weakly dominated strategies, and so we consider *iterated admissibility*—the process of maximal iterated elimination of weakly dominated strategies.[6] We will show that under the two information structures, iterated admissibility leaves essentially *unique* (but different) outcomes.[7]

It should also be noted that unlike Nash equilibria, iterated admissibility does not rely on common knowledge of the strategies.

It is convenient to define two cutoff beliefs, denoted as ρ^ℓ and ρ^b, and the corresponding times, denoted as T^ℓ and T^b. First, let ρ^ℓ be the cutoff belief at which a lender's flow payoff becomes zero, that is,

$$\lambda\rho^\ell x + c - k = 0.$$

[6]As is well known, the outcome of iterated elimination of weakly dominated strategies depends on the order of elimination. To avoid this issue, iterated admissibility eliminates as many strategies as possible in every round.

[7]It can be easily verified that the outcome is supported as a subgame perfect Nash equilibrium (in case of Transparency) or as a sequential equilibrium that satisfies some forward induction refinements like D1 (in case of Privacy).

Later, in Lemma 2, it will be verified that under Assumption 1, $\rho^\ell < \rho_0$ and so $\rho^\ell < 1$.

Then the following lemma immediately follows from the fact that the lenders are one-period-lived.

Lemma 1. *It is (weakly) optimal for a lender to lend (respectively, not to lend) if her belief is above the cutoff value ρ^ℓ (respectively, below ρ^ℓ).*

Of course, how the belief of a lender is formed depends on the information structure and, under Privacy, the strategies of the borrower and the other lenders as well.

Then let T^ℓ be the time at which the belief of the borrower (and under Transparency, the belief of a lender as well) reaches ρ^ℓ if trade has always occurred and no success has arrived. By setting $\rho_t = \rho^\ell$ and $t = T^\ell$ in (1), we have

$$T^\ell = \frac{1}{\lambda} \log \left(\frac{\rho_0}{1-\rho_0} \times \frac{1-\rho^\ell}{\rho^\ell} \right).$$

Next, let $W^b(T)$ be the value function of the borrower under the circumstance in which the borrower unilaterally terminates trade at time T, that is,

$$\begin{aligned} W^b(T) \;&=\; (1-\rho_0+\rho_0 e^{-\lambda T}) \int_0^T e^{-rt}(-c)\mathrm{d}t \\ &\quad +\rho_0 \int_0^T \left\{ e^{-rs}(1-x) + \int_s^\infty e^{-rt}\lambda(1-x)\mathrm{d}t + \int_0^\infty e^{-rt}(-c)\mathrm{d}t \right\} \lambda e^{-\lambda s}\mathrm{d}s. \end{aligned}$$

The value function of the borrower W^b is analogous to that of the social planner (2), with the cost k replaced by c and the gain 1 replaced by $1-x$.

Because the cost-benefit structure is similar to $W(T)$, we can show that $W^b(T)$ is also single-peaked and has a unique maximizer. We denote the maximizer as T^b, and the belief of the borrower at time T^b as ρ^b. A routine calculation shows that

$$\rho^b = \frac{c}{\lambda(1-x) + (\lambda/r)\left\{\lambda(1-x) - c\right\}}$$

and

$$T^b = \frac{1}{\lambda} \log \left(\frac{\rho_0}{1-\rho_0} \times \frac{1-\rho^b}{\rho^b} \right).$$

If $c = 0$, the borrower never wants to stop. That is, $\lim_{c\to 0} \rho^b = 0$ and $\lim_{c\to 0} T^b = \infty$.

In the following lemma, we show two things. (*i*) If a lender has the prior belief ρ_0, she is willing to lend. (*ii*) If the borrower can unilaterally terminates trade and if the borrower's cost c is small, a lender wants to stop earlier than the borrower does.

Lemma 2. *(i) Under Assumption 1, we have*

$$\rho_0 > \rho^\ell.$$

(ii) Under Assumption 2, we have

$$\rho^\ell > \rho^b.$$

Note that we do not use Assumption 2 for Part (*i*) and Assumption 1 for Part (*ii*), respectively. The proof of Part (*i*) is immediate, and that of (*ii*) is in Appendix A.2. Because $\rho^b < \rho^\ell$, we must have $\rho^b < 1$ and also $T^b > T^\ell$.

3.2 Transparency

Under Transparency, both the borrower and the lenders observe the entire history, so their beliefs evolve in the same way. That is, there is no information asymmetry. In this case, by Lemma 1, the lenders are willing to lend until the (common) belief reaches ρ^ℓ and then they stop there.

Consider the strategies of the lenders for which they continue to lend until T^ℓ and they do not lend thereafter. The borrower chooses an optimal stopping time against the lenders' strategies. Thus, the borrower chooses T to maximize his value function $W^b(T)$ subject to $T \le T^\ell$. Notice that when the constraint $T \le T^\ell$ is binding, the solution to the constrained problem is different from T^b that simply maximizes $W^b(T)$.[8] Because W^b is single-peaked, the solution to the constrained problem is $\min\{T^b, T^\ell\}$. Because $T^b > T^\ell$, we must have $\min\{T^b, T^\ell\} = T^\ell$.

[8]If Assumption 2 does not hold, the constraint $T \le T^\ell$ never binds. In this case, trade stops at time T^b.

To summarize, we have the following.

Proposition 1. *Under Transparency, the outcome of iterated admissibility is essentially unique. In the outcome, the agents trade until time T^ℓ and stop if no success has arrived until that time.*

A formal proof can be found in Appendix B.

3.3 Privacy

We turn to the case of Privacy. First, notice the fact that the borrower always wants to borrow after a single success. Conditional on that the borrower wishes to borrow, the most pessimistic belief that a lender can have is such that the borrower wishes to borrow at any point of time however many times he has failed consecutively. Of course, if the borrower stops borrowing after a certain number of consecutive failures, a lender can have a more optimistic belief about his type.

The next lemma says that the most pessimistic belief is equal to the prior ρ_0.

Lemma 3. *Consider the borrower who wishes to borrow at period t no matter what happened in the past. Then the belief of lender t is ρ_0.*

The proof is in Appendix A.3. By Lemma 2, we have $\rho_0 > \rho^\ell$, and so, by Lemma 1, the lender always wants to lend. Thus, under Privacy, the borrower is the one who terminates trade. To summarize, we have

Proposition 2. *Under Privacy, the outcome of iterated admissibility is essentially unique. In the outcome, the agents trade until T^b and stop if no success has arrived until that time.*

Again, a formal proof can be found in Appendix B.

4 Transparency vs Privacy

Recall that T^*, T^ℓ, and T^b are the stopping time in the first best, that under Transparency (Proposition 1), and that under Privacy (Proposition 2), respectively. Then, our main result is stated as follows.

Theorem 1.

(i) Under Transparency, trade stops too early, that is, $T^\ell < T^*$.

(ii) Under Privacy, trade stops later than under Transparency, that is, $T^\ell < T^b$.

(iii) A higher social welfare is achieved under Privacy, that is, $W(T^\ell) < W(T^b)$.

Here, "too early" (respectively, "too late") means that trade stops earlier (respectively, later) than in the first best. Under Privacy, trade may stop too early or too late, depending on c. That is, there is a $\tilde{c}$ such that $T^b > T^*$ (respectively, $T^b < T^*$) for all $c < \tilde{c}$ (respectively, $c > \tilde{c}$).

Under Transparency, it is a lender who terminates trade, and she chooses to stop lending too early. The reason is the presence of an *informational externality* similar to that in Keller, Rady, and Cripps (2005). Recall that the only difference between the first best and the outcome under Transparency is that in the first best, the planner maximizes social welfare, whereas in the outcome under Transparency, each lender maximizes her own payoff. Information about the past outcomes is beneficial to future lenders, because with that, they can estimate the probability of the borrower's type more precisely. However, because each lender is short-lived, she does not internalize this externality. Hence, under Transparency, a lender stops lending too early.

Under Privacy, on the other hand, it is the borrower who terminates the trade. Assumption 2 implies that when the lenders can observe the past investment outcomes (which occurs under Transparency), they want to terminate trade earlier than the borrower wants to (by Lemma 2).[9] This means that trade terminates earlier under Trans-

[9] If Assumption 2 does not hold, the borrower is the one who terminates trade under Transparency

parency than under Privacy.

Why is a higher social welfare achieved under Privacy than under Transparency? When trade stops too early even under Privacy (that is, $T^\ell < T^b < T^*$), this immediately follows from the fact that W is single-peaked. Below, we provide an intuition for the other case $T^\ell < T^* < T^b$. For this, we look at the worst possible allocation under Privacy, which is attained when $c = 0$ and so the borrower wants to continue forever—farthest from T^*. We argue that even this worst allocation is welfare superior to the allocation under Transparency.

First, notice that the allocations from time 0 to time T^ℓ are the same between the two information structures. Now, suppose that the borrower has not succeeded until time T^ℓ, and take the view point of the social planner, whose belief is equal to ρ^ℓ at time T^ℓ. Suppose that the planner has to choose from only two options at time T^ℓ—whether the planner must stop at time T^ℓ or must continue forever thereafter.

The former option achieves exactly the same allocation as that under Transparency, whereas the latter achieves the allocation under Privacy when $c = 0$. The expected discounted continuation social value from the former option is 0, whereas that from the latter option is $\lambda\rho^\ell - k$. Now we argue that $\lambda\rho^\ell - k > 0$.

The intuition is as follows. Notice that $\lambda\rho^\ell - k$ is also equal to the *flow* social surplus at time T^ℓ. Under Transparency, the lender at time T^ℓ is just indifferent between lending and not lending. Thus, the (flow) surplus to the lender is 0. On the other hand, the flow surplus to the borrower is still positive when c is (sufficiently close to) zero. The lender's and the borrower's flow surpluses add up to $\lambda\rho^\ell - k$, and hence, it must be positive.

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too. In this case, there is of course no difference between the outcomes of the two information structures. See also footnote 8.

The Relation between c and W

Under Privacy, social welfare is non-monotone in c (see Figure 1). It is because under Privacy, the borrower is the one who stops trade. Notice that the borrower's unconstrained stopping time T^b, which is the stopping time under Privacy, is decreasing in c. When c is so small that $T^b > T^*$, the borrower stops trade too late. When c is so large that $T^b < T^*$, the borrower stops trade too early.

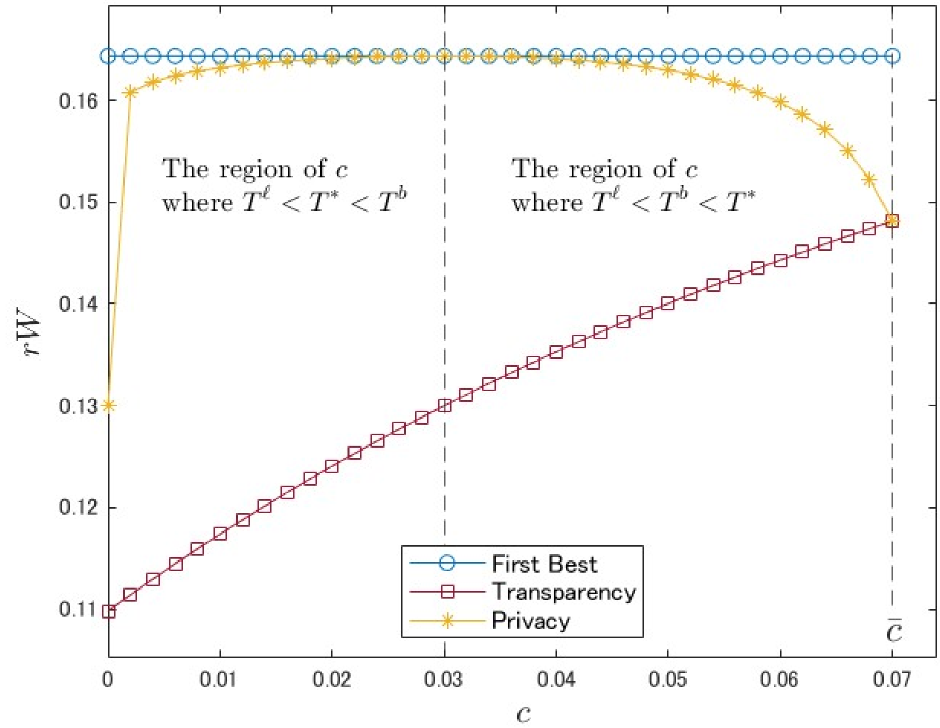

Figure 1: Failure Cost and Social Welfare. The Figure shows the relation between c and rW for each of Transparency, Privacy, and the first best. The parameters are taken as $k = 0.15$, $r = 0.02$, $\lambda = 0.4$, $x = 0.8$, and $\rho_0 = 0.7$.

When $c = k(1 - x)$ such that $T^b = T^*$, the borrower stops trade at the socially best timing. This is because when $c = k(1 - x)$, the cost-benefit ratio of the planner $k/1$ is exactly the same as that of the borrower $c/(1 - x)$.

4.1 Proof of Theorem 1

Now, we give a formal proof of the theorem. Part (*i*) follows from a routine calculation, and part (*ii*) immediately follows from Lemma 2. When $T^\ell < T^b < T^*$, Part (*iii*) immediately follows from the fact that W is single-peaked. Thus, we will show that Part (*iii*) follows when $T^\ell < T^* < T^b$.

By substituting T^ℓ into (3), we get social welfare under Transparency as

$$rW(T^\ell) = z(\rho_0) - z(\rho^\ell)\left(\frac{1-\rho_0}{1-\rho^\ell}\right)^{1+r/\lambda}\left(\frac{\rho^\ell}{\rho_0}\right)^{r/\lambda}$$

Similarly, by substituting T^b into (3), we get social welfare under Privacy as

$$rW(T^b) = z(\rho_0) - z(\rho^b)\left(\frac{1-\rho_0}{1-\rho^b}\right)^{1+r/\lambda}\left(\frac{\rho^b}{\rho_0}\right)^{r/\lambda}.$$

Note that $T^* < T^b$ if and only if $c < k(1-x)$, and hence, we have

$$z(\rho^\ell) = \lambda\left(\frac{k-c}{\lambda x}\right) - k = \frac{k(1-x)-c}{x} > 0.$$

The result now follows from the fact that $z(\rho^\ell) > 0 > z(\rho^*) > z(\rho^b)$. □

A Omitted Proofs

A.1 Characterization of the First Best

Let ρ^* be the belief of the planner at time T^*. Then we show the following statement.

Proposition 3. *The planner's value function $W(T)$ is single-peaked, and the cutoff belief ρ^* is given by*

$$\rho^* = \frac{k}{\lambda + (\lambda/r)(\lambda - k)}. \tag{4}$$

The flow value of social welfare in the first best is given by

$$rW(T^*) = z(\rho_0) - z(\rho^*)\left(\frac{1-\rho_0}{1-\rho^*}\right)^{1+\frac{r}{\lambda}}\left(\frac{\rho^*}{\rho_0}\right)^{\frac{r}{\lambda}}.$$

Proof of Proposition 3. Taking the derivative of (2), we have

$$W'(T) = e^{-rT} \times \left[(1-\rho_0)(-k) + \rho_0 e^{-\lambda T}\left(1+\frac{\lambda}{r}\right)(\lambda - k)\right].$$

Let

$$f(T) = \left[(1-\rho_0)(-k) + \rho_0 e^{-\lambda T}\left(1+\frac{\lambda}{r}\right)(\lambda - k)\right].$$

Note that Assumption 1 implies that $\lambda - k > 0$, and so, $f(T)$ is decreasing in T. Moreover, we have $f(0) > 0$ by Assumption 1, and $\lim_{T\to\infty} f(T) < 0$. Thus $W(T)$ is single-peaked.

The maximizer T^* satisfies

$$T^* = \frac{1}{\lambda}\log\left(\frac{\rho_0}{1-\rho_0} \times \frac{(1+\lambda/r)(\lambda-k)}{k}\right).$$

Then, by setting $t = T^*$ and $\rho_t = \rho^*$ in (1), we have (4).

Finally, notice from (1) that

$$T^* = \frac{1}{\lambda}\log\left(\frac{\rho_0}{1-\rho_0} \times \frac{1-\rho^*}{\rho^*}\right).$$

Thus, we have

$$\begin{aligned}
&W(T^*) \\
= \; & \frac{-k}{r}\left\{1 - \frac{1-\rho_0}{1-\rho^*}\left(\frac{1-\rho_0}{1-\rho^*} \times \frac{\rho^*}{\rho_0}\right)^{r/\lambda}\right\} + \frac{\lambda\rho_0}{r}\left\{1 - \frac{1-\rho_0}{1-\rho^*}\left(\frac{1-\rho_0}{1-\rho^*} \times \frac{\rho^*}{\rho_0}\right)^{1+r/\lambda}\right\} \\
= \; & \frac{z(\rho_0)}{r} - \frac{z(\rho^*)}{r}\left(\frac{1-\rho_0}{1-\rho^*}\right)^{1+r/\lambda}\left(\frac{\rho^*}{\rho_0}\right)^{r/\lambda},
\end{aligned}$$

which completes the proof. □

A.2 Proof of Lemma 2

A routine calculation shows $\rho^b = \rho^\ell$ at $c = \bar{c}$. To see this, by setting $\rho^b = \rho^\ell$, we have the following quadratic equation of c,

$$c^2 - [r + k + \lambda(1-x)]c + k(r+\lambda)(1-x) = 0.$$

One of the solutions does not satisfy $c < k$. Thus, we have a unique solution $\bar{c}$ as

$$\bar{c} = \frac{k + r + \lambda(1-x) - \sqrt{\{k + r + \lambda(1-x)\}^2 - 4k(1-x)(r+\lambda)}}{2}.$$

The lemma now follows from the fact that ρ^b is increasing in c while ρ^ℓ is decreasing in c. □

A.3 Proof of Lemma 3

We show that if the belief starts with some ρ, the posterior is equal to ρ. In this period, there are two possibilities: Trade occurs or no trade occurs. If there is no trade, there will be no belief updating, and hence, the posterior remains ρ. If there is trade, the posterior is calculated in the following way.

$$\begin{cases} 1 & \text{if the borrower succeeds,} \\ \dfrac{\rho(1-\lambda \mathrm{d}t)}{1-\rho+\rho(1-\lambda \mathrm{d}t)} & \text{if the borrower fails.} \end{cases}$$

Because the borrower succeeds with probability $\lambda\rho\mathrm{d}t$, the posterior is unconditionally equal to

$$\lambda\rho\mathrm{d}t \times 1 + (1-\lambda\rho\mathrm{d}t) \times \frac{\rho(1-\lambda \mathrm{d}t)}{1-\rho+\rho(1-\lambda \mathrm{d}t)},$$

which is again equal to ρ. Thus, if we start from the prior ρ_0, the posterior remains to be ρ_0. □

B Iterated Admissible Set

B.1 Formal Model

Here we prove that the outcome of iterated admissibility is unique. We start by formally defining the game form. In each period, the borrower first chooses from $\{Y, N\}$, and then observing this, the lender also chooses from $\{Y, N\}$. Trade occurs if and only if they both choose Y.

The set of possible outcomes in period t consists of N_b, N_ℓ, S and F. The event N_b means that the borrower chooses N, while the event N_ℓ means that the borrower chooses Y and then the lender chooses N. Trade does not occur in these two events. The event S means that trade occurs and the borrower succeeds. Finally, the event F means that trade occurs and the borrower fails. Let $\mathcal{H}^t = \{N_b, N_\ell, S, F\}^t$ be the set of histories up to period t.

We can denote the strategies as follows. Because the borrower can observe the history under either information structure (Transparency or Privacy), his strategy $\boldsymbol{\sigma}^b = (\sigma_t^b)_{t\geq 0}$ is a collection of mappings

$$\sigma_t^b : \mathcal{H}^{t-1} \to \{Y, N\}.$$

The lenders' strategies depend on the information structure. Under Transparency, the lenders can observe the history. Thus, the strategy of lender t, denoted as σ_t^I, is a mapping

$$\sigma_t^I : \mathcal{H}^{t-1} \to \{Y, N\}.$$

Under Privacy, the lenders can only observe the calendar time. Thus, the set of strategies of lender t is simply $\{Y, N\}$.[10]

Finally, let

$$\hat{\rho}_t : \mathcal{H}^{t-1} \to [0, 1]$$

be the belief of the borrower (and under Transparency, the lenders as well) following a history $h^{t-1} \in \mathcal{H}^{t-1}$. We assume that the agents are all Bayesian.

Solution concept

Here we consider the outcome of *iterated admissible* strategies—strategies that survive the process of maximal iterated elimination of weakly dominated strategies.

[10]It is easy to verify that the same outcome can be obtained if a lender chooses first, if a lender and the borrower choose simultaneously, or if lenders cannot distinguish N_b and N_ℓ in case of Transparency.

B.2 Transparency

Note that under Transparency, the game is a perfect information one. Also, recall that there is no information asymmetry. That is, given a publicly observable history $h^{t-1} \in \mathcal{H}^{t-1}$, the belief of any agent is $\hat{\rho}_t(h^{t-1})$, and this is common knowledge. We shall show Proposition 1.

Round 1.

Lenders. Consider lender t's strategy $\hat{\sigma}_t^I$ such that

$$\hat{\sigma}_t^I(h^{t-1}) = \begin{cases} Y & \text{if } \hat{\rho}_t(h^{t-1}) > \rho^\ell, \\ N & \text{if } \hat{\rho}_t(h^{t-1}) < \rho^\ell. \end{cases}$$

Then $\hat{\sigma}_t^I$ weakly dominates any other strategy. To see this, notice first that if the borrower chooses N, the lenders are indifferent between any two strategies. If the borrower chooses Y, a lender's expected payoff depends on her belief $\hat{\rho}_t(h^{t-1})$. It is strictly optimal for a lender to choose Y if and only if her belief is above ρ^ℓ.

Borrower. Let Σ^{b1} be the set of the borrower's admissible strategies (that is, strategies that are not weakly dominated). By definition of the elimination process, all the strategies outside Σ^{b1} are eliminated in this round.

As is argued in Section 3.2, the strategy of the borrower $\hat{\boldsymbol{\sigma}}^b = (\hat{\sigma}_t^b)_{t \geq 0}$ defined as

$$\hat{\sigma}_t^b(h^{t-1}) = \begin{cases} Y & \text{if } t < T^b \text{ or if } \hat{\rho}_t(h^{t-1}) = 1, \\ N & \text{otherwise.} \end{cases} \tag{5}$$

is the *unique* best response to the lenders' strategies that always choose Y (regardless of the history), and so, $\hat{\boldsymbol{\sigma}}^b \in \Sigma^{b1}$.

Round 2.

Lenders. Because $\hat{\sigma}_t^I$ weakly dominates any other strategy, no further elimination is possible.

Borrower. Because W^b is increasing for all $T < T^b$ and also $T^\ell < T^b$ (by Lemma 2), the strategy $\hat{\boldsymbol{\sigma}}^b$ yields the highest payoff against $\hat{\sigma}_t^I$. Thus, $\hat{\boldsymbol{\sigma}}^b$ survives this round too.

Outcome

Now we reached the fixed point. The behavior of the agents is uniquely identified except for a lender with belief $\hat{\rho}_t(h^{t-1}) = \rho^\ell$ and the borrower with belief $\hat{\rho}_t(h^{t-1}) = \rho^b$.[11] In the discrete time model, such a lender and borrower do not exist for generic parameters. Moreover, the effects of such exceptional cases, if they exist, on any object of interest become negligible as $\mathrm{d}t \to 0$. In this essentially unique outcome, if no success has arrived, trade occurs until T^ℓ and stops there. Of course, after a single success, the agents always choose to trade.

B.3 Privacy

Note that under Privacy, information asymmetry emerges endogenously as the game proceeds. The history is private information of the borrower, and the lenders only know the calender time. We shall prove Proposition 2.

Round 1.

Lenders. Always choosing Y is included in the lenders' admissible strategies. This is because always choosing Y is the *unique* best response to the borrower's strategy that chooses Y only after success.[12]

Borrower. The same as in Round 1 under Transparency. We eliminate all the strategies outside Σ^{b1}. Note that for any $\boldsymbol{\sigma}^{b1} \in \Sigma^{b1}$, if the borrower succeeds, he will choose Y after that.

[11] We regard T^b as the time at which the belief reaches ρ^b in the limit as $\mathrm{d}t \to 0$.

[12] For $t < T^\ell$, choosing Y is conditionally dominant.

Round 2.

Lenders. By Round 1, the borrower will choose Y after he made a single success. Given this, a lender's most pessimistic belief is such that the borrower also chooses Y even if he has never succeeded. By Lemma 3, this most pessimistic belief is equal to the prior ρ_0. Because $\rho_0 > \rho^\ell$ by Lemma 2, Lemma 1 implies that it is dominant for the lenders to always choose Y.

Borrower. Note that the lenders' strategies of always choosing Y survived the first round. Since $\hat{\boldsymbol{\sigma}}^b$ defined in (5) is the unique best response to these, $\hat{\boldsymbol{\sigma}}^b$ survives in this round.

Round 3.

Lenders. No further elimination is possible because it is iteratively dominant for a lender to always choose Y (Round 2).

Borrower. Note that $\hat{\boldsymbol{\sigma}}^b$ defined (5) is the unique best response to the lenders' strategies of always choosing Y. Given that the lenders always choose Y (Round 2), this is the only strategy that survives this round—all the other strategies are eliminated.

Outcome

Because the strategy profile that survives Round 3 is unique, no further elimination is possible for both parties. That is, we identified the iterated admissible set. The resulting outcome is essentially unique (but the same qualification applies as in case of Transparency). In this essentially unique outcome, if no success has arrived, trade occurs until T^b and stops there. Of course, after a single success, the agents always trade.

B.4 Remarks on Conditional Dominance

Note that under Transparency, weak dominance can be strengthened to conditional dominance (see Shimoji and Watson, 1998). That is, from the process of iterated elimination of conditionally dominated strategies, we get the same outcome. The lenders' strategies in Round 1 are conditionally dominant. No strategy of the borrower is eliminated in Round 1 (because the borrower is indifferent between any two strategies if the lenders always choose N). However, any strategy that leads to a different outcome will still be eliminated in Round 2.

This is not the case under Privacy. Again, no strategy of the borrower is conditionally dominated, and can be eliminated in Round 1. In particular, the borrower's strategy that chooses N after a success survives. Given this, no further elimination is possible.

C Ex Post Bargaining

In Section 2, we assumed that the borrower can commit to repaying an exogenous amount x when he succeeds. In this section, we shall replace these assumptions by the following two assumptions, and show that relaxing the assumptions does not change our results.

First, we assume that there is no commitment. The borrower can choose to renege on his debt when he succeeds. If he reneges, he can receive 1 unit of the consumption good solely by himself.

Second, we assume that when the borrower succeeds, the borrower and a lender split 1 unit of the consumption good via ex-post Nash bargaining. Ex-post Nash bargaining is commonly assumed in model of bilateral meetings (see, for example, Duffie, Gârleanu, and Pedersen, 2005).

Importantly, we assume that the lenders can keep track of whether the borrower has repaid or not in the past. If this information is not observable, the borrower always has an incentive to renege. If this information is observable, the lenders can punish the reneging borrower by excluding him from the credit market.

More precisely, we consider the following strategy about reneging. If the borrower succeeds, the borrower repays some amount x (which is endogenized below) and the lender participates in the bargaining process. If the borrower does not repay x, the borrower chooses not to repay x forever thereafter and the future lenders choose not to lend.

Now, we shall describe the borrower's incentive not to renege. Notice that when the borrower succeeds, the lenders have no incentives to stop under either information structure. So, in the borrower's continuation payoff, we can suppose that trade occurs forever, under either information structure.

If the borrower repays x (which is to be endogenized below), he obtains a continuation payoff of

$$r(1-x) + \lambda(1-x) - c.$$

On the other hand, if he reneges on his debt x, he obtains a continuation payoff of r, because he is excluded from the market. The difference between the two is

$$-rx + \lambda(1-x) - c.$$

If a lender accepts x, she obtains x. If she walks away from bargaining with the borrower, she obtains 0. The difference between the two is x.

Let θ be the lender's bargaining power. Then, ex-post Nash bargaining solves

$$\max_{x\in[0,1]} \quad x^{\theta}\left[-rx - c + \lambda(1-x)\right]^{1-\theta}.$$

The solution to the bargaining problem is given by

$$x = \left(\frac{\lambda - c}{r+\lambda}\right)\theta. \tag{6}$$

We have $x \in (0,1)$ because Assumption 1 guarantees $\lambda > c$.

For x given by (6), the borrower's incentive to repay holds automatically, that is,

$$-rx + \lambda(1 - x) - c \geq 0.$$

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