The Canon Institute for Global Studies

CIGS Working Paper Series No. 23-013E

Inflation and entry costs in a monetary search model

Ryoji Hiraguchi (Meiji University)

Keiichiro Kobayashi (Keio University)

2023.8

General Incorporated Foundation

The Canon Institute for Global Studies

一般財団法人　キヤノングローバル戦略研究所

Phone: +81-3-6213-0550　　https://cigs.canon/

Inflation and entry costs in a monetary search model

Ryoji Hiraguchi*[†], Keiichiro Kobayashi [‡§]

August 4, 2023

Abstract

In this study, we construct a variant of the Lagos-Wright monetary model in which both buyers and sellers optimally decide whether to enter decentralized market by paying fixed entry costs. In the decentralized market, the sellers produce the intermediate inputs which are necessary to produce the general good traded in the centralized market. We show that the Friedman rule of setting nominal interest rate to zero may not be optimal. The optimal inflation rate is derived explicitly for specific functional forms. It is shown that the optimal inflation rate is lower for lower buyer entry costs, because the lower entry costs generate the congestion of buyers which must be compensated for by lower cost of money holdings. It is also shown that the optimal inflation is lower for higher seller entry costs. These results may explain why the secular decline in inflation has been observed in recent decades when the emergence and growth of Internet usage has lowered shopping costs for buyers.

Keywords: Optimal monetary policy; entry cost; competitive pricing; low inflation.

JEL classification code: E13; E42; E52.

*Faculty of Political Science and Economics, Meiji University. 1-1 Kanda-surugadai, Chiyoda-ku, Tokyo, Japan. E-mail: hiraguchi@meiji.ac.jp. Tel: 81-43-290-2399.

†The Canon Institute for Global Studies, Japan

‡Faculty of Economics, Keio University. 2-15-45 Mita, Minato-ku, Tokyo, Japan. Email: kobayasi@econ.keio.ac.jp Tel: 81-3-5418-6703.

§The Canon Institute for Global Studies, Japan

1 Introduction

Free entry is a fundamental property of a market economy. What is the effect of free entry on the monetary phenomena? How do changes in entry costs affect inflation? To answer these questions, we construct a variant of the Lagos-Wright monetary model (Lagos and Wright 2005) in which both buyers and sellers make optimal entry decisions whereas they must pay fixed costs when they enter the decentralized market, in which money is necessary for transactions. To the best of our knowledge, this study is the first attempt at constructing a model in which all agents make entry decisions. Constructing such a model is the first contribution of this study. The second contribution of this study is deriving an optimal inflation rate explicitly for specific functional forms. We show that the optimal inflation is lower for a lower relative cost of buyers' entry. In other words, the optimal inflation is lower when buyers' entry costs are lower, or sellers' entry costs are higher. This may explain why the secular declines in inflation have been observed worldwide in recent decades when the emergence and growth in internet usage lowers shopping costs for buyers, as the shopping costs can be interpreted as buyer entry costs in our model.

As discussed below, it is natural to assume that all agents, that is, both buyers and sellers, can make entry decisions. However, in most of the recent monetary search models built by Lagos and Wright (2005), there is no entry decision or only sellers or buyers make the entry decisions. No monetary theory exists yet in which both buyers and sellers make entry decisions. As we see below, it is difficult to generalize the Lagos-Wright model such that both buyers and sellers make entry decisions. We overcome this difficulty by introducing a simple production chain flow from a decentralized to a centralized market.

Difficulty in a monetary model with entry decision: Generally, entering a market is costly for both buyers and sellers. Imagine how street markets and flea markets are open or how people sell and buy in shopping districts. Sellers must also set up shops by expending fixed costs. Even when sellers own their shops, they must expend fixed costs regularly to prepare for daily operations. Buyers must spend time and effort visiting the marketplace and search for goods or services they want to buy the most. Thus, it is natural to model any market such that both buyers and sellers optimally make entry decisions by expending their entry costs, respectively.

However, it is difficult to assume that all agents make entry decisions in the Lagos-Wright model. Difficulties arise when the matching function is assumed to be a constant return to scale (CRS). The CRS matching function is the dominant assumption in the monetary search literature. Under CRS matching technology, the matching probabilities for buyers and sellers are both determined by an identical variable: the ratio between the buyer and seller measures. Then, we have two equations for only one unknown, that is, the free-entry conditions for buyers and sellers to be solved only for the buyer-seller ratio.[1] Thus, too many conditions must be satisfied in a steady state. Therefore, the interior solution for the steady-state equilibrium cannot exist, except for the knife-edge case in which the money growth rate coincides with a particular value. Except for the knife-edge case, the monetary equilibrium becomes a corner solution in which all sellers or all buyers enter the market. This difficulty can be resolved if the following assumptions are made: the matching function is either increasing returns to scale (IRS) or decreasing returns to scale (DRS), because the matching probabilities are functions of the two variables- that is, the measures of sellers and buyers- in these cases. Thus, when the matching function is IRS or DRS, the internal solution for steady-state equilibrium can exist as there are two equations for the two unknowns. However, assuming the IRS or DRS matching function is not standard in the theoretical literature and is not fully supported by empirical evidence. [2] In this study, we overcome this difficulty by positing

[1]More precisely, the steady state in the Lagos-Wright model with free entries of buyers and sellers is determined using the following four equations for three unknowns in the case of the CRS matching function. The four equations are the free-entry conditions for buyers, for sellers, Euler equation for money holdings, and the optimality condition to determine the terms of trade in a decentralized market. The three unknown variables are the value of money in a centralized market, the output quantity in the decentralized market, and the ratio of buyers to sellers.

[2]Even in a model with an IRS or DRS matching function, another difficulty arises when entry costs are identical for buyers and for sellers. Given the identical costs, a model with both buyers and sellers making entry decisions renders trivial and unrealistic welfare implication that a monetary policy is irrelevant to the social welfare. This is because the free-entry conditions for buyers and sellers imply that they obtain

technological constraint that the output in the decentralized market is not consumed; however, it is taken to the centralized market and is used as an intermediate input in the production of final goods in the centralized market. This setting enables the internal solution for a steady-state equilibrium to exist in which both buyers and sellers make their entry decisions. This is not only due to the ratio of buyers to sellers but also the measure of buyers (and sellers) enters production of the final good. This implies that two unknowns exist (the measures of buyers and sellers) for the two equations (the free-entry conditions for buyers and sellers). Because social welfare is linked to the final output in a centralized market that varies with the monetary policy, the monetary policies have a substantial effect on social welfare.

Implication on effects of the internet on inflation: For a specific set of functional forms that appears natural and plausible, we derive the optimal monetary policy in the steady state, that is, the optimal nominal interest rate or inflation rate. We find that the optimal monetary policy deviates from the Friedman rule. The novelty of our findings is that the optimal inflation rate is an increasing function of the relative entry cost of buyers to sellers. Thus, the optimal inflation rate is lower for a lower entry cost of buyers or a higher entry cost of sellers. This result sheds light on the mechanism of declining inflation rates in major economies over the last two or three decades. The last three decades have been exactly the period when the Internet usage has grown at a breathtaking speed worldwide, and new e-commerce businesses have grown dramatically. Widespread trading on the Internet has decreased shopping costs for buyers and increased the number of matches. Our theoretical results imply the following: technological progress that reduces buyers' shopping costs caused a secular decline in inflation rates worldwide in the early 21st century. This is because decreased shopping costs can be interpreted as a decrease in buyers' entry costs, which predicts that optimal inflation is lower for a lower entry cost

zero expected utility in the equilibrium because the measures for entering buyers and sellers are given by the condition that the expected welfare gains of entry equal the entry costs. Thus, regardless of monetary policy, the social surplus of entering a decentralized market is zero. The proposed model avoids this trivial problem. It cananalyze the welfare effects of the monetary policy.

for buyers.

Related literature: Recent literature on monetary search models, called the third generation Models, were built based on a seminal study by Lagos and Wright (2005). Concerning the entry decisions of agents, no model exists in which all agents make an entry decision. Lagos and Wright (2005) assume that there is no entry decision for either buyers or sellers. Rocheteau and Wright (2005) analyze a model in which only sellers make entry decisions. Liu et al. (2011) study a model in which buyers, not sellers, make entry decisions by paying fixed costs. Nosal (2008) analyzes a model in which buyers can choose whether to trade when they are successfully matched with sellers, whereas agents do not make entry decisions. In these models, in which either sellers or buyers make entry decisions, the Friedman rule is not optimal in general.. Our result is similar to theirs in this regard. However, we confirm the suboptimality of the Friedman rule in the model, where all the agents make entry decisions. Another novelty of this study is that we explicitly derive the optimal monetary policy, given a certain set of functional forms. We also explicitly show the relationship between the optimal inflation and entry costs; that is, the optimal inflation is lower for - lower the relative costs of the buyer entries.

Even in the literature on search theory, there are few models in which both sellers and buyers make entry decisions. An exception is Wasmer and Weil (2004), who study search and matching in financial and labor markets. Bankers and firms match in the financial market and both are subject to free-entry conditions. In the labor market, firms and workers meet and only firms are subject to free entry conditions. We consider that the financial and labor markets in Wasmer and Weil (2004) correspond to the DM and the CM in our model, respectively. However, neither money nor monetary policies exist in their model.

Our results imply that an increase in internet usage may have caused the secular decline in the inflation rate in recent decades. Several empirical studies support the hypothesis that an increase in Internet usage helped decrease inflation. Yi and Choi (2005), Friesenbichler (2018), Koester et al. (2021), and Çoban (2022) show empirical support for the hypothesis that an increase in internet usage reduces the inflation. Goolsbee and Klenow (2018) show that prices are lower when goods are traded on the Internet, rather than when they are traded in real market.

The remainder of this paper is organized as follows. The model is described in Section 2. Section 3 presents a welfare analysis in which the optimal monetary policy or optimal inflation rate is explicitly derived. Section 4 concludes the paper. The proofs of the propositions and lemmas are provided in Appendix.

2 Model

2.1 Setup

The setup is very similar to that of Lagos and Wright (2005). Time is discrete and flows from $t = 0$ to $+\infty$. There is a continuum of the infinitely lived agents with a measure of one. They consist of both buyers and sellers. The measure of buyers (and sellers) equals 0.5. Each date is divided into day and night. The day market (DM) is decentralized and the night market (CM) is centralized. In the DM, the buyer and seller trade goods if they match successfully. Individuals are anonymous in the DM, and trade in the DM is mediated by money, which is intrinsically useless, perfectly divisible, and storable, and provided by the central bank. Based on the literature, we assume that the agents in the DM can use the money carried from the CM of the previous period. In addition, we assume that both buyers and sellers enter the DM by paying the respective fixed costs, and the probabilities of matching for buyers and for sellers are determined endogenously. They decide in the current CM whether to enter the DM in the next period, whereas they pay for entry costs when entering the DM. Fig.1 describes the timing of events in a period.

The model has three goods: special goods, intermediate goods, and general goods. The agents consume only general goods. They are produced and traded in the CM and their production needs labor and intermediate goods. Let $F(Q, H)$ denote the production function of the general goods, where Q denote the quantity of the intermediate good.

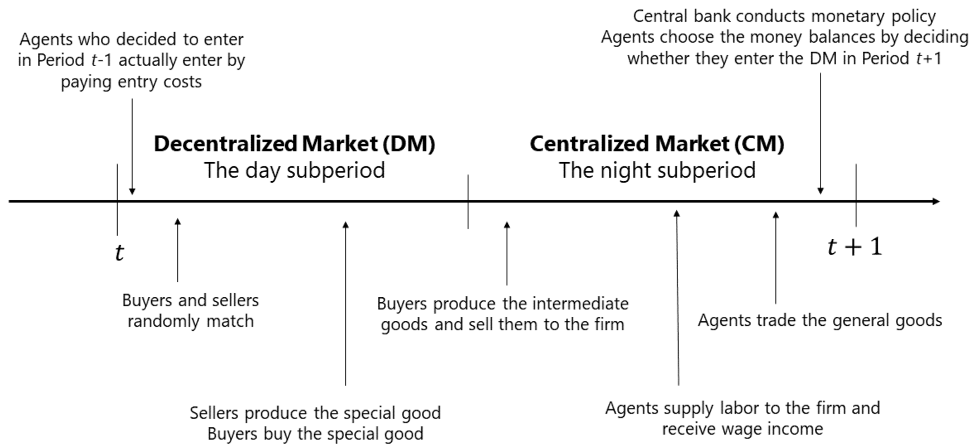

Figure 1: Timing of events in a period

H denotes labor supply. The general goods are produced by firms owned by sellers and buyers, whereas they earn zero profit in the perfect competition. In the DM, the seller produces and sells special goods to the buyer. The buyer then takes them to the CM, and produces an intermediate good from special good. In the CM, the buyer sells intermediate good to firms that produce general goods. Additionally, let $Q = g(q)$ denote production function of the intermediate good, where q denotes the quantity of the special good. The production function g satisfies $g' > 0$, $g'' < 0$, and $g'(0) = +\infty$. In the following, we assume $g(q) = q^\theta$ where $\theta \in (0, 1)$. To enter the DM, the buyer must pay k_b units of utility cost. Similarly, to enter the DM, the seller must pay k_s units of utility cost. Let e and σ denote the measures of buyers and sellers entering the DM, respectively. These satisfy $0 \leq e \leq 1/2$ and $0 \leq \sigma \leq 1/2$. The matching function in the DM is given by $N(e, \sigma)$. Function N has a constant returns to scale and is strictly increasing and strictly concave. In the following, we focus on the interior equilibrium, where only a subset of buyers and sellers enter the DM; that is, $0 < e < 1/2$ and $0 < \sigma < 1/2$. In the DM, each buyer matches with a seller with probability $\pi_b(z) = N(1, z)$, and for each seller, the probability of meeting with a buyer is $\pi_s(z) = N(1, z)/z$, where $z = \sigma/e$.

In the DM, the matched buyer obtains q units of the special good and the matched seller loses utility $c(q)$ by producing q output units. The function c satisfies $c(0) = 0$, $c' > 0$, and $c'' > 0$. For simplicity, we use competitive pricing rather than bargaining in terms of pricing in the DM. [3] Let p denote the price of the special good in terms of the general good.

In the CM, individuals sell intermediate goods to the general-good firms if they have, supply labor, receive wage income, and buy general good. Also, the individuals determine whether to enter the DM in the next period and adjust money balances. The price mechanisms of the CM are also competitive. Let w denote the wage rates in terms of general goods. Additionally, let R denote the price for the intermediate good in terms of the general good. In the CM, each agent obtains utility $U(C)$ from consuming C units of general goods and suffers from the linear disutility H by supplying H units of labor.

A central bank controls the money supply M by setting its growth rate $\mu - 1$. We assume that in the initial period, the government randomly chooses M_0 units of buyers and gives them one unit of money each, where $M_0 > 0$. As only buyers with money enter the DM, we have $e_0 \leq M_0$.

2.2 Buyer

We follow Lagos and Wright (2005) and focus on degenerate stationary equilibrium, in which the level of consumption is the same across all agents and output is constant. We index the variables for the next period as $+1$. We solve the model backward and first investigate the CM.

Let $V_b(m)$ denote the value function of a buyer holding m units of money at the beginning of each period. Let $W_b(m, q)$ denote the value function of the buyer in the CM who holds m units of money and q units of special goods. The buyer can sell $g(q)$ units of intermediate goods to general-goods firms at the price R. In the CM, buyers choose consumption of the general good C_b and the amount of money to be carried to the next

[3] In the literature on the Lagos-Wright framework, competitive pricing is used in Aruoba, Waller and Wright (2011), Berentsen, Camera and Waller (2005, 2007), Lagos and Rocheteau (2005), and Rocheteau and Wright (2005).

period m_{+1} to solve

$$\begin{aligned} W_b(m,q) &= \max_{C_b, m_{+1}} [U(C_b) - H_b + \delta V_b(m_{+1})], \\ \text{s.t.} \;\; C_b &= wH_b + \phi(m - m_{+1} + T) + Rg(q), \end{aligned}$$

where $\delta > 0$ is the discount factor, H_b is labor supply, ϕ is the value of money in terms of the general good, and T is the transfer from the government or taxes, if $T < 0$ in terms of money. The value function is rewritten as

$$W_b(m,q) = \frac{\phi}{w} m + \frac{R}{w} g(q) + W_b(0,0),$$

where $W_b(0,0)$ is equal to

$$W_b(0,0) = \max_{m_{+1}} \left[-\frac{\phi}{w} m_{+1} + \delta V_b(m_{+1}) \right] + \max_{C_b} \left[U(C_b) - \frac{C_b}{w} \right] + \frac{\phi}{w} T, \tag{1}$$

The consumption of the general good C_b is determined using the following first-order conditions:

$$U'(C_b) = \frac{1}{w}.$$

Subsequently, we investigate the equilibrium condition of the DM. Let $V_b^E(m)$ denote the value function of buyers who decided to enter the DM. Similarly, let $V_b^N(m)$ denote the value function for buyers who do not enter the DM. These value functions are written as

$$\begin{aligned} V_b^E(m) &= -k_b + \pi_b \max_{pq \le \phi m} W_b \left(-\frac{p}{\phi} q + m, q \right) + (1 - \pi_b) W_b(m, 0), \\ V_b^N(m) &= W_b(m, 0), \end{aligned}$$

where $\pi_b = \pi_b(z)$ and p is the price of the special good in terms of the general good. The nominal price of a special good is given by p/ϕ. The value function of the buyer at the beginning of each period is

$$V_b(m) = \max \left\{ V_b^E(m), V_b^N(m) \right\}. \tag{2}$$

The welfare maximization problem for the choice of m_{+1} is described in (1). It can be written as follows. [4]

$$\max_{m_{+1}} \left[\max \left\{ -\frac{\phi}{w} m_{+1} + \delta V_b^E(m_{+1}), -\frac{\phi}{w} m_{+1} + \delta V_b^N(m_{+1}) \right\} \right]. \tag{3}$$

[4] In general, we have

$\max_x [\max\{f(x) + g(x), f(x) + h(x)\}] = \max_x [f(x) + \max\{g(x), h(x)\}]$

There may be two solutions to the above maximization that correspond to entering the next-period DM and not entering. We are interested in interior equilibria in which individuals are indifferent between entering the next-period DM and not entering it. In such an equilibrium, we have

$$\max_{m_{+1}} \left[-\frac{\phi}{w} m_{+1} + \delta V_b^E(m_{+1}) \right] = \max_{m_{+1}} \left[-\frac{\phi}{w} m_{+1} + \delta V_b^N(m_{+1}) \right], \tag{4}$$

where we denote the solution on the left-hand side and the solution on the right-hand side by m_{+1}^E and m_{+1}^N, respectively, and $m_{+1}^E \neq m_{+1}^N$ in general. The buyers who choose m_{+1}^E enter the next-period DM and those who choose m_{+1}^N do not. Throughout this study, we focus on equilibria in which nominal interest rate i is strictly positive[5] where i is defined as follows:

$$i \equiv \frac{\phi/w}{\delta \phi_{+1}/w_{+1}} - 1,$$

The constraint on money binds for the agent who enters the DM:

$$pq = \phi m. \tag{5}$$

The value function of the buyer entering the DM, V_b^E is written as

$$V_b^E(m) = -k_b + \pi_b \frac{R}{w} g\left(\frac{\phi m}{p}\right) + (1-\pi_b)\frac{\phi m}{w} + W_b(0,0). \tag{6}$$

The first-order condition on the left-hand side of (4) is written as

$$\frac{\phi}{w} = \delta \frac{\phi_{+1}}{w_{+1}} \left\{ \pi_{b,+1} \frac{R_{+1}}{p_{+1}} g'\left(\frac{\phi_{+1} m_{+1}}{p_{+1}}\right) + 1 - \pi_{b,+1} \right\}. \tag{7}$$

From the definition of nominal interest rate, we have

$$i = \pi_{b,+1} \left(\frac{R_{+1}}{p_{+1}} g'\left(q_{+1}\right) - 1 \right) \tag{8}$$

The right-hand side of (4) satisfies

$$-\frac{\phi}{w} m_{+1} + \delta V_b^N(m_{+1}) = -\left(1 - \delta \frac{\phi_{+1}/w_{+1}}{\phi/w}\right) \frac{\phi}{w} m_{+1} + W_b(0,0) \leq W_b(0,0),$$

[5]We consider the Friedman rule as a limiting case in which i converges to zero from above, i.e., $i \to 0+$.

where the last inequality holds, because the nominal interest rate is positive.[6] Thus, the buyer who does not enter the DM holds no money, that is, $m^N_{+1} = 0$. Therefore (4) is simplified as follows:

$$k_b = -\frac{\phi}{w\delta} m_{+1} + \pi_{b,+1} \frac{R_{+1}}{w_{+1}} g\left(q_{+1}\right) + \left(1 - \pi_{b,+1}\right) \frac{\phi_{+1} m_{+1}}{w_{+1}}, \tag{9}$$

where m_{+1} is the money balance for the entering buyer, m^E_{+1}.

As $m = pq/\phi$ and $i = \frac{\phi/w}{\delta\phi_{+1}/w_{+1}} - 1$, (9) can be written as

$$k_b = -i \frac{p_{+1} q_{+1}}{w_{+1}} + \pi_{b,+1} \left(\frac{R_{+1}}{w_{+1}} g\left(q_{+1}\right) - \frac{p_{+1} q_{+1}}{w_{+1}} \right). \tag{10}$$

Substituting (8) into this equation yields

$$k_b = \frac{R_{+1}}{w_{+1}} g\left(q_{+1}\right) \pi_{b,+1} \left\{ 1 - \frac{q_{+1} g'\left(q_{+1}\right)}{g\left(q_{+1}\right)} \right\}. \tag{11}$$

2.3 Seller

Let $W_s(m)$ denote the seller's value function in the CM with m dollars. Additionally, $V_s(m)$ denotes the seller's value at the beginning of each period. The seller solves

$$W_s(m) = \max_{C_s, H_s, m_{+1}} \{U(C_s) - H_s + \delta V_s(m_{+1})\}, \text{ s.t. } \; C_s = wH_s + \phi(m - m_{+1}). \tag{12}$$

It is shown that

$$W_s(m) = \frac{\phi}{w} m + W_s(0).$$

The value of C_s is given by

$$U'(C_s) = \frac{1}{w}.$$

Subsequently, we investigate the equilibrium condition of the DM. Let $V^E_s(m)$ denote the value function of seller who decides to enter the DM. Similarly, we let $V^N_s(m)$ denote the value function of sellers who decide not to enter the DM. These value functions are

[6] Throughout this study, we assume that money growth is given, such that this inequality holds. Unless this inequality holds, there exists no equilibrium because, in that case, buyers who do not enter would choose $m_{+1} = +\infty$ to obtain $V^N_b(m_{+1}) = +\infty$.

expressed as follows:

$$\begin{aligned} V_s^E(m) &= -k_s + \pi_s \max_q \{-c(q) + W_s(\phi^{-1}pq + m)\} + (1-\pi_s)W_s(m), \\ V_s^N(m) &= W_s(m), \end{aligned}$$

where $\pi_s = \pi_s(z)$. The seller's value function at the beginning of each period is

$$V_s(m) = \max\left\{V_s^E(m), V_s^N(m)\right\}. \tag{13}$$

The function $V_s^E(m)$ is simplified as

$$V_s^E(m) = -k_s + \pi_s \max_q \left\{-c(q) + \frac{p}{w}q\right\} + \frac{\phi}{w}m + W_s(0).$$

Note that $V_s(m) = \frac{\phi}{w}m+$(other terms). This equation together with (12) implies that the seller does not carry money into the next period; that is, $m_{+1} = 0$. This is because the seller cannot use money in the next DM and it is costly to carry money from the current CM to the next because the nominal interest rate is assumed to be positive.

We are interested in the interior equilibrium in which the sellers are indifferent between entering the DM and not entering. In such an equilibrium, we have $V_s^E(0) = V_s^N(0)$, which implies that

$$k_s = \pi_s \max_q \left\{-c(q) + \frac{p}{w}q\right\}. \tag{14}$$

The FOC for the DM problem and free-entry condition are respectively

$$wc'(q) = p, \tag{15}$$

$$k_s = \pi_s \left\{-c(q) + c'(q)q\right\}. \tag{16}$$

2.4 Feasibility conditions

In the equilibrium, the total amount of intermediate goods is

$$Q = N(e,\sigma)g(q), \tag{17}$$

where $\sigma = ez$. In the CM, all agents choose the same consumption level $C(= C_b = C_s)$ defined by

$$U'(C) = \frac{1}{w}. \tag{18}$$

The resource constraint is

$$F(Q,H)=C. \tag{19}$$

where $H=(H_b+H_s)/2$ is the total labor supply in the CM. The factor prices w and R are determined competitively:

$$\begin{aligned} w &= F_H(Q,H), & (20)\\ R &= F_Q(Q,H). & (21)\end{aligned}$$

Because the number of buyers entering the DM is e, the equilibrium condition on money is

$$M=em. \tag{22}$$

where m is the buyer's nominal balance entering the DM.

2.5 Initial period

In the initial period, only buyers who receive (one unit of) money from the government can enter a DM. In the following, we focus on the equilibria where all the buyers with money enter the DM in the initial period.

$$e_0=M_0. \tag{23}$$

Buyers enter the DM if their surplus from entering the DM, $V_b^E(1)-V_b^N(1)$, is strictly positive. As we show in Appendix A.1, this occurs if

$$-k_b+\pi_{b,0}\left(\frac{R_0}{w_0}g\left(\frac{\phi_0}{p_0}\right)-\frac{\phi_0}{w_0}\right)>0. \tag{24}$$

We define the competitive equilibrium as follows:

Definition 1 *The competitive equilibrium is the set of prices* $\{\phi_t,w_t,p_t,R_t\}$ *and allocation* $\{q_t,e_t,z_t,Q_t,H_t,C_t,m_t\}$ *that evolves according to (5), (7), (11), (15), (16),(17), (18), (19), (20), (21) and (22), given that the sequence of money supply* $\{M_t\}_{t=0}^{\infty}$ *is exogenously determined and the variables in the initial period satisfy (23) and (24).*

2.6 Stationary equilibrium

The next proposition characterizes the steady-state allocation and proves existence of a steady state under a specific government policy.

Proposition 1 *In the steady state where M_t grows at a constant rate of μ, where $\mu > \delta$, the allocation (q, e, z, Q, H, C) is determined by (19) and*

$$\frac{\mu}{\delta} - 1 = \pi_b(z) \left(\frac{g'(q)}{c'(q)} \frac{F_Q(Q,H)}{F_H(Q,H)} - 1 \right), \tag{25}$$

$$k_b = \pi_b(z) g\,(q) \frac{F_Q(Q,H)}{F_H(Q,H)} \left(1 - \frac{qg'(q)}{g(q)} \right), \tag{26}$$

$$k_s = \pi_s(z)(-c(q) + qc'(q)), \tag{27}$$

$$Q = eN(1,z)g(q), \tag{28}$$

$$F_H(Q,H)U'(F(Q,H)) = 1, \tag{29}$$

where $\pi_b(z) = N(1,z)$ and $\pi_s(z) = N(1,z)/z$. This steady state can be realized from the initial period if the government chooses $M_t = e\mu^t$ for $t \geq 1$.

Proof. See Appendix. ■

Once we determine the steady-state allocation, the stationary price (w, ϕ, p, R) are determined uniquely, as shown in Appendix A.1. As in the previous section, we focus on the equilibrium in which all buyers who hold money enter the DM during the initial period. Such a steady state exists if (24) holds true. Appendix A.1 shows that this inequality is satisfied. Thus, the stationary allocation defined above can be implemented as competitive equilibrium.

Stability of the equilibrium: The equilibrium values of e and σ, which are determined by the free-entry conditions $V_b^E(m) = V_b^N(0)$ and $V_s^E(0) = V_s^N(0)$, are stable for the following reasons. If e is slightly greater than the equilibrium value, then , the buyer's matching probability decreases slightly. Then, given that market prices are invariant, the welfare of the buyer entering the DM, V_b^E, is less than the welfare of the buyer not entering the DM, V_b^N. In this case, the number of entering buyers decreases and the value of e returns to the equilibrium. On the other hand, if e is less than its equilibrium value, $V_b^E > V_b^N$ and more buyers begin to enter the DM. Thus, e increases and returns to the equilibrium. A similar mechanism applies to sellers. Therefore, the steady-state equilibrium is stable.

Hereafter, we put the following assumptions for functional forms:

Assumption 1 *$F(Q,H) = Q^\alpha H^{1-\alpha}$, $N(e,\sigma) = Ae^\beta \sigma^{1-\beta}$, $c(y) = y^\psi$, $U(c) = \frac{c^{1-\rho}-1}{1-\rho}$ and $g(y) = y^\theta$ where $\psi > 1$, $\alpha \in (0,1)$, $\beta \in (0,1)$, $\theta \in (0,1)$, and $\rho \in (0,1)$.*

Under Assumption 1, $\pi_b = N/e = Az^{1-\beta}$ and $\pi_s = N/\sigma = Az^{-\beta}$, where $z = \sigma/e$. Let $k = k_b/k_s$ denote the ratio of buyer's entry cost relative to that of the seller.

Proposition 2 *Under Assumption 1, the equilibrium level of z is determined by*

$$\frac{\mu}{\delta} - 1 = i(z;k), \tag{30}$$

where the function $i(z;k)$ is a decreasing function of z and is defined as

$$i(z;k) = A\left\{z^{-\beta}\frac{\psi-1}{\psi}\frac{\theta}{1-\theta}k - z^{1-\beta}\right\}. \tag{31}$$

We have $q = A^{-1/\psi}Bz^{\beta/\psi}$, $e = \Gamma z^\epsilon$ and $H = \eta e$, where $B = (\frac{k_s}{\psi-1})^{1/\psi} > 0$, $\Gamma = (1-\alpha)^{1/\rho}A^{(1-\theta/\psi)\alpha\zeta}B^{\theta\alpha\zeta}\eta^{-\zeta\alpha-1} > 0$, $\zeta = 1/\rho - 1 > 0$, $\epsilon = \alpha\zeta\left(1-\beta+\beta\frac{\theta}{\psi}\right) > 0$, and $\eta = \frac{1-\alpha}{\alpha(1-\theta)}k_b > 0$. Given that k_b and k_s are fixed an increase in the nominal interest rate reduces z, q, e, σ, H and Q.

Proof. See Appendix. ■

In the following, we sometimes write Γ as $\Gamma(A)$ to ensure that Γ depends on matching parameter A. We note the parameter k as an argument of $i(z;k)$, as we analyze how changes in k affect value of i in what follows. This proposition indicates that a higher nominal interest rate or higher inflation slows down the overall economic activities, including the entry of all agents and amount of production. Thus, entries and outputs are maximized by the Friedman rule, which sets the nominal interest rate at zero. However, in the next section, we show that the Friedman rule does not necessarily maximize social welfare. In some cases, the optimal monetary policy is to set nominal interest rate at a positive value.

We consider the Friedman rule as a limiting case in which nominal interest rate converges to zero: $\mu/\delta - 1 \to 0+$. Under the Friedman rule (30) implies that $z^{-\beta}\frac{\psi-1}{\psi}\frac{\theta}{1-\theta}k - z^{1-\beta} = 0$ and then z is equal to

$$z^{FR} \equiv \frac{\psi - 1}{\psi}\frac{\theta}{1-\theta}k.$$

3 Welfare

3.1 Steady-state welfare

Social welfare in the steady state is expressed as

$$S(e, \sigma, q) = -k_s\sigma - k_b e + U(F(Q, H)) - Nc(q) - H. \tag{32}$$

where $Q = Ng(q)$ and $N = N(e, \sigma)$.

The next proposition characterizes the stationary welfare as a function of z.

Proposition 3 *The stationary welfare in the competitive equilibrium is a function of the buyer-seller ratio z:*

$$s(z) = \Gamma k^s \left\{ \frac{1 + \alpha\zeta\theta}{\alpha\zeta(1-\theta)}kz^{\epsilon} - \frac{\psi}{\psi - 1}z^{\epsilon+1} \right\}. \tag{33}$$

Proof. See Appendix. ■

Function $s(z)$ is maximized when z is equal to

$$z^*(k) = \nu_0 k, \tag{34}$$

where $\nu_0 = \frac{1+\alpha\zeta\theta}{\alpha\zeta(1-\theta)}\frac{\psi-1}{\psi}\frac{\epsilon}{\epsilon+1} > 0$.

3.2 Optimal monetary policy and entry cost

We define $i^*(k) \equiv i(z^*(k); k)$ as the nominal interest rate at $z = z^*(k)$, although it can be negative at this point. Thus,

$$i^*(k) = \nu_1 k^{1-\beta}, \tag{35}$$

where $\nu_1 = A(\nu_0)^{-\beta}(\frac{\psi-1}{\psi}\frac{\theta}{1-\theta} - \nu_0)$. Nominal interest rate that maximizes welfare in the steady state is $\max\{0, i^*(k)\}$. This is positive if and only if $\nu_1 > 0$ or, equivalently, $\frac{\psi-1}{\psi}\frac{\theta}{1-\theta} > \nu_0$. The following lemma shows that it holds if θ is sufficiently high.

Lemma 1 *The constant ν_1 is strictly positive if and only if*

$$\theta > \frac{1-\beta}{1-\beta/\psi}. \tag{36}$$

Proof. See Appendix. ■

On the right-hand side of the above inequality, $\frac{1-\beta}{1-\beta/\psi}$ is less than one because $\beta \in (0, 1)$ and $\psi > 1$. Therefore, it is satisfied provided that θ is sufficiently close to one. In the following, we assume that (36) holds. As long as the inequality holds, coefficient ν_1 is strictly positive. In that case, the function $i^*(k)$ is an increasing function of k. We also put the following inequality to ensure that the matching probabilities $\pi_b = Az^{1-\beta}$ and $\pi_s = Az^{-\beta}$ are less than one and the equilibrium number of entrants $e = \Gamma z^\epsilon$ and $\sigma = \Gamma z^{\epsilon+1}$ are less than 1/2 for all $i \in (0, i^*(k)]$. Note that since $z^{FR} > z^*(k)$, the conditions above are satisfied if the parameter A satisfies the following assumption.

Assumption 2 *The parameter A on the matching function is sufficiently small, such that*

$$A < \min\left\{(z^{FR})^{\beta-1}, z^*(k)^\beta, A_1, A_2\right\}$$

where the coefficients A_1 and A_2 are such that

$$\begin{aligned} \Gamma(A_1)(z^{FR})^\epsilon &= 1/2, \\ \Gamma(A_2)(z^{FR})^{\epsilon+1} &= 1/2. \end{aligned}$$

, respectively and $\Gamma(A) = (1-\alpha)^{1/\rho} A^{(1-\theta/\psi)\alpha\zeta} B^{\theta\alpha\zeta} \eta^{-\zeta\alpha-1} > 0$.

We have the following proposition.

Proposition 4 *Suppose Assumptions 1 and 2, and (36) holds. Then the Friedman rule is suboptimal. The optimal nominal interest rate $i^*(k) > 0$ is an increasing function of the relative entry cost of buyer $k = \frac{k_b}{k_s}$.*

Why is the Friedman rule suboptimal? This proposition states that the Friedman rule is suboptimal for a certain range of parameter values. The condition (36) can be satisfied in various cases, such as $\theta < 1$, $\beta < 1$ and $\psi > 1$. One simple explanation of suboptimality of the Friedman rule in an economy with entries is the congestion externality, a buyer's entry decreases the probability of matching for other buyers, and increases that for the sellers. See Rocheteau and Wright (2005), Liu, Wang and Wright (2011), and Berentsen and Waller (2015). [7] This external effect is not internalized in the decision-making of an individual buyer in competitive equilibrium. Thus, a reduction in the number of entering buyers e can improve social welfare if the congestion externality is high. As an increase in the nominal interest rate reduces buyers' entry e, by increasing the opportunity cost of holding money, the deviation from the Friedman rule may become optimal policy.

3.2.1 Response of optimal interest rate to changes in entry costs

Proposition 4 states that the optimal interest rate or the inflation rate is higher in an economy in which the buyers' cost of entry is higher or seller entry costs are lower. In this section, we present a simplified explanation of the optimal interest rate's response to a change in the buyers' or sellers' entry costs. Considering the optimal response of i, we focus on the changes in e and σ in response to changes in k_b or k_s.

First, suppose that k_b increases but k_s does not. Proposition 4 implies that the optimal interest rate should become higher. This intuition can be written as follows: an

[7] Some authors distinguish between the effect of one buyer's entry on other buyers, and that on sellers. The former is called the congestion effect and the the latter is called the thick-market effect (Rocheteau and Wright 2005; and Shimer and Lones 2001). In this study, we did not distinguish between them and call both congestion externality.

increase in the entry cost k_b reduces the entry of buyers and leads to an increase in the matching probability for buyers; an increase in matching probability increases the expected gain of buyer entry, leading to many buyer entries that exacerbate the congestion externality. Thus, the central bank's optimal response is to increase the costs of holding money i to reduce the buyers' entry. This intuition can be explained using the following equations: (25) indicates that $i = \pi_b\{(R/w)(g'(q)/c'(q)) - 1\}$ and (26) implies that $k_b = \pi_b g(q)(R/w)(1 - qg'(q)/g(q))$. The first equation is the Euler equation for money holdings and the second represents the free-entry condition. An increase in the entry cost $k_b$decreases the buyers' entry e and increases π_b. For simplicity, we assume that the optimal amounts of q and H do not change. Then, the decrease in e reduces the total number of matches and quantity of the intermediate good Q, leading to an increase in the the relative price, R/w. In other words, the free-entry condition implies that an increase in k_b increases $\pi_b(R/w)$. Now, let us consider the Euler equation above. An increase in $\pi_b(R/w)$ indicates an increase in gains from money holdings, which must be balanced in equilibrium with cost of money holdings i. Therefore, an increase in k_b assuming that the optimal q and H are invariant, induces an increase in optimal interest rate i.

Second, suppose that k_s increases but k_b does not. Proposition 4 implies that the optimal interest rate should become lower. How can this result be explained? Increase in the entry cost of the sellers, k_s, decreases the entry of sellers and leads to a decrease in the matching probability of the buyers. This decrease reduces the expected gain of buyer entries, leading to too few buyer entries, in terms of congestion externality. Thus, the central bank's optimal response is to decrease the cost of holding money i to increase buyers' entry. This intuition can be explained using the following equations. The sellers' free entry condition (27) can only be satisfied only if $\pi_s(z)$ becomes larger in response to increases in k_s. To increase π_s measure of the sellers, σ, should be very small if the measure of the buyers, e, does not change. The central bank can now increase π_s by increasing e through decreasing i. The buyers' entry e increases if the central bank decreases i because i is the opportunity cost of holding money, and a decrease in i indicates an increase in the gain of entry for a buyer. Thus, a decrease in i can make σ not so small and diversify the impact of the increase in k_s to an increase in e. Thus, social welfare should improve when i reduces in response to the higher k_s than when i is invariant. [8] The buyers' entry e does not necessarily increase with k_s, where e is an equilibrium value in the optimal steady state in which the central bank chooses $i = i^*(k)$ because general equilibrium effects exist. The following lemma summarizes the responses of the equilibrium values of variables in the optimal steady state to the changes in k_b and k_s.

Proposition 5 *We consider the optimal steady state in which the central bank chooses $i = i^*(k)$. The equilibrium values of e, σ, and π_s decrease, and z, q, and π_b increase in k_b in the optimal steady state. The equilibrium values of e, σ, z and π_b decrease, and q and π_s increase in k_s.*

Proof. See Appendix. ■

4 Conclusion

In this study, we construct a monetary search model in which all the agents make entry decision. One feature of our model is that goods produced in decentralized markets are used as production inputs in the centralized market. We demonstrate that the Friedman rule is suboptimal and explicitly derive the optimal nominal interest rate or optimal inflation rate for a certain set of functional forms to show that the optimal inflation rate is lower for a lower cost of buyers' entry or a higher cost of sellers' entry.

This result suggests an explanation for a secular decline in the inflation in the last decades. Interpreting the cost of entry for buyers as shopping costs, our result can be rephrased as the optimal inflation is lower in economies with lower shopping costs. Over the last several decades, the Internet has dramatically declined the shopping costs for the buyers. Our theory suggests the following possible explanations for low inflation: optimal

[8] Given that q is invariant, social welfare $S(e, \sigma, q)$, defined in (32), is a concave function of e and σ. Therefore, the optimal response to a change in constraint k^b or k^s should be a change in both e and σ. A change in either only e or only σ cannot be an optimal response.

inflation rates decrease as the shopping costs for buyers decrease, owing to widening the usage of information technology and the Internet.

Many points remain to analyze in this theory. One is that our theory suggests that a lower entry cost for sellers may increase the optimal inflation rate. We must explore the nature of the technological progress in recent decades to determine how these technological changes affect the buyers' and sellers' entry costs, and how they affect the optimal inflation. One more theoretical point has been not fully handled in this paper: the demand externality. The entry of buyers may affect the aggregate demand in a way that increases demand positively affects seller production. In that case, the demand externality may induce a higher nominal interest rate or a higher inflation is associated with higher output. These are examples of the agenda for future research that may deepen our understanding of the relationship between inflation and agents' entry.

Acknowledgement We thank Makoto Watanabe for his valuable comments.

Appendix

In Appendix, we provide proofs of the propositions and lemmas.

A Proofs

A.1 Proof of Proposition 1

We first prove (25)–(29) and then show the existence of the steady state under a specific government policy.

We first prove (25). Because $\mu = \phi/\phi_{+1}$ in the steady state, (7) implies

$$\frac{\mu}{\delta} - 1 = \pi_b \left(\frac{R}{p} g'(q) - 1 \right). \tag{37}$$

From (15) and (20), we have $p = wc'(q) = F_H(Q, H)c'(q)$. Substituting this equality and (21) into (37) yields (25).

Subsequently, we demonstrate that (26). Condition (11), together with (20) and (21), yields (26) for the steady state. The remaining equations (27), (28), and (29) are the same as expressed in (16), (17), and (18).

In the latter half of this Appendix, we show that the steady state can be realized, given that the government adopts the policy specified below. Let (q, e, z, Q, H, C) be expressed by (25)–(29) and (19). Suppose that all agents expect that money stock M_t and prices evolve as follows:

$$\begin{aligned} M_t &= e\mu^t \\ R &= F_Q(Q, H), \\ w &= F_H(Q, H), \\ p &= c'(q)w, \\ \phi_t &= \frac{eqc'(q)w}{M_t}, \\ P_t &= p/\phi_t. \end{aligned}$$

Suppose the government randomly chooses e buyers and provides each of them one unit of money during the initial period. At time 0, the value function of the buyer with money entering the DM solves

$$V_b^E(1) = -k_b + \pi_b \max_{P_0 q \leq 1} W_b(-P_0 q + 1, q) + (1 - \pi_b) W_b(1, 0).$$

where $W_b(m, q) = \frac{\phi}{w} m + \frac{R}{w} g(q) + K$. Thus, it is simplified as

$$V_b^E(1) = -k_b + \pi_b \max_{P_0 q \leq 1} \left(\frac{\phi}{w}(1 - P_0 q) + \frac{R}{w} g(q) \right) + (1 - \pi_b) \frac{\phi}{w} + W_b(0, 0).$$

Given that the nominal interest rate is positive, that is, $\mu > \delta$, we have $p < R g'(q)$ from (37). Thus, the constraint on money binds at time 0, implying that $q_0 = 1/P_0 = q$.

On the other hand, value function of the buyer with money who does not enter the DM is $W_b(1, 0)$. The difference between the two value functions is

$$V_b^E(1) - W_b(1, 0) = -k_b + \pi_b \left(\frac{R}{w} g(q) - \frac{\phi P_0}{w} q \right)$$

Equation (10), together with $i = \mu/\delta - 1$, implies that in the steady state

$$k_b = -\frac{pq}{w} \left(\frac{\mu}{\delta} - 1 \right) + \pi_b \left\{ \frac{R}{w} g(q) - \frac{p}{w} q \right\}. \tag{38}$$

From (38), we have

$$V_b^E(1) - W_b(1, 0) = \frac{p}{w} q \left(\frac{\mu}{\delta} - 1 \right) > 0.$$

Therefore, all buyers with money enter the DM. Thus, $e_0 = e$. Given above price expectations, the economy remains in the steady state where $e_t = e$ for all $t \geq 1$.

A.2 Proof of Proposition 2

Under Assumption 1, $\pi_b = Az^{1-\beta}$ and $\pi_s = Az^{-\beta}$ where $z = \sigma/e$. (25)-(29) can be re-expressed as follows:

$$\frac{\mu}{\delta} - 1 = Az^{1-\beta}\left(q^{\theta-\psi}\frac{H}{Q}\frac{\alpha}{1-\alpha}\frac{\theta}{\psi} - 1\right), \tag{39}$$

$$k_b = \frac{H}{e}\frac{QF_Q}{HF_H}\left(1 - \frac{qg'(q)}{g(q)}\right) = \frac{H}{e}\frac{\alpha}{1-\alpha}(1-\theta), \tag{40}$$

$$k_s = Az^{-\beta}(\psi - 1)q^{\psi}, \tag{41}$$

$$Q = Aez^{1-\beta}q^{\theta}, \tag{42}$$

$$F(Q,H)^{1-\rho} = \frac{H}{1-\alpha}. \tag{43}$$

Here

$$q^{\theta-\psi}\frac{H}{Q} = q^{\theta-\psi}\frac{1}{Az^{1-\beta}q^{\theta}}\frac{H}{e} = \frac{Az^{-\beta}(\psi-1)}{Ak^{s}}\frac{1}{z^{1-\beta}}\frac{(1-\alpha)k_b}{\alpha(1-\theta)} = z^{-1}(\psi-1)\frac{(1-\alpha)k}{\alpha(1-\theta)}.$$

Therefore

$$q^{\theta-\psi}\frac{H}{Q}\frac{\alpha}{1-\alpha}\frac{\theta}{\psi} = z^{-1}\frac{\psi-1}{\psi}\frac{\theta}{1-\theta}k. \tag{44}$$

Thus, we obtain (30) and (31). From (41), we have $q = A^{-1/\psi}Bz^{\beta/\psi}$ where $B = (\frac{k_s}{\psi-1})^{1/\psi}$. From (40), we have $H = \eta e$ where $\eta = \frac{1-\alpha}{\alpha}\frac{k_b}{1-\theta}$. From (43), we have $\{eF(Az^{1-\beta}q^{\theta}, \eta)\}^{1-\rho} = \frac{\eta e}{1-\alpha}$. Thus,

$$e = \left(\frac{1-\alpha}{\eta}\right)^{1/\rho}F(AB^{\theta}z^{1-\beta+\theta\beta/\psi}, \eta)^{\zeta} = \Gamma z^{\epsilon}, \tag{45}$$

where $\Gamma = (1-\alpha)^{1/\rho}(A^{1-\theta/\psi}B^{\theta})^{\alpha\zeta}\eta^{-\alpha\zeta-1}$ and $\epsilon = \alpha\zeta(1-\beta+\beta\frac{\theta}{\psi})$. Thus, we have $\sigma = ez = \Gamma z^{\epsilon+1}$. Clearly, we have $\partial i(z;k)/\partial z < 0$. It is straightforwardly shown that variables $e = \Gamma z^{\epsilon}$, $\sigma = \Gamma z^{\epsilon+1}$, and $q = A^{-\psi}Bz^{\beta/\psi}$ are lower for a higher i, given that the value of k is invariant.

A.3 Proof of Proposition 3

The social welfare is $S = -\sigma k_s - ek_b + U(F(Q,H)) - Nc(q) - H$. From (40), we have $H = k_b\frac{1-\alpha}{\alpha}\frac{1}{1-\theta}e$. From (43), we can express the third term of S, $U(F(Q,H))$ as

$$U(F(Q,H)) = \frac{H}{(1-\alpha)(1-\rho)} = \frac{1}{\alpha}\frac{1}{1-\rho}\frac{1}{1-\theta}k_be. \tag{46}$$

Finally, from (41), the fourth term of S, $Nc(q)$ can be written as

$$Nc(q) = \sigma\pi_s c(q) = \frac{\sigma}{\psi - 1} k_s. \tag{47}$$

Thus, the stationary welfare S can be written as

$$S = \frac{1}{1-\theta}\left(\frac{1}{\alpha\zeta} + \theta\right) k_b e - \frac{\psi}{\psi - 1} k_s \sigma.$$

As $e = \Gamma z^{\epsilon}$ and $\sigma = \Gamma z^{\epsilon+1}$, S can be rewritten as a function of z; that is, $s(z)$ defined by (33).

A.4 Proof of Lemma 1

The Friedman rule is suboptimal if and only if $\frac{\psi-1}{\psi}\frac{\theta}{1-\theta} > \nu_0 = (\frac{1}{\alpha\zeta} + \theta)\frac{1}{1-\theta}\frac{\psi-1}{\psi}\frac{\epsilon}{\epsilon+1} > 0$. The inequality can be simplified as

$$\left(\frac{1}{\alpha\zeta} + \theta\right)\frac{\epsilon}{\epsilon + 1} < \theta.$$

which can be further simplified as

$$\epsilon < \zeta\alpha\theta.$$

Because $\epsilon = \alpha\zeta\left(1 - \beta + \beta\frac{\theta}{\psi}\right)$, it is rewritten as (36).

A.5 Proof of Proposition 5

First, we provide the proof of responses to changes in k_b. It is obvious from (34), z increases in k_b as $k = k_b/k_s$. Since π_b increases in z, it increases in k_b in the optimal steady state where $z = z^*(k) \propto k_b$. Similarly, π_s decreases in k_b as it decreases in z. As B is independent of k_b, $q = Bz^{\beta/\psi}$ increases in k_b.

The proof of e decreasing in k_b is as follows. Proposition 2 implies $e = \Gamma z^{\epsilon}$, where $\Gamma = \Gamma_1(k_b)^{-\alpha\zeta-1}$, where Γ_1 is a constant that is independent of k_b. In the optimal steady state where $z \propto k_b$, e can be rewritten as

$$e = \Gamma_2(k_b)^{-\alpha\zeta-1+\epsilon},$$

where Γ_2 is a constant that is independent of k_b. Since $\epsilon = \alpha\zeta(1-\beta+\beta\theta/\psi)$, we have

$$-\alpha\zeta - 1 + \epsilon = -1 - \alpha\beta\zeta(1-\theta/\psi) < 0.$$

Therefore, e in the optimal steady state decreases in k_b. The seller's entry $\sigma = ez \propto (k_b)^{-\alpha\zeta+\epsilon}$ also decreases in k_b, because $-\alpha\zeta + \epsilon = -\alpha\beta\zeta(1-\theta/\psi) < 0$.

Second, we provide the proof of responses to changes in k_s. z decreases in k_s. Thus, π_b decreased and π_s increases with increasing k_s. As $B \propto (k_s)^{\frac{1}{\psi}}$ and $z \propto (k_s)^{-1}$, we have

$$q = B(z)^{\frac{\beta}{\psi}} \propto (k_s)^{\frac{1}{\psi}}(k_s)^{-\frac{\beta}{\psi}} = (k_s)^{\frac{1-\beta}{\psi}},$$

which implies that q increases as k_s. Since $\Gamma \propto B^{\alpha\theta\zeta} \propto (k_s)^{\alpha\theta\zeta/\psi}$, we obtain

$$e = \Gamma z^{\epsilon} \propto (k_s)^{\alpha\zeta\theta/\psi}(k_s)^{-\epsilon} = (k_s)^{-\alpha\zeta(1-\theta/\psi)(1-\beta)},$$

which implies that e decreases in k_s. Similarly,

$$\sigma = \Gamma z^{\epsilon+1} \propto (k_s)^{-\alpha\zeta(1-\theta/\psi)(1-\beta)-1},$$

which implies that σ decreases in k_s.

References

Aruoba, S. B., C. Waller and R.Wright (2011). Money and capital, Journal of Monetary Economics 58(2): 98–116.

Berentsen, A., G. Camera and C. Waller (2005). The distribution of money balances and the non-neutrality of money. International Economic Review 46: 465–487.

Berentsen, A., G. Camera and C. Waller (2007). Money, credit and banking. Journal of Economic Theory 135: 171–195.

Berentsen, A. and C. Waller (2015). Optimal stabilization policy with search externalities. Macroeconomic Dynamics 19(3): 669–700.

Çoban, M. N. (2022). The effect of the internet on inflation: A research on ASEAN-5 countries. Journal of ASEAN Studies 10(1): 61-79.

Friesenbichler, K. (2018). Inflation and broadband revisited: Evidence from an OECD Panel. A replication study of Yi and Choi (Journal of Policy Modeling, 2005). International Journal for Re-Views in Empirical Economics (IREE) 2 (2018-1):1-21.

Goolsbee, A. D., and P. J. Klenow (2018). Internet Rising, Prices Falling: Measuring Inflation in a World of E-Commerce. American Economic Review, Papers and Proceedings 108: 488-492.

Koester, G., E. Lis, C. Nickel, C. Osbat and F. Smets (2021). Understanding low inflation in the euro area from 2013 to 2019: cyclical and structural drivers. ECB Strategy Review Occasional Paper Series No. 280.

Lagos, R. and G. Rocheteau (2005). Inflation, output and welfare. International Economic Review 46: 495–522.

Lagos, R., and R. Wright (2005). A unified framework for monetary theory and policy analysis. Journal of Political Economy 113: 463-484.

Liu,L., L. Wang and R. Wright (2011). On the 'hot potato' effect of inflation. Macroeconomic Dynamics 15: 191-216.

Nosal, E. (2008). Search, welfare and the 'hot potato' effect of inflation. Federal Reserve Bank of Chicago, Policy Discussion Paper Series, PDP 2008-8.

Rocheteau, G., and R. Wright (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. Econometrica 73: 175–202.

Shimer, R. and S. Lones (2001). Matching, search, and heterogeneity. The B.E. Journal of Macroeconomics 1(1): 1-18.

Wasmer, E. and P. Weil (2004). The Macroeconomics of labor and credit market imperfections. American Economic Review 94 (4): 944-963.

Yi, M. H., and C. Choi (2005). The effect of the Internet on inflation: Panel data evidence. Journal of Policy Modeling 27 (7): 885-889.