Intergenerational expectations and deflationary equilibrium

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Motivation (1/3)

- We observe decade-long deflationary stagnation
  - The last three decades in Japan
  - Great Recession in the United States
- Puzzling fact:

  Coexistence of deflation and increase in gov't debt (including money)
- Can we have the coexistence as an equilibrium outcome?
  - Total value of debt grows indefinitely
  - Transversality condition (TVC) is not satisfied $\cdots$?
    - TVC: the PDV of debt in the future converges to zero

Motivation (2/3)

- Popular prescription to escape from deflationary recession
  - Increasing gov't debt by monetary and fiscal expansion is effective
    - Krugman (1998); Bernanke (2000); Benhabib, et al. (2002); Eggertsson and Woodford (2003); Auerbach and Obstfeld (2005)
  - All these theories depend on the premise that TVC must be satisfied in equilibrium. The logic goes as follows.
    - Suppose that the gov't makes debt grows at a sufficiently high rate
    - Then, TVC is violated, if deflation continues permanently.
    - As TVC must be satisfied in equilibrium, deflation cannot continue.
  - Now our experience in the last decades seems inconsistent with this logic.

Motivation (3/3)

- Neo-Fisherian explanation
  - Fisher equation implies the low interest rate policy makes deflationary expectations, given that
    - TVC is satisfied because people believe that tax will be increased sufficiently in the future
    - Cochrane (2017), Schmit-Grohe and Uribe (2017)
  - The mainstream does not agree with Neo-Fisherians, because
    - the government can easily commit to the irresponsible policy (Krugman 1998), i.e., no tax increase, and violate TVC

Research Question

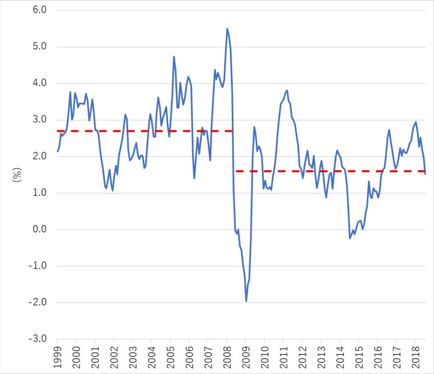
- Can government-debt expansion with the low nominal interest rate (reflationary policy) realize a higher inflation?
  - Answer: Not necessarily.

- Can the economy get stuck in a deflationary equilibrium, given that the government commits itself to the reflationary policy and the commitment is fully trusted?
  - Answer: Yes, it can, even if people believe there will be no tax increase.

- Can the government debt keep growing indefinitely in the deflationary equilibrium?
  - Answer: Yes, it can. ($\Leftrightarrow$ Finite upper bound in rational bubble models)

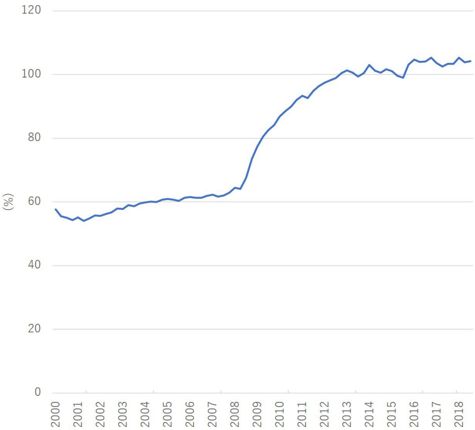
Disinflation in the United States

Figure: CPI inflation rate in US economy



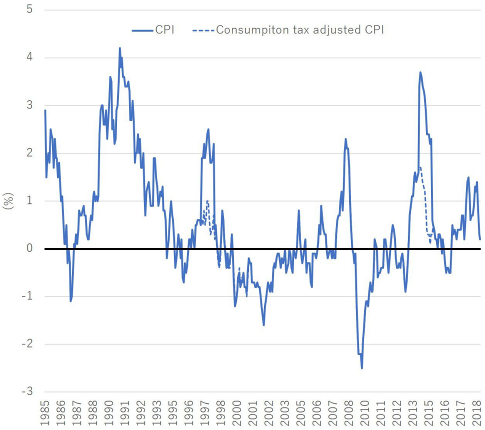
Debt growth in the United States

Figure: Government debt to GDP ratio in the United States



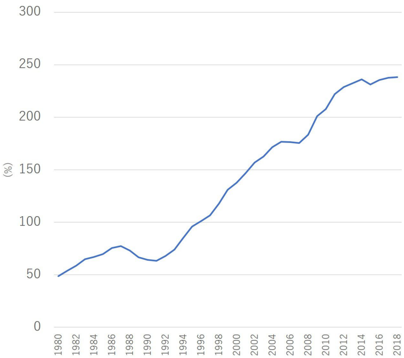
Deflation in Japan

Figure: CPI inflation rate in Japan



Debt growth in Japan

Figure: Debt to GDP ratio in Japan



Summary

- We show deflation with growing debt can be an equilibrium
  - Intergenerational altruism: assets are bequeathed indefinitely
  - (Bubbly) expectations: current generation is happy with money as they believe the future generation will be happy with money
  - TVC is not necessarily satisfied in equilibrium
- Extreme monetary easing may not be effective in fighting deflation
  - Same as the Neo-Fisherian argument: $1 + \pi_t = (1 + i_t)\beta$ with $i_t = 0$ implies $\pi_t < 0$.
  - Secular stagnation can be a steady state generated by the policy $i_t = 0$.

Empirical support

1. TVC is violated in Japan (Doi, 2004)
   - The Bohn condition is not satisfied in Japan for 1965–2000
     - Bohn: When the debt increases, the primary balance should be improved
   - The Bohn condition is a sufficient condition for TVC
2. Intergenerational altruism is present in Japan (Horioka, 2008)
   - The survey shows that 70 percent of the inheritees (parents) are subjectively altruistic to their inheritors (children)
   - The same survey shows that the inheritors feel that 20–30 percent of their inheritees are altruistic
3. Bequest motive increased the household savings, since the mid-2000s in Japan (Hamaaki and Hori, 2018)
   - Zero interest rate since the early 2000s.

Setting (1/2)

- A closed economy with the representative household and the government
- The economy is deterministic. Time is discrete: $t = 0, 1, 2, \cdots, \infty$,
- Representative household with intergenerational altruism
  - are endowed with output y_t every period,
  - the generation t lives only for period t and die at the end of t,
  - period utility for the generation t is $U(c_t)$,
  - the lifetime utility of generation t is $V_t = U(c_t) + \beta W_{t+1}$, where
    - W_{t+1} is the generation t's expectation on the lifetime utility of generation $t+1$
    - β is the degree of altruism (and also the time discount factor)

    - Note: the model can be generalized such that each generation lives N periods, with

$$V_t = \left\{ \sum_{j=0}^{N-1} \beta^j U(c_{t+j}) \right\} + \beta^N W_{t+N}$$

    - We focus on the case where $N = 1$.

Setting (2/2)

- Government issues the nominal bond B^s_{t+1}, i.e., the economy is cashless.
  - bond evolves by
    $$(1+\pi_{t+1})b^s_{t+1} = (1+i_t)b^s_t + \tau_t,$$
    where $b^s_t = \frac{B^s_t}{P_t}$ and $1+\pi_{t+1} = \frac{P_{t+1}}{P_t}$,
  - Gov't conducts fiscal policy, that decides the real transfers (or lump-sum taxes): $\{\tau_{t+j}\}_{j=0}^{\infty}$, assuming that τ_t satisfies
    $$\underline{\tau} \leq \tau_t \leq \bar{\tau},$$
    where $\underline{\tau} < 0 < \bar{\tau}$.
  - Gov't conducts monetary policy, that decides the nominal interest rates: $\{i_{t+j}\}_{j=0}^{\infty}$,
    - i_t satisfies $i_t \geq 0$

Optimization

- Household takes z_t as given , where $z_t = \{y_{t+j}, \tau_{t+j}, i_{t+j}, P_{t+j}\}_{j=0}^{\infty}$
- Household solves the following problem, given z_t and b_t, where $b_t = \frac{B_t}{P_t}$ and B_t is the nominal bond holdings at the beginning of t:

$$V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta W(b_{t+1}; z_{t+1}),$$

$$\text{s. t.} \qquad c_t + (1+\pi_{t+1})b_{t+1} \leq y_t + (1+i_t)b_t + \tau_t$$

where $W(b_{t+1}; z_{t+1})$ is the generation t's expectation on the generation $t+1$'s lifetime utility.

- Equilibrium conditions

$$c_t \leq y_t,$$

$$b_{t+1} \leq b_{t+1}^s.$$

Intergenerational rationality

- Intergenerational rationality: consistency of value functions for all generations

$$W(b;z) = V(b;z)$$

  - Given the generation t's expectation that *generation $t+1$'s lifetime utility is $V(b_{t+1};z_{t+1})$,* the lifetime utility of generation t becomes $V(b_t;z_t)$.

- Intergenerationaly-rational household solves, given z_t and b_t,

$$\underbrace{V(b_t;z_t)}_{\text{my utility}} = \max_{c_t,b_{t+1}} U(c_t) + \beta \underbrace{V(b_{t+1};z_{t+1}),}_{\text{my expectation on my child's utility}}$$

$$\text{s. t.} \qquad c_t + (1+\pi_{t+1})b_{t+1} \leq y + (1+i_t)b_t + \tau_t$$

- This is a standard Bellman equation!

Standard recursive macroeconomics

Sequential problem:

$$\text{(SP)} \qquad \max \sum_{j=0}^{\infty} \beta^j U(c_{t+j}),$$

$$\text{s. t.} \qquad c_t + (1+\pi_{t+1})b_{t+1} \leq y_t + (1+i_t)b_t + \tau_t$$

Recursive problem

$$\text{(RP)} \qquad \underbrace{V(b_t; z_t)}_{\text{my utility at } t} = \max U(c_t) + \beta \underbrace{V(b_{t+1}; z_{t+1})}_{\text{my utility at } t+1},$$

$$\text{s. t.} \qquad c_t + (1+\pi_{t+1})b_{t+1} \leq y_t + (1+i_t)b_t + \tau_t.$$

- The nature decides the objective function: $\sum_{j=0}^{\infty} \beta^j U(c_{t+j})$.
- Usual interpretation: (SP) is the original problem. (RP) is reformulation for convenience.
  - TVC is necessary for the optimality in (SP)

$$\text{TVC:} \qquad \lim_{t\to\infty} \beta^t \bar{\lambda}_t b_t = 0.$$

New interpretation as intergenerational problem

$$\text{(RP)} \quad \underbrace{V(b_t; z_t)}_{\text{my utility}} = \max U(c_t) + \beta \underbrace{V(b_{t+1}; z_{t+1})}_{\text{my expectation on my child's utility}},$$

$$\text{s. t.} \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t.$$

$$\text{(SP)} \quad \max \sum_{j=0}^{\infty} \beta^j U(c_{t+j}),$$

$$\text{s. t.} \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t$$

- Our interpretation:
  (RP) is the original problem. (SP) is not true description of the economy.
  - TVC need not to be satisfied
- Expectation $V(b_{t+1}; z_{t+1})$ is endogenous, based not only on the nature but also on the social norm or social convention.

Definition: Intergenerationaly-Rational Expectations Equilibrium (IREE)

A set of quantities $\{c_t, b_{t+1}\}$ and prices $\{P_t\}$ that satisfies

1. quantities solve (RP), given $b_t = b_t^s$ and z_t,
2. value function $V(b; z)$ is well-defined,
3. resource constraints are satisfied.

$$\text{(RP)} \quad V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta V(b_{t+1}; z_{t+1}),$$

$$\text{s. t.} \qquad c_t + (1 + \pi_{t+1}) b_{t+1} \leq y_t + (1 + i_t) b_t + \tau_t. \qquad (\lambda_t)$$

We will show:

- The solution to (RP) may or may not satisfy TVC, $\lim_{t\to\infty} \beta^t \lambda_t (1 + i_t) b_t = 0$
- In equilibrium,
  - for some policy schedule $\{\tau_{t+j}, i_{t+j}\}_{j=0}^{\infty}$,
    TVC is satisfied and the equilibrium is the usual rational expectations equilibrium (REE). It is also IREE.
  - for other policy schedule $\{\tau_{t+j}, i_{t+j}\}_{j=0}^{\infty}$,
    TVC is not satisfied and the equilibrium is IREE, but not REE.

The solution to the Bellman equation

$$\text{(RP)} \quad V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta V(b_{t+1}; z_{t+1}),$$

$$\text{s. t.} \qquad c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t \qquad (\lambda_t)$$

- Constraints and conditions for the household:

$$c_t = y_t + (1 + i_t)b_t - (1 + \pi_{t+1})b_{t+1} + \tau_t,$$

$$\lambda_t = U'(c_t),$$

$$(1 + \pi_{t+1})\lambda_t = \beta V'(b_{t+1}; z_{t+1}),$$

$$V'(b_t; z_t) = (1 + i_t)\lambda_t.$$

which implies

$$(1 + \pi_{t+1})\lambda_t = \beta(1 + i_{t+1})\lambda_{t+1}. \tag{1}$$

- Given $\{i_t, \tau_t, \pi_t\}_{t=0}^{\infty}$, the household decides $\{c_t, b_{t+1}\}_{t=0}^{\infty}$.
- Equilibrium conditions:

$$c_t = y_t,$$

$$b_t = b_t^s.$$

- The equilibrium inflation $\{\pi_{t+j}\}_{j=1}^{\infty}$ is determined by (1) and $c_t = y_t$.

The fundamental solution to the Bellman equation

- The fundamental solution $V^f(b;z)$ is given as the solution to

$$V^f(b_t;z_t) = \max_b U(y_t + (1+i_t)b_t - (1+\pi_{t+1})b + \tau_t) + \beta V^f(b;z_{t+1}).$$

- Define

$$V_t^* = \sum_{j=0}^{\infty} \beta^j U(y_{t+j}).$$

- We know that the fundamental equilibrium such that $V(b;z) = V^f(b;z)$, $c_t = y_t$, and $b_t = b_t^s$, exists if

$$\lim_{t\to\infty} \beta^t \bar{\lambda}_t (1+i_t) b_t^s = 0,$$

  where $\bar{\lambda}_t = U'(y_t)$.

- If the fundamental equilibrium exists, then

$$V^f(b_t^s;z_t) = V_t^*.$$

The bubbly solution to the Bellman equation

$$\text{(RP)} \quad V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta V(b_{t+1}; z_{t+1}),$$

$$\text{s. t.} \qquad c_t + (1 + \pi_{t+1}) b_{t+1} \leq y_t + (1 + i_t) b_t + \tau_t \qquad (\lambda_t)$$

- can guess and verify the value function with a bubble term:

$$V(b_t; z_t) = V^b(b_t; z_t) \equiv V_t^* + (1 + i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} \tau_{t+j} = V_t^* + (1 + i_t)\bar{\lambda}_t (b_t - b_t^s) + \beta^{-t} X^s,$$

$$b_{t+1} = B(b_t; z_t) \equiv \frac{(1 + i_t) b_t + \tau_t}{1 + \pi_{t+1}},$$

$$c_t = C(b_t; z_t) \equiv y_t.$$

where $\bar{\lambda}_t \equiv U'(y_t)$ and $X^s = \lim_{t\to\infty} \beta^t \bar{\lambda}_t b_t^s$.

- We can verify the above guess: Given that the FOC (1) is satisfied,
  - $V^b(b_t; z_t)$ satisfies all FOCs and envelope condition,
  - it is shown that $V^b(b_t; z_t) = U(y_t) + \beta V^b(B(b_t, z_t); z_{t+1})$,
  - $V^b(b_t; z_t)$ is well-defined
  - TVC can be violated: $\lim_{t\to\infty} \beta^t \bar{\lambda}_t b_t^s \neq 0$.

Policy to be assessed

- The economy was initially in the zero inflation steady state.
- **Government commitment:**

  The government make the following commitment in period 0.

  - As long as $\pi_t \leq 0$, the government sets $i_t = \tau_t = 0$,
  - if $\pi_t > 0$, the government sets

$$i_t = i^* = \beta^{-1}(1 + \pi^*) - 1,$$
$$\tau_t = \tau^* = -i^* \hat{b}.$$

  - The policy (i^*, τ^*) is consistent with the inflation target $\pi^* > 0$.

Standard prediction: immediate inflation

- Suppose that the equilibrium should be REE.
- The inflation target is immediately attained.

- Given the government commitment, P_0 cannot be P_{-1}.
  - Suppose $P_0 = P_{-1}$. Then, $\pi_0 = 0$.
  - Then, the government chooses $i_0 = 0$.
  - Then, $\pi_1 = \beta(1 + i_0) - 1 = \beta - 1 < 0$. Thus, the government chooses $i_1 = 0$.
  - By induction, $i_t = 0$ and $\pi_t = \beta - 1$ for all $t \geq 1$.
  - Then, the TVC is violated: $\lim_{t \to \infty} \beta^t b_t = \lim_{t \to \infty} \beta^t \frac{B_0}{(1+\pi)^t} = B_0$.
  - As TVC should be satisfied in REE, P_0 cannot be P_{-1}.
- The only possible equilibrium is $P_0 \geq (1 + \pi^*)P_{-1}$ and $\pi_t = \pi^*$ for all $t \geq 0$.

Deflationary equilibrium: an unintended consequence

- Suppose that the equilibrium is IREE, not REE.
- In this case, the steady state with permanent deflation can be an equilibrium.
  - $P_0 = P_{-1}$
  - $i_t = 0$ for all $t \geq 0$.

- In the steady state, $c_t = y$, $\lambda_t = \bar{\lambda}_t = U'(y)$, and $\frac{1+i}{1+\pi_{t+1}} = \beta^{-1}$. (Fisher equation)
- Real value of nominal bond evolves by $b_t = \beta^{-t} b_0 \rightarrow \infty$.
- If the nominal rate is zero, $i = 0$, then $\frac{P_{t+1}}{P_t} = 1 + \pi_t = \beta < 1$. (Deflation)

- TVC is violated:

$$\lim_{t\to\infty} \beta^t \lambda_t b_t = \lim_{t\to\infty} \beta^t \times U'(y)\beta^{-t} b_0 = U'(y) b_0 > 0.$$

Deflationary equilibrium: Policy implications

- IREE: The TVC can be violated in equilibrium
  - TVC is the key for the usual logic to escape from deflation.
    1. If deflation continues under monetary easing ($i = 0$), then TVC will be violated
    2. TVC must be satisfied in equilibrium
    3. Therefore, deflation will stop, only if monetary easing continues
  - IREE indicates this logic may not be correct.
    Deflation can continue under $i_t = 0$ in the IREE.
- Extreme monetary easing ($i = 0$) may induce persistent deflation, as
  $1 + \pi_t = (1 + i_t)\beta$
- Government debt can grow indefinitely
  - Along this equilibrium path, TVC is violated.

Two notes on the bubbly solution

1. The same argument holds for the model with capital, k_t.
   - There should be unique equilibrium path $\{k_t\}_{t=0}^{\infty}$ that satisfies the resource constraints $c_t \geq 0$ and $k_t \geq 0$ for all t.
   - There is the unique fundamental value function: $V^f(k_t; z_t)$
   - Define

$$V^b(k_t, b_t; z_t) = V^f(k_t; z_t) + (1 + i_t)\bar{\lambda}_t(b_t - b_t^s) + \beta^{-t}X^s.$$

   Then, $V^b(k_t, b_t; z_t)$ is the bubbly solution.

2. Two types households can coexist in equilibrium:
   Type-F whose value is $V^f(b_t; z_t)$ and Type-B whose value is $V^b(b_t; z_t)$
   - They can coexists, as long as the measure of Type-F is not too large.
   - Type-F consume more and Type-B consume less.
   - TVC is satisfied for Type-F.
   - TVC is not satisfied for Type-B.

Sequential formulation of bubbly solution

- We will consider a sequential problem, the solution to which is

$$V^b(b_t; z_t) = V_t^* + (1+i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j}\tau_{t+j} = (1+i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j [U(y_{t+j}) + \bar{\lambda}_{t+j}\tau_{t+j}]$$

- Suppose that, given b_t, the generation t chooses c_t and $\varepsilon_t = (1+\pi_{t+1})b_{t+1} - (1+i_t)b_t$ to maximize

$$(1+i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j [U(c_{t+j}) + \bar{\lambda}_{t+j}\varepsilon_{t+j}]$$

- $V^b(b; z)$ is the solution to the sequential problem with intertemporal budget:

$$(1+i_t)\bar{\lambda}_t b_t + \max_{c_{t+j}, \varepsilon_{t+j}} \sum_{j=0}^{\infty} \beta^j [U(c_{t+j}) + \bar{\lambda}_{t+j}\varepsilon_{t+j}],$$

$$\text{s. t.} \quad \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j}(c_{t+j} + \varepsilon_{t+j}) \leq \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j}(y_{t+j} + \tau_{t+j}),$$

with the equilibrium conditions: $c_t \leq y_t$ and $\varepsilon_t \leq \tau_t$.

Sequential formulation of bubbly solution

- (1) implies

$$(1+i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} \varepsilon_{t+j} = \lim_{n\to\infty} \beta^n \bar{\lambda}_{t+n} (1+i_{t+n}) b_{t+n},$$

$$(1+i_t)\bar{\lambda}_t b_t^s + \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} \tau_{t+j} = \lim_{n\to\infty} \beta^n \bar{\lambda}_{t+n} (1+i_{t+n}) b_{t+n}^s.$$

- Define $X \equiv \lim_{t\to\infty} \beta^t \bar{\lambda}_t (1+i_t) b_t$ and $X^s \equiv \lim_{t\to\infty} \beta^t \bar{\lambda}_t (1+i_t) b_t^s$.
- Note that $X = X(b_0; z_0)$ is the PDV of the remaining debt in the infinite future, which is evaluated in period 0.
- Then, $V^b(b, z)$ is the solution to the bubbly sequential problem:

(SP: B)
$$\max_{c_{t+j}, X} \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\} + \beta^{-t} X,$$

s. t.
$$\left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} c_{t+j} \right\} + \beta^{-t} X \leq \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} y_{t+j} \right\} + \beta^{-t} X^s + (1+i_t)\bar{\lambda}_t (b_t - b_t^s),$$

with the equilibrium conditions: $c_t \leq y_t$ and $X \leq X^s$.

Relationship between (RP), (SP: B) and (SP: F)

Consider Recursive problem (RP), Bubbly problem (SP: B), Fundamental problem (SP: F).

(RP)
$$V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta V(b_{t+1}; z_{t+1}),$$
$$\text{s. t.} \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t \qquad (\lambda_t)$$

(SP: B)
$$\max_{c_{t+j}, X} \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\} + \beta^{-t} X,$$
$$\text{s. t.} \quad \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} c_{t+j} \right\} + \beta^{-t} X \leq \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} y_{t+j} \right\} + \beta^{-t} X^s + (1 + i_t)\bar{\lambda}_t (b_t - b_t^s),$$

(SP: F)
$$\max_{c_{t+j}, X} \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\},$$
$$\text{s. t.} \quad \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} c_{t+j} \right\} + \beta^{-t} X \leq \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} y_{t+j} \right\} + \beta^{-t} X^s + (1 + i_t)\bar{\lambda}_t (b_t - b_t^s),$$

with the equilibrium conditions, $X \leq X^s$ and $c_t \leq y_t$.

Intergenerational rationality: redux

- Intergenerational rationality: consistency of value function $V(b_t; z_t)$ for all generations t
- Objective function for the bubbly problem (SP: B):

$$\left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\} + \beta^{-t} X,$$

where X is the PDV of the remaining debt in the infinite future.

- Intergenerational economy: $X > 0$ is possible, because
  - generation t
    - has the belief that *"only the last generation of infinite future obtains utility* $\lim_{t\to\infty} \beta^{-t} X$ *from remaining debt."*
    - $\beta^{-t-1} X$ in expectation is given not by the nature, but social norm or social convention.
  - This belief is not irrational: $\forall t\ (< \infty)$, generation t cannot refute the belief.
    - Similar to Abreu and Gul (2000)

A reduced-form New Keynesian model with government bond

Intergenerational problem with labor l_t and bond b_t.

Cashless economy with no capital:

$$\text{(RP)} \quad V(b_t; z_t) = \max_{c_t, l_t, b_{t+1}} U(c_t, l_t) + \beta V(b_{t+1}; z_{t+1}),$$

$$\text{s. t.} \quad c_t + (1 + \pi_{t+1}) b_{t+1} \le w_t l_t + (1 + i_t) b_t + \tau_t + d_t, \qquad (\lambda_t)$$

- Supply side of the economy is given by
  - production technology (PT): $y_t = A l_t$
  - New Keynesian Phillips Curve (NKPC): $\pi_t = \beta \pi_{t+1} + \kappa(y_t - y^f)$

Definition: IREE in a NK model

Set of allocations $\{c_t, y_t, l_t, b_t\}_{t=0}^{\infty}$ and prices $\{p_t, w_t\}_{t=0}^{\infty}$ that satisfies

1. $\{c_t, l_t, b_t\}_{t=0}^{\infty}$ is the solution to (RP). $V(b_t; z_t)$ is well-defined.
2. prices $\{p_t, w_t\}_{t=0}^{\infty}$ clears the goods market, labor market, and bond market
3. (PT) and (NKPC) are satisfied.

Stationary equilibrium

- All variables are pinned down by π or i, where $1+\pi = \beta(1+i)$

$$P_t = (1+\pi)^t P_0$$
$$c(\pi) = y(\pi) = y^f + \frac{(1-\beta)\pi}{\kappa},$$
$$l(\pi) = \frac{y(\pi)}{A},$$
$$w = -\frac{U_l(y(\pi), l(\pi))}{U_c(y(\pi), l(\pi))}$$

- Can guess and verify that

$$V(b_t, z_t) = \hat{V}(\pi) + \underbrace{\lambda(\pi)(1+i)\, b_t + C_t^b}_{\text{bubble term}}$$

is well-defined and solves (RP), satisfying all constraints, where $\hat{V}(\pi) = \frac{1}{1-\beta} U(y(\pi), l(\pi))$, $\lambda(\pi) = U_c(y(\pi), l(\pi))$, and $C_t^b = \sum_{j=0}^{\infty} \beta^j \lambda(\pi) \tau_{t+j} < +\infty$, as τ_t is bounded.

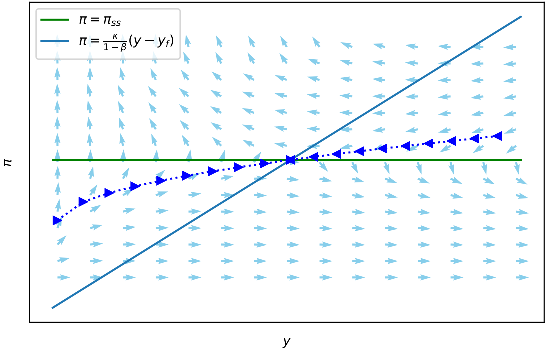
Transition dynamics

- Equilibrium path is given by $\{y_t, \pi_{t+1}\}$, which is determined by

$$\text{(NKPC)} \qquad \pi_t = \beta\pi_{t+1} + \kappa(y_t - y^f),$$

$$\text{(FOCs)} \qquad 1 + \pi_{t+1} = (1+i)\beta \frac{U_c\left(y_{t+1}, \frac{y_{t+1}}{A}\right)}{U_c\left(y_t, \frac{y_t}{A}\right)}$$

- Phase diagram



Deflationary stagnation with $i = 0$

- Suppose that central bank set $i = \tau = 0$
- Then, $1 + \pi = \beta(1 + i) = \beta$

$$P_t = \beta^t P_0, \qquad \text{(Deflation)}$$

$$c = y = y_z \equiv y^f - \frac{(1-\beta)^2}{\kappa}, \qquad \text{(Stagnation)}$$

- Value function is

$$V(b_t) = \hat{V}(\pi) + \underbrace{\lambda b_t}_{\text{bubble term}}$$

- TVC is violated in the IREE with $i = 0$ and growing b_t

$$\lim_{t\to\infty} \beta^t U_c(c_t, l_t) b_t = \lim_{t\to\infty} \beta^t U_c(y_z, l_z) b_0 = U_c(y_z, l_z) b_0 > 0$$

- The equilibrium is not REE, but it is IREE

Summary

- We show deflation with growing debt can be an equilibrium
  - Intergenerational altruism: assets are bequeathed indefinitely
  - (Bubbly) expectations: current generation is happy with money as they believe the future generation will be happy with money
    - The expectations on the future generation is irrefutable, as long as they are consistent with the current generation.
  - TVC is not necessarily satisfied in equilibrium
- Extreme monetary easing may not be effective in fighting deflation
  - Same as the Neo-Fisherian: $1 + \pi_t = (1 + i_t)\beta$ with $i_t = 0$ implies $\pi_t < 0$.
  - Secular stagnation can be a steady state generated by the policy $i_t = 0$.
- Government debt, as a bubble, can grow indefinitely in equilibrium
  - The bubble may collapse ⇒ Sudden inflation (i.e., debt crisis)

Future Research

1. Theoretical implications
    - Asset price bubbles
    - Endogenous heterogeneity in preferences and beliefs
    - Further study on TVC and bubbles
2. Empirical and quantitative implications
    - Consistency with the data on
        - the money demand during deflationary period
        - the government debt
        - intergenerational altruism

Appendix:
Alternative interpretation of the intergenerational rationality

- Suppose that $\varepsilon_t - \bar{\varepsilon}_t$ gives the utility as a social status, where $\bar{\varepsilon}_t$ is the social level of ε_t. (Cole, Mailath, and Postlewaite 1992)
- We assume that the utility of the social status is $\lambda_t(\varepsilon_t - \bar{\varepsilon}_t)$.
- (SP: B) is equivalent to :

$$(1+i_t)\bar{\lambda}_t b_t + \max_{c_{t+j},\varepsilon_{t+j}} \sum_{j=0}^{\infty} \beta^j [U(c_{t+j}) + \bar{\lambda}_{t+j}(\varepsilon_{t+j} - \bar{\varepsilon}_{t+j})],$$

$$\text{s. t.} \quad \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j}(c_{t+j} + \varepsilon_{t+j}) \le \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j}(y_{t+j} + \tau_{t+j}).$$

- It is rewritten as

(SP: B)
$$\max_{c_{t+j},X} \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\} + \beta^{-t}(X - \bar{X}),$$

$$\text{s. t.} \quad \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} c_{t+j} \right\} + \beta^{-t} X \le \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} y_{t+j} \right\} + \beta^{-t} X^s + (1+i_t)\bar{\lambda}_t (b_t - b_t^s),$$

with the equilibrium conditions: $c_t \le y_t$ and $X \le X^s$, where $\bar{X}$ is the social level of X.