Rational Bubbles and Middlemen

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Motivation

Bubbles:

- Continuous price increases, interrupted by a sudden market crash
- Chains of intermediaries engaged in flipping

Examples: Dutch tulip mania (1634-7); Mississippi Bubble (1719-20); South Sea Bubble (1720); Roaring Twenties followed by the 1920 crash; Housing bubble preceded the 2008 financial crisis

$\Longrightarrow$ Explore for a (simple) framework of bubbles that features the above

Our Approach

- Why would a smart person hold an asset they know is overpriced?
  - they're hoping to sell it to another person just before the bubble bursts
- Why would that other smart person buy an asset that's about to collapse?
  - Bubbles are impossible
  - They expect the overpricing to grow forever
  - Our answer: finite horizon, identifying exactly the timing of bubble burst

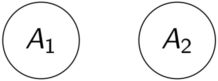
Our Approach

Implications:

- The intuition of market participants, “if they want to ride a bubble, they must carefully time the point at which they sell to a “greater fool”, and so, get out of the bubble”
- Booms turn into euphoria as “rational exuberance morphs into irrational exuberance”
  Charles P. Kindleberger (1978)
  “Manias, Panics, and Crashes: A History of Financial Crises”

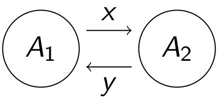
Illustrative example

- Suppose there are two agents, A_1 and A_2



- And two goods—goods x and y

Illustrative example



- Good y can be produced (at a certain cost) and consumed by both agents
- Good x is owned by agent A_1, but consumed only by A_2

Illustrative example

- The consumption value of good x is stochastic
- Specifically, the value

$$V = \begin{cases} v & \text{with some probability} \\ 0 & \text{with the remaining probability} \end{cases}$$

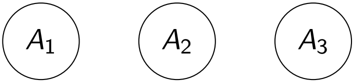
where $v > 0$

Illustrative example

- Obviously, bubble never occur
- That is, consider a case where
  - $V = 0$, that is the value of object x is 0
  - And all agents know this
- In this case, trade doesn't occur
  - A_2 rejects to produce any positive amount of good y to get good x

Illustrative example

- Now suppose the trade can be done through a middleman (flipper)
- In particular, there are three agents, A_1, A_2 and A_3



Illustrative example

As before, two goods, x and y

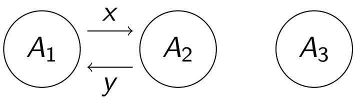
- Good x is now owned by A_1 and can be consumed only by A_3
- Good y can be produced and consumed by all agents
- The consumption value of good x

$$V = \begin{cases} v & \text{with some probability} \\ 0 & \text{with the remaining probability} \end{cases}$$

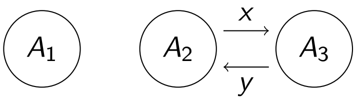
Illustrative example

Trading protocol is similar as before:

- First A_1 and A_2 can trade goods x and y



- If the trade occurs, then A_2 and A_3 can trade goods x and y



Illustrative example

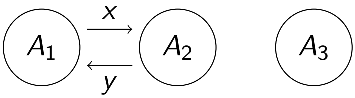
- Now suppose as before
  - $V = 0$, that is the value of object x is 0
  - And all agents know this
- Can good x ever be traded with good y?
- Can bubble occur?

Illustrative example

- Yes!
- There are certain cases in which good x is traded for good y, *although everyone knows* the consumption value of x is 0
- Specifically suppose A_2 is a fool who (mistakenly) believes that A_3 is a greater fool than he is
  - That is, A_2 puts high probability on the event than A_3 does on the event that x has value
  - Consistent with all agents knowing the value of x is 0
- In this case...

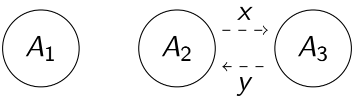
Illustrative example

Then A_2 is still willing to trade with A_1



Illustrative example

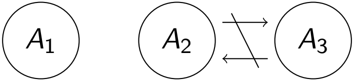
Hoping to trade with A_3



- Recall A_2 does NOT know that A_3 knows $V = 0$

Illustrative example

Unfortunately for A_2, A_3 refuses the trade



- A_3 knows good x has no value
- A_2 turns out to be the greatest fool who cannot find a greater fool

Bubble

Middlemen (flippers) are a source of bubbles

- End users care about the quality of an asset
- Middlemen don't
  - Downstream middlemen only care about how end users think about the asset
  - Upstream middlemen only care about how down stream middlemen think about the asset

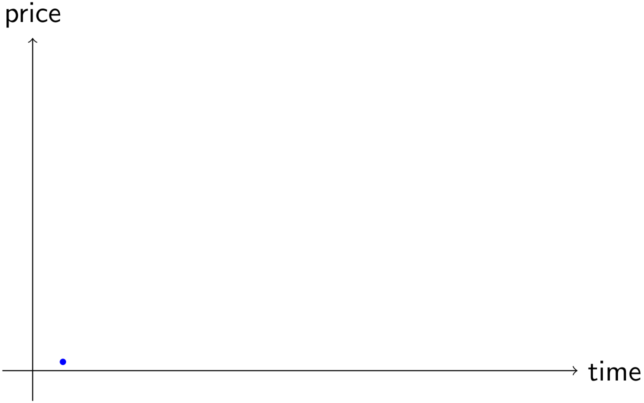
Paper

Based upon this observation

- We construct a tractable model of bubbles in an economy with flippers
  - An object with no value is traded although everyone knows that it has no value
  - A fool buys the object, hoping to find a greater fool who buys the object from him
- Bubble occurs in the unique equilibrium
- The model describes the life of a bubble

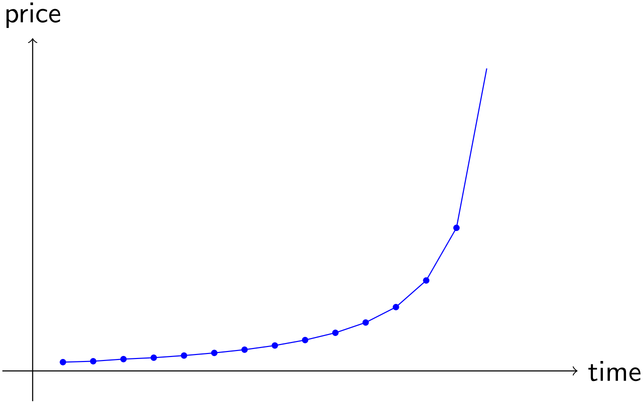
Price path

An object without fundamental value is traded at a positive price



Price path

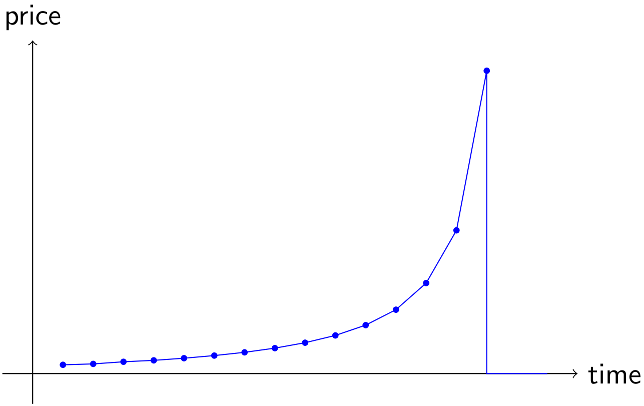
Price of the object increases—and accelerates—as time passes



While the fundamental of the economy does NOT grow

Price path

And someday, it bursts



Paper

And

- Provide a simple condition for which bubble is detrimental
- Show bubble-bursting policy (Conlon, 2015) does not affect welfare
- Information increases size of bubble
  - Not information on fundamentals, but information on knowledge of the other agents

Fools

We do NOT assume irrational agents nor heterogeneous priors

- Fools are not irrational, but ignorant people

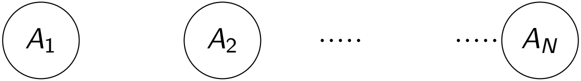
The Model

Objects

- Two goods—x and y
- Good x is durable and indivisible
- Good y is perishable and divisible

Environment

N agents, A_1, A_2,..., A_N



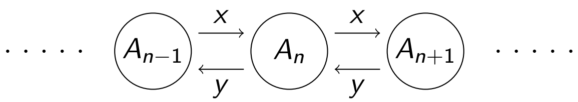
Environment

- Good x is owned by A_1 and can be consumed only by A_N
  - The consumption value of good x

$$V = \begin{cases} v > 0 & \text{with some probability} \\ 0 & \text{with remaining probability} \end{cases}$$

- Good y can be produced and consumed by all agents
  - The cost of producing $\hat{y}$ units of good y is $\hat{y}$
  - The utility of consuming $\hat{y}$ units of good y is $\kappa\hat{y}$

Environment



- Agent A_{n-1} and A_{n+1} can trade only through A_n
  - First A_{n-1} and A_n can (if both want) exchange x and some amount of good y
  - Conditional on the trade between A_{n-1} and A_n, A_n and A_{n+1} can exchange x and some amount of good y
- The amount of y is determined by Nash bargaining

Knowledge

- Introduce type space
  - Each type describes who knows what
- In a way reminiscent of Rubinstein's Email game
  - Rather schematic
  - A way to help illustrating the relevant knowledge structure

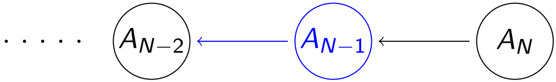
Knowledge

- If $V = 0$, A_N gets a signal s_N with some probability
- Thus, if A_N gets s_N, then he knows that $V = 0$
  - If not, A_N becomes optimistic about the value of good x

Knowledge

- If A_N gets the signal s_N, then he sends a signal ("rumor") s_{N-1} to A_{N-1}
- The "rumor" reaches A_{N-1} with some probability
- Thus, if A_{N-1} gets s_{N-1}, then he knows that A_N knows $V = 0$

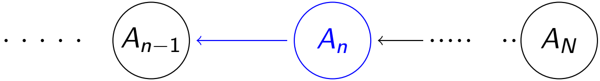
Knowledge



- If A_{N-1} gets the signal s_{N-1}, then he sends a signal ("rumor") s_{N-2} to A_{N-2}
- The "rumor" reaches A_{N-2} with some probability
- Thus, if A_{N-2} gets a signal s_{N-2}, then he knows that A_{N-1} knows that A_N knows $V = 0$

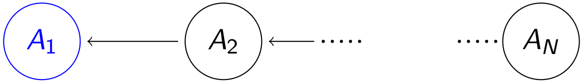
Knowledge

In general



- If A_n gets the signal s_n, then he sends a signal ("rumor") s_{n-1} to A_{n-1}
- The "rumor" reaches A_{n-1} with some probability
- Thus, if A_{n-1} gets a signal s_{n-1}, then he knows that A_n knows that ... that A_N knows that $V = 0$

Knowledge



- If A_1 gets the signal s_1, the process stops

Knowledge

- Finally, assume all but A_N always know the value of x

Type space

Formally, the set of the state of the world

$$\Omega = \{\omega_v, \omega_\phi, \omega_N, ..., \omega_1\}$$

where

- ω_v means $V = v$
- ω_ϕ means $V = 0$ and no agents get a signal
- ω_n means $V = 0$ and agent n is the last one to get a signal

Partition

Partition of

- A_N is
$$\{\underbrace{\{\omega_v, \omega_\phi\}}_{\text{no signal}}, \underbrace{\{\omega_N, ..\omega_1\}}_{\text{signal}}\}$$
- A_n is
$$\{\underbrace{\{\omega_v\}}_{V=v}, \underbrace{\{\omega_\phi, .., \omega_{n+1}\}}_{V=0,\ \text{no signal}}, \underbrace{\{\omega_n, ..., \omega_1\}}_{V=0,\ \text{signal}}\}$$

Prior

- Prior distribution μ on Ω
- Homogeneous prior—μ is common knowledge

Price

- Price (the amount of good y) is determined by Nash barganing
  - Outside option is 0
  - The value of good x is unknown, but the expected value is common knowledge
  - Can be generalized
  - Let θ be the bargaining power of A_n in trade betweenA_n and A_{n+1}
- Price of each pair is NOT observed by outsiders
  - Over-the-couter market

Timing

1. Nature determines V
2. Signals ("rumors") are send, and a type is determined
3. Actual trades start

Main result

Definition

We say bubble occurs if

- Everyone knows the value of good x is 0
- And yet good x is exchanged with positive amount of good y

Main result

Theorem

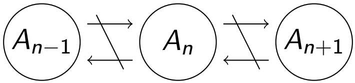
The equilibrium is unique. In the equilibrium, a bubble occurs when $\omega \in \{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$*. Moreover, a bubble bursts for sure.*

Backward induction

Backward induction

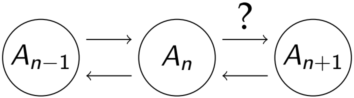
- Clearly, A_N buys good x if and only if he doesn't get a signal
  - If he gets a signal, he knows x has no value
  - If he hasn't, his expected value of good x is positive, and hence willing to produce some amount of good y
- Suppose that A_{n+1} buys good x if and only if he doesn't get a signal
- Given this, how should A_n behave?

Optimal behavior of A_n



- If A_n gets a signal, then A_{n+1} also gets a signal
- Induction hypothesis: A_{n+1} will reject the trade
- Optimal not to buy x

Optimal behavior of A_n



- If A_n doesn't get a signal, $\omega \in \{\omega_{n+1}, \omega_{n+2}, ..., \omega_\phi\}$
- Two possibilities:
  1. A_{n+1} also doesn't get a signal, that is, $\omega \in \{\omega_{n+2}, ..., \omega_\phi\}$
  2. A_{n+1} gets a signal, that is, $\omega = \omega_{n+1}$
- Induction hypothesis:
  1. A_{n+1} buys x when $\omega \in \{\omega_{n+2}, ..., \omega_\phi\}$
  2. A_{n+1} doesn't buy x when $\omega = \omega_{n+1}$
- Since there is a chance that A_{n+1} buys good x, A_n is willing to buy good x

Price

The exact price is given as follows: Define $(\hat{y}_n)_{n=1}^{N-1}$ by: For $N-1$,

$$\hat{y}_{N-1} = \theta v_e$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}_n = \theta \kappa \psi_{n+1} \hat{y}_{n+1}$$

Example: $N = 3$ and Uniform $\mu(\omega)$

- At state ω_3, bubbles occur.
  - More precisely, A_1 and A_2 exchange x and

    $$\frac{1}{4}\kappa v$$

    units of good y
  - Then A_2 and A_3 of course do not trade
- recall partition of A_2

  $$\mathcal{P}_2 = \{\{\omega_v\}, \{\omega_\phi, \omega_3\}, \{\omega_2, \omega_1\}\}$$

  so that at ω_3, from A_2's point of view, the state is either ω_ϕ or ω_3
  - He puts the same probability in each state

Example: $N = 3$ and Uniform $\mu(\omega)$

Recall A_3's partition

$$\mathcal{P}_3 = \{\{\omega_v, \omega_\phi\}, \{\omega_3, \omega_2, \omega_1\}\}$$

- At ω_ϕ, A_3 doesn't know whether $V = 0$ or v
  - Recall true state of the world is ω_3
  - But importantly, A_2 assigns probability 1/2 to the event ω_ϕ
  - Thus what happens at ω_ϕ matters a lot
- And so A_3 accepts a trade as long as

$$\hat{y}_3 \leq \frac{1}{2} \times 0 + \frac{1}{2} \times v = \frac{v}{2}$$

Example: $N = 3$ and Uniform $\mu(\omega)$

Then, from middleman A_2's point of view...

- At ω_3, A_3 refuses the trade. A_2 gets 0 by having good x
- But at ω_ϕ, A_3 accepts the trade. This implies, at ω_ϕ, A_2 gets

$$\frac{\kappa v}{2}$$

  by having good x
  - Note that from $v/2$ units of good y, an agents gets utility $\kappa v/2$
- Since he assigns the same probability to each event, his expected value of having good x is

$$\frac{1}{2} \times 0 + \frac{1}{2} \times \frac{\kappa v}{2} = \frac{\kappa v}{4}$$

- He accepts a trade if

$$\hat{y}_2 \leq \frac{\kappa v}{4}$$

Example: $N = 3$ and Uniform $\mu(\omega)$

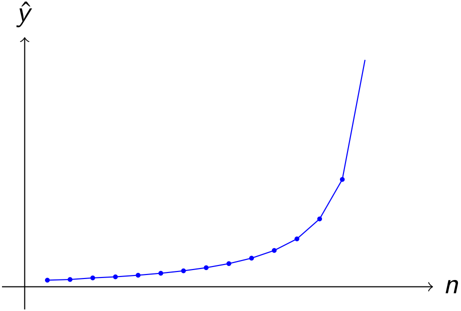
- In words, at ω_3, A_2 doesn't know whether he
  - can find a greater fool
  - or not—he is the greatest fool
- And unfortunately, A_2 turns out to be the greatest fool

Price Path

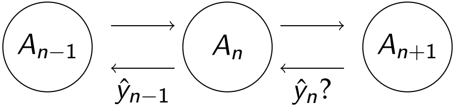
Price

Price of good x is

- Always increasing
- Accelerating unless prior distribution is extreme
  - Satisfied when, for example, in each step the signal is lost with the same probability

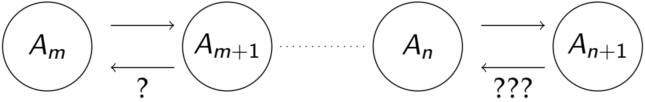


Increasing



- Why increasing?
- Agent A_n always faces a risk that A_{n+1} rejects the trade
  - That is, A_n may be the greatest fool who fails to find a greater fool
- To compensate this, price must increase

Accelerating



- Why accelerating?
- When $m < n$, the risk that A_n faces is higher than that A_m faces
  - Why so? Will see
- To compensate this, price must accelerate

Accelerating

- Why it is the case that when $m < n$, the risk that A_n faces is higher than that A_m faces?
- Given that A_n doesn't get a signal, the probability that A_{n+1} does not get a signal is

$$\psi_n = 1 - \frac{\mu(\omega_{n+1})}{\mu(\omega_{n+1}) + \mu(\omega_{n+2}) + ... + \mu(\omega_{\phi})}$$

- The probability is decreasing in n
  - To get an idea, suppose that μ is uniform so that for each $\omega, \omega' \in \Omega$, $\mu(\omega) = \mu(\omega')$
  - Then
$$\psi_n = 1 - \frac{1}{N - n + 1}$$
  - ψ_n is decreasing in n

Welfare/ Probability of Bubble

Welfare

Welfare implication

- Consider the interim stage where planner knows $V = 0$
- When $\kappa > 1$, bubble improves welfare
- But when $\kappa < 1$, bubble is detrimental

Probability of bubble

- How likely (ex ante) does a bubble occur?
- The probability can be arbitrarily close to 1
- Recall bubble occurs at states $\{\omega_N, ..., \omega_3\}$
- With uniform distribution $(\mu(\omega) = 1/((N+2))$ the probability is
$$1 - \frac{4}{N+2}$$
- As $N \to \infty$, the probability goes to 1
- Note that the ex ante probability that good x has value is very small

Applications

Bubble-bursting policy

- Should a central bank burst bubble?
- Suppose it knows that the asset is worthless if and only if all agents know, that is,

$$\mathcal{P}_{CB} := \{\{\omega_v, \omega_\phi\}, \{\omega_N, ..\omega_3\}\} = \mathcal{P}_N$$

- And it can release the information to burst the bubble
- Should it adopt such a policy?

Bubble-bursting policy

Trade-off when $\kappa < 1$ (the other case is opposite), bubble-bursting policy is

- Good when $\omega \in \{\omega_N, ..\omega_3\}$
  - Without policy, bubble occurs while detrimental
  - With policy, announcement follows and bubble doesn't occur
- Bad when $\omega = \omega_\phi$
  - Without policy, agents A_n put positive probability that he is the greatest fool
  - With policy, agents $A_n, n \neq N$ now know that he cannot be the greatest fool
    - They all know that A_N doesn't get the signal and so will "buy" good x
    - The inaction of the central bank affects agents' beliefs
  - Thus, policy <u>increases</u> price
- Neutral when $\omega \in \{\omega_v, \omega_2, \omega_1\}$

Announcement

Surprisingly, these two effects *completely* offset each other!

Proposition

The bubble-bursting policy has no effect on ex ante welfare.

Bubble and information

- In the model, flippers' information is "fine"
  - Everyone has a chance to get a signal
  - This is why, everyone can be the greatest fool
- What if information is "coarser"?
  - That is, $A_n, n \neq N$ never gets a signal
- What happens to the size of bubble?
- Information enhances bubble, that is...

Bubble and information

- $\hat{y}_n$ is the price when information is finer
- y_n^0 is that when coarser

Proposition

$$\hat{y}_n > y_n^0$$

Conclusion

A tractable model of bubble

- Flippers cause bubbles
- Bubble occurs in an unique backward induction outcome