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Masaru Inaba (Senshu University / The Canon Institute for Global Studies)
Kengo Nutahara (Senshu University / The Canon Institute for Global Studies)
Daichi Shirai (Tohoku Gakuin University / The Canon Institute for Global Studies)

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一般財団法人　キヤノングローバル戦略研究所
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Sources of Inequality and Business Cycles: Evidence from the US and Japan

Masaru Inaba[*]
Senshu University and CIGS

Kengo Nutahara[†]
Senshu University and CIGS

Daichi Shirai[‡]
Tohoku Gakuin University and CIGS

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[*] E-mail: imasaru@isc.senshu-u.ac.jp

[†] E-mail: nutti@isc.senshu-u.ac.jp

[‡] E-mail: shirai.daichi@mail.tohoku-gakuin.ac.jp

Abstract

The main objectives of this paper are twofold. The first is identifying the sources of inequality and business cycle fluctuations in the US and Japan. The second is investigating the effects of reducing inequality on business cycles. We develop a tractable heterogeneous-agent business cycle model with unconstrained (U) and hand-to-mouth (HtM) households. We also introduce wedges, which imply various types of distortions in economic activities, into the model following the business cycle accounting approach and estimate them by the Bayesian method. We focus on consumption inequality as inequality. We find that, in the US, the labor market distortions specific to the U households have significant impacts on both business cycles and consumption inequality, while the primary source of business cycles is the distortion in aggregate productivity, and that of consumption inequality is the labor market distortion specific to the HtM. In contrast, we find that no common factors significantly impact both business cycles and consumption inequality in Japan. In Japan, the key for business cycles is the distortion in aggregate productivity, and that for consumption inequality is the labor market distortions specific to U and HtM households. We also investigate the effects of reductions in consumption inequality on business cycle volatility through two types of experiments: (1) removing labor market distortions specific to two types of households, which are primary sources of consumption inequality, and (2) redistribution policy. Removing the labor market distortions increases output growth volatility in the US while it reduces in Japan. Removing cyclical consumption inequality by redistribution policy reduces output growth volatility in the US and increases in Japan. In contrast, reducing the level of consumption inequality from the estimated steady-state levels increases output volatility in both countries. However, the relation between the level of consumption inequality and output growth volatility is not monotonic. If consumption inequality is quite severe, a reduction in consumption inequality reduces output growth volatility.

Keywords: Inequality; consumption inequality; business cycle accounting; wedges; distortions; hand-to-mouth

JEL codes: E25; E32; E37

1 Introduction

Inequality and business cycles are two important aspects of modern macroeconomics, each with significant implications for social welfare. However, the relation between these two phenomena is complex. While inequality is often a consequence of macroeconomic dynamics, it can also affect the business cycle through various channels.

This paper seeks to accomplish two main research objectives. First, we aim to identify the driving sources of inequality and business cycles in the US and Japan. In particular, we investigate whether there are common factors that significantly impact both inequality and business cycles. Second, we explore the effects of reducing inequality on business cycles. Our focus is on whether reducing inequality increases or decreases business cycle volatility.

To reach these research objectives, we develop a tractable heterogenous-agent business cycle model with unconstrained (U) households and hand-to-mouth (HtM) households. While U households are the standard representative permanent income households, HtM households consume their entire disposable income in each period. Each household faces idiosyncratic risks of being HtM from U and those of being U from HtM, as employed by Bilbiie (2020, 2021), and Bilbiie, Primiceri, and Tambalotti (2022).

We also introduce, into the model, time-varying "wedges," which resemble aggregate productivity, distortionary taxes on investment and labor income, and government consumption, following the business cycle accounting (BCA) approach proposed by Chari, Kehoe, and McGrattan (2002, 2007). These wedges can be interpreted as distortions in the economy; the efficiency wedge is for distortion in aggregate productivity, the investment wedge is for distortion in investment activity, the labor wedge is for labor market distortions, and the government wedge is for distortion in the resource constraint. While our model is relatively simple and describes only the real side, introducing these wedges enables the model to cover various detailed models, including monetary ones. Because of the heterogeneity among households, we introduce three types of labor wedges: neutral to two types of households, U-specific and HtM-specific labor wedges.

In this paper, we focus on consumption inequality as inequality. This contrasts recent studies by Bilbiie et al. (2022) and Bayer, Born, and Luetticke (2023), who focus on income inequality and wealth distributions in the US to investigate the relation between inequality and business cycles. As Attanasio and Pistaferri (2016) and Meyer and Sullivan (2023) point out, consumption inequality is a good indicator of economic well-being because social welfare is measured by consumption and leisure. Then, consumption inequality is an important topic, as well as income inequality and wealth distribution. We construct the data on consumption inequality by using the Consumer Expenditure Surveys of the Bureau of Labor Statistics for the US and the Family Income and Expenditure Survey (*Kakei Chosa*) of the Ministry of Internal Affairs and Communications for Japan.

By the Bayesian estimations of the model, we find that, in the US, the U-specific labor wedge significantly impacts both business cycles and consumption inequality, while the primary source of business cycles is the efficiency wedge, and that of consumption inequality is the HtM-specific labor wedge. On the other hand, we find that no common factors significantly impact both business cycles and consumption inequality in Japan. In Japan, the key for business cycles is the distortion in aggregate productivity, and that for consumption inequality is the labor market distortions specific to U and HtM households. The most significant contributor to consumption inequality is the HtM in the US, while it is the U in Japan.

To investigate the relation between inequality and business cycles, we conduct two counterfactual simulations and analyze the effects of reducing consumption inequality on output growth volatility. The first is by removing the effects of the U-specific and HtM-specific labor wedges, which are the primary driving sources of consumption inequality in both countries. We find that it increases the output growth volatility in the US, whereas it reduces the output growth volatility in Japan. The second is by reducing consumption inequality through redistribution policies. We find that removing cyclical consumption inequality reduces the output growth volatility in the US, whereas it increases in Japan. We also find that reducing the level of consumption inequality from the estimated steady-state levels increases the output growth volatility in both countries. However, the relation between the level of consumption inequality and output growth volatility is not monotonic. If consumption inequality is quite severe, a reduction in consumption inequality reduces output growth volatility.

Related literature: Many researchers have studied the sources of business cycle fluctuations. In particular, King and Rebelo (1999) and Hayashi and Prescott (2002) emphasize the importance of aggregate productivity shock. Since the seminal work of Smets and Wouters (2003, 2007), estimations of medium-scale DSGE models using the Bayesian method have become popular in investigating the source of business cycles. Recent studies emphasize the importance of investment shocks as well as productivity shocks in business cycles, including Justiniano, Primiceri, and Tambalotti (2010); Justiniano, Primiceri and Tambalotti (2011), and Kaihatsu and Kurozumi (2014a) for the US, and Sugo and Ueda (2008), Hirose and Kurozumi (2012), Kaihatsu and Kurozumi (2014b), and Inaba, Nutahara, and Shirai (2022) for Japan.

The relation between inequality and business cycles by the estimation of a heterogeneous-agent New-Keynesian (HANK) model has been studied by Bayer et al. (2023) and Bilbiie et al. (2022). Bayer et al. (2023) emphasize that including data on inequality measures in observations is important for the estimation results. Bilbiie et al. (2022) is closely related to our paper since they also investigate the effects of reductions in consumption inequality on business cycles in their models. There are two main differences between their papers and ours. The first is on the models; they develop medium-scale HANK models with many frictions and shocks, while our model is relatively simple with time-varying wedges following the BCA approach. The second is that they use income inequality measures as an observable variable in estimations, while we use the data on consumption inequality.

The paper by Berger, Bocola, and Dovis (2019) is also related to ours. They show that a class of HANK models can be equivalently represented as a representative-agent NK model with a time-varying discount factor wedge, as in the BCA approach. By measuring their wedges by micro data and estimating their model, they show the importance of imperfect risk-sharing among households in business cycles.

While we employ a DSGE model to investigate the relation between business cycles and inequality, some researchers conduct time-series analyses by VAR models. De Giorgi and Gambetti (2017) and Geiger, Mayer, and Scharler (2020) investigate the effects of macro shocks on consumption inequality in the US. Inui, Sudo and Yamada (2017) investigate the impact of monetary policy shocks on inequality in Japan. Theophilopoulou (2022) studies the effects of uncertainty shocks on inequality in the UK.

We employ a kind of the BCA approach proposed by Chari et al. (2002, 2007). We extend the prototype model of BCA to a heterogenous-agent one and introduce additional wedges. BCA has been applied to different periods in various countries. The results obtained by BCA vary depending on the country and the sample period. The efficiency wedge is found as the most important driving force by Chari et al. (2002, 2007) for the US during the Great Depression, by Kobayashi and Inaba (2006) and Otsu (2011) for Japan during the 1990s, by Šustek (2011) for the US during 1959–2004, by Gerth and Otsu (2018) for the European Great Recession, by Chakraborty and Otsu (2013) for Brazil and Russia in the late 2000s, by Cho and Doblas-Madrid (2013) for the East Asian Crisis, and by Brinca, Chari, Kehoe, and McGrattan (2016) for many OECD countries during the Great Recession. The labor wedge is found as the primary driving source of business cycles by Brinca et al. (2016) for France, the UK, Belgium, and New Zealand in the 1980s and the US during the Great Recession. Chakraborty and Otsu (2013) and Brinca et al. (2016) find that the investment wedge plays a dominant role in China and India in the late 2000s and in Spain, Ireland, and Iceland during the Great Recession.

Introducing HtM households into the model has a long tradition in macroeconomics, as done by Campbell and Mankiw (1989), Galí, López-Salido and Vallés (2007), Bilbiie (2008), Kaplan and Violante (2014), and Bilbiie et al. (2022). On the empirical side, Kaplan, Violante, and Weidner (2014) estimate the share of HtM households in the total households and analyze demographic characteristics and portfolio composition for the US, Canada, Australia, the UK, Germany, France, Italy, and Spain. Hara, Unayama, and Weidner (2016) estimate them for Japan.

Finally, our paper is related to the literature on measuring consumption inequality. As pointed out by Attanasio and Pistaferri (2016) and Meyer and Sullivan (2023), consumption inequality provides a better indicator of economic well-being because social welfare is measured by consumption and leisure. In the US, the measurement of consumption inequality and the relation between consumption inequality and income inequality have been studied by many researchers, including Krueger and Perri (2006), Attanasio, Battistin, and Ichimura (2007), Blundell, Pistaferri, and Preston (2008), Heathcote, Perri, and Violante (2010), Attanasio and Pistaferri (2014), Attanasio, Hurst, and Pistaferri (2014), Aguiar and Bils (2015), and Meyer and Sullivan (2023). In Japan, there is also an accumulation of empirical analyses of consumption inequality, including papers by Ohtake and Saito (1998), Abe and Yamada (2009), Yamada (2012), and Inui et al. (2017). Lise, Sudo, Suzuki, Yamada, and Yamada (2014) and Higa (2019) show the widening of consumption inequality in recent years in Japan.

Structure of the rest of the paper: The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 explains the data for estimations and our estimation strategy. Section 4 explains the main estimation results. The driving sources of inequality and business cycles are shown. Section 5 examines the effects of reductions of consumption inequality on business cycle volatility through two types of counterfactual simulations. Finally, Section 6 presents the conclusion.

2 The Model

2.1 Households

There are two types of households: Unconstrained (U) and Hand-to-mouth (HtM). The total population is one, and the number of HtM households is θ.

The U households are the standard representative permanent income households. All U households pool their income and consume the same amount C_t^U. They supply labor L_t^U given the wage rate W_t. They also invest X_t^U, and their capital holding at the end of period t is K_t^U. The rental rate of capital is R_t.

Each U households face an idiosyncratic risk of being HtM at the next period at probability $1-s$, as employed by Bilbiie (2020, 2021), and Bilbiie et al. (2022). This risk engages precautionary saving of U households. Then, the U households maximize the following utility V_t^U

$$V_t^U = u(C_t^U, L_t^U) + \beta E_t \left[sV_{t+1}^U + (1-s)V_{t+1}^H \right], \tag{1}$$

where $u(C_t^U, L_t^U)$ is the instantaneous utility, and V_t^H is the utility of HtM.

The budget constraint of U households is

$$C_t^U + (1+\tau_t^X)X_t^U \leq sR_tK_{t-1}^U + (1-\tau_t^L-\tau_t^{LU})W_tL_t^U + T_t^U, \tag{2}$$

where τ_t^X, τ_t^L, and τ_t^{LU} denotes the investment wedge, the neutral labor wedge, and the U-specific labor wedge, respectively. T_t^U is the lump-sum tax to the U households.

As explained by Chari et al. (2007), these wedges, which resemble distortionary taxes on investment and labor supply activities, can be interpreted as reduced forms of distortions in the economy. We consider the neutral labor wedge, which affects both households and household-type specific ones since the model has two households. All members of the U household at period $t-1$ have the claim of the rental rate of capital at the period t. Then, the income from capital holding is multiplied by s in the budget constraint.

The evolution of the capital stock is given by

$$K_t^U = (1-\delta)K_{t-1}^U + X_t^U. \tag{3}$$

HtM households consume their entire disposable income. As in U households, all HtM households pool their income and consume the same amount C_t^H. They supply labor L_t^H given the wage rate W_t. Unlike U households, HtM households cannot invest. Then, the budget constraint of HtM households is

$$C_t^H = (1-s)\frac{1-\theta}{\theta}R_t K_{t-1}^U + (1-\tau_t^L-\tau_t^{LH})W_t L_t^H + T_t^H, \tag{4}$$

where τ_t^{LH} is the HtM-specific labor wedge, and T_t^H is the lump-sum tax to HtM households. The capital income comes from HtM households that were U households in the previous period.

The first-order necessary conditions of two households are given by

$$\begin{aligned}(1+\tau_t^X)\Lambda_t^U = \beta E_t \{ & s\Lambda_{t+1}^U \left[(1+\tau_{t+1}^X)(1-\delta) + sR_{t+1}\right] \\ & +(1-s)\Lambda_{t+1}^H(1-s)\frac{1-\theta}{\theta}R_{t+1}\},\end{aligned} \tag{5}$$

$$u_L(C_t^U, L_t^U) = \Lambda_t^U(1-\tau_t^L-\tau_t^{LU})W_t, \tag{6}$$

$$u_L(C_t^H, L_t^H) = \Lambda_t^H(1-\tau_t^L-\tau_t^{LH})W_t, \tag{7}$$

where Λ_t^U and $\Lambda_t^H$are the marginal utilities. The instantaneous utility function is specified as

$$u(C_t^i, L_t^i) = \log(C_t^i) + \xi\log(1-L_t^i),$$

for $i = U$ and H.

2.2 Firms

Firms are perfectly competitive. Firms produce output Y_t using labor input L_t and capital service K_t. The production function is given by

$$Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha}, \tag{8}$$

where $\alpha \in (0,1)$ is the cost share of capital, and Z_t is the efficiency wedge, which resembles aggregate productivity.

The marginal productivity conditions are given by

$$R_t = \alpha \frac{Y_t}{K_t}, \tag{9}$$

$$W_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{10}$$

2.3 Fiscal policy

The government spends G_t using lump-sum taxes of two households: T_t^U and T_t^H. Then, the government budget constraint is given by

$$G_t = (1-\theta)T_t^U + \theta T_t^H. \tag{11}$$

The government spending G_t is given by

$$G_t = Z_t g_t, \tag{12}$$

where g_t is the government wedge. In the model, G_t is the government spending, but to fit the data, we consider G_t also includes the net exports in the empirical part following Chari et al. (2007).

The lump-sum taxes are exogenous for households, and they are given by

$$T_t^U = \tau_t^X X_t^U + (\tau^L + \tau^{LU})W_t L_t^U - \frac{1}{1-\theta}G_t, \tag{13}$$

$$T_t^H = (\tau^L + \tau^{LH})W_t L_t^H. \tag{14}$$

In this setting, the government spending G_t is financed by the lump-sum tax to the U household. This setting works as a redistribution policy from the U (non-poor) to HtM (poor).

2.4 Aggregations and the market clearing conditions:

In our model, only U households invest and hold capital. Then, the aggregate investment X_t and the aggregate capital, which is equal to capital service K_t at equilibrium, are given by

$$X_t = (1-\theta)X_t^U, \tag{15}$$

$$K_t = (1-\theta)K_{t-1}^U. \tag{16}$$

The aggregate consumption C_t and the aggregate labor L_t are the weighted sum of those of U and HtM. Then, they are given by

$$C_t = (1-\theta)C_t^U + \theta C_t^H, \tag{17}$$

$$L_t = (1-\theta)L_t^U + \theta L_t^H. \tag{18}$$

Finally, the resource constraint is given by

$$Y_t = C_t + X_t + G_t. \tag{19}$$

2.5 Exogenous shocks and equilibrium conditions

There are six exogenous wedges in the model. They follow the AR(1) processes:[1]

$$z_t = (1-\rho_z)z + \rho_z z_{t-1} + \varepsilon_t^z, \tag{20}$$

$$\tau_t^X = (1-\rho_X)\tau^X + \rho_X \tau_{t-1}^X + \varepsilon_t^X, \tag{21}$$

$$\tau_t^L = (1-\rho_L)\tau^L + \rho_L \tau_{t-1}^L + \varepsilon_t^L, \tag{22}$$

$$\tau_t^{LU} = (1-\rho_{LU})\tau^{LU} + \rho_{LU} \tau_{t-1}^{LU} + \varepsilon_t^{LU}, \tag{23}$$

$$\tau_t^{LH} = (1-\rho_{LH})\tau^{LH} + \rho_{LH} \tau_{t-1}^{LH} + \varepsilon_t^{LH}, \tag{24}$$

$$g_t = (1-\rho_g)g + \rho_g g_{t-1} + \varepsilon_t^g, \tag{25}$$

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[1] In the business cycle accounting literature, it is often that the wedges follow the following VAR(1) process. In this paper, we assume AR(1) process for simplicity of analysis. An extension to VAR(1) specification of wedges is considered in Appendix E. In addition, Chari et al. (2007) define the efficiency wedge is on the level of aggregate productivity, while we define the efficiency wedge as the growth rate of aggregate productivity. This setting eliminates arbitrariness in taking the trend and increases the fit of the data.

where $z_t = Z_t/Z_{t-1}$, and z, τ^X, τ^L, τ^{LU}, τ^{LH}, and g are the steady-state values. For $j = z$, X, L, LU, LH, and g, ρ_j is the persistent parameter and ε_t^j is an i.i.d. structural shock to a wedge.

The model is detrended by the efficiency wedge Z_t. The detrended equilibrium conditions are log-linearized around a balanced growth path. Appendix A shows the log-linearized equilibrium system.

2.6 Interpretations of wedges

Our model is a tractable two-agent business cycle model with various exogenous wedges. However, our simple model can cover various detailed models by adjusting wedges.

The efficiency wedge resembles aggregate productivity in the production function. Then, the efficiency wedge would cover distortions from misallocation among the multi-sector model. In addition, Chari et al. (2007) show that equilibrium allocation generated by an input-financing friction model employed by Kiyotaki and Moore (1997) is covered by their prototype model with the efficiency wedge.

The labor wedges would cover the distortions of labor market behaviors and distortionary labor income taxations. As in Equations (6) and (7), the labor wedge captures the distortions on the consumption–leisure decision. Chari et al. (2007) show that their prototype model with the labor wedge covers a sticky-wage model à la Erceg, Henderson, and Levin (2000). In sticky-price models, the real marginal cost of firms fluctuates, generating the gap between the wage and the marginal product of labor. As shown by Šustek (2011), the labor wedge would capture this gap. If these distortions are common among two types of households, the neutral labor wedge works. If the distortions are specific to U or HtM, U-specific or HtM-specific labor wedge works. In other words, our model is real, but it can capture monetary models by adjusting the labor wedge. Wage markup shock is supposed as an important source in business cycles as shown by Smets and Wouters (2007). Since the wage markup generates the deviation of the real wage rate from the marginal product of labor, the labor wedge can capture its effect.

The investment wedge covers the distortions of investment activities in addition to distortionary investment taxation. As in Equation (5), the investment wedge captures the distortions on the intertemporal decision. Chari et al. (2007) show that their prototype model with the investment wedge can cover the models with financial frictions à la Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999). In sticky-price models, the real marginal cost of firms fluctuates, generating the gap between the rental rate of capital and the marginal product of capital. Then, the investment wedge would capture this gap, as shown by Šustek (2011). Bilbiie et al. (2022) emphasize the importance of risk to switch from the U to the HtM $(1 - s)$ as countercyclical in business cycles. This risk distorts intertemporal decisions, then the investment wedge would be able to capture it in our model.

Finally, as shown by Chari et al. (2007), the government wedge covers a model with an open economy model with a sudden stop in addition to wasteful government consumption. Then, while our model is a closed economy, our model can cover open economy models.

3 Data and Estimation Strategy

Data: We use five observable variables. Four of them are quarterly macro series: growth rates of real GDP per capita Y_t, real consumption per capita C_t, real investment per capita X_t, and labor supply L_t. The last one is inequality, which is annual. As a measure of inequality, we focus on consumption inequality between HtM and aggregate consumption: C_t^H/C_t. A smaller value of C_t^H/C_t means severe consumption inequality. If amounts of consumption are identical among households, then C_t^H/C_t is one.

According to the empirical facts, the fraction of HtM households is about 20%, and their income is lower than non-HtM. Then, as a proxy of C_t^H/C_t, we calculate the ratio of consumption of the lowest 20 % to that of all consumer units in the "Quintiles of income before taxes" of the Consumer Expenditure Surveys (CEX) of BLS for the US. For Japan, we use the Family Income and Expenditure Survey (*Kakei Chosa*) of the Ministry of Internal Affairs and Communications. This data is annual.

The sample period for the US is from 1984:Q1 to 2019:Q4, and that for Japan is from 1980:Q2 to 2019:Q4. The availability of GDP and consumption inequality data determines the sample period. Since our model is log-linearized, we stopped the sample period at 2019:Q4 to avoid the significant fluctuation caused by Covid-19. The data construction details are in Appendix B.

Table 1: Fixed Parameter Values

	Parameter	Value
β	Discount factor	0.99
α	Cost-share of capital	0.37
δ	Depreciation rate of capital	0.025
θ	Share of HtM HH	0.2
$1-s$	Transition rate from U to HtM	$1-0.987$
$\frac{G}{Y}$	Steady-state value of G_t/Y_t	0.1645 (US)
		0.2439 (JPN)

Estimation strategy: The observation equations for macro variables (Y_t, C_t, X_t ,and L_t) are

$$
\begin{aligned}
100\Delta \log Y_t &= 100(z-1) + (\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t), \\
100\Delta \log C_t &= 100(z-1) + (\hat{c}_t - \hat{c}_{t-1} + \hat{z}_t), \\
100\Delta \log X_t &= 100(z-1) + (\hat{x}^U_t - \hat{x}^U_{t-1} + \hat{z}_t), \\
100 \log L_t &= 100 \log L + \hat{L}_t,
\end{aligned}
$$

where z and L denote steady-state values, and $\hat{v}_t$ is the log-deviation from its steady-state value: $\hat{v}_t = \log(v_t) - \log(v)$ for $v = c$, z, y, x^U, and L. The data of the consumption inequality C^H_t/C_t is annual. We follow Pfeifer (2021) to model the observation equation of consumption inequality with this mixed frequency. It is given by

$$
100\log\left(\frac{C^H_t}{C_t}\right)_{\text{annual}} = 100\log\left(\frac{C^H}{C}\right) + \frac{1}{1+z+z^2+z^3}\left[(\hat{c}^H_{t-3} - \hat{c}_{t-3}) + z(\hat{c}^H_{t-2} - \hat{c}_{t-2}) + z^2(\hat{c}^H_{t-1} - \hat{c}_{t-1}) + z^3(\hat{c}^H_t - \hat{c}_t)\right],
$$

where C^H/C is the steady-state consumption inequality.[2]

Table 1 shows the fixed parameter values. The model is specified to be quarterly. The discount factor β is 0.99. The cost share of capital α is 0.37. The depreciation rate of capital δ is 0.025. These values are standard in the literature. The share of HtM households

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[2] Appendix C explains the derivation of this observation equation.

Table 2: Prior Distribution

	Parameter	Distribution	Mean	SD
τ^X	Steady-state investment wedge	Gamma	0.5	0.2
$100(z-1)$	Steady-state output growth	Gamma	0.4017 (US)	0.2
		Gamma	0.3112 (JPN)	0.2
$\frac{C^H}{C}$	Steady-state consumption inequality	Gamma	0.5788 (US)	0.05
		Gamma	0.7080 (JPN)	0.05
L	Steady-state total labor supply	Gamma	0.1973 (US)	0.05
		Gamma	0.2129 (JPN)	0.05
ρ_z	Persistence of efficiency wedge	Beta	0.5	0.2
ρ_x	Persistence of investment wedge	Beta	0.5	0.2
ρ_L	Persistence of neutral labor wedge	Beta	0.5	0.2
ρ_{LU}	Persistence of U-specific labor wedge	Beta	0.5	0.2
ρ_{LH}	Persistence of HtM-specific labor wedge	Beta	0.5	0.2
ρ_g	Persistence of government wedge	Beta	0.5	0.2
σ_z	SD of efficiency wedge shock	IG	0.5	Inf
σ_x	SD of investment wedge shock	IG	0.5	Inf
σ_L	SD of neutral labor wedge shock	IG	0.5	Inf
σ_{LU}	SD of U-specific labor wedge shock	IG	0.5	Inf
σ_{LH}	SD of HtM-specific labor wedge shock	IG	0.5	Inf
σ_g	SD of government wedge shock	IG	0.5	Inf

θ is 0.2. The transition rate from U to HtM $1-s$ is $1-0.987$. These two values are from the paper by Bilbiie et al. (2022). Finally, the steady-state share of government consumption in GDP G/Y is set as the data mean: 0.1645 for the US and 0.2439 for Japan.

The Bayesian estimation is employed by using Dynare. The prior distributions of the parameters are presented in Tables 2. For the steady-state investment wedge τ^X, growth rate $100(z-1)$, labor supply L, consumption inequality C^H/C, the prior distribution is the gamma. The means of the prior distribution except for τ^X are set at the sample mean. Since there is no prior information on τ^X, we set the mean of τ^X as 0.5 so that the shape of the posterior mode is close to the log-likelihood kernel in the mode check plots in Dynare. For the persistence parameters of structural shocks, the prior distribution is the beta distribution with a mean of 0.5 and a standard deviation of 0.2.

Following standard Bayesian likelihood approaches, we use the Kalman filter to evaluate the likelihood function of the log-linearized equilibrium system and the Metropolis–Hastings algorithm to generate draws from the posterior distribution of the deep parameters.

4 Empirical Results

4.1 Posterior Estimates

The posterior estimates are presented in Table 3. Many posterior estimates are close between the US and Japan. Judging by the credible intervals, τ^X is higher in the US, implying that the distortion in investment activity is higher in the US. In addition, C^H/C is higher in Japan than in the US, implying that consumption inequality is less severe in Japan than in the US. The persistence parameter of the efficiency wedge is higher in Japan, while the persistence of the HtM labor wedge is much lower in Japan. The standard deviations of efficiency, neutral labor, and government wedges are larger in Japan than in the US.

4.2 Driving sources of inequality and business cycles

Table 4 shows the variance decompositions of the growth rates of output $(100\Delta \log Y_t)$, consumption $(100\Delta \log C_t)$, and investment $(100\Delta \log X_t)$, labor supply $(100 \log L_t)$, and consumption inequality $(\hat{c}_t^H - \hat{c}_t)$. We focus on consumption inequality generated in the model $(\hat{c}_t^H - \hat{c}_t)$ rather than the observation data since the observation data is annual.

In the US, the efficiency wedge plays the dominant role in output growth, accounting for more than 70%. The U-specific labor wedge is the second, accounting for about 15%. For consumption growth, the efficiency wedge is still the most significant factor, and 50% is accounted for. The U-specific and HtM-specific labor wedges also have substantial roles in consumption growth; each wedge accounts for about 15%. In the case of investment

Table 3: Posterior Estimates

	US		Japan	
	Mean	90% credible interval	Mean	90% credible interval
τ^X	1.1202	[0.8730 , 1.3724]	0.5673	[0.3322 , 0.8041]
$100(z-1)$	0.3566	[0.2280 , 0.4808]	0.4342	[0.2243 , 0.6475]
$\frac{C^H}{C}$	0.5706	[0.5378 , 0.6072]	0.7122	[0.6790 , 0.7436]
L	0.2000	[0.1888 , 0.2094]	0.2208	[0.2041 , 0.2364]
ρ_z	0.0788	[0.0421 , 0.1158]	0.1922	[0.1454 , 0.2412]
ρ_x	0.9493	[0.9178 , 0.9810]	0.9727	[0.9551 , 0.9918]
ρ_L	0.8359	[0.6659 , 0.9890]	0.9916	[0.9847 , 0.9997]
ρ_{LU}	0.9854	[0.9741 , 0.9991]	0.9905	[0.9829 , 0.9990]
ρ_{LH}	0.8706	[0.8055 , 0.9369]	0.4500	[0.2646 , 0.6359]
ρ_g	0.9885	[0.9797 , 0.9979]	0.9773	[0.9636 , 0.9916]
σ_z	0.9835	[0.8841 , 1.0758]	1.6196	[1.4745 , 1.7714]
σ_x	0.3863	[0.3067 , 0.4696]	0.5537	[0.4586 , 0.6420]
σ_L	0.1907	[0.1046 , 0.2821]	0.6791	[0.5066 , 0.8369]
σ_{LU}	0.7115	[0.6186 , 0.8110]	0.5893	[0.3998 , 0.7649]
σ_{LH}	3.4108	[2.8603 , 3.9387]	3.5238	[2.9183 , 4.1320]
σ_g	1.5785	[1.4204 , 1.7267]	2.1127	[1.9144 , 2.3123]

growth, the investment wedge is the most significant contributor, and the efficiency wedge is the second. The total labor supply is mainly accounted for by the U-specific labor wedge (about 65%) and the government wedge (about 20%). Except for total labor supply, the role of the efficiency wedge is crucial in these variables. On the other hand, consumption inequality is explained by household-specific labor wedges: about 75% by the HtM-specific labor wedge and about 20% by the U-specific labor wedge. In contrast, the efficiency wedge has almost no contribution to consumption inequality.

In Japan, the efficiency wedge is the main contributor to output, consumption, and investment growth, as in the US. Unlike the US, the role of the U-specific labor wedge is

Table 4: Variance Decompositions

US

	z	τ^L	τ^{LH}	τ^{LU}	τ^x	g
$\Delta \log Y_t$	71.38	2.40	4.78	15.41	2.23	3.80
$\Delta \log C_t$	56.36	0.73	14.94	16.45	5.13	6.39
$\Delta \log X_t$	31.10	7.30	0.09	15.21	45.04	1.25
$\log L_t$	3.52	1.18	3.47	65.79	1.79	24.24
$\hat{c}_t^H - \hat{c}_t$	0.08	0.01	75.63	23.67	0.04	0.57

Japan

	z	τ^L	τ^{LH}	τ^{LU}	τ^x	g
$\Delta \log Y_t$	81.11	5.94	1.72	3.31	1.98	5.93
$\Delta \log C_t$	55.47	8.15	16.66	4.20	4.60	10.92
$\Delta \log X_t$	48.03	6.72	3.50	4.18	36.46	1.11
$\log L_t$	10.10	43.47	0.33	21.04	2.26	22.81
$\hat{c}_t^H - \hat{c}_t$	1.45	0.19	44.91	49.85	0.32	3.28

Note: Infinity-horizon forecast error variance decompositions are performed.

minor to them. The investment wedge significantly affects investment growth, as in the US. Unlike the US, the neutral labor wedge accounts for about 40% of the total labor supply. The U-specific labor wedge also significantly affects the total labor supply, accounting for about 20%. Consumption inequality is explained by household-specific labor wedges, as in the US. However, Japan's primary source of consumption inequality is the U-specific labor wedge, whereas it is the HtM-specific labor wedge in the US.

According to our results, household-specific labor wedges are essential for consumption inequality in the US and Japan. Bayer et al. (2023) and Bilbiie et al. (2022) find that markup shock is an important source of income inequality and wealth distribution dynamics. Since the markup shock distorts the factor prices, including the wage, our labor wedges might capture a similar effect. If so, our result implies that markup shock specific to each household is crucial, not neutral to two types of households.

We focus on the impulse response functions to interpret the mechanism behind the results of the variance decomposition. Figure 1 shows the estimated impulse responses to a one standard deviation shock to the wedges on output growth ($100\Delta \log Y_t$), consumptions of two households, and consumption inequality ($\hat{c}_t^H - \hat{c}_t$) in the US. For output growth, the response to the efficiency wedge shock is the largest among shocks. This result is consistent with our finding that the main driver for output growth is the efficiency wedge in the variance decomposition. The second largest response is to the U-specific labor wedge shock, which is also in line with the variance decomposition.

On the other hand, the consumption responses to the efficiency wedge shock are very close for the two types of households. Thus, only a slight consumption inequality is caused by the efficiency wedge shock. In the case of investment, neutral labor, and government wedges, the differences in consumption responses between the two types of households are also slight. The consumption responses differ significantly from the U-specific and HtM-specific labor wedge shocks; these significantly contribute to consumption inequality in the variance decomposition. Remarkably, the impact of the HtM-specific labor wedge is the most significant to consumption inequality. In the variance decomposition, the U-specific labor wedge accounts for about 15% of output growth and about 20% of consumption inequality. The impulse responses imply that the U-specific labor wedge shock decreases output growth and increases consumption inequality ($\hat{c}_t^H - \hat{c}_t$). So then, erasing the effects of the U-specific labor wedge would positively impact the business cycles while widening the consumption gap.

Figure 2 is the analogue of Figure 1 in Japan. As in the US, output growth responds most significantly to efficiency wedge shocks, and its magnitude is much larger than in the US case. Unlike the US, the magnitudes of the neutral labor wedge and the government wedge on output growth are almost the same as those of the U-specific labor wedge. On the other hand, in the US, the magnitude of the U-specific labor wedge on output growth is more significant than those of the neutral labor and the government wedges. This difference is consistent with the variance decomposition, which shows that the contributions of the neutral labor wedge and the government wedge to output growth are more significant than that of the U-specific labor wedge in Japan.

In Japan, the consumption response to the HtM-specific labor wedge is less persistent

Figure 1: Impulse Responses to a One Standard Deviation Shock to Wedges (1): US

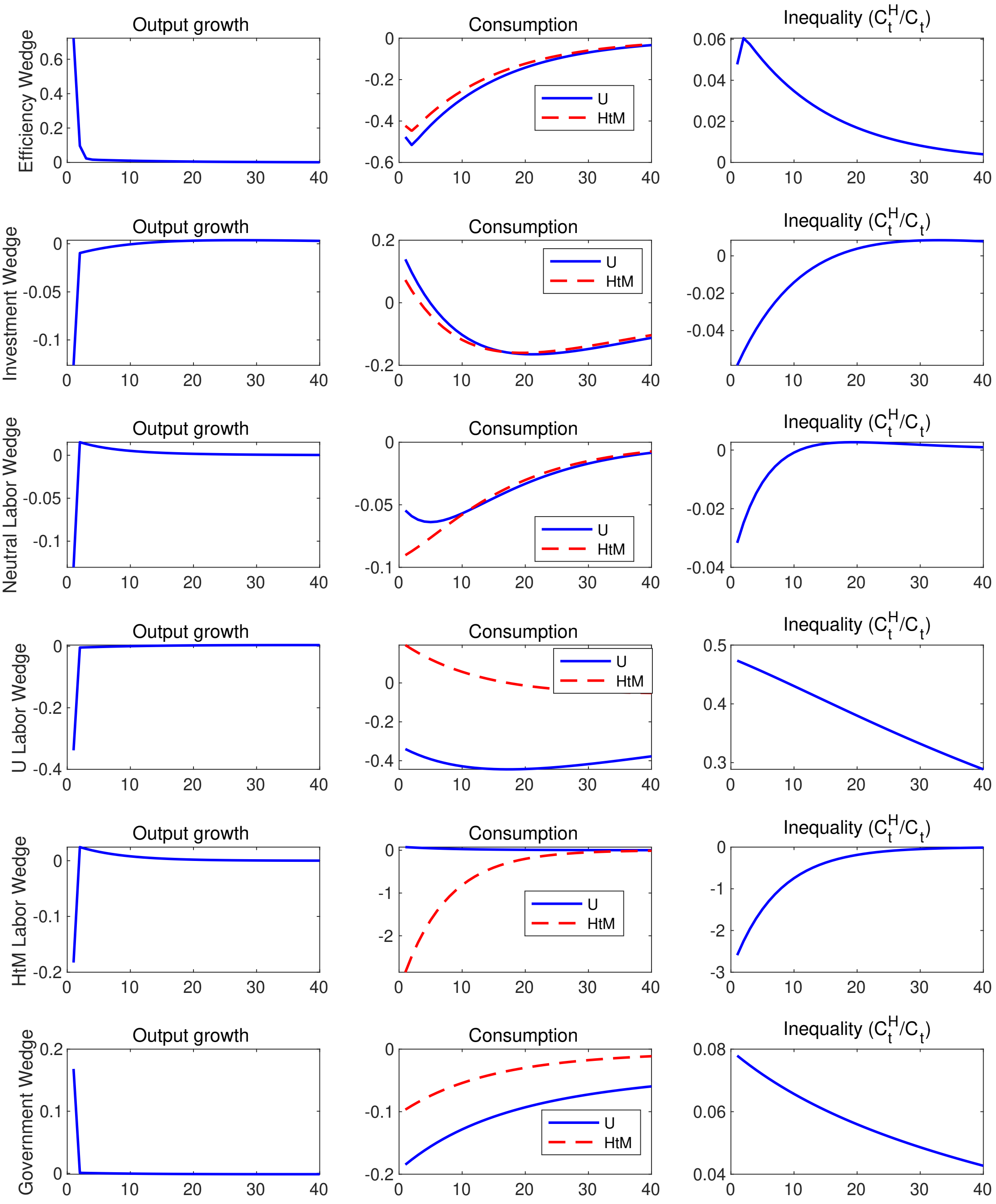
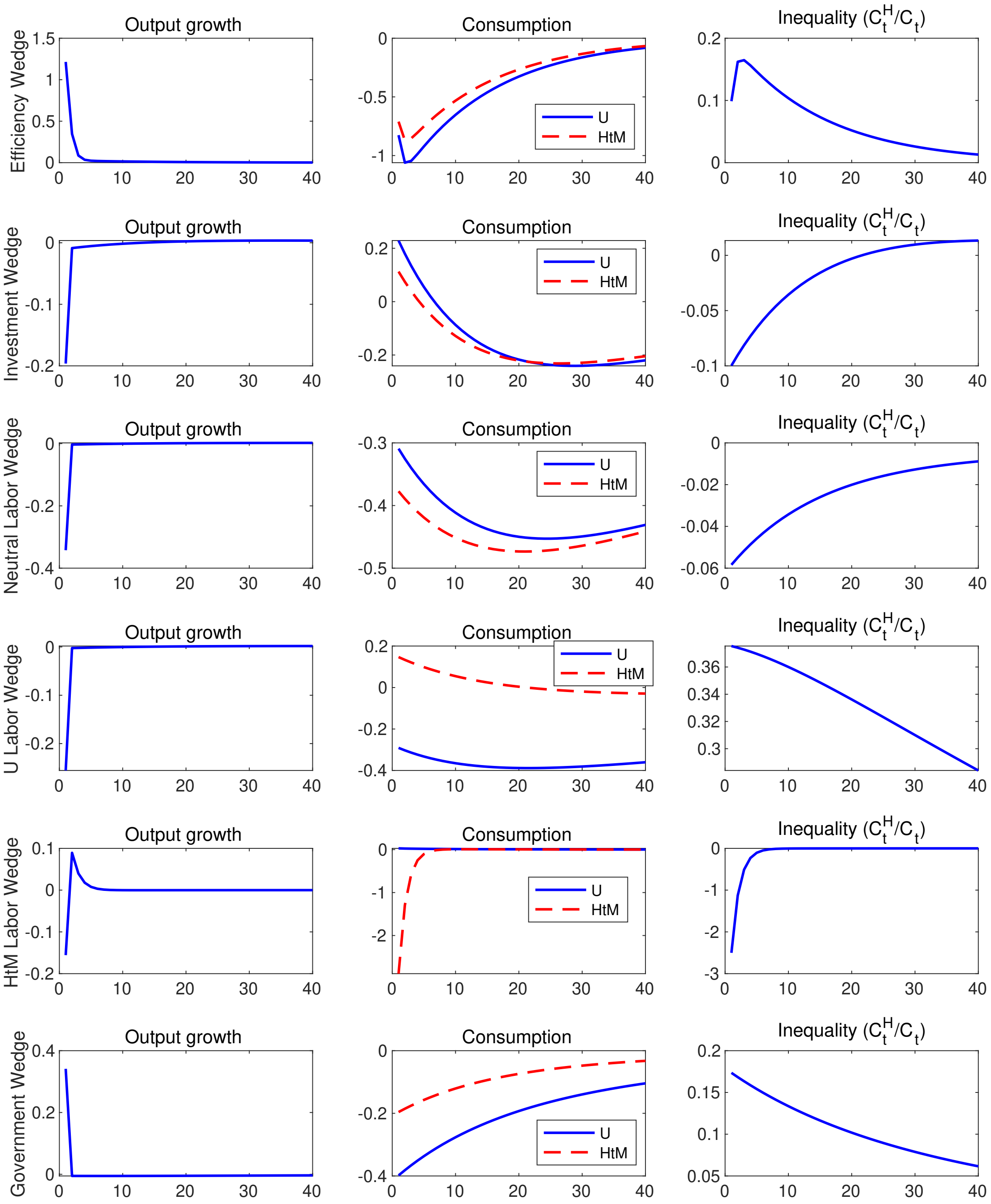


Figure 2: Impulse Responses to a One Standard Deviation Shock to Wedges (2): Japan



than in the US. Then, the gap in consumption between the two types of households disappears quickly in Japan. In the US, on the other hand, the HtM-specific labor wedge has a long-lasting impact on consumption inequality. This difference in the shapes of these impulse responses can be interpreted as creating the fact that the U-specific labor wedge explains more consumption inequality in Japan, whereas the HtM-specific labor wedge is the major contributor to consumption inequality in the US.

What is the central message of these results? In the US, the labor market distortions specific to the U households significantly impact business cycles and consumption inequality, while the primary source of business cycles is the distortion in aggregate productivity (i.e., the efficiency wedge), and that of consumption inequality is the labor market distortion specific to the HtM household, the poor. Then, the labor market distortions specific to the U households are essential to understand the relation between business cycles and consumption inequality in the US. On the other hand, we find that no common factors significantly impact both business cycles and consumption inequality in Japan. In business cycles, the distortion in aggregate productivity is dominant, and the labor market distortions are less critical. The key to understanding consumption inequality is the U households: non-poor households. In both countries, the effects of labor market distortions common to the two household types (i.e., the neutral labor wedge) are insignificant except for aggregate labor supply. In particular, the effects on output growth and consumption inequality are minor.

We report the variance decomposition of the aggregate labor wedge if we consider a representative agent economy in Appendix D.

5 Does a Reduction in Inequality Increase or Reduce the Volatility of Business Cycles?

This section analyzes how reducing consumption inequality would affect the business cycle based on the estimates in Section 4. Here, we consider the following two types of counterfactual simulations. The first is by eliminating the effects of the U-specific and HtM-specific labor wedges, which are the main drivers of consumption inequality in the variance decomposition. Since each labor wedge implies distortions in the labor market,

Table 5: Output Growth Volatilities by Reducing Consumption Inequality (1): Removing the effects of the U-specific and the HtM-specific Labor Wedges

	Data	(1)	(2)	(3)
US	0.583	0.643	0.720	0.783
JP	1.023	1.017	1.003	1.006

Note: Standard deviations of $100\Delta \log Y_t$ are reported.

(1) τ_t^{LH} is constant at the estimated steady-state level.

(2) τ_t^{LU} is constant at the estimated steady-state level.

(3) τ_t^{LH} and τ_t^{LU} are constant at their estimated steady-state levels.

we can analyze what happens to the business cycle if the government removes those distortions and reduces consumption inequality. The second is by reducing consumption inequality through redistribution policies by the government. In this approach, we examine the effects of government policies while the market distortions represented by the wedges are left. Bilbiie et al. (2022) also employ a similar method to our second strategy.

5.1 Eliminating the effects of the U-specific and the HtM-specific labor wedges

In this experiment, we perform a counterfactual experiment in which we fix the values of either the U-specific or the HtM-specific labor wedges or both at their steady-state values while the other wedges are the same as the estimated ones.

Table 5 shows the standard deviations of output growth. Case (1) is where the HtM-specific labor wedge τ_t^{LH} is constant at the estimated steady-state level. Case (2) is where the U-specific labor wedge τ_t^{LU} is constant at the estimated steady-state level. Case (3) is where the HtM-specific and U-specific labor wedges are constant at their estimated steady-state levels.

In the US, eliminating the effects of the U-specific and the HtM-specific labor wedges increases output growth volatility. In contrast, it reduces the volatility of output growth in

Japan. This difference comes from the correlations between wedges and output growth. In the US, the U-specific and the HtM labor wedges are positively correlated with output growth: $corr(\tau_t^{LH}, \Delta \log Y_t) = 0.25$ and $corr(\tau_t^{LU}, \Delta \log Y_t) = 0.12$. In contrast, in Japan, they are negatively correlated with output growth: $corr(\tau_t^{LH}, \Delta \log Y_t) = -0.12$ and $corr(\tau_t^{LU}, \Delta \log Y_t) = -0.26$. In other words, the labor market distortions specific to these households are severe in a boom in the US, while they are severe in a recession in Japan. These wedges have adverse effects on output growth. Thus, removing the effects of these wedges increases the volatility of the output growth in the US, whereas it reduces in Japan.

The magnitudes of removing the effects of these wedges are quantitatively small in Japan. This would be because the contributions of τ_t^{LH} and τ_t^{LU} are small in the variance decomposition. On the other hand, in the US, the U-specific labor wedge is the second contributor to output growth and removing wedges significantly impacts output growth volatility.

5.2 Redistribution policy

In the second experiment, we change the lump-sum taxes to U and HtM, Equations (13) and (14), to achieve constant C_t^H/C_t. To implement counterfactual simulations, we replace the HtM budget constraint in the equilibrium system with the equation where C_t^H/C_t is constant.

Table 6 shows the standard deviations of output growth. Case (1) is where C_t^H/C_t is constant at the estimated steady-state level. Case (2) is where C_t^H/C_t is constant at its maximum steady-state level. In the US, $C_t^H/C_t = 0.744$, and in Japan, $C_t^H/C_t = 0.884$. Our numerical simulations imply that, in our model, $C_t^H/C_t = 1$ cannot be achieved by the redistribution policy. This result would be because the lump-sum transfer to HtM becomes enormous. The income effect from this transfer makes HtM reduce its labor supply to zero, and the assumption of interim solutions does not hold. Case (3) is of the model without HtM: the case of the number of HtM θ is zero, and the risk of being of $1 - s$ is zero. While Case (3) is not achieved by redistribution policy, we investigate it as the case of perfect consumption equality.

Case (1) shows that in the US, eliminating fluctuations of consumption inequality

Table 6: Output Growth Volatilities by Reducing Consumption Inequality (2): Redistribution Policy

	Data	(1)	(2)	(3)
US	0.583	0.547	0.589	0.641
JP	1.023	1.153	1.231	1.002

Note: Standard deviations of $100\Delta \log Y_t$ are reported.

(1) C_t^H/C_t is constant at the estimated steady-state level.

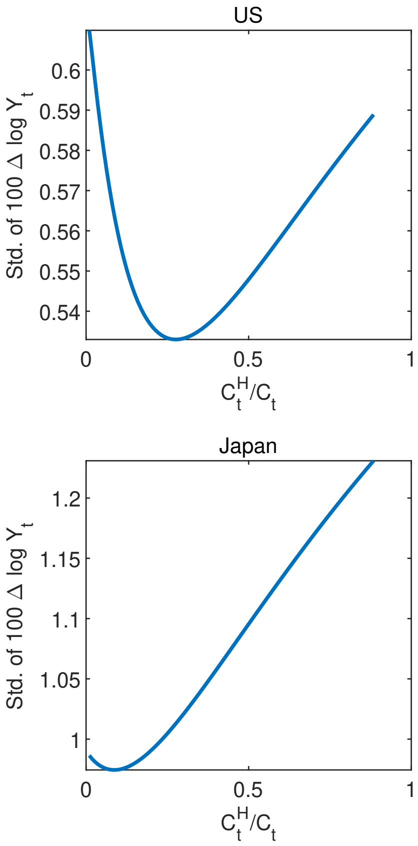
(2) C_t^H/C_t is constant at its maximum level. (US: 0.744, Japan: 0.884)

(3) Model without HtM ($\theta = 0$, $s = 1$): Perfect consumption equality

reduces output growth volatility, whereas it increases output growth volatility in Japan. The US case is consistent with the result of Bilbiie et al. (2022), who also find that eliminating fluctuations of consumption inequality reduces output growth volatility. Our results show that it is different in Japan. Case (2) shows that reducing the steady-state consumption inequality (increasing in C_t^H/C_t) from the estimated level increases output growth volatilities in both countries. This result is in contrast to that of Bilbiie et al. (2022). In their model, reducing steady-state consumption inequality decreases output growth volatilities. Case (3) shows that output growth volatility becomes more volatile if HtM does not exist in the US, whereas it becomes less volatile in Japan. The intuitions of this difference would be similar to those of our first experiment by removing the effects of τ_t^{LH}. The effects of removing the HtM households entail those of removing the HtM-specific labor wedge. In Table 5, removing the effects of τ_t^{LH} increases output growth volatility in the US, whereas it reduces output growth volatility in Japan. As explained in the previous subsection, it comes from the correlation of τ_t^{LH} and output growth.

Comparing Cases (1) and (2) in Table 6, an increase in C_t^H/C_t from the estimated steady-state level increases the volatility of output growth both in the US and Japan. Is this a general property? Figure 3 shows the volatility of output growth under various values of C_t^H/C_t. According to this figure, the relation between C_t^H/C_t and output growth volatility is not monotonic. If C_t^H/C_t is quite low and consumption inequality is severe, increasing C_t^H/C_t reduces output growth volatility. However, if consumption inequality is mild to

Figure 3: Output Growth Volatilities and C_t^H/C_t under Redistribution Policy



Note: In the US, the minimum standard deviation of output growth is 0.533 at $C_t^H/C_t = 0.275$. In Japan, the minimum is 0.974 at $C_t^H/C_t = 0.09$.

some extent, increasing C_t^H/C_t increases the volatility of output growth. In the US, the minimum standard deviation of output growth is 0.533 at $C_t^H/C_t = 0.275$. In Japan, the minimum is 0.974 at $C_t^H/C_t = 0.09$.

In Appendix E, we consider the case where the wedges follow the VAR(1) process. Even if the wedges are VAR(1) and the shocks are correlated, the main implication of this section doesn't change.

6 Concluding Remarks

In this paper, we have investigated the sources of inequality and business cycles in the US and Japan, using a tractable heterogeneous-agent business cycle model with U and HtM households. Following the BCA approach, we introduced six “wedges,” interpreted as distortions in the economy: efficiency, neutral labor, U-specific labor, HtM-specific labor, investment, and government wedges. While our model is simple, including these wedges enables the model to cover various detailed models. We have highlighted consumption inequality as inequality.

We have found that, in the US, the U-specific labor wedge significantly impacts both business cycles and consumption inequality, while the primary source of business cycles is the efficiency wedge, and that of consumption inequality is the HtM-specific labor wedge. On the other hand, we have found that no common factors significantly impact both business cycles and consumption inequality in Japan. In Japan, the key for business cycles is the distortion in aggregate productivity, and that for consumption inequality is the labor market distortions specific to U and HtM households. The most significant contributor to consumption inequality is the HtM (poor) in the US, whereas it is the U (non-poor) in Japan.

We have also investigated the relation between inequality and business cycles. Especially we have conducted two types of counterfactual simulations to analyze the effects of the reduction in consumption inequality on output growth volatility. The first is by removing the effects of the U-specific and HtM-specific labor wedges, which are the primary driving sources of consumption inequality. We have found that it increases the output growth volatility in the US, whereas it reduces the output growth volatility in Japan. The second is by reducing consumption inequality through redistribution policies. We have found that removing cyclical consumption inequality reduces the output growth volatility in the US, whereas it increases in Japan. We also have found that reducing the level of consumption inequality from the estimated steady-state levels increases the output growth volatility in both countries. However, the relation between the level of consumption inequality and output growth volatility is not monotonic. If consumption inequality is quite severe, a reduction in consumption inequality reduces output growth volatility.

Our model is tractable and simple, and there are remaining future tasks. Nonetheless, we have found a complex relation between inequality and the business cycle, and our work has a certain contribution to this field.

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A Equilibrium System

The equilibrium system of the model is given by

$$
\begin{aligned}
&\Lambda_t^U = \frac{1}{C_t^U},\\
&(1+\tau_t^X)\Lambda_t^U = \beta E_t \Big\{ s\Lambda_{t+1}^U \left[(1+\tau_{t+1}^X)(1-\delta) + sR_{t+1}\right]\\
&\qquad\qquad + \Lambda_{t+1}^H (1-s)^2 \frac{1-\theta}{\theta} R_{t+1} \Big\},\\
&\frac{1}{1-L_t^U} = \Lambda_t^U (1-\tau_t^L-\tau_t^{LU}) W_t,\\
&\Lambda_t^H = \frac{1}{C_t^H},\\
&\frac{1}{1-L_t^H} = \Lambda_t^H (1-\tau_t^L-\tau_t^{LH}) W_t,\\
&C_t^H = W_t L_t^H + (1-s)\frac{1-\theta}{\theta} R_t K_{t-1}^U,\\
&K_t^U = (1-\delta)K_{t-1}^U + X_t^U,\\
&Y_t = \left[K_t\right]^{\alpha} \left(Z_t L_t\right)^{1-\alpha},\\
&R_t = \alpha \frac{Y_t}{K_t},\\
&W_t = (1-\alpha)\frac{Y_t}{L_t},\\
&Y_t = C_t + X_t + G_t,\\
&K_t = (1-\theta)K_{t-1}^U,\\
&X_t = (1-\theta)X_t^U,\\
&C_t = (1-\theta)C_t^U + \theta C_t^H,\\
&L_t = (1-\theta)L_t^U + \theta L_t^H.
\end{aligned}
$$

Let the detrended variables as

$$
\begin{aligned}
&c_t = \frac{C_t}{Z_t}, \qquad c_t^U = \frac{C_t^U}{Z_t}, \qquad c_t^H = \frac{C_t^H}{Z_t}, \qquad \lambda_t^H = \Lambda_t^H Z_t, \qquad \lambda_t^H = \Lambda_t^H Z_t,\\
&x_t = \frac{X_t}{Z_t}, \qquad x_t^U = \frac{X_t^U}{Z_t}, \qquad k_t = \frac{K_t}{Z_t}, \qquad k_t^U = \frac{K_t^U}{Z_t}, \qquad y_t = \frac{Y_t}{Z_t}, \qquad w_t = \frac{W_t}{Z_t},
\end{aligned}
$$

and $z_t = Z_t/Z_{t-1}$.

Then, the detrended equilibrium system is given by

$$
\begin{aligned}
&\lambda_t^U = \frac{1}{c_t^U}, \\
&(1+\tau_t^X)\lambda_t^U = \beta E_t \{ s\frac{\lambda_{t+1}^U}{z_{t+1}} \left[(1+\tau_{t+1}^X)(1-\delta) + sR_{t+1} \right] \\
&\qquad\qquad + \frac{\lambda_{t+1}^H}{z_{t+1}}(1-s)^2 \frac{1-\theta}{\theta} R_{t+1} \}, \\
&\xi \frac{1}{1-L_t^U} = \lambda_t^U (1-\tau_t^L - \tau_t^{LU}) w_t, \\
&\xi \frac{1}{1-L_t^H} = \lambda_t^H (1-\tau_t^L - \tau_t^{LH}) w_t, \\
&c_t^H = w_t L_t^H + (1-s)\frac{1-\theta}{\theta} R_t \frac{k_{t-1}^U}{z_t}, \\
&\lambda_t^H = \frac{1}{c_t^H}, \\
&k_t^U = (1-\delta)\frac{k_{t-1}^U}{z_t} + x_t^U, \\
&y_t = (k_t)^\alpha L_t^{1-\alpha}, \\
&R_t = \alpha \frac{y_t}{k_t}, \\
&w_t = (1-\alpha)\frac{y_t}{L_t}, \\
&y_t = c_t + x_t + g_t, \\
&k_t = (1-\theta)\frac{k_{t-1}^U}{z_t}, \\
&x_t = (1-\theta) x_t^U, \\
&c_t = (1-\theta) c_t^U + \theta c_t^H, \\
&L_t = (1-\theta) L_t^U + \theta L_t^H.
\end{aligned}
$$

At a balanced growth path, the equilibrium system becomes

$$
\begin{aligned}
&\lambda^U = \frac{1}{c^U}, \\
&(1+\tau^X)\lambda^U = \beta \{ s\frac{\lambda^U}{z} \left[(1+\tau^X)(1-\delta) + sR \right] + \frac{\lambda^H}{z}(1-s)^2 \frac{1-\theta}{\theta} R \}, \\
&\xi \frac{1}{1-L^U} = \lambda^U (1-\tau^L - \tau^{LU}) w, \\
&\xi \frac{1}{1-L^H} = \lambda^H (1-\tau^L - \tau^{LH}) w,
\end{aligned}
$$

$$
\begin{aligned}
&c^H = wL^H + (1-s)\frac{1-\theta}{\theta}R\frac{k^U}{z},\\
&\lambda^H = \frac{1}{c^H},\\
&\left[1-\frac{1-\delta}{z}\right]k^U = x^U,\\
&y = k^\alpha L^{1-\alpha},\\
&R = \alpha\frac{y}{k},\\
&w = (1-\alpha)\frac{y}{L},\\
&y = c + x + g,\\
&k = (1-\theta)\frac{k^U}{z},\\
&x = (1-\theta)x^U,\\
&c = (1-\theta)c^U + \theta c^H,\\
&L = (1-\theta)L^U + \theta L^H.
\end{aligned}
$$

If c^H/c and L are known, this balanced growth path can be computed as follows. First, if c^H/c is known, then

$$
\begin{aligned}
\frac{\lambda^H}{\lambda^U} &= \frac{c^U}{c^H}\\
&= \frac{\left(\frac{c^H}{c}\right)^{-1}-\theta}{1-\theta}.
\end{aligned}
$$

R can be calculated by the Euler equation as

$$
R = \frac{(1+\tau^X) - \frac{\beta s}{z}(1+\tau^X)(1-\delta)}{\frac{\beta s^2}{z} + \frac{\beta(1-s)^2}{z}\left(\frac{\lambda^H}{\lambda^U}\right)\left(\frac{1-\theta}{\theta}\right)}.
$$

Given the value of R, the following values are calculated as

$$
\begin{aligned}
&\frac{k}{y} = \frac{\alpha}{R},\\
&\frac{x}{y} = \left[1-\frac{1-\delta}{z}\right]\left(\frac{k}{y}\right)z,\\
&\frac{c}{y} = 1-\frac{x}{y}-\frac{g}{y},\\
&\frac{y}{L} = \left(\frac{k}{y}\right)^{\frac{\alpha}{1-\alpha}},\\
&w = (1-\alpha)\left(\frac{y}{L}\right).
\end{aligned}
$$

If L is known, then

$$\begin{aligned} y &= \left(\frac{y}{L}\right) L, \\ c &= \left(\frac{c}{y}\right) y, \\ k &= \left(\frac{k}{y}\right) y, \\ k^U &= \frac{kz}{1-\theta}. \end{aligned}$$

Finally, since c^H/c is given, then

$$\begin{aligned} c^H &= \left(\frac{c^H}{c}\right) c, \\ L^H &= \frac{c^H - (1-s)\frac{1-\theta}{\theta} R \frac{k^U}{z}}{w}, \\ L^U &= \frac{L - \theta L^H}{1-\theta}, \\ c^U &= \frac{c - \theta c^H}{1-\theta}. \end{aligned}$$

The ratios of the labor disutility parameter ξ and labor wedge are given by

$$\begin{aligned} \frac{\xi}{(1-\tau^L-\tau^{LU})} &= \lambda^U(1-L^U)w, \\ \frac{\xi}{(1-\tau^L-\tau^{LH})} &= \lambda^H(1-L^H)w. \end{aligned}$$

Let $\hat{v}_t$ denote the log-deviation from its steady-state: $\hat{v}_t = \log(v_t) - \log(v)$. In the case of investment and labor wedges, $\tilde{\tau}_t^J$ is defined as the difference from its steady-state: $\tilde{\tau}_t^J = \tau_t^J - \tau^J$ for $J = X$, L, LU, and LH.

Then, the log-linearized equilibrium system is given by

$$\begin{aligned} &\hat{\lambda}_t^U = -\hat{c}_t^U, \\ &\tilde{\tau}_t^X + \hat{\lambda}_t^U, = \frac{\beta s(1-\delta)}{z}\left(E_t\hat{\lambda}_{t+1}^U - E_t\hat{z}_{t+1} + E_t\tilde{\tau}_{t+1}^X\right) \\ &+ \frac{\beta s^2 R}{z(1+\tau^X)}\left(E_t\hat{\lambda}_{t+1}^U + E_t\hat{R}_{t+1} - E_t\hat{z}_{t+1}\right) \\ &+ \left(1 - \frac{\beta s(1-\delta)}{z} - \frac{\beta s^2 R}{z(1+\tau^X)}\right)\left(E_t\hat{\lambda}_{t+1}^H - E_t\hat{z}_{t+1} + E_t\hat{R}_{t+1}\right), \\ &\frac{L^U}{1-L^U}\hat{L}_t^U = \widehat{\lambda}_t^U - \tilde{\tau}_t^L - \tilde{\tau}_t^{LU} + \hat{w}_t, \\ &\frac{L^H}{1-L^H}\hat{L}_t^H = \widehat{\lambda}_t^H - \tilde{\tau}_t^L - \tilde{\tau}_t^{LH} + \hat{w}_t, \end{aligned}$$

$$
\begin{aligned}
\hat{c}_t^H &= \frac{wL^H}{c^H}(\hat{w}_t + \hat{L}_t^H) + \left(1 - \frac{wL^H}{c^H}\right)\left[\hat{R}_t + \hat{k}_{t-1}^U - \hat{z}_t\right], \\
\hat{\lambda}_t^H &= -\hat{c}_t^H, \\
\hat{k}_t^U &= \frac{1-\delta}{z}(\hat{k}_{t-1}^U - \hat{z}_t) + \left[1 - \frac{1-\delta}{z}\right]\hat{x}_t^U, \\
\hat{y}_t &= \alpha(\hat{k}_{t-1}^U - \hat{z}_t) + (1-\alpha)\hat{L}_t, \\
\hat{R}_t &= \hat{y}_t - \hat{k}_{t-1}^U + \hat{z}_t, \\
\hat{w}_t &= \hat{y}_t - \hat{L}_t, \\
\hat{y}_t &= \frac{c}{y}\hat{c}_t + \frac{(1-\theta)x^U}{y}\hat{x}_t^U + \frac{g}{y}\hat{g}_t, \\
\hat{c}_t &= \left(1 - \frac{\theta c^H}{c}\right)\hat{c}_t^U + \frac{\theta c^H}{c}\hat{c}_t^H, \\
\hat{L}_t &= \left(1 - \frac{\theta L^H}{L}\right)\hat{L}_t^U + \frac{\theta L^H}{L}\hat{L}_t^H.
\end{aligned}
$$

The evolution of wedges is given by

$$
\begin{aligned}
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \varepsilon_t^z, \\
\tilde{\tau}_t^X &= \rho_X \tilde{\tau}_{t-1}^X + \varepsilon_t^X, \\
\tilde{\tau}_t^L &= \rho_L \tilde{\tau}_{t-1}^L + \varepsilon_t^L, \\
\tilde{\tau}_t^{LU} &= \rho_{LU} \tilde{\tau}_{t-1}^{LU} + \varepsilon_t^{LU}, \\
\tilde{\tau}_t^{LH} &= \rho_{LH} \upsilon_{t-1}^{LH} + \varepsilon_t^{LH}, \\
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \varepsilon_t^g.
\end{aligned}
$$

B Data Appendix

The data sources are summarized in Table A1.

B.1 US Data

GDP, consumption, investment, and government spending are all obtained in nominal values and deflated using the GDP deflator. These data are seasonally adjusted quarterly series. All per capita variables are obtained by dividing by labor force population. We use Personal Consumption Expenditures as consumption. Investment is a series of Gross

Table A1: Data Sources

Variables	Sources (FRED codes)
Quarterly series	US: NIPA tables
	Japan: ESRI, Quarterly Estimates of the GDP
Labor force population	US: CNP16OV
	Japan: MIC, Labour Force Survey
Average hours worked per worker	US: PRS85006023
	Japan: Ministry of Health, Labour and Welfare, Monthly Labor Survey
The employed persons	US: CE16OV
	Japan: MIC, Labour Force Survey
Consumption inequality	US: Consumer Expenditure Survey (See below)
	Japan: MIC, Family Income and Expenditure Survey

Variables for consumption inequality for the US	Sources (FRED codes)
Number of people in consumer units: all	CXU980010LB0101M
Number of people in consumer units: lowest 20%	CXU980010LB0102M
Total average annual expenditures: all	CXUTOTALEXPLB0101M
Total average annual expenditures: lowest 20%	CXUTOTALEXPLB0102M
Expenditures: vehicle insurance: all	CXU500110LB0101M
Expenditures: vehicle insurance: lowest 20%	CXU500110LB0102M
Expenditures: Health Insurance: all	CXUHLTHINSRLB0101M
Expenditures: Health Insurance: lowest 20%	CXUHLTHINSRLB0102M
Expenditures: personal insurance and pensions: all	CXUINSPENSNLB0101M
Expenditures: personal insurance and pensions: lowest 20%	CXUINSPENSNLB0102M

Notes: For the US, all data are taken from the FRED Database available through the Federal Reserve Bank of St. Louis. FRED codes are reported in the last column.

Private Domestic Investment. The government spending is defined as the sum of Government Consumption Expenditures and Gross Investment, and Net Exports of Goods and

Services. The reason for including net exports of goods and services as an item of government expenditure is to assume a closed economy model following Chari et al. (2007). The GDP deflator is from Implicit Price Deflators for Gross Domestic Product.

Labor supply L_t, following Hayashi and Prescott (2002) and Kobayashi and Inaba (2006), is constructed by

$$L_t = \frac{\text{Weekly average hours worked per employed person} \times \text{Employed person}}{24 \times 7 \times \text{Labor force}}.$$

For the data of the consumption inequality C_t^H/C_t, we calculate the ratio of consumption of the lowest 20 % to that of all consumer units in the "Quintiles of income before taxes." We use average monthly consumption expenditures per household by quintile in annual income before taxes. We use adult-equivalent consumption expenditures as the observables for C_t^H and C_t. The adult-equivalent consumption expenditure of households is defined by dividing the average consumption expenditure by the square root of the number of household members according to the square root scale, which is also used by OECD. To make the definitions consistent with consumption in the model, we deduct "expenditures for vehicle insurance," "health insurance," and "personal insurance and pensions" from average annual expenditures.

B.2 Japanese Data

The data for Japan are constructed basically in the same way as for the US data. We use Private Consumption as consumption. Investment is defined as Gross Capital Formation by the private sector, which consists of Private Residential Investment, Private Nonresidential Investment, and Change in Private Inventories. The government spending is defined as the sum of Government Consumption, Public Investment, Changes in Public Inventries, and Net Exports. The Economic and Social Research Institute (ESRI) releases a series on the 2008 SNA basis for years from 1994 to 2022. In addition, the ESRI provides a series on the 2008 SNA basis for years from 1980 to 1993 as provisional estimates. To connect the data for the entire period, we splice the nominal variables and GDP deflators using the overlapping years.

The aggregate labor L_t, following Hayashi and Prescott (2002) and Kobayashi and

Inaba (2006), is constructed by

$$L_t = \frac{\text{Monthly average hours worked per employed person} \times \text{Employed person}}{24 \times 7 \times 4 \times \text{Labor force}}.$$

For the consumption inequality C_t^H/C_t, we use average monthly consumption expenditures per household by quintile in annual income before taxes for working households with two or more members. We also use adult-equivalent consumption expenditures as well as US data. The Ministry of Internal Affairs and Communications (MIC) releases consistent series for periods 1863–1975, 1976–2007, and 2008–2021, respectively. To connect the data for the entire period, we splice data using the overlapping years.

C Derivation of the Observation Equation of Consumption Inequality

The available data on consumption inequality is annual. Suppose the period t is the 4th quarter of a year. The annual consumption of HtM ($HtMC$) is given by $C_{t-3}^H + C_{t-2}^H + C_{t-1}^H + C_t^H$. The aggregate consumption (AC) is given by $C_{t-3} + C_{t-2} + C_{t-1} + C_t$. The consumption inequality is $HtMC/AC$.

First, dividing the both numerator and denominator by Z_t, it is obtained

$$\begin{aligned}
\frac{HtMC}{AC} &= \frac{C_{t-3}^H + C_{t-2}^H + C_{t-1}^H + C_t^H}{C_{t-3} + C_{t-2} + C_{t-1} + C_t} \\
&= \frac{\frac{C_{t-3}^H}{Z_{t-3}} \times \frac{Z_{t-3}}{Z_{t-2}} \times \frac{Z_{t-2}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_{t-2}^H}{Z_{t-2}} \times \frac{Z_{t-2}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_{t-1}^H}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_t^H}{Z_t}}{\frac{C_{t-3}}{Z_{t-3}} \times \frac{Z_{t-3}}{Z_{t-2}} \times \frac{Z_{t-2}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_{t-2}}{Z_{t-2}} \times \frac{Z_{t-2}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_{t-1}}{Z_{t-1}} \times \frac{Z_{t-1}}{Z_t} + \frac{C_t}{Z_t}} \\
&= \frac{\frac{c_{t-3}^H}{z_{t-2}z_{t-1}z_t} + \frac{c_{t-2}^H}{z_{t-1}z_t} + \frac{c_{t-1}^H}{z_t} + c_t^H}{\frac{c_{t-3}}{z_{t-2}z_{t-1}z_t} + \frac{c_{t-2}}{z_{t-1}z_t} + \frac{c_{t-1}}{z_t} + c_t} \\
&= \frac{c_{t-3}^H + c_{t-2}^H z_{t-2} + c_{t-1}^H z_{t-1} z_{t-2} + c_t^H z_t z_{t-1} z_{t-2}}{c_{t-3} + c_{t-2} z_{t-2} + c_{t-1} z_{t-1} z_{t-2} + c_t z_t z_{t-1} z_{t-2}}.
\end{aligned}$$

Log-linearizing this equation yields

$$\widehat{\left(\frac{HtMC}{AC}\right)} = \widehat{HtMC} - \widehat{AC}.$$

Since the log-linearization of the numerator and denominator imply

$$\widehat{HtMC} = \frac{1}{1 + z + z^2 + z^3}\left[\hat{c}_{t-3}^H + z(\hat{c}_{t-2}^H + \hat{z}_{t-2}) + z^2(\hat{c}_{t-1}^H + \hat{z}_{t-1} + \hat{z}_{t-2}) + z^3(\hat{c}_t^H + \hat{z}_t + \hat{z}_{t-1} + \hat{z}_{t-2})\right]$$

$$\widehat{AC} = \frac{1}{1 + z + z^2 + z^3}\left[\hat{c}_{t-3} + z(\hat{c}_{t-2} + \hat{z}_{t-2}) + z^2(\hat{c}_{t-1} + \hat{z}_{t-1} + \hat{z}_{t-2}) + z^3(\hat{c}_t + \hat{z}_t + \hat{z}_{t-1} + \hat{z}_{t-2})\right],$$

then, the annual consumption inequality is given by

$$\left(\frac{\widehat{HtMC}}{AC}\right) = \frac{1}{1+z+z^2+z^3}\left[(\hat{c}^H_{t-3} - \hat{c}_{t-3}) + z(\hat{c}^H_{t-2} - \hat{c}_{t-2}) + z^2(\hat{c}^H_{t-1} - \hat{c}_{t-1}) + z^3(\hat{c}^H_t - \hat{c}_t)\right].$$

D Sources of Aggregate Labor Wedge

The importance of the labor wedge in the business cycle is pointed out by many researchers, including Chari et al. (2002, 2007), Kobayashi and Inaba (2006), Shimer (2009), and Karabarbounis (2014). Our model has heterogeneous households and multiple labor wedges, the neutral, U-specific, and HtM-specific labor wedges. What is the source of the aggregate labor wedge in our model?

The aggregate labor wedge is defined by

$$\xi \frac{C_t}{1-L_t} = (\text{Aggregate Labor Wedge})_t \times (1-\alpha)\frac{Y_t}{L_t}.$$

This definition is consistent with the representative agent model where the instantaneous utility function of the representative household is specified as

$$u(C_t, L_t) = \log(C_t) + \xi \log(1-L_t),$$

and the production function is Cobb-Douglas, as employed by Chari et al. (2002) and Chari et al. (2007).

Table A2 shows the results of the variance decomposition of the growth rate of the aggregate labor wedge. Since our observable variables in the estimation include GDP and consumption growth rates, we focus on the growth rate of the aggregate labor wedge.

The results from the baseline model tell us as follows. In the US, 60% of the aggregate labor wedge is accounted for by the U-specific labor wedge, and the HtM-specific labor wedge accounts for about 30%. The contribution of the neutral labor wedge is minor. In Japan, the neutral and the HtM-specific labor wedges each account for about 40%, and the U-specific labor wedge accounts for about 20%. In both countries, the contributions of efficiency, investment, and government wedges are small.

The contribution of the HtM-specific labor wedge in the US is close to that of Japan. The difference is that in the US, the contribution of the U-specific labor wedge is significant,

Table A2: Variance Decompositions on Aggregate Labor Wedge

	z	τ^{L}	τ^{LH}	τ^{LU}	τ^{x}	g
US	0.01	6.12	33.25	60.61	0.01	0.01
Japan	0.01	40.86	37.15	21.95	0.01	0.02

Note: Infinity-horizon forecast error variance decompositions are performed.

and that of the neutral labor wedge is very small. In Japan, on the contrary, the U-specific labor wedge and the neutral labor wedge significantly contribute.

These results imply that to understand the aggregate labor wedge in the US, the labor market distortions specific to HtM (poor) and U (non-poor) are important, and distortions related to the overall labor market are insignificant. In contrast, in Japan, in addition to household-specific distortions, we should focus on the distortions in the overall labor market.

E The Case Where Wedges Follow the VAR(1) Process

In this appendix, we consider the case where the wedges follow the VAR(1) process. Let $\boldsymbol{s}_t$ and $\boldsymbol{\varepsilon}_t$ denote the vector of wedges and shocks such that $\boldsymbol{s}_t = [z_t, \tau_t^x, \tau_t^L, \tau_t^{LH}, \tau_t^{LU}, g_t]'$ and $\boldsymbol{\varepsilon}_t = [\varepsilon_t^z, \varepsilon_t^x, \varepsilon_t^L, \varepsilon_t^{LH}, \varepsilon_t^{LU}, \varepsilon_t^g]'$. Then, the wedge $\boldsymbol{s}_t$ evolves according the following VAR(1) process:

$$\boldsymbol{s}_t = \boldsymbol{V}\boldsymbol{s}_{t-1} + \boldsymbol{\varepsilon}_t.$$

We specify the VAR(1) coefficient matrix $\boldsymbol{V}$ as

$$\boldsymbol{V} = \begin{bmatrix} \rho_z & \rho_{zx} & \rho_{zL} & \rho_{zLH} & \rho_{zLU} & 0 \\ \rho_{xz} & \rho_x & \rho_{xL} & \rho_{xLH} & \rho_{xLU} & 0 \\ \rho_{Lz} & \rho_{Lx} & \rho_L & 0 & 0 & 0 \\ \rho_{LHz} & \rho_{LHx} & 0 & \rho_{LH} & 0 & 0 \\ \rho_{LUz} & \rho_{LUx} & 0 & 0 & \rho_{LU} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_g \end{bmatrix}.$$

Table A3: Posterior Estimates: Model with VAR(1) Wedge

	US		Japan	
	Mean	90% credible interval	Mean	90% credible interval
τ^X	1.0389	[0.7818 , 1.2861]	0.5543	[0.2809 , 0.8225]
$100(z-1)$	0.2800	[0.0994 , 0.4489]	0.2407	[0.0411 , 0.4105]
$\frac{C^H}{C}$	0.5804	[0.5661 , 0.5952]	0.7200	[0.7034 , 0.7400]
L	0.1961	[0.1916 , 0.2003]	0.2093	[0.2016 , 0.2169]
ρ_z	0.1427	[0.0395 , 0.2445]	0.0665	[0.0106 , 0.1140]
ρ_x	0.9087	[0.8406 , 0.9826]	0.6248	[0.4030 , 0.8248]
ρ_L	0.8107	[0.7123 , 0.9193]	0.9804	[0.9661 , 0.9955]
ρ_{LU}	0.8884	[0.8126 , 0.9737]	0.8041	[0.6143 , 0.9783]
ρ_{LH}	0.8507	[0.7850 , 0.9235]	0.7485	[0.6161 , 0.8833]
ρ_g	0.9764	[0.9622 , 0.9919]	0.9733	[0.9602 , 0.9880]
ρ_{zx}	-0.5914	[-0.8777 , -0.3441]	0.0822	[-0.3071 , 0.4243]
ρ_{zL}	0.0490	[-0.1201 , 0.2088]	-0.0681	[-0.0990 , -0.0381]
ρ_{zLU}	-0.2559	[-0.3847 , -0.1196]	-0.0747	[-0.3050 , 0.1734]
ρ_{zLH}	-0.0156	[-0.0337 , 0.0028]	0.0445	[0.0073 , 0.0841]
ρ_{xz}	0.0977	[0.0288 , 0.1640]	0.1005	[0.0387 , 0.1627]
ρ_{xL}	-0.0061	[-0.0402 , 0.0260]	0.0223	[-0.0079 , 0.0588]
ρ_{xLU}	0.0038	[-0.0524 , 0.0620]	0.2605	[0.1061 , 0.4073]
ρ_{xLH}	-0.0061	[-0.0148 , 0.0026]	0.0184	[-0.0024 , 0.0401]
ρ_{Lz}	0.0627	[-0.0878 , 0.2137]	-0.0759	[-0.2455 , 0.0780]
ρ_{Lx}	-0.7016	[-0.9516 , -0.4581]	-0.0765	[-0.1684 , 0.0072]
ρ_{LUz}	-0.1900	[-0.3513 , -0.0193]	-0.1493	[-0.3218 , -0.0080]
ρ_{LUx}	-0.0833	[-0.2947 , 0.1281]	0.2442	[-0.0100 , 0.4720]
ρ_{LHz}	0.1475	[-0.2473 , 0.5421]	0.3146	[0.0051 , 0.6729]
ρ_{LHx}	-0.3004	[-0.6817 , 0.0638]	-0.1731	[-0.4369 , 0.0965]

Table A4: Posterior Estimates: Model with VAR(1) Wedge

	US		Japan	
	Mean	90% credible interval	Mean	90% credible interval
σ_z	1.0271	[0.9190 , 1.1436]	1.5209	[1.3648 , 1.6604]
σ_x	0.3118	[0.1942 , 0.4316]	0.2334	[0.1447 , 0.3162]
σ_L	0.1650	[0.1066 , 0.2205]	0.8228	[0.6217 , 1.0033]
σ_{LU}	0.5163	[0.4131 , 0.6192]	0.5781	[0.3383 , 0.8873]
σ_{LH}	3.1857	[2.7068 , 3.6515]	2.4231	[1.7768 , 3.0337]
σ_g	1.5924	[1.4374 , 1.7500]	2.1094	[1.9104 , 2.2958]
σ_{zx}	0.2200	[-0.0212 , 0.4669]	0.1131	[-0.3257 , 0.5522]
σ_{zL}	0.1431	[-0.3372 , 0.6034]	0.2642	[0.0285 , 0.5052]
σ_{zLU}	0.3694	[0.1783 , 0.5766]	0.0701	[-0.2902 , 0.4388]
σ_{zLH}	0.4943	[0.3628 , 0.6303]	-0.1710	[-0.4059 , 0.0554]
σ_{xL}	-0.2775	[-0.7804 , 0.2092]	0.0074	[-0.4341 , 0.4468]
σ_{xLU}	-0.2530	[-0.5362 , -0.0004]	0.0567	[-0.3981 , 0.5092]
σ_{xLH}	0.0616	[-0.1755 , 0.2922]	0.1039	[-0.3402 , 0.5520]

Following Chari et al. (2007), we assume that the government wedge is independent of other wedges. We also assume that each of the three labor wedges is independent of the other two. In this specification, we allow the correlations among shocks. We assume that the government wedge shock is independent of other wedge shocks and that each of the three labor wedge shocks is independent of the other two.

In the estimation, we set the prior distribution of the off-diagonal elements of $\boldsymbol{V}$, ρ_{ij}, is Normal distribution with mean zero, and the standard deviation is 0.3. The prior distribution of the correlations between wedge shocks i and j, σ_{ij}, is the generalized Beta distribution with support [-1,1], mean zero, and the standard deviation is 0.3. The prior distributions of other parameters are the same as in Table 2.

The posterior estimates are in Tables A3 and A4. σ_{ij} is the correlation between wedge i and j. Judging by the credible intervals, the efficiency wedge shock positively correlates

Table A5: Output Growth Volatilities by Reducing Consumption Inequality in Model with VAR(1) Wedges (1): Removing the effects of U-specific and HtM-specific Labor Wedges

	Data	(1)	(2)	(3)
US	0.583	0.642	0.614	0.673
JP	1.023	0.954	1.007	0.939

Note: Standard deviations of $100\Delta \log Y_t$ are reported.

(1) τ_t^{LH} is constant at the estimated steady-state level.

(2) τ_t^{LU} is constant at the estimated steady-state level.

(3) τ_t^{LH} and τ_t^{LU} are constant at their estimated steady-state levels.

with U-specific and HtM-specific labor wedge shock in the US. In Japan, the efficiency wedge positively correlates with the neutral labor wedge.

In this specification of the wedges, the variance decomposition is not appropriate for investigating the driving sources of business cycles and inequality since the shocks are correlated. In addition, since a wedge shock affects other wedges through the VAR(1) process, it is unclear the relation between the importance of a wedge shock and the importance of the wedge itself. Then, we focus on the effects of reducing consumption inequality on business cycle volatility as in Section 5.

Table A5 is the analogue of Table 5. This table shows how removing U-specific and HtM-specific labor wedges affects output growth volatility. Case (1) is where the HtM-specific labor wedge τ_t^{LH} is constant at the estimated steady-state level. Case (2) is where the U-specific labor wedge τ_t^{LU} is constant at the estimated steady-state level. Case (3) is where the HtM-specific and U-specific labor wedges are constant at their estimated steady-state levels. As in Table 5, fixing the levels of U-specific and HtM-specific labor wedges increases output growth volatility, whereas it decreases in Japan. This difference comes from the correlations between wedges and output growth, as in the AR(1) case. In the US, the U-specific and the HtM labor wedges are positively correlated with output growth: $corr(\tau_t^{LH}, \Delta \log Y_t) = 0.25$ and $corr(\tau_t^{LU}, \Delta \log Y_t) = 0.08$. In contrast, in Japan, they are negatively correlated with output growth: $corr(\tau_t^{LH}, \Delta \log Y_t) = -0.03$ and $corr(\tau_t^{LU}, \Delta \log Y_t) = -0.14$.

Table A6: Output Growth Volatilities by Reducing Consumption Inequality in Model with VAR(1) Wedges (2): Redistribution Policy

	Data	(1)	(2)	(3)
US	0.583	0.530	0.561	0.674
JP	1.023	1.183	1.246	0.985

Note: Standard deviations of $100\Delta \log Y_t$ are reported.

(1) C_t^H/C_t is constant at the estimated steady-state level.

(2) C_t^H/C_t is constant at its maximum level. (US: 0.752, Japan: 0.886)

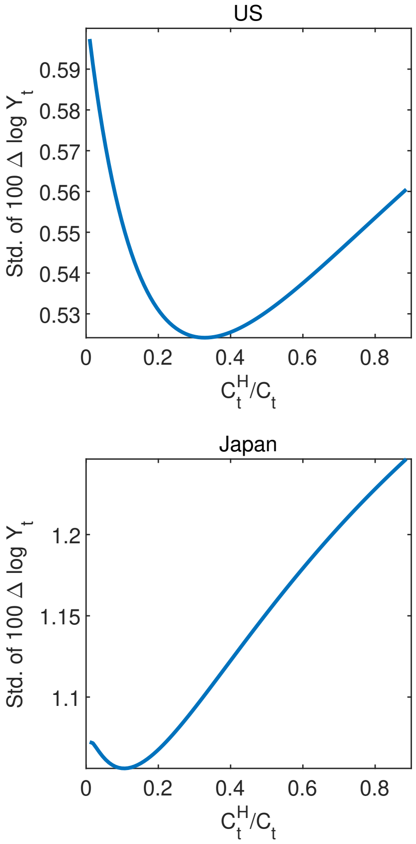
(3) Model without HtM ($\theta = 0, s = 1$): Perfect consumption equality

Table A6 is the analogue of Table 6. This table shows the effects of the redistribution policy on output growth volatility. Case (1) is where C_t^H/C_t is constant at the estimated steady-state level. Case (2) is where C_t^H/C_t is constant at its maximum steady-state level. In the US, $C_t^H/C_t = 0.752$, and in Japan, $C_t^H/C_t = 0.886$. Case (1) shows that in the US, eliminating fluctuations of consumption inequality reduces output growth volatility, whereas it increases output growth volatility in Japan. Case (2) shows that reducing the steady-state consumption inequality (increasing in C_t^H/C_t) from the estimated level increases output growth volatilities in both countries. Case (3) shows that output growth volatility becomes more volatile if HtM does not exist in the US, whereas it becomes less volatile in Japan. These results are consistent with those of Table 6.

Figure A1 is the analogue of Figure 3. As in Figure 3, the relation between C_t^H/C_t and output growth volatility is not monotonic. If C_t^H/C_t is quite low and consumption inequality is severe, increasing C_t^H/C_t reduces output growth volatility. However, if consumption inequality is mild to some extent, increasing C_t^H/C_t increases the volatility of output growth. In the US, the minimum standard deviation of output growth is 0.5241 at $C_t^H/C_t = 0.3286$. In Japan, the minimum is 1.056 at $C_t^H/C_t = 0.1073$.

These results imply that the main implication of Section 5 is robust to the case where the wedges follow the VAR(1) process and the shocks are correlated.

Figure A1: Output Growth Volatilities and C_t^H/C_t under Redistribution Policy



Note: In the US, the minimum standard deviation of output growth is 0.5241 at $C_t^H/C_t = 0.3286$. In Japan, the minimum is 1.056 at $C_t^H/C_t = 0.1073$.