CIGS The Canon Institute for Global Studies

CIGS Working Paper Series No. 24-004E

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Keiichiro Kobayashi (*Keio University, CIGS, RIETI*)

September 02, 2024

General Incorporated Foundation
The Canon Institute for Global Studies
一般財団法人 キヤノングローバル戦略研究所
Phone: +81-3-6213-0550 https://cigs.canon/

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Abstract

We propose a tractable model of financial crises to demonstrate that corporate debt restructuring can promote economic recovery. The model can replicate the following empirical regularities: Credit-fueled asset-price booms end up with collapses, followed by deep and persistent recessions with productivity declines. Risk-shifting firms amplify the booms and busts of asset prices by purchasing the assets by borrowed money. Resultant debt overhang lowers productivity and output by discouraging borrowing firms from expending additional efforts. This inefficiency is aggravated by the spillover effect in the monopolistic competition. Larger asset-price booms are followed by deeper and more persistent recessions. The ex-post government subsidy to lenders for implementing debt relief can improve the borrowers' productivity and increase the lenders' payoff and social welfare, without inducing time inconsistency.

Key words: Financial crisis, love-for-variety, zombie lending, the debt Laffer curve.

JEL Classification: E02, G01, G33

1 Introduction

Recent studies show the following empirical regularities about booms and busts of asset prices: when the asset-price boom is fueled with an increase in credit, the asset boom tends to end up with bust, followed by a deep and persistent recession with lower observed total factor productivity (TFP). See, for example, Jordà, Schularick and Taylor (2015). We propose a parsimonious model to replicate these empirical regularities, which is also tractable in analyzing policy interventions. Our model may be one of few attempts to give an integrated explanation for the movements of asset prices, credit and productivity in the

*I am grateful to Tomohiro Hirano for fruitful discussions so many times about this project. I thank Gadi Barlevy, Toni Braun, Simone Lenzu, Tomoyuki Nakajima, Matthias Schlegl, Petr Sedláček, Makoto Watanabe, Tack Yun and seminar participants at Kyoto, Waseda, CIGS 2023 Year End Macroeconomics conference, and JEA 2024 Spring meeting for various insightful and helpful comments. The previous title of this paper was "Asset-Price Collapse and Macroeconomic Debt Overhang."

†Keio University, CIGS, RIETI

whole cycle of a financial crisis. In particular, a unique feature of our model is to relate the productivity declines after the asset-price burst to debt overhang. *Debt overhang* in this study indicates the situation that the stipulated amount of debt is larger than the repayable amount and the repayment is yet to be settled.

Why we focus on debt overhang? As we will see, our debt overhang theory of financial crisis provides a new perspective consistent with the observation that persistent stagnation after the crisis tends to be accompanied by the contraction of borrowers' demand for credit, rather than the tightening of the credit supply. A typical example is the stagnant three decades of Japan since the 1990s. After the bursting of asset price bubbles at the beginning of the 1990s, Japan has suffered from persistently weak demand for credit in the corporate sector. Another reason to raise a debt overhang theory is that it provides a new perspective on policy interventions. The standard prescription in the literature is to close and liquidate the zombie firms, who suffer from debt overhang, because they are regarded as intrinsically unproductive. Our theory implies they may not be intrinsically inefficient, and the reduction of debt overhang could restore the productivity of borrowers and also increase lenders' payoffs and social welfare.

What we do in this paper is the following. We construct a simple two-period model, in which we unify the model of risk-shifting booms of asset prices (Allen and Gale 2000; Allen, Barlevy and Gale 2022) and the model of macroeconomic debt overhang due to spillover effect through aggregate output (Lamont 1995). Our theory differs from these two models in that it can account for productivity declines and deep and persistent stagnation in the aftermath of financial crises, by focusing on discouraged entries. One key ingredient is our assumption that the risky asset, the price of which can be driven up by risk-shifting, is also used as an input for production by borrowers. In the first period, there arrives the news that the productivity of the asset becomes either A_H or A_M in the second period, where $A_H \gg A_M$. The parameter A_H represents the degree of optimistic expectation in period 1, concerning the productivity of the asset in period 2. When the ex-ante optimism is small (i.e., A_H is small), the price of the asset is low and there are no debt overhang and no recession in equilibrium. We call this situation the Normal Equilibrium (NE). When the ex-ante optimism is large (i.e., A_H is large), there emerges the Debt Overhang Equilibrium (DOE) where the asset price is higher and debt is larger initially. It then becomes debt overhang if the productivity of the asset turns out to be low. The debt overhang is accompanied by recession with lower productivity.

In the DOE, the asset price is driven up by firms who buy the asset by borrowed money. The borrowers bid up the asset price because they can push the cost on the lenders by defaulting on the debt, when the productivity of the asset turns out to be low (A_M). This is the *risk-shifting boom* of asset prices (Allen and Gale 2000; Allen, Barlevy and Gale 2022). In period 2, the borrower cannot repay the full amount of debt when the productivity of the asset turns out to be low. The TFP declines disproportionately because debt overhang discourages borrowing firms from expending efforts in production activity. They are discouraged because the lenders cannot commit to reward their effort as the lenders have the legitimate right to take all as long as debt is larger than the borrowers revenue. This is what we call the lack of lender's commitment (Kobayashi, Nakajima and Takahashi 2023). The borrowers choose to exit, that is, not to expend additional effort because they know they will get nothing, as the lenders take all. In addition to the debt overhang due to the lack of lenders' commitment, there exist a spillover effect through shrinkage of aggregate output in our economy of the monopolistic competition, which we call the *aggregate output externality*. We define the aggregate output externality as the effect of an exit (or entry) of one firm that decreases (or increases) the other firms' revenues by reducing (or increasing) the aggregate demand. This externality discourages a firm from continuing production when some other firms exit due to debt overhang. In our model, the aggregate output externality decreases the aggregate productivity due to the "love-for-variety" structure.[1] It is also shown that a larger asset-price boom may lead to a deeper recession: When the asset-price boom is larger in the first period, the resulting debt overhang becomes larger, leading to a larger number of exiting firms (varieties), which implies a lower aggregate productivity due to the love-for-variety. In the extended model where new-born firms can enter the economy, it is shown that a larger boom-and-bust leads to a more persistent stagnation in which fewer or no new firms enter the economy. This is because the expected profits for new entrants are negatively affected by exits of incumbent firms, and they become lower in the deeper recession after the larger asset boom. This mechanism simply explains the persistence of the post-crisis stagnation in our two-period model.[2]

Using this model, we compare ex-ante and ex-post policy interventions, including monetary policy. In particular, we emphasize the ex-post debt restructuring or debt relief. A policy intervention to incentivize restructuring of debt overhang may increase the recovery of debt for lenders and also improve productivity and social welfare. The result that the lenders are better off by reducing the face value of debt is the same as the classical argument of debt overhang or the debt Laffer curve (Sachs 1988; Krugman 1988), which is about the sovereign debt, while our focus in this paper is on corporate debt similar to Kobayashi et al. (2023). As argued in these studies, lenders may know that restructuring

[1] Lamont (1995) argue that the investment is reduced by the macroeconomic debt overhang due to the spillover effect, that is similar to the aggregate output externality in our model. The difference is the following: in Lamont's model, the spillover effect discourages the investment and does not change productivity because there is no exit of firms in his model, while the output externality causes endogenous productivity declines in our model because firms can exit the market.

[2] In the appendix we also provide a simple infinite time-horizon model and confirm the persistence of ex-post recessions after large asset booms.

of debt overhang increases their payoff, and reduce debt on their own. In other words, the debt Laffer curve arguments in Sachs (1988), Krugman (1988), and Kobayashi et al.(2023) do not imply the necessity of policy intervention, because the lenders themselves can choose the efficient amount of debt restructuring in their models, unless there exist exogenous frictions. However, because there exists the aggregate output externality in our setup, the amount of debt reduction without policy intervention is smaller than the socially optimal level. Because of this output externality, a policy intervention to encourage debt reduction is welfare improving. This policy implication is one novelty of our study. To facilitate debt restructuring, the government can subsidize the lenders to partially compensate the loss of debt write-off so that the optimal amount of debt reduction is realized. Our result that debt relief improves productivity of the borrowers can be seen as complementary to that of Caballero, Hoshi and Kashyap (2008). They stress that zombie firms with debt overhang are intrinsically inefficient and should be liquidated. Our result points to the possibility that zombie firms may be able to become productive if their debts are forgiven. We also show that ex-post policy to encourage debt restructuring does not necessarily distort ex-ante incentives, that is, the time-inconsistency problem may not arise when the ex-post policy is subsidy to the lenders while the ex-ante allocation is decided by the borrowers.

The rest of the paper is organized as follows. Next section reviews the related literature. In Section 3, we describe the setting of the baseline model. Section 4 specifies the equilibrium and shows that a larger asset boom causes a deeper recession. Section 5 discusses the policy implications. In Section 6, we analyze the extended model, where new-born firms can enter the economy after the asset-price collapse, and show that a larger asset boom leads to a more persistent stagnation. Section 7 concludes.

2 Literature

2.1 Empirical regularities

There is a large empirical literature that report empirical regularities concerning asset-price and credit booms and their effects on the subsequent economic growth. Our model is an attempt to give an integrated account for the empirical regularities of the crisis cycle reported by the following literature. Jordà, Schularick, and Taylor (2015), who analyze data of 17 countries for the past 140 years and show that the asset-price booms fueled by credit booms tend to end up with financial crisis, followed by deep and persistent recession. Greenwood, Hanson, Shleifer and Sørensen (2022) also report that a rapid growth in private credit and asset prices predicts a financial crisis.

Credit booms in the short-run can predict the crisis.[3] Schularick and Taylor (2012)

[3] Credit deepening in the long-run leads to higher long-term economic growth (King and Levine 1993).

analyze data on 14 countries for 140 years and report that credit booms tend to lead to financial crises. Verner (2019) reports based on the data of 143 countries for 60 years that credit booms in the short-run usually lead to financial crises. Giroud and Mueller (2021) find that a firm leverage boom predicts a boom-bust cycle of employment. Krishnamurthy and Muir (2024) report that credit grows high with too low credit spread in pre-crisis periods. They also report that a severe and protracted recession tends to follow the crisis.

Greenwood, Hanson, Shleifer and Sørensen (2022) emphasize that corporate debt buildups have adverse effects on the economy, as well as household debt. As our model raises the prospect that corporate debt may be a significant driver of a financial crisis, this point is noteworthy. The dominant view in the literature since the Global Financial Crisis (GFC) has been that the household debt is the driver of financial crisis, and less attention has been paid to the corporate debt. Recently, however, there have emerged empirical studies that emphasize the importance of the corporate debt (Greenwood et al. 2022; Jordà et al. 2022; Sever 2023; Ivashina et al. 2024). Jordà et al. (2022) argue that corporate debt has a little power in predicting crises and it predicts slow recovery only in the countries with inefficient bankruptcy procedures. Ivashina et al. (2024) with data of bank loans in 115 countries over the period 1940–2014 show that the corporate debt accounts for the vast majority of nonperforming loans after the crisis and predicts slower recovery in average countries. They argue that the difference from Jordà et al. (2022) is due to coverage of data and definitions of corporate debt and crisis events. In particular, Ivashina et al. (2024) focus on the bank loans to firms, while Jordà et al. (2022) include corporate bond, which seems uncorrelated with crises. Kornejew, Lian, Ma, Ottonello, and Perez (2024) document that business credit booms are often followed by severe declines in output in environments with poorly functioning business bankruptcy. They also construct a model that accounts for declines in output in the aftermath of non-fundamental credit booms, where efficient bankruptcy systems can mitigate the declines. The difference from our model is that Kornejew et al. (2024) do not have asset prices in their model and they do not analyze policies concerning debt restructuring explicitly.

There are studies that point to distinction between good credit booms with high economic growth and bad credit booms with low growth (Gorton and Ordoñez 2020). Müller and Verner (2023) report, based on the data of 116 countries for 80 years, that bad credit booms are mostly debt booms in non-tradable sector.[4]

[4] Although our focus in this paper is not on whether the asset booms are caused by credit-supply or credit-demand shocks, we note that Verner (2019) reports that the short-run credit booms are usually driven by credit-supply expansion. Justiniano, Primiceri and Tambalotti (2019) also argue that the empirical facts about the housing boom preceding the Great Recession are consistent with the explanation that the boom was caused by an increase in credit supply, not in credit demand. Adverse effect of credit supply shock is also reported by Mian, Sufi and Verner (2017). They show that a credit supply shock induces a decrease in the interest rate and an increase in household debt with consumption boom, followed by persistently

It is also well known that financial crises tend to be followed by persistent productivity slowdown. Duval, Hong and Timmer (2020) argue that financial frictions might have caused the productivity slowdown during the Great Recession. Adler et al. (2017) report that productivity growth fell sharply after the GFC in 2008.[5] Related literature is on the great depressions, a decade-long deep recessions observed in the 20th century. It is said that deep and persistent productivity declines are the major cause of the great depressions (Hayashi and Prescott 2002, Kehoe and Prescott 2002). Our paper is also related to the literature on the Secular Stagnation, e.g., Rachel and Summers (2019) and Eggertsson, Mehrotra and Robbins (2019). In this literature, changes in the aggregate productivity is taken as given exogenously, while our model provides an endogenous explanation for persistent stagnation of productivity (see Section 6).

2.2 Theoretical ingredients

Our theory is a new attempt to integrate the following theories of risk-shifting asset booms, debt overhang, and aggregate output externalities.

Risk-shifting effect on asset prices: This study is related to the literature on risk-shifting booms of asset prices, which are theoretically analyzed by Allen and Gale (2000) and Allen, Barlevy and Gale (2022). They demonstrate that asset-price booms can be driven by risk shifting by investors who buy the asset with borrowed money. In their models, the cost of default is exogenous and no policy response is possible ex-post, whereas in our model the ex-post debt reduction can reduce the inefficiency. The risk shifting from the firms to the lenders (households) in our model is possible due to the technological constraint that only firms can produce output, and the households cannot produce anything from capital.

Debt overhang and aggregate output externality: Our study is related to the broad literature of debt overhang. As Kobayashi, Nakajima and Takahashi (2023) argue, debt overhang can be categorized into two types. The first type of debt overhang is due to the lack of borrowers' commitment, and the second type is due to the lack of lenders' commitment. The debt overhang in this paper is the second type. We choose the second type because it seems consistent with our experience in the lost decades in Japan, associated with a persistent shrinkage of credit demand. The first type of debt overhang is analyzed by, e.g., Albuquerque and Hopenhayn (2004), Kovrijnykh and Szentes (2007), and Aguiar, Amador, and Gopinath (2009). In these models, the inefficiency is generated

lower GDP growth.

[5]There is an opposite view that labor productivity increased in GFC. See Lazear, Shaw and Stanton (2013) who argue that people tend to work harder during recessions.

from the lenders' offer of back-loading payoff schedule to the borrowers in order to prevent the borrowers' default at the early stage. The second type of debt overhang is argued in macroeconomics by Sachs (1988), Krugman (1988), Occhino and Pescatori (2015), and Kobayashi, Nakajima, and Takahashi (2023). In the second type, the inefficiency arises because borrowers choose not to expend effort as the lenders cannot commit to reward their effort. The lack of lenders' commitment is caused by the fact that the lenders have legitimate right to take all when the amount of debt is larger than the borrowers' revenues. In this case, the lenders cannot credibly commit to give positive amounts to the borrowers to reward their efforts. Anticipating that the lenders will take all, the borrowers refrain from expending effort and make their production inefficient. In our model, the inefficiency of debt overhang is aggravated by the aggregate output externality, which is a spillover in the monopolistic competition. This spillover effect is argued by Lamont (1995) in the context of debt overhang (see also, e.g., Blanchard and Kiyotaki 1987). A difference between our model and Lamont's model is that there are no exits of firms and the aggregate productivity is invariant in his model, whereas exits of firms endogenously lower the productivity in our model due to the love-for-variety structure. See also Philippon (2010) for multiple equilibria due to the similar spillover of debt overhang for households and banks.[6] The aggregate output externality in our model works through the shortage of the aggregate demand, which is similar to Illing, Ono and Schlegl (2018). The difference is that the demand shortage in their model is due to unlimited liquidity preference, which is exogenously assumed, while the demand shortage in our model is caused by debt overhang.

2.3 Theoretical studies on financial crises and policy responses

This paper is related to the vast literature on financial crises and the policy responses. We can clarify the difference of our model from the existing studies in three aspects: The source of inefficiencies, the nature of inefficiencies, and the policy interventions. First, concerning the source of inefficiency, the literature primarily focus on pecuniary externality due to borrowing constraints (Aguiar and Amador 2011; Benigno et al. 2023; Bianchi 2011, 2016; Bianchi and Mendoza 2010; Farhi, Golosov, and Tsyvinski 2009; Gertler, Kiyotaki, and Queralto 2012; Lorenzoni 2008; Lorenzoni and Werning 2019) or coordination failure such as bank runs (Diamond and Dybvig 1983; Gertler and Kiyotaki 2015; Keister 2016; Keister and Narasiman 2016). On the other hand, the source of inefficiency in our model is debt overhang, which can emerge from various reasons such as news shocks, asset bubbles and overconfidence, even if pecuniary externality or coordination failure are nonexistent. Second, concerning the nature of propagation of inefficiencies, many exist-

[6] See also Occhino (2017) for a simplified model of multiple equilibria due to debt overhang.

ing models feature allocative inefficiencies in consumption allocation (Bianchi 2011; Chari and Kehoe 2016; Farhi, Golosov, and Tsyvinski 2009; Jeanne and Korinek 2020; Keister 2016) or inefficient production due to increases in the cost of credit, that is, the credit crunch (Bianchi 2016; Bianchi and Mendoza 2010; Gertler, Kiyotaki, and Queralto 2012; Lorenzoni 2008). In contrast to them, our model features inefficient production due to shortage of the aggregate demand. Third, concerning the policy interventions, the existing literature primarily focus on the trade-off that the bailout policy induces between ex-ante incentive and ex-post efficiency, that is, the time inconsistency (Bianchi 2016; Chari and Kehoe 2016; Green 2010; Keister 2016: Keister and Narasiman 2016). Chari and Kehoe (2016) argue that bailouts can be welfare reducing because of the time inconsistency, while Bianchi (2016), Green (2010), Keister (2016), and Keister and Narasiman (2016) make the case that welfare improving effects of bailout policies overwhelm the adverse effects of time inconsistency. It is shown in our model that the time inconsistency of ex-post policy disappears and only welfare-improving effects survive under some circumstances where ex-post policy is subsidy to lenders and ex-ante allocation is decided by borrowers.

2.4 Zombie lending

Our theory is very closely related to the growing literature on the zombie lending or evergreening in the wake of financial crises. Zombie lending is the bank lending to non-viable firms due to distorted bank incentives. The pioneering works by Peek and Rosengren (2005) and Caballero, Hoshi and Kashyap (2008) report the proliferation of zombie lending in Japan during the 1990s. Acharya, Lenzu and Wang (2024) and the references therein analyze models of zombie lending and report related empirical findings.[7] Acharya et al. (2024) emphasize that the accommodative government policy can distort bank incentive and induce zombie lending, leading to persistent stagnation. Our model complements to the existing literature of zombie lending in the following three respects. First, even without distortionary policy, large debt overhang can induce persistent stagnation in our model. Second, Acharya et al.(2024) implies that removal of distortionary policy is welfare improving, while introduction of active policy intervention is necessary to mitigate the aggregate output externality in our model. Third, most of the literature assume that zombie firms are intrinsically inefficient and that their exits improve productivity and welfare through mitigating congestion, while our model implies that zombie firms may be

[7] The literature usually defines the zombie firms as firms that are kept afloat with subsidized loans from the lenders. Recently Rocheteau (2024) raises the possibility that the equity shares of firms with negative net present values (NPV) can be positively priced in the market without any subsidy. He calls these firms zombies and shows that they can exist if the equity shares of zombie firms provide liquidity under the environment with high liquidity demand. Since Rocheteau (2024) assumes that the negative NPV of zombie firms are exogenously given, they are not in our interest in this paper.

able to restore efficiency by reducing their debt burden. Nakamura and Fukuda (2013) could be a supportive evidence for our theory. They report that significant portion of zombie firms in non-tradeable sector in Japan that had difficulties in repaying debt in the 1990s have recovered and become productive in the 2000s, implying that debt-ridden zombie firms may not have been intrinsically unproductive. Becker and Ivashina (2022) empirically show that inefficient bankruptcy procedures amplify inefficiency of zombie lending. Kornejew et al. (2024) support their arguments empirically and theoretically. These results may also support our argument on the welfare improving effect of debt reduction.

3 Model

The model is a two-period closed economy, where households and firms are inhabited. In period 1, firms buy capital from households on credit, that is, they promise to pay consumer goods to households in period 2 in exchange for receiving capital in period 1. Firms install capital for specialization though its productivity, which is an aggregate shock, has not been revealed yet. In period 2, the productivity of capital is revealed. After the productivity is revealed, the lending households have a chance to restructure the firms' debt, given that the firms are still able to exit the market and default on the restructured debt. The production and consumption take place only in period 2. Social welfare is maximized when the total output in period 2 is maximized. In Section 6, we extend the model by introducing new entries of new-born firms in period 2.

3.1 Setup

There are two periods, period 1 and period 2, in the economy. There inhabits a unit mass of identical households and each household owns a firm. Thus, the measure of the firms is also unity. The firms can produce the intermediate goods from capital only in period 2, and the intermediate goods are aggregated into the consumer goods. The households can consume the consumer goods only in period 2. Each household is endowed with K units of capital at the beginning of period 1. The total amount of capital in the economy is thus K. Firms can produce consumer goods from capital, while households cannot produce anything. In period 1, firms choose the amount of capital, k, where $k \leq K$, to use for production in S-sector which is explained below shortly. Each firm has to buy k from (another) household and install k in period 1 to prepare for production in S-sector (S-production) in period 2. As firms have nothing to pay for k in period 1, they issue debt D to buy k. That is, a firm purchases k units of capital from a household in exchange for a promise to pay $D \equiv Qk$ units of period-2 consumer goods to the household, where Q is the price of capital in terms of period-2 consumer good. We simply posit that debt contract is the optimal contract in this economy, meaning that it is implicitly assumed that there exist asymmetric information and agency problems a lá Townsend (1979) or Gale and Helwig (1985).

Production technologies: Initially in period 1, all firms are in S-sector, which stands for "Specialized production." They install capital in period 1 for production in period 2. In period 2, lending households can reduce debt D to $\hat{D}$ ($\leq D$) using a costly debt-restructuring technology (see the next paragraph titled "Debt-restructuring technology"). Then, the firms can choose whether to produce output in S-sector or to exit S-sector. The exited firms move to C-sector, which stands for "Common production." After producing output in S- or C-sector, the firms repay $\hat{D}$ if revenues are larger than $\hat{D}$. If revenues are smaller than $\hat{D}$, they repay all revenues to the lenders and default on the remaining debt.

- **S-sector:** In S-sector, each firm produces specialized intermediate goods in the monopolistically competitive market. Productivity parameter in S-sector, A_s, is common for all firms. A_s is stochastic and revealed at the beginning of period 2. There are two states $s \in \{M, H\}$ in period 2. The state s becomes $s = H$, where $A_s = A_H$, with probability p_H, and becomes $s = M$, where $A_s = A_M$, with probability $p_M = 1 - p_H$. We consider the case where $A_M \ll A_H$ and $p_H \ll 1$. The state M is the medium or "normal" state, whereas H is the high or "good" state. Given the realization of A_s in period 2, firm i, where $i \in [0, 1]$, produces the intermediate goods

  $$y_i = A_s k_i,$$

  where k_i is the amount of capital that firm i installed in period 1. To use k_i for production in S-sector, firm i must install k_i in period 1, and no more capital can be added in period 2. The consumption goods Y_S is produced from the intermediate goods y_i by the Dixit-Stiglitz aggregator:

  $$Y_S = \left(\int_0^n y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

  where σ is the elasticity of substitution with $\sigma > 1$ and $n \in [0, 1]$ is the number of remaining firms in S-sector, which is endogenously decided as a result of firms' choice of exit at the beginning of period 2. The firms who exit S-sector goes to C-sector.

- **C-sector:** In C-sector, a firm can produce $A_L k$ units of consumption goods from k units of capital in period 2. The firms need not install capital in period 1 for production in C-sector. Households can sell capital in period 2 for the use of C-sector, or firms in S-sector can move to C-sector and can use their capital k for production in C-sector in period 2, although they were installed in period 1 for production in S-sector. Productivity parameter in C-sector, A_L, is deterministic and satisfies

$$0 < A_L < A_M \ll A_H.$$

C-sector is a perfectly competitive market and firms do not have monopoly power there. In the symmetric equilibrium where $k_i = k$ for all i, the total output in C-sector, Y_C, is given by

$$Y_C = A_L(K - nk),$$

where n is the number of S-sector firms, k is the amount of capital per one S-sector firm, and thus nk is the total amount of capital used in S-sector.

- **Fixed utility cost:** We assume a fixed utility cost is necessary to undertake S-production. The firm wants compensation for the utility cost when it produces output in S-sector. We make the following assumption:

  Assumption 1. The firm needs to expend an infinitesimally small fixed utility cost in period 2 when it produces output in S-sector, while no utility cost is necessary to produce output in C-sector. The consumption equivalence of the fixed utility cost of S-sector is ε in terms of period-2 consumer goods, where

$$0 < \varepsilon \ll A_L K.$$

Debt-restructuring technology: In period 2, after the state s and the aggregate productivity in S-sector A_s are revealed and before production takes place, the lending households are given a chance to reduce debt. When a lender i ($\in [0, 1]$) reduces the debt from D to $\hat{D}$ ($\leq D$), where they are measured in terms of period-2 consumer goods, she has to pay the dead-weight cost:

$$z_i(D - \hat{D})^{\phi}, \tag{1}$$

in terms of the period-2 consumer goods, where $\phi \geq 1$ and the cost parameter z_i distributes over $[0, z_{\max}]$ with the cumulative distribution function $F(z)$ and the density function $f(z) = F'(z)$. For simplicity of the analysis, we assume that z_i is revealed in period 2, and all lenders have the identical expectations $\Pr(z_i \leq z) = F(z)$ in period 1 about their own z_i. The dependence of debt-restructuring cost on the stipulated amount D is a plausible assumption, whose microfoundation can be given by a bargaining game inside the bank, such as the one in Appendix A, in which some participants never give up their claim of D. The lender's debt restructuring (described later in Lemma 1) under technology (1) can be seen as the reduced form of the outcome of the game in Appendix A. As we will see in the following analysis of equilibrium, there exists an equilibrium value $\bar{z}$ such that lenders i with $z_i \in [0, \bar{z}]$ restructure their D to $\hat{D}$. With the debt restructuring, the total output in S-sector becomes

$$Y_S^{DR} = Y_S - \left(\int_0^{\bar{z}} z_i dF(z_i) \right) (D - \hat{D})^{\phi}.$$

Total consumption in the economy, Y, is then given by

$$Y = Y_S^{DR} + Y_C.$$

We describe the decision making of the model backward.

3.2 Decision making in period 2

In the previous period (period 1), capital stock of each firm k and the debt for each firm $D = Qk$ were already determined. In period 2, the debt D is restructured to $\hat{D}_i$ by lender $i \in [0, 1]$ and the borrowing firm i decide whether to exit. We use the same subscript for a lender and her borrower. What is to be determined in period 2 is the amount of restructured debt $\hat{D}_i$ for $0 \leq i \leq 1$ and the number of continuing firms in S-sector, n.

The demand function for firm i's good is given as the solution to $\max_{y_i} Y_S - \int_0^n p_i y_i di$, where $Y_S = \left(\int_0^n y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$ and p_i is the price of the intermediate good i. The first order condition (FOC) implies

$$p = Y_S^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}}.$$

In a symmetric equilibrium where each firm uses the identical amount of capital $k_i = \bar{k}$, the aggregate output in S-sector is given by

$$Y_S = n^{\frac{\sigma}{\sigma-1}} A_s \bar{k},$$

where $n^{\frac{\sigma}{\sigma-1}} A_s$ is the total factor productivity (TFP) in S-sector, which is increasing in n. Revenue of a firm in S-sector is

$$py = Y_S^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}} = n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}} \equiv \pi(n, A_s, k),$$

where k is the firm's capital and $\bar{k}$ is the social level of capital per firm in S-sector. Here we use π as the abbreviation of $\pi(n, A_s, k)$ flexibly. In the symmetric equilibrium where $k = \bar{k}$, the revenue is $\pi = n^{\frac{1}{\sigma-1}} A_s \bar{k}$.

Firms' exit decision: Given that the debt is restructured to $\hat{D}_i$, the Free Entry Condition (FEC) for firm i who chooses whether to continue operations in S-sector is written as

$$\pi(n, A_s, k) - \hat{D}_i \geq \varepsilon. \tag{2}$$

The firm continues to operate in S-sector and repay $\hat{D}_i$ if (2) is satisfied. The firm with $\pi - \hat{D} < \varepsilon$ has two options, i.e., either to earn $\pi(n, A_s, k)$ in S-sector and repay $\min\{\pi(n, A_s, k),\ \hat{D}_i\}$ to the lender, or to move to C-sector to produce $A_L k$ units of consumer good and repay all of them to the lender.[8] Since $\pi - \hat{D}_i < \varepsilon$, the firm obtains $\max\{0, \pi - \hat{D}_i\} < \varepsilon$ if it operates in S-sector. Since the firm pays the utility cost ε to produce output in S-sector (Assumption 1), the payoff for the firm becomes $\max\{0, \pi - \hat{D}_i\} - \varepsilon < 0$ if it operates in S-sector, whereas the payoff becomes $0 = \max\{0, A_L k - \hat{D}_i\}$ if it operates in C-sector. Therefore, the firm with $\pi - \hat{D} < \varepsilon$ exits S-sector and goes to C-sector. In sum, the revenue of firm i can be given by $y(\hat{D}_i)$, where

$$y(\hat{D}_i) = \begin{cases} \pi(n, A_s, k) - \varepsilon, & \text{if} \quad \pi(n, A_s, k) \geq \hat{D}_i + \varepsilon, \\ A_L k, & \text{if} \quad \pi(n, A_s, k) < \hat{D}_i + \varepsilon. \end{cases} \tag{3}$$

Let $N(n)$ be the measure of firms who satisfies (2), where n is given in $\pi(n, A_s, k)$. In equilibrium, the rational expectations, i.e., $N(n) = n$ must holds. Since $N(n) = n$ may have multiple solutions, the equilibrium values of n can be multiple. For example, $n = 0$ is always an equilibrium value, as $N(0) = 0$ because $\pi(0, A_s, k) = 0 < \hat{D}_i + \varepsilon$ for any $\hat{D}_i \geq 0$. We make the following assumption that agents are optimistic to eliminate the possibility of multiple equilibria due to pure coordination failure of expectations.

Assumption 2. When there exist multiple values of n, which satisfies $N(n) = n$, the expectations of households and firms are coordinated such that the largest value of n prevails as the commonly-held expectation in equilibrium.

This assumption says that the macroeconomic expectations are coordinated such that the most optimistic one among all feasible expectations prevails.

Lenders' decision on debt restructuring: Taking n as given and anticipating firms' exit decision (3), the lender i solves the following debt restructuring problem to maximize her profit.

$$\max_{\hat{D}} \left[\min\{\hat{D}, y(\hat{D})\} - z_i (D - \hat{D})^{\phi} \right], \quad \text{s.t.} \quad \hat{D} \leq D. \tag{4}$$

The solution is given explicitly, as follows. If $D \leq \pi(n, A_s, k) - \varepsilon$, then the lender chooses $\hat{D} = D$, and the firm earns $\pi(n, A_s, k)$ and repay D. In the case where $D > \pi(n, A_s, k) - \varepsilon$, consider the lender i whose z_i satisfies

$$\pi - \varepsilon - z_i (D - \pi + \varepsilon)^{\phi} \geq A_L k, \tag{5}$$

which is rewritten as

$$z_i \leq \frac{\pi - \varepsilon - A_L k}{(D - \pi + \varepsilon)^{\phi}}. \tag{6}$$

[8] Without loss of generality, we can focus on the case where $A_L k \leq \hat{D}_i$.

This lender i restructures the debt to $\hat{D} = \pi - \varepsilon$, and the firm i earns π to repay $\pi - \varepsilon$ to the lender. The lender with z_i that is larger than $\frac{\pi-\varepsilon-A_L k}{(D-\pi+\varepsilon)^\phi}$ does not restructure the debt, i.e., $\hat{D} = D$, and the firm goes to C-sector to earn $A_L k$ and repay all $A_L k$ to the lender. In sum, we have proven the following lemma.

Lemma 1. *When $D > \pi - \varepsilon$, the lenders choose $\hat{D}$ such that*

- $\hat{D} = \pi - \varepsilon$, *if z_i satisfies* (6),
- $\hat{D} = D$, *if z_i does not satisfy* (6).

Once $D > \pi - \varepsilon$, the borrowing firms obtain nothing (except for the compensation of utility cost ε).

Debt overhang effect: Firm i's decision to exit S-sector is inefficient. This is because the exiting firm's capital cannot be used efficiently in S-sector with productivity A_H or A_M, but is used inefficiently in C-sector with the lowest productivity A_L. This individual inefficiency for an exiting firm can be called debt overhang effect, which is the inefficiency caused by the lack of lender's commitment in the following sense (Kobayashi, Nakajima and Takahashi 2022): When $\pi - \hat{D} < \varepsilon$, the firm would have chosen to continue operations in S-sector if the lender could promised to give ε to the firm to compensate the utility cost (Assumption 1); but, the lender cannot credibly commit to give ε because the lender has the legitimate right to take $\hat{D}$ and leave $\pi - \hat{D}$ $(< \varepsilon)$ to the borrower. The borrower precisely anticipates that the lender will take more than $\pi - \varepsilon$, and chooses to exit S-sector to save the utility cost ε. In sum, the inefficiency of debt overhang is caused by the lack of lender's commitment, which is that the lender cannot credibly commit to make the repayment strictly less than the contractual amount of debt $\hat{D}$.

Aggregate output externality: In addition to the inefficient use of capital for the exiting firm itself, the exit of the firm has a negative externality on the other firms. The exit of one firm reduces the other firms' expected revenues of operating in S-sector by reducing the aggregate output Y_S, because the revenue of a firm π depends on Y_S: $\pi = py = Y_S^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}}$. Since $Y_S = n^{\frac{\sigma}{\sigma-1}} A_s \bar{k}$, we can also rephrase this result as debt overhang decreases the TFP of S-sector, $n^{\frac{\sigma}{\sigma-1}} A_s$, by decreasing the equilibrium value of n. As this negative effect works through reducing the aggregate output Y_S, we call it the aggregate output externality in this paper. It is similar to the spillover effect in Lamont (1995), whereas the external effect that the aggregate output affects the value of n is present in our model, while it is not in Lamont's model.

3.3 Decision making in period 1

Firms promise to pay $D(k) = Qk$ units of consumer goods in period 2 in exchange for receiving k in period 1. The firms install k in period 1 for the S-production in period 2. There are two unknowns in period 1: Q and k, which are given by two conditions: the FOC with respect to k for the maximization of the firms' expected profit, and the participation condition (PC) for households' selling capital.

Borrower's problem: Firms know that the lenders' decision making in period 2 implies that a firm obtains zero if $\pi(n, A_s, k) - D(k) < \varepsilon$ in period 2, as shown in Lemma 1. Knowing this and taking n as given, the firms in period 1 solve

$$\max_k E[\max\{\pi(n, A_s, k) - \varepsilon - D(k),\ 0\}], \tag{7}$$

where $E[\ \cdot\ ]$ is the unconditional expectation. The FOC with respect to k is

$$E\left[\left(\frac{\sigma-1}{\sigma}\right) n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{-\frac{1}{\sigma}} - Q \ \middle|\ \text{ND}\right] = 0, \tag{8}$$

where $E[\ \cdot \mid \text{ND}]$ is the expectation conditional on that debt overhang does not occur, i.e., $\pi(n, A_s, k) - D(k) \geq \varepsilon$.[9] The FOC must hold with equality since otherwise k goes to 0 or $+\infty$. In equilibrium where $k = \bar{k}$, this condition implies

$$Q = \left(\frac{\sigma-1}{\sigma}\right) E[n_s^{\frac{1}{\sigma-1}} A_s \mid \text{ND}]. \tag{9}$$

When the price Q is given by (9), the quantity of capital k is determined as $k = \bar{k}$ by (8), while $\bar{k}$ is determined by the supply, i.e., $\bar{k} = K$, in the symmetric equilibrium where the strict inequality holds in the PC (see the next paragraph).

Lender's problem: The households (lenders) maximize the expected value of their consumption in period 2, given that their choice is either to sell capital K to the firms in exchange for the risky debt or to hold the capital and sell it in the next period for the use in C-sector. The households' choice is limited to the two options because they are subject to the technological constraint that they cannot produce output in S-sector nor C-sector. Thus, the households' decision-making in period 1 is degenerated such that they sell the

[9] Condition that $\pi(n, A_s, k) - \varepsilon - D(k) \geq 0$ gives the threshold $A(Q, k)$ such that the debt overhang does not occur if and only if $A_s \geq A(Q, k)$. With our discrete setting that $A_s \in \{A_M, A_H\}$, it is easily shown that the FOC (8) can be rewritten as

$$E\left[\left(\frac{\sigma-1}{\sigma}\right) n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{-\frac{1}{\sigma}} - Q \ \middle|\ A_s \geq A(Q,k)\right] = 0,$$

with $A(Q, k) = A_M$ or $A(Q, k) = A_H$. In the case where the value of A_s distributes continuously, it can be easily shown that the FOC (8) is also given by the above equation, where $A(Q, k)$ is chosen from the continuous distribution. See Appendix C for the details.

capital to the firms if the following participation condition (PC) is satisfied, and they hold the capital until period 2 if the PC is not satisfied. The PC for households' selling capital is given as follows. On one hand, the household can obtain ρQ units of period-2 consumer good by selling one unit of capital in period 1 in exchange for the debt that matures in period 2, where ρ is the expected value of recovery rate of debt, which is given endogenously (see the next paragraph). On the other hand, when the household does not sell one unit of capital in period 1, she can obtain A_L units of period-2 consumer good by selling it in period 2 as an input to C-sector, because the capital is used in S-sector only if it is sold to a firm and is installed for specialization in period 1. Thus the PC is

$$\rho Q \geq A_L. \tag{10}$$

If the inequality in PC is strict (>), then all capital K is sold to the firms in period 1:

$$k = K.$$

If the PC holds with equality (=), then $k \leq K$. If the PC does not hold ($\rho Q < A_L$), then $k = 0$, and all capital is used in C-sector.

Recovery rate of debt: In the case of no debt overhang, the recovery rate of debt is 1. In the case of debt overhang, i.e., $\pi - D < \varepsilon$, the recovery rate is lower than one. The expected value of recovery rate is

$$\rho = \frac{R - \Gamma}{D},$$

where R is the expected value of debt repayment and Γ is the expected value of debt restructuring cost. The value of ρ for the DOE is given in (40) in Appendix B.

3.4 Social optimum

We can consider the problem for the social planner who chooses k, the amount of capital installed in period 1 for S-sector, and n, the number of remaining firms in period 2 in S-sector facing the realization of $A_s \in \{A_M, A_H\}$. We measure the social welfare by $E[C - n\varepsilon]$, where C is the household consumption. We know $C = Y$. Since $A_L < A_M \ll A_H$ and the total production in S-sector is $Y_S = n^{\frac{\sigma}{\sigma-1}} A_s k$, production in S-sector is always more efficient than production in C-sector if $n = 1$. Thus, the socially optimal allocation is obviously $k = K$ and $n = 1$.

4 Equilibrium

In this paper, we focus on the equilibrium where all capital is sold to firms in period 1: $k = K$, by assuming that the parameter region is such that the PC holds with strict inequality in equilibrium: $\rho Q > A_L$. Since there are only two states ($s = M$ and $s = H$)

in period 2, it is sufficient to check the existence of two possible equilibria: the Normal Equilibrium (NE), where debt overhang never occurs, and the Debt Overhang Equilibrium (DOE), where debt overhang occurs when $A_s = A_M$ and does not occur when $A_s = A_H$. We will see that the NE exists if A_H is not so large, while the DOE emerges and the NE ceases to exist if A_H is sufficiently large. Both NE and DOE could coexist for moderate values of A_H. We clarify the condition for the existence of multiple equilibria in footnote 14.

4.1 Normal Equilibrium

We clarify the conditions for existence of the Normal Equilibrium (NE) where debt overhang does not occur in any state, $A_s = A_H$ or $A_s = A_M$. Define $\xi = p_H(A_H/A_M)+1-p_H$. In the NE with $k = K$ and $n = 1$, (9) implies

$$Q^N = \left(\frac{\sigma-1}{\sigma}\right)\xi A_M,$$

and $D^N = Q^N K$, where the superscript N denotes the Normal. We also define an infinitesimally small number $\bar{\varepsilon}$ by $\varepsilon = \bar{\varepsilon} A_M K \sigma^{-1}$. Then, we have the following proposition.

Proposition 2. *Suppose that A_L sufficiently small and A_H not too large, so that the following three conditions are satisfied:*

$$A_H < \left(\frac{1}{(\sigma-1)p_H}+1\right)A_M - \left(\frac{\sigma}{\sigma-1}\right)\frac{\varepsilon}{p_H K}, \tag{11}$$

$$A_L < \left(\frac{\sigma-1}{\sigma}\right)A_M, \tag{12}$$

$$A_H < \frac{(1-p_H)}{\left((1-(1-p_H)\bar{\varepsilon})^{-\frac{1}{\sigma}} - p_H^{\frac{\sigma-1}{\sigma}}\right)p_H^{\frac{1}{\sigma}}}A_M. \tag{13}$$

Then, there exists the Normal Equilibrium where $n = 1$ and $k = K$, and the debt is always repaid fully. The asset price is $Q^N = \left(\frac{\sigma-1}{\sigma}\right)\xi A_M$ and the debt is $D^N = Q^N K$.

Note that these conditions (11), (12), (13) are sufficient conditions for the existence of NE. Proof is provided in Appendix B. Note also that in the limit $\varepsilon \to 0$, (13) can be rewritten as $A_H < \frac{1-p_H}{(1-p_H^{\frac{\sigma-1}{\sigma}})p_H^{\frac{1}{\sigma}}}A_M$. The intuition of this proposition is as follows for infinitesimally small ε and $\bar{\varepsilon}$: If A_H is not too large, the asset price is not too high and the debt is not too large, leading to no default in the state M. In the Normal Equilibrium, the TFP is either A_M or A_H, which is strictly bigger than A_L. As $k = K$ and $n = 1$ in all states, the Normal Equilibrium is socially optimal. The ex-ante social welfare is measured by $W = E[Y - n\varepsilon]$. In the NE, the welfare W^N is given by

$$W^N = [p_H A_H + (1-p_H)A_M]K - \varepsilon,$$

which is socially optimal.

4.2 Debt Overhang Equilibrium

First, in Section 4.2.1, we specify the nature of the Debt Overhang Equilibrium (DOE) where debt overhang occurs when $A_s = A_M$, and does not occur when $A_s = A_H$, on the premise that the DOE exists. Second, in Section 4.2.2, we then clarify the (sufficient) condition for its existence. We focus on the parameter region where $\rho Q > A_L$ so that $k = K$. The parameter region is to be specified later in Proposition 4.

4.2.1 Nature of Debt Overhang Equilibrium

Now, suppose that the DOE exists. Since it must be the case that $n = 1$ when debt overhang does not occur, i.e., $\pi - D \geq \varepsilon$, the FOC (9) implies that the asset price must be

$$Q^B = \left(\frac{\sigma - 1}{\sigma}\right) A_H,$$

where the superscript B denotes the Boom of asset prices. Since the expected value of the productivity of the capital is ξA_M and $Q^N = \left(\frac{\sigma-1}{\sigma}\right) \xi A_M$, the asset price in DOE, Q^B, is higher than the "fundamental" price Q^N. In other words, the firms bid up the price to Q^B because they are willing to buy the capital at a higher price as they only care about the state of no debt overhang, i.e., $s = H$, and they do not care about the lenders' loss from their default at $s = M$.

The number of firms in S-sector is $n = 1$ for $s = H$, and n is endogenously determined for $s = M$ by the lenders' decisions on debt restructuring in period 2.

Equilibrium value of n when $A_s = A_M$: When A_M is realized, the firms cannot pay D, and the lenders decide whether to restructure the debt. As we argued in Section 3.2, the lender i takes n as given and restructures the debt when the following condition, which is equivalent to (6), is satisfied in the DOE where $k = K$, $Q^B = [(\sigma - 1)/\sigma]A_H$, and $\pi = n^{\frac{1}{\sigma-1}} A_M K$:

$$n^{\frac{1}{\sigma-1}} A_M K - \varepsilon - z_i \left[\left(\frac{\sigma - 1}{\sigma}\right) A_H K - n^{\frac{1}{\sigma-1}} A_M K + \varepsilon\right]^\phi \geq A_L K. \tag{14}$$

This condition is rewritten as

$$z_i \leq \bar{z}, \tag{15}$$

where $\bar{z} = \hat{G}(n) \equiv \max\{0,\ \min\{z_{\max},\ G(n)\}\}$ and

$$G(n) \equiv \frac{n^{\frac{1}{\sigma-1}} A_M - \varepsilon' - A_L}{\left[\left(\frac{\sigma-1}{\sigma}\right) A_H - n^{\frac{1}{\sigma-1}} A_M + \varepsilon'\right]^\phi K^{\phi-1}}, \tag{16}$$

where $\varepsilon' = \varepsilon/K$. Since lender i, with $z_i \leq \bar{z}$, restructures debt to $\hat{D} = n^{\frac{1}{\sigma-1}} A_M K - \varepsilon$ and the borrowing firm i continues operation in S-sector, the equilibrium value of n is given by

$$n = F(\bar{z}).$$

These two conditions imply that the equilibrium value of n is determined by

$$n = F(\hat{G}(n)). \tag{17}$$

Note that there may exist multiple values of n that satisfy (17). Assumption 2 guarantees that the largest n among the solutions to (17) is selected as an equilibrium value of n.

Larger boom leads to deeper recession: We consider the graphs of $n = F(z)$ and $z = \hat{G}(n)$ in the (n, z) space of Figure 1, where the horizontal axis is n-axis and the vertical axis is z-axis. We denote the equilibrium values by $(n^e, \bar{z}^e)$. Suppose A_H is small enough such that $G(1) > z_{\max}$. In this case, Assumption 2 implies that $\bar{z} = z_{\max}$ and $n = 1$. All lenders restructure debt and all capital is used in production in S-sector. Suppose A_H is large such that $G(1) < z_{\max}$. In this case, there are two possibilities: (P1) The graphs of $z = G(n)$ and $n = F(z)$ have no intersections, or (P2) they have intersections.

- In the case (P1), no lenders reduce debt and $\bar{z}^e = n^e = 0$ in equilibrium.[10] All capital are used in C-sector and total production is $Y = A_L K$.
- In the case (P2), the equilibrium value of n^e, which corresponds to the rightmost intersection of $n = F(z)$ and $z = G(n)$, is smaller than 1 and it is graphically shown that n^e is smaller for a larger A_H. See Figure 1.[11] The intuitive explanation is as follows. A larger A_H makes the debt D larger, implying that the debt restructuring cost is also larger. Condition (14) implies that the larger debt makes the threshold value $\bar{z}$ lower and the number of remaining firms, $n = F(\bar{z})$, smaller.
- Since $G(\bar{n}) = 0$ for any A_H, where $\bar{n} = \{(A_L + \varepsilon')/A_M\}^{\sigma-1}$, the following claim is shown graphically:

[10]The proof is the following. The graph of $n = F(z)$ is always above that of $z = G(n)$ in the case (P1), meaning that, for any given n, firms' exit decision implies that the number of firms remaining in S-sector is strictly smaller than n, except for the case of $n = 0$. Thus, $n = \bar{z} = 0$ is the sole equilibrium.

[11]The proof is as follows. It is graphically confirmed in Figure 1 that $z = G(n)$ intersects $n = F(z)$ from above to below as n increases at the largest intersection n^e, because $G(1) < z_{\max}$. This means that when A_H increases the intersection n^e shifts to the left. This is because $z = G(n)$ shifts lower as A_H increases and the cumulative distribution function $F(z)$ is monotonically increasing in z. Therefore, we can conclude that n^e is smaller for a larger A_H.

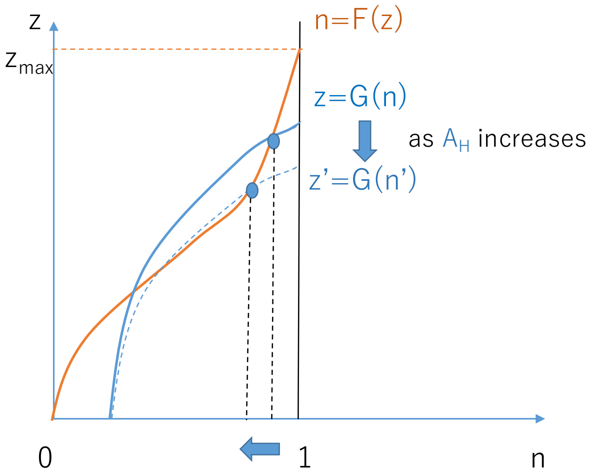

Figure 1: Larger boom (A_H) leads to smaller n

Claim 1. Suppose that the case (P2) is realized for a certain value $A_H = A_H^c$. Then, there exists a threshold $\hat{A}_H$ that is larger than A_H^c such that (P2) is realized and $n^e > 0$ for any $A_H \in [A_H^c, \hat{A}_H]$, whereas, for $A_H > \hat{A}_H$, (P1) is realized and $n^e = 0$.

Figure 1 implies that the graph of $z = G(n)$ shifts down as A_H increases. The intersection of the two graphs suddenly disappears when A_H exceeds a certain threshold $\hat{A}_H$ from below to above.[12] In other words, if A_H exceeds $\hat{A}_H$, then n^e jump down from a positive value to zero.

Both cases (P1) and (P2) imply that a larger A_H leads to a lower n^e. A larger A_H can be interpreted as a larger asset boom, while a smaller n^e can be interpreted as a deeper recession or lower productivity. Thus, we can interpret that a larger boom ex-ante leads to a deeper recession ex-post. Here we can confirm the following lemma.

Lemma 3. *In the state M, n^e is smaller for a larger A_H. The total output in state M, $Y(A_M)$, which is defined in the following proof, decreases as n decreases. Thus, the total output $Y(A_M)$ is smaller for a larger A_H.*

Proof. As shown in footnote 11, n^e is lower for a larger A_H. Given the equilibrium values

[12] The proof is as follows. For $n \in (0, \bar{n}]$, it is the case that $F^{-1}(n) > 0 = G(\bar{n}) = \hat{G}(n)$. For $n \in (\bar{n}, 1]$, we can make $F^{-1}(\bar{n}) > G(n)$, by making A_H sufficiently large. Therefore, $F^{-1}(n) > G(n)$ for all $n \in (0, 1]$ for a sufficiently large A_H. Continuity of $G(n)$ with respect to A_H implies that the claim holds.

of n and $\bar{z}$, that satisfy $n = F(\bar{z})$, the total output in S-sector[13] is given by

$$\begin{aligned} Y_S^{DR} &= n^{\frac{\sigma}{\sigma-1}} A_M K - \left[\left(\frac{\sigma-1}{\sigma} \right) A_H - n^{\frac{1}{\sigma-1}} A_M + \varepsilon' \right]^{\phi} K^{\phi} \int_0^{\bar{z}} z dF(z) \\ &= \int_0^{\bar{z}} y_S(n,z) dF(z), \end{aligned} \tag{18}$$

where

$$y_S(n,z) = \left\{ n^{\frac{1}{\sigma-1}} A_M K - \left[\left(\frac{\sigma-1}{\sigma} \right) A_H - n^{\frac{1}{\sigma-1}} A_M + \varepsilon' \right]^{\phi} K^{\phi} z \right\}. \tag{19}$$

By definition of $\bar{z}$, we have $y_S(n,\bar{z}) = A_L K$ and $y_S(n,z)$ is decreasing in z, implying $Y_S^{DR} > n A_L K$. Thus,

$$Y(A_M) \equiv Y_S^{DR} + Y_C, \tag{20}$$

where $Y_C = (1-n) A_L K$, satisfies $Y(A_M) > A_L K$. Noting $n = F(\bar{z})$ and $y_S(n,\bar{z}) = A_L K$, differentiate $Y(A_M)$ with $\bar{z}$ to get

$$\begin{aligned} \frac{dY(A_M)}{d\bar{z}} =& \frac{dY_S^{DR}}{d\bar{z}} + \frac{dY_C}{d\bar{z}} \\ =& y_S(n,\bar{z}) f(\bar{z}) + \int_0^{\bar{z}} \left[\frac{\partial y_S(n,z)}{\partial n} \right] f(\bar{z}) dF(z) - A_L K f(\bar{z}) \\ =& \int_0^{\bar{z}} \left[\frac{\partial y_S(n,z)}{\partial n} \right] f(\bar{z}) dF(z). \end{aligned}$$

The definition (19) implies that $\frac{\partial y_S(n,z)}{\partial n} > 0$, and $\frac{dn}{d\bar{z}} = f(\bar{z}) > 0$. Thus we obtain

$$\frac{dY(A_M)}{dn} > 0.$$

Therefore, $Y(A_M)$ decreases as n decreases. In other words, $Y(A_M)$ decreases as A_H increases. □

On the aggregate productivity: The result that output in the ex-post recession is lower for a larger ex-ante asset boom is also shown by Allen, Barlevy and Gale (2022). Comparing our result with theirs makes clear the difference. Their result is derived from the exogenous assumption that cost of default is increasing in the amount of defaulted debt. In our model, we also assume the exogenous cost of debt restructuring (see Appendix A for the microfoundation). This mechanism of a larger ex-ante boom making a lower ex-post output is the same in both their paper and ours. In addition to this, it is shown in our model that the total production ($n^{\frac{\sigma}{\sigma-1}} A_M K$) and the total factor productivity in S-sector ($n^{\frac{\sigma}{\sigma-1}} A_M$) are increasing in n. As stated in Lemma 3, a larger ex-ante boom leads

[13] The total output would be slightly changed if we adopt the microfoundation in Appendix A. The necessary modification is described in Appendix A.

to a lower n in our model, implying that the lower total factor productivity and total production. On the other hand, in Allen, Barlevy and Gale (2022), there is no mechanism that an ex-ante boom leads to a lower productivity. The decreases in the aggregate productivity in our model is the adverse effect of the aggregate output externality, which may be a unique feature of our model.

4.2.2 Existence of Debt Overhang Equilibrium

In the following proposition, we specify the sufficient condition for the existence of the DOE.

Proposition 4. *The Debt Overhang Equilibrium exists if A_H is sufficiently large and satisfy*

$$A_H > \left(\frac{1}{(\sigma - 1)p_H} + 1 \right) A_M. \tag{21}$$

In this equilibrium, $k = K$, $Q^B = (\frac{\sigma-1}{\sigma})A_H$, and $D^B = Q^B K$. The number of firms in S-sector is $n = 1$ if $A_s = A_H$, and it is n^e, which is the largest solution to (17)*, if $A_s = A_M$.*

Proof is given in Appendix B. Note that condition (21) is not compatible with condition (11), and it is equivalent to $A_M < Q^N$, meaning that the NE cannot exist. This is because debt overhang is inevitable if the asset price is Q^N and A_s turns out to be A_M, as the revenue $(A_M K - \varepsilon)$ would be strictly smaller than debt $D = Q^N K$. Therefore when (21) holds the NE cannot exist. It may be possible that both the NE and the DOE coexist for A_H that satisfies (11). Although the conditions for multiple equilibria are specified in the footnote 14, we focus in what follows on the case where condition (21) is satisfied.[14]

Asset boom impairs the ex-ante welfare: In the DOE, the ex-ante welfare is

$$W^B = p_H A_H K + (1 - p_H)[Y(A_M) - n\varepsilon].$$

[14] In this footnote we specify the condition for existence of multiple equilibria. For simplicity we focus on the case where $\varepsilon \to 0$ in this footnote. We assume (12) is satisfied. The conditions for existence of the NE are (11) and (13), and for existence of the DOE are (35) and the negation of (39). Combining these conditions, we can say that both the NE and DOE can exist if the parameters satisfy

$$\max\left\{ \left[\frac{1}{(\sigma-1)p_H} + 1 \right] n^{\frac{1}{\sigma-1}}, \ \frac{\sigma}{\sigma-1} \right\} < \frac{A_H}{A_M} < \min\left\{ \frac{1}{(\sigma-1)p_H} + 1, \ \frac{1-p_H}{\left(1 - p_H^{\frac{\sigma-1}{\sigma}}\right) p_H^{\frac{1}{\sigma}}} \right\}$$

where n is the biggest value that solves $n = F(G(n))$. For example, if $p_H = 10^{-3}$, $\sigma = 3$, and K is very large, then, as $n = 0$ is the solution to $n = F(G(n))$, the condition for existence of multiple equilibria reduces to $1.5 < \frac{A_H}{A_M} < 10$, which can be satisfied by infinitely many pairs of A_M and A_H.

As we see that $Y(A_M) < A_M K$ and $Y(A_M)$ decreases as n decreases, it is obvious that $W^B < W^N$ for a sufficiently small ε. Whether or not W^B is decreasing in A_H is ambiguous because the first term ($A_H K$) is increasing in A_H, while the second term ($Y(A_M)$) is decreasing. However, as we see in Claim 1 an infinitesimal increase in A_H from $\hat{A}_H$ makes n^e jump down from a positive value to zero. It means that an infinitesimal increase in A_H can lead to a big jump down of W^B from $p_H A_H K + (1 - p_H)[Y(A_M) - n\varepsilon]$ to $p_H A_H K + (1 - p_H) A_L K$. In the end, we can say that a small increase in A_H decreases the social welfare W^B in the neighborhood of $A_H = \hat{A}_H$. Therefore, it can be said that a larger asset-price boom may impair the ex-ante social welfare by making the ex-post recession deeper.

5 Policy responses

Our model enables us to assess ex-ante and ex-post policy interventions to the asset-price boom and macroeconomic debt overhang. In this section, we consider the case where (21) is satisfied, so that the equilibrium is the DOE. In other words, we consider the case where there arrives a news shock in period 1 that the productivity of capital A_H can be extremely high in period 2. In Section 5.1, we first establish a benchmark policy that completely suppress the risk-shifting asset price boom. Then, we analyze ex-post subsidy to lenders for debt restructuring in Section 5.2, and ex-ante macroprudential policy in Section 5.3. Finally, in Section 5.4, we will argue about monetary policy in a modified model, in which nominal money is introduced as a unit of account.

The analysis in this section can be summarized in the following four points. First, the benchmark policy, that is the subsidy to borrowers to prevent debt overhang from occurring, can completely suppress the risk-shifting booms and attain the social optimum, though such a generous subsidy is difficult to implement. Second, ex-post subsidy to lenders for debt restructuring is welfare improving (i.e., ex-post optimal). In contrast to the existing literature, the ex-post policy does not cause time inconsistency in our model as long as the participation constraint for lenders $\rho Q > A_L$ is satisfied with strict inequality. This is because the subsidy is to lenders, not to debt-ridden borrowers. Third, ex-ante imposition of borrowing limit is socially optimal, while finding the optimal borrowing limits for individual firms is not likely to be feasible in reality. Fourth, an ex-post monetary easing can be welfare improving if it can make inflation higher, as the inflation reduces the burden of debt overhang.

Parameters of debt restructuring cost: Before going on to the policy analysis, we confirm how $\{z_i\}_{i=0}^{1}$, which are the cost parameters of debt restructuring, affect the efficiency of the outcome. Obviously, reduction of z_i's encourages the lenders to restructure the debt overhang, leading to an increase in n, that improves social welfare. This is similar to Kornejew et al. (2024) in that efficient bankruptcy systems increase output of defaulted firms. A key difference is the aggregate output externality is present in our model, and not in theirs. The externality plays a crucial role in the policy implications in this section.

5.1 Benchmark: complete suppression of risk shifting booms

This subsection is based on discussion by Watanabe (2024). The asset price Q^B in the DOE is higher than the fundamental price (Q^N) because the firms can default on their debt and they maximize $E[\max\{\pi - \varepsilon - D,\ 0\}]$ to solve (7). If the government can make the firms maximize $E[\pi - \varepsilon - D]$, instead of $E[\max\{\pi - \varepsilon - D,\ 0\}]$, then the FOC of (7) with respect to k would imply that the asset price equals the fundamental value (Q^N), and the risk-shifting boom of asset price is completely suppressed. It is straightforward from this argument to have the following claim.

Claim 2. Suppose the government credibly announces in period 1 that it will give any borrower a sufficient amount of subsidy in period 2 to enable the borrower to pay the stipulated debt entirely. In this case, the asset price becomes the fundamental price (Q^N).

This claim holds for any subsidy in general, as long as it can prevent the default, while it may be contingent on the realization of A_s but should be independent of the choice variable k. When the parameters satisfy (21), the government actually pays a positive amount of subsidy ex-post in state $s = M$. This is because (21) implies that the debt is larger than the revenue in the state $s = M$, that is, $D = Q^N K > A_M K = \pi$.

The above reasoning concerning Claim 2 is basically given by Watanabe (2024) and it is so strong that any risk-shifting asset booms can be completely suppressed by the same policy intervention in the existing models such as Allen and Gale (2000) and Allen, Barlevy and Gale (2022).[15] However, the subsidy to make any borrower never default on

[15] The borrower subsidy that is financed by a lump-sum tax leads to the equilibrium where the fundamental price of the risky asset is realized in Allen and Gale (2002):

$$r = f'(B - \bar{P}),$$
$$\bar{P} = \frac{1}{r}\left[\int_0^{R_{\max}} Rh(R)dR - c'(1)\right],$$

where $\bar{P}$ is the fundamental asset price and r is the loan rate, which is equal to the safe rate in the no-default equilibrium. The same is true for Allen, Barlevy and Gale (2022), in which the safe rate R and the fundamental asset price $\bar{p}^D$ are realized in the no-default equilibrium with borrower subsidy, where

$$R = \rho(\bar{p}^D),$$
$$1 + R = \frac{(1-\pi)(D + \bar{p}^D) + \pi(d + p^d)}{\bar{p}^D}.$$

their debt may not be realistic as a policy recommendation. Ex-ante moral hazard by the borrowers and the resultant amount of subsidy necessitated would be unthinkably huge. So I describe the borrower subsidy here as a theoretical possibility, and move on to other policy tools in what follows.

5.2 Ex-post debt restructuring

The inefficiency of debt overhang emerges when the state turns out to be M, in the Debt Overhang Equilibrium. In this subsection, we focus on period 2 of the DOE, when $A_s = A_M$ is realized. We define the *ex-post optimal policy* as follows.

Definition 1. There is a chance of government intervention at the beginning of period 2 after the aggregate shock $A_s = A_M$ is revealed and before lenders restructure the debt and borrowers produce outputs. A policy intervention at this stage is ex-post optimal if it maximizes the total output.

Ex-post problem for social planner: Given the debt overhang $D = Q^B K$, the social planner would maximize the total output (20), by solving the following optimization problem:

$$\max_{\bar{z}} \; n^{\frac{\sigma}{\sigma-1}} A_M K - n\varepsilon - \left[\left(\frac{\sigma-1}{\sigma}\right) A_H + \varepsilon' - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi} K^{\phi} \int_0^{\bar{z}} z dF(z) + (1-n) A_L K, \tag{22}$$

$$\text{s.t.} \quad n = F(\bar{z}).$$

The constraint $n = F(\bar{z})$ is imposed as the social planner internalizes the aggregate output externality. The optimal value $\bar{z}^o$ is given by the FOC of the above problem:

$$n^{\frac{1}{\sigma-1}} A_M K - \varepsilon - \bar{z}\left[\left(\frac{\sigma-1}{\sigma}\right) A_H + \varepsilon' - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi} K^{\phi} + T(n,\bar{z}) \geq A_L K, \tag{23}$$

where

$$T(n,\bar{z}) = \frac{n^{\frac{1}{\sigma-1}} A_M K}{\sigma-1} + \frac{\phi n^{\frac{2-\sigma}{\sigma-1}} A_M}{\sigma-1}\left[\left(\frac{\sigma-1}{\sigma}\right) A_H + \varepsilon' - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi-1} K^{\phi} \int_0^{\bar{z}} z dF(z).$$

The solution is $(\bar{z}^o, n^o) = (z_{\max}, 1)$ if the inequality of the FOC is strict ($>$), while $\bar{z}^o < z_{\max}$ and $n^o < 1$ if the FOC holds with equality.

Ex-post optimal policy: Notice that the value of $\bar{z}$ is determined by (5) in a competitive equilibrium without government interventions. The condition (5) can be rewritten as follows to determine $\bar{z}$:

$$n^{\frac{1}{\sigma-1}} A_M K - \varepsilon - \bar{z}\left[\left(\frac{\sigma-1}{\sigma}\right) A_H + \varepsilon' - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi} K^{\phi} \geq A_L K, \tag{24}$$

where the left-hand side is the lender's profit of restructuring debt overhang, while the right-hand side is what the lender can get if she does not restructure the debt. This condition and $n = F(\bar{z})$ determine the equilibrium value $(\bar{z}^e, n^e)$ without policy intervention. We focus on the case where (24) holds with equality without loss of generality.[16] Comparing (24) with (23), we have the following proposition.[17]

Proposition 5. *The government can realize the optimal allocation $n^o = F(\bar{z}^o)$ by giving the subsidy, the schedule of which is $T(n, \bar{z})$, to the lenders who restructure the debt.*

Proof. Given the subsidy $T(n, \bar{z})$, the optimal exit decision by firms implies that the equilibrium $(n, \bar{z})$ is determined by (23) and $n = F(\bar{z})$. If there exist multiple solutions, Assumption 2 guarantees that the largest possible n (and $\bar{z}$) is realized in equilibrium. □

This ex-post subsidy for debt restructuring can improve social welfare by internalizing the aggregate output externality. The aggregate output externality can be seen as one example of externalities caused by the financial crisis, which can be resolved by debt restructuring, such as the counterparty risk among borrowing firms or the free-rider problem among lenders who have claims on the same borrower and want to free ride on the other lenders' debt restructuring. Our result demonstrates that an ex-post government intervention to enhance debt reduction can improve welfare by mitigating serious externalities of financial crises.

Equilibrium with anticipated ex-post interventions: What happens if the lenders and borrowers expect in period 1 that government subsidy $T(n, \bar{z})$ will be given in period 2 when debt overhang occurs? The answer is that nothing changes except that n becomes n^o when debt overhang occurs. Given that the subsidy is for lenders, not borrowers, the firms obtain nothing when they have debt overhang, i.e., $\pi - \varepsilon < D$, as in the case without subsidy, which is shown in Lemma 1.[18] We can show as follows that the equilibrium does not change with anticipation of ex-post policy intervention. First, the ex-post debt restructuring policy affects the allocation only in the state where debt overhang occurs. Second, as long as the participation condition for lenders, $\rho Q \geq A_L$, continues to hold with strict inequality, the decision making by firms in period 1 is irrelevant to the anticipation about what happens in the state $s = M$ in period 2 where debt overhang occurs because the firms do not care about the debt-overhang state, where they obtain nothing

[16] If the inequality of (24) is strict (>), the equilibrium value is $(\bar{z}^e, n^e) = (z_{\max}, 1)$, which is socially optimal.

[17] In Appendix A, we provide a slight modification associated with adoption of the microfoundation for debt-restructuring technology.

[18] Note that the restructured debt is the same amount, i.e., $\hat{D} = \pi - \varepsilon$, with or without the subsidy to the lenders.

anyway. The conditions for existence of the NE are not affected by the anticipation of the government intervention, and thus Proposition 2 still holds. Concerning the DOE, we have the following proposition that shows the DOE is identical in period 1 no matter whether the ex-post policy interventions are expected or not.

Proposition 6. *We assume parameters satisfy* (21)*. Suppose all agents expect that the government gives subsidy with the schedule $T(n, \bar{z})$ to the lenders, conditional on undertaking debt restructuring, if $D = D^B$ and $A_s = A_M$. Then, there exists the Debt Overhang Equilibrium, where $k = K$, $Q^B = \left(\frac{\sigma-1}{\sigma}\right) A_H$, and $D^B = Q^B K$. These values are the same as those in Proposition 4. If $A_s = A_H$, D^B is fully repaid and $n = 1$, while if $A_s = A_M$, the debt overhang occurs and $(\bar{z}^o, n^o)$ are realized.*

Proof. The expectations of government intervention affects only ρ, which changes the participation condition (PC) for households' selling capital: $\rho Q > A_L$. Given our assumption on parameters (21) and (41), it is obvious from the proof of Proposition 4 that the PC holds with strict inequality, even when the government intervention is anticipated. Therefore, nothing changes in conditions for equilibrium. □

5.3 Ex-ante macroprudential policy: Borrowing limit

It is easily shown that an imposition of the appropriately designed borrowing limit can modify the equilibrium in such a way that no default occurs when $A_s = A_M$. Suppose that the financial regulator imposes the borrowing constraint in period 1 that each firm's debt D cannot exceed $\bar{D}$, where

$$A_L K < \bar{D} \leq A_M K - \varepsilon.$$

In this case, the asset price in equilibrium becomes $Q = \bar{D}/K$, and the PC is satisfied: $\rho Q = Q > A_L$. Each firm buys K units of capital in period 1, and when A_s turns out to be A_M in period 2, the firms can pay the debt $\bar{D}$, because their earnings are $A_M K$, given $n = 1$. There is no default and no exit from S-sector. The allocation, $k = K$ and $n = 1$, is socially optimal.[19] It may be practically difficult to find the appropriate level of $\bar{D}$ for individual firms in reality. Moreover, the optimality of the borrowing limit is crucially based on the assumption that A_s is a binary variable, i.e., $A_s \in \{A_M, A_H\}$. If A_s takes on a continuous value as in Appendix C, the borrowing limit cannot prevent debt overhang from occurring with a positive probability, though it may improve social welfare to some extent.

[19] For some parameter values, both the NE and DOE coexist. In this case, for any $\bar{D}' \in (Q^N K, Q^B K)$, if we set the ex-ante borrowing limit at $\bar{D}'$, then the economy goes to the NE, leaving the borrowing constraint $D \leq \bar{D}'$ nonbinding.

5.4 Monetary policy in a model with nominal variables

In this subsection, we analyze monetary policy. When the inefficiency is caused by debt overhang and its spillover, conventional monetary easing, i.e., lowering the interest rate, may not have a direct effect to restore efficiency. However, we can show that an ex-post inflation may reduce debt overhang and restore the efficiency.

Here, we modify our model by adding money. Money is just a unit of account used both in period 1 and period 2, and we assume that the quantity of money supplied is zero. Debt contract is made in terms of money. In period 1, a firm purchases k units of capital in exchange for debt $Q'k$, where Q' is the asset price in terms of money in period 1. Here the debt evolves at the loan rate $1+I$ and the firm is obliged to repay $D' = (1+I)Q'k$ in terms of money in period 2 to the lender household. We can define P_s as the price of period-2 consumer goods in terms of money in the state s, where $s \in \{M, H\}$. Then, the real burden of debt is $D_s = (1+I)Q'/P_s$ in terms of period-2 consumer goods.

We assume that the central bank can set the nominal rate I and the nominal price levels P_s. Setting the nominal rate I in period 1 is ex-ante monetary policy, whereas setting P_s for $s \in \{M, H\}$ is ex-post monetary policy. We assume that the values of P_s is anticipated by firms and households in period 1.[20] We will assess ex-ante and ex-post policies respectively.

Given I and P_s, a firm in period 1 maximizes the expected profit:

$$\max_k E[\max\{\pi - \varepsilon - D,\ 0\}],$$

where $\pi \equiv p(y)y = n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}}$ and $D_s = (1+I)Q'k/P_s$. FOC wrt k at $k = \bar{k}$ decides $(1+I)Q'$ by

$$E\left[P_s^{-1} | \text{ND}\right]\ (1+I)Q' = E[n^{\frac{1}{\sigma-1}} A_s\ |\text{ND}] \left(\frac{\sigma-1}{\sigma}\right)$$

The real burden of debt overhang D_s at the state $s \in \{M, H\}$ is

$$D_s = \frac{(1+I)Q'k}{P_s} = \frac{E[n^{\frac{1}{\sigma-1}} A_s\ |\text{ND}]}{E\left[P_s^{-1}|\text{ND}\right]} \left(\frac{\sigma-1}{\sigma}\right) P_s^{-1} k.$$

In this modified model, we focus on the DOE where debt overhang ($\pi - \varepsilon - D < 0$) does not occur in the state H and occurs in the state M. Thus, since $n_H = 1$ and $E[P^{-1}|\text{ND}] = P_H^{-1}$, we have

$$(1+I)Q' = \left(\frac{\sigma-1}{\sigma}\right) A_H P_H,$$

$$D_H = \left(\frac{\sigma-1}{\sigma}\right) A_H K, \tag{25}$$

$$D_M = \left(\frac{\sigma-1}{\sigma}\right) A_H \frac{P_H}{P_M} K. \tag{26}$$

[20] Our results in this subsection hold qualitatively unchanged, even if the central bank can set the totally unexpected values of P_s.

Ex-ante monetary policy: We assume period-2 prices (P_H and P_M) are fixed, because they are choice variables for ex-post monetary policy, not ex-ante monetary policy. Since $(1+I)Q' = \left(\frac{\sigma-1}{\sigma}\right) A_H P_H$ in the DOE, a change in I is exactly offset by the corresponding change in Q' so that $(1+I)Q'$ is unchanged. (25) and (26) indicate that the nominal rate I is irrelevant to the real debt burden D_s and to the decision-makings by lenders and firms in both period 1 and period 2. It is obvious from this that ex-ante monetary policy, i.e., a change in I, has no effect on equilibrium allocation.

Ex-post monetary policy: Central bank decides period-2 prices, P_s for $s \in \{M, H\}$. We do not specify how central bank implement P_s, and just assume that it can decide P_s. This assumption is a shortcut for the description of monetary policy. We focus on the debt-overhang state $s = M$ in period 2, where lenders restructure debt to choose $\bar{z}$ and n^e. As (26) indicates, higher P_M for state M reduces real burden of debt $D_M = \frac{(1+I)Q'K}{P_M} = \left(\frac{\sigma-1}{\sigma}\right) A_H P_H K P_M^{-1}$, and shifts the graph of $\bar{z} = G(n) = \frac{\pi(n)-A_L K}{(D-\varepsilon-\pi(n))^\phi}$ upward in Figure 1, increasing $\bar{z}$ and n^e in equilibrium. Higher P_M at state M is interpreted as ex-post monetary easing. Therefore, the ex-post monetary easing, whether anticipated or unanticipated, can reduce the real debt burden D_M and increase efficiency and output.[21] This policy implication holds on the premise that the central bank can control the price level. There may be also other policy interventions such as tax/subsidy on C-sector.[22]

6 Sclerosis or Secular Stagnation

One of the empirical regularities of financial crises that we wanted to explain is that a persistent and decade-long recession often follows a huge asset-price decline. An extended version of our two-period model can explain the basic mechanism of this persistence, as

[21] If P_M is sufficiently large, it makes $D_M = (1+I)Q'K/P_M$ so small that debt overhang never occurs. Then, the first best allocation is attained, as long as the participation condition for lending households, $E[\rho_s(1+I)Q'/P_s] > A_L$, is satisfied. To make policy analysis more realistic, we can assume exogenous nominal rigidity that P_M cannot exceed a certain upper limit, and therefore the debt overhang occurs in the state A_M.

[22] I thank Tack Yun for pointing to the policy issues of monetary policy and the tax/subsidy in C-sector. Consider a business income tax on firms in C-sector: $\tau A_L k$ for producing $A_L k$. With this policy, the effective productivity in C-sector becomes $(1-\tau)A_L$. An increase in τ increases n^e by shifting the graph of $\bar{z} = G(n)$ upward in Figure 1, where

$$\bar{z} = G(n) = \frac{n^{\frac{1}{\sigma-1}} A_M - (1-\tau)A_L}{\left[\left(\frac{\sigma-1}{\sigma}\right) A_H - n^{\frac{1}{\sigma-1}} A_M\right]^\phi K^{\phi-1}}.$$

The tax on C-sector, τ, may be welfare improving, given that tax revenue is transferred back to the households in a lump-sum fashion. The interpretation of the tax on C-sector is not straightforward, though, because $A_L k$ can be interpreted as a fire-sale value of the asset k. The above argument may imply that subsidy to facilitate the fire sale, i.e., a negative value of τ, is welfare reducing.

we demonstrate it in this section. Before going on to the details of the extended version of the model, we summarize the intuition in advance: Suppose that there exist new-born firms in period 2 who can potentially enter this economy. They can enter the economy by paying a fixed entry cost, and they can buy capital K and produce output in S-sector. We consider period 2 of the DOE where the productivity of capital is turned out to be A_M and debt overhang occurs. If many new firms enter the economy, the output will increase. In this case, the recession is short-lived. If no one or very few new firms enter the economy, we say, the recession is persistent. We can easily see as follows that many firms enter when the debt overhang is small and no firms enter when the debt overhang is large. This is due to the aggregate output externality: the expected revenue for a new comer $\pi = n^{\frac{1}{\sigma-1}} A_M K$ is proportional to $n^{\frac{1}{\sigma-1}}$, where n is the number of remaining incumbent firms. When debt overhang is small, n is large and the expected revenue for a potential entrant exceeds the entry cost. When debt overhang is large, n is small and the expected revenue for a potential entrant is less than the entry cost. Then, the new firm chooses not to enter when debt overhang is large. In sum, we can explain the mechanism of persistence as follows: a large asset-price boom is often followed by huge bust and debt overhang, which in turn depresses the new entry and leads the economy into a persistent recession. The persistent recession after the asset price collapse can be called "sclerosis" (Acharya, Lenzu and Wang 2024) or secular stagnation. On the other hand, the recession that follows a small asset boom is shallow and short-lived as there are many new entrants.

The policy implication of the extended model is basically the same as Section 5. In particular, our result implies that the policy intervention to subsidize debt restructuring may be able to attain the fast economic recovery without going through a deep recession (see also the infinite-horizon model in Appendix D).

6.1 Extended model – Larger boom leads to more persistent recession

We extend our model by adding the following assumption. Assumption 3 is a common knowledge for all agents in both period 1 and period 2.

Assumption 3. In period 2, the measure λ of new firms are created, where $0 < \lambda < 1$. The new firms are owned by randomly selected λ households. A new firm can buy capital k in a lump sum at price Q_M from an incumbent firm, where k is all capital that the incumbent firm installed in period 1 and Q_M is the spot price of capital in terms of the period-2 consumer goods. We assume for simplicity that the new firms are subject to no financial frictions and they can pay $Q_M k$ to the incumbent firms and also lose the dead-weight loss γk as the entry cost, where

$$A_M - \gamma - \varepsilon' > A_L.$$

After purchasing k, the new firm enters S-sector and produces and sells output to obtain the revenue

$$\pi = \{n + e(n)\}^{\frac{1}{\sigma-1}} A_M K^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}},$$

where $e(n)$ is the measure of new entrants in period 2, where $0 \leq e(n) \leq \lambda$. The values of $e(n)$ and Q_M are equilibrium outcomes.

In this extended model, we analyze how the entry decisions of new firms are affected by the size of A_H. First, we define $\underline{A}_H$, $\underline{\underline{A}}_H$ and $\bar{A}_H$ as follows.

Definition 2. $\underline{A}_H$ is the value of A_H that satisfies $\hat{H}(A_H) = z_\lambda$, where

$$\hat{H}(A_H) = \max\{0, \ \min\{z_{\max}, H(A_H)\}\},$$
$$H(A_H) \equiv \frac{\gamma}{\left[\left(\frac{\sigma-1}{\sigma}\right) A_H - A_M + \varepsilon'\right]^{\phi} K^{\phi-1}},$$

and z_λ is defined by $\lambda = 1 - F(z_\lambda)$. Define $n(A_H)$ as the largest solution of

$$n = F(\hat{G}(n + \lambda)),$$

for a given value of A_H. Define $\underline{\underline{A}}_H$ by $\underline{\underline{A}}_H = \max\{\underline{A}_H, \underline{A}'_H\}$, where $\underline{A}'_H$ is the solution to

$$n(A_H) = 1 - \lambda.$$

$\bar{A}_H$ is the value of A_H that satisfies the following equation:

$$n(A_H) = \left(\frac{A_L + \gamma + \varepsilon'}{A_M}\right)^{(\sigma-1)} - \lambda.$$

Given the above extension of the model, Proposition 4, that specifies the features of the DOE, is modified as follows. This modified proposition says that for a small A_H, many new firms enter S-sector, so that the total number of firms in S-sector is $n + e(n) > n$ and no recession or a shallow recession occurs, and that for a large A_H, no firms enter S-sector and a deep recession occurs. We can interpret this result of entries that the recession is shallow and short-lived for a small A_H and it is deep and persistent for a large A_H.

Proposition 7. *We focus on the case where condition* (21) *is satisfied. In the Debt Overhang Equilibrium, the equilibrium values in period 1 are* $k = K$, $Q^B = (\frac{\sigma-1}{\sigma})A_H$, *and* $D^B = Q^B K$. *The number of firms in S-sector is* $n = 1$ *if* $A_s = A_H$ *in period 2. In the debt-overhang state where* $A_s = A_M$ *in period 2, the equilibrium becomes one of the following three cases according to the value of* A_H.

- ***No recession*** *for* $A_H \leq \underline{A}_H$*: The number of remaining firms* n *is given by* $n = F(\hat{H}(A_H))$, *and* $n \geq 1 - \lambda$. *The number of new entries* $e(n)$ *is given by* $e(n) = 1 - n$ *so that the total number of S-sector firms is* $n + e(n) = 1$. *Price of capital in period 2 becomes* $Q_M = A_M - \gamma - \varepsilon'$.

- ***Shallow recession*** *for* $A_H \in (\underline{\underline{A}}_H, \bar{A}_H]$*: This case exists only if* $\underline{\underline{A}}_H < \bar{A}_H$*.* n *is given by* $n = F(\hat{G}(n+\lambda))$*, and* $n < 1-\lambda$*. The number of new entries* $e(n) = \lambda$*. The total number of firms in S-sector is* $n + e(n) = n + \lambda \; < 1$*.* Q_M *is given by* $Q_M = A_L$*.*

- ***Persistent recession*** *for* $A_H > \bar{A}_H$*:* n *is given by* $n = F(\hat{G}(n))$*, and* $e(n) = 0$*. The total number of firms in S-sector is* n*.* Q_M *is given by* $Q_M = A_L$*.*

Proof is given in Appendix B. Broadly speaking, this proposition says that the entry of new-born firms in period 2 implies that no recession occurs if debt overhang is small (i.e., A_H is small) and that persistent recession occurs if debt overhang is large (i.e., A_H is large). In Appendix B.3, right after the proof of Proposition 7, we describe the detailed classification of the equilibrium according to the sizes of $\underline{A}_H$, $\underline{\underline{A}}_H$, and $\bar{A}_H$.

6.2 Policy implications from the extended model

The previous subsection demonstrated that a large debt overhang subsequent to a large asset-price boom makes the stagnation persistent by discouraging entries of new firms. Policy implication of this extended model is that the ex-post optimal policy is to give sufficient subsidy that incentivize the lenders to implement debt restructuring and increase n, so that the new firms become willing to enter, i.e. $(\min\{1, n+\lambda\})^{\frac{1}{\sigma-1}} A_M K - Q_M K - \gamma K \geq 0$. Therefore, policy intervention to encourage the lender to restructure the debt is welfare improving in this model. This view could be interpreted as complementary to that in Acharya et al. (2024). Acharya et al. (2024) view that the persistent stagnation can result from the distortionary policy that facilitate zombie lending, which is a subsidy to the banks that extend and rollover the loans to the nonviable firms. Acharya et al. (2024) argues that the government policy that rewards the lenders for continuing to lend debt overhang makes the stagnation persistent, while we argue that the government policy that rewards the lenders for reducing debt overhang can stop the persistent stagnation. An implicit policy implication of Acharya et al. (2024) is that stopping the inefficient policy intervention may be sufficient to improve welfare. A value added of our argument to theirs is to indicate that stopping the policy to rewards the lenders for zombie lending may not be enough, as we show in the previous subsection that the persistent stagnation can occur because of the aggregate output externality, even without inefficient government policy. What is emphasized in our model is that it may be necessary for economic recovery to implement an active policy intervention that rewards the lenders for debt restructuring.

6.3 Infinite-horizon model

In Appendix D, we confirm that the results in this section can be generalized in a simple infinite horizon model. We show in Appendix D that a large debt overhang makes no entry and generates persistent losses in productivity and output for an extended period of time. The government discounts the future outputs with the time preference factor β (< 1). In our model, whatever the value of β is, the optimal policy is to give a subsidy to lenders for restructuring debt overhang and to make the economic recovery as fast as possible. In our model, there is no trade-off between the V-shaped recovery (i.e., the deep recession with fast recovery) and the L-shaped stagnation (i.e., the shallow and persistent stagnation). In Acharya, Lenzu and Wang (2024), the policymakers must choose either the V-shaped recovery or the L-shaped stagnation, because the zombie firms are assumed to be intrinsically inefficient: if the zombie firms are liquidated, the rush of bankruptcies makes a deep and short-term recession (V-shape), whereas if they are kept afloat by subsidy, their inefficiency makes a shallow and persistent stagnation (L-shape).

Our theory implies that policy intervention that encourages debt restructuring by the lenders attains the fast economic recovery without going through a deep (and short-lived) recession, because the zombie firms in our model can become productive once their burdens of debt are lifted. This result may be remarkable because policymakers usually argue on the premise that the trade-off between the V-shaped recovery and the L-shaped stagnation is inevitable. Our result implies that it may not be.

7 Conclusion

We demonstrated that the model of risk-shifting booms of asset prices and ex-post debt overhang can replicate empirical regularities of financial crises, i.e., credit-fueled asset boom tends to end up with the bust, followed by a deep and persistent recession with productivity declines. We focus on debt overhang as a main driver of the inefficiencies in the aftermath of a financial crisis. It is also shown that a larger asset-price boom leads to a deeper and more persistent recession ex-post. As the inefficiency of debt overhang is aggravated by aggregate output externality, ex-post policy intervention that incentivizes debt restructuring increases the aggregate productivity and output, and improves the social welfare. An example is an ex-post subsidy to the lenders for restructuring the debt overhang. We also showed that time inconsistency may not appear even when the ex-post subsidy is anticipated. The tradeoff between the V-shaped recovery (i.e., short-term but deep recession) and the L-shaped stagnation (i.e., shallow but persistent stagnation) may not be inevitable, because timely and appropriate debt relief encouraged by policy intervention can achieve economic recovery without going through a deep recession. These results may shed some light on the relevance of the various policy responses to financial crises that may be worth analyzing further theoretically and quantitatively in the future.

Appendix A: Microfoundation for debt-restructuring technology

In the main text of this paper, the lenders solve the problem (4), taking as given the debt-restructuring technology (1) and the borrower's exit decision (3). The solution is given in Lemma 1. Here we provide a microfounded model of the decision-making problem for a lender that replicates this result.

A.1 Setting

Although a lender is a household in the main text of this paper, we assume only in this Appendix that the lender i consists of a unit mass of households who equally share the same amount of total credit D to the firm i, for $i \in [0,1]$. We call the lender the bank, and the households the depositors. Since the measure of depositors (households) in one bank is unity, each household has the right to receive D. The bank i can offer firm i the reduction of her debt from D to $\hat{D}$ if all depositors in bank i agree. The depositors in bank i are divided into two groups: the *hawkish* depositors with measure λ_i and the *dovish* depositors with measure $1-\lambda_i$. The variable λ_i is not known to depositors initially and is revealed in the midst of the bargaining. The hawkish depositors have the full bargaining power and maximize their payoffs, whereas the dovish depositors accept any offers from the hawkish depositors as long as their payoff is weakly better off than when the bargaining breaks down.

Bargaining: We consider the bargaining between the hawkish and dovish depositors in period 2, when the productivity of the asset is revealed to be A_M and λ_i is not revealed yet. In the following, we assume that the hawkish (dovish) depositors act as one agent in the bargaining. The hawkish depositors make a take-it-or-leave-it (TIOLI) offer $(\hat{D}, q)$ to the dovish depositors, where $\hat{D}$ is the amount of the restructured debt and q is the amount of period-2 good to be transferred from the dovish depositors to each hawkish depositor. After $(\hat{D}, q)$ is fixed, λ_i is revealed and the dovish depositors choose whether to accept the TIOLI offer or not. If the dovish depositors refuse the TIOLI offer, the debt is not restructured and the repayment from firm i becomes $A_L K$, which is equally distributed to all depositors. Thus, the hawkish depositors choose $(\hat{D}, q)$ to maximize the expected value of their payoff per capita:

$$\Psi(\hat{D}, q) \equiv P(q)(y(\hat{D}) + q) + (1 - P(q))A_L K, \tag{27}$$

where $y(\hat{D})$ is given in (3), and $P(q)$ is the probability that the dovish depositors accept the TIOLI offer, which is

$$P(q) \equiv \Pr[y(\hat{D}) - z_i q^{\phi} \geq A_L K], \tag{28}$$

where

$$z_i \equiv \frac{\lambda_i}{1-\lambda_i}.$$

The definition of $P(q)$, (28), is explained as follows. We assume that a transfer q to one hawkish depositor generates a dead-weight loss of $q^\phi - q$, where $\phi \geq 1$. Thus, the transfer including the dead-weight loss from dovish to hawkish depositors is $\lambda_i q^\phi$ in total, and is $z_i q^\phi$ per one dovish depositor. The payoff of the success of the bargaining for a dovish depositor is, therefore, $y(\hat{D}) - z_i q^\phi$. A dovish depositor accepts the TIOLI offer iff $y(\hat{D}) - z_i q^\phi \geq A_L K$. As we see in the main text, the random variable z_i follows the CDF, $F(z)$. Therefore, (28) can be rewritten as

$$P(q) = F\left(\frac{y(\hat{D}) - A_L K}{q^\phi}\right).$$

Dependence on the stipulated debt D: So far, there has been no dependence on the stipulated amount of debt D. The payoff for the hawkish depositor of the successful bargaining is $y(\hat{D}) + q$, whereas she is given the legitimate right to take D by the debt contract, but she has no right to take more than D. Thus, it is reasonable to assume that there is no resistance when the hawkish depositor offers q as long as $y(\hat{D}) + q \leq D$, while there arise prohibitively high coordination costs when she offers q bigger than $D - y(\hat{D})$. Given this assumption, the hawkish depositor's problem becomes

$$\max_{\hat{D},q} \Psi(\hat{D}, q), \tag{29}$$

$$\text{s.t.} \quad q \leq D - y(\hat{D}). \tag{30}$$

A.2 Equilibrium

Given the borrower's exit decision (3), the hawkish depositor optimally chooses

$$\hat{D} = y(\hat{D}) = \pi - \varepsilon,$$

that maximizes the total surplus for both the hawkish and dovish depositors. We assume that, for all $q \in [0, D - \pi + \varepsilon]$,

$$\Psi'(q) \equiv \frac{\partial \Psi}{\partial q}(\pi - \varepsilon, q) > 0. \tag{31}$$

We discuss the plausibility of (31) later. Given that (31) holds, the constraint (30) binds and the solution is

$$q = D - \pi + \varepsilon.$$

This bargaining solution $(\hat{D}, q) = (\pi - \varepsilon, D - \pi + \varepsilon)$ implies the following equilibrium:

- If z_i satisfies (6), the bargaining is successful and bank i offers $\hat{D} = \pi - \varepsilon$ to firm i, and

- if z_i does not satisfy (6), the bargaining fails and bank i offers no restructuring ($\hat{D} = D$) to firm i.

This result is identical to Lemma 1. The plausibility of the assumption (31) may need to be examined carefully. Denoting $C \equiv \pi - \varepsilon - A_L K$, $\Psi'(q)$ is calculated as follows:

$$\Psi'(q) = F(Cq^{-\phi}) - f(Cq^{-\phi})(C+q)C\phi q^{-(\phi+1)}.$$

We pick one example to satisfy (31) as follows. Define $\bar{q} \equiv Q^B K - \pi + \varepsilon$.

Claim 3. Let z_i follows the exponential distribution with $F(z) = 1 - e^{-\xi z}$ and $f(z) = \xi e^{-\xi z}$, where $\xi > 0$. For a sufficiently large ξ, $\Psi'(q)$ satisfies $\Psi'(q) > 0$ for all $q \in (0, \bar{q}]$.

Proof. In the case where $F(z) = 1 - e^{-\xi z}$,

$$\Psi'(q) = 1 - e^{-\xi C q^{-\phi}} - \xi e^{-\xi C q^{-\phi}}(C+q)C\phi q^{-(\phi+1)}.$$

We have

$$\lim_{q \to 0+} \Psi'(q) = 1, \quad \text{and} \quad \lim_{q \to \infty} \Psi'(q) = 0.$$

With some algebra, we have

$$\begin{aligned}\Psi''(q) &\equiv \frac{\partial^2}{\partial q^2}\Psi(\pi - \varepsilon, q) \\ &= -\Lambda(q) q^{-2(\phi+1)} e^{-\xi C q^{-\phi}}.\end{aligned}$$

where

$$\Lambda(q) \equiv \xi^2 C^3 \phi^2 + \xi^2 C^2 \phi^2 q - \xi C\phi(\phi-1)q^{\phi+1} - \xi C^2 \phi(\phi+1) q^{\phi}.$$

For $0 < q \leq \bar{q}$, we have

$$\begin{aligned}\Lambda(q) &> \xi^2 C^3 \phi^2 + \xi^2 C^2 \phi^2 \times 0 - \xi C\phi(\phi-1)\bar{q}^{\phi+1} - \xi C^2 \phi(\phi+1)\bar{q}^{\phi} \\ &= \xi^2 C^3 \phi^2 - \xi C\phi(\phi-1)\bar{q}^{\phi+1} - \xi C^2 \phi(\phi+1)\bar{q}^{\phi}.\end{aligned}$$

The right-hand side is positive if

$$\xi > \frac{\phi-1}{C^2\phi}\bar{q}^{\phi+1} + \frac{\phi+1}{C\phi}\bar{q}^{\phi}. \tag{32}$$

Suppose ξ is sufficiently large so that (32) is satisfied. Then $\Lambda(q) > 0$ and thus $\Psi''(q) < 0$ for all $q \in (0, \bar{q}]$. Therefore, $\Psi'(q) \geq \Psi'(\bar{q})$ for $q \in (0, \bar{q}]$. The value of $\Psi'(\bar{q})$ can be written as a function $\psi(\xi)$ of ξ as follows:

$$\Psi'(\bar{q}) = \psi(\xi) = 1 - (1 + a\xi)e^{-b\xi},$$

where $a > 0$ and $b > 0$ are constant. As $\lim_{\xi \to \infty} \psi(\xi) = 1$, it is obvious that $\Psi'(\bar{q}) = \psi(\xi) > 0$ if ξ is sufficiently large. We have shown that $\Psi'(q) \geq \Psi'(\bar{q}) > 0$ for a sufficiently large ξ. Thus, it has been proven that condition (31) can hold for some parameter region. □

This example demonstrates that (31) holds in an appropriate setting.

A.3 On necessary modifications for total output and optimal policy

If we adopt the bargaining model in this appendix as a microfoundation for the main text, we need to modify the expressions for total output (18) and social planner's problem (22) as follows. In the bargaining model, the cost for dovish depositors $z_i q^\phi$ is divided into the transfer to the hawkish depositors, $z_i q$, and the dead-weight loss, $z_i(q^\phi - q)$, whereas, in the model in the main text, all the cost $z_i q^\phi$ is the dead-weight loss. In the bargaining model, the transfer to the hawkish depositors is not lost and can be consumed. Thus the total output (18) should be modified to

$$Y_S^{DR} = n^{\frac{\sigma}{\sigma-1}} A_M K - [Q(n)^\phi - Q(n)] \int_0^{\bar{z}} z dF(z),$$
$$\text{where} \quad Q(n) \equiv \left[\left(\frac{\sigma - 1}{\sigma} \right) A_H - n^{\frac{1}{\sigma-1}} A_M + \varepsilon' \right] K$$

We consider that all gains of hawkish and dovish depositors are merged in the representative household and we measure the social welfare by total output. Thus the ex-post problem for social planner (22) should be modified to

$$\max_{\bar{z}} \; n^{\frac{\sigma}{\sigma-1}} A_M K - n\varepsilon - [Q(n)^\phi - Q(n)] \int_0^{\bar{z}} z dF(z) + (1-n) A_L K,$$
$$\text{s.t.} \;\; n = F(\bar{z}).$$

The FOC implies that the ex-post optimal policy is given in Proposition 5, in which the subsidy should be modified from $T(n, \bar{z})$ to

$$T(n, \bar{z}) - \frac{n^{\frac{2-\sigma}{\sigma-1}}}{\sigma - 1} A_M K \int_0^{\bar{z}} z dF(z) + \bar{z} Q(n).$$

Appendix B: Proofs

B.1 Proof of Proposition 2

The condition for no debt overhang, $\pi - D - \varepsilon > 0$, in period 2 at $n = 1$ and $A_s = A_M$ is

$$\left[1 - \left(\frac{\sigma - 1}{\sigma} \right) \xi \right] A_M K > \varepsilon,$$

which is rewritten as (11). The PC for selling capital is satisfied with strict inequality if $\rho Q^N = Q^N > A_L$, where $\rho = 1$ because no default occurs in the NE. This condition is satisfied if $\left(\frac{\sigma-1}{\sigma}\right) \xi A_M > A_L$. Since $\xi > 1$ the sufficient condition for $Q^N > A_L$ is (12). We focus on the parameter region where (12) is satisfied.

To complete the proof of existence of the NE, we need to show there is no deviation. In the NE, a firm could deviate in a way that it increases k to a certain value, k_d, such that it cannot repay $D_d = Q^N k_d$, when $A_s = A_M$, and it repays D_d only when $A_s = A_H$.[23] For the existence of the NE, it is necessary to confirm this deviation is not profitable. The expected profits for a firm when it does not deviate is $E[\pi^N - \varepsilon - D^N] = \xi A_M K/\sigma - \varepsilon$. The expected profits for a deviating firm is $E[\pi_d - \varepsilon - D_d \mid \text{ND}] = p_H\{A_H K^{\frac{1}{\sigma}} k_d^{\frac{\sigma-1}{\sigma}} - \varepsilon - Q^N k_d\}$. It is maximized by $k_d = \left(\frac{A_H}{\xi A_M}\right)^\sigma K$ and the maximized value of profits from deviation is

$$E[\pi_d - \varepsilon - D_d \mid \text{ND}] = p_H \frac{(\xi A_M)^{1-\sigma} A_H^\sigma}{\sigma} K - p_H \varepsilon$$

The condition for no deviation is $E[\pi^N - \varepsilon - D^N] > E[\pi_d - \varepsilon - D_d \mid \text{ND}]$. This condition can be rewritten as

$$(\xi A_M)^\sigma > p_H A_H^\sigma + \frac{\sigma}{K} \frac{(1-p_H)\varepsilon}{(\xi A_M)^{1-\sigma}}.$$

By definition, we have

$$\varepsilon < \bar{\varepsilon} \xi A_M K \sigma^{-1}.$$

These two condition implies the sufficient condition for $E[\pi^N - \varepsilon - D^N] > E[\pi_d - \varepsilon - D_d \mid \text{ND}]$ is

$$\{1 - (1-p_H)\bar{\varepsilon}\}(\xi A_M)^\sigma > p_H A_H^\sigma,$$

which is rewritten as (13). This condition is satisfied if A_H is not so large.[24]

B.2 Proof of Proposition 4

For the existence of the DOE, the following conditions must be satisfied:

$$\left[1 - \left(\frac{\sigma-1}{\sigma}\right)\right] A_H K > \varepsilon, \tag{33}$$

$$\left[A_M - \left(\frac{\sigma-1}{\sigma}\right) A_H\right] K < \varepsilon, \tag{34}$$

where (33) says there is no default if $A_s = A_H$, and (34) says that a firm cannot fully repay the debt even if all other firms stay in S-sector, if $A_s = A_M$. The first condition

[23] Note that choosing k_d, which is larger than K, is feasible for an individual firm. This is because firms are subject to no quantity constraint, and they can choose any quantity under the market price Q^N. Although the optimal choice of quantity is K under the price Q^N, the firms can choose a larger amount at will.

[24] We could be interested in whether the deviated firm actually default on D_d when $A_s = A_M$, that is, whether $\pi(1, A_M, k_d) - Qk_d < \varepsilon$. But this inequality is not necessary for the existence of the NE. Suppose $\pi(1, A_M, k_d) - Qk_d < \varepsilon$ is satisfied. In this case, the deviation is feasible and is not profitable as long as (13) is satisfied. Suppose $\pi(1, A_M, k_d) - Qk_d \geq \varepsilon$. In this case, the optimal deviation with default is not feasible and therefore the NE can exist stably. As (13) is the sufficient condition for no deviation, we assume this condition is satisfied.

is always satisfied as ε is infinitesimally small. The second condition is satisfied for any $\varepsilon > 0$ if A_H is so large that

$$\frac{A_H}{A_M} > \frac{\sigma}{\sigma - 1}. \tag{35}$$

Another necessary condition for existence of DOE is that the firms have no incentive to deviate from the equilibrium. Now, we specify the condition for no deviation. The expected profit for a firm in the DOE is

$$p_H(A_H K - \varepsilon - D^B) = \frac{p_H A_H}{\sigma} K - p_H \varepsilon.$$

Suppose that a firm considers to deviate from the DOE by reducing k to k_d so that it does not default on $D_d = Q^B k_d$ when $A_s = A_M$. The optimization problem for a deviating firm is

$$\max_{k_d} \ [p_H A_H + (1 - p_H) n^{\frac{1}{\sigma-1}} A_M] K^{\frac{1}{\sigma}} k_d^{\frac{\sigma}{\sigma-1}} - \left(\frac{\sigma - 1}{\sigma}\right) A_H k_d - \varepsilon, \tag{36}$$

$$\text{s.t.} \ \ n^{\frac{1}{\sigma-1}} A_M K^{\frac{1}{\sigma}} k_d^{\frac{\sigma-1}{\sigma}} - \varepsilon - \left(\frac{\sigma - 1}{\sigma}\right) A_H k_d \geq 0. \tag{37}$$

The condition (37) says that k_d is chosen such that the firm does not default on the debt when $A_s = A_M$. The solution to (36) on the premise that (37) is nonbinding is

$$k_d = \left[p_H + (1 - p_H) n^{\frac{1}{\sigma-1}} \frac{A_M}{A_H}\right]^{\sigma} K. \tag{38}$$

Substituting (38) into (37), it is shown that (37) is equivalent to

$$\left[1 - \left(\frac{\sigma - 1}{\sigma}\right)(1 - p_H)\right] n^{\frac{1}{\sigma-1}} A_M - \varepsilon'' \geq \left(\frac{\sigma - 1}{\sigma}\right) p_H A_H, \tag{39}$$

at the solution (38), where $\varepsilon'' = \varepsilon K^{-1}[p_H + (1 - p_H) n^{\frac{1}{\sigma-1}} (A_M/A_H)]^{1-\sigma}$. If (39) is violated, the profit of the firm at A_M is negative, implying that (36) at k_d that satisfies (38) is smaller than the profit with default on the debt at A_M. With any k, the profit with default at A_M is weakly smaller than the profit of no deviation, i.e., $p_H A_H K/\sigma - p_H \varepsilon$, which is the maximized profit with default at A_M. Therefore, the deviation is not more profitable than no deviation, if (39) is violated. Thus, the sufficient condition for no deviation is

$$\left[1 - \left(\frac{\sigma - 1}{\sigma}\right)(1 - p_H)\right] n^{\frac{1}{\sigma-1}} A_M - \varepsilon'' < \left(\frac{\sigma - 1}{\sigma}\right) p_H A_H.$$

Since $\varepsilon'' > 0$ and $n \leq 1$, the sufficient condition for the above condition is $[1 - (\sigma - 1)\sigma^{-1}(1 - p_H)]A_M < (\sigma - 1)\sigma^{-1} p_H A_H$, which is equivalent to $A_M < Q^N$, and can be rewritten as (21). When this condition is satisfied, (35) is automatically satisfied, because $(\sigma - 1)^{-1} p_H^{-1} + 1 = [\sigma/(\sigma - 1) - (1 - p_H)] p_H^{-1} > \sigma/(\sigma - 1)$ for any $p_H \in (0, 1)$ and $\sigma > 1$.

What to be done finally is to specify the parameter region where $\rho Q^B > A_L$ is satisfied. Note that

$$\rho = p_H + (1 - p_H) \frac{Y(A_M) - \varepsilon F(\bar{z})}{Q^B K}. \tag{40}$$

Lender's optimal decision on debt restructuring means that $Y(A_M) - \varepsilon F(\bar{z}) > A_L K$, as shown in (5). Therefore, $\rho > p_H + (1-p_H)\frac{A_L}{Q^B}$, and the sufficient condition for $\rho Q^B > A_L$ is given by $[p_H + (1-p_H)\frac{A_L}{Q^B}]Q^B > A_L$, which can be rewritten as

$$\frac{A_H}{A_L} > \frac{\sigma}{\sigma - 1}, \tag{41}$$

which is automatically satisfied if (35) is satisfied. Thus, $\rho Q^B > A_L$ if (21) is satisfied, as we saw (35) holds if (21) is satisfied.

B.3 Proof of Proposition 7

The number of entering firms $e(n)$ cannot exceed the number of new firms, that is, $e(n) \leq \lambda$. Most importantly, $e(n)$ cannot exceed $1-n$, because a newly entering firm can buy the capital from an incumbent firm in a lump sum, implying that $e(n)K \leq (1-n)K$, because we know that only C-sector firms (who moved from S-sector to C-sector because of the debt overhang) will sell their capital and each incumbent in C-sector holds K units of capital.[25] Thus, we have

$$e(n) \leq \min\{1-n,\ \lambda\}.$$

First, we can derive the following claim:

Claim 4. The equilibrium entry $e(n)$ is either 0 or $\min\{1-n, \lambda\}$,

Proof. The free entry condition for a new firm is

$$(n+e)^{\frac{1}{\sigma-1}} A_M K - Q_M K - \gamma K \geq 0, \tag{42}$$

where e is the measure of newly entering firms. Suppose that (42) is satisfied for $e \in [0,\ \min\{1-n, \lambda\})$. Now we show e cannot be the equilibrium value $e(n)$. If this e is the equilibrium value, it must be the case that there exist new firms who do not enter S-sector even though they can, and their measure is $\min\{1-n, \lambda\} - e > 0$. This is because the left-hand side of (42) is increasing in e. But it is a contradiction because these firms can and want to enter as long as (42) is satisfied. Thus, if (42) is satisfied for any value of $e \in [0, \min\{1-n, \lambda\})$, then all new firms up to measure $\min\{1-n, \lambda\}$ will enter, so that the equilibrium value of e becomes $e(n) = \min\{1-n, \lambda\}$. As the left-hand side of (42) is increasing in e, (42) is satisfied for $e = \min\{1-n, \lambda\}$. In the case where (42) is not satisfied for all $e \in [0,\ \min\{1-n, \lambda\}]$, it is obvious that the equilibrium value of e is given by $e(n) = 0$, because no new firms choose to enter. Thus we have proven that $e(n)$ is either 0 or $\min\{1-n, \lambda\}$. □

[25] The new firms can purchase capital from the incumbents in S-sector, but they will not sell their capital because the new firms cannot offer a price that the incumbents are better off as the new firms have to pay the entry cost γK.

Next, we prove the first bullet of the proposition. We specify the value of A_H that makes the equilibrium values $e(n) = 1 - n < \lambda$ and $Q_M > A_L$. In this equilibrium, the total number of firms in S-sector is $n + e(n) = 1$ and the new firms are indifferent between entering and not entering, because $1 - n$ firms enter and $\lambda - 1 + n > 0$ firms do not enter. Since the payoff of entering is $\pi - \varepsilon - Q_M K - \gamma K = A_M K - \varepsilon - Q_M K - \gamma K$ and the payoff of not entering is 0, the free entry condition for new firms is $(A_M - \varepsilon' - Q_M - \gamma)K = 0$, which implies

$$Q_M = A_M - \gamma - \varepsilon',$$

where $\varepsilon' = \varepsilon / K$. $Q_M = A_M - \gamma - \varepsilon' > A_L$ by Assumption 3. Given this price, let us consider the debt restructuring decision by the lenders. The lenders can obtain $(A_M - \gamma - \varepsilon')K$ when they do not restructure the debt and sell the firms at the price $Q_M K$, while they get $\pi - \varepsilon - z_i(D - \pi + \varepsilon)^\phi$ by restructuring the debt, where $\pi = A_M K$. Therefore, the lenders choose to restructure the debt if $\pi - \varepsilon - z_i(D - \pi + \varepsilon)^\phi \geq (A_M - \gamma - \varepsilon')K$. This condition is equal to $z_i \leq \bar{z}$, with the modified definition: $\bar{z} = \hat{H}(A_H)$. This is because the condition $\pi - \varepsilon - z_i(D - \pi + \varepsilon)^\phi \geq (A_M - \gamma - \varepsilon')K$ with $\pi = A_M K$ can be rewritten as

$$z_i \leq \hat{H}(A_H).$$

The monotonicity of $\hat{H}(A_H)$ implies that $1 - n = 1 - F(\hat{H}(A_H)) \leq \lambda$ if and only if $A_H \leq \underline{A}_H$. Thus, we have proven the first bullet of the proposition.

Now we move on to the proof of the second bullet of the proposition. We specify the value of A_H that makes n satisfy $\lambda < 1 - n$. Claim 4 implies that $e(n)$ is either 0 or λ if $\lambda < 1 - n$. Here we specify the value of A_H that makes $e(n) = \lambda$. Obviously, from the above argument, the necessary condition for the equilibrium value of n to satisfy $\lambda < 1 - n$ is $A_H > \underline{A}_H$. Suppose $\lambda < 1 - n$. In this case, as the new firms buy capital in a lump sum from the incumbents, the new firms can buy up to λK, while the incumbents in C-sector want to sell $(1 - n)K$ $(> \lambda K)$. Thus the price Q_M is driven down to $Q_M = A_L$, at which the sellers are indifferent between selling or not selling. On the premise that $e(n) = \lambda$, the number of remaining incumbent firms n is decided by

$$n = F(\hat{G}(n + \lambda)),$$

for a given A_H. We denote the solution to the above equation by $n(A_H)$. By definition of $\underline{\underline{A}}_H$ and as $n(A_H)$ decreases monotonically, the condition $\lambda < 1 - n(A_H)$ is satisfied for $A_H > \underline{\underline{A}}_H$. Note that if $\underline{A}_H < \underline{\underline{A}}_H$, there exists no equilibrium for $A_H \in (\underline{A}_H, \underline{\underline{A}}_H]$. This is because $A_H > \underline{A}_H$ implies $\lambda < 1 - n$ and $A_H \leq \underline{\underline{A}}_H$ implies $\lambda \geq 1 - n(A_H)$, which contradict with each other because $n = n(A_H)$ must hold in equilibrium. Now we focus on the case where $A_H > \underline{\underline{A}}_H$. In order to have $e(n) = \lambda$, the following free entry condition for new firms must be satisfied for $n = n(A_H)$:

$$(n + \lambda)^{\frac{1}{\sigma - 1}} A_M K - \varepsilon - A_L K - \gamma K \geq 0.$$

This condition is satisfied if $A_H \le \bar{A}_H$ by definition and monotonicity of $F(\hat{G}(n+\lambda))$ in A_H. Here we have proven that $e(n) = \lambda < 1-n$ if $A_H \in (\underline{\underline{A}}_H, \bar{A}_H]$. It is the second bullet of the proposition.

Next, we consider the third bullet point of the proposition. If $A_H > \bar{A}_H$, it is obvious by definition that

$$(n+\lambda)^{\frac{1}{\sigma-1}} A_M K - \varepsilon - A_L K - \gamma K < 0,$$

for $n(A_H)$. This means that a new firm chooses not to enter S-sector, even if all the other new firms enter S-sector, and that in equilibrium there is no new entrant, i.e., $e(n) = 0$. The third bullet of the proposition has been proven. *(End of Proof)*

Detailed classification of the equilibrium: The equilibrium can be classified according to the sizes of the three parameters, $\underline{A}_H$, $\underline{\underline{A}}_H$, and $\bar{A}_H$:

Note that $\underline{A}_H \le \underline{\underline{A}}_H$ always holds by definition of $\underline{\underline{A}}_H$, i.e., Definition 2. Thus, it is sufficient to consider the following three cases:

- Case where $\underline{A}_H \le \underline{\underline{A}}_H \le \bar{A}_H$. In this case, the equilibrium becomes no recession (i.e., $e(n) = 1-n$) for $A_H \le \underline{A}_H$, shallow recession (i.e., $e(n) = \lambda$ with $\lambda < 1-n$) for $A_H \in [\underline{\underline{A}}_H, \bar{A}_H]$, and persistent recession (i.e., $e(n) = 0$) for $A_H > \bar{A}_H$. There exists no equilibrium for $A_H \in (\underline{A}_H, \underline{\underline{A}}_H]$, if the set $(\underline{A}_H, \underline{\underline{A}}_H]$ is non empty.[26]

- Case where $\underline{A}_H \le \bar{A}_H \le \underline{\underline{A}}_H$. In this case, the equilibrium becomes no recession (i.e., $e(n) = 1-n$) for $A_H \le \underline{A}_H$, and persistent recession (i.e., $e(n) = 0$) for $A_H > \bar{A}_H$. There exists no equilibrium for $A_H \in (\underline{A}_H, \bar{A}_H]$, if the set $(\underline{A}_H, \bar{A}_H]$ is non empty because $A_H \le \underline{\underline{A}}_H$ for $A_H \in (\underline{A}_H, \bar{A}_H]$.[27]

- Case where $\bar{A}_H \le \underline{A}_H \le \underline{\underline{A}}_H$. In this case, the equilibrium becomes no recession (i.e., $e(n) = 1-n$) for $A_H \le \underline{A}_H$, and the persistent recession (i.e., $e(n) = 0$) for $A_H > \bar{A}_H$. Multiple equilibria (i.e., no recession and persistent recession) coexist for $A_H \in (\bar{A}_H, \underline{A}_H]$, if the set $(\bar{A}_H, \underline{A}_H]$ is non empty.

Appendix C: Continuous distribution of A_s

As we notified in footnote 9, here we describe the risk shifting in an extended case where the productivity parameter A_s is not a binary variable but a continuous variable. Suppose that $A_s \in [0,\ A_{\max}]$, and the distribution function is $\Theta(A)$, i.e., $\Pr(A_s \le A) = \Theta(A)$. The threshold $A(Q,k)$ is given by the solution to $\pi(n, A, k) = Qk + \varepsilon$. Then, given Q, the firm

[26]See the proof of the second bullet point of Proposition 7 above.

[27]This is easily proven by applying the argument in the proof of the second bullet of Proposition 7 above.

in period 1 solves

$$\max_k \int_{A(Q,k)}^{A_{\max}} \{\pi(n, A, k) - Qk - \varepsilon\} d\Theta(A),$$

as the firm can default on the debt Qk when $\pi - Qk - \varepsilon < 0$. Noting that $\pi(n, A(Q, k), k) - Qk - \varepsilon = 0$, the FOC wrt k can be written as

$$\int_{A(Q,k)}^{A_{\max}} \left\{ \left(\frac{\sigma - 1}{\sigma} \right) n^{\frac{1}{\sigma-1}} A \bar{k}^{\frac{1}{\sigma}} k^{-\frac{1}{\sigma}} - Q \right\} d\Theta(A) = 0.$$

This condition decides Q. Since $k = \bar{k}$ and $n = 1$ for $A > A(Q, K)$ in equilibrium, we have

$$Q = \left(\frac{\sigma - 1}{\sigma} \right) E[A \mid A \geq A(Q, k)]$$

The condition $\pi(n, A(Q, k), k) - Qk - \varepsilon = 0$ can be written in the equilibrium where $n = 1$ and $k = K$ as

$$Q = A(Q, K) - \varepsilon',$$

where $\varepsilon' = \varepsilon / K$. The two variables Q and $A(Q, K)$ are determined by the above conditions. In what follows, we write $\underline{A} \equiv A(Q, K)$ and $\Phi(\underline{A}) \equiv E[A | A \geq \underline{A}]$. The variables Q and $\underline{A}$ are determined by the above two conditions, which are rewritten as

$$Q = \left(\frac{\sigma - 1}{\sigma} \right) \Phi(\underline{A}), \tag{43}$$

$$Q = \underline{A} - \varepsilon'. \tag{44}$$

Note that (43) decides Q from $\underline{A}$ and (44) decides $\underline{A}$ from Q. We consider the graphs of (43) and (44) in the $(\underline{A}, Q)$-space, where the horizontal axis is $\underline{A}$-axis and the vertical axis is Q-axis. Since $\lim_{A \to A_{\max}} \Phi(A) = A_{\max}$, the graph of (43) becomes lower than the graph of (44) in the neighborhood of $\underline{A} = A_{\max}$, as $\frac{\sigma - 1}{\sigma} < 1$ and ε' is small. Since $\Phi(0) > 0$, the graph of (43) is above that of (44) in the neighborhood of $\underline{A} = 0$. Thus, we can see graphically that there exists at least one intersection of (43) and (44), implying that there exists at least one equilibrium. The number of intersections can be multiple and in that case we have multiple equilibria.

In the case of multiple solutions, it is shown as follows that the rightmost intersection in the $(\underline{A}, Q)$-space is a stable equilibrium in the following sense. The stability of equilibrium against a small perturbation can be evaluated by considering how $(\underline{A}, Q)$ are decided by (43) and (44). If, in the $(\underline{A}, Q)$-space, the graph of (43) intersects (44) from above to below as $\underline{A}$ increases, then the intersection is a stable equilibrium, because (43) decides the response of Q to $\underline{A}$ and (44) decides the response of $\underline{A}$ to Q. Therefore, the rightmost intersection is a stable equilibrium against small perturbations in $\underline{A}$ and Q. In particular, if the intersection is unique, it is a stable and unique equilibrium. The equilibrium value of Q is higher than the fundamental value $Q^F = \int_0^{A_{\max}} (1 - \sigma^{-1}) A d\Theta(A)$.

Appendix D: An infinite-horizon model

Here we describe an infinite horizon model, which is an extension of the 2-period model in Section 6.

D.1 Setting

The model is an infinite horizon economy with $t = 0, 1, 2, \cdots, \infty$. A fixed amount of the risky asset (capital), K, is owned by the unit mass of households. Investment in the risky asset takes place in period 0, and production and consumption take place from period 1 on. Productivity of the asset $A_t \in \{A_M, A_H\}$ is uncertain in period 0 and it is revealed in period 1. Thus, either $A_1 = A_H$ or $A_1 = A_M$. A_t is invariant from period 1 on: $A_t = A_1$ for all $t \geq 2$. The rate of time preference β is $\beta \in (0, 1)$, and social welfare is $\sum_{t=0}^{\infty} \beta^t C_t$, where C_t is total consumption of the houesholds in period t. As $C_t = Y_t$, where Y_t is the total output in period t, the social welfare is rewritten as $\sum_{t=0}^{\infty} \beta^t Y_t$. The safe rate is $1 + r_t = 1 + r = \beta^{-1}$. Initially, a unit mass of firms exist in period 0. We call them the *incumbent firms*. Every period from period 1 on, *new-born firms* with measure λ are born and decide whether or not to enter S-sector. If not entering S-sector, the new-born firms just cease to exist with their payoff being zero. There exists a technological constraint that the measure of firms (or varieties) in S-sector cannot be bigger than 1. (It should be less than or equal to 1.)

Incumbent firms optimally choose capital k in period 0. They obtain k in period 0 by promising to pay $D_1 = (1 + r)Q_0 k$ in terms of consumer goods in period 1. The debt D_1 becomes debt overhang in period 1 if $A_1 = A_M$. The lenders can restructure debt by paying the cost of debt restructuring from period 1 on. (A lender can choose period t (≥ 1), when it restructures the debt overhang of its borrower.)

Since the lenders maximize their payoff by debt restructuring, they will give the incumbent firms only ε to let them work in S-sector or give them nothing to let them work in C-sector. The lenders can sell the capital of incumbent firms to the new-born firms. In this case, the incumbent firms (= managers) get nothing. Thus, the incumbent firms get nothing once they have the debt overhang, whether or not the lenders restructure the debt.

Debt restructuring cost for a lender is $z_i \Delta^{\phi}$, where $\Delta = D_1 - V$ and V is the value that the lender can recover by debt restructuring. (V is an equilibrium outcome.) Without debt restructuring nor selling asset to a new-born firm, the incumbent firm stays in C-sector to produce and repay $A_L k$. Thus, D_t can grow:

$$D_{t+1} = (1 + r)(D_t - A_L k), \quad \text{for } t \geq 1,$$

but we assume for simplicity that the value of D_t does not affect the cost of debt restructuring in period t. The cost of debt restructuring is time-invariant, and it is $z_i \Delta^{\phi}$, where

$\Delta = D_1 - V$, for all $t \geq 2$. (Lender i can reduce the debt D_t to V_t by paying $z_i \Delta^\phi$ in period $t(\geq 2)$.)

We assume that each incumbent firm decides the amount of capital k optimally in period 0. From period 1 on, however, capital of a firm is traded in a lump sum. Anyone who wants to buy capital can either buy K units in a lump sum from a seller or buy 0 units.

Every period t (≥ 1), new-born firms with measure λ can enter S-sector by paying the entry cost γK to operate K units of capital. The timing is the same as in Section 6.1. A new firm can enter the economy at the beginning of period t and buys capital K from the lender of a debt overhang firm (C-sector firm). The new entrant can produce output in S-sector in period t by using K that she purchased in the same period t. It needs to pay the utility cost ε to operate in S-sector.

When the lender of a C-sector firm sells capital K to a new entrant, the C-sector firm just exits (or is liquidated). We assume for simplicity there is no cost of exit or liquidation for the lender or the C-sector firm. (Introducing the cost would not change the result qualitatively.)

D.2 Equilibrium

Now we specify the equilibrium dynamics. We will first describe the dynamics from period 1 on, when the uncertainty $A_1 \in \{A_M, A_H\}$ is revealed. In the end, we will describe the decision problem of period 0.

The case with $A_1 = A_H$: First, we consider the case where A_1 is revealed to be $A_1 = A_H$ in period 1. We specify the price of capital Q_t from period 1 on. It is reasonable to assume $Q_t = Q_H$, a constant. As $A_1 = A_H$, there is no default on the debt. Firms receive revenues from selling goods (π) and assets ($Q_H K$) and pay dividends ($\pi - \varepsilon + Q_1 K - D_1$). For period t (≥ 1), there is no uncertainty, and the incumbent firms repeat the same operation, that is, to buy K with borrowing and produce output in S-sector and payout dividends next period. Since we assume capital is traded in a lump sum, a firm borrows $Q_H K$ and buy K units of capital in period t, and earns $A_H K - \varepsilon + Q_H K$ in period $t+1$ and repay the debt $(1+r)Q_H K$. The profit of a firm in period $t+1$ for $t \geq 1$ is

$$A_H K - \varepsilon + Q_H K - (1+r)Q_H K.$$

Price $Q_H K$ is the price of a firm and we assume there are many potential firm managers who want to buy the firms. Thus, the zero profit condition ($A_H K - \varepsilon + Q_H K - (1 + r)Q_H K = 0$) should be satisfied, and so

$$Q_H = \frac{A_H - \varepsilon'}{r} = \frac{\beta(A_H - \varepsilon')}{1-\beta},$$

where $\varepsilon' = \varepsilon/K$. Note that there is no monopolistic profits from period 1 on because the asset K is traded in a lump-sum.

Case with $A_1 = A_M$: Next, we consider the case $A_s = A_M$ for $t \geq 1$. In this case, debt D_1 cannot be repaid entirely (debt overhang). The equilibrium is specified by $\{n_t, e_t\}_{t=1}^{\infty}$, where n_t is the measure of the incumbent firms (firms who has existed from period 0) who operate in S-sector in period t. Their debt overhang has been restructured between period 1 and period t. The variable e_t is the measure of firms who have entered S-sector during the periods between period 1 and period t. There are two types of equilibrium:

- Case 1 (Short-term recession): There exists T such that $n_{t+1} \geq n_t$ for $t = 1, 2, \cdots, T-1$, and $n_t = n_T$ for $t \geq T+1$, and that $e_t = t \times \lambda$ for $t = 1, 2, \cdots, T-1$, $e_T = 1 - n_T$, and $e_t = e_T$ for $t \geq T+1$. Therefore, $n_t + e_t = 1$ for all $t \geq T$. In period t, n_t incumbent firms and e_t new-born firms operate in S-sector and $1 - n_t$ incumbent firms operate in C-sector. T and $\{n_t, e_t\}_{t=1}^{\infty}$ must satisfy in equilibrium that

$$n_{T-1} + \lambda(T-1) < 1, \quad \text{and} \quad n_T + \lambda(T-1) < 1 \leq n_T + \lambda T.$$

- Case 2 (Deep and persistent stagnation): $n_t = n_1 < 1$ and $e_t = 0$ for all $t \geq 1$. The total number of firms that operate in S-sector is n_1 for all $t \geq 1$, which is smaller than one. $1 - n_1$ firms operate in C-sector for all t.

D.2.1 Case 1: Short-term recession

In what follows, we specify the values of equilibrium variables in short-term recession and prove the existence in Proposition 8 below. Take (T, n_1, Δ) as given for now. The values of these variables will be specified later in equilibrium. Then, we can set $e_t = \lambda t$ for $t = 1, 2, \cdots, T-1$, and $e_T = 1 - n_T$, where n_T will be also specified in equilibrium. At the beginning of every period t, there are three options for lender i ($\in [0,\ 1]$) of debt overhang:

- Debt restructuring to get

$$(n_t + e_t)^{\frac{1}{\sigma-1}} A_M K - \varepsilon + V_t - z_t \Delta^{\phi}, \tag{45}$$

  where V_t is the value of the firm that continues operation in S-sector from t on.

- Staying in C-sector to get

$$A_L K + Q_t^L K, \quad \text{for} \quad t \leq T,$$

  where Q_t^L is the unit price of capital that is used in C-sector next period.

- Selling the capital K to a new-born firm at the price:

$$A_L K + Q_t^L K, \quad \text{for } t = 1, 2, \cdots, T-1, \text{ and}$$
$$A_M K - \varepsilon + V_T - \gamma K, \quad \text{for } t = T. \tag{46}$$

  This is because for $t = 1, 2, \cdots, T-1$, there are $1 - n_t - e_t$ lenders who have to stay in C-sector, although they want to sell the asset of incumbent firms to the new-born firms. The competition among lenders drives down the price of the incumbent firms such that the lenders become indifferent between selling the firm and staying in C-sector, i.e., $A_L K + Q_t^L K$. For $t = T$, there are new-born firms that want to buy the incumbent firms but cannot. The competition among the new-born firms drives up the price such that the new-born firms are indifferent between entering and not-entering in period T, i.e., $A_M K - \varepsilon + V_T - \gamma K$.

The variables V_t and Q_t^L are given by

$$V_t = \beta \left[(n_{t+1} + e_{t+1})^{\frac{1}{\sigma-1}} A_M K - \varepsilon + V_{t+1} \right],$$
$$Q_t^L = \beta(A_L + Q_{t+1}^L), \text{ for } 1 \leq t \leq T-1,$$
$$Q_T^L = \frac{\beta(A_M - \varepsilon')}{1-\beta} - \gamma.$$

The value of V_T is given by $V_T = \beta(A_M K - \varepsilon + V_T)$, i.e., $V_T = \beta(A_M K - \varepsilon)/(1-\beta)$. This is because $V_t = V_T$ for all $t \geq T$, as $n_t + e_t = 1$ for $t \geq T$. The value of Q_T^L is given by $Q_T^L + \gamma = Q_M$, where $Q_M K = \beta(A_M K - \varepsilon + Q_M K)$, thus $Q_M = \beta(A_M - \varepsilon')/(1-\beta) = V_T/K$. The condition $Q_T^L = Q_M - \gamma$ means that all remaining capital in C-sector is sold to the new firms in period T.

Now, given (T, n_1, Δ), we specify $\{n_t, \bar{z}_t\}_{t=1}^{\infty}$, where $n_t = F(\bar{z}_t)$ for $t \geq 2$. For $t \leq T$, there exists $\bar{z}_t$ such that the lender with the cost parameter $z_i \leq \bar{z}_t$ restructure the debt overhang. For $t \leq T-1$, the lender with $z_i = \bar{z}_t$ must be indifferent between debt restructuring and staying in C-sector. The condition is that $\bar{z}_t = \max\{0, \min\{z_{\max}, \max\{\bar{z}_{t-1}, \hat{z}_t\}\}\}$ and $n_t = F(\bar{z}_t)$, where $\hat{z}_t$ is given by

$$(n_t + e_t)^{\frac{1}{\sigma-1}} A_M K - \varepsilon + V_t - \hat{z}_t \Delta^{\phi} = A_L K + Q_t^L K.$$

Thus, for $t = 1, 2, \cdots, T-1$, the values of $\hat{z}_t$ are given by

$$\hat{z}_t = \{(n_t + t\lambda)^{\frac{1}{\sigma-1}} A_M K - \varepsilon + V_t - A_L K - Q_t^L K\} \Delta^{-\phi}. \tag{47}$$

We take $\max\{\bar{z}_{t-1}, \hat{z}_t\}$ because lenders who restructured the debt in period $t-1$ cannot undo it in period t. For $t = 1$, $\bar{z}_1 = \max\{0, \min\{z_{\max}, \hat{z}_1\}\}$, where $\hat{z}_1$ is given by (47) at $t = 1$, because we take n_1 as given for now. For $t = 2, 3, \cdots, T-1$, the values of n_t is given by $n_t = F(\bar{z}_t)$. For $t = T$, the value of $\hat{z}_T$ is given by the condition that the lenders are indifferent between debt restructuring and selling capital to new-born firm, i.e., (45) equals (46) at $t = T$, implying that

$$\hat{z}_T = \frac{\gamma K}{\Delta^{\phi}},$$

and $n_T = F(\bar{z}_T)$, where $\bar{z}_T = \max\{0, \min\{z_{\max}, \max\{\bar{z}_{T-1}, \hat{z}_T\}\}\}$.

Next we specify (T, n_1, Δ) as follows. First, given (n_1, Δ), the value of T is determined by the condition

$$(T-1)\lambda < 1 - n_T \leq T\lambda.$$

Then, given Δ, the value of n_1 is decided by

$$n_1 = F(\bar{z}_1).$$

Now, given Δ, we have shown that the values can be determined for $\{n_t, e_t, V_t, Q_t^L\}_{t=1}^{\infty}$ for given A_H. Next we specify the equilibrium value of Δ by solving the period 0 problem for an incumbent firm. The variable k is to be chosen only in period 0.

$$\max_k \mathbb{E}_0[\max\{0,\ (n_1 + e_1)^{\frac{1}{\sigma-1}} A_1 \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}} - \varepsilon + Q_1 k - (1+r)Q_0 k\}],$$

where Q_0 and debt $D_1 = (1+r)Q_0 k$ are to be determined. Since we consider DOE, the firm only cares the state $A_1 = A_H$. Thus, the firm solves

$$\max_k p_H\{A_H \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}} - \varepsilon + Q_H k - (1+r)Q_0 k\},$$

where we can assume either that the firm managers live for only two periods or that they live forever because $Q_H K$ is the future value of a firm in state A_H. The FOC implies

$$(1+r)Q_0 = \left(\frac{\sigma-1}{\sigma}\right) A_H + Q_H = \left(\frac{1}{1-\beta} - \frac{1}{\sigma}\right) A_H - \frac{\beta\varepsilon'}{1-\beta}.$$

Therefore,

$$D_1 = (1+r)Q_0 K = \left(\frac{1}{1-\beta} - \frac{1}{\sigma}\right) A_H K - \frac{\beta\varepsilon}{1-\beta}, \tag{48}$$

and the amount of debt that should be restructured is given by

$$D_1 - V_1(\Delta),$$

where $V_1(\Delta)$ is the value from the above sequence $\{V_t\}_{t=1}^{\infty}$, given Δ. The rational expectations imply that the following equilibrium condition must be satisfied:

$$\Delta = D_1 - V_1(\Delta), \tag{49}$$

which pins down the equilibrium value of Δ. Here we specify the equilibrium values of all variables in Case 1 (Short-term recession) for a given value of A_H.

Finally, we need to check whether the free entry condition for the new-born firms is satisfied in period 1 of this equilibrium or not. If it is satisfied, the short-term recession is actually the equilibrium. If it is not satisfied in period 1, then the short-term recession cannot be the equilibrium, and the equilibrium would be the persistent stagnation where $n_t = n_1$ and $e_t = 0$ for all $t \geq 1$.

Let us denote n_1 of Case 1 for a given A_H by $n(A_H)$. The free entry condition for a new-born firm in period 1 is :

$$(n(A_H) + \lambda)^{\frac{1}{\sigma-1}} A_M K - \varepsilon + V_1(A_H) - A_L K - Q_1^L K - \gamma K \geq 0, \tag{50}$$

where $V_1(A_H)$ is the value of V_1 in Case 1 equilibrium for the given value of A_H. Note that a new-born firm needs to pay $A_L K + Q_1^L K$, because it purchases K before production takes place in period 1. We define $\bar{A}_H^{\infty}$ as the value of A_H that makes the Left Hand Side of (50) equals zero. Note that $n(A_H)$ and $V_1(A_H)$ are decreasing in A_H, because an increase in A_H affects n_1 and V_1 only through increasing the debt D_1. We have proven the following proposition:

Proposition 8. *Suppose $A_H \leq \bar{A}_H^{\infty}$. In the case that the productivity of capital turns out to be A_M in period 1, the economy follows the short-term recession (Case 1).*

D.2.2 Case 2: Deep and persistent stagnation

We will show in this subsubsection that there exists $\overline{\overline{A}}_H^{\infty}$ such that if $A_H > \overline{\overline{A}}_H^{\infty}$, the equilibrium of Case 2 exists.

First, we specify the equilibrium variables of Case 2 equilibrium, on the premise that there exists Case 2 equilibrium, where $e_t = 0$ for all $t \geq 1$. In the end, we will show the condition for the existence of Case 2 equilibrium. Since there is no entry of new-born firms, all variables are invariant from period 1 on. Given the number of debt restructuring n_1, the variables are

$$Q_t^L = Q^L = \frac{\beta A_L}{1-\beta},$$

$$V_t = V(n_1) = \frac{\beta(n_1^{\frac{1}{\sigma-1}} A_M K - \varepsilon)}{1-\beta},$$

$$\Delta = D_1 - V,$$

where D_1 is given by (48). The number of firms operating in S-sector, n_1, is given by $n_1 = F(\bar{z})$, where the threshold $\bar{z}$ is the largest z that satisfies

$$n_1^{\frac{1}{\sigma-1}} A_M K - \varepsilon + V(n_1) - z\Delta^{\phi} \geq A_L K + Q_L K,$$

which is given by $\bar{z} = \hat{J}(n_1)$, where $\hat{J}(n) = \max\{0, \ \min\{z_{\max}, \ J(n)\}\}$, and

$$J(n) = \left(\frac{1-\beta}{K}\right)^{\phi-1} \frac{n^{\frac{1}{\sigma-1}} A_M - \varepsilon' - A_L}{[(1-(1-\beta)\sigma^{-1})A_H - \beta n^{\frac{1}{\sigma-1}} A_M]^{\phi}}.$$

Thus, n_1 is the largest value of n that satisfies $n = F(\hat{J}(n))$. Here we have specified the variables in Case 2 (the deep and persistent stagnation).

To have $e_t = 0$, we need the following no entry condition for the new-born firms:

$$(n_1 + \lambda)^{\frac{1}{\sigma-1}} A_M K - \varepsilon + V(n_1) - (A_L K + Q_L K) - \gamma K < 0. \tag{51}$$

This condition implies that a new-born firm will not enter S-sector, even if all the other new firms enter. Since n_1 is a decreasing in A_H, there exists a threshold value $\overline{\overline{A}}_H^\infty$ such that the condition (51) is satisfied iff $A_H > \overline{\overline{A}}_H^\infty$. Thus we have shown the following proposition.

Proposition 9. *Suppose $A_H > \overline{\overline{A}}_H^\infty$. In the case that the productivity of capital turns out to be A_M in period 1, the economy falls into the deep and persistent stagnation (Case 2).*

D2.3 Equilibrium for intermediate value of A_H

We have shown that if the debt overhang is small (i.e., $A_H \leq \bar{A}_H^\infty$), the economy goes through the short-term recession, whereas if the debt overhang is large (i.e., $A_H > \overline{\overline{A}}_H^\infty$), it goes through the deep and persistent stagnation. Here we examine the case where A_H takes on an intermediate value.

If $\bar{A}_H^\infty > \overline{\overline{A}}_H^\infty$, we have multiple equilibria: For $A_H \in (\overline{\overline{A}}_H^\infty,\ \bar{A}_H^\infty]$, the short-term recession (Case 1) and the deep and persistent stagnation (Case 2) can coexist for the same value of A_H.

We can show that there exists no equilibrium if $\bar{A}_H^\infty < \overline{\overline{A}}_H^\infty$. The reasoning is as follows. Suppose $\bar{A}_H^\infty < \overline{\overline{A}}_H^\infty$, and suppose that there exists an equilibrium for $A_H \in (\bar{A}_H^\infty, \overline{\overline{A}}_H^\infty)$. Then, neither Case 1 nor Case 2 holds for $A_H \in (\bar{A}_H^\infty,\ \overline{\overline{A}}_H^\infty)$, meaning that this equilibrium must satisfy the free entry condition in period 1 and $n_t + e_t < 1$ for all $t \geq 1$. Given that $n_t + e_t < 1$ forever, it must be the case that $Q_t^L = Q_L = \beta A_L/(1-\beta)$. Then, the free entry condition must be satisfied for all $t \geq 1$, once it is satisfied in period 1. Thus, $e_t = \min\{1 - n_t,\ t\lambda\}$ for all $t \geq 1$. Therefore, it must be the case that $n_T + e_T = 1$ for a finite T. Then, it contradicts the assumption that $n_t + e_t < 1$ for all $t \geq 1$. Therefore, the equilibrium cannot exist for $A_H \in (\bar{A}_H^\infty,\ \overline{\overline{A}}_H^\infty)$, if $\bar{A}_H^\infty < \overline{\overline{A}}_H^\infty$.

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