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Expectations-driven productivity in the layered markets

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Expectations-driven productivity in the layered markets

(Preliminary and incomplete.)

Keiichiro Kobayashi*

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Abstract

This paper explores a hypothesis that the macroeconomic expectations may affect the aggregate productivity, even in the business cycle frequencies. The economy consists of the layered markets, in which firms engage in monopolistic competition with free entry. The firms form the division of labor and produce varieties of goods, while the number of varieties determines the productivity. The number of varieties in one market is determined in equilibrium, given the expectations on the number of varieties in another market. Coordination of the expectations between the layered markets generates multiple equilibria, corresponding to high and low productivity. A policy that works on the expectations may change economic organizations and the observed state of technology.

Key words: Division of labor, monopolistic competition, multiple equilibria.

JEL Classification: E23, E30, O40.

1 Introduction

> The division of labor is limited by the extent of the market.
>
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>
> Adam Smith (1776) *Wealth of Nations*

What are the sources of productivity shocks in business cycle frequencies? Can pessimistic and optimistic expectations affect the productivity by changing actions and coordination of economic agents? We study a theoretical hypothesis of endogenous changes in the aggregate productivity in response to macroeconomic expectations. We consider the model of the Dixit-Stiglitz monopolistic competition, where the productivity is determined by the number of the varieties of goods that can vary in accordance with the number of

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firms' entries. Multiple equilibria can emerge in the economy with layered markets of monopolistic competition.

The number of varieties can be interpreted as a degree of specialization or the division of labor. The extent of the market in the well-known theorem of Adam Smith quoted in the above epigraph is also represented by the number of varieties in the demand market. In this paper, we formalize Smith's theorem as a two-way interaction that the number of varieties in the demand market (i.e., the extent of the market) affects the number of varieties in the supply market (the division of labor) and vice versa. This interaction thus affects the aggregate productivity.

The key innovation of the model is the layered-market structure in which several markets with monopolistic competition are layered and agents in one market take the prices and the numbers of varieties in other markets as given. In the rational expectations equilibrium in the layered markets, multiple equilibria can emerge. In one equilibrium, a large number of firms enter, given the expectations that the number of varieties in the other market is large, because an increase in the number of varieties in the other market raises the profits of entrants in this market under the layered production. As a result, the numbers of varieties are large in both markets and the aggregate productivity is high. In another equilibrium, the numbers of varieties are small in both markets and the productivity is low. Thus, fluctuations in the macroeconomic expectations in the layered markets can possibly change the observed productivity frequently over time.

Our theory demonstrates that the equilibrium switching between the multiple equilibria may cause the changes in productivity, and it sheds some new lights on the relationship between the expectations and the productivity: the optimistic expectations on the prospects of the economy may raise the aggregate productivity, whereas the pessimism may have the opposite effect.

The rest of the paper is organized as follows. The literature is reviewed in Section 2. The setup of the model is provided in Section 3. The multiplicity and stability of the equilibria are characterized in Section 4. Section 5 provides the concluding remarks.

2 Literature

Our work is related to the literature of endogenous changes in productivity in the medium-term business cycles that is analyzed by Evans, Honkapohja, and Romer (1998) and Comin and Gertler (2006). Evans et al. use the Dixit-Stiglitz model of monopolistic competition to formulate that an increase in the number of varieties of capital goods increases the aggregate productivity. Multiple equilibria arise from the intertemporal expectations on the future returns on investments in their model, while in our model multiplicity arises from the simultaneous expectations on the contemporaneous actions of other agents. "Animal spirits" affect the investment decisions in Evans et al., whereas the optimistic or pessimistic expectations affect simultaneous division of labor in our model. While Evans et al. explain the medium-term shift of the trend growth rate as a shift between high and low growth equilibria, the negative productivity shocks in the short-run are left unexplained. In our model, a decrease in the productivity in the short-run can be expressed as a shift from the high to low equilibrium. Comin and Gertler also use the model of Dixit-Stiglitz monopolistic competition to show that endogenous R&D investment, driven by exogenous shocks on wage markups, changes the number of varieties, leading to an endogenous productivity fluctuations in the medium-term. In their model, productivity is not driven by the expectations, but by the fundamental shocks, though it is non-technical, while the productivity is expectations-driven in our model. Though our model does not allow the (wage) markups to vary as in Comin and Gertler (2006), it would be easily extended so as to replicate the countercyclical markups.[1]

The vast literature on the monopolistic competition and the aggregate demand externality (e.g., Blanchard and Kiyotaki [1987], Farhi and Werning [2016]) is related to this paper, while the literature focus on the external effect of the nominal rigidities on the aggregate demand, and does not focus on the productivity. Our model is a real model that does not include nominal rigidities, and focuses on the the external effect of the market size on the entry decisions of firms in the other markets.

The literature on coordination failure and multiple equilibria is closely related (Cooper and John [1988]). There is a strategic complementarity between the numbers of varieties in the layered markets in our model. In our model, the complementarity in the firm entry generates multiplicity in productivity, while multiple equilibria in the existing literature, e.g., Kiyotaki (1988) and Gali (1996), correspond to the multiplicity in investments, not in productivity.

Finally, our model is related to the long tradition of the literature on the division of labor and the extent of the market (e.g., Stigler [1951]). The original theorem of Adam Smith cited in the epigraph is an impressive statement that the size of the demand determines the productivity of the supply, represented by the division of labor, which must be a recurrent observation in economic progress in all times and places. The recent literature, e.g., Becker and Murphy (1992), however, emphasize the importance of coordination costs and general knowledge in forming the division of labor, implying that the nature of the supply side is crucial. Our model integrates both insights: the size of the demand affects

[1] As Comin and Gertler (2006) did, we can assume that the elasticity of substitutions among the intermediate goods is decreasing in the number of varieties. In this case, the markup is low (high) in the high (low) productivity equilibrium, which is associated with a large (small) number of varieties. Thus, in the extended model, the markup would be countercyclical. We could introduce the wage markup in the similar manner to show it is also countercyclical.

the division of labor in the supply market, whereas the productivity in the supply market, which can be interpreted as reduction in coordination costs or an increase in general knowledge, raises the division of labor in the demand market.

3 Model

We consider a static model of the following layered production of Dixit-Stiglitz technology. Given the expectations on prices and the numbers of varieties in the layered markets, firms decide whether to enter the markets. For clarity of analysis, we assume that there are three layers of production in the economy: L1, L2, and L3.

L1 is the layer for production of the final goods, Y, from the downstream intermediate inputs, y_i, $i \in [0, N]$, where N is the number of varieties of the downstream intermediate goods, which is endogenously determined in equilibrium. L2 is the layer for production of the downstream intermediate goods, y_i, from the upstream intermediate inputs, z_{ji}, $j \in [0, M]$, where M is the number of varieties, which is also endogenously determined in equilibrium. L3 is the layer for production of the upstream intermediate goods, z_{ji}, from labor input, l_{ji}. The consumer's problem is

$$\max \ln C + \gamma \ln(1 - L), \tag{1}$$

$$\text{s.t. } C \leq wL + \Pi + c, \tag{2}$$

where C is consumption, w is the wage rate, L is the labor supply, Π is the total dividend from the firms, and c is an endowment, which is given exogenously. In equilibrium,

$$C = Y + c,$$
$$L = \int_0^N \int_0^M l_{ji} di dj,$$
$$\Pi = \int_0^N \pi_{yi} di + \int_0^M \pi_{zj} dj,$$

where π_{yi} is the profit of a producer of the downstream goods, while π_{zj} is that of the upstream goods.

3.1 Final goods production (L1)

Production technology is

$$Y = \left(\int_0^N y_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Cost minimization problem is

$$\min_{y_i} \int_0^N p_i y_i di, \quad \text{s.t. } \left(\int_0^N y_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \geq Y,$$

where p_i is the price of y_i. The solution is

$$y_i = \left(\frac{p_i}{P}\right)^{-\varepsilon} Y,$$
$$\text{where } P \equiv \left(\int_0^N p_i^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

By definition, $P = N^{-\frac{1}{\varepsilon-1}} p$ in the symmetric equilibrium.

3.2 Downstream intermediate goods production (L2)

Production technology for y_i is

$$y_i = \left(\int_0^M z_{ji}^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}.$$

Cost minimization problem is

$$\min_{z_{ji}} \int_0^M q_j z_{ji} dj, \quad \text{s.t.} \quad \left(\int_0^M z_{ji}^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} \geq y_i,$$

where q_j is the price of the j-th goods, z_{ji}. The solution is

$$z_{ji} = \left(\frac{q_j}{Q}\right)^{-\sigma} y_i,$$
$$\text{where} \quad Q \equiv \left(\int_0^M q_j^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}.$$

In the symmetric equilibrium, $Q = M^{-\frac{1}{\sigma-1}} q$ by definition, and the profit maximization for a downstream intermediate producer is

$$\begin{aligned}
\pi_{yi} &= \max_{p_i} \; p_i y_i - \int_0^M q_j z_{ji} dj \\
&= \max_{p} \; p y(p) - Mq \left(\frac{q}{Q}\right)^{-\sigma} y(p) \\
&= \max_{p} \; \frac{Y}{P^{\varepsilon}} \left[p^{1-\varepsilon} - M \frac{q^{1-\sigma}}{Q^{-\sigma}} p^{-\varepsilon}\right],
\end{aligned}$$

which can be denoted as π_y by omitting the subscript i in the symmetric equilibrium. The solution is

$$p = \left(\frac{\varepsilon}{\varepsilon - 1}\right) qM \left(\frac{q}{Q}\right)^{-\sigma}. \tag{3}$$

The equilibrium value of N is determined by the free entry condition:

$$\pi_y = \kappa,$$

where κ is the fixed cost for establishing a new firm, in terms of the final goods. We assume for simplicity that the fixed cost κ is not a dead-weight loss, but a transfer to the houeshold, so that the total output Y is not affected by the total payment of the fixed cost κN in equilibrium.

3.3 Upstream intermediate goods production (L3)

Production technology is

$$z_{ji} = l_{ji},$$

where l_{ji} is the labor input. Given the demand $z_{ji} = (q_j/Q)^{-\sigma} y_i$, an upstream intermediate producer solves the following problem in the symmetric equilibrium:

$$\begin{aligned}
\pi_{zj} =& \max_{q_j} \ q_j \int_0^N z_{ji} di - w \int_0^N l_{ji} di \\
=& \max_{q} \left(\int_0^N y_i di \right) \left(\frac{q}{Q} \right)^{-\sigma} (q - w),
\end{aligned}$$

which can be denoted as π_z by omitting the subscript j in the symmetric equilibrium. The solution in the symmetric equilibrium is $q = \frac{\sigma}{\sigma-1} w$, which implies

$$\begin{aligned}
Q &= M^{-\frac{1}{\sigma-1}} \frac{\sigma}{\sigma - 1} w, \\
z_{ji} = l_{ji} &= \left(\frac{q}{Q} \right)^{-\sigma} \left(\frac{p}{P} \right)^{-\varepsilon} Y = M^{-\frac{\sigma}{\sigma-1}} N^{-\frac{\varepsilon}{\varepsilon-1}} Y.
\end{aligned}$$

Similar to the downstream market, the equilibrium value of M is determined by the free entry condition:

$$\pi_z = \kappa,$$

where κ is not a dead-weight loss, but is a transfer from the firm to the households.

4 Equilibrium

In the symmetric equilibrium, we have $p = PN^{\frac{1}{\varepsilon-1}}$, and $q = QM^{\frac{1}{\sigma-1}}$. Given the solution for the upstream firm $q = \frac{\sigma}{\sigma-1} w$, the solution for the downstream firm (3) implies that the price of downstream intermediate good is

$$p = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \left(\frac{\sigma}{\sigma - 1} \right) w M^{-\frac{1}{\sigma-1}},$$

which then implies

$$P = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \left(\frac{\sigma}{\sigma - 1} \right) w M^{-\frac{1}{\sigma-1}} N^{-\frac{1}{\varepsilon-1}}. \tag{4}$$

The profit of the upstream intermediate producer is

$$\pi_z = \frac{wY}{(\sigma - 1) N^{\frac{1}{\varepsilon-1}} M^{\frac{\sigma}{\sigma-1}}}.$$

The profit of the downstream intermediate producer is

$$\pi_y = \frac{\sigma w Y}{(\varepsilon - 1)(\sigma - 1) N^{\frac{\varepsilon}{\varepsilon-1}} M^{\frac{1}{\sigma-1}}}.$$

The total labor supply is

$$L = \int_0^N \int_0^M l_{ji} dj di = M^{-\frac{1}{\sigma-1}} N^{-\frac{1}{\varepsilon-1}} Y.$$

Therefore, the aggregate production can be written as

$$Y = M^{\frac{1}{\sigma-1}} N^{\frac{1}{\varepsilon-1}} L, \tag{5}$$

implying that the aggregate productivity is $M^{\frac{1}{\sigma-1}} N^{\frac{1}{\varepsilon-1}}$. Consumer's utility maximization implies

$$\frac{\gamma(Y+c)}{1 - M^{-\frac{1}{\sigma-1}} N^{-\frac{1}{\varepsilon-1}} Y} = w \tag{6}$$

Since P is the unit cost of Y, we impose the restriction:

$$P = 1, \tag{7}$$

and use Y as the numeraire. Then, (4) implies that

$$w = \left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\sigma-1}{\sigma}\right) M^{\frac{1}{\sigma-1}} N^{\frac{1}{\varepsilon-1}}. \tag{8}$$

Thus, (6) implies

$$Y = \xi M^{\frac{1}{\sigma-1}} N^{\frac{1}{\varepsilon-1}} - \eta, \tag{9}$$

where

$$\phi \equiv \left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\sigma-1}{\sigma}\right), \qquad \xi \equiv \frac{\phi}{\phi+\gamma}, \qquad \text{and} \qquad \eta \equiv \frac{\gamma c}{\phi+\gamma}.$$

The profit of upstream producer can be rewritten as

$$\pi_z = \frac{\phi\xi}{\sigma-1} N^{\frac{1}{\varepsilon-1}} M^{\frac{2-\sigma}{\sigma-1}} - \frac{\eta\phi}{\sigma-1} M^{-1}, \tag{10}$$

and also the profit of downstream producer can be rewritten as

$$\pi_y = \frac{\sigma\phi\xi}{(\varepsilon-1)(\sigma-1)} N^{\frac{2-\varepsilon}{\varepsilon-1}} M^{\frac{1}{\sigma-1}} - \frac{\sigma\eta\phi}{(\varepsilon-1)(\sigma-1)} N^{-1}. \tag{11}$$

The equilibrium values of the varieties, $(M,\ N)$, are determined by the free entry condition: $\pi_z = \kappa$ and $\pi_y = \kappa$.

4.1 Multiple equilibria

We set the parameter restrictions on the elasticity of substitution, ε and σ:

$$\varepsilon > 2, \qquad \sigma > 2, \qquad \text{and} \qquad (\varepsilon-2)(\sigma-2) - 1 > 0. \tag{12}$$

This restriction is plausible as the elasticity of substitution among intermediate goods is set at around 6 in the New Keynesian literature (Nutahara [2021], Khan, Phaneuf and Victor [2020]). This value makes the markup rate around 20% in the New Keynesian models, and the micro evidence shows that the industry-level markup rates are actually around 20% (De Loecker and Warzynski [2012]).

For simplicity of exposition, we change the variables and define

$$m \equiv M^{\frac{1}{\sigma-1}},$$
$$n \equiv N^{\frac{1}{\varepsilon-1}}.$$

Note that the aggregate productivity is mn, as the aggregate production can be written as $Y = mnL$. Free entry conditions, $\pi_z = \kappa$ and $\pi_y = \kappa$, are rewritten as

$$\pi_z(m,n) \equiv anm^{2-\sigma} - bm^{1-\sigma} = \kappa, \tag{13}$$
$$\pi_y(m,n) \equiv \frac{\sigma}{\varepsilon - 1}[amn^{2-\varepsilon} - bn^{1-\varepsilon}] = \kappa, \tag{14}$$

where

$$a \equiv \frac{\phi\xi}{\sigma - 1}, \qquad \text{and} \qquad b \equiv \frac{\eta\phi}{\sigma - 1} = \frac{\gamma\phi c}{(\sigma-1)(\phi+\gamma)}.$$

The competitive equilibrium in this economy is the set of prices $(P = 1, Q, p, q, w)$ and quantities (Y, y, z, l, m, n) such that the quantities solve for the consumer's problem and the profit maximization for the firms in the upstream and downstream markets, and all markets clear. The equilibrium values of all variables are determined, once the pair (m, n) is given. Thus, for brevity, we call (m, n) the equilibrium in what follows.

Definition 1. The equilibrium is the pair (m, n) that solves (13) and (14).

We have to note that the free entry condition implies that if $\pi_z < \kappa$, then no firms enter the upstream market and m becomes zero, and that if $\pi_y < \kappa$, then no firms enter the downstream market and n becomes zero. Thus, there may exist a singular equilibrium, which corresponds to the corner solution, i.e., $m = n = 0$. To avoid unnecessary complications in our analysis, we simply exclude the corner solution from the equilibrium by assuming the following.

Assumption 1. There exists a shared belief in this economy that in equilibrium there exist non-zero entries both in the upstream and downstream markets, i.e.,

$$m > 0, \qquad \text{and} \qquad n > 0. \tag{15}$$

The following proposition demonstrates the multiplicity of the equilibria.

Proposition 1. *Suppose that the parameters satisfy the condition:*

$$H(\tilde{m}) < 0, \tag{16}$$

where

$$H(m) \equiv \frac{b}{a} - \frac{(\varepsilon-2)(\sigma-2)-1}{(\varepsilon-1)(\sigma-1)} \left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} m^{\frac{\varepsilon+\sigma-2}{\varepsilon-1}}, \tag{17}$$

$$\tilde{m} = \left[\frac{\left(\frac{\varepsilon+\sigma-2}{\varepsilon-1}\right)\left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} a}{(\sigma-1)\kappa}\right]^{\frac{\varepsilon-1}{(\varepsilon-2)(\sigma-2)-1}}. \tag{18}$$

Then, there exist two equilibria, (m_l, n_l) and (m_h, n_h), where $m_l < m_h$ and $n_l < n_h$. If $H(\tilde{m}) = 0$, the equilibrium is unique. If $H(\tilde{m}) > 0$ there exists no equilibrium.

The condition for multiplicity (16) is satisfied if the outside endowment, c, is sufficiently small. As c becomes small, the parameter b becomes small proportionately, while all other parameter values are fixed, (16) is obviously satisfied.

All proofs of propositions are given in the Appendix. The outline of the proof is the following. In the Appendix, we define a continuously differentiable function $F(m)$, where the solution to $F(m) = 0$ equals the solution to (13) and (14). $F(m)$ satisfies that $F(0) = \frac{b}{a} > 0$ and $\lim_{m\to\infty} F(m) = \infty$. In the Appendix, it is shown that $F'(m) = 0$ has two solutions $m = 0$, $\tilde{m}$, and $F'(m) < 0$ for $0 < m < \tilde{m}$ and $F'(m) > 0$ for $m > \tilde{m}$. Thus we can say that $F(\tilde{m})$ is the minimum value of $F(m)$. It is also shown that $H(\tilde{m}) = F(\tilde{m})$. Thus, the condition $H(\tilde{m}) < 0$ means that the minimum value of $F(m)$ at $\tilde{m} > 0$ is negative, which implies, together with the fact that $F(0) > 0$ and $\lim_{m\to\infty} F(m) = \infty$, that $F(m) = 0$ has two solutions.

Intuitive explanation: When the productivity in the downstream market (n) is expected to be small, then the demand for the upstream goods is expected to be small. Thus, fewer firms enter the upstream market because the expected profit π_z is small, and division of labor is established to a lesser degree (i.e., m is small). On the other hand, given the expectations of the low productivity in the upstream market (m) that implies the scarcity of the upstream goods, fewer firms enter the downstream market (i.e., n is small), because the expected profit π_y is small. Thus, the equilibrium with small m and n is supported by the rational expectations. As the output Y is produced from labor L by the aggregate production function (5), i.e., $Y = mnL$, the aggregate productivity of the economy (mn) is low in this equilibrium. There exists another equilibrium, which is described as follows. Given the productivity in the downstream market (n) and hence the size of the demand is expected to be large, more firms enter the upstream goods market (i.e., m is large), whereas given the expectations of the high productivity in the upstream market (m) and hence the abundance of the upstream goods, more firms enter the downstream market (i.e., n is large). Thus, the equilibrium with large m and n is also supported by the rational expectations. The aggregate productivity is high in this equilibrium.

4.2 Nash stability of the equilibria

For conveniece of notation, we denote an equilibrium by (m_e, n_e). We have shown the multiplicity of the equilibria, $(m_e, n_e) = (m_l, n_l)$ and $(m_e, n_e) = (m_h, l_h)$. Next, we need to examine the stability of them. We define the following concept of the Nash stability.

Definition 2. The equilibrium, (m_e, n_e), is Nash stable, if the inequalities, $\frac{\partial}{\partial m}\pi_z(m_e, n_e) < 0$ and $\frac{\partial}{\partial n}\pi_y(m_e, n_e) < 0$, are both satisfied.

This condition means that the firms have no incentive to increase or decrease the number of firms in the upstream market m from m_e by changing their entry decisions, given the number of firms in the downstream market n_e, and vice versa. Of course the Nash stability defined above is connected with the concept of the Nash equilibrium. Although m or n are not directly chosen by the firms, their choice whether to enter the market decides m and n indirectly. In this sense, we can say that the firms in the upstream (downstream) market choose m (n). The intuition of the Nash stability can be explained as follows: The "equilibrium response" of the upstream (downstream) firms is to choose $m = m_e$ ($n = n_e$), in response to the downstream (upstream) firms' choice n_e (m_e). To clarify this intuition, we define the equilibrium response of the firms as follows.

The equilibrium response of the firms: Figure 1 shows the graph of $\pi_z(m, n) = \kappa$, or (13), in the (m, n)-space.

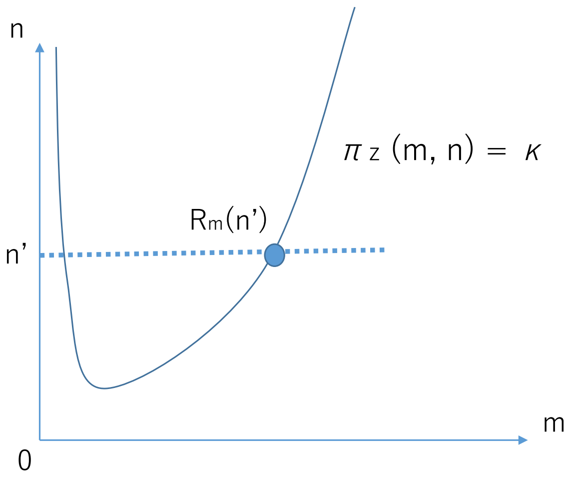

Figure 1

We define in what follows the equilibrium response $m = R_m(n')$ of the upstream firms to the fixed value of $n = n'$. We first define $l_m(n')$ and $h_m(n')$ as the two values of m that solve $\pi_z(m, n') = \kappa$, where $l_m(n') \leq h_m(n')$. We show that $R_m(n') = h_m(n')$ as follows. Since $\pi_z(m, n)$ is continuously differentiable, we have the following lemma:

Lemma 2. *Given a fixed value of n', the equation $\pi_z(m, n') = \kappa$ has at most two solutions, $l_m(n')$ and $h_m(n')$, with $l_m(n') \leq h_m(n')$, while there is no solution if κ is large enough. $\pi_z(m, n')$ satisfies that $\pi_z(m, n') < \kappa$ for $m < l_m(n')$ and $m > h_m(n')$, and $\pi_z(m, n') > \kappa$ for $l_m(n') < m < h_m(n')$. $\frac{\partial}{\partial m}\pi_z(m, n')$ satisfies that $\frac{\partial}{\partial m}\pi_z(l_m(n'), n') > 0$ and $\frac{\partial}{\partial m}\pi_z(h_m(n'), n') < 0$ if $l_m(n') < h_m(n')$, and $\frac{\partial}{\partial m}\pi_z(h_m(n'), n') = 0$ if $l_m(n') = h_m(n')$.*

Given n', the free entry condition implies that only $m = l_m(n')$ or $m = h_m(n')$ can be the equilibrium response of the firms in the upstream market. Under the (implicit) assumption that a firm is an atomic agent, a firm's deviation of the entry decision does not affect the value of m at all and, therefore, both $l_m(n')$ and $h_m(n')$ can be the equilibrium values of m. Here, to consider the stability in an more realistic environment, we add the following assumption that a firm's entry decision infinitesimally affects the value of m:

Assumption 2. If a firm enters the upstream market, the number of firms changes from m to $m + \delta_m$, where $\delta_m > 0$ is an infinitesimally small number. If a firm enters the downstream market, the number of firms changes from n to $n + \delta_n$, where $\delta_n > 0$ is an infinitesimally small number.

Under this assumption, $m = l_m(n')$ is unstable because new entry of firms inevitably occurs at $m = l_m(n')$ as an entry of a new firm gives the entrant a positive profit: $\pi_z(l_m(n') + \delta_m, n') - \kappa \approx \frac{\partial}{\partial m}\pi_z(l_m(n'), n') \times \delta_m > 0$. On the other hand, $m = h_m(n')$ is stable because no new entry occurs at this point as a new entry of a firm would give the entrant a negative profit: $\pi_z(h_m(n') + \delta_m, n') - \kappa \approx \frac{\partial}{\partial m}\pi_z(h_m(n'), n') \times \delta_m < 0$. Thus, the equilibrium response of m to the given value of n' is only $h_m(n')$, under Assumption 2. We have shown that

$$R_m(n') = h_m(n'),$$

if there exists the solution to $\pi_z(m, n') = \kappa$, and $R_m(n')$ does not exist otherwise. Note that the free entry implies that there would exist the corner solution that corresponds to $\pi_z < \kappa$, i.e., $R_m(n') = 0$, which we eliminate by Assumption 1. Similar argument holds for the downstream market. Figure 2 shows the graph of $\pi_y(m, n) = \kappa$, i.e., (14).

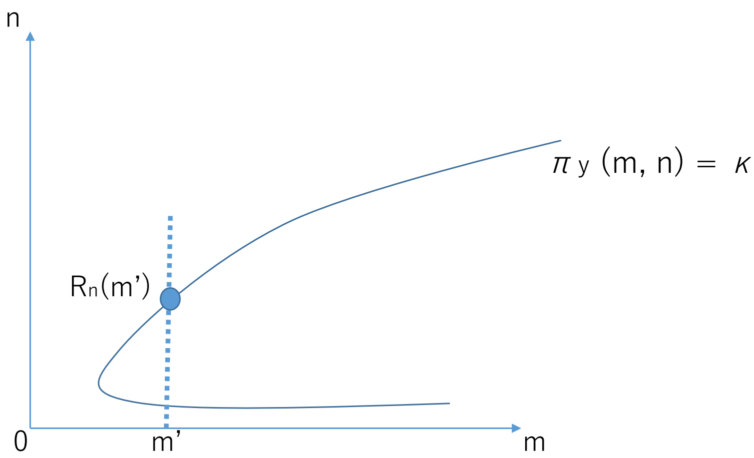

Figure 2

Lemma 3. *Given a fixed value of m', the equation $\pi_y(m', n) = \kappa$ has at most two solutions, $l_n(m')$ and $h_n(m')$, with $l_n(m') \leq h_n(m')$, while there is no solution if κ is large enough. $\pi_y(m', n)$ satisfies that $\pi_y(m', n) < \kappa$ for $n < l_n(m')$ and $n > h_n(m')$, and $\pi_y(m', n) > \kappa$ for $l_n(m') < n < h_n(m')$. $\frac{\partial}{\partial n}\pi_y(m', n)$ satisfies that $\frac{\partial}{\partial n}\pi_y(m', l_n(m')) > 0$ and $\frac{\partial}{\partial n}\pi_z(m', h_n(m')) < 0$ if $l_n(m') < h_n(m')$, and $\frac{\partial}{\partial n}\pi_y(m', h_n(m')) = 0$ if $l_n(m') = h_n(m')$.*

Under Assumption 2, the equilibrium response n of the downstream firms to the fixed value of m', $R_n(m')$, is given by

$$R_n(m') = h_n(m'),$$

if there exists the solution to $\pi_y(m', n) = \kappa$, and $R_n(m')$ does not exist otherwise. Figure 3 combines the graphs of (13) and (14) to show $m = R_m(n)$ and $n = R_n(m)$. There are two intersections of $\pi_z(m, n) = \kappa$ and $\pi_y(m, n) = \kappa$, which are the equilibria, $(m_e, n_e) = (m_l, n_l)$ and (m_h, n_h).

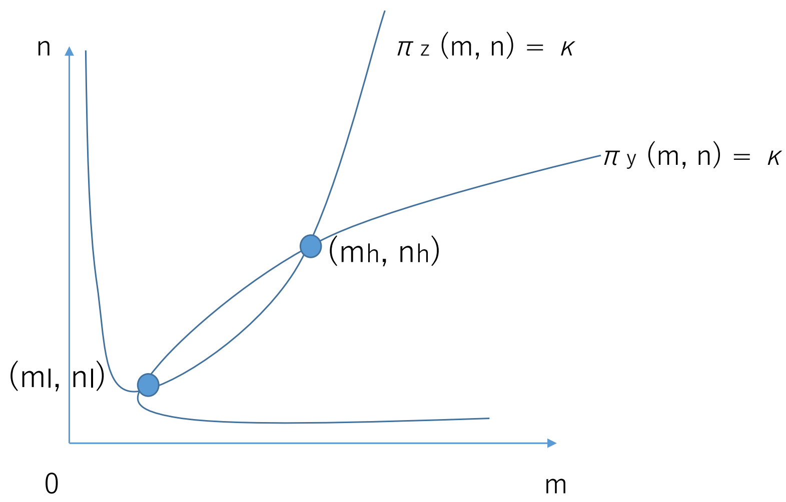

Figure 3

The above definition of the Nash stability can be rewritten as follows:

Definition 2′. An equilibrium (m_e, n_e) is Nash stable, if the equilibrium responses in the both markets satisfy that $R_m(n_e) = m_e$ and $R_n(m_e) = n_e$.

Proposition 4. *In the case of multiple equilibria, both* (m_l, n_l) *and* (m_h, n_h) *are Nash stable, if the parameters satisfy the following condition:*

$$(m_l)^{\frac{\varepsilon+\sigma-2}{\varepsilon-1}} > \frac{(\sigma-1)b}{(\sigma-2)\left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} a}, \tag{19}$$

$$(n_l)^{\frac{\varepsilon+\sigma-2}{\varepsilon-1}} > \frac{(\varepsilon-1)b}{(\varepsilon-2)\left(\frac{\varepsilon-1}{\sigma}\right)^{\frac{1}{\sigma-1}} a}. \tag{20}$$

Can these conditions, (19) and (20), be satisfied for a reasonable range of parameters? The answer is yes, and we construct an example of a range of parameters as follows.

We consider various values for parameter c, the outside endowment, given that all other parameter values are fixed. Suppose that c is chosen so that the value of parameter b satisfies $H(\tilde{m}) = 0$. Note that the value of c affects the value of b only, and does not affect any other parameters. Denote the value of b that satisfies $H(\tilde{m}) = 0$ by b_m. For $b < b_m$, condition (16) is satisfied and the multiple equilibria emerge. We also consider the value of b (and c) that satisfies (19) and (20), given that the values of all other parameters are fixed. As we show in the Appendix, there exist b_z and b_y such that (19) is satisfied for $b > b_z$ and (20) is satisfied for $b > b_y$. We can prove the following lemma.

Proposition 5. *There exists positive real numbers, b_m, b_y, and b_z, that satisfy $0 < \max\{b_z, b_y\} < b_m$, and for $b \in (\max\ \{b_z, b_y\}, b_m)$, there exist two Nash stable equilibria, (m_l, n_l) and (m_h, n_h). For $b \in (0, \max\{b_z, b_y\})$, there exists only one Nash stable equilibrium (m_h, n_h).*

This proposition can be graphically interpreted in the (m, n)-space as follows: For $b \in (\max\{b_z, b_y\}, b_m)$, the functions $m = R_m(n)$ and $n = R_n(m)$ have two intersections, which are (m_l, n_l) and (m_h, n_h), and both of them are Nash stable (Figure 3); For $b \in (0, \max\{b_z, b_y\})$, the functions $m = R_m(n)$ and $n = R_n(m)$ have only one intersection, which is (m_h, n_h).

As the variables (m, n) are flow variables such as consumption in the growth model, any positive values can be chosen and the economy instantaneously jumps to one of the possible equilibria. Thus, under Assumption 2, the Nash stable equilibrium is the only equilibrium that can be realized in the model, and Proposition 5 says that there exist two equilibria for a range of the parameter values.

Numerical example: The case where $\varepsilon = \sigma = 6$ is shown in the Appendix. In this case,

$$\frac{\max\{b_y, b_z\}}{b_m} = 0.882.$$

For the outside endowment c that makes b satisfy $0.882 \times b_m < b < b_m$, there exist two Nash-stable equilibria that correspond to high and low productivities.

5 Conclusion

We have theoretically shown that the macroeconomic expectations may affect the aggregate productivity. Changes in economic organizations that determine the productivity such as the division of labor in one market are induced by changes in the expectations on the extent of the other markets. These changes can be modeled as switching among the multiple equilibria due to sunspot shocks that may occur in the business cycle frequencies.

The economy consists of layered markets, in which firms engage in monopolistic competition with free entry. They form the division of labor that determines the productivity, which is indicated by the number of varieties of the intermediate goods in the market. The number of varieties in one market is determined in equilibrium, given the expectations on the number of varieties in another market. As a result of coordination of the expectations on the numbers of varieties between the layered markets, there emerge multiple equilibria, corresponding to high and low productivity, respectively: More (Less) firms engage in the division of labor and the number of varieties in the market becomes large (small), given the expectations that the number of varieties in the other market is large (small). This is because under the layered production, the profits of entrants in the market is larger (smaller), as the number of varieties in the neighboring market is larger (smaller).

It is confirmed that the multiplicity is robustly observed for a wide range of parameter values. This model provides a theoretical hypothesis that the technology shocks in the business cycles may be manifestation of the equilibrium switching due to changes in the macroeconomic expectations. Policy implications could be broad, since our result implies that a policy that works on the expectations may change the economic fundamentals and the observed state of technology. That is, the optimistic or pessimistic expectations may change the aggregate productivity in a positive or negative directions. Awaiting future research is to further clarify the nature of the expectations-driven productivity and better control the business cycles and economic growth.

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Appendix

Proofs of propositions are given in this Appendix.

Proof of Proposition 1

What is to show is that there exits two solutions to simultaneous equations (13) and (14). These equations imply $m^{\sigma-1} = \frac{\varepsilon-1}{\sigma} n^{\varepsilon-1}$, and (13) can be rewritten as $F(m) = 0$, where

$$F(m) \equiv \frac{\kappa}{a} m^{\sigma-1} - \left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} m^{\frac{\varepsilon+\sigma-2}{\varepsilon-1}} + \frac{b}{a}$$

The parameter restriction (12) implies that $F'(m) = 0$ has two solutions $m = 0$, and $\tilde{m}$, and that $F'(m) < 0$ for $0 < m < \tilde{m}$ and $F'(m) > 0$ for $m > \tilde{m}$. Therefore, $F(\tilde{m}) = \min_{m \geq 0} F(m)$. The necessary and sufficient condition for the existence of two solutions for (13) and (14) is $F(\tilde{m}) < 0$. The first-order condition $F'(\tilde{m}) = 0$ that holds at $m = \tilde{m}$ implies that $F(\tilde{m}) = H(\tilde{m})$.

Proof of Lemma 2

Since

$$\frac{\partial}{\partial m} \pi_z(m, n') = m^{-\sigma}[(\sigma-1)b - (\sigma-2)an'm],$$

$\pi_z(m, n') - \kappa$ is increasing in m for $m \in (0, \frac{(\sigma-1)b}{(\sigma-2)an'})$, and decreasing in m for $m > \frac{(\sigma-1)b}{(\sigma-2)an'}$. The lemma follows from this fact straightforwardly.

Proof of Lemma 3

Since

$$\frac{\partial}{\partial n} \pi_y(m', n) = \frac{\sigma}{(\varepsilon-1)n^{\varepsilon}}[(\varepsilon-1)b - (\varepsilon-2)am'n],$$

$\pi_y(m', n) - \kappa$ is increasing in n for $n \in (0, \frac{(\varepsilon-1)b}{(\varepsilon-2)am'})$, and decreasing in n for $n > \frac{(\varepsilon-1)b}{(\varepsilon-2)am'}$. The lemma follows from this fact straightforwardly.

Proof of Proposition 4

The condition for the Nash stability is that both $\frac{\partial}{\partial m}\pi_z(m_e, n_e) < 0$ and $\frac{\partial}{\partial n}\pi_y(m_e, n_e) < 0$ are satisfied at the equilibrium values of (m_e, n_e). The definitions of $\pi_z(m, n)$ and $\pi_y(m, n)$ in (13) and (14), together with $m_e^{\sigma-1} = \frac{\varepsilon-1}{\sigma} n_e^{\varepsilon-1}$, imply that these inequalities can be rewritten as

$$(m_e)^{\frac{\varepsilon+\sigma-2}{\varepsilon-1}} > \frac{(\sigma-1)b}{(\sigma-2)\left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} a},$$

$$(n_e)^{\frac{\varepsilon+\sigma-2}{\varepsilon-1}} > \frac{(\varepsilon-1)b}{(\varepsilon-2)\left(\frac{\varepsilon-1}{\sigma}\right)^{\frac{1}{\sigma-1}} a}.$$

As we know that $m_l < m_h$ and $n_l < n_h$, it is straightforward that (m_l, n_l) and (m_h, n_h) are both Nash stable if (19) and (20) are satisfied.

Proof of Proposition 5

Here we regard b as a variable and all other parameters are fixed. We redefine $F(m)$ as a function of m and b, and denote it by $F(m,b)$:

$$F(m,b) \equiv \frac{\kappa}{a}m^{\sigma-1} - \left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} m^{\frac{\varepsilon+\sigma-2}{\varepsilon-1}} + \frac{b}{a}$$

The condition (16) can be rewritten as

$$\tilde{m} > m^*(b),$$

where

$$m^*(b) \equiv \left[\frac{(\varepsilon-1)(\sigma-1)b}{[(\varepsilon-2)(\sigma-2)-1]\left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} a}\right]^{\frac{\varepsilon-1}{\varepsilon+\sigma-2}}.$$

This condition for b can be rewritten as $b < b_m$, where b_m is the solution to $\tilde{m} = m^*(b)$, that is,

$$b_m = \frac{[(\varepsilon-2)(\sigma-2)-1]\left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} a}{(\varepsilon-1)(\sigma-1)}\tilde{m}^{\frac{\varepsilon+\sigma-2}{\varepsilon-1}}.$$

The condition (19) can be rewritten as

$$m_l(b) > m^{**}(b),$$

where $m_l(b)$ is the smaller solution to $F(m,b)=0$, and

$$m^{**}(b) \equiv \left[\frac{(\sigma-1)b}{(\sigma-2)\left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} a}\right]^{\frac{\varepsilon-1}{\varepsilon+\sigma-2}}.$$

Obviously, $m^{**}(b) < m^*(b)$ for all $b > 0$. For $b = b_m$, the equation $F(m, b_m) = 0$ has a unique solution and $m_l(b_m) = m_h(b_m) = m^*(b_m) = \tilde{m}$ and $m^{**}(b_m) < m^*(b_m) = \tilde{m}$. Since $F(\tilde{m}, b)$ is the minimum value of $F(m,b)$, given any b, and $F(\tilde{m}, b_m) = 0$, we know

$$F(m^{**}(b_m), b_m) > 0. \tag{21}$$

Now we define b_z as the solution to $F(m^{**}(b), b) = 0$. Thus,

$$b_z = [(\sigma-2)\kappa]^{\frac{-(\varepsilon+\sigma-2)}{(\varepsilon-2)(\sigma-2)-1}} \left[\frac{(\sigma-2)\left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} a}{\sigma-1}\right]^{\frac{(\varepsilon-1)(\sigma-1)}{(\varepsilon-2)(\sigma-2)-1}}.$$

The parameter restriction (12) implies that

$$F(m^{**}(b), b) > 0 \qquad \Longleftrightarrow \qquad b > b_z. \tag{22}$$

The conditions (21) and (22) implies that

$$b_z < b_m. \tag{23}$$

For $b \in (b_z, b_m)$, we have $F(m^{**}(b), b) > 0$ and $m^{**}(b) < m^{**}(b_m) < m^*(b_m) = \tilde{m}$. Since $F(\tilde{m}, b) < 0$ is the minimum value of $F(m, b)$, the fact that $F(m^{**}(b), b) > 0$ implies that $m^{**}(b) < m_l(b) < \tilde{m}$. Thus conditions (16) and (19) are satisfied for $b \in (b_z, b_m)$.

The condition (20), together with $m^{\sigma-1} = \frac{\varepsilon-1}{\sigma} n^{\varepsilon-1}$, can be rewritten as

$$m_l(b) > m^{***}(b),$$

where

$$m^{***}(b) \equiv \left[\frac{(\varepsilon-1)b}{(\varepsilon-2)\left(\frac{\sigma}{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} a} \right]^{\frac{\varepsilon-1}{\varepsilon+\sigma-2}}.$$

Obviously, $m^{***}(b) < m^*(b)$ for all $b > 0$. Thus, for $b = b_m$, $m^{***}(b_m) < m^*(b_m) = \tilde{m}$. Since $F(\tilde{m}, b)$ is the minimum value of $F(m, b)$, given any b, and $F(\tilde{m}, b_m) = 0$, we know

$$F(m^{***}(b_m), b_m) > 0. \tag{24}$$

Now we define b_y as the solution to $F(m^{***}(b), b) = 0$. Thus,

$$b_y = \left[\frac{\sigma}{(\varepsilon-1)(\varepsilon-2)\kappa} \right]^{\frac{(\varepsilon+\sigma-2)}{(\varepsilon-2)(\sigma-2)-1}} \left[\frac{(\varepsilon-2)\left(\frac{\varepsilon-1}{\sigma}\right)^{\frac{1}{\sigma-1}} a}{\sigma-1} \right]^{\frac{(\varepsilon-1)(\sigma-1)}{(\varepsilon-2)(\sigma-2)-1}}.$$

The parameter restriction (12) implies that

$$F(m^{***}(b), b) > 0 \qquad \Longleftrightarrow \qquad b > b_y. \tag{25}$$

The conditions (24) and (25) implies that

$$b_y < b_m. \tag{26}$$

For $b \in (b_y, b_m)$, we have $F(m^{***}(b), b) > 0$ and $m^{***}(b) < m^{***}(b_m) < m^*(b_m) = \tilde{m}$. Since $F(\tilde{m}, b) < 0$ is the minimum value of $F(m, b)$, the fact that $F(m^{***}(b), b) > 0$ implies that $m^{***}(b) < m_l(b) < \tilde{m}$. Thus conditions (16) and (20) are satisfied for $b \in (b_y, b_m)$.

Therefore, the open interval, $(\max\{b_z, b_y\}, b_m)$, is not a null set, and the condition for multiplicity (16) and for the Nash stability for both equilibria, (19) and (20), are all satisfied for $b \in (\max\{b_z, b_y\}, b_m)$.

Numerical example: In the case where $\varepsilon = \sigma = 6$, we have that

$$\frac{b_z}{b_m} = \frac{4}{3 \times 2^{\frac{2}{3}}} = 0.8399,$$
$$\frac{b_y}{b_m} = \frac{4 \times \left(\frac{3}{5}\right)^{\frac{2}{3}} \left(\frac{5}{6}\right)^{\frac{2}{5}}}{3} = 0.8817.$$

Thus, $\frac{\max\{b_y, b_z\}}{b_m} = 0.8817$.

Finally, we confirm that the high equilibrium (m_h, n_h) is always Nash stable. For b that satisfies $b < b_m$, $F(\tilde{m}) < 0$ and there exists the high equilibrium (m_h, n_h) that solves $F(m_h) = 0$ and satisfies $m_h \geq \tilde{m}$. As $m^*(b) > \max\{m^{**}(b), m^{***}(b)\}$ for all b and $\tilde{m} > m^*(b)$ for $b < b_m$, it is the case that $m_h > \max\{m^{**}(b), m^{***}(b)\}$ for $b < b_m$. This inequality implies that both $\frac{\partial}{\partial m}\pi_z(m_h, n_h) < 0$ and $\frac{\partial}{\partial n}\pi_y(m_h, n_h) < 0$ are satisfied for $b < b_m$, meaning that the high equilibrium (m_h, n_h) is always Nash stable if it exists.