Opacity: Insurance and Fragility*

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December 4, 2019

Abstract

What are the effects of banks holding opaque, complex assets? Should regulators require bank assets to be more transparent? I study these questions in a model of financial intermediation where opacity determines how long the realized value of an asset remains unknown. By allowing a bank to sell assets before the realization is known, opacity provides insurance to the bank's depositors. However, higher opacity also increases depositors' incentives to join a bank run. In choosing the level of opacity, therefore, a bank faces a trade-off between providing insurance and increasing fragility. If depositors can accurately observe the level of opacity, banks will choose the socially-efficient level. If depositors are unable to observe this choice, however, banks will have an incentive to become overly opaque and regulation to limit opacity can improve welfare.

Keywords: Opacity, Bank runs, Insurance, Banking regulation

JEL classification: G01, G21, G28

*I am grateful to James Barth, Roberto Chang, Todd Keister, Nobuhiro Kiyotaki, Yang Li, Loretta Mester, Yuliyan Mitkov, Tomas Sjöström, Eugene White, Shengxing Zhang and seminar participants at Rutgers University, the Bank of Canada, the Federal Reserve Board, the Halle Institute for Economic Research, Instituto Tecnológico Autónomo de México (ITAM), Wesleyan University, the 4th Annual Chapman Conference on Money and Finance, and the 16th Annual Vienna Macroeconomic Workshop for their helpful comments. All errors remain my own.

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1 Introduction

Just over a decade ago, the economy was in the midst of the global financial crisis. An important feature of this crisis was widespread *runs* in which depositors and other creditors withdrew funds from a variety of shadow banking arrangements.[1] One such arrangement was Asset-Backed Commercial Paper (ABCP) conduits, some of which invested funds into complex assets whose value was difficult to assess in a timely manner. This type of *opacity* is blamed for causing or at least exacerbating the global financial crisis. The Dodd-Frank Wall Street Reform and Consumer Protection Act was introduced in 2010 to "to promote the financial stability of the United States by improving accountability and transparency in the financial system." Subsequently, new rules were stipulated, including stronger prudential standards for financial firms that use derivatives and a prohibition on commercial banks from sponsoring and investing hedge funds. It is, however, said that banks have been historically and purposefully opaque. This opacity enables banks to issue *information insensitive* liabilities by keeping asset qualities unknown and isolating the valuation of liabilities from the risk of assets.[2] Doing so allows bank liabilities to be a stable medium of exchange and a store of value. This role of opacity is an important feature not only of traditional commercial banks but also of shadow banks.[3] For example, an ABCP conduit issues information insensitive liabilities in the form of commercial paper backed by Asset-Backed Securities, Mortgage-Backed Securities, or derivatives that may be highly complex and risky. This disparity between these two views raises a fundamental question: should the banking system be transparent or opaque?.

This paper addresses the question by constructing a version of the Diamond and Dybvig (1983) model of financial intermediation that illustrates the costs and benefits of opacity in a unified framework. In particular, I study an environment with financial markets and fundamental uncertainty as in Allen and Gale (1998) and with limited commitment as in Ennis and Keister (2009). I add the ability of a bank to make its assets opaque in the sense that it will take time

[1]See Gorton and Metrick (2012) for further details.

[2]See Gorton and Metrick (2012), Holmström (2015) and Dang, Gorton, Holmström, and Ordoñez (2017).

[3]Gorton, Lewellen, and Metrick (2012) show that the main provider of these information insensitive instruments has shifted from commercial banks issuing demand deposits to shadow banks.

to discern the true value of the assets. Until the true state is known, the bank's assets will trade in financial markets based on their expected payoff. By choosing the level of opacity, the bank determines how many of its depositors will be paid while its assets remain information insensitive in this sense. The bank's assets mature in the long-term and yield the realized return, which implies that the bank's repayments to its creditors in the long-term are necessarily contingent on the realized return. Opacity, therefore, can make the bank's liabilities information insensitive only in the short-term. This fact, in turn, affects depositors' decisions on when to withdraw. I show that while opacity is a way to provide insurance to the bank's depositors, it may at the same time worsen financial fragility. I use the model to derive the optimal level of opacity and discuss the conditions under which regulation that limits opacity is desirable.

In practice, opacity is interpreted as the complexity of the bank's asset. Derivatives and asset-backed securities tend to be more complex and harder to assess, and even financial firms themselves may have difficulty assessing their asset qualities.[4] For this reason, I assume *symmetric information*: neither the bank nor depositors and outside investors have information on the asset quality during the information insensitive period.[5] Assets can be structured with various degree of complexity and, hence, I assume the choice of opacity is a continuous variable.

I begin my analysis by showing that opacity generates a risk-sharing opportunity in the spirit of the classic Hirshleifer (1971) effect. The asset price depends on the expected asset return until the realized return is known and, hence, opacity provides depositors with insurance against fundamental uncertainty. In other words, opacity allows the bank to transfer the asset-return risk from risk-averse depositors to risk-neutral investors. A higher level of opacity insures more depositors from the uncertainty in the short-term. The repayments made to depositors who wait to withdraw are made using matured assets and, hence, are still exposed to uncertainty. My first contribution is to discover a novel mechanism through which this type of insurance raises the possibility of a self-fulfilling bank run. The insurance offered by opacity is available only to

[4]Ben Bernanke, the chair of the Federal Reserve Board at that time, testified that "Financial firms sometimes found it quite difficult to fully assess their own net derivatives exposures... The associated uncertainties helped fuel losses of confidence that contributed importantly to the liquidity problems" in September 2010.

[5]There are papers supposing asymmetric information such that bank has more information. See , for instance, Bouvard, Chaigneau, and Motta (2015) or Faria-e Castro, Martinez, and Philippon (2017).

depositors who withdraw before the asset returns are known. This limited availability of insurance enhances depositors' incentive to withdraw early and, as a result, a bank run is more likely to occur.

My second contribution is to derive the optimal level of opacity. In choosing a level of opacity, the bank faces a trade-off between providing insurance and increasing its susceptibility to a run. The optimal level of opacity depends on the extent to which outside investors discount future consumption and on the volatility of asset returns. When asset returns are more volatile, the insurance is more beneficial and a higher level of opacity is optimal. My third contribution is to show that when the choice of opacity is unobservable by depositors, regulating opacity can improve the allocation of resources and financial stability. Depositors may have difficulty evaluating the details of complex structures of derivatives or asset-backed securities. I find that, in this case, the bank will choose the highest possible level of opacity. As a result, the associated expected utility of depositors will be lower, and fragility will be higher than the optimal. Introducing a regulatory limit on opacity can then improve welfare. This analysis provides a novel justification for regulating opacity.

Related literature: My paper contributes to a growing literature on opacity and financial stability. My paper is the first to study how opacity itself makes depositors more likely to panic and thereby show that higher opacity is always worse for financial stability. Existing papers on opacity in theoretical models of bank runs or roll-over risk conclude that opacity enhances or has mixed effects on financial stability. The key differences in these studies are the assumption of asymmetric information and the focus on information-driven bank runs. Parlatore (2015) builds a global game model of bank runs based on Goldstein and Pauzner (2005) and shows that transparency increases the economy's vulnerability to bank runs. She interprets the precision of private signals about fundamentals as opacity. In her environment, transparency means precise information about the fundamental state, which enhances the strategic complementarity of depositors' withdrawal decisions. She shows that a lower precision, or opacity, reduces the risk of bank runs by removing possible coordination incentives. Bouvard et al. (2015) and Ahnert and Nelson (2016) also study the roll-over behavior of a bank's creditors in a global game and show that transparency reduces the likelihood of bank runs at solvent banks during crises but increases fragility during normal times. Chen and Hasan (2006) studies the transparency and contagion of bank runs. They build a version of the Diamond-Dybvig model with two banks in which their investment returns are random but correlated. Their results show that transparency may increase the chance of contagion when a run occurs at a bank. In contrast with all of these papers, my paper studies an environment with symmetric information and it is the *opacity itself* that increases the run susceptibility.

My analysis also contributes to a growing literature on information and risk-sharing in financial intermediation. A popular idea in this literature is that less information can enhance risk-sharing in financial intermediation. My paper is the first to discover such a mechanism increases financial fragility. My paper shares the idea that opacity improves risk-sharing with Kaplan (2006) and Dang et al. (2017). Kaplan (2006) extends Diamond and Dybvig (1983) to include risky investment and compares two types of deposit contracts: the middle-period repayments are contingent (transparent) or non-contingent (opaque) on the realization. In contrast, I measure opacity by the time it takes to verify asset returns and, hence, the degree of opacity is continuous. He studies the optimal banking contract and shows that a non-contingent contract generates risk-sharing effects. The idea that less information can improve welfare originated in Hirshleifer (1971). While Kaplan (2006) assumes that policymakers can prevent bank runs costlessly with commitment, my paper studies an environment with limited commitment and, hence, runs possibly occur. Dang et al. (2017) study the effects of opacity on roll-over behavior in a model of financial intermediation. They show that a bank can provide a fixed amount of goods, also known as safe liquidity, independent of the realized return of its assets if the returns are unobservable, or opaque. My paper shares the idea of Hirshleifer effects with these papers but goes further in studying how this type of risk-sharing mechanism affects not only the allocation of resources but also financial fragility. Also, I introduce financial markets where the bank can liquidate its investment as in Allen and Gale (1998), and the opacity affects the liquidation value. Kaplan (2006) and Dang et al. (2017) do not have mechanisms that opacity affects the liquidation value of assets. I will show that this asset price is a source of the Hirshleifer effect but also a source of bank runs.

The idea that opacity enhances financial stability is often studied with the risk-taking behavior of banks. Jungherr (2018) characterizes the optimal level of opacity in an environment where opacity reduces the risk of bank runs but encourages banks to take an excess risk. He shows that when asset returns are correlated, banks choose higher opacity than the socially optimal level in order to hide information about their portfolio. Cordella and Yeyati (1998), Hyytinen and Takalo (2002) and Moreno and Takalo (2016) also show that transparency may enhance the bank's risk-taking and increases the chance of a bank failure. Shapiro and Skeie (2015) study the optimal disclosure about bailout policies in resolving a bank. A higher willingness to bail out reduces run incentives of depositors but leads to riskier behavior of the bank. In my model, the bank does not have a portfolio choice, and the bank's risk-taking is not the source of fragility.

Adverse selection is another growing idea in studying opacity, together with runs. Faria-e Castro et al. (2017) model disclosure by combining the ideas of bank runs, competitive financial markets as in Allen and Gale (1998) and the Bayesian persuasion approach. My paper also combines the idea of bank runs and competitive financial markets, but supposes symmetric information. They study how asymmetric information drives adverse selection in financial markets, but disclosure negatively affects runs or roll-over risk, characterizing the optimal use of disclosure. My paper also introduces a financial market in which a bank can trade its assets, but one of the key assumptions of the paper is that the assets will be traded at a discounted pooling price.

Opacity is a popular idea in discussions of disclosing stress test results as well. Goldstein and Sapra (2014) review this literature and show that opacity is preferred in a majority of studies. Goldstein and Leitner (2015) emphasize the Hirshleifer effect to characterize optimal information disclosure. Alvarez and Barlevy (2015) study a mandatory disclosure of bank's balance sheet in an environment where banks are interconnected. They show that the mandatory disclosure of a bank's balance sheet may reassure not only its creditors but also other banks' creditors by reducing concerns of contagion, but it loses an opportunity of risk-sharing. These works suppose that a regulator or bank has more information about bank's assets, whereas my paper studies an environment of symmetric information.

The rest of the paper is organized as follows: Section 2 introduces the model environment and the definition of equilibrium and financial fragility. Section 3 derives the equilibrium condition for a bank run and analyzes the effect of increasing opacity on financial fragility. Section 4 characterizes the optimal level of opacity subject to the trade-off between risk-sharing and fragility. I study the case where the choice of opacity is unobservable in Section 5 and then conclude.

2 The Model

The analysis is based on a version of Diamond and Dybvig (1983) augmented to include a choice regarding the transparency of a bank's asset. The model also includes financial markets to trade assets as in Allen and Gale (1998) and the limited commitment features of Ennis and Keister (2009). This section describes the model environment including agents, technologies, financial markets and information structure.

2.1 The environment

Depositors: There are three periods, labeled $t = 0, 1, 2$, and a continuum of depositors, indexed by $i \in [0, 1]$. Each depositor has preferences given by

$$u(c_1^i, c_2^i; \omega_i) = \frac{(c_1^i + \omega_i c_2^i)^{1-\gamma}}{1-\gamma} \tag{1}$$

where c_t^i expresses consumption of the goods in period t. The coefficient of relative risk aversion γ is greater than 1. The parameter ω^i is a binominal random variable with support $\Omega \equiv \{0, 1\}$, which is realized in period 1 and privately observed by each depositor. If $\omega^i = 1$, depositor i is patient, while she is impatient if $\omega^i = 0$. Each depositor is chosen to be impatient with a known probability $\pi \in (0, 1)$, and the fraction of impatient depositors in each location is equal to π.

Technology: Each depositor is endowed with one unit of goods at the beginning of period 0. There is a single, constant-returns-to-scale technology for transforming this endowment into consumption in the last period. Goods invested in period 0 mature in period 2 and yield a random return R_z where $z \in Z = \{b, g\}$. Let n_g denote the probability of the good return R_g and let $n_b = 1 - n_g$ denote the probability of the bad return $R_b < R_g$. The state z realizes at the beginning of period 1.

Investors: The investment can be securitized and traded in period 1 as an *asset* in a competitive asset market, and a large number of wealthy risk-neutral investors may purchase them. These investors have endowments in period 1 and preferences are given by

$$g(c_1^f, c_2^f; \rho) \equiv c_1^f + \rho c_2^f, \tag{2}$$

where c_t^f is the period-t consumption of investor f. The parameter $\rho < 1$ captures differences in preferences of investors relative to depositors. Investors' endowments are large enough that their preferences are never constrained. This setup drives that, given an expected return $\mathbb{E}R$ and information at hand, the asset is valued as $p = \rho\mathbb{E}R$ by these investors.[6]

Financial intermediation: The investment technology is operated at a central location, in which depositors pool and invest resources together in period 0 to insure individual liquidity uncertainty. This intermediation technology can be interpreted as a financial intermediary, or *bank*. At the beginning of period 1, each depositor learns her type and either contacts the bank to withdraw funds in period 1 or waits until period 2 to withdraw. Depositors are isolated from each other in periods 1 and 2, and they cannot engage in trade. Upon withdrawal, a depositor must immediately consume what is given. Repayments follow a *sequential service constraint* as in Wallace (1988) and Peck and Shell (2003). Depositors who choose to withdraw in period 1 are assumed to arrive in random order and each repayment is made sequentially. Similarly to Peck and Shell (2003), when making her choice, a depositor does not know her position in the order of withdrawals.

Bank opacity: The bank can make its securitized assets opaque in the sense that it will take time to reveal the realization. A degree of opacity of its assets is determined in period 0, denoted by $\theta \in [0, \pi]$, and this degree of opacity is known to depositors. The opacity is measured by the number of withdrawals that will occur before the return on the bank's assets becomes known: Before θ withdrawals, nobody knows the realization of asset returns. After θ

[6]Introducing risk-neutral agents is a common way to simplify and make the model tractable. See, for example, Allen and Gale (1998) and Ordoñez (2018).

withdrawals, everybody knows the realization. In other words, for simplicity, I measure the time needed for investors and others to figure out the asset quality by the number of depositors who withdraw before this event happens.

2.2 Decentralized economy

The intermediation technology is operated by a bank. The bank behaves competitively and act to maximize the expected utility of their depositors at all times. In period 0, a bank receives deposits and makes an investment. The bank securitizes its investment and chooses a degree of opacity θ of the asset anticipating subsequent events, and this chosen degree of opacity is common knowledge. Actual amounts of payments will depend on a non-cooperative simultaneous-move game in which the bank chooses a repayment strategy and depositors choose a withdrawal strategy at the beginning of period 1. The bank anticipates an equilibrium path to be played in choosing its degree of opacity in period 0.

Limited commitment: A bank correctly anticipates that a fraction π of its depositors will be impatient, but it does not observe whether a given depositor is patient or impatient. Payments are, therefore, contingent not on a depositor's type but on the other available information at the time of the withdrawal. At π withdrawals, the bank can make an inference about whether any patient depositors have withdrawn or not. A *bank run* is defined as withdrawals by a positive measure of patient depositors. The bank reacts when it recognizes that a run is underway and I assume the run stops at this point as in Ennis and Keister (2009), suggesting that a bank's reaction restores confidence in that bank.[7] Given that the run stops, the bank allocates its remaining consumption efficiently making repayments to the remaining impatient depositors in period 1 and to the remaining patient depositors in period 2. The assumption of noncommitment is crucial to prohibit the bank from preventing self-fulfilling runs by means of time-inconsistent policy.[8]

[7]This reaction can be, for example, a resolution with haircut. This assumption can be generalized by having more rounds of coordination failure. See Ennis and Keister (2010) for the details. Having multiple coordination failure, however, does not change main mechanisms in this model and results remain unchanged qualitatively.

[8]Diamond and Dybvig (1983) shows that by pre-committing to a payment schedule, e.g. deposit freeze, the bank could prevent equilibrium bank runs. However, suspending payments and giving zero consumption to remaining

Repayment plan: The bank sets a state-contingent repayment plan by the beginning of period 1. The repayment made by the bank in period 1 can be summarized by the function

$$c : [0, \pi + \pi(1 - \pi)] \times \{b, g\} \mapsto \mathbb{R}^2_+ \tag{3}$$

where the number $c_z(\mu)$ is the payment to μ-th depositor withdrawing in period 1 in state z. Opacity together with sequential service implies $c_b(\mu) = c_g(\mu)$ for $\mu = [0, \theta]$. When the bank does not have a bank run, only impatient depositors withdraw in period 1. In such a case, the repayment plan in period 1 is subject to the feasibility constraint:

$$\int_0^{\pi} c_z(\mu) d\mu \leq \theta p_u + (\pi - \theta) p_z, \forall z. \tag{4}$$

where p_u is the discounted expected return of asset such that $p_u = n_b p_b + n_g p_g$. The asset trades at the price p_u during the first θ withdrawals and at the price p_z afterward. In the case of a run, some patient depositors are among the first π withdrawals. Although the run stops after π withdrawals are made, there are still $\pi(1 - \pi)$ impatient depositors who will need to withdraw in period 1. The bank repays these impatient depositors, and the number of repayments in period 1 will be at most $\pi + \pi(1 - \pi)$. The remaining patient depositors who have chosen to withdraw in period 1 choose to wait until period 2 at this point. In period 2, the payment to these remaining patient depositors will be made by equally dividing the remaining resources. In this case, the repayment plan in period 1 is subject to the following feasibility constraint:

$$\int_0^{\pi + \pi(1-\pi)} c_z(\mu) d\mu \leq \theta p_u + (\pi + \pi(1 - \pi) - \theta) p_z, \forall z. \tag{5}$$

Withdrawal plan: Depositors choose a contingent withdrawal plan at the same time the bank makes its decision. A depositor's withdrawal plan is conditioned on both her type and an extrinsic sunspot variable $s \in S = [0, 1]$ that is unobservable to the bank.[9] Let y_i denote the

impatient depositors after a fraction π of depositors have withdrawn are time-inconsistent in the spirit of Kydland and Prescott (1977). Ennis and Keister (2009), on the other hand, show that bank runs can occur in equilibrium if the bank fails to commit to their initial payment schedules.

[9] See, for example, the discussion in Diamond and Dybvig (1983), Cooper and Ross (1998) and Peck and Shell (2003).

withdrawal strategy for depositor i, that is,

$$y_i : \Omega \times S \mapsto \{0, 1\},$$

where $y_i(\omega_i, s) = 0$ corresponds to withdrawal in period 1 and $y_i(\omega_i, s) = 1$ corresponds to withdrawal in period 2. A bank run, therefore, occurs if $y_i(1, s) = 0$ for a positive measure of patient depositors. Let y denote the profile of withdrawal plans for all depositors.

Expected payoffs: Given θ, the strategies (c, y) determine a level of consumption that each depositor receives in every possible case as a function of her position in the withdrawal order. Rewriting (1) so that (c_1^i, c_2^i) are functions of θ, the depositor i's preferences are contingent on both ω_i and θ such that $u(c_1^i, c_2^i; \omega_i, \theta)$. Let $v(c, (y_i, y_{-i}), \theta)$ denote the expected utility of depositor i as a function of her chosen strategy y_i, that is

$$v_i(c, y; \theta) = \mathbb{E}[u(c_1^i, c_2^i; \omega_i, \theta)], \tag{6}$$

where the expectation $\mathbb{E}$ is over ω_i and her position in the order of withdrawals.[10] The bank chooses the repayment plan c to maximize the expected utility of depositors:

$$U(c, y; \theta) = \int_0^1 v_i(c, y_i; \theta) di. \tag{7}$$

The bank chooses θ to maximize the depositors' expected utilities by anticipating its effects on the equilibrium to be played in the game.

2.3 Timeline

The timing of events is summarized in Figure 1. In period 0, depositors deposit their endowments into the bank, the bank chooses the degree of opacity, and the period ends. This choice of opacity immediately becomes public information.[11] At the beginning of period 1, each depositor decides a contingent withdrawal plan based on her type and sunspot state. At the same time, the bank sets

[10]See Appendix for a explicit expression for (c_1^i, c_2^i).

[11]I will study another case in which depositors cannot directly observe the choice in Section 5.

a contingent repayment plan based on the realization of the sunspot state and fundamental state. After they make contingent plans, the state of the world is realized. Depositor i learns her type ω^i and the realized sunspot state and takes action accordingly to her plan. The depositors who have chosen to withdraw in period 1 are randomly assigned positions in the queue at the bank. The bank begins redeeming deposits withdrawn by depositors from the front of the line sequentially. To make these repayments, the bank sells assets in the financial market. These assets are valued by investors based on their expected payoff. The asset quality is observed after θ withdrawals, at which point assets are valued based on the realized returns. Observing the realization of the fundamental state, the bank changes the amount of repayment accordingly to the repayment plan. When π withdrawals are made, the bank can infer the realization of the sunspot state by whether an additional withdrawal occurs or not. When runs are underway, the bank's reaction halts further withdrawals by patient depositors. The remaining impatient depositors receive repayments. In period 2, the investment mature, and the bank repays remaining depositors.

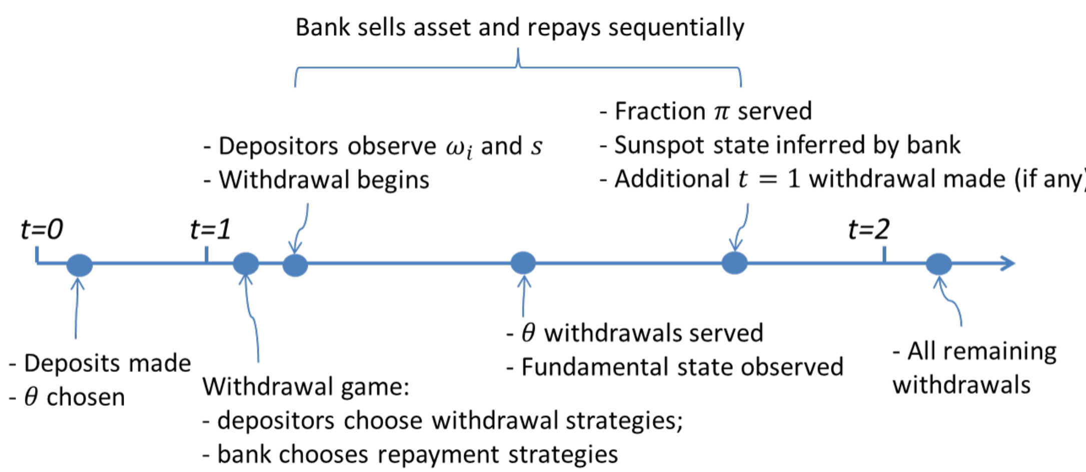

Figure 1: Timeline

2.4 Discussion

In addition to representing a deposit-taking bank, the environment can be interpreted as a model of certain types of roll-over crises. In the case of ABCP conduits, institutional investors purchase ABCP issued by the conduit in period 0 and make a roll-over decision in period 1. Such debts are often backed by opaque assets that are difficult to evaluate in a timely manner. A large number of institutional investors may not roll-over in period 1, causing a roll-over crisis. Sequential service in this context implies that institutional investors are isolated from each other and make a roll-over decision at different points in time. When an institutional investor makes a decision, she does not know how many of others have already done so.

Opacity in the model is the length of time in which the asset quality is not known, which can be interpreted as a measure of the difficulty of assessing the asset quality. An example of such an opaque asset is Level 3 assets classified by the Financial Accounting Standards. In this classification, there are three types of assets based on the difficulty of asset valuation. A Level 1 asset is the easiest asset to evaluate, and the valuation is based on observable market prices. A Level 2 asset needs valuation models to determine its price, and the models are observable. The valuation of Level 3 assets also requires models, but the models are not observable. Evaluating Level 3 assets takes more steps than evaluating other types, and the additional step requires additional time to determine the price. Some policymakers have argued that financial institutions could not assess their asset quality promptly during the recent crisis period (see footnote 4). Based on this fact, I measure the difficulty by the length of time it takes to evaluate the asset quality in the model. The longer the quality of the bank's assets remains unknown, the more investors who will be able to withdraw during that period.

I separate the idea of opacity from risk diversification. Some assets may become more opaque as a by-product of diversifying its underlying assets, but risk diversification does not necessarily cause opacity. For instance, by disclosing the information about its underlying assets, asset-backed securities become transparent and easy to evaluate. Therefore in the model, the asset return R_z is not dependent on θ, and the focus on opacity itself helps keep analyses tractable.

I restrict the choice of opacity to be in the interval $[0, \pi]$, but it is possible to generalize this interval. This assumption is useful for maintaining tractability in limited commitment framework given that I assume a run stops after π withdrawals and the bank's reaction. I will show that raising the level of opacity has a trade-off and the cost is increasing financial fragility. However, there is no cost to raise the level of opacity when $\theta > \pi$, because runs stop at π withdrawals. I could generalize the choice set for θ by allowing multiple waves of runs to occur as in Ennis and Keister (2010). This generalization allows the bank to respond to runs, but runs may not stop. By doing so, I can expand the range for opacity levels while still keeping the trade-off. However, the current assumption does not change the results qualitatively and can deliver the main ideas without unnecessary complications.

3 Equilibrium in the withdrawal game

Depositors and the bank choose their withdrawal strategies and a repayment plan, respectively, at the same time in period 1. In this simultaneous move game, a depositor's strategy is y_i and she aims to maximize $v(c, (y_i, y_{-i}); \theta)$. The bank's strategy is c and it aims to maximize $U(c, y; \theta)$. An equilibrium of this game is then defined as follows:

Definition 1. Given θ, an *equilibrium of the withdrawal game* is profile of strategies (c^*, y^*) such that

1. $v_i(c^*, (y_i^*(s), y_{-i}^*(s)); \theta) \geq v_i(c^*, (y_i(s), y_{-i}^*(s)); \theta)$ for all s, for all y_i, for all i
2. $U(c^*, y^*(s); \theta) \geq U(c, y^*(s); \theta)$ for all c

Notice that the bank takes the strategies of depositors as given and chooses the best response to these strategies, and a change in c does not influence the behavior of depositors. In addition, the payoffs of this game depend on θ. Let $\mathcal{Y}(\theta)$ denote the set of equilibria of the game associated with the choice θ such that

$$\mathcal{Y}(\theta) = \{(c^*, y^*) \mid \theta\}. \tag{8}$$

In this section, I study equilibrium in the withdrawal game given the bank's choice of opacity and show how opacity affects equilibrium outcomes.

3.1 A cutoff strategy profile

I am interested in how likely it is that a bank run will occur.A standard approach in the literature is to ask values what conditions the following strategy profile is a part of equilibrium.[12] Denote $\hat{y}_i(\omega_i, s)$ be a *cutoff strategy profile* such that

$$\hat{y}_i(\omega_i, s; q) = \begin{Bmatrix} \omega_i \\ 0 \end{Bmatrix} \text{ if } s \begin{Bmatrix} \geq \\ < \end{Bmatrix} q \text{ for some } q \in [0,1], \forall i. \tag{9}$$

In this strategy profile, impatient depositors withdraw at period 1 and patient depositors withdraw in period 2 if the sunspot state is $s < q$, but both types of depositors withdraw in period 1 if the sunspot state $s \geq q$. Notice that, since s is uniformly distributed on $[0, 1]$, the value q can be interpreted as the probability of a run. To find equilibrium probability of bank runs, I first derive conditions under which a run equilibrium exists by (i) studying the bank's best response to this q-strategy profile and (ii) verifying whether this profile is part of the equilibrium.

3.2 The best-response allocation

In period 1, the bank chooses a repayment plan, which pays c_1 to the first θ depositors and state-contingent amounts after θ withdrawals. At the θ-th withdrawal, the fundamental state is revealed, and the bank switches to paying an amount c_{1z} to each of the following withdrawal until the π-th withdrawals. Once π withdrawals have been made, the bank will be able to infer the sunspot state by observing whether an additional withdrawal is requested or not. By this point of time, all uncertainty has been resolved. If a run has occurred, the bank gives a common amount of c^R_{1z} to each of the remaining impatient depositors. Letting π_s be the remaining impatient depositors, the profile (9) generates $\pi_{s \geq q} = 0$ and $\pi_{s<q} = \pi(1 - \pi)$. In period 2, each of the remaining patient depositors will receive a common amount c^N_{2z} if $s \geq q$ and c^R_{2z} if $s < q$. Given θ

[12] See, for example, Cooper and Ross (1998), Peck and Shell (2003) and Ennis and Keister (2010).

and q, I will below characterize these consumption levels to solve:

$$\max_{[c_1,\{c_{1z},c_{1z}^N,c_{2z}^N,c_{2z}^R\}_{z=b,g}]} \theta u(c_1)+\Sigma_z n_z\Big[(\pi-\theta)u(c_{1z})+(1-q)(1-\pi)u(c_{2z}^N) \\ +q(1-\pi)[\pi u(c_{1z}^R)+(1-\pi)u(c_{2z}^R)]\Big] \tag{10}$$

subject to

$$(1-\pi)\frac{c_{2z}^N}{R_z} = 1-\theta\frac{c_1}{p_u}-(\pi-\theta)\frac{c_{1z}}{p_z}, \tag{11}$$

$$\pi(1-\pi)\frac{c_{1z}^R}{p_z}+(1-\pi)^2\frac{c_{2z}^R}{R_z} = 1-\theta\frac{c_1}{p_u}-(\pi-\theta)\frac{c_{1z}}{p_z}, \forall h. \tag{12}$$

The first constraint corresponds to the resource constraint in $s<q$, and the second constraint corresponds to the resource constraint in $s \geq q$. When $s<q$, the bank must continue to serve the additional $\pi(1-\pi)$ depositors after π withdrawals at period 1. The right-hand side of each of the constraints is the remaining resource after π withdrawals. Letting λ_z^N and λ_z^R denote the multiplier on the first constraint and the second constraint in the fundamental state z respectively, the solution to the problem is characterized by the first-order conditions:

$$\lambda_g^N+\lambda_g^R+\lambda_b^N+\lambda_b^R = p_u u'(c_1), \tag{13}$$

$$\lambda_z^N+\lambda_z^R = n_z p_z u'(c_{1z}), \tag{14}$$

$$\lambda_z^N = n_z p_z u'(c_{1z}^N) = n_z R_z u'(c_{2z}^N), \tag{15}$$

$$\lambda_z^R = n_z p_z u'(c_{1z}^R) = n_z R_z u'(c_{2z}^R), \forall z. \tag{16}$$

The last two conditions imply that $c_{1z}^N < c_{2z}^N$ and $c_{1z}^R < c_{2z}^R$ always hold, because $p_z < R_z, \forall z$. The best-response allocation to profile (9) is summarized by a vector

$$\mathbf{c}^* \equiv \{c_1^*, \{c_{1z}^*, c_{1z}^{N*}, c_{2z}^{N*}, c_{2z}^{R*}\}_{z=b,g}\}, \tag{17}$$

that is characterized by the conditions above. This best-response allocation is determined based on a combination of (θ, q). The expected utility of depositors evaluated at $\mathbf{c}^*$ has the following two features. First, the expected utility of depositors is monotonically increasing in the level of opacity holding q fixed.

Proposition 1. For any $q \in [0, 1]$, $U(c^*, \hat{y}(q); \theta)$ is monotonically increasing in θ.

This proposition captures the insurance benefit provided by opacity. Recall that θ represents the number of withdrawals that are made before the asset quality is known. A higher θ means that more depositors are insured from the risk in the bank's assets. Given q, the expected utility of depositors increases over θ to the extent of their risk-aversion. Second, holding the level of opacity fixed, the expected utility of depositors decreases in q.

Proposition 2. For any $\theta \in [0, \pi]$, $U(c^*, \hat{y}(q); \theta)$ is monotonically decreasing in q.

The intuition behind this result is as follows: The bank anticipates that runs are more likely to occur as q increases. In other words, the expected number of period-1 withdrawals increases. The bank, then, plans to liquidate more assets in period 1. However, the asset yields less in period 1 than in period 2, which reduces the available resources and consequently, welfare.

3.3 Equilibrium bank runs

I will now study when the cutoff strategy profile is a part of an equilibrium in the withdrawal game for a given value of θ. Let $\mathcal{Q}$ be a set of values of q such that $\hat{y}$ is indeed part of an equilibrium, that is,

$$\mathcal{Q}(\theta) = \{q : \hat{y}(q) \in \mathcal{Y}(\theta)\}. \tag{18}$$

To calculate the equilibrium probability of bank runs, I compare expected payoffs of depositors in the sunspot state $s < q$ as in Keister (2016) and Li (2017). Since impatient depositors do not value consumption in period 2, they strictly prefer to withdraw in period 1. I only have to consider the actions of patient depositors to find the set $\mathcal{Q}(\theta)$. Patient depositors receive c_{2z}^N or c_{2z}^R, which depends on the sunspot state and the fundamental state if she waits until period 2. She receives, however, c_1 or c_{1z} in any sunspot state if she withdraws in period 1. The expected payoffs by withdrawing in period 1 or 2 are respectively

$$\mathbb{E}u(c_{1k}) = \frac{\theta}{\pi}u(c_1) + \left(1 - \frac{\theta}{\pi}\right)\Sigma_z n_z u(c_{1z}) \tag{19}$$

$$\mathbb{E}u(c_{2z}^R) = \Sigma_z n_z u(c_{2z}^R) \tag{20}$$

where k denotes her position in the order of withdrawals. The next result follows from (14)-(16) in a straightforward way.

Lemma 1. The best-response allocation $\mathbf{c}^*$ satisfies that:

1. $\mathbb{E}u(c_{1k})$ is monotonically decreasing in q,
2. $\mathbb{E}u(c_{2z}^R)$ is monotonically increasing in q.

The intuition behind this lemma is that the bank becomes more conservative as q increases. When a run is more likely to occur, the expected number of period-1 withdrawals increases. In response, the bank gives less consumption to period-1 withdrawers, which gives patient depositors less incentive to run. When q is sufficiently high, patient depositors may not have an incentive to withdraw in period 1. Let $barq$ denote the maximum value of q, such that, given θ, profile (9) is an equilibrium strategy:

$$\bar{q} = argmax\{q \in \mathcal{Q}(\theta)\}. \tag{21}$$

In this paper, I use $\bar{q}$ as the measure of financial fragility.[13] The maximum equilibrium probability, $\bar{q}$, satisfies $\mathbb{E}u(c_{1k}(\bar{q})) = \mathbb{E}u(c_{2z}^R(\bar{q}))$, and Lemma 1 assures that there always exists an unique value of $\bar{q}$. When $q > \bar{q}$, withdrawing in period 1 is not a best response for a depositor. If $\bar{q} = 1$, there is always an equilibrium in which a patient depositor certainly withdraws in period 1 in any sunspot state such that $y_i(\omega_i, s) = 0$ for all i for all s. Before I study how θ influences $\bar{q}$, I summarize the comparative statics for $\bar{q}$ holding θ fixed.

Proposition 3. Given θ, financial fragility as measured by $\bar{q}$ increases when

[13] I study the minimum equilibrium probability of a bank run in the appendix, which shows all of the qualitative results in this paper still hold if I use the minimum equilibrium probability of a bank run as the measure of financial fragility.

- the investors discount the asset returns more
  - ρ increases,
- the risk in the investment return increases,
  - R_g increases,
  - R_b decreases,
  - n becomes closer to $\frac{1}{2}$.

This proposition shows how parameters influence the benefits and costs of withdrawing in period 1. The investor's discount rate captures the cost. In particular, the marginal rate of substitution between consumption in period 1 and period 2 depends on ρ, as shown in (14)-(16). A higher ρ increases expected payoffs in period 1 relative to period 2. It is straightforward that lower costs encourage a depositor to run on the bank. The other parameters influence the value of insurance. When the bank's assets are riskier, a depositor has a higher incentive to join a run and financial fragility worsens even more.

3.4 The impact of opacity

I now ask: how does an increase in opacity affect financial fragility? To investigate this question, I need to study how opacity influences the expected payoffs in the game. Before the asset return is known, the bank meets withdrawals by selling assets at the discounted expected value, p_u. The repayments to the first θ withdrawals, therefore, do not depend on the realization of asset returns, and these depositors are isolated from the risk in their bank's assets (this is the Hirshleifer effect).

This risk-transfer is, however, available only when the asset return is not known. A depositor is insured from the risk only if she withdraws early enough in period 1. Otherwise, her repayment depends on the realization of asset returns. Because the order of withdrawals is random, the possibility of withdrawing funds before the return is known depends on θ. While a higher θ insures more depositors against the risk in the bank's assets, it also implies that a depositor has a higher chance of withdrawing funds before the return is known.

The limited availability of risk-transfer increases a patient depositor's incentives to withdraw in period 1, and this incentive depends on the level of θ. A larger θ increases the possibility of withdrawing before the return is known, which, in turn, raises expected payoffs from withdrawing in period 1, or a bank run. Therefore, as the bank's assets become more opaque, a bank run is more likely to occur.

Proposition 4. $\bar{q}$ is monotonically increasing in θ.

This result is illustrated in Figure 2, which depicts $\bar{q}$ as a function of θ given $(\gamma, \pi, n, R_g, R_b, \rho) = (2, 0.5, 0.5, 2, 1, 0.9)$. Notice that $\bar{q}$ remains at zero for a while as θ increases, which implies that the chance of obtaining insurance is sufficiently low for a patient depositor. Joining a run is, therefore, too risky for her because she has a high chance of receiving c_{1z}^R which is exposed to the risk and is even smaller than c_{2z}^R. As θ increases, an attempt to obtain insurance becomes more attractive, and the incentive to join a run becomes stronger.

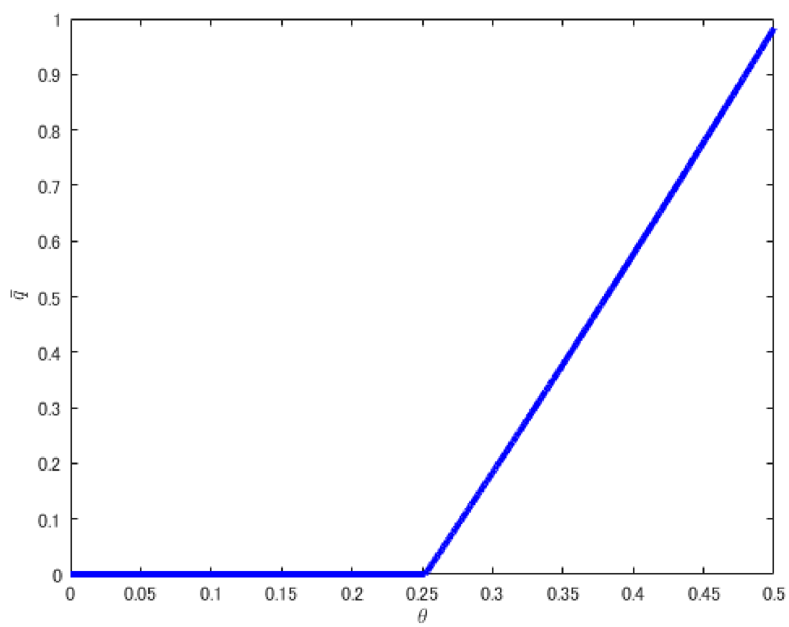

Figure 2: Higher opacity leads to more fragility.

3.5 Discussion

It is worth emphasizing that a higher θ benefits the remaining depositors after θ withdrawals. A higher θ allows the bank to insure more depositors from the risk, and then the bank changes the consumption given to each of depositors in a way that the remaining depositors can receive more goods. This substitution effect increases expected payoffs from withdrawing in period 1 and period 2. However, Proposition 4 shows that higher opacity always raises the expected payoff from withdrawing in period 1 more than period 2, because the increase of the probability of obtaining the insurance is the dominant effect.

It is also worth noting that a run can occur even if the expected amount of goods repaid in period 2 is larger than the one repaid before θ in period 1. In other words, there exists sets of parameters in which $c_1 < \mathbb{E}c_{2z}^R$ but $\mathbb{E}u(c_{1k}) \geq \mathbb{E}u(c_{2z}^R)$. This is because the repayments in period 2 have risk. When ρ is small, the asset price becomes lower, and these inequalities are more likely to hold.

An interesting feature of Proposition 4 is that depositors become sensitive in the absence of information and may run on the bank, increasing financial fragility. The existing literature supposes that depositors make a withdrawal decision after they receive partial or full information about the realization of asset returns. They focus on bank runs driven by information and show that opacity helps financial stability by preventing depositors from identifying unsound banks. I study the strategic aspect of bank runs. My model allows depositors to make a withdrawal decision before they learn the realization of the asset returns. This environment enables me to study self-fulfilling bank runs together with opacity, studies of which have been lacking in the literature. This distinction of the focus between information-driven runs and self-fulfilling runs explains the difference in the results.

This feature of the model is reminiscent of the housing market crash in 2007. A wide range of intermediation arrangements were issuing short-term debt to fund complex financial assets like a more complex version of mortgage-backed securities. These assets turned out to be difficult to value when house prices declined. The associated uncertainty worsened losses of confidence, followed by bank runs at Northern Rock and a variety of shadow banking arrangements in the

United States. My model indicates that this feature is an essential element of financial crises. Depositors attempt to withdraw early to insure themselves from the risk in the bank's assets. When the uncertainty is prolonged, the depositors become more nervous, and this tendency becomes stronger. In this way, my model illustrates how the opacity of a bank's assets can make bank runs more likely to occur.

4 Optimal opacity

In this section, I study a bank's choice of the degree of opacity in period 0. When the bank chooses θ, it creates a withdrawal game based on that level of opacity. The bank must form a belief about which equilibrium of the withdrawal game will be played for each possible value of θ. I suppose that the bank expects the worst equilibrium with regards to welfare associated with that value of θ to be played. I desire find the optimal degree of opacity under this robust control type of assumption.

4.1 Worst scenario

A bank faces uncertainty about the realization of the sunspot variable and, hence, about whether a run will occur. I suppose that the bank chooses θ to maximize its depositors' welfare in the worst case within the set $\mathcal{Q}(\theta)$. In other words, the bank chooses θ by solving $max_\theta min_{q \in \mathcal{Q}(\theta)} U(c^*, \hat{y}(q); \theta)$. Using Proposition 2, it is straightforward to show that the worst case scenario in the welfare perspective is $\bar{q}$. In particular, a run equilibrium exists if and only if $q \leq \bar{q}$ and the expected utility of depositors $U(c^*, \hat{y}(q); \theta)$, evaluated at $\mathbf{c}^*$, is decreasing in q, and hence $\bar{q} \in argmin\, U(c^*, y^*; \theta)$.

Recall that the expected utility of depositors (7) is the function of (c, y) given θ. In this approach, (c, y), and hence the function U, are determined by $(\theta, \bar{q}(\theta))$. Define $\mathcal{W}(\theta, \bar{q}(\theta))$ such that

$$\mathcal{W}(\theta, \bar{q}(\theta)) = U(c^*(\theta), \hat{y}(\bar{q}(\theta)); \theta). \tag{22}$$

The first argument captures the direct effect of opacity, which is providing insurance, or the Hirshleifer effect. The second argument shows the indirect effect, which is worsened financial fragility, or the fragility effect. The optimal level of opacity will be determined by the trade-off between the Hirschleifer effect and fragility effect. The solution to this problem characterizes an equilibrium in this overall economy together with the strategy profiles in the subsequent withdrawal game.

Definition 2. An *equilibrium with an observable degree of opacity* is (c^*, y^*, θ^*) such that

1. $(c^*, y^*) \in \mathcal{Y}(\theta^*)$
2. $\theta^* \in argmax\, \mathcal{W}(\theta, \bar{q}(\theta))$.

4.2 Banking problem

My interest is in how the interaction between depositors' withdrawal decisions and the bank's repayment plan depends on the level of opacity θ. The bank chooses the level of opacity in period 0 to maximize the expected utility of depositors. Notice that the function (7) depends on θ. The bank chooses θ by anticipating equilibrium outcomes in the withdrawal game and this choice of θ is immediately observable to depositors. The bank's problem is, therefore,

$$max_\theta \; \mathcal{W}(\theta, \bar{q}(\theta)). \tag{23}$$

4.3 Optimal opacity

I will now characterize the optimal level of θ. A higher level of opacity means that more depositors receive the certain value c_1 instead of the random variable c_{1z}. This also, however, increases financial fragility $\bar{q}$. The higher $\bar{q}$ means that the bank expects a higher number of period-1 withdrawals. To redeem more deposits in period 1, the bank plans to liquidate more assets that yield only a fraction of ρ of its long-term return, which is detrimental to welfare.

Proposition 5. The optimal level of opacity is: $\theta^* = \left(\frac{\chi}{1-\chi}\right)\left\{\left(\pi(\gamma-1)\psi^{-1}\rho^{-\frac{1}{\gamma}}\right)^{\frac{1-\gamma}{\gamma}} - 1\right\}$, where $\chi = \left(\frac{n_g p_g^{1-\gamma} + n_b p_b^{1-\gamma}}{(n_g p_g + n_b p_b)^{1-\gamma}}\right)^{\frac{1}{\gamma}} > 0$, and $\psi = \left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\} > 0$.

Opacity has a non-monotonic effect on welfare. The insurance benefit raises welfare when θ is small, but eventually, an increase in $\bar{q}$ becomes significant. At some point, the fragility cost dominates the insurance benefits and diminishes the welfare. The optimal level of opacity balances the insurance benefit and the fragility cost.

Proposition 6. The optimal opacity becomes smaller when

- the investors discount the asset returns more
  - ρ increases,
- the risk in the investment return increases,
  - R_g increases,
  - R_b decreases,
  - n becomes closer to $\frac{1}{2}$.

An increase in the investors' discount rate drives the asset prices up, which in turn raises the bank's repayments in period 1. When ρ is larger, an increase in θ attracts the depositors to withdraw in period 1. In other words, depositors become more sensitive to the level of opacity. When the fragility effect is larger, the bank should reduce the level of opacity. The other comparative statistics study how the level of the asset's risk influences the optimal level of opacity. When the asset becomes riskier, the insurance is more tempting to depositors. While the higher risk makes the insurance more valuable, it makes depositors more nervous about the risk. In such a situation, depositors react to the opacity more sensitively, which raises the cost of increasing the level of opacity. Therefore, when the bank's asset is riskier, less opacity is socially optimal. These results hold for any levels of the parameters, and the proof is presented in the appendix.

4.4 Discussion

Endogenizing a bank's choice of opacity in this framework is nontrivial because there are often multiple equilibria associated with each choice. This issue is not a new problem in studying self-fulfilling bank runs. In order to solve this problem, I borrowed ideas from the literature on robust control and assumed that the worst outcome consistent with equilibrium would occur for each possible choice of opacity. In the appendix, I show that this approach can be generalized to some extent. In particular, the qualitative results remain unchanged even if I assume that the best outcome consistent with equilibrium will occur for each possible choice of opacity. Assuming any linear combination of the worst and best outcomes to occur, therefore, does not change the qualitative results. Another common approach in the literature is to use the global game approach in selecting an equilibrium.[14] One prominent distinction is that my approach allows the bank's repayments to be flexible while the global game approach typically assumes a rigid repayment plan to desire the uniqueness. A flexible repayment plan is useful to avoid the time-inconsistency problem and to study shadow banking arrangements.

Proposition 6 is perhaps somewhat surprising. While one may assume that the bank would be less concerned about bank runs when liquidity in financial markets is greater, Proposition 6 disputes this by demonstrating that the bank ought to be more conservative about the fragility cost in deciding the level of opacity. This is because an investor's discount rate can be interpreted as the level of liquidity in financial markets, consequently suggesting that the bank should reduce opacity when the market has greater liquidity because depositors are more sensitive to opacity under such conditions. Therefore, the bank should be less concerned about fragility cost when liquidity dries up in financial markets, and a greater level of opacity is desirable.

5 Regulating opacity

In the previous analyses, I supposed that depositors could directly observe their bank's choice of θ. However, depositors may still find it difficult to know which of the assets would take longer to investigate. Assuming that such information is difficult to ascertain, a depositor in this model chooses a withdrawal strategy without directly knowing θ. Depositors can still make inferences and understand the bank's incentives, but the bank cannot credibly reveal its choice and cannot directly influence the depositors' behavior.

[14]See, for example, Goldstein and Pauzner (2005).

5.1 Modified withdrawal game

In this environment, the bank's choice of θ becomes a part of the simultaneous-move game. The bank chooses (c, θ) at the same time to maximize $U(c, y, \theta)$. Depositors choose y_i as before to maximize $v_i(c, y, \theta)$. An equilibrium of this game is defined as follows:

Definition 3. An *equilibrium of the modified withdrawal game* is profile of strategies $(c^{**}, y^{**}, \theta^{**})$ such that

1. $v_i(c^{**}, (y_i^{**}(s), y^{**}-i(s)), \theta^{**}) \geq v_i(c^{**}, (y_i(s), y_{-i}^{**}(s)), \theta^{**})$ for all s, for all y_i, for all i

2. $U(c^{**}, y^{**}(s), \theta^{**}) \geq U(c, y^{**}(s), \theta)$ for all c and for all θ

The only difference from the original game is that θ is chosen to maximize the expected utility of depositors in the game.

5.2 The best response allocation

Similarly to Section 3.2, I consider the cutoff strategy profile (9) and study the best response of the bank, finding the maximum value of q that is in an equilibrium. Given q, the bank will choose opacity θ and the consumption levels c to solve:

$$\max_{[\theta, c_1, \{c_{1z}, c_{1z}^N, c_{2z}^N, c_{2z}^R\}_{z=b,g}]} \theta u(c_1) + \Sigma_z n_z \Big[(\pi - \theta)u(c_{1z}) + (1-q)(1-\pi)u(c_{2z}^N) + q(1-\pi)[\pi u(c_{1z}^R) + (1-\pi)u(c_{2z}^R)]\Big] \tag{24}$$

subject to

$$(1-\pi)\frac{c_{2z}^N}{R_z} = 1 - \theta\frac{c_1}{p_u} - (\pi - \theta)\frac{c_{1z}}{p_z}, \tag{25}$$

$$\pi(1-\pi)\frac{c_{1z}^R}{p_z} + (1-\pi)^2\frac{c_{2z}^R}{R_z} = 1 - \theta\frac{c_1}{p_u} - (\pi - \theta)\frac{c_{1z}}{p_z}, \forall h. \tag{26}$$

The only difference from Section 3.2 is that θ is now a one of the choice variables. The solution to this problem is characterized by the first-order conditions (14)-(16) and the following first-order condition of θ:

$$u(c_1) - \Sigma_z u(c_{1z}) \geq \Sigma_z(\lambda_z^N + \lambda_z^R)\left(\frac{c_1}{p_u} - \frac{c_{1z}}{p_z}\right). \tag{27}$$

By these conditions, I can establish the following proposition:

Proposition 7. The bank's dominant strategy is $\theta^{**} = \pi$.

The solution has a feature that the bank always chooses maximal opacity. The bank has an incentive to raise θ as much as possible because it still provides insurances to depositors, but does not directly influence the behavior of depositors. The strategy profile (9) is, then, a part of equilibrium when $q \in \mathcal{Q}(\pi)$. Following Section 4, I consider the worst case scenario over $q \in \mathcal{Q}(\theta)$. Proposition 2 implies the best response allocation has a feature that $U(c^*, \hat{y}, \pi)$ is decreasing in q. In the worst scenario, the expected utility of depositors is therefore evaluated at $\bar{q}(\theta)$ and will be $\mathcal{W}(\pi, \bar{q}(\pi)) \leq \mathcal{W}(\theta^*, \bar{q}(\theta^*))$. This inequality shows that equilibrium outcomes may be worse for depositors than Section 4.

5.3 Regulation

A natural question is how policymakers can improve the expected utility of depositors. The source of inefficiency is that the bank becomes overly opaque. One way to improve welfare is to impose an observable upper bound on θ so that $\theta \in [0, \theta^*]$.

Proposition 8. When θ is unobservable, an observable upper bound on opacity such that $\theta \leq \theta^*$ can implement the constrained efficient allocation.

While such a cap does not change the incentives of the bank, it can shrink the bank's choice set. The bank 's conditional dominant strategy is now θ^*. Depositors still anticipate that the bank will be maximally opaque, but they know that the maximum level is only θ^*. The outcome is, then, constrained efficient.

Proposition 6 implies that policymakers should regulate opacity more strictly when the liquidity in financial markets is high, when the bank's assets are riskier, or when the asset price is more volatile. An example of such a regulation is to limit asset classes of the bank's investment. For instance, if θ^* corresponds to Level 2 assets in the Financial Accounting Standards, prohibiting the bank from investing in Level 3 assets improves financial stability and welfare. More precise levels of measurement for opacity based on the length of time it takes to investigate asset quality would perhaps be more useful in designing regulation.

6 Conclusion

The idea that opaque financial assets either caused or greatly amplified the financial crisis of 2007-8 has led to a range of proposals to increase the transparency of assets in the financial system. At the same time, there is a counterargument that requiring financial institutions to be more transparent would undermine a fundamental feature of banking. In order to study whether regulation to increase transparency would be desirable, I have presented a model with three key features: (i) the opacity of a bank's assets determines the amount of time that elapses before anyone can accurately determine the assets' realized value; (ii) the bank can sell assets before the realized value is known to anyone; and (iii) depositors are more likely to run on the bank when withdrawing early is more attractive.

It follows from (ii) that opacity brings an insurance benefit: the more assets the bank can sell before their realized value is known, the less consumption risk its depositors face. Requiring transparency would thus lower welfare through a Hirshleifer (1971) effect in which having more information can destroy insurance possibilities. At the same time, however, I showed that opacity also has a cost in terms of financial stability. In particular, higher opacity makes a bank more fragile in the sense that it introduces equilibria in which a self-fulfilling bank run is more likely to occur. This result is new in the literature and formalizes the concerns people often express about opaque assets. By doing so, I established the trade-off that the literature had not studied before.

Endogenizing a bank's choice of opacity in this framework is not straightforward because there are multiple equilibria associated with each choice. I focused on the worst outcome consistent with equilibrium for each possible choice of opacity. The socially-optimal level of opacity balances the insurance benefit with the fragility cost described above. When the financial market discounts the bank's assets less, or when the asset is riskier, depositors become more sensitive to the opacity. In such a situation, raising the level of opacity is more costly, and greater transparency will be desirable from the social point of view. Whether or not regulation is required to achieve this outcome depends on what information a bank's depositors can observe. If they are sufficiently well informed about their bank's choices, competition will drive banks to choose the optimal level. If not, banks will have an incentive to become overly opaque, and regulation to limit opacity would improve welfare.

The primary takeaway of the article is that the opacity of a bank's assets indeed increases financial fragility as in the policymakers' claim while cautioning policymakers not to go too far. Requiring transparency would destroy banks' opportunities to provide insurance to depositors against the risk in the bank's assets.

I conclude by noting potentially promising directions for future research. First, government guarantees are a popular policy to prevent banking panics and may complement the provision of safe liquidity. However, guarantees in bad times may distort banks' incentives, and there could also be various formulations of guarantees. It is then unclear how guarantees interplay with opacity and what scheme of guarantees is efficient. Second, investors may discount the uncertainty of asset returns. When they can not distinguish banks having good or bad assets, adverse selection problems may arise, and asset prices may be severely discounted before the returns are known. A lower pooling price reduces the insurance benefit but also reduce incentives to run on the bank. It is ambiguous which of these competing effects dominate the other. Studies on these extensions would be interesting future research.

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A Expected individual consumption

I here denote the expected individual consumption explicitly. Suppose a measure Ψ of patient depositors follow run strategy such that

$$\Psi(y) = \int_0^1 \mathbb{1}_{(y_i(1,s<q)=0)} di.$$

If $y_i(\omega_i, s) = \omega_i$,

$$\begin{aligned}
c_1^i =&(1-q)\left(\frac{\theta}{\pi}u(c_1) + \left(1-\frac{\theta}{\pi}\right)\left(n_g u(c_{1g}^N) + n_b u(c_{1b}^N)\right)\right) \\
&+ q\Bigg[\frac{\theta}{\pi+\Psi(1-\pi)}u(c_1) + \frac{\pi-\theta}{\pi+\Psi(1-\pi)}\left(n_g u(c_{1g}) + n_b u(c_{1b})\right) \\
&+ \left(1-\frac{\pi}{\pi+\Psi(1-\pi)}\right)\left(n_g u(c_{1g}^R) + n_b u(c_{1b}^R)\right)\Bigg], \\
c_2^i =&(1-q)\left(n_g u(c_{2g}^N) + n_b u(c_{2b}^N)\right) + q\left(n_g u(c_{2g}^R) + n_b u(c_{2b}^R)\right).
\end{aligned}$$

If $y_i(\omega_i, s) = \begin{Bmatrix} \omega_i \\ 0 \end{Bmatrix}$ if $s \begin{Bmatrix} \geq \\ < \end{Bmatrix} q$,

$$\begin{aligned}
c_1^i =&(1-q)\left(\frac{\theta}{\pi}u(c_1) + \left(1-\frac{\theta}{\pi}\right)\left(n_g u(c_{1g}^N) + n_b u(c_{1b}^N)\right)\right) \\
&+ q\left[\frac{\theta}{\pi+\Psi(1-\pi)}u(c_1) + \frac{\pi-\theta}{\pi+\Psi(1-\pi)}\left(n_g u(c_{1g}) + n_b u(c_{1b})\right)\right], \\
c_2^i =&(1-q)\left(n_g u(c_{2g}^N) + n_b u(c_{2b}^N)\right) + q\left(1-\frac{\pi}{\pi+\Psi(1-\pi)}\right)\left(n_g u(c_{2g}^R) + n_b u(c_{2b}^R)\right).
\end{aligned}$$

B Full solution to the bank's problem (Section 3.2)

I here summarize the full characterization of the solution to the banking problem. Through constraints (11)-(12) and the first-order conditions (14)-(16), it is straightforward to derive each consumption variables given θ as below.

$$
\begin{aligned}
c_1(\theta) =& \frac{p_u}{\theta + \eta\chi} && (28)\\
c_{1z}(\theta) =& \frac{p_z(1-\theta\frac{c_1}{p_u})}{\eta}, && (29)\\
c^N_{2z}(\theta) =& \frac{R_z(1-\theta\frac{c_1}{p_u})(\frac{\Lambda^{\frac{1}{\gamma}}}{\eta})}{\eta}, && (30)\\
c^R_{1z}(\theta) =& \frac{p_h(1-\theta\frac{c_1}{p_u})(\frac{\Lambda^{\frac{1}{\gamma}}}{\eta})}{\pi(1-\pi)+(1-\pi)^2(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}}, && (31)\\
c^R_{2z}(\theta) =& \frac{p_h(1-\theta\frac{c_1}{p_u})(\frac{\Lambda^{\frac{1}{\gamma}}}{\eta})}{\pi(1-\pi)+(1-\pi)^2(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}}\left(\frac{1}{\rho}\right)^{\frac{1}{\gamma}}, \forall h \in b, g && (32)
\end{aligned}
$$

where

$$
\begin{aligned}
\eta =& (\pi - \theta) + \Lambda^{\frac{1}{\gamma}} > 0,\\
\Lambda =& (1-q)\left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} + q\left\{\pi(1-\pi)+(1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} > 0.
\end{aligned}
$$

The last inequality implies the first inequality because its first term is positive ($\theta \in [0, \pi]$).

C The lower bound of q in equilibria

I now turn my attention to the greatest lower bound of q such that

$$
\underline{q} = argmin\{q \in \mathcal{Q}(\theta)\}, \tag{33}
$$

by solving for q such that $\mathbb{E}u(c^N_{2z}) = \mathbb{E}u(c_{1k})$ holds. Lemma 1 assures that there always exists an unique value for $\underline{q}$ as well. This threshold value, $\underline{q}$, is the minimum value of q in which patient depositors prefer to wait until period 2 when all other patient depositors wait until period 2. When q is small, the bank becomes aggressive to give consumption in period 1, and patient depositors may withdraw in period 1 whatever the sunspot state is. This threshold $\underline{q}$ has the following feature: a no-run strategy profile, such that $y_i(\omega_i, s) = \omega_i$ for all i for all s, is a part of equilibrium if and only if $\underline{q} = 0$.[15] In comparing $\underline{q}$ and $\bar{q}$, they have the following relation:

Proposition 9. $\underline{q}\begin{Bmatrix} = \\ > \end{Bmatrix}0$ if and only if $\bar{q}\begin{Bmatrix} < \\ = \end{Bmatrix}1$.

This proposition means that there are two cases: (i) when $\underline{q} = 0$, there always exists a no-run equilibrium and co-exists with equilibria in which runs occur with probability $q \le \bar{q}$ and (ii) when $\underline{q} > 0$, there always exists an equilibrium in which runs certainly occur and co-exists with equilibria in which runs occur with probability $q > \underline{q}$. Proposition 3-8 still hold even if I use $\underline{q}$ as the measure of financial fragility instead of $\bar{q}$.

In Diamond and Dybvig (1983), Cooper and Ross (1998), Ennis and Keister (2010) and many others, there always exists a no-run equilibrium such that $\underline{q} = 0$. On the other hand, Peck and Shell (2003) and Shell and Zhang (2018) introduce a preference parameter in such a way that patient depositors value short-term consumption more and show that a no-run equilibrium may not exist in some cases, which corresponds to $\underline{q} > 0$. My model shows that, even without an additional preference parameter, a no-run equilibrium may not exist through uncertainty and insurance.

D Proofs for selected results

Proposition 1. The bank's best response to $\hat{y}(\theta)$ is summarized in the vector $\mathbf{c}^*$ in which each consumption variable is derived in Section B. Substituting (28)-(32) into the objective function (10), I can derive the objective function $U(c^*, \hat{y}(q); \theta)$ as a function of θ such that

$$U(c^*, \hat{y}(q); \theta) = \left(\frac{1}{1-\gamma}\right) p_u^{1-\gamma} \big(\theta + \eta\chi\big)^{\gamma}. \tag{34}$$

[15] In Ennis and Keister (2010), Li (2017) and many others, the value of $\underline{q}$ is always at 0 and a no-run equilibrium always exists.

Taking a derivative of θ,

$$\frac{\partial U(c^*, \hat{y}(q); \theta)}{\partial \theta} = \underbrace{\left(\frac{\gamma}{1-\gamma}\right)}_{\text{negative}} \underbrace{p_u^{1-\gamma}\{\theta + \eta\chi\}^{\gamma-1}}_{\text{positive}} \underbrace{(1-\chi)}_{\text{negative}} > 0.$$

The second group is straightforward as $\eta > 0$ and $\chi > 0$. The sign of the third group is implied by the convexity such that

$$\chi = \left(\frac{n_g p_g^{1-\gamma} + n_b p_b^{1-\gamma}}{(n_g p_g + n_b p_b)^{1-\gamma}}\right)^{\frac{1}{\gamma}} > 1.$$

□

Proposition 2. Recall the objective function (34). Holding q fixed, I take a derivative of θ:

$$\frac{\partial U(c^*, \hat{y}(q); \theta)}{\partial q} = \underbrace{\left(\frac{\gamma}{1-\gamma}\right)}_{\text{negative}} \underbrace{p_u^{1-\gamma}\{\theta + \eta\chi\}^{\gamma-1}}_{\text{positive}} \underbrace{\chi\frac{1}{\gamma}\frac{\partial \Lambda}{\partial q}}_{\text{positive}} > 0.$$

The sign of $\frac{\partial \Lambda}{\partial q}$ is positive because

$$\left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\} > \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}. \tag{35}$$

□

Lemma 1. I first show that the expected payoffs in period 1 (19) is monotonically decreasing in q. Recall the expected payoff in period 1:

$$\begin{aligned}\mathbb{E}u(c_{1k}) =& \frac{\theta}{\pi}u(c_1) + \left(1 - \frac{\theta}{\pi}\right)\Sigma_z n_z u(c_{1z}) \\ =& \left(\frac{P_u^{1-\gamma}}{1-\gamma}\right)\left(\frac{1}{\theta + \eta\chi}\right)^{1-\gamma}\left(\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\chi\right)\end{aligned}$$

Taking a derivative of q,

$$\frac{\partial \mathbb{E}u(c_{1k})}{\partial q} = (-1)\underbrace{\left(\frac{1}{\theta+\eta\chi}\right)^{1-\gamma}\chi\left\{\frac{\theta}{\pi}p_u^{1-\gamma}+\left(1-\frac{\theta}{\pi}\right)\Delta\chi\right\}}_{\text{positive}}\underbrace{\frac{\partial\eta}{\partial q}}_{\text{positive}} < 0.$$

The positive sign of $\frac{\partial\eta}{\partial q}$ is because $\frac{\partial\Lambda}{\partial q}$. Similarly, I next show that the expected payoff (20) is monotonically increasing in q. Letting $A = \frac{1}{\pi(1-\pi)+(1-\pi)^2(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}}(\frac{1}{\rho})^{\frac{1}{\gamma}} > 0$,

$$\frac{\partial \mathbb{E}u(c_{2z}^R)}{\partial q} = (n_g p_g^{1-\gamma}+n_b p_b^{1-\gamma})A^{1-\gamma}\chi^{1-\gamma}\left(\frac{1}{\theta+\eta\chi}\right)^{-\gamma}\Lambda^{\frac{1-2\gamma}{\gamma}}\left\{\chi\Lambda\frac{\partial\eta}{\partial q}+\left(\frac{1}{\theta+\eta\chi}\right)\left(\frac{1}{\gamma}\right)\frac{\partial\Lambda}{\partial q}\right\} > 0,$$

where all of the terms are positive. □

Proposition 3. I first find $\bar{q}$ such that $\mathbb{E}u(c_{1k}(\bar{q})) = \mathbb{E}u(c_{2z}^R(\bar{q}))$ holds. Equating $\mathbb{E}u(c_{1k})$ and $\mathbb{E}u(c_{2z}^R)$,

$$\left\{\frac{\theta}{\pi}+\left(1-\frac{\theta}{\pi}\right)\chi\right\}^{\frac{\gamma}{1-\gamma}}\left\{\pi(1-\pi)+(1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}\chi^{\frac{-\gamma}{1-\gamma}}\rho = \Lambda.$$

By substituting Λ and solving for q,

$$\bar{q} = \frac{\rho\left\{\frac{\theta}{\pi}\chi^{-1}+\left(1-\frac{\theta}{\pi}\right)\right\}^{\frac{\gamma}{1-\gamma}}\left\{\pi(1-\pi)+(1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}-\left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}{\left\{\pi(1-\pi)+(1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}-\left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}. \tag{36}$$

When R_g increases, when R_b decreases, or when n is closer to $\frac{1}{2}$, the relevant term is only the first term of the numerator. Each of these changes raises a benefit of risk-sharing through a decrease in χ, which pushes $\bar{q}$ up. Parameter ρ has effects on the risk-sharing between good and bad fundamental states and the risk-sharing between period 1 and period 2, and hence they appear every terms. Suppose ρ increase, then effects on $\bar{q}$ depend on $\rho\left\{\frac{\theta}{\pi}\chi^{-1}+\left(1-\frac{\theta}{\pi}\right)\right\}^{\frac{\gamma}{1-\gamma}}$. This entire term is monotonically increasing when ρ becomes larger, and hence the numerator will increase more than the denominator, which, in turn, raises $\bar{q}$. □

Proposition 4. I below find $\frac{\partial\bar{q}}{\partial\theta}$ by differentiating (36) with respect to θ. Notice that the denominator of (36) is positive and θ appears only in the first term of the numerator. Differentiating $\bar{q}$

by θ gives:

$$\frac{\partial \bar{q}}{\partial \theta} = \underbrace{\frac{\gamma}{1-\gamma}}_{\text{negative}} \underbrace{\left(1-\chi\right)}_{\text{negative}} \underbrace{\left(\frac{1}{\pi}\right)\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\chi\right\}^{\frac{1}{1-\gamma}} Z}_{\text{positive}} > 0, \tag{37}$$

where $Z = \frac{\left\{\pi(1-\pi)+(1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}\chi^{-1}\rho}{\left\{\pi(1-\pi)+(1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} - \left\{(1-\pi)(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}} > 0.$ □

Proposition 5. I solve for the optimal level of opacity by differentiating the objective function (34) with respect to θ. I substitute (36 into q, and then solve for $\theta \in [0, \pi]$:

$$max_{\theta\in[0,\pi]} \ \left(\frac{1}{1-\gamma}\right) {p_u}^{1-\gamma}\left\{\theta(1-\chi) + \pi\chi + \left(\frac{\theta}{\pi}(1-\chi)+\chi\right)^{\frac{1}{1-\gamma}} \psi\rho^{\frac{1}{\gamma}}\chi^{-\frac{1}{1-\gamma}}\right\}^{\gamma}. \tag{38}$$

When $\frac{\partial U(c^*, \hat{y}(q(\theta)); \theta)}{\partial \theta} = 0$, the first-order condition is

$$(1-\chi) + \left(\frac{1}{1-\gamma}\right)\frac{1}{\pi}(1-\chi)\left\{\frac{\theta}{\pi}(1-\chi)+\chi\right\}^{\frac{\gamma}{1-\gamma}} \psi\rho^{\frac{1}{\gamma}}\chi^{-\frac{\gamma}{1-\gamma}} = 0,$$

which characterize $\theta^* = \left(\frac{\chi}{1-\chi}\right)\left\{\left(\pi(\gamma-1)\psi^{-1}\rho^{-\frac{1}{\gamma}}\right)^{\frac{1-\gamma}{\gamma}} - 1\right\}$. Note that $\left(\frac{\chi}{1-\chi}\right) < 0$, because $\chi > 1$. □

Proposition 6. To see the effect of an increase in ρ, I take differentiate θ^* with respect to ρ:

$$\begin{aligned} \frac{\partial \theta^*}{\partial \rho} &= \underbrace{\left(\frac{\chi}{1-\chi}\right)}_{\text{negative}} \underbrace{\pi\left\{\pi(\gamma-1)\right\}^{\frac{1-\gamma}{\gamma}}}_{\text{positive}} \underbrace{\left(\frac{1-\gamma}{\gamma}\right)}_{\text{negative}} \times \\ &\quad \left\{\underbrace{(-1)\psi^{-\frac{1}{\gamma}}(1-\pi)^2\left(\frac{\gamma-1}{\gamma}\right)\rho^{-\frac{1}{\gamma^2}}}_{\text{negative}} + \underbrace{\psi^{-1}\left(-\frac{1}{\gamma}\right)\rho^{\frac{-\gamma^2+\gamma-1}{\gamma^2}}}_{\text{negative}}\right\} < 0. \end{aligned} \tag{39}$$

When R_g increases, when R_b decreases, or when n is closer to $\frac{1}{2}$, the relevant term is only χ and χ increases. In order to prove the second comparative statics, it suffices to show $\frac{\partial \theta^*}{\chi} > 0$. When $\theta^* \in [0, 1]$,

$$\frac{\partial \theta^*}{\partial \chi} = \underbrace{\left\{ \left(\pi(\gamma-1)\psi^{-1}\rho^{-\frac{1}{\gamma}} \right)^{\frac{1-\gamma}{\gamma}} - 1 \right\}}_{\text{negative}} \underbrace{\frac{\partial}{\partial \chi}\left(\frac{\chi}{1-\chi} \right)}_{\text{positive}} < 0. \tag{40}$$

$\square$

Proposition 7. I first characterize the solution to the modified banking problem. The objective function (24) and the set of constraints (25)-(26) remain unchanged from the bank's problem in Section 4, but here is one more choice variable, θ. The solution is characterized by the resource constraint (25)-(26), the first-order conditions (14)-(16) and (27). Combining these equation, I characterize optimal consumption levels by θ and I formulate the optimization problem as a function of θ:

$$U(c^{**}(\theta), \hat{y}(q), \theta) = max_{\theta \in [0,\pi]} \frac{p_u^{1-\gamma}}{1-\gamma} \big(\theta + \eta\chi\big)^{\gamma}.$$

I differentiate this objective function by θ:

$$\frac{\partial U(c^{**}(\theta), \hat{y}(q), \theta)}{\partial \theta} = \underbrace{\frac{\gamma}{1-\gamma}}_{\text{negative}} \underbrace{p_u^{1-\gamma}\bigg(\theta + \eta\chi\bigg)^{\gamma-1}}_{\text{positive}} \underbrace{\bigg(1-\chi\bigg)}_{\text{negative}}, \tag{41}$$

where the sign of the second last and last group are implied by Proof of Proposition 1. Therefore, $U(c^{**}(\theta), \hat{y}(q), \theta)$ is monotonically increasing in θ given $\hat{y}(q)\forall q$. The optimal level of θ will be at the maximum level $\theta^{**} = \pi$. $\square$

Proposition 8. Proof of Proposition 7 shows thatθ^{**} in this environment is the maximum feasible θ. By limiting the bank's choice set from $[0, \pi]$ to $[0, \theta^*]$, θ^{**} is still at the corner solution but at the same level to θ^*. Since the objective function and the set of constraints are the same one as in Section 4, $\mathcal{W}(\theta^*, \bar{q}(\theta^*)) \geq \mathcal{W}(\pi, \bar{q}(\pi))$. $\square$

Proposition 9. I derive $\underline{q}$ such that $\mathbb{E}u(c_{1k}) = \mathbb{E}u(c_{2z}^N)$ holds. Equating $\mathbb{E}u(c_{1k})$ and $\mathbb{E}u(c_{2z}^N)$,

$$\left\{ \frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\chi \right\}^{\frac{\gamma}{1-\gamma}} (1-\pi)^{\gamma} \chi^{\frac{-\gamma}{1-\gamma}} \rho = \Lambda. \tag{42}$$

By substituting Λ and solving for q,

$$\underline{q} = \frac{\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\chi\right\}^{\frac{\gamma}{1-\gamma}} (1-\pi)^{\gamma} \chi^{-1} \rho^{\gamma} - \left\{(1-\pi)(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}{\left\{\pi(1-\pi) + (1-\pi)^2 \left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} - \left\{(1-\pi)(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}$$

Conditions $\bar{q} \begin{Bmatrix} \leq \\ > \end{Bmatrix} 1$ and $\underline{q} \begin{Bmatrix} \geq \\ < \end{Bmatrix} 0$ reduce to the same criteria:

$$\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\chi\right\}^{\frac{\gamma}{1-\gamma}} \chi^{\frac{-\gamma}{1-\gamma}} \rho \begin{Bmatrix} \geq \\ < \end{Bmatrix} 1. \tag{43}$$

$\square$