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Trend inflation, asset prices and monetary policy*

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Abstract

The main objective of this paper is to investigate monetary policy response to asset price in a sticky price economy where the trend inflation rate is non-zero. We find that monetary policy response to asset price is helpful for achieving equilibrium determinacy if the trend inflation is negative (i.e., deflation) and sufficiently low. If this is not the case, monetary policy response to asset price becomes a source of equilibrium indeterminacy. We also find that monetary policy response to asset price can be helpful for equilibrium determinacy even if the trend inflation is positive in the case where the nominal wage is also sticky, and the parameter values are consistent with recent micro evidence.

Keywords: Trend inflation; asset prices; equilibrium indeterminacy; monetary policy; deflation; sticky price

JEL classifications: E31; E32; E52; E58

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1 Introduction

The central bank's stance with regard to asset price fluctuations is one of the classic policy considerations. The Japanese economy experienced a boom-bust cycle of asset prices during the period from the latter half of the 1980s to the 1990s. In the U.S., a large fluctuation of asset prices occurred during the period prior to and post the Great Recession. This highlights the need for the central bank to focus more on the fluctuation of asset prices.

Seminal work on this topic has been done by Carlstrom and Fuerst (2007). They found that equilibrium indeterminacy arises if monetary policy responds to the share prices, as asset price, in a standard sticky price model. Economic fluctuations are caused by non-fundamental expectational shocks under the indeterminacy situation. Equilibrium indeterminacy then implies a difficulty in predicting what happens in an economy after a shock. Therefore, a monetary policy causing equilibrium indeterminacy should not be adopted by the central bank.

Existing research on this theme, including Carlstrom and Fuerst (2007), assumes that the trend inflation is zero, while the actual trend inflation is not zero. In many developed countries, the trend inflation rate is about 2%. However, the trend inflation rate in the Japanese economy might be negative considering that Japan has suffered from long-term deflation since the late 1990s. It would thus be important to analyze this theme using a model with non-zero trend inflation.

The main objective of this paper is to study the effect of monetary policy response to the share price in a sticky price economy with non-zero trend inflation. We develop a sticky price model with shares of firms and non-zero trend inflation. Following Carlstrom and Fuerst (2007), the share price is focused on as the asset price, reflecting firm profit. We find that monetary policy response to asset price is helpful for achieving equilibrium determinacy if the trend inflation is negative (i.e., deflation) and sufficiently low; or else, monetary policy response to asset price is a source of equilibrium indeterminacy.

We also find that monetary policy response to asset price can be helpful for achieving equilibrium determinacy even if the trend inflation is positive in the case where the nominal wage is also sticky and where the parameter values on the sticky wage are consistent with the recent micro evidence. Our results imply that the central bank's response to asset price is justified in an environment characterized by low trend inflation, such as that in Japan in the context of long-term deflation.

To understand the results, the focus should be on the relationship between inflation and asset price. It is well known that the central bank should fight inflation for equilibrium determinacy in a standard sticky price model. If an increase in inflation decreases asset price, monetary policy response to asset price weakens the overall reaction of the central bank to inflation. Carlstrom and Fuerst's (2007) indeterminacy result comes from that an increase in inflation causes the decline of the firm's profit and the share price.

Non-zero trend inflation changes this relationship between inflation and share price through price dispersion. Price dispersion itself has negative effects on the firm's profit and the share price. An increase in inflation increases price dispersion if the trend inflation is positive. Consequently, the negative effect of an increase in inflation on the share price is strengthened. In contrast, an increase in inflation decreases price dispersion if the trend inflation is negative. Consequently, an increase in inflation works as a channel that increases the firm's profit and share prices in this case.

The sticky wage functions as this channel, whereby an increase in inflation increases the firm's profit and share price. Thus, even if trend inflation is positive, monetary policy response to the share price can be helpful for achieving equilibrium determinacy. This mechanism has already been found by Carlstrom and Fuerst(2007). However, the negative effect of an increase in inflation on the share price through the sticky price dominates the positive effect through the sticky wage under the parameter values employed by Carlstrom and Fuerst(2007). We find that the positive effect through the sticky wage can dominate the negative one through the sticky price under the parameter values, which are consistent with recent micro evidence.

There are two strands of literature with regard to the analysis of monetary policy in response to asset price fluctuations. One is the evaluation of a policy from the viewpoint of equilibrium determinacy, as in the present paper. Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) find that a monetary policy responding to asset prices is a source of equilibrium indeterminacy. Bullard and Schaling (2002) use one-period claims to random nominal quantities as the asset, and Carlstrom and Fuerst (2007) use share. Nutahara (2014) investigates the difference in the effects of monetary policy responses to the share price and to the capital price. Nutahara (2015) focuses on the effect of credit market imperfection on monetary policy response to asset price. The second strand is the evaluation of a policy from the viewpoint of welfare or of variances in output and inflation. Bernanke and Gertler (2001), Gilchrist and Leahy (2002), Iacoviello (2005), and Faia and Monacelli (2007) employ this approach. These papers assume that the trend inflation is zero.

The role of non-zero trend inflation is addressed by several studies, including those of Acari (2004), Amano et al. (2009), Ascari and Ropele (2009), Ascari and Sbordone (2014), Kurozumi (2014), Kurozumi and Zandweghe (2017), Phaneuf and Victor (2019), and Khan, Phaneuf and Victor (2020). Ascari and Ropele (2009), Kurozumi and Zandweghe (2017) and Khan, Phaneuf and Victor (2020) closely relate to the current study as they focus on equilibrium determinacy in a sticky price economy with non-zero trend inflation. However, they do not consider monetary policy response to asset price. The main contribution of this paper is to bridge the roles of non-zero trend inflation and monetary policy response to the asset price.

The remainder of the paper is organized as follows: Section 2 introduces the baseline model. Section 3 shows the main result and its interpretation. Section 4 extends the model by introducing sticky wages, and shows that monetary policy response is helpful for equilibrium determinacy even if the trend inflation rate is positive. Section 5 concludes.

2 The Baseline Model

In the baseline model, nominal prices are sticky, but nominal wages are flexible. The nominal wage stickiness is introduced in the extended model in Section 4.

Final-good firms: The final-good market is perfectly competitive. The final-good firm produces a final good Y_t using intermediate goods $Y_t(j)$. The production function is given by

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta_P - 1}{\theta_P}} dj \right]^{\frac{\theta_P}{\theta_P - 1}}, \tag{1}$$

where θ_P is the elasticity of substitution among intermediate goods. Letting $P_t(j)$ denote the price of intermediate goods $Y_t(j)$, the profit maximization problem is to maximize

$$P_t Y_t - \int_0^1 P_t(j) Y_t(j), \tag{2}$$

subject to the equation (1). The first-order condition implies the demand function of intermediate goods j as follows:

$$Y_t(j) = \left[\frac{P_t(j)}{P_t} \right]^{-\theta_P} Y_t. \tag{3}$$

The aggregate price level is given by

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta_P} dj \right)^{\frac{1}{1-\theta_P}}. \tag{4}$$

Intermediate-good firms: The intermediate-good market is monopolistically competitive. The intermediate-good firm indexed by j produces differentiated intermediate good $Y_t(j)$ using labor input $L_t(j)$. The production function is given by

$$Y_t(j) = L_t(j). \tag{5}$$

Letting W_t denote the real wage rate, the cost minimization problem is to minimize

$$W_t L_t(j) \tag{6}$$

subject to the equation (5). The first-order condition implies

$$W_t = MC_t, \tag{7}$$

where MC_t is the Lagrange multiplier of the constraint and it can be interpreted as the real marginal cost of the intermediate-good firm.

Sticky prices are introduced as in Calvo (1983). At every period, a fraction $\xi_P \in [0,1]$ of intermediate-good firms can reoptimize their prices. The remainder of the firms do not change their prices. This is consistent with micro evidence by Nakamura and Steinsson (2008). The objective function of the intermediate-good firms that reoptimize their prices at period t is

$$E_t \sum_{s=0}^{\infty} (\beta\xi_P)^s \left(\frac{\Lambda_{t+s}}{\Lambda_t}\right) \left[\left(\frac{P_t(j)}{P_{t+s}}\right) Y_{t+s}(j) - TC(Y_{t+s}(j))\right], \tag{8}$$

where Λ_t is the marginal utility of consumption of households, $TC(\cdot)$ is the total cost function, and $\beta^s \frac{\Lambda_{t+s}}{\Lambda_t}$ is the stochastic discount factor. The demand function for $Y_{t+s}(j)$ is given by the equation (5).

The reoptimized price P_t^o is the same for all intermediate-good firms. The first-order condition for reoptimized price P_t^o is

$$1 = \frac{E_t \sum_{s=0}^{\infty} (\beta\xi_P)^s \theta_P MC_{t+s} \Lambda_{t+s} Y_{t+s} \left[\frac{P_t^0}{P_{t+s}}\right]^{-\theta_P}}{E_t \sum_{s=0}^{\infty} (\beta\xi_P)^s (\theta_P - 1) \Lambda_{t+s} Y_{t+s} \left[\frac{P_t^0}{P_{t+s}}\right]^{1-\theta_P}}. \tag{9}$$

Letting $\pi_t^{\#} = P_t^o / P_{t-1}$ denote the reset price inflation, the equation (9) is written as

$$\pi_t^{\#} = \left(\frac{\theta_P}{\theta_P - 1}\right) \pi_t \left(\frac{x_{1,t}^P}{x_{2,t}^P}\right), \tag{10}$$

where

$$x_{1,t}^P = \Lambda_t MC_t Y_t + \beta\xi_P E_t \pi_{t+1}^{\theta_P} x_{1,t+1}^P, \tag{11}$$

$$x_{2,t}^P = \Lambda_t Y_t + \beta\xi_P E_t \pi_{t+1}^{\theta_P - 1} x_{2,t+1}^P. \tag{12}$$

Household: The household indexed by $h \in [0,1]$ consumes $C_t(h)$, holds safe asset $B_t(h)$ and share of capital $S_t(h)$, and supplies labor service $\ell_t(h)$.

The utility function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t(h)^{1-\sigma} - 1}{1-\sigma} - \frac{L_t(h)^{1+\chi}}{1+\chi} \right], \tag{13}$$

where $\beta \in (0,1)$ denotes the discount factor; $\sigma > 0$, the elasticity of intertemporal substitution; and $\chi > 0$, the inverse of the labor supply.

The budget constraint is

$$P_t C_t(h) + B_{t+1}(h) + P_t Q_t S_{t+1}(h) \leq R_t B_t(h) + P_t W_t L_t(h) + P_t(Q_t + D_t) S_{t+1}(h) + T_t, \tag{14}$$

where P_t denotes the price level; C_t consumption; B_t nominal bond holding; Q_t (real) asset price; S_t share holding; R_t gross nominal interest rate from bond holding; W_t real wage; and T_t transfer from the government.

Because of complete insurance markets, the decisions of $C_t(h)$, $\ell_t(h)$, $B_t(h)$, and $S_t(h)$ are the same for all households; the first-order conditions are then given by

$$\Lambda_t = C_t^{-\sigma} \tag{15}$$

$$\chi L_t = \Lambda_t W_t \tag{16}$$

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} \cdot \frac{R_t}{\pi_{t+1}} \right], \tag{17}$$

$$\Lambda_t Q_t = \beta E_t \left[\Lambda_{t+1} \cdot (Q_{t+1} + D_{t+1}) \right], \tag{18}$$

where Λ_t is the marginal utility of consumption; and $\pi_{t+1} = P_{t+1}/P_t$, the gross price inflation rate. The equation (18) is rewritten as

$$Q_t = E_t \left[\frac{Q_{t+1} + D_{t+1}}{R_t/\pi_{t+1}} \right]. \tag{19}$$

Central bank: The central bank controls the nominal interest rate R_t following an extended Taylor rule:

$$R_t = \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{Q_t}{Q} \right)^{\phi_Q}, \tag{20}$$

where $\phi_\pi \geq 0$ and $\phi_Q \geq 0$ denote the central bank's stance with regard to inflation and asset price. If $\phi_Q > 0$, monetary policy responds to asset price fluctuation.[1]

Aggregations and market clearing conditions: The aggregate price level P_t is the weighted average of the reset price P_t^O and the past price level P_{t-1}:

$$P_t^{1-\theta_P} = (1-\xi_P)P_t^O + \xi_P P_{t-1}^{1-\theta_P}, \tag{21}$$

and it is rewritten as

$$\pi_t^{1-\theta_P} = (1-\xi_P)(\pi_t^{\#})^{1-\theta_P} + \xi_P. \tag{22}$$

The good-market clearing condition is

$$Y_t = C_t. \tag{23}$$

The aggregate production function is given by

$$Y_t = \Delta_t^P L_t, \tag{24}$$

where Δ_t^P denotes the price dispersion defined as

$$\Delta_t^P = \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\theta_P} df. \tag{25}$$

The evolution of Δ_t^P is given by

$$\Delta_t^P = (1-\xi_P)\left(\frac{\pi_t^{\#}}{\pi_t}\right)^{-\theta_P} + \xi_P \pi_t^{\theta_P} \Delta_{t-1}^P. \tag{26}$$

The supply of corporate share is constant, and the market clearing condition of share is given by

$$S_t = \bar{S}. \tag{27}$$

[1]For simplicity of analysis, the term on output is omitted from our monetary policy rule. Even in the case where monetary policy responds to output, our main result does not change qualitatively.

As employed by Carlstrom and Fuerst (2007), the monopolistic rent of intermediate-good firms is paid to households as dividend, and is given by

$$D_t = \int_0^1 \left[\left(\frac{P_t(j)}{P_t} \right) Y_t(j) - MC_t Y_t(j) \right] dj, \tag{28}$$

in the current model. By the equations (3), (4), (24), and (25), the equation (28) is rewritten as

$$\begin{aligned} D_t &= \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{1-\theta_P} dj\, Y_t - MC_t Y_t \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\theta_P} dj \\ &= (1 - MC_t \Delta_t^P) Y_t. \end{aligned} \tag{29}$$

The equation (29) implies that the change in the price dispersion Δ_t^P affects the dividend D_t that affects the asset price Q_t as in the equation (19).

Equilibrium conditions: The equilibrium is a set of prices and quantities (C_t, Y_t, L_t, Λ_t, W_t, π_t, $\pi_t^{\#}$, Q_t D_t, R_t, MC_t, Δ_t^P, $x_{1,t}^P$, $x_{2,t}^P$), that satisfies the equations (7), (10), (11), (12), (15), (16), (17), (19), (20), (22), (23), (24), (26), and (29).

The log-linearized equilibrium system is given by

$$\hat{\Lambda}_t = -\sigma \hat{C}_t, \tag{30}$$

$$-\hat{\Lambda}_t + \chi \hat{L}_t = \hat{W}_t, \tag{31}$$

$$\hat{\Lambda}_t = \hat{\Lambda}_{t+1} - \hat{\pi}_{t+1} + \hat{R}_t, \tag{32}$$

$$\hat{Q}_t = \beta \hat{Q}_{t+1} + (1-\beta)\hat{D}_{t+1} + \hat{\pi}_{t+1} - \hat{R}_t, \tag{33}$$

$$\hat{C}_t = \hat{Y}_t, \tag{34}$$

$$\hat{W}_t = \widehat{MC}_t, \tag{35}$$

$$\hat{Y}_t = \hat{L}_t - \hat{\Delta}_t^P, \tag{36}$$

$$\hat{\pi}_t^{\#} = \hat{\pi}_t + \hat{x}_{1,t}^P - \hat{x}_{2,t}^P, \tag{37}$$

$$\hat{x}_{1,t}^P = (1 - \beta\xi_P \pi^{\theta_P})(\hat{\Lambda}_t + \widehat{MC}_t + \hat{Y}_t) + \beta\xi_P \pi^{\theta_P} [\theta_P \hat{\pi}_{t+1} + \hat{x}_{1,t+1}^P], \tag{38}$$

$$\hat{x}^P_{2,t} = (1-\beta\xi_P\pi^{\theta_P-1})(\hat{\Lambda}_t + \hat{Y}_t) + \beta\xi_P\pi^{\theta_P-1}[(\theta_P-1)\hat{\pi}_{t+1} + \hat{x}^P_{2,t+1}], \tag{39}$$

$$\hat{\pi}_t = (1-\xi_P\pi^{\theta_P-1})\hat{\pi}^{\#}_t, \tag{40}$$

$$\hat{\Delta}^P_t = \theta_P\hat{\pi}_t - \theta_P(1-\xi_P\pi^{\theta_P})\hat{\pi}^{\#}_t + \xi_P\pi^{\theta_P}\hat{\Delta}^P_{t-1}, \tag{41}$$

$$\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_Q\hat{Q}_t, \tag{42}$$

$$\hat{D}_t = \hat{Y}_t - \frac{MC\Delta^P}{1-MC\Delta^P}(\widehat{MC}_t + \hat{\Delta}^P_t), \tag{43}$$

where $\hat{A}_t$ denotes the log-deviation from its steady-state value: $\hat{A}_t = \log(A_t) - \log(A)$. The steady-state values of the real marginal cost MC and the price dispersion Δ^P are given by

$$MC = \frac{\pi^{\#}}{\pi}\left(\frac{\theta_P-1}{\theta_P}\right)\left(\frac{1-\xi_P\beta\pi^{\theta_P-1}}{1-\xi_P\beta\pi^{\theta_P}}\right), \tag{44}$$

$$\Delta^P = \frac{(1-\xi_P)(\pi^{\#})^{-\theta_P}\pi^{\theta_P}}{1-\xi_P\pi^{\theta_P}}, \tag{45}$$

where

$$\pi^{\#} = \left(\frac{\pi^{1-\theta_P}-\xi_P}{1-\xi_P}\right)^{\frac{1}{1-\theta_P}}. \tag{46}$$

This equilibrium system can be simplified as follows:

$$-\sigma\hat{Y}_t = -\sigma\hat{Y}_{t+1} - \hat{\pi}_{t+1} + \hat{R}_t, \tag{47}$$

$$\hat{Q}_t = \beta\hat{Q}_{t+1} + (1-\beta)\hat{D}_{t+1} + \hat{\pi}_{t+1} - \hat{R}_t, \tag{48}$$

$$\frac{\xi_P\pi^{\theta_P-1}}{1-\xi_P\pi^{\theta_P-1}}\hat{\pi}_t = \hat{x}^P_{1,t} - \hat{x}^P_{2,t}, \tag{49}$$

$$\hat{x}^P_{1,t} = (1-\beta\xi_P\pi^{\theta_P})[(1+\chi)\hat{Y}_t + \chi\hat{\Delta}^P_t] + \beta\xi_P\pi^{\theta_P}[\theta_P\hat{\pi}_{t+1} + \hat{x}^P_{1,t+1}], \tag{50}$$

$$\hat{x}^P_{2,t} = (1-\beta\xi_P\pi^{\theta_P-1})(1-\sigma)\hat{Y}_t + \beta\xi_P\pi^{\theta_P-1}[(\theta_P-1)\hat{\pi}_{t+1} + \hat{x}^P_{2,t+1}], \tag{51}$$

$$\hat{\Delta}^P_t = \frac{\theta_P\xi_P(\pi^{\theta_P} - \pi^{\theta_P-1})}{1-\xi_P\pi^{\theta_P-1}}\hat{\pi}_t + \xi_P\pi^{\theta_P}\hat{\Delta}^P_{t-1}, \tag{52}$$

$$\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_Q\hat{Q}_t, \tag{53}$$

$$\hat{D}_t = \hat{Y}_t - \frac{MC\Delta^P}{1-MC\Delta^P}[(\sigma+\chi)\hat{Y}_t + (1+\chi)\hat{\Delta}^P_t]. \tag{54}$$

3 Main Results

We employ numerical methods to consider the determinacy region of equilibrium because the baseline model is too complex to solve analytically.

3.1 Parameter values

The parameter values are those taken as standard in the literature. The model period is one quarter. The discount factor β is set to be 0.99 such that the annual real interest rate is 4%. The relative risk aversion σ is set to be 1. The Frisch elasticity of labor supply χ is set to be 1. The elasticity of substitution among intermediate-good θ_P is 6. Under this value, the steady state markup rate with zero inflation steady state is 20%, as in Khan, Phaneuf and Victor (2020). This markup rate is consistent with micro evidence from De Loecker and Warzynski (2012). The reset price probability $1 - \xi_P$ is 0.34, following Khan, Phaneuf and Victor (2020). This value is consistent with estimates by Smets and Wouters (2007) and micro evidence from Nakamura and Steinsson (2008).

3.2 Main results

Figure 1 shows the determinacy region of equilibrium if the trend inflation is positive ($\pi = 1.01$). The horizontal axis is the central bank's stance with regard to inflation ϕ_π, and the vertical axis is the central bank's stance with regard to the share price ϕ_q. The region with diamonds indicates equilibrium determinacy, and the other region indicates either equilibrium indeterminacy.

Figure 1 implies that an increase in ϕ_q shrinks the determinacy region of ϕ_π. The monetary policy response to share prices is a source of equilibrium indeterminacy in this case. This aligns with the results of Carlstrom and Fuerst (2007) and Nutahara (2014) highlighting the focus on the steady state with zero inflation.

[Figure 1]

Figure 2 is the analogue of Figure 1 if the trend inflation is negative ($\pi = 0.99$). Contrary to Figure 1, an increase in ϕ_q enlarges the determinacy region of ϕ_π. The monetary policy response to share prices benefits achieving equilibrium determinacy in this case. The result presented in Figure 2 is thus unique, and it contrasts with previous studies, including that of Carlstrom and Fuerst (2007) and Nutahara (2014), that focus on zero trend inflation and monetary policy response to share prices causing equilibrium indeterminacy.

[Figure 2]

3.3 Interpretation

As discussed by Carlstrom and Fuerst (2007) and Nutahara (2014, 2015), it is useful to consider the effects of permanent increase in inflation on the asset price. It is well known that the central bank should increase nominal interest rate more than the increase in inflation for equilibrium determinacy. This is known as the Taylor principle. If the trend inflation is not zero, it is known that the Taylor principle no longer guarantees equilibrium determinacy, as shown by Ascari and Ropele (2009). Even in that case, the central bank should increase the nominal interest rate sufficiently for equilibrium determinacy.

The key factor is the effect of a permanent increase in inflation on the asset price. If an increase in inflation increases the asset price, monetary policy response to asset price is beneficial from the viewpoint of equilibrium determinacy. This is because monetary policy response to asset price strengthens the overall reaction of the central bank to inflation. On the other hand if an increase in inflation decreases the asset price, monetary policy response to asset price is a source of equilibrium indeterminacy because it weakens the overall reaction of the central bank to inflation.

Let us suppose a one-percent permanent increase in inflation rate: $\hat{\pi}_t = \hat{\pi}_{t+1} = \hat{\pi}$. By the equation (52), it is obtained

$$\hat{\Delta}^P = \frac{\theta_P \xi_P (\pi^{\theta_P} - \pi^{\theta_{P-1}})}{(1 - \xi_P \pi^{\theta_P - 1})(1 - \xi_P \pi^{\theta_P})} \hat{\pi}. \tag{55}$$

This equation implies that the price dispersion $\hat{\Delta}^P$ increases if the trend inflation is positive $\pi > 1$, and the price dispersion decreases if the trend inflation is negative $\pi < 1$. By the equations (49), (50) and (51), the increase in the dividend $\hat{D}$ (to a permanent increase in inflation) is given by

$$\hat{D} = \Phi(\pi, \beta, \sigma, \chi, \xi_P, \theta_P)\hat{\pi}, \tag{56}$$

where $\Phi(\pi, \beta, \sigma, \chi, \xi_P, \theta_P)$ is the coefficient of $\hat{\pi}$, and it is function of π, β, σ, χ, ξ_P and θ_P. Since the functional form of $\Phi(\pi, \beta, \sigma, \chi, \xi_P, \theta_P)$ is very complex, we employ the numerical result here.

Figure 3 shows the effects of permanent increase in inflation on the dividend $\hat{D}$ using the equation (56). The horizontal axis is the steady state inflation rate (trend inflation rate) π, and the vertical axis is the rate on the increase in $\hat{D}$ caused by a one-percent permanent increase in inflation rate. The parameter values are the same as those explained in section 3.1.

[Figure 3]

Figure 3 implies that the higher the trend inflation, the lower the effect of permanent increase in inflation on the dividend. Specifically, the effect on the dividend becomes positive if the trend inflation rate is sufficiently low. The threshold value is about $\pi = 0.995$ given other parameter values. As in the equation (19) and its log-linearized version (48), the share price is a discounted sum of the dividend. Consequently, the increase (decrease) in the dividend implies the increase (decrease) in the share price.

The effect of a permanent increase in inflation on the share price depends on the trend inflation rate. In particular, if the trend inflation is negative and sufficiently low, a permanent increase in inflation increases the share price. In this case, monetary policy response to share price is helpful for achieving equilibrium determinacy.

The threshold inflation rate $\pi = 0.995$ implies 2% deflation per year. This trend inflation rate seems to be unrealistic. For example, Hirose (2018) estimates the trend inflation rate for the Japanese economy during the period from 1999 to 2013, to be about -1.2% per year. In the next section, the model is extended by introducing sticky wage. It shows that monetary policy response to share price is justified even if the trend inflation rate is positive.

4 Introducing Sticky Wages

The result in Section 3 is based on the model where only nominal price is sticky. Additionally, we introduce here the nominal wage stickiness as well as nominal price stickiness.

4.1 Sticky prices–wages model

In the extended model, nominal wage stickiness is introduced. In this version, the household indexed by $h \in [0, 1]$ supplies differentiated labor service $\ell_t(h)$ to the intermediate-good firms. The labor market is monopolistically competitive. The intermediate-good firm f aggregates their labor inputs $L_t(f, h)$ according to the following technology:

$$L_t(f) = \left[\int_0^1 \ell_t(f, h)^{\frac{\theta_W - 1}{\theta_W}} dh \right]^{\frac{\theta_W}{\theta_W - 1}}, \tag{57}$$

where θ_W is the elasticity of substitution between labor supplies. The cost minimization of the intermediate-good firm and the aggregation over intermediate-good firms imply the demand function of labor $\ell_t(h)$ as

$$\ell_t(h) = \left[\frac{NW_t(h)}{P_t W_t} \right]^{-\theta_W} L_t, \tag{58}$$

where $NW_t(h)$ is the nominal wage of the household j.

The Calvo-type sticky wages are introduced as in the study by Erceg, Henderson and Levine (2000). At every period, a fraction $\xi_W \in [0, 1]$ of households can reoptimize their nominal wages. The rest of households do not change their nominal wages. This is consistent with micro evidence rendered by Barattieri, Basu and Gottschalk (2014). The utility function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t(h)^{1-\sigma} - 1}{1-\sigma} - \frac{\ell_t(h)^{1+\chi}}{1+\chi} \right], \tag{59}$$

as in the baseline model. The objective function of the nominal wage setting problem is

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j \left[\Lambda_{t+j} \ell_{t+j}(h) \left(\frac{NW_t(h)}{P_{t+j}} \right) - \frac{\ell_{t+j}(h)^{1+\chi}}{1+\chi} \right], \tag{60}$$

and the labor demand function is given by

$$\ell_{t+j}(h) = \left[\frac{NW_t(h)}{P_{t+j} W_{t+j}} \right]^{-\theta_W} L_{t+j}. \tag{61}$$

The reoptimized wage NW_t^o is the same for all households. The first-order condition for reoptimized wage NW_t^o is

$$1 = \frac{E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j \theta_W \left(L_{t+j} \left[\frac{NW_t^0}{P_{t+j} W_{t+j}} \right]^{-\theta_W} \right)^{1+\chi}}{E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j (\theta_W - 1) \Lambda_{t+j} W_{t+j} L_{t+j} \left[\frac{NW_t^o}{P_{t+j} W_{t+j}} \right]^{1-\theta_W}}. \tag{62}$$

Letting $W_t^\# = NW_t^o / P_t$ denote the reset real wage, the equation (62) is written as

$$W_t^\# = \left(\frac{\theta_W}{\theta_W - 1} \right) \left(\frac{x_{1,t}^W}{x_{2,t}^W} \right), \tag{63}$$

where

$$x_{1,t}^W = (W_t)^{\theta_W(1+\chi)} L_t^{1+\chi} + \beta \xi_W E_t (\pi_{t+1})^{\theta_W(1+\chi)} x_{1,t+1}^W, \tag{64}$$

$$x_{2,t}^W = \Lambda_t (W_t)^{\theta_W} L_t + \beta \xi_W E_t (\pi_{t+1})^{\theta_W - 1} x_{2,t+1}^W. \tag{65}$$

The aggregate real wage level W_t is the weighted average of the reset real wage and the past real wage:

$$W_t^{1-\theta_W} = (1-\xi_W)(W_t^{\#})^{1-\theta_W} + \xi_W W_{t-1}^{1-\theta_W} \pi_t^{\theta_W - 1}. \tag{66}$$

The labor market clearing condition is given by

$$N_t = \Delta_t^W L_t, \tag{67}$$

where $N_t = \int_0^1 \ell_t(h)dh$ is observable aggregate labor supply, and Δ_t^W denotes the wage dispersion, that is defined as

$$\Delta_t^W = \int_0^1 \left(\frac{W_t(h)}{W_t}\right)^{-\theta_W} dh. \tag{68}$$

The evolution of Δ_t^W is given by

$$\Delta_t^W = (1-\xi_W)\left(\frac{W_t^{\#}}{W_t}\right)^{-\theta_W} + \xi_W \pi_t^{\theta_W} \left(\frac{W_{t-1}}{W_t}\right)^{-\theta_W} \Delta_{t-1}^W. \tag{69}$$

Equilibrium conditions: The equilibrium is a set of prices and quantities (C_t, Y_t, L_t, N_t, Λ_t, W_t, $W_t^{\#}$, π_t, $\pi_t^{\#}$, Q_t D_t, R_t, MC_t, Δ_t^P, Δ_t^W, $x_{1,t}^P$, $x_{2,t}^P$, $x_{1,t}^W$, $x_{1,t}^W$), that satisfies the equations (7), (10), (11), (12), (15), (17), (19), (20), (22), (23), (24), (26), (29). (63), (64), (65), (66), (67), and (68).

The log-linearized equilibrium system is given by

$$\hat{\Lambda}_t = -\sigma \hat{C}_t, \tag{70}$$

$$\hat{\Lambda}_t = \hat{\Lambda}_{t+1} - \hat{\pi}_{t+1} + \hat{R}_t, \tag{71}$$

$$\hat{Q}_t = \beta \hat{Q}_{t+1} + (1-\beta)\hat{D}_{t+1} + \hat{\pi}_{t+1} - \hat{R}_t, \tag{72}$$

$$\hat{C}_t = \hat{Y}_t, \tag{73}$$

$$\hat{W}_t = \widehat{MC}_t, \tag{74}$$

$$\hat{Y}_t = \hat{L}_t - \hat{\Delta}_t^P, \tag{75}$$

$$\hat{\pi}_t^{\#} = \hat{\pi}_t + \hat{x}_{1,t}^P - \hat{x}_{2,t}^P, \tag{76}$$

$$\hat{x}_{1,t}^P = (1 - \beta\xi_P\pi^{\theta_P})(\hat{\Lambda}_t + \widehat{MC}_t + \hat{Y}_t) + \beta\xi_P\pi^{\theta_P}[\theta_P\hat{\pi}_{t+1} + \hat{x}_{1,t+1}^P], \tag{77}$$

$$\hat{x}_{2,t}^P = (1 - \beta\xi_P\pi^{\theta_P-1})(\hat{\Lambda}_t + \hat{Y}_t) + \beta\xi_P\pi^{\theta_P-1}[(\theta_P - 1)\hat{\pi}_{t+1} + \hat{x}_{2,t+1}^P], \tag{78}$$

$$\hat{\pi}_t = (1 - \xi_P\pi^{\theta_P-1})\hat{\pi}_t^{\#}, \tag{79}$$

$$\hat{\Delta}_t^P = \theta_P\hat{\pi}_t - \theta_P(1 - \xi_P\pi^{\theta_P})\hat{\pi}_t^{\#} + \xi_P\pi^{\theta_P}\hat{\Delta}_{t-1}^P, \tag{80}$$

$$(1 + \theta_W\chi)\, \hat{W}_t^{\#} = \hat{x}_{1,t}^W - \hat{x}_{2,t}^W, \tag{81}$$

$$\hat{x}_{1,t}^W = [1 - \beta\xi_W\pi^{\theta_W(1+\chi)}][\theta_W(1+\chi)\hat{W}_t + (1+\chi)\hat{L}_t] + \beta\xi_W\pi^{\theta_W(1+\chi)}[\theta_W(1+\chi)\hat{\pi}_{t+1} + \hat{x}_{1,t+1}^W], \tag{82}$$

$$\hat{x}_{2,t}^W = [1 - \beta\xi_W\pi^{\theta_W-1}][\hat{\Lambda}_t + \theta_W\hat{W}_t + \hat{L}_t] + \beta\xi_W\pi^{\theta_W-1}[(\theta_W - 1)\hat{\pi}_{t+1} + \hat{x}_{2,t+1}^W], \tag{83}$$

$$\hat{W}_t = (1 - \xi_W\pi^{\theta_W-1})\hat{W}_t^{\#} - \xi_W\pi^{\theta_W-1}(\hat{\pi}_t - \hat{W}_{t-1}), \tag{84}$$

$$\hat{N}_t = \hat{\Delta}_t^W + \hat{L}_t, \tag{85}$$

$$\hat{\Delta}_t^W = -\theta_W(1 - \xi_W\pi^{\theta_W})(\hat{W}_t^{\#} - \hat{W}_t) + \xi_W\pi^{\theta_W}[\theta_W(\hat{\pi}_t - \hat{W}_{t-1} + \hat{W}_t) + \hat{\Delta}_{t-1}^W], \tag{86}$$

$$\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_Q\hat{Q}_t, \tag{87}$$

$$\hat{D}_t = \hat{Y}_t - \frac{MC\Delta^P}{1 - MC\Delta^P}(\widehat{MC}_t + \hat{\Delta}_t^P), \tag{88}$$

where $\hat{A}_t$ denotes the log-deviation from its steady-state value: $\hat{A}_t = \log(A_t) - \log(A)$. The steady-state values of the real marginal cost MC and the price dispersion Δ^P are given by

$$MC = \frac{\pi^{\#}}{\pi}\left(\frac{\theta_P - 1}{\theta_P}\right)\left(\frac{1 - \xi_P\beta\pi^{\theta_P-1}}{1 - \xi_P\beta\pi^{\theta_P}}\right), \tag{89}$$

$$\Delta^P = \frac{(1 - \xi_P)(\pi^{\#})^{-\theta_P}\pi^{\theta_P}}{1 - \xi_P\pi^{\theta_P}}, \tag{90}$$

where

$$\pi^{\#} = \left(\frac{\pi^{1-\theta_P} - \xi_P}{1 - \xi_P}\right)^{\frac{1}{1-\theta_P}}. \tag{91}$$

4.2 Determinacy region

As in Section 3, we employ the numerical method to investigate the determinacy region. The reset wage probability $1 - \zeta_W$ is set to be 0.25 following Khan, Phaneuf and Victor (2020). This is consistent with Smets and Wouters (2007) and Barattieri, Basu and Gottschalk (2014). The elasticity of substitution among labor θ_W is set to be 6 following Khan, Phaneuf and Victor (2020). The rests of the parameter values are the same as those in Section 3.1.

Figure 4 shows the determinacy region of equilibrium if the trend inflation is positive but sufficiently low ($\pi = 1.0001$). The horizontal axis is the central bank's stance with regard to inflation ϕ_π, and the vertical axis is the central bank's stance with regard to the share price ϕ_q. The region with diamonds indicates equilibrium determinacy, and the other region indicates either equilibrium indeterminacy. Figure 4 implies that an increase in ϕ_q enlarges the determinacy region of ϕ_π. Thus, the monetary policy response to share prices is beneficial for equilibrium determinacy in this case.

[Figure 4]

This result can be interpreted by the central bank's stance to inflation, as in Section 3.3. The key is the effect of a permanent increase in inflation on the asset price. Figure 5 shows the effects of permanent increase in inflation on the dividend $\hat{D}$ in the sticky price-wage economy. The horizontal axis is the steady state inflation rate (trend inflation rate) π, and the vertical axis is the rate of the increase in $\hat{D}$ caused by one-percent permanent increase in inflation rate. Figure 5 implies that the higher the trend inflation, the lower the effect of permanent increase in inflation on the dividend. In this case, the effect on the dividend can be positive even if the trend inflation rate is positive (but sufficiently low). The threshold value is about $\pi = 1.0004$ given the parameter values. Therefore, even if the trend inflation is positive (but sufficiently low), a permanent increase in inflation increases the share price. It therefore implies that monetary policy response to share price is helpful for achieving equilibrium determinacy under low inflation environment.

[Figure 5]

Carlstrom and Fuerst (2007) find that sticky wage does work as a mechanism where a permanent increase in inflation increases the share price. The wage is the cost for firms, and its stickiness implies that the firm's profit and share price increase if the inflation rate increases. However, under the parameter values employed by Carlstrom and Fuerst (2007), a permanent increase in inflation decreases the share price in the sticky price-wage economy with zero trend inflation. This is because the negative effect of an increase in inflation on the share price through sticky price dominates the positive effect through sticky wage. In contrast, under our parameter values, consistent with the recent literature, a permanent increase in inflation decreases the share price in the sticky price-wage economy even if the trend inflation rate is positive. It implies that the positive effect of an increase in inflation on the share price dominates the negative effect from the sticky price.

5 Concluding Remarks

The main objective of this paper is to investigate the role of non-zero trend inflation for monetary policy response to asset price fluctuation in a sticky price economy. In previously published research, the trend inflation is assumed to be zero, and monetary policy response to the share price is found causing equilibrium indeterminacy. We have found that a monetary policy response to the share price is helpful for achieving equilibrium determinacy if the trend inflation is negative (i.e., deflation) and sufficiently low; otherwise, monetary policy response to asset price becomes a cause of equilibrium indeterminacy. We also find that monetary policy response to the share price can be helpful for equilibrium determinacy even if the trend inflation is positive but sufficiently low in the case where the nominal wage is also sticky and the parameter values are consistent with the recent micro evidence.

The key is the relationship between inflation and asset price. If an increase in inflation decreases the asset price, monetary policy response to asset price becomes a source of equilibrium indeterminacy. If the increase in inflation does not decrease the asset price, the monetary policy response to asset price becomes helpful for equilibrium determinacy. We find that non-zero trend inflation affects this relationship through the change in price dispersion.

Our results imply that the central bank's response to asset price is justified in an environment of low trend inflation, such as that of the long-term deflation in Japan.

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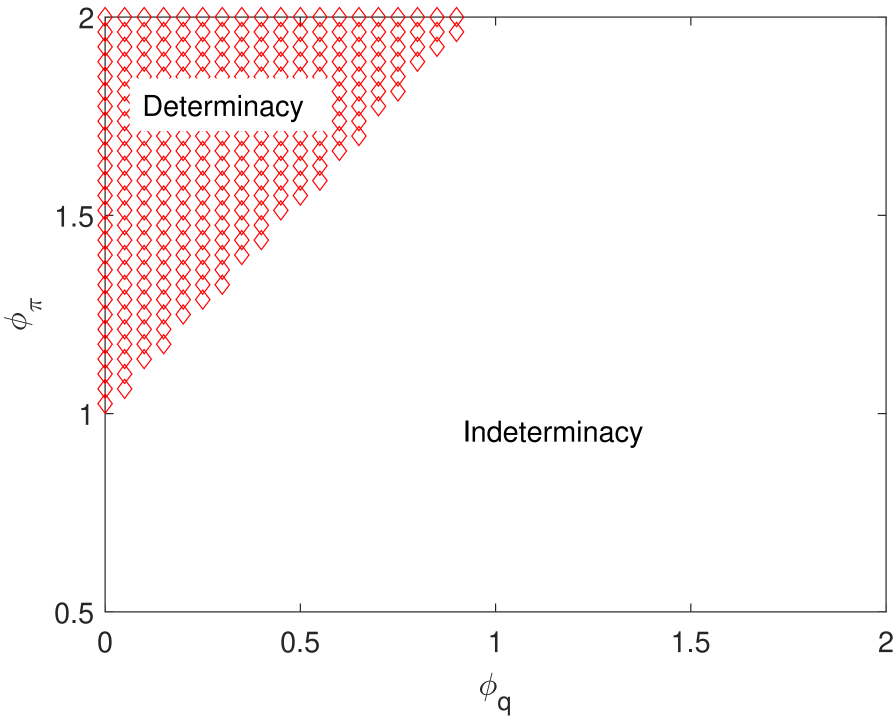
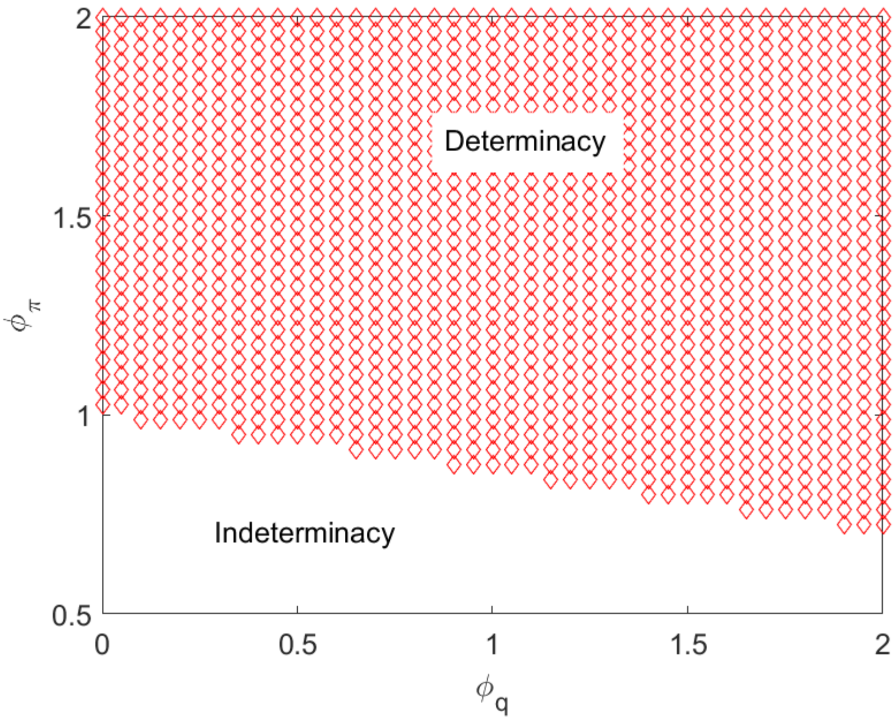
Figure 1: Determinacy region in the sticky price economy with positive trend inflation: $\pi = 1.01$



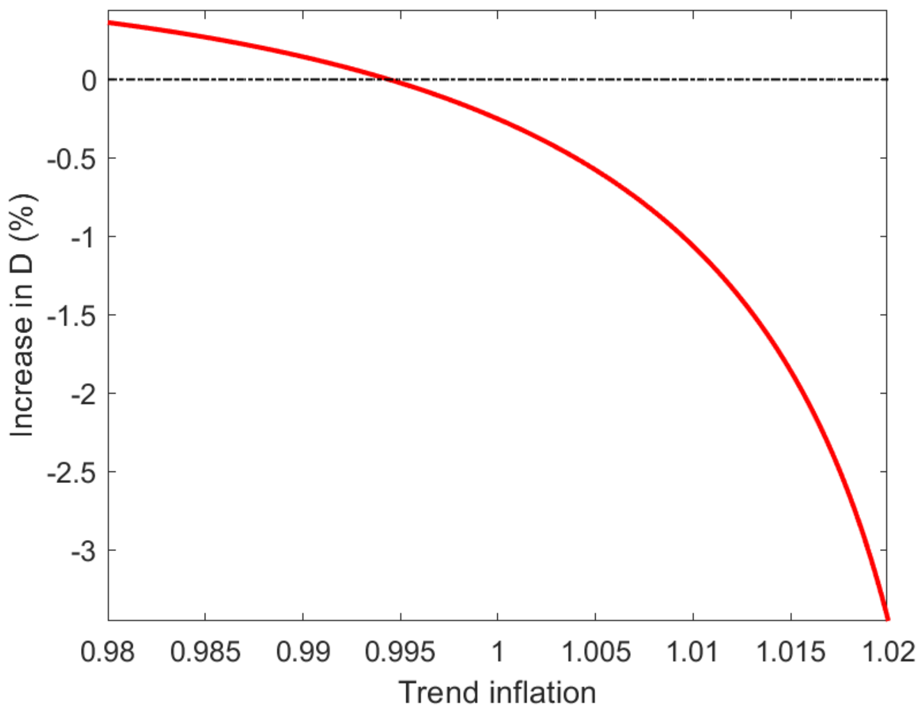
NOTE: The horizontal axis is the central bank's stance to inflation ϕ_π, and the vertical axis is the central bank's stance to the share price ϕ_q. The region with diamonds indicates equilibrium determinacy, and the other region indicates either equilibrium indeterminacy.

Figure 2: Determinacy region in the sticky price economy with negative trend inflation: $\pi = 0.99$



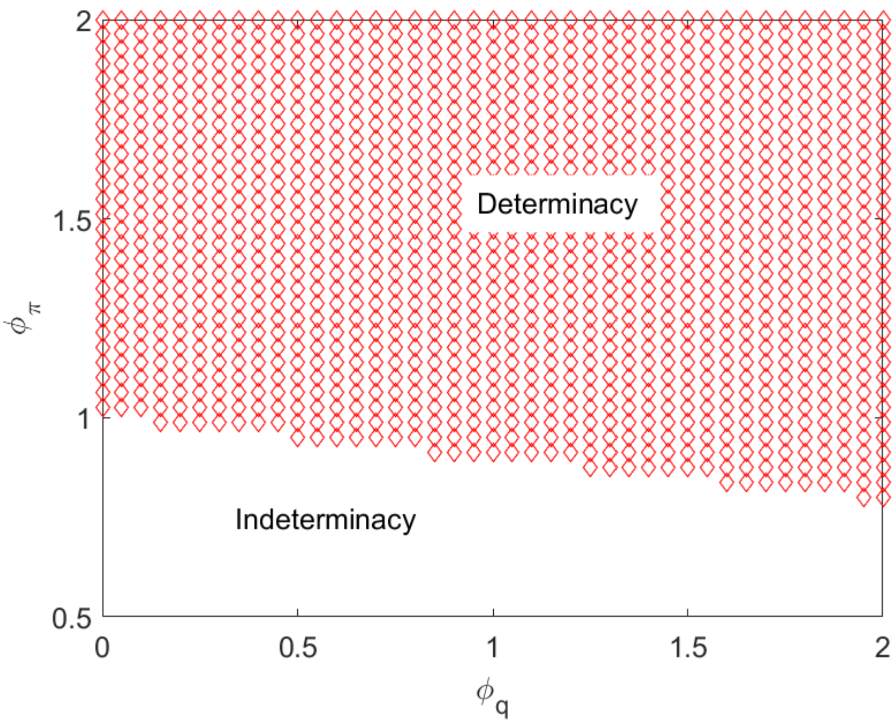
NOTE: The horizontal axis is the central bank's stance to inflation ϕ_π, and the vertical axis is the central bank's stance to the share price ϕ_q. The region with diamonds indicates equilibrium determinacy, and the other region indicates either equilibrium indeterminacy.

Figure 3: Effects of permanent increase in inflation on the dividend $\hat{D}$ (1): Baseline



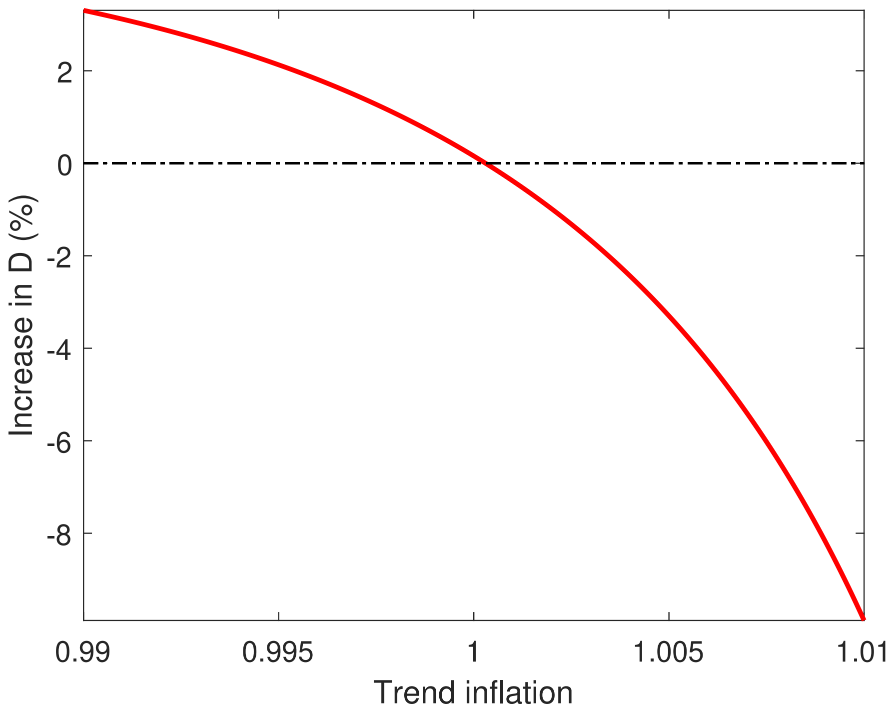
NOTE: The horizontal axis is the steady state inflation rate (trend inflation rate) π, and the vertical axis is the rate on the increase in $\hat{D}$ caused by one-percent permanent increase in inflation rate.

Figure 4: Determinacy region in the sticky price-wage economy with positive trend inflation: $\pi = 1.0001$



NOTE: The horizontal axis is the central bank's stance to inflation ϕ_π, and the vertical axis is the central bank's stance to the share price ϕ_q. The region with diamonds indicates equilibrium determinacy, and the other region indicates either equilibrium indeterminacy.

Figure 5: Effects of permanent increase in inflation on the dividend $\hat{D}$ (2): Sticky price-wage economy



NOTE: The horizontal axis is the steady state inflation rate (trend inflation rate) π, and the vertical axis is the rate on the increase in $\hat{D}$ caused by one-percent permanent increase in inflation rate.