Corporate Finance and Monetary Policy*

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This version: May 2017

Abstract

We develop a general equilibrium model where entrepreneurs finance random investment opportunities using trade credit, bank-issued assets, or currency. They search for bank funding in over-the-counter markets where loan sizes, interest rates, and down payments are negotiated bilaterally. The theory generates pass through from nominal interest rates to real lending rates depending on market microstructure, policy, regulation, and firm characteristics. Higher banks' bargaining power or reduced frictions, e.g., raise pass through but weaken transmission to investment. The structure of interest rates arises from liquidity, regulatory, and intermediation premia, and depends on policy. Numerical examples illustrate key quantitative points.

JEL Classification Nos: D83, E32, E51

Keywords: corporate finance, monetary policy, banking, interest rates

*We thank the editor, John Leahy, and four anonymous referees for comments. We also thank Francesca Carapella, Etienne Wasmer, and Sebastien Lotz for input, as well as participants at the 2015 WAMS-LAEF Workshop in Sydney, 2016 Money, Banking, and Asset Markets Conference at UW-Madison, 2016 Search and Matching Conference in Amsterdam, Spring 2016 Midwest Macro Meetings at Purdue, 2016 West Coast Search and Matching Workshop at the San Francisco Fed, 2016 Conference on Liquidity and Market Frictions at the Bank of France, 2016 SED Meetings in Toulouse, the Bank of Canada, University of Iowa, Dallas Fed, University of Hawaii, University of Michigan, U.C. Irvine, and U.C. Los Angeles. We are grateful to Mark Leary for providing data on firms' cash holdings. Wright acknowledges support from the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business. The usual disclaimers apply. Emails: grochete@uci.edu; rwright@bus.wisc.edu; cmzhang@purdue.edu.

It is commonly thought and taught that central banks influence economic activity through their impact on short-term nominal rates, which gets passed through to the real rates at which firms and households borrow. We illustrate the empirical relationship between these different interest rates in Figure 1.[1] While perhaps appealing heuristically, modeling the transmission mechanism rigorously has proved illusive. This project builds on recent advances in monetary economics, as surveyed in Lagos et al. (2017) and Rocheteau and Nosal (2017), to develop a general equilibrium model of firms' investment and financing choices that helps us understand the channels through which policy affects interest rates, liquidity, bank lending, and output.

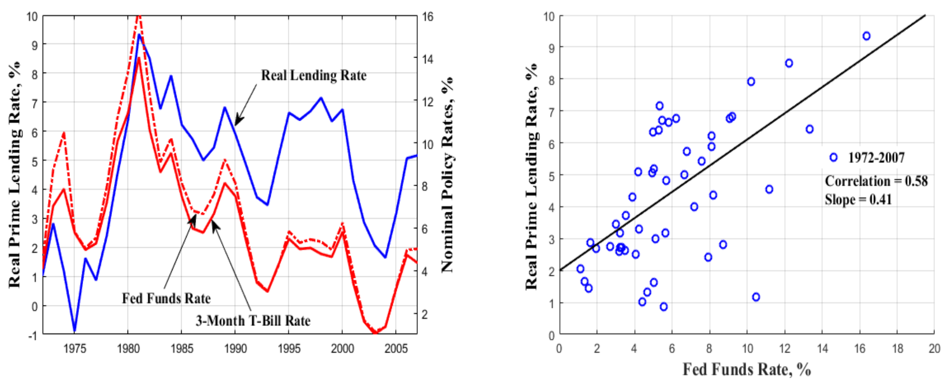

Figure 1: Real prime lending rate vs. short-term nominal rates

Our approach combines a theory of financial intermediation, whereby banks acquire illiquid loans in exchange for liquid short-term liabilities through decentralized trade, and a theory of money demand by firms facing idiosyncratic uncertainty. The transmission mechanism operates as follows. A lower nominal rate induces firms to ramp up cash holdings, in accordance with the evidence on firms' money demand, shown in the top panels of Figure 2. Firms with large amounts of cash rely less on external funding to finance investments, allowing them to negotiate lower real

[1] The real lending rate is computed as the bank prime lending rate from which we subtract a measure of anticipated inflation following the methodology developed in Hamilton et al. (2015). The fitted line in the right panel of Figure 1 is estimated using OLS.

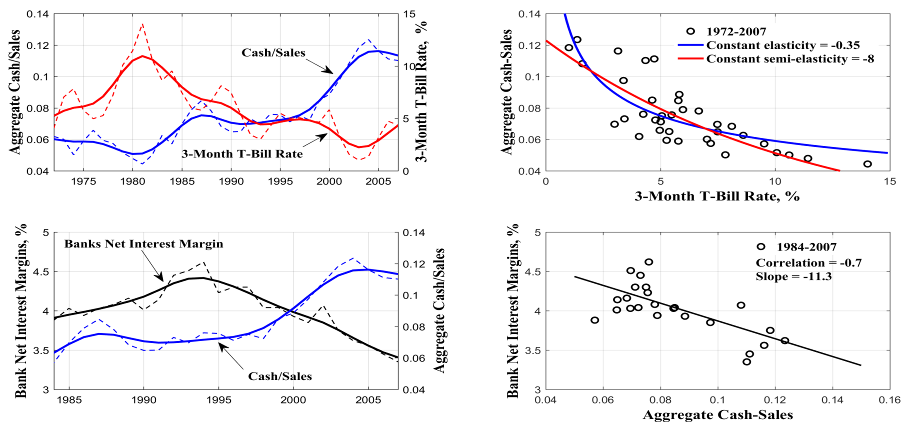

Figure 2: Firms' Money Demand (top); Banks' Net Interest Margins (bottom)

lending rates and reduce banks' share of the surplus, in accordance with the negative correlation between banks' net interest margins and firms' cash/sales ratio, as shown in the bottom panels of Figure 2.[2] The theory makes precise predictions for the determinants of pass through, namely, policy instruments (the interbank or money growth rate), regulations (reserve and capital requirements), credit market microstructure (matching efficiency, bargaining power or entry costs), and firms' characteristics (borrowing capacity or capital intensity). If access to bank loans improves, e.g., pass through to the real lending rate increases, but transmission to aggregate investment weakens.

The model also generates a rich structure of returns, including real and nominal yields on overnight rates in the interbank market, on liquid bonds, on illiquid bonds, and on corporate lending. Interest spreads correspond to distinct liquidity, regulatory, and intermediation premia that are all influenced by policy. To illustrate the subtleties in pass through, as money growth increases, real rates of return on monetary assets decrease, the real return on illiquid bonds are unaffected, and the

[2] Figure 2 uses Compustat data from Graham and Leary (2016) for unregulated non-financial firms. The annual data is HP filtered with a smoothing parameter of 6.25. The fitted curves in the top right panel are estimated using log-log and semi-log specifications as in Lucas (2000). The bottom panel uses data on banks' net interest margins for the U.S. from the FFIEC. Net interest margins are defined as the spread between what banks receive on interest-earning assets and what they pay on interest-paying liabilities, divided by total interest-earning assets.

real lending rate increases. These findings are in sharp contrast with many models, with one interest rate typically interpreted as both a Fed choice and the cost of investment. To show how this matters quantitatively, we provide calibrated examples, the results of which are consistent with pass through and money demand as illustrated in Figures 1 and 2, as well as empirical work on the differentiated effects of monetary policy across industries.

1 Preview

In the model economy, entrepreneurs receive random opportunities to invest, but might not be able to get sufficient credit from suppliers of capital goods. Hence they may use either retained earnings held in liquid assets (internal finance) or loans from banks (external finance), as shown in Figure 3. Banks have a comparative advantage in monitoring and enforcement, and so their liabilities can serve as payment instruments. Realistically, bank loans are made in an over-the-counter (OTC) market featuring search and bargaining. Loan contracts are negotiated between entrepreneurs and banks, in terms of the interest rate, loan size, and down payment. Due to limited commitment/enforcement, only a fraction of investment returns are pledgeable, and getting a loan is a time-consuming process that is not always successful; hence, in accordance with the evidence, credit has both an intensive margin – the size of loans – and extensive margin – the ease of obtaining loans.[3]

We begin with a special case where only external finance is possible. The real lending rate banks charge depends on the internal rate of return of investments, banks' bargaining power, and the availability of alternative means of finance such as trade credit. In order to link the lending rate to monetary policy, we allow entrepreneurs to accumulate outside money, the opportunity cost of which is the nominal

[3] Having both an intensive and an extensive margin is consistent with evidence from the Joint Small Business Credit Survey (2014). Among firms that applied for loans, 33% received what they requested, 21% received less, and 44% were denied. Also, to be clear, our concern is not with firms' choice to issue equity or bonds to acquire new capital (even though we do introduce corporate bonds in Section 5.3); it is with the choice between using liquid retained earnings and bank or trade credit.

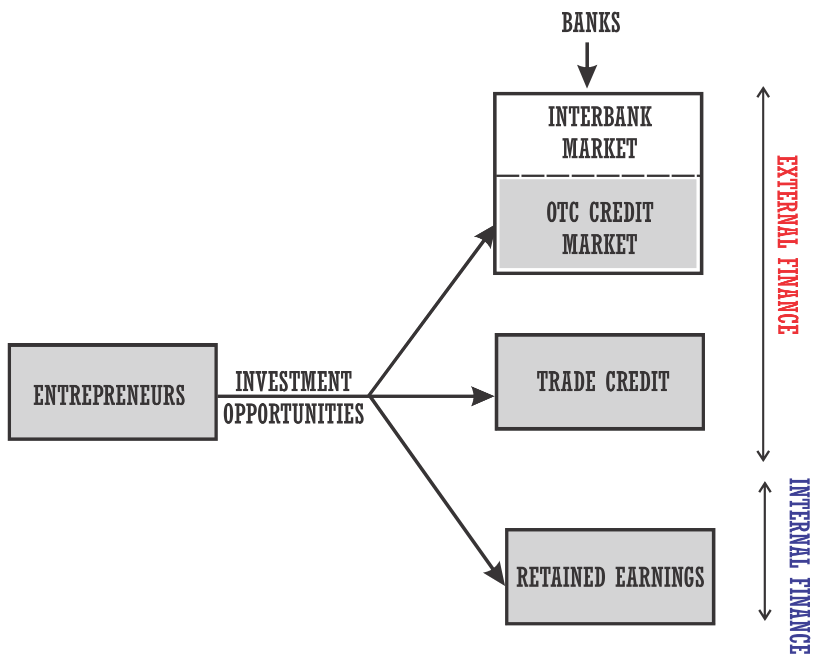

Figure 3: Sketch of the model

interest rate on an illiquid bond. This generates coexistence of money (internal finance) and various forms of credit (external finance) under broad conditions. Money held by firms has two roles: an insurance function, allowing them to finance investment internally; and a strategic function, allowing them to negotiate better loans. Consistent with the evidence, firms' money demand increases with idiosyncratic risk and decreases with pledgeability.[4]

A key result concerns pass through from the nominal policy rate to the real loan rate, as an increase in the former raises the latter by affecting the cost of liquid retained earnings. Perhaps surprisingly, higher interest rate pass through need not imply stronger transmission to aggregate investment. An increase in banks' bargaining power, e.g., raises pass through but makes money demand less elastic with respect to nominal interest rates, thereby weakening transmission. Moreover, for low interest rates transmission occurs exclusively through internally-financed investment, while

[4]For recent surveys of the empirical literature on corporate liquidity management, see Sánchez and Yurdagul (2013), Campello (2015), and Graham and Leary (2016). In addition to transaction and precautionary motives, these studies also attribute large corporate cash holdings to tax reasons that we abstract from here.

for higher interest rates it also occurs through bank-financed investment, and this channel generates a financing multiplier that increases with pledgeability.

To illustrate how these findings matter quantitatively, we calibrate the model and show it generates realistic pass through and transmission. We also quantify the importance of liquidity constraints for the output sensitivity to policy and show it is large. Also consistent with the evidence, firm heterogeneity generates differential effects of policy, where an increase in the policy rate has a larger effect on firms that are more reliant on internal finance and are more capital intensive.

We also extend the model to include different policy instruments, like an interbank rate or reserve and capital requirements. This gives a realistic structure of returns on interbank loans, corporate loans, liquid bonds and illiquid bonds. While the real rates of return on money and liquid bonds are less than the rate of time preference, due to liquidity or regulatory premia, the real lending rate is larger than its level under perfect enforcement due to an intermediation premium. An increase in money growth reduces the returns on monetary assets but raises the real lending rate. We also study open market operations and their effects on interest spreads. If the supply of bonds is low, the nominal yield on liquid bonds can be negative or the economy can fall into a liquidity trap depending on regulations. These results all shed new light on monetary policy and its relation to corporate finance.

In terms of the literature, the dynamic general equilibrium approach to liquidity builds on Lagos and Wright (2005) and the extension in Rocheteau and Wright (2005). However, this paper concerns entrepreneurs financing investment, rather than the usual situation with households financing consumption.[5] The credit market here is similar to Wasmer and Weil (2004), although we are more explicit about frictions, formalize the role of money, have both internal and external finance, and

[5]There are a few notable exceptions, e.g., Rocheteau and Nosal (2017, Chapter 15) and Rocheteau and Wright (2013, Section 7.2) where firms trade capital in an OTC market, Silveira and Wright (2010) or Chiu et al. (2015), where firms trade ideas. Recent New Monetarist models of household credit in goods markets include Sanches and Williamson (2010), Gu et al. (2016), and Lotz and Zhang (2016). Related papers on intermediation include Cavalcanti and Wallace (1999), Gu et al. (2013), Donaldson et al. (2016) and Huang (2016), which all have banks as endogenous institutions arising from explicit frictions, with their liabilities facilitating third-party transactions.

endogenize loan size. The determination of the loan size and lending rate is similar to the theory of intermediation premia (bid-ask spreads) in OTC financial markets by Duffie et al. (2005) and Lagos and Rocheteau (2009). The extension with bank entry is related to Atkeson et al. (2015), where banks trade derivative swap contracts in an OTC market. Of course the overall approach is related to the literature on pledgeability following Kiyotaki and Moore (1997), Holmstrom and Tirole (1998, 2011) and Tirole (2006).[6]

Bolton and Freixas (2006) also analyze monetary policy and corporate finance, but do not actually have money in their model, and simply take the real interest rate as a policy tool. A survey of the very different New Keynesian approach is found in Bernanke et al. (1999), focusing on nominal rigidities and the effects of policy on the cost of borrowing and amplification through balance sheet effects. The most relevant example is Gerali et al. (2010) who introduce an imperfectly competitive banking sector where banks are subject to capital requirements; they obtain incomplete pass through by assuming banks face costs of adjusting retail rates. Our model, in contrast, generates pass through in the absence of nominal rigidities or regulation. There are also models with a competitive banking sector where a spread between deposit and lending rates arise due to collateral requirements or agency problems between households and banks (e.g., Goodfriend and McCallum 2007, Christiano et al. 2008). In contrast to these approaches, having an OTC loan market lets us delve further into details of the transmission mechanism, and to show how search frictions and market power matter. In that spirit, Malamud and Schrimpf (2017) develop a money-in-the-utility-function model where intermediaries possess a technology to issue and trade general state-contingent claims, and customers can only trade such claims through bilateral meetings with intermediaries, and show how monetary policy redistributes wealth by influencing intermediation rents.

[6] New Monetarist work emphasizing pledgeability includes Lagos (2010), Venkateswaran and Wright (2013), Williamson (2012, 2015) and Rocheteau and Rodriguez-Lopez (2014). Related work in finance includes DeMarzo and Fishman (2007) and Biais et al. (2007). Also, while Bernanke et al. (1999) and Holmstrom and Tirole (1998) rationalize limited pledgeability using moral hazard, we provide alternative foundations using limited commitment in a Supplemental Appendix.

The rest of the paper is organized as follows. Sections 2 and 3 present the environment and derive preliminary results. Section 4 studies the case with only external finance. Section 5 adds internal finance. Section 6 presents some calibrated examples. By way of extensions, Section 7 introduces an interbank market and regulation, and Section 8 analyzes bank entry. Section 9 concludes. A few proofs are relegated to the Appendix, while several Supplemental Appendices not for publication describe alternative formulations and technical details.

2 Environment

Each period $t = 1, 2, ...$ is divided into two stages: first, there is a competitive market for capital and an OTC market for banking services; second, there is a frictionless market where agents settle debts and trade final goods and assets. This environment is ideal for our purposes because at its core is an asynchronicity between expenditures and receipts crucial for any analysis of money or credit. Into this setting, we introduce three types of agents, $j = e, s, b$. Type e agents are *entrepreneurs* in need of capital; type s are *suppliers* that can provide this capital; and type b are *banks* that are discussed in detail below. The measure of entrepreneurs is 1, the measure of suppliers is irrelevant due to constant returns, and the measure of banks is captured by matching probabilities as explained below. All agents have linear utility over a numéraire good c, where $c > 0$ ($c < 0$) is interpreted as consumption (production). They discount across periods according to $\beta = 1/(1 + \rho)$, $\rho > 0$.

In stage 1, capital k is produced by suppliers at unit cost. In stage 2, entrepreneurs transform k acquired in stage 1 into $f(k)$ units of c, where $f(0) = 0$, $f'(0) = \infty$, $f'(\infty) = 0$, and $f'(k) > 0 > f''(k)\ \forall k > 0$.[7] For simplicity, k fully depreciates at the end of a period.[8] Entrepreneurs face two types of idiosyncratic uncertainty: one is related to investment opportunities, as in Kiyotaki and Moore

[7] Note that k could be any input in the production process, including intermediate goods, physical capital, and even labor.

[8] This assumption rules out claims on capital circulating across periods. In a Supplemental Appendix, we study a version of the model with long-lived capital.

(1997); the other is related to financing opportunities, as in Wasmer and Weil (2004). Specifically, in the first stage, each entrepreneur has an investment opportunity with probability λ, in which case he can operate technology f. Simultaneously, the entrepreneur participates in an OTC market where he meets a banker who is willing to give him a loan with probability α. Assuming independence, $\alpha\lambda$ is the probability an entrepreneur has an investment opportunity and access to funds, while $\lambda(1-\alpha)$ is the probability he has an investment opportunity but no such access.

As regards the enforcement of debt, post production an entrepreneur can renege, but creditors have partial recourse: they can recover $\chi f(k)$, with $\chi \leq 1$ representing the fraction of output that is pledgeable. Here χ is a primitive capturing properties of output and capital, like portability and tangibility, plus institutions including the legal system, but it can also be derived rigorously from information and commitment frictions (see the Supplemental Appendix). The resources recovered in the case of default can vary with the creditor: if it is a supplier, pledgeable output is $\chi_s f(k)$, which might be the resale value of some intermediate good that can be repossessed without formal bankruptcy; if the entrepreneur is also in debt to a bank, pledgeable output is $\chi_b f(k)$, with $\chi_b \geq \chi_s$ because banks can enforce larger repayments.[9] Pledgeable output is generally divided between suppliers and bankers depending on institutional details, e.g. seniority according to bankruptcy law; our assumption is that bank debt is more senior.

Limited pledgeability generates a demand for outside liquidity in the form of fiat currency or inside liquidity in the form of short-term bank liabilities. The fiat money supply evolves according to $M_{t+1} = (1+\pi) M_t$, where π is the rate of monetary expansion (contraction if $\pi < 0$) implemented by lump sum transfers to (taxes on) entrepreneurs. The price of money in terms of numéraire is $q_{m,t}$, and in stationary equilibrium $q_{m,t} = (1+\pi)q_{m,t+1}$, so π is also inflation. The real gross rate of return of money is $1+r_m = q_{m,t+1}/q_{m,t} = 1/(1+\pi)$, and as usual we impose $\pi > \beta - 1$. Let

[9] There is ample evidence that businesses use bank and trade credit as alternative means of financing investments (e.g., Petersen and Rajan 1997; Mach and Wolken 2006).

$A_{m,t} = q_{m,t}M_t$ be aggregate real balances. Banks issue intraperiod liabilities, like notes or demand deposits, in stage 1 and commit to redeem them in stage 2. Banks' commitment here is assumed, but can be endogenized as in Gu et al. (2013) and Huang (2016). It is also convenient to make bank liabilities perfectly recognizable within a period, but counterfeitable thereafter, to preclude them circulating across periods, which is not crucial but does ease the presentation.[10]

There is a fixed supply A_g of one-period government bonds that pay to the bearer 1 unit of numéraire in stage 2, although not much changes if they instead pay out dollars. To simplify the presentation, through most of the paper these bonds, in contrast to bank-issued assets, cannot be used as a medium of exchange – say, they are book-keeping entries, not tangible assets that can pass between agents – but we relax this in Section 5.4. The price of a newly-issued bond in stage 2 is q_g, its real return is $r_g = 1/q_g - 1$, and its nominal return is $i_g = (1+\pi)/q_g - 1$. Banks can trade money and bonds, and make intra-period loans to each other, again with commitment, in a competitive interbank market in stage 1. These trades, which can be interpreted as overnight loans in the Fed Funds market, play no role until regulatory requirements are introduced in Section 7.

3 Preliminary Results

Consider an entrepreneur at the beginning of stage 2 with k units of capital, and financial wealth ω denominated in numéraire, including real balances a_m, plus government bonds a_g, minus debts. His value function satisfies

$$W^e(k,\omega) = \max_{c,\hat{a}_m,\hat{a}_g} \{c + \beta V^e(\hat{a}_m, \hat{a}_g)\} \text{ st } c = f(k) + \omega + T - \frac{\hat{a}_m}{1+r_m} - \frac{\hat{a}_g}{1+r_g},$$

where T denotes transfers minus taxes, and $V^e(\hat{a}_m, \hat{a}_g)$ is the continuation value with a new portfolio $(\hat{a}_m, \hat{a}_g)$ in stage 1 next period. The constraint indicates the change in financial wealth, $\hat{a}_m/(1+r_m) + \hat{a}_g/(1+r_g) - \omega$, is covered by retained

[10] For recent analyses of recognizability and liquidity in closely related environments, see Lester et al. (2012) or Li et al. (2012).

earnings, $f(k)+T-c$. Eliminating c using the constraint, we get

$$W^e(k,\omega)=f(k)+\omega+T+\max_{\hat{a}_m,\hat{a}_g\geq 0}\left\{-\frac{\hat{a}_m}{1+r_m}-\frac{\hat{a}_g}{1+r_g}+\beta V^e(\hat{a}_m,\hat{a}_g)\right\}. \tag{1}$$

Hence, W^e is linear in wealth, and the choice of $(\hat{a}_m,\hat{a}_g)$ is independent of (k,ω).

In stage 1,

$$V^e(\hat{a}_m,\hat{a}_g)=\mathbb{E}W^e\left(k,\hat{a}_m+\hat{a}_g-q_k k-\phi\right),$$

where q_k is the price of k. Thus, the entrepreneur purchases k at cost $q_k k$, pays ϕ for banking services, and $q_k k+\phi$ is subtracted from his stage 2 wealth. Expectations are with respect to (k,ϕ) and depend on whether he has an investment opportunity (if not, $k=\phi=0$) and whether he has access to bank lending (if not, $\phi=0$). Substituting V^e into (1), we reduce the choice of real balances to

$$\max_{\hat{a}_m,\hat{a}_g\geq 0}\left\{-i\hat{a}_m-\left(\frac{i-i_g}{1+i_g}\right)\hat{a}_g+\mathbb{E}\left[f(k)-q_k k-\phi\right]\right\}, \tag{2}$$

where $i\equiv(1+\pi)(1+\rho)-1$ and (k,ϕ) is a function of $(\hat{a}_m,\hat{a}_g)$. If bonds are not pledgeable, which is the case throughout most of the paper, then (k,ϕ) is independent of $\hat{a}_g$ and (2) implies $i_g=i$, i.e., i is the nominal rate on illiquid bonds.

The value function of a supplier with financial wealth ω in stage 2 is

$$W^s(\omega)=\omega+\max_{\hat{a}_m,\hat{a}_g\geq 0}\left\{-\frac{\hat{a}_m}{1+r_m}-\frac{\hat{a}_g}{1+r_g}+\beta V^s(\hat{a}_m,\hat{a}_g)\right\}. \tag{3}$$

In the capital market in stage 1,

$$V^s(\hat{a}_m,\hat{a}_g)=\max_{k\geq 0}\left\{-k+W^s(\hat{a}_m+\hat{a}_g+q_k k)\right\}.$$

Thus, he produces k units of capital at a unit cost and sells them at price q_k so that his wealth increases by $q_k k$. Using the linearity of W^s, if the capital market is active then $q_k=1$ and $V^s(\hat{a}_m,\hat{a}_g)=W^s(\hat{a}_m+\hat{a}_g)$. Moreover, his portfolio problem is simply $\max_{\hat{a}_m,\hat{a}_g}\left\{-i\hat{a}_m-(i-i_g)\,\hat{a}_g/(1+i_g)\right\}$. Given $i\geq 0$ and $i_g\leq i$, suppliers have no strict incentive to hold cash or bonds.

Finally, the stage-2 value function of a bank is $W^b(\omega)$, analogous to (3), and the stage-1 value function is $V^b(\hat{a}_m,\hat{a}_g)=\mathbb{E}W^b\left(\hat{a}_m+\hat{a}_g+\Pi\right)$ where Π are intra-period profits from bank intermediation activities.

4 External Finance

To begin we study nonmonetary economies or, equivalently, the financing problems of entrepreneurs without cash, in order to focus on external finance and the determination of lending rates.

4.1 Trade credit

Without banks, entrepreneurs must rely on trade credit, as in the left panel of Figure 4. Such credit is subject to $\psi = q_k k \le \chi_s f(k)$, since an entrepreneur cannot credibly pledge more than a fraction χ_s of his output. Hence, an entrepreneur with financial wealth ω solves

$$\max_{k,\psi} W^e(k, \omega - \psi) \text{ st } \psi = q_k k \le \chi_s f(k). \tag{4}$$

Using $q_k = 1$ and the linearity of W^e, this reduces to

$$\Delta(\chi_s) \equiv \max_{k \ge 0} \{f(k) - k\} \text{ st } k \le \chi_s f(k). \tag{5}$$

There are two cases. If $k \le \chi_s f(k)$ is slack, then $k = k^*$, where $f'(k^*) = 1$. This first-best outcome obtains when $\chi_s \ge \chi_s^* = k^*/f(k^*)$. If $k \le \chi_s f(k)$ binds, then k is the largest nonnegative solution to $\chi_s f(k) = k$. This second-best outcome obtains when $\chi_s < \chi_s^*$, and implies k is increasing in χ_s. We define the interest rate on trade credit as $r_s \equiv q_k - 1$. In the absence of discounting across stages, $r_s = 0$.

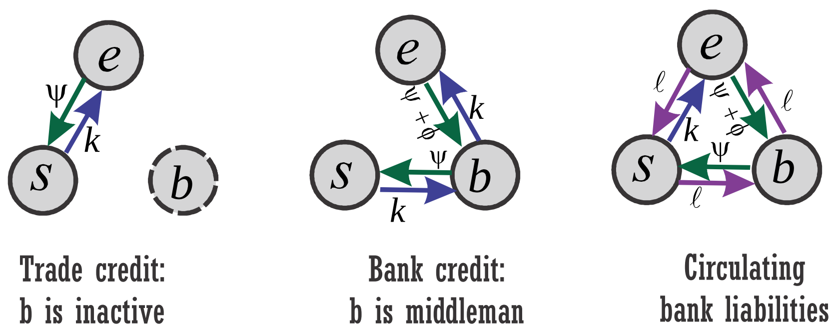

Figure 4: Transaction patterns

4.2 Bank credit

Now suppose trade credit is not viable – say, $\chi_s = 0$ – and consider banking. If an entrepreneur with an investment opportunity meets a bank, there are gains from trade, since banks can credibly promise payment to the supplier, and enforce payment from the entrepreneur up to the limit implied by χ_b. For this service, the bank charges the entrepreneur a fee, ϕ. Figure 4 shows two ways to achieve the same outcome. In the middle panel, the bank gets k from the supplier in exchange for a promise $\psi = q_k k$, then gives k to the entrepreneur in exchange for a promise $\psi + \phi$. In the right panel, the bank extends a loan to the entrepreneur by crediting his deposit account the amount ℓ, i.e., there is a swap between an illiquid loan and liquid short-term liabilities. Then the entrepreneur transfers his deposit claim to the supplier, who redeems it for ψ in stage 2, while the entrepreneur settles by returning $\psi + \phi$ to the bank. This arrangement uses deposit claims as inside money.[11]

A loan contract is a pair (ψ, ϕ), where $\psi = q_k k$. The terms are negotiated bilaterally, and if an agreement is reached, the entrepreneur's payoff is $W^e(k, \omega^e - \psi - \phi)$ while the bank's is $W^b(\omega^b + \phi)$. This implies individual surpluses

$$\begin{aligned} S^e &\equiv W^e(k, \omega^e - \psi - \phi) - W^e(0, \omega^e) = f(k) - \psi - \phi \\ S^b &\equiv W^b(\omega^b + \phi) - W^b(\omega^b) = \phi, \end{aligned}$$

and total surplus $S^e + S^b = f(k) - k$. One can check the maximum surplus a bank can get is $\chi_b f(\hat{k}) - \hat{k} \leq f(\hat{k}) - \hat{k}$, where $\hat{k}$ solves $\chi_b f'(\hat{k}) = 1$. Notice that k cannot be below $\hat{k}$, as then we could raise the surplus of both parties.[12]

The Nash bargaining solution, where $\theta \in (0, 1)$ is bank's share, is given by

$$(k, \phi) \in \arg\max \left[f(k) - k - \phi\right]^{1-\theta} \phi^\theta \quad \text{st} \quad k + \phi \leq \chi_b f(k). \tag{6}$$

[11] For some issues, the difference between the middle panel and right panel is not important, but there are scenarios where it might matter – e.g., if physical transfers of k are spatially or temporally separated, having a transferable asset can be essential, as in related models going back to Kiyotaki and Wright (1989). In Duffie et al. (2005), intermediaries (brokers) buy and sell assets on behalf of investors; there, the trading arrangement resembles the middle panel.

[12] The bargaining set is not convex, but that actually does not matter for generalized Nash bargaining. A Supplemental Appendix provides both a detailed characterization of the Pareto frontier and strategic foundations for Nash using an alternating offer bargaining game.

This leads to the following results (proofs of formal results are in the Appendix):

Proposition 1 *If* $\chi_b \geq \chi_b^* \equiv [(1-\theta)k^* + \theta f(k^*)] / f(k^*)$, *then the solution to (6) is*

$$\phi = \theta [f(k^*) - k^*] \tag{7}$$

$$k = k^*. \tag{8}$$

If $\chi_b < \chi_b^*$, *then the unique* $(\phi, k) \in \mathbb{R}_+ \times \left[\hat{k}, k^*\right]$ *solves*

$$\phi = \chi_b f(k) - k \tag{9}$$

$$\frac{k}{f(k)} = \frac{\chi_b f'(k) - \theta}{(1-\theta) f'(k)}. \tag{10}$$

If the constraint does not bind, ϕ is independent of χ_b, but increases with θ and $f(k^*) - k^*$. We define the lending rate as the payment to the bank divided by the loan size, $r_b = \phi/k$, which can be interpreted as an intermediation premium over the frictionless rate, $r_s = 0$, and is proportional to the average return,

$$r_b = \frac{\theta [f(k^*) - k^*]}{k^*}. \tag{11}$$

If $\chi_b < \chi_b^*$, the constraint binds and k is increasing in χ_b, with $k(0) = 0$ and $k(\chi_b^*) = k^*$. Also, $\partial k/\partial\theta < 0$ and $\partial\phi/\partial\theta > 0$, so banks with more bargaining power charge higher fees and make smaller loans. In this case, the lending rate is

$$r_b = \frac{\chi_b f(k)}{k} - 1 = \frac{\theta [1 - \chi_b f'(k)]}{\chi_b f'(k) - \theta}. \tag{12}$$

One can check $\partial r_b/\partial\theta > 0$, but $\partial r_b/\partial\chi_b$ is ambiguous – e.g., if $f(k) = zk^\gamma$, then $r_b = \theta(1-\gamma)/\gamma$ is independent of χ_b.

4.3 Trade and bank credit

Without a bank, an entrepreneur can pledge a fraction χ_s to a supplier; with a bank, he can pledge up to $\chi_b > \chi_s$. Bank credit is essential if $\chi_s < \chi_s^* = k^*/f(k^*)$, since then trade credit alone cannot implement the first best. In this case, a measure

$\lambda(1-\alpha)$ of investment projects are financed with only trade credit while $\lambda\alpha$ use bank credit. The loan contract solves the bargaining problem

$$\max_{k,\phi}\left[f(k)-k-\phi-\Delta(\chi_s)\right]^{1-\theta}\phi^{\theta}\text{ st } k+\phi\leq\chi_b f(k),$$

where $\Delta(\chi_s)$ is the entrepreneur's disagreement point.

If $\chi_b \geq \chi_b^* \equiv \left[(1-\theta)k^*+\theta f(k^*)-\theta\Delta(\chi_s)\right]/f(k^*)$, the solution is $k=k^*$ and $\phi=\theta\left[f(k^*)-k^*-\Delta(\chi_s)\right]$, and the bank lending rate is

$$r_b=\frac{\phi}{k^*}=\frac{\theta\left[f(k^*)-k^*-\Delta(\chi_s)\right]}{k^*}.$$

Notice $\partial r_b/\partial\chi_s<0$, and $r_b\to 0$ as $\chi_s\to\chi_s^*$.[13] Intuitively, the outside option of trade credit lets firms negotiate better terms. If $\chi_b<\chi_b^*$, then (k,ϕ) solve

$$\frac{(1-\chi_b)f'(k)}{(1-\chi_b)f(k)-\Delta(\chi_s)} = \frac{\theta}{1-\theta}\frac{1-\chi_b f'(k)}{\chi_b f(k)-k} \tag{13}$$

$$\phi = \chi_b f(k)-k. \tag{14}$$

There is a unique $k\in\left[\hat{k},k^*\right]$ solving (13), and it increases with χ_b and χ_s. Other implications can be derived, but the time has come to introduce money.

5 Internal Finance

Now let entrepreneurs accumulate cash in stage 2 to finance investments in the next stage 1. This is internal finance, defined as the use of retained earnings to pay for new capital, with the following features emphasized by Bernanke et al. (1996): it is an immediate funding source, has no explicit interest payments, and sidesteps the need for third parties. This allows us to study the transmission of monetary policy by which a change in the opportunity cost of holding cash, i, affects lending and investment. To ease the exposition, we first set $\chi_s=0$.

[13] We assume a banked entrepreneur obtains all financing from the bank, e.g., because these loans have higher seniority. Alternatively, the bank could provide a first loan of size $\ell=k^*-\chi_s f(k^*)$ that the entrepreneur could use to obtain a second loan directly from the supplier of size $k^*-\ell=\chi_s f(k^*)$. This does not affect the real allocation but changes the denominator of r_b.

5.1 Monetary equilibrium

Consider an entrepreneur in stage 1 with an investment opportunity but no access to banking. Then $k \le a_m^e$, where a_m^e is real money balances, and his profit is

$$\Delta^m(a_m^e) = f(k^m) - k^m \text{ where } k^m = \min\{a_m^e, k^*\}. \tag{15}$$

Notice $\Delta^m(a_m^e)$ is increasing and strictly concave for all $a_m^e < k^*$.

Consider next a banked entrepreneur, where loan contracts now specify an investment level k, a down payment d, and the bank's fee ϕ. If the loan negotiations are unsuccessful, the entrepreneur can purchase k with cash and get $\Delta^m(a_m^e)$, so his surplus from the loan is $f(k) - k - \phi - \Delta^m(a_m^e)$. The bargaining problem is

$$\max_{k,d,\phi} \left[f(k) - k - \phi - \Delta^m(a_m^e)\right]^{1-\theta} \phi^\theta \text{ st } k - d + \phi \le \chi_b f(k) \text{ and } d \le a_m^e. \tag{16}$$

With internal and external finance, what was previously called the pledgeability constraint is now called a liquidity constraint, reflecting credit plus cash. If the constraint does not bind, d is not uniquely determined, but k and ϕ are, so we select the solution with the highest d, i.e., $d = \min\{k^*, a_m^e\}$.

Lemma 1 *There exists a unique solution to (16) such that $d = \min\{k^*, a_m^e\}$ and it is continuous in a_m^e. There is $a^* < k^*$, where $a^* > 0$ if $\chi_b < \chi_b^*$, such that the following is true. If $a_m^e \ge a^*$ then the solution to (16) is*

$$k^c = k^* \tag{17}$$

$$\phi^c = \theta\left[f(k^*) - k^* - \Delta^m(a_m^e)\right]. \tag{18}$$

If $a_m^e < a^*$, $(\phi^c, k^c) \in \mathbb{R}_+ \times \left(\hat{k}, k^*\right)$ solves

$$\frac{a_m^e + \chi_b f(k^c) - k^c}{(1-\chi_b) f(k^c) - a_m^e - \Delta^m(a_m^e)} = \frac{\theta}{1-\theta} \frac{1 - \chi_b f'(k^c)}{(1-\chi_b) f'(k^c)} \tag{19}$$

$$k^c + \phi^c = a_m^e + \chi_b f(k^c). \tag{20}$$

If $a_m^e \ge a^*$, the liquidity constraint does not bind and $\partial\phi^c/\partial a_m^e < 0$, so by having more cash in hand, the entrepreneur reduces payments to the bank and increases profit. If $a_m^e < a^*$ and the liquidity constraint binds, $\partial k^c/\partial a_m^e > 0$. Hence, $\partial\left[a_m^e + \chi_b f(k^c)\right]/\partial a_m^e > 1$, which says that by accumulating a dollar, a firm raises its financing capacity by more than a dollar.

The lending rate, $r_b \equiv \phi^c/(k^c - a_m^e)$, also depends on the entrepreneur's cash position,

$$r_b = \begin{cases} \dfrac{\theta\left[f(k^*) - k^* - \Delta^m(a_m^e)\right]}{k^* - a_m^e} & \text{if } a_m^e \in [a^*, k^*) \\ \dfrac{\chi_b f(k^c)}{k^c - a_m^e} - 1 & \text{if } a_m^e < a^*. \end{cases} \tag{21}$$

In that case, $\partial r_b/\partial a_m^e < 0$ and $\lim_{a_m^e \nearrow k^*} r_b = 0$. The fact that r_b decreases with a_m^e is instrumental in creating pass through from the nominal policy rate to the real lending rate, as discussed below.

Lemma 2 *The entrepreneur's choice of money balances is a solution to*

$$\max_{a_m^e \geq 0}\left\{-ia_m^e + \lambda\left(1-\alpha\right)\Delta^m(a_m^e) + \lambda\alpha\Delta^c(a_m^e)\right\}, \tag{22}$$

where $\Delta^c(a_m^e) \equiv f(k^c) - k^c - \phi^c$ can be written as follows:

$$\Delta^c(a_m^e) = \begin{cases} (1-\theta)\left[f(k^*) - k^*\right] + \theta\Delta^m(a_m^e) & \textit{if } \ a_m^e \geq a^* \\ (1-\chi_b) f\left[k^c(a_m^e)\right] - a_m^e & \textit{if } a_m^e < a^*. \end{cases}$$

From (22), the entrepreneur maximizes his expected profits from an investment opportunity net of the cost of holding money. The profits of a banked investor are $\Delta^c(a_m^e)$. If $a_m^e \geq k^*$, the banked entrepreneur finances k^* without credit, so $\Delta^c(a_m^e) = f(k^*) - k^*$. If $a_m^e \in [a^*, k^*)$, the entrepreneur can still finance k^*, but only by using bank credit as well as cash, and the bank captures a fraction θ of the surplus. Now $\Delta^c(a_m^e)$ increases with a_m^e. If $a_m^e < a^*$, the liquidity constraint binds and the entrepreneur's surplus equals his nonpledgeable output net of real balances.[14] An equilibrium with internal and external finance is a list (k^m, k^c, a_m^e, r_b) solving (15), (16), (21), (22), $a_m^e > 0$ and $k^c - a_m^e > 0$.

[14]Under Nash bargaining, $\Delta^c(a_m^e)$ can be nonmonotone when the liquidity constraint binds, which can possibly lead to a solution that is discontinuous in parameters.

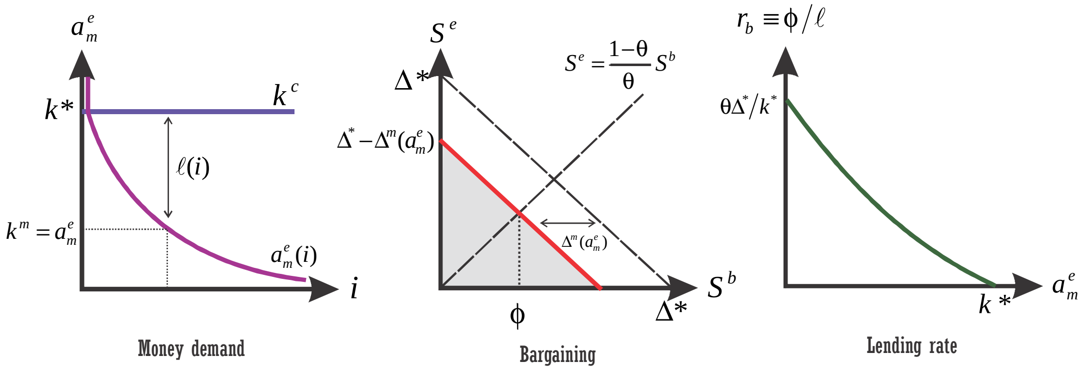

Figure 5: Determination of equilibrium when $\chi_b = 1$

Proposition 2 *For all $i > 0$ and $\chi_b > 0$, if $\lambda(1-\alpha) > 0$ or $\lambda\alpha\theta > 0$ then there exists an equilibrium where internal and external finance coexist. Equilibrium is unique if $\chi_b \geq \chi_b^*$, $\theta \in \{0, 1\}$, or i is small; even without these conditions, it is unique for generic parameters.*

Proposition 2 says fiat money is valued if some entrepreneurs are unbanked, $\alpha < 1$, or banks have some bargaining power, $\theta > 0$. As long as $i > 0$, entrepreneurs are not perfectly liquid with respect to investment opportunities, and then money and bank credit coexist. Figure 5 illustrates the determination of equilibrium when $\chi_b = 1$. The left panel shows money demand, $a_m^e(i)$, and the demand for bank loans, $\ell(i)$. Given a_m^e, the middle panel represents the negotiation over ϕ, where $\Delta^* = f(k^*) - k^*$. Note the Pareto frontier shifts inward as entrepreneurs hold more real balances. Since output is fully pledgeable, the Pareto frontier is linear and the bank gets a constant share θ of the surplus $\Delta^* - \Delta^m(a_m^e)$. Given ϕ and ℓ, the right panel determines the lending rate.

5.2 The transmission mechanism

To characterize the determinants of pass through from the nominal policy rate to the real lending rate, first, consider equilibria where $k^c = k^*$, which obtains if $\chi_b \geq \chi_b^*$, or if i is low. The FOC associated with (22) is

$$i = \lambda \left[1 - \alpha(1-\theta)\right] \left[f'(k^m) - 1\right]. \tag{23}$$

Not surprisingly, $\partial k^m / \partial i < 0$ and $\partial k^m / \partial \lambda > 0$. A more novel feature of firms' money demand is how it depends on credit market frictions, $\partial k^m / \partial \alpha < 0$ and $\partial k^m / \partial \theta > 0$. As bank credit becomes less readily available, or more expensive because banks have more bargaining power, entrepreneurs hold more cash. This is true even if they have access to bank loans with certainty, $\alpha = 1$, since it reduces the rent captured by banks when $\theta > 0$, a strategic motive for holding cash not in other models.

The real lending rate in this case, where $k^c = k^*$, is

$$r_b = \frac{\theta \left\{\left[f(k^*) - k^*\right] - \left[f(k^m) - k^m\right]\right\}}{k^* - k^m}, \tag{24}$$

implying $\partial r_b / \partial i > 0$. This is a key implication of the theory: the policy rate i affects firms' internal funds and hence the bargaining solution, including the real rate r_b.

Proposition 3 *When $i/\lambda \left[1 - \alpha(1-\theta)\right]$ is small, the pass through from i to r_b is approximated by*

$$r_b \approx \frac{\theta i}{2\lambda \left[1 - \alpha(1-\theta)\right]}. \tag{25}$$

This shows there is pass through from i to r_b given $\theta > 0$. Changes in θ have two effects: a direct effect, as r_b increases with θ for a given a_m^e; and an indirect effect, as entrepreneurs hold more real balances to reduce payments to banks.[15] From (25), the first effect dominates. As α increases, entrepreneurs reduce money balances, but this weakens their outside option and allows banks higher r_b. Notice pass through is positive even without search, $\alpha = 1$. In this case, an increase in i raises r_b but does not affect aggregate investment, which is at its first-best level; instead it merely alters the corporate finance mix and redistributes profit from firms to banks. Finally, higher λ lowers pass through by raising money demand. If λ varies over the

[15] The first effect is analogous to the bid-ask spread increasing with dealers' bargaining power in Duffie et al. (2005); the second corresponds to the portfolio response in the generalization of that model by Lagos and Rocheteau (2009).

business cycle, e.g., recessions are associated with fewer opportunities to invest, and our model predicts asymmetric pass through in expansions and recessions.

Proposition 4 *For low $i/\lambda\left[1-\alpha(1-\theta)\right]$, the transmission to investment is approximated by:*

$$k^m \approx k^* + \frac{i}{f''(k^*)\lambda\left[1-\alpha(1-\theta)\right]} \tag{26}$$

$$k^c = k^* \tag{27}$$

$$K \approx \lambda k^* + \frac{(1-\alpha)i}{f''(k^*)\left[1-\alpha(1-\theta)\right]} \tag{28}$$

where $K \equiv \lambda\left[\alpha k^c + (1-\alpha)k^m\right]$ is aggregate investment. The effect on aggregate lending, $L \equiv \alpha\lambda\left(k^* - a_m^e\right)$, is

$$L \approx \frac{-\alpha i}{f''(k^*)\left[1-\alpha(1-\theta)\right]}. \tag{29}$$

According to (28), aggregate investment decreases with the nominal rate, consistent with textbook discussions. However, the transmission mechanism here comes from explicit frictions, including financial considerations, and not nominal rigidities. Also, the strength of the mechanism depends on characteristics of credit markets – e.g., $|\partial K/\partial i|$ is larger when α and θ are lower. Thus, a decrease in α changes the mix between k^c and k^m, so more investment is financed with cash. This effect strengthens the transmission of i to K, because k^m depends on i, while k^c does not when the liquidity constraint is slack. Lower α also makes k^m less sensitive to changes in i, which tends to weaken transmission, though one can show the first effect dominates. Transmission mechanism is weaker when banks have more bargaining power because this leads entrepreneurs to hold more cash and makes money demand less elastic.

The next result is that a change in fundamentals that increases pass through does not necessarily imply a stronger transmission mechanism.

Corollary 1 *Suppose i is close to 0 and $\alpha < 1$. An increase in α or θ raises $\partial r_b/\partial i$ but reduces $|\partial K/\partial i|$. An increase in λ reduces $\partial r_b/\partial i$ and has no effect on $|\partial K/\partial i|$.*

Consider next the case where the liquidity constraint binds, $k^c < k^*$. Suppose for now that $\theta = 0$, so k^c solves $a_m^e + \chi_b f(k^c) = k^c$ (we consider $\theta > 0$ in Section 6). Then

$$\frac{\partial \Delta^c(a_m^e)}{\partial a_m^e} = \frac{f'(k^c) - 1}{1 - \chi_b f'(k^c)}. \tag{30}$$

By financing an additional unit of k the entrepreneur raises his surplus by $f'(k^c) - 1$. The denominator in (30) is a financing multiplier that says one unit of k raises pledgeable output by $\chi_b f'(k^c)$, thereby increasing entrepreneurs' financing capacity. From (22), the choice of a_m^e implies

$$(1-\alpha) f'(k^m) + \alpha \frac{(1-\chi_b) f'(k^c)}{1 - \chi_b f'(k^c)} = 1 + \frac{i}{\lambda}. \tag{31}$$

This has a unique solution, and $\partial a_m^e / \partial i < 0$. Letting $\bar{k} \equiv k^* - \chi_b f(k^*)$ and $\bar{\imath} \equiv \lambda (1-\alpha) \left[f'(\bar{k}) - 1 \right]$, we have:

Proposition 5 *Assume $\theta = 0$ and $\chi_b < \chi_b^*$. For all $i > \bar{\imath}$ the liquidity constraint binds. For $i - \bar{\imath} > 0$ but small,*

$$k^c - k^* \approx \frac{k^m - \bar{k}}{1 - \chi_b} \approx \frac{(i - \bar{\imath})}{D} \tag{32}$$

where $D < 0$. Aggregate lending is

$$L \approx \alpha \lambda \left[k^* - \bar{k} + \frac{\chi_b (i - \bar{\imath})}{D} \right]. \tag{33}$$

From Propositions 3 and 5, transmission changes qualitatively as i increases above $\bar{\imath}$. For low i policy affects internally- but not bank-financed investment. As i rises above $\bar{\imath}$ the pledgeability constraint binds and banked-financed investment decreases by more than internally-financed investment, as determined by the multiplier $(1-\chi_b)^{-1}$. From (33), aggregate L decreases with i as entrepreneurs economize on cash, reducing pledgeable output and loan size. In the special case $\alpha = 1$, $\bar{\imath} = 0$ and the liquidity constraint binds for all $i > 0$, given $\chi_b < \chi_b^*$. From (31) k^c solves $f'(k^c) = (i + \lambda) / (\lambda + i\chi_b)$. So, $\partial k^c / \partial i|_{i=0^+} = (1 - \chi_b)/\lambda f''(k^*)$, and investment is more responsive to policy as χ_b decreases. We summarize all this below:

Corollary 2 *Assume $\theta = 0$ and $\chi_b < \chi_b^*$. For all $i < \bar{\imath}$, $\partial k^m / \partial i < \partial k^c / \partial i = 0$ and $\partial L / \partial i > 0$. For all $i > \bar{\imath}$, $\partial k^c / \partial i < \partial k^m / \partial i < 0$ and $\partial L / \partial i < 0$.*

5.3 Trade credit and corporate bonds

If we re-introduce trade credit with $\chi_s > 0$, the surplus of an unbanked entrepreneur, $\Delta^m(a_m^e; \chi_s)$, is given by (15) except now $k^m \leq a_m^e + \chi_s f(k^m)$. Emulating the above logic, $\partial\phi/\partial i$ decreases with χ_s, showing lower pass through from i to banks' interest margin. Moreover, for low i,

$$k^m \approx \frac{(1-\chi_s)\, i}{\lambda\,[1-\alpha(1-\theta)]\, f''(k^*)} + k^*.$$

As χ_s increases, the transmission mechanism weakens.

Alternatively, suppose $\chi_s = 0$ but entrepreneurs can borrow and lend money in a competitive market after the realization of the investment and banking shocks.[16] Corporate bonds are repaid in stage 2 with yield $i_e \geq 0$. For simplicity, assume entrepreneurs can pledge their full output to either banks or other entrepreneurs, but not suppliers. The surplus for an unbanked entrepreneur is $\Delta^m = \max_k \{f(k) - (1+i_e)k\}$, which implies $f'(k^m) = 1 + i_e$, and his net debt is $k^m - a_m^e$. The bargaining problem between the bank and the entrepreneur is

$$\max_{k,\ell,\phi} [f(k) - k - \phi - (k-\ell)i_e - \Delta^m]^{1-\theta}\, \phi^\theta,$$

where $\ell \leq k$ is the bank loan and $k - \ell$ is the amount borrowed in the corporate bond market.

For all $i_e > 0$ the solution is $k = \ell = k^*$ and $\phi = \theta\,[f(k^*) - k^* - \Delta^m]$. Banks finance all the investment of matched entrepreneurs so their cash can be lent to the unbanked. Entrepreneurs are indifferent about carrying money to the corporate bond market if $i_e = i$ and market clearing requires $a_m^e = \lambda(1-\alpha)k^m$. Hence there is full pass through from the policy rate to the corporate bond rate. By reallocating liquidity across investors the corporate bonds market raises k^m and reduces ϕ. So, a common theme of this section is that the presence of alternative sources of external financing raises investment by unbanked entrepreneurs, reduces interest payments

[16] The idea of allowing agents to reallocate liquidity after the idiosyncratic risk, which was suggested by a referee, follows Berentsen et al. (2007). One can also enrich the model by introducing limited commitment or participation in the corporate bond market.

to banks, and makes aggregate investment less sensitive to changes in monetary policy.

5.4 Structure of interest rates

We now assume that government bonds are partially liquid: an entrepreneur with a portfolio (a_m^e, a_g^e) in stage 1 can trade up to $a_m^e + \chi_g a_g^e$ in exchange for k where $\chi_g \in [0, 1]$. Moreover, to make the lending rate comparable to the interest rate on one-period bonds, consider bank loan contracts with one period maturity, motivated by assuming investment opportunities in period t pay off in $t + 1$.[17]

The real return of an illiquid bond (i.e., one with $\chi_g = 0$) is ρ, which one might call the natural real interest rate in a frictionless economy. The spreads between illiquid bonds, liquid bonds, and money are given by

$$\frac{1}{\chi_g}\frac{\rho - r_g}{1 + r_g} = \frac{\rho - r_m}{1 + r_m} = i. \tag{34}$$

The spread between illiquid and liquid bonds is $\chi_g i$, which is increasing with bond pledgeability. The spread between the real rate on a one-period bank loan and the rate of time preference is

$$\frac{r_b - \rho}{1 + \rho} \approx \frac{\theta i}{2\lambda\left[1 - \alpha(1 - \theta)\right]}, \tag{35}$$

where the RHS is identical to (25).

The real rates on money and liquid bonds are less than the natural rate, $r_m \leq r_g \leq \rho$, the differences representing liquidity premia on assets that serves as financing instruments. In contrast, the real rate on a bank loan is greater than the natural rate, $r_b > \rho$, the difference being an intermediation premium due to banks swap illiquid entrepreneurs' IOUs for liquid bank liabilities. The effects of a change in i on the distribution of returns are summarized as follows:

[17] We sketch the derivations here as they are similar to those above; details are in the online appendix.

Proposition 6 *An increase in i reduces the real returns of monetary assets, r_m and r_g, for all $\chi_g > 0$; it does not affect the real return on illiquid bonds, $r_g = \rho$ if $\chi_g = 0$; it raises the real lending rate, r_b.*

This illustrates a key difference between our model and those where claims on capital can serve as media of exchange (e.g., Lagos and Rocheteau 2008). In those models, an increase in i raises the liquidity premium of claims on capital, which raises k through a Tobin effect and reduces $f'(k)$. Here i raises the intermediation premium on bank loans, which reduces k when the liquidity constraint binds.[18]

6 Calibrated Examples

We now calibrate a simple version of the model using annual U.S. data between 1958 and 2007. The production function is $f(k) = k^{\gamma}$ with $\gamma = 1/3$. We interpret i as the 3-month T-bill secondary market rate, which averages 5.4% per year over the sample. The associated real rate is $\rho = 2\%$ (computed using the method in Hamilton et al. 2015). To focus on bank credit, as a benchmark, set $\chi_s = 0$ and $\chi_b = 1$.[19] We determine later the minimum value of χ_b consistent with a non-binding liquidity constraint and explore variation around that value.

The semi-elasticity of money demand is a key target for calibration:

$$\frac{\partial \log(k^m)}{\partial i} = \frac{1}{(\gamma - 1)\left\{\lambda\left[1 - \alpha(1-\theta)\right] + i\right\}}.$$

We use Lucas's (2000) estimate of -7 from aggregate data, consistent with the estimate using data for firms in Figure 2. This implies $\lambda\left[1 - \alpha(1-\theta)\right] = 0.16$. To obtain bargaining power θ, we target the average difference between the real prime

[18] As suggested by a referee, we could easily extend our model to have both types of financing as follows. A fraction of firms issue one-period corporate bonds that are partially pledgeable. Their rate of return, which includes a liquidity premium, is determined as in (34). Other firms do not have access to the corporate bond market and hence rely on bank credit.

[19] One could think of this version as one where some investment opportunities can only be financed with bank credit whereas others can be financed with trade credit or other forms of external finance under perfect enforcement. The second type of investment would not affect money demand or bank credit.

Table 1: Parameter Values

Parameter	Value	Target
γ	0.33	Fixed
i	0.054	3 month T-bill rate (nominal)
ρ	0.02	3 month T-bill rate (real)
θ	0.16	real lending rate
α	0.9	loan application acceptance rate
λ	0.66	semi-elasticity of money demand

rate and the real 3-month T-bill rate in Figure 1. We interpret this spread, which is 2.4% over the sample period, as r_b in the model.[20] From (24), $\theta = 0.16$.

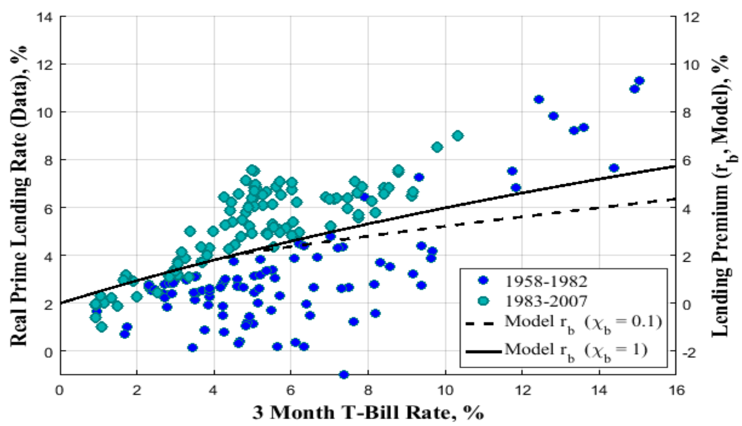

Figure 6: Interest Rate Pass Through, 1958 to 2007

We interpret α as the probability of a loan application being accepted – i.e., it is not that it is hard to find a bank, but it may be hard to find one willing to finance your project, similar to the way some people interpret job search. The 2003 Survey of Small Business Finances reports the fraction of firms with their most recent loan application accepted is 0.9. Hence, $\alpha = 0.9$, which gives $\lambda = 0.16/\left[1 - \alpha(1-\theta)\right] = 0.66$. Finally, we check that at the average i of 5.4%, the liquidity constraint does not bind for $\chi_b \geq 0.12$.

[20] The prime rate is a base interest rate set by commercial banks for many types of loans, including loans to small businesses and credit card loans. It is published by the Federal Reserve H.15 statistical release.

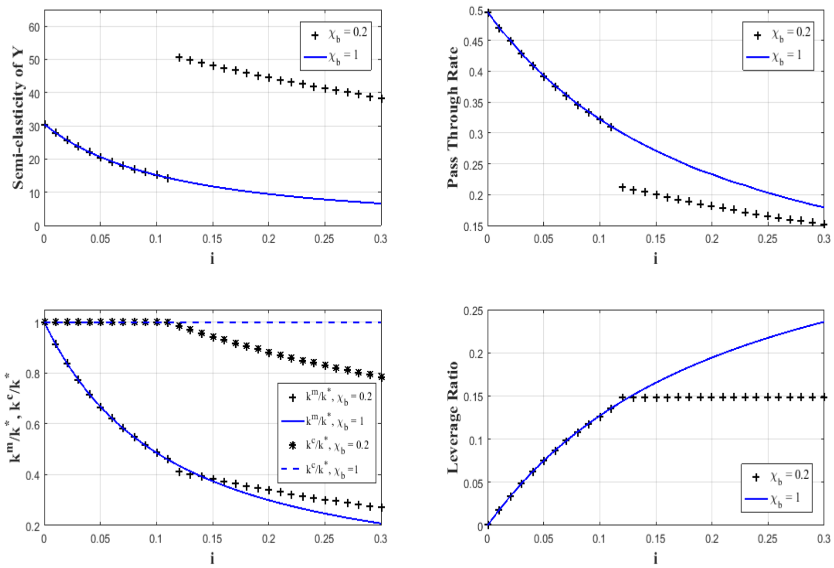

Figure 7: Monetary Policy Transmission for Different χ_b

Figure 6 shows the real lending rate from the model for two values of χ_b (right axis) and the data (left axis). In accordance with (35), the difference between the two axes is $\rho = 2\%$. The higher pass through in the second half of the sample can be explained with lower search frictions in the credit market or a higher bargaining power of banks. This is consistent with Adão and Silva (2016), who find the impact of monetary policy on real interest rates has increased from 1980 to 2013.

Figure 7 illustrates transmission. In the top left panel, we compute the absolute value of the semi-elasticity of output, given by $Y \equiv \lambda[\alpha f(k^c) + (1-\alpha) f(k^m)]$. For $i = 5.4\%$, the output semi-elasticity is about 20, which means a one percentage point increase in i reduces aggregate output by about 0.2%. When $\chi_b = 0.2$, the liquidity constraint binds for all $i > \bar{\imath} = 11\%$, in which case semi-elasticity rises from about 10 to 50 at $i = \bar{\imath}$. So a binding liquidity constraint entails powerful amplification of monetary policy.[21] The top right panel shows the pass through rate decreases with i, with a discrete jump when the liquidity constraint binds.

[21] These magnitudes are within the range found by Dedola and Lippi (2005) and Barth and Ramey (2002).

The bottom left panel shows k^m/k^* and k^c/k^* fall with i, with k^c/k^* being steeper than k^m/k^* when $i > \bar{\imath}$, in accordance with Corollary 2. The bottom right panel plots the average leverage ratio across firms,

$$\Lambda = \frac{\alpha\lambda\left(k^c - k^m + \phi\right)}{f(k^c) - (k^c - k^m + \phi)}$$

where the numerator is debt including interest, and the denominator is equity measured as output net of debt. Leverage increases with i since entrepreneurs hold less cash and ask for bigger loans when i is high, and is constant at $\chi_b/(1-\chi_b)$ when the liquidity constraint binds.

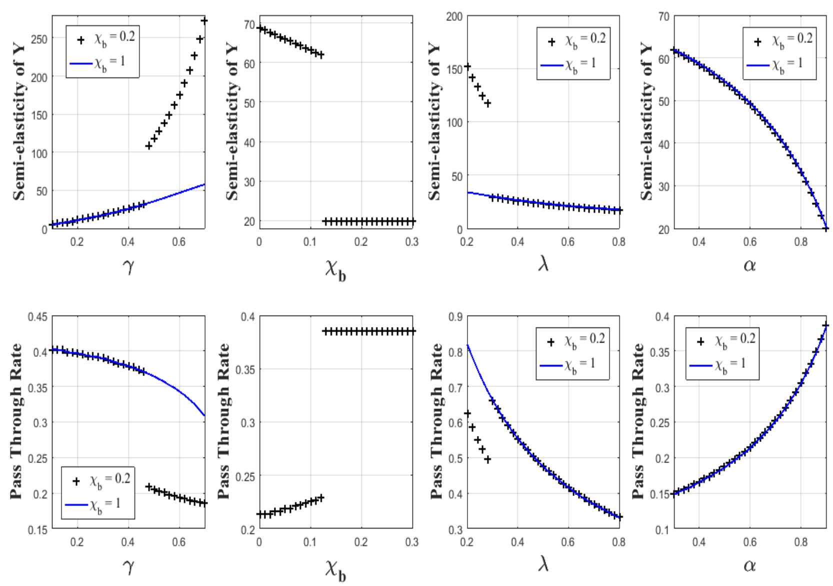

Figure 8: Output Semi-Elasticity and Pass Through

One can introduce heterogeneity across firms to ask how different industries respond to policy. The top panels of Figure 8 show the output semi-elasticity against four firm characteristics: output elasticity, γ; pledgeability, χ_b; frequency of investment opportunities, λ; and access to banks, α. As shown, firms or industries with higher γ, and lower χ_b, λ, or α, are more sensitive to changes in i. Notice γ gives the largest variation, as the output semi-elasticity goes from 0 to 250 – i.e., a one percentage point increase in i can reduce Y up to 2.5%. This range is broadly consistent with those found in the literature (see Dedola and Lippi, 2005). The bottom rows shows pass through against the same firm characteristics. The correlation between pass through and output semi-elasticity is negative when firms differ with respect to γ, χ_b or α, and positive when they differ by λ.

While more could be done quantitatively, these exercises illustrate how the theory is basically consistent with the evidence, and provides an indication of the size of the effects. Given this we proceed to extensions of the baseline theory to make it more realistic and policy relevant.

7 An Interbank Market

We showed that for low interest rates, a change in i only affects investment by unbanked entrepreneurs, which makes monetary policy less potent when frictions in the credit market vanish. Here we show that the potency of policy can be restored by adding a reserve requirement, whereby a fraction $\nu \in [0,1]$ of bank liabilities must be backed by fiat money.[22] Hence, a bank that issues ℓ in deposit claims must hold $\nu\ell$ in real balances until the claims are redeemed in stage 2. We also open an interbank market for short-term loans of reserves in stage 1, after shocks and meetings are realized, where policy can intervene to affect the interest rate on these loans, i_f.

7.1 The interbank rate

Assuming $f'(a_m^e) \geq 1 + \nu i_f$, so there are gains from trade, loan contracts solve a bargaining problem similar to (16), where the bank surplus is $\Pi = \phi - \nu i_f (k - d)$. We focus on equilibria where the liquidity constraint does not bind (e.g., $\chi_b = 1$). In this case (k^c, ϕ^c) solves

$$
\begin{aligned}
f'(k^c) &= 1 + \nu i_f, & (36)\\
\phi^c &= (1-\theta)\nu i_f \left(k^c - a_m^e\right) + \theta \left[f(k^c) - k^c - \Delta^m(a_m^e)\right]. & (37)
\end{aligned}
$$

[22] There is no claim such regulations are part of an optimal arrangement; we take them as given. For related formalizations see, e.g., Gomis-Porqueras (2002) or Bech and Monnet (2015).

There are two novelties. First, $\nu i_f > 0$ acts as a tax on bank-financed investment. Second, it raises ϕ^c as long as $\theta < 1$. Dividing (37) by loan size,

$$r_b = (1-\theta)\nu i_f + \theta \left[\frac{f(k^c) - k^c - \Delta^m(a_m^e)}{k^c - a_m^e} \right]. \tag{38}$$

The first component of the lending rate is the cost due to the reserve requirement; the second reflects the bank's surplus.

Entrepreneurs' demand for real balances solves a generalized version of (23),

$$f'(k^m) = 1 + \frac{i - \alpha\lambda(1-\theta)\nu i_f}{\lambda\left[1-\alpha(1-\theta)\right]}. \tag{39}$$

Now $f'(k^m) > 1 + \nu i_f$ holds if $i/\lambda > \nu i_f$, i.e., loans are useful if the average cost of holding real balances is larger than the regulatory cost from issuing bank liabilities. Relative to (23), the term $\alpha\lambda(1-\theta)\nu i_f$ means the reserve requirement raises the bank's cost of issuing liabilities, part of which is passed on to the entrepreneur.

The aggregate demand for reserves is $\alpha\lambda\nu\left(k^c - k^m\right)$, which decreases with i_f. Banks supply $\hat{a}_m^b$ to maximize $-(1+\pi)\hat{a}_m^b + \beta(1+i_f)\hat{a}_m^b = (i_f - i)\,\beta\hat{a}_m^b$. Without intervention by policy, $i_f = i$. Alternatively, the monetary authority can set $i_f < i$, in which case it supplies all of the loans.

Proposition 7 *For small $i/\lambda\left[1-\alpha(1-\theta)\right]$ and $i > \lambda\nu i_f$,*

$$r_b \approx \nu i_f + \frac{\theta\left(i - \lambda\nu i_f\right)}{2\lambda\left[1-\alpha(1-\theta)\right]}. \tag{40}$$

According to (40), the real lending rate depends on the structure of the nominal rates i and i_f. Each rate has a distinct pass through that depends on regulation and the structure of the credit market. The interbank rate has high pass through relative to the bond rate when bank's bargaining power is low, search frictions in the credit market are high, and the reserve ratio is high. If the monetary authority does not intervene in the interbank market then $i_f = i$, and

$$r_b \approx \left\{ \nu + \frac{\theta\left(1-\lambda\nu\right)}{2\lambda\left[1-\alpha(1-\theta)\right]} \right\} i,$$

so pass through increases with ν.

Proposition 8 *For small* $i/\lambda\left[1-\alpha(1-\theta)\right]$ *and* $i > \lambda\nu i_f$,

$$k^m \approx k^* + \frac{i - \alpha\lambda(1-\theta)\nu i_f}{f''(k^*)\lambda\left[1-\alpha(1-\theta)\right]} \quad (41)$$

$$k^c \approx k^* + \frac{\nu i_f}{f''(k^*)} \quad (42)$$

$$K \approx \lambda k^* + \frac{(1-\alpha)i + \alpha\lambda\theta\nu i_f}{f''(k^*)\left[1-\alpha(1-\theta)\right]}. \quad (43)$$

Moreover, $|\partial K/\partial i_f| > |\partial K/\partial i|$ *if* $\alpha/(1-\alpha) > (\nu\lambda\theta)^{-1}$.

Policy can use different interest rates to target investment according to financing methods. The interbank rate negatively affects bank-financed investment, while the bond rate negatively affects internally-financed investment. Notice k^m increases with i_f since a higher cost of bank credit raises entrepreneurs' demand for cash. If frictions in the credit market are small, i.e., α is close to 1, a change of the interbank rate in the presence of a small reserve requirement has a larger effect on aggregate investment than a change in the bond rate of the same magnitude.

7.2 Open market operations

We now describe the effects of unanticipated open market operations, or OMOs, in the interbank market.[23] If agents anticipate no intervention then $i_f = i$. Moreover, $\lambda\nu < 1$ so there is a demand for bank loans and reserves. Consider a one-time, unanticipated increase in the money supply by μM, where $\mu > 0$ is small, engineered by buying government bonds held by banks. Since bonds have no regulatory role, in this case, only the change in M is relevant.

Suppose the economy returns to steady state in stage 2 with q_m scaled down by $1+\mu$. As a result, we have $a_m^{e\prime} = a_m^e/(1+\mu)$, where a prime denotes a variable at the time of the OMO, and $a_m^{e\prime} + \hat{A}_m^{b\prime} = a_m^e + \hat{A}_m^b$. Hence, the change in the supply of reserves in the interbank market is $\hat{A}_m^{b\prime} - \hat{A}_m^b = \mu a_m^e/(1+\mu)$, while the change in

[23] Williamson (2012) and Rocheteau et al. (2015) study OMOs in great detail in models of consumer finance.

demand is $\alpha\lambda\nu\left[k^{c\prime} - k^c + \mu a_m^e/(1+\mu)\right]$. Market clearing implies

$$k^{c\prime} - k^c = \frac{(1-\alpha\lambda\nu)\,\mu}{\alpha\lambda\nu(1+\mu)} a_m^e. \tag{44}$$

By redistributing liquidity from entrepreneurs to banks, the OMO raises k^c but reduces k^m.

Proposition 9 *Assume small $i/\lambda\left[1-\alpha(1-\theta)\right]$ and $i > \lambda\nu i_f$. Consider an unanticipated injection of $\mu M > 0$ reserves in the interbank market. For small μ, the change in the interbank rate is*

$$i_f' - i_f \approx \frac{f''(k^*)\,(1-\alpha\lambda\nu)\,\mu}{\alpha\lambda\nu^2(1+\mu)} a_m^e. \tag{45}$$

It is passed to the real lending rate according to

$$r_b' - r_b \approx \left[1 - \frac{\theta}{2\,(1-\alpha\lambda\nu)}\right] \nu\left(i_f' - i_f\right). \tag{46}$$

The overall change in investment is

$$K' - K = \left(\frac{1-\lambda\nu}{\nu}\right) \frac{\mu}{1+\mu} a_m^e. \tag{47}$$

From (45) the OMO reduces the cost of borrowing reserves, $i_f' < i_f$, thus providing incentives for banks to extend loans. Perhaps surprisingly, pass through is negative if $\theta > 2\,(1-\alpha\lambda\nu)$. This is because the money injection reduces entrepreneurs' real balances, thereby weakening their bargaining position. From (47) the change in aggregate investment is positive if $\lambda\nu < 1$. Intuitively, money is more effective at financing investment when held by banks, because they can leverage liquid assets, by issuing liabilities, and through the interbank market can reallocate liquidity towards banks with lending opportunities.

8 Bank Entry

We now endogenize the measure of banks, b, and hence access to loans. The matching probability for an entrepreneur is $\alpha(b)$, and for a bank is $\alpha(b)/b$. As standard, $\alpha(b)$ is increasing and concave, with $\alpha(0) = 0$, $\alpha(\infty) = 1$, and $\alpha'(0) = 1$. We justify the role of banks when i approaches 0 by assuming a fraction n of entrepreneurs can access the second-stage market for real balances (as in Section 5), while the remaining $1-n$ cannot (as in Section 4).[24] Hence, there is a nondegenerate distribution of real balances among entrepreneurs at the beginning of each period: a fraction n hold a_m^e and $1-n$ hold 0.

In order to participate in the credit market at stage 1, banks must incur a cost $\bar{\varsigma} > 0$, and are required to hold $\bar{a}_g > 0$ worth of government bonds purchased in stage 2. This assumption captures liquidity coverage ratios or capital requirements imposed on banks. During the free-banking era, e.g., banks had to buy eligible government bonds to deposit with the state authority. Since 2014, regulations in Basel III require banks to hold a minimum level of liquid assets.

The effective entry cost of banks is

$$\varsigma = \bar{\varsigma} + \bar{a}_g \left[(1+\rho)q_g - 1\right] = \bar{\varsigma} + \bar{a}_g \left(\frac{i-i_g}{1+i_g}\right). \tag{48}$$

The second term in (48) is the spread between illiquid and liquid bonds, as derived in Section 3.[25] The expected interest payment received by a bank is $\Phi = n\phi_1 + (1-n)\phi_0$, where ϕ_0 is given by (6) and ϕ_1 is given by (16). Hence, the payoff of an entering bank is $V^b = -\varsigma + \lambda\Phi\alpha(b)/b + \beta\max\{V^b, 0\}$, and free entry means $V^b \leq 0$, or $-\varsigma + \lambda\Phi\alpha(b)/b \leq 0$, with an equality if $b > 0$. A positive solution exists if $\lambda\Phi > \varsigma$.

Assume $\chi_b \geq \chi_b^*$ so that $k^c = k^*$. Then equilibrium is a list (a_m^e, b, i_g) solving

$$-\bar{\varsigma} - \bar{a}_g \left(\frac{i-i_g}{1+i_g}\right) + \frac{\alpha(b)}{b}\lambda\theta\left[f(k^*) - k^* - n\Delta^m(a_m^e)\right] \leq 0, \;\; \text{"="} \text{ if } b > 0 \tag{49}$$

$$f'(a_m^e) = 1 + \frac{i}{\lambda\left[1-\alpha(b)(1-\theta)\right]} \tag{50}$$

$$b\bar{a}_g \leq A_g, \quad \text{"="} \;\text{ if } i_g < i. \tag{51}$$

[24] For instance, a fraction $1-n$ of entrepreneurs are born at the beginning of the period, and therefore cannot accumulate real balances before they enter the capital market (related to a specification in Williamson 2012).

[25] Wasmer and Weil (2004) conjecture higher search costs for banks could be induced by tighter monetary policy; (48) confirms this by showing that what matters for bank entry is the spread, $i - i_g$, which can be affected by OMOs.

These conditions correspond to free entry, money demand, and market clearing. The left panel of Figure 9 depicts equilibrium absent regulation, $\bar{a}_g = 0$. The curve BE corresponds to (49), and is downward sloping because banks' profits fall when entrepreneurs hold more cash. The curve MD corresponds to (50) represented in (b, a_m^e) space. It slopes downward since the probability of being unbanked falls with b, which reduces entrepreneurs' demand for money. At the Friedman rule, $i = 0$, MD is horizontal and $b = b^*$ solves $b^*/\alpha(b^*) = \lambda\theta(1-n)\left[f(k^*) - k^*\right]/\bar{\varsigma}$. This means $b^* > 0$ if $\bar{\varsigma} < \lambda\theta(1-n)\left[f(k^*) - k^*\right]$, which we impose here.[26]

We now describe the effects of two policies: changes in the money growth rate; and sales of government bonds in stage 2.

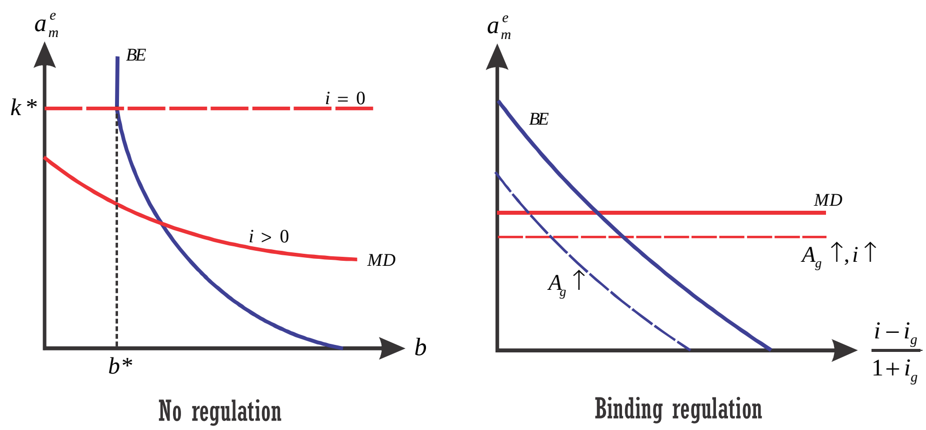

Figure 9: Equilibrium with entry

Proposition 10 *For small i, equilibrium is unique. There exist $\underline{A}_g(i) > 0$ and $\bar{A}_g(i) > \underline{A}_g(i)$ such that:*

1. For all $A_g > \bar{A}_g$, $i_g = i$ and OMOs are ineffective, $\partial a_m^e/\partial A_g = \partial b/\partial A_g = 0$. Moreover, $\partial a_m^e/\partial i < 0$ and $\partial b/\partial i > 0$.

2. For all $A_g < \bar{A}_g$, $i_g < i$ and OMOs are effective, $\partial i_g/\partial A_g > 0$, $\partial a_m^e/\partial A_g < 0$, and $\partial b/\partial A_g > 0$. Moreover, $\partial a_m^e/\partial i < 0$, $\partial b/\partial i = 0$, and $(i - i_g)/(1 + i_g)$ increases with i.

[26] We mention that a departure from $i = 0$ can be optimal here due to search externalities in the credit market, as discussed in an extension by Rocheteau et al. (2016).

3. For all $A_g < \underline{A}_g$, $i_g < 0$.

If A_g is sufficiently abundant, the spread between illiquid and liquid bonds is zero and equilibrium is determined as in Figure 9. Then (small) changes in the bond supply have no effect on bank entry or money demand. However, money growth reduces $\Delta^m(a_m^e)$ and leads to higher interest payments and more banks as the MD curve shifts down. The effect is second-order when i is small, so that pass through and transmission are similar to Proposition 3.

If A_g is not so large, the interest rate on bonds eligible for entry falls below i. Then the measure of banks is pinned down by regulation and the supply of bonds, $b = A_g/\bar{a}_g$. This is represented in the right panel of Figure 9, where the endogenous variable from (49) is the regulatory premium on bonds. The MD curve is horizontal as the relevant spread for money demand is between cash and illiquid bonds, i. In contrast, BE is downward sloping because the bond premium banks are willing to pay to enter rises as entrepreneurs hold less cash. As A_g increases, b increases, which raises entrepreneurs' probability of obtaining a bank loan and reduces a_m^e as MD shifts down. From (49), an increase in A_g reduces the regulatory premium on bonds BE shifts downward. For small i, the shift in BE dominates so that i_g increases and a_m^e decreases. An increase in the money growth rate reduces entrepreneur's money demand, MD shifts down, which leads to a higher regulatory premium on bonds.

Finally, the model generates negative interest rates on government bonds if i is small and A_g is low.[27] Banks are willing to acquire such bonds at a price above face value to satisfy regulatory requirements. If banks can satisfy the requirement with something, like currency, we obtain the following result:

Corollary 3 *Suppose banks can hold $\bar{a}_g$ in government bonds or fiat money. There is $\underline{A}_g(i) > 0$ such that for all $A_g < \underline{A}_g$, the economy is in a liquidity trap, $i_g = 0$. In*

[27] Negative nominal rates have become more prevalent in the low inflation environment following the Great Recession. In February 2016, Bloomberg estimated that nearly 30% of developed-country government bonds were traded on a negative yield (The Economist, February 2016). Rocheteau et al. (2015) discuss negative nominal rates, as well as liquidity traps and OMOs in more detail, but again, in a simpler model.

this case, $\partial b/\partial i < 0$.

In a liquidity trap banks hold both money and government bonds to satisfy the requirement $\bar{a}_g = a_g^b + a_m^b$. By market clearing $ba_g^b = A_g$, which implies $a_m^b = \bar{a}_g - A_g/b$. A change in A_g is offset by a change in a_m^b and hence has no effect on equilibrium. However, an increase in the money growth rate raises i, which has a first-order effect on the entry cost $\varsigma = \bar{\varsigma} + i\bar{a}_g$, and that reduces b. This is a case where policy affects the economy differently in and out of a liquidity trap.

9 Conclusion

This paper has developed a theory of the monetary transmission mechanism by analyzing corporate finance through a New Monetarist lens (Lagos et al. 2017; Rocheteau and Nosal 2017). The environment has entrepreneurs with random investment and financing opportunities, as in Kiyotaki and Moore (1997) and Wasmer and Weil (2004), respectively. Different from much of the macro literature, our credit market is decentralized, and loan contracts are negotiated bilaterally. A main contribution of the theory was to show how changes in nominal policy rates affect real lending rates. We also showed how pass through depends on policy, regulations, market microstructure, and firm characteristics. Improved access to bank credit, e.g., can lead to greater pass through but weaker transmission. Also, while many macro models used in policy analysis have a single interest rate, we have a rich structure of yields, including those in the interbank market, on different bonds, and on corporate lending rate, with spreads depending on liquidity, regulatory, and intermediation premia.

Our corporate finance approach to the transmission mechanism opens new research avenues. For one, while we assume interactions between banks and firms are transitory, in reality there are long-term lending relationships (Bolton et al. 2016). Rocheteau et al. (2016) extend our model to accommodate relationship lending and characterize the optimal response of policy in a financial crisis. Also, we focused on external finance mainly through bank and trade credit, but one can further study equity claims (e.g., Lagos 2010; Williamson 2012; Lester et al. 2012) or frictional bond markets (e.g., Feldhutter 2012; He and Milbradt 2014). Also, we focused on equilibria with degenerate distributions of money holdings, but extensions of our model can incorporate portfolio heterogeneity and the life cycle of firms (Adão and Silva 2016). Another idea is to replace random search and bargaining with directed search and price posting, which would allow one to naturally introduce informational asymmetries (e.g., as in Guerrieri et al. 2011). Finally, one could combine the previous New Monetarist literature on consumer finance with our approach to corporate finance for an integrated theory of money (liquidity) demand by households and firms.

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Appendix: Proofs

Proof of Proposition 1. If $k+\phi \leq \chi_b f(k)$ does not bind, then the solution to (6) is such that k maximizes the match surplus, $f(k)-k$, while ϕ shares the surplus according to bargaining powers. This gives $k=k^*$ and (7). Substituting k and ϕ by their expressions into the pledgeability constraint gives $k^*+\theta\left[f(k^*)-k^*\right] \leq \chi_b f(k^*)$, i.e., $\chi_b \geq \chi_b^*$. If the pledgeability constraint binds, then ϕ solves (9) and (6) becomes:

$$k \in \arg\max \left[f(k)\right]^{1-\theta}\left[\chi_b f(k)-k\right]^{\theta}.$$

The FOC gives (10). The LHS of (10), $k/f(k)$, is increasing in k from 0 to ∞, where the limits are obtained by L'Hopital's rule. The RHS,

$$\frac{\chi_b}{(1-\theta)}-\frac{\theta}{(1-\theta)f'(k)},$$

is decreasing for all $k \geq 0$. The RHS evaluated at k^*, $(\chi_b-\theta)/(1-\theta)$, is smaller than the LHS provided $\chi_b < \chi_b^*$. Moreover, at $k=\hat{k}$, the RHS is χ_b, which exceeds the LHS since $\chi_b f(\hat{k})-\hat{k}=\max_{k\geq 0}\{\chi_b f(k)-k\}>0$ for all $\chi_b>0$. Hence, there is a unique solution $k \in \left[\hat{k},k^*\right]$ to (10). ■

Proof of Lemma 1. The Nash bargaining problem (16) can be reexpressed as

$$\max_{(k,\phi)\in\mathcal{A}(a_m^e)} \left[f(k)-k-\phi-\Delta^m(a_m^e)\right]^{1-\theta}\phi^{\theta},$$

where the terms of the loan contract, (k,ϕ), are admissible if they belong to the compact, convex, and non-empty set:

$$\mathcal{A}(a_m^e) \equiv \left\{(k,\phi)\in[0,k^*]\times\mathbb{R}_+ : \phi \leq f(k)-k-\Delta^m(a_m^e) \text{ and } \phi \leq \chi_b f(k)-k+a_m^e\right\}.$$

The first inequality requires that the entrepreneur's surplus is non-negative and the second inequality follows from the liquidity constraint. The objective of the Nash bargaining problem and the correspondence $\mathcal{A}(a_m^e)$ are continuous, hence, by the Theorem of the Maximum, the set of solutions is non-empty and upper-hemi continuous in a_m^e. For all $(k,\phi)\in$Int$[\mathcal{A}(a_m^e)]$, the Nash product is strictly jointly concave, hence the solution is unique and it is continuous in a_m^e. The rest of the proof is identical to the one of Proposition 1 where a^* is the value for a_m^e such that the liquidity constraint holds at equality when $k = k^*$, i.e.,

$$(1-\theta)a^* + \theta f(a^*) = (\chi_b^* - \chi_b)\, f(k^*).$$

In order to establish that there is a unique solution $k \in (\hat{k}, k^*]$ to (19) notice that the RHS is increasing from 0 to $\theta/(1-\theta)$ while the LHS is decreasing from

$$\frac{a_m^e + \chi_b f(\hat{k}) - \hat{k}}{(1-\chi_b)f(\hat{k}) - a_m^e - \Delta^m(a_m^e)} > 0,$$

to

$$\frac{a_m^e + \chi_b f(k^*) - k^*}{(1-\chi_b)f(k^*) - a_m^e - \Delta^m(a_m^e)} < \frac{\theta}{1-\theta},$$

by the definition of a^* and the assumption that $a_m^e < a^*$. ■

Proof of Lemma 2. The problem (22) follows directly from (2) where $i_g = i$ since bonds are illiquid and $f(k) - k - \phi = \Delta^m(a_m^e)$ in the event the entrepreneur receives an investment opportunity but is unbanked, with probability $\lambda(1-\alpha)$, and $f(k) - k - \phi = f(k^c) - k^c - \phi^c$ in the event the entrepreneur receives an investment opportunity and is matched with a bank, with probability $\alpha\lambda$. The surplus $\Delta^c(a_m^e)$ is obtained from the solution to the bargaining problem in Lemma 1. ■

Proof of Proposition 2. Equilibrium has a recursive structure. First, (22) determines $a_m^e \in [0, k^*]$. Then (15) and (16) determine k^m and k^c. Finally, r_b comes from (21). In order to show the equilibrium is unique, we establish that there is a unique solution, a_m^e, to (22) rewritten as $\max_{a_m^e \geq 0} J(a_m^e; i)$ where

$$J(a_m^e; i) \equiv -i a_m^e + \lambda\,(1-\alpha)\,\Delta^m(a_m^e) + \lambda\alpha\Delta^c(a_m^e).$$

With no loss in generality, we can restrict the choice for a_m^e to the compact interval $[0, k^*]$. Indeed, if $a_m^e > k^*$ then $J'(a_m^e; i) = -i < 0$. The objective, $J(a_m^e; i)$, is continuous in a_m^e. Therefore, by the Theorem of the Maximum, a solution exists and $\max_{a_m^e \in [0,k^*]} J(a_m^e; i)$ is continuous in i.

Part 1. Conditions for a concave problem.

While $J(a_m^e)$ is not strictly concave in general, one can impose conditions on fundamentals such that it is. If $\chi_b \geq \chi_b^*$ then $a^* < 0$ and, substituting $\Delta^c(a_m^e)$ by its expression given by Lemma 2, we obtain

$$J(a_m^e) = -ia_m^e + \lambda\left[1 - \alpha(1-\theta)\right]\Delta^m(a_m^e) + \lambda\alpha(1-\theta)\left[f(k^*) - k^*\right], \qquad (52)$$

which is strictly concave for all $a_m^e < k^*$. If $\theta = 0$,

$$\Delta^{c\prime}(a_m^e) = \frac{f'(k) - 1}{1 - \chi_b f'(k)} \text{ if } a_m^e < a^*,$$

where $k = \chi_b f(k) + a_m^e$. Hence, $\Delta^c(a_m^e)$ is concave, and strictly concave if $a_m^e < a^*$. If $\theta = 1$, $\Delta^c(a_m^e) = \Delta^m(a_m^e)$ and $J(a_m^e) = -ia_m^e + \lambda\Delta^m(a_m^e)$, which is strictly concave for all $a_m^e < k^*$.

Part 2. Uniqueness for low i.

Suppose now that $\chi_b < \chi_b^*$ and $\theta \in (0,1)$. We prove that the solution is unique when i is close to 0. If $i = 0$ then $a_m^e = k^*$ and $J(a_m^e; 0)$ attains its upper bound $\lambda\left[f(k^*) - k^*\right]$. For $i > 0$ close to 0 we prove that

$$\arg\max_{a_m^e \geq 0} J(a_m^e; i) = \arg\max_{a_m^e \in [a^*, k^*]} \left\{-ia_m^e + \lambda\left[1 - \alpha(1-\theta)\right]\Delta^m(a_m^e)\right\},$$

where the RHS is a singleton by the strict concavity of $\Delta^m(a_m^e)$. In words, we can restrict the choice for a_m^e to the interval $[a^*, k^*]$ over which the objective is concave. In order to show this result we use that

$$J(a_m^e; i) \leq -ia_m^e + \lambda(1-\alpha)\Delta^m(a_m^e) + \lambda\alpha\left[f(k^*) - k^*\right], \text{ for all } a_m^e \in [0, k^*],$$

where the RHS is a concave function of a_m^e. It follows that

$$\max_{a_m^e \in [0,a^*]} J(a_m^e; i) \leq \max_{a_m^e \in [0,a^*]} \left\{-ia_m^e + \lambda(1-\alpha)\Delta^m(a_m^e)\right\} + \alpha\lambda\left[f(k^*) - k^*\right].$$

There is a $\bar{\imath} = \lambda(1-\alpha)\Delta^{m\prime}(a^*) > 0$ such that $\arg\max_{a_m^e \in [0,a^*]} \left\{-ia_m^e + \lambda(1-\alpha)\Delta^m(a_m^e)\right\} = a^*$ for all $i < \bar{\imath}$. Hence,

$$\max_{a_m^e \in [0,a^*]} J(a_m^e; i) \leq -ia^* + \lambda(1-\alpha)\Delta^m(a^*) + \alpha\lambda\left[f(k^*) - k^*\right] \text{ for all } i < \bar{\imath}.$$

The RHS is continuous in i and it approaches $\lambda(1-\alpha)\Delta^m(a^*) + \alpha\lambda\left[f(k^*) - k^*\right] < \lambda\left[f(k^*) - k^*\right]$ when i tends to 0. Using that $\max_{a_m^e \in [0,k^*]} J(a_m^e; i)$ is continuous in i, it follows that $\arg\max_{a_m^e \geq 0} J(a_m^e; i) = \arg\max_{a_m^e \in [a^*, k^*]} J(a_m^e; i)$ for i close to 0, and therefore it is unique.

Part 3. Generic uniqueness. When i is not close to 0, we can prove generic

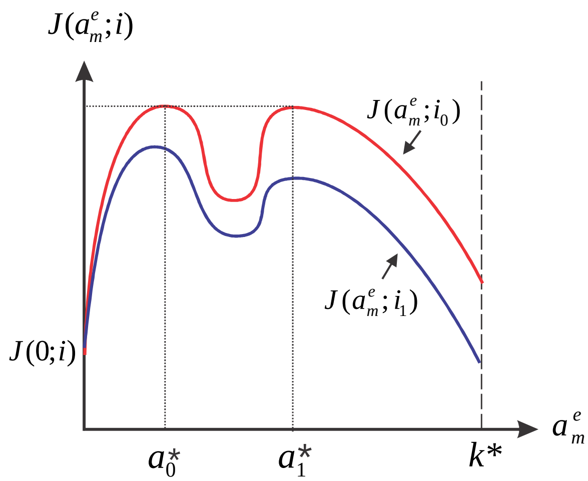

Figure 10: Generic uniqueness of the equilibrium

uniqueness by adapting the proof from Gu and Wright (2016). The function $J(a_m^e; i)$ is continuous and differentiable everywhere except maybe at $a_m^e = a^*$. Suppose there is more than one solution to $\max_{a_m^e \geq 0} J(a_m^e; i)$. For the sake of argument, suppose there are two solutions denoted a_0^* and $a_1^* > a_0^*$. We now argue that this multiplicity of solutions is not robust to a small perturbation in i making the multiplicity non-generic. To see this, note that $\partial J(a_m^e; i)/\partial i = -a_m^e$ which is decreasing in a_m^e. Hence, a small increase in i eliminates the solution in the neighborhood of a_1^*. We illustrate this argument in Figure 10 by representing $J(a_m^e; i)$ for two values of i, i_0 and $i_1 > i_0$. At $i = i_0$, there are two solutions, $J(a_0^*; i_0) = J(a_1^*; i_0)$, but only the lowest remains as i is increased to i_1.

Part 4. Coexistence of internal and external finance.

We first prove that $\Delta^{c\prime}(0) = +\infty$ for all $\theta > 0$. If $\chi_b > \chi_b^*$ then $a^* < 0$ and from Lemma 2 $\Delta^{c\prime}(0) = \theta \Delta^{m\prime}(0) = +\infty$. Suppose next that $\chi_b < \chi_b^*$. We can rewrite (19) as

$$(1 - \chi_b) f(k^c) = a_m^e + \Delta^m(a_m^e) + \frac{1-\theta}{\theta} \frac{(1-\chi_b) f'(k^c)}{1 - \chi_b f'(k^c)} \left[a_m^e + \chi_b f(k^c) - k^c\right]. \quad (53)$$

Denote

$$\Psi(k^c) \equiv \frac{1-\theta}{\theta}\frac{(1-\chi_b)f'(k^c)}{1-\chi_b f'(k^c)}\left[\chi_b f(k^c) - k^c\right],$$

which is decreasing in k^c for all $k^c \in \left(\hat{k}, k^*\right)$ and $\theta \in (0,1)$. By differentiating (53) we obtain

$$\left.\frac{\partial k^c}{\partial a_m^e}\right|_{a_m^e=0^+} = \left[(1-\chi_b)f'(k_0^c) - \Psi'(k_0^c)\right]^{-1}\left[1 + \Delta^{m\prime}(0) + \frac{1-\theta}{\theta}\frac{(1-\chi_b)f'(k_0^c)}{1-\chi_b f'(k_0^c)}\right],$$

where $k_0^c \in \left(\hat{k}, k^*\right)$ is the solution to (10). Given that $\Delta^{m\prime}(0) = +\infty$ it follows that:

$$\left.\frac{\partial k^c}{\partial a_m^e}\right|_{a_m^e=0^+} = \frac{\Delta^{m\prime}(0)}{(1-\chi_b)f'(k_0^c) - \Psi'(k_0^c)} = +\infty.$$

From Lemma 2

$$\Delta^{c\prime}(0) = (1-\chi_b)f'\left(k_0^c\right)\left.\frac{\partial k^c}{\partial a_m^e}\right|_{a_m^e=0^+} - 1 = +\infty.$$

Using that $\Delta^{m\prime}(0) = +\infty$ and $\Delta^{c\prime}(0) = +\infty$ for all $\theta > 0$, there exists a positive solution to $\max_{a_m^e \geq 0} J(a_m^e)$ if $\lambda(1-\alpha) > 0$ or $\lambda\alpha\theta > 0$. Finally, we check that $k^c > k^m$ for all $i > 0$. If the liquidity constraint does not bind, (23) implies $k^m < k^*$ and from (17) $k^c = k^*$. If the liquidity constraint does bind, then from (19) $(1-\chi_b)f(k^c) - a_m^e - \Delta^m(a_m^e) > 0$, which implies $k^c > k^m$ for all $\chi_b > 0$. ■

Proof of Propositions 3 and 4. A second-order Taylor series expansion of $f(k^m) - k^m$ in the neighborhood of k^* is:

$$f(k^m) - k^m \approx f(k^*) - k^* + \frac{f''(k^*)}{2}(k^m - k^*)^2.$$

Substituting $f(k^m) - k^m$ by its approximation into (24) gives

$$r_b \approx \frac{\theta f''(k^*)(k^m - k^*)}{2}. \tag{54}$$

Substituting $f'(k^m)$ by its first-order approximation in the neighborhood of k^*, $1 + f''(k^*)(k^m - k^*)$, into (23) gives

$$k^m - k^* \approx \frac{i}{f''(k^*)\lambda\left[1-\alpha(1-\theta)\right]}, \tag{55}$$

which corresponds to (26). Substituting $k^m - k^*$ from (55) into (54) gives (25). We obtain (28) by replacing k^c and k^m by their expressions given by (27) and (26) into $K = \lambda\left[\alpha k^c + (1-\alpha)k^m\right]$. Similarly, (29) is obtained by substituting the individual loan size, $k^* - k^m$, by the expression given by (55) into $L = \lambda\alpha(k^* - k^m)$. ■

Proof of Proposition 5. We linearize $k^m + \chi_b f(k^c) = k^c$ and (31) in the neighborhood of $(k^m, k^c) = (\bar{k}, k^*)$ to obtain:

$$\begin{aligned} k^m - \bar{k} - (1-\chi_b)\,(k^c - k^*) &= 0 \\ \lambda\,(1-\alpha)\,f''(\bar{k})(k^m - \bar{k}) + \lambda\alpha\frac{f''(k^*)}{1-\chi_b}(k^c - k^*) &= i - \bar{\imath}. \end{aligned}$$

The solution is

$$\begin{pmatrix} k^m - \bar{k} \\ k^c - k^* \end{pmatrix} = \frac{1}{D}\begin{pmatrix} 1-\chi_b \\ 1 \end{pmatrix}(i - \bar{\imath}),$$

where $D = \lambda\alpha f''(k^*)/(1-\chi_b) + (1-\chi_b)\,\lambda\,(1-\alpha)\,f''(\bar{k})$. We obtain (33) by replacing k^c and k^m by their expressions given by (32) into $L = \alpha\lambda(k^c - k^m)$. ■

Proof of Propositions 7 and 8. From (38) the real lending rate is:

$$r_b = (1-\theta)\nu i_f + \theta\left[\frac{f(k^c) - k^c - f(k^m) + k^m}{k^c - a_m^e}\right]. \tag{56}$$

A second-order approximation of $f(k) - k$ in the neighborhood of k^* is:

$$f(k) - k \approx f(k^*) - k^* + f''(k^*)\frac{(k-k^*)^2}{2}.$$

Substituting this expression into (56) we obtain:

$$\begin{aligned} r_b &\approx (1-\theta)\nu i_f + \frac{\theta}{2}f''(k^*)\left[\frac{(k^c - k^*)^2 - (k^m - k^*)^2}{k^c - a_m^e}\right] \\ &\approx (1-\theta)\nu i_f + \frac{\theta}{2}f''(k^*)(k^c - k^* + k^m - k^*), \end{aligned} \tag{57}$$

where we have used that $k^m = a_m^e$ in equilibrium. Assuming k^c and k^m are close to k^*, a first-order approximation of (36) and (39) gives (41) and (42). We replace $k^c - k^*$ and $k^m - k^*$ by their expressions from (41) and (42) into (57) to obtain:

$$r_b \approx \left(1 - \frac{\theta}{2}\right)\nu i_f + \frac{\theta}{2}\left(\frac{i - \alpha\lambda(1-\theta)\nu i_f}{\lambda\left[1 - \alpha(1-\theta)\right]}\right) = \nu i_f + \frac{\theta\,(i - \lambda\nu i_f)}{2\lambda\left[1-\alpha(1-\theta)\right]},$$

which corresponds to (40). We obtain (43) by substituting k^c and k^m by their expressions given by (41) and (42) into $K = \lambda\left[\alpha k^c + (1-\alpha)k^m\right]$. ■

Proof of Proposition 9. Using a first-order approximation for small i_f,

$$k^c = f'^{-1}\left(1+\nu i_f\right) \approx k^* + \frac{\nu i_f}{f''(k^*)}. \tag{58}$$

It follows that

$$k^{c\prime} - k^c \approx \frac{\nu\left(i_f' - i_f\right)}{f''(k^*)}.$$

The demand for reserves from a bank matched with an entrepreneur with an investment opportunity is $\nu(k^c - a_m^e)$. There are $\alpha\lambda$ such banks. Hence, the change in the aggregate demand for reserves is

$$\alpha\lambda\nu\left[(k^{c\prime} - a_m^{e\prime}) - (k^c - a_m^e)\right] = \alpha\lambda\nu\left[\frac{\nu}{f''(k^*)}(i_f' - i_f) - (a_m^{e\prime} - a_m^e)\right].$$

The change in the entrepreneur's real balances is $(a_m^{e\prime} - a_m^e) = -\mu a_m^e/(1+\mu)$. The change in the supply of reserves is $\mu q_{m,t}' M_t = \mu a_m^e/(1+\mu)$. Hence, by market clearing,

$$\alpha\lambda\nu\left[\frac{\nu}{f''(k^*)}(i_f' - i_f) + \frac{\mu a_m^e}{1+\mu}\right] = \frac{\mu a_m^e}{1+\mu},$$

the change in the interbank market rate is such that

$$\frac{\alpha\lambda\nu^2}{f''(k^*)}(i_f' - i_f) = (1-\alpha\lambda\nu)\frac{\mu}{1+\mu}a_m^e.$$

This gives (45). From (57)

$$\begin{aligned}
r_b &\approx (1-\theta)\nu i_f + \theta\frac{f''(k^*)}{2}\left[(k^c - k^*) + (a_m^e - k^*)\right] \\
&= (1-\theta)\nu i_f + \theta\frac{f''(k^*)}{2}\left[\frac{\nu i_f}{f''(k^*)} + (a_m^e - k^*)\right] \\
&= \left(1-\frac{\theta}{2}\right)\nu i_f + \theta\frac{f''(k^*)}{2}\left(a_m^e - k^*\right),
\end{aligned}$$

where in the second line we substituted $k^c - k^*$ by its expression given by (58). Hence, the change in the lending rate is

$$\begin{aligned}
r_b' - r_b &= \left(1-\frac{\theta}{2}\right)\nu(i_f' - i_f) + \theta\frac{f''(k^*)}{2}(a_m^{e\prime} - a_m^e) \\
&= \left(1-\frac{\theta}{2}\right)\nu(i_f' - i_f) - \theta\frac{f''(k^*)}{2}\frac{\mu}{1+\mu}a_m^e
\end{aligned}$$

where we used that $(a_m^{e\prime} - a_m^e) = -\mu a_m^e/(1+\mu)$. Substituting the second term on the RHS by its expression given by (45) we obtain (46). Change in aggregate investment is $K' - K = \alpha\lambda(k^{c\prime} - k^c) + (1-\alpha)\lambda(k_m' - k_m)$. Substituting $k^{c\prime} - k^c$ by its expression given by (44) and $k_m' - k_m = -\mu a_m^e/(1+\mu)$ we obtain (47). ■

Proof of Proposition 10. Equations (49) and (50) can be reexpressed as $\Gamma(b; i, i_g) \leq 0$, with an equality if $b > 0$, where

$$\begin{aligned}\Gamma(b; i, i_g) \equiv \frac{\alpha(b)}{b}\lambda\theta\left\{f(k^*) - k^* - n\Delta^m \circ f'^{-1}\left[1 + \frac{i}{\lambda\left[1 - \alpha(b)(1-\theta)\right]}\right]\right\} \\ - \bar{\varsigma} - \bar{a}_g\left(\frac{i - i_g}{1 + i_g}\right).\end{aligned} \tag{59}$$

As i approaches 0, $\Gamma'(b; i, i_g)$ converges uniformly to

$$\frac{\left[\alpha'(b)b/\alpha(b) - 1\right]}{b}\frac{\alpha(b)}{b}\lambda\theta(1-n)\left[f(k^*) - k^*\right] < 0 \text{ for all } b,$$

where we used that $\alpha'(b)b < \alpha(b)$ by the strict concavity of $\alpha(b)$ and the assumption $\alpha(0) = 0$. Hence, for small i, $\Gamma(b; i, i_g)$ is a decreasing and continuous function of b and there is a unique solution, denoted $b(i, i_g)$, to $\Gamma(b; i, i_g) \leq 0$ ("=" if $b > 0$). See left panel of Figure 11 for the case where the solution is interior. By the Implicit Function Theorem ($\Gamma(b; i, i_g)$ is continuously differentiable and $\Gamma'(b) \neq 0$), $b(i, i_g)$ is continuous in i_g. Using that $\Gamma(b; i, i_g)$ is increasing in i_g, $b(i, i_g)$ is nondecreasing in i_g for all $i_g < i$, and it is strictly increasing if $b > 0$. In Figure 11, $\Gamma(b)$ shifts upward as i_g increases. Moreover from (59), $\Gamma(b; i, i) > \Gamma(b; 0, 0)$ and hence $b(i, i) > b^*$. The aggregate demand correspondence for bonds is

$$A_g^d(i_g) = \begin{cases} [\bar{A}_g(i), +\infty) & \text{if } i_g = i \\ \{b(i, i_g)\bar{a}_g\} & \phantom{\text{if }} i_g < i, \end{cases}$$

where $\bar{A}_g(i) = \bar{a}_g b(i, i) > \bar{a}_g b^* > 0$. See the right panel of Figure 11. We obtained $A_g^d(i)$ by using the fact that if $i_g = i$, banks are willing to hold bonds in excess of the regulatory requirement. In contrast, if $i_g < i$ then holding bonds is costly so that banks that enter only hold $\bar{a}_g$ and the aggregate demand for bonds is $b(i, i_g)\bar{a}_g$. Given that $+\infty \in A_g^d(i)$ and $\{0\} \in A_g^d(i_g)$ for i_g sufficiently low, since $\lim_{i_g \to -\infty} \varsigma = +\infty$, there is a unique $i_g \leq i$ such that the market-clearing condition, $\{A_g\} \in$

$A_g^d(i_g)$, holds. See right panel of Figure 11. Finally, let $\underline{A}_g(i) = \bar{a}_g b(i,0)$. Since by assumption $b^* = b(0,0) > 0$, for small i, $\underline{A}_g(i) > 0$. For all $A_g < \underline{A}_g(i)$, $i_g < 0$. Using that $b(i,0) < b(i,i)$, $\underline{A}_g(i) < \bar{A}_g(i)$.

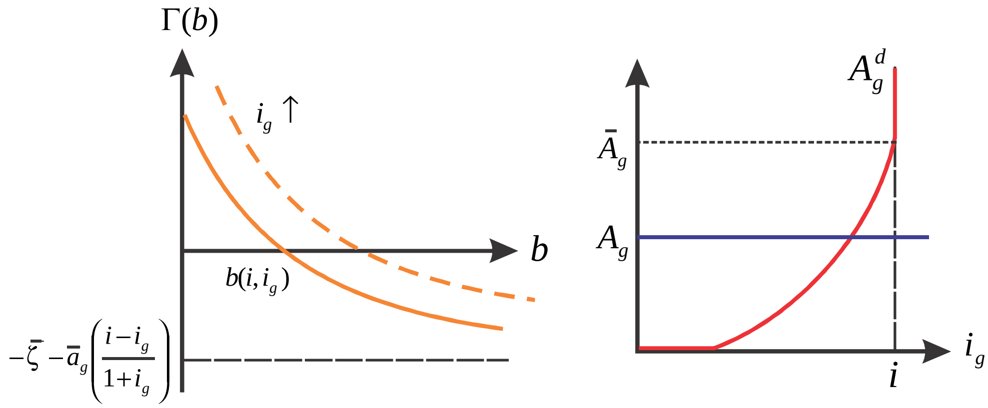

Figure 11: Equilibrium with entry

We now turn to comparative statics. If $A_g > \bar{A}_g$, then $i_g = i$ and $b = b(i,i)$ is independent of A_g. From (59), $\Gamma(b; i, i)$ is decreasing in i. Hence, $\partial b/\partial i > 0$. If $A_g < \bar{A}_g$, then $b = A_g/\bar{a}_g$ is increasing with A_g but it is independent of i. With a slight abuse of notation, the entry condition can be rewritten as:

$$\Gamma\left(\frac{A_g}{\bar{a}_g}, i, \frac{i - i_g}{1 + i_g}\right) = 0,$$

which determines the spread $(i - i_g) / (1 + i_g)$. From (59), Γ is increasing with i and decreasing with $(i - i_g) / (1 + i_g)$. Hence, the equilibrium spread is increasing in i and decreasing in A_g. From (50), a_m^e is decreasing with A_g. ■

Supplemental Appendices (not for publication)

A1. The bargaining set

In a match between an entrepreneur and a bank, the surpluses are $S^e = f(k) - k - \phi$ and $S^b = \phi$. If the pledgeability constraint is slack, the surplus is maximized at $f(k^*) - k^*$. Then the frontier is linear, $S^e + S^b = f(k^*) - k^*$, as in the right panel of Figure 12. The constraint is slack if $\phi \leq \chi_b f(k^*) - k^*$. Hence, the frontier has a linear portion iff $\chi_b \geq k^*/f(k^*)$, and is entirely linear if $f(k^*) - k^* \leq \chi_b f(k^*) - k^*$, which only occurs when $\chi_b = 1$.

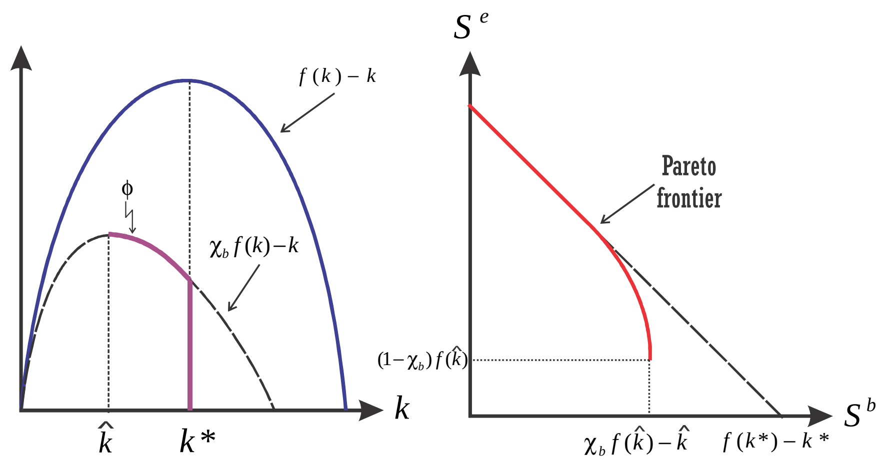

Figure 12: Pareto frontier for bank loans

If the pledgeability constraint binds, then $\phi = \chi_b f(k) - k$, as in the left panel of Figure 12. Take a pair (ϕ, k) below the curve $\chi_b f(k) - k$ such that $k < k^*$. By raising k, S^e increases. Moreover, $k \geq \hat{k} = \arg\max [\chi_b f(k) - k]$, since otherwise one could raise $\phi = \chi_b f(k) - k$ and increase both surpluses. Hence, the frontier when the constraint binds is

$$\left\{(S^e, S^b) \in \mathbb{R}_+^2 : S^e = (1 - \chi_b) f(k),\ S^b = \chi_b f(k) - k,\ k \in \left[\hat{k}, \bar{k}\right]\right\},$$

where $\bar{k} = k^*$ if $\chi_b f(k^*) \geq k^*$, and $\bar{k}$ is the largest solution to $\chi_b f(\bar{k}) - \bar{k} = 0$ otherwise. It is easy to check the frontier is downward sloping, $\partial S^e/\partial S^b < 0$, and $\partial S^e/\partial S^b \to -\infty$ as $k \to \hat{k}$. If $\chi_b f(k^*) \geq k^*$ then $\partial S^e/\partial S^b \to -1$ as $k \to k^*$.

The bargaining set is not convex since the point on the frontier that maximizes S^b, $\chi_b f(\hat{k}) - \hat{k}$, is above the horizontal axis. Hence, the entrepreneur enjoys a positive surplus, $(1- \chi_b) f(\hat{k})$, due to limited pledgeability.

A2. An alternative bargaining solution

As an alternative to the Nash solution, many recent models use Kalai's proportional bargaining solution, which is this context is given by:

$$\max_{\phi,k} S^b = \phi \text{ st } S^e \geq \frac{1-\theta}{\theta} S^b \text{ and } k + \phi \leq \chi_b f(k).$$

Thus, a bank chooses (ϕ, k) to maximize S^b subject to the entrepreneur getting at least a fraction $1- \theta$ of the total surplus. In fact, the strict proportional solution requires a strict equality in the first constraint; we use an inequality to guarantee existence despite nonconvexity of the bargaining set, which formally corresponds to the lexicographic proportional solution. Provided $\chi_b \geq \chi_b^*$, the pledgeability constraint is slack and Kalai coincides with Nash. If the constraint binds, k solves

$$(\chi_b - \theta) f(k) = (1-\theta) k \text{ if } \chi_b > \theta \text{ and } k \geq \hat{k};\ k = \hat{k} \text{ otherwise.}$$

Thus, the solution $k \geq \hat{k}$ splits the surplus so the bank gets a share θ of the surplus and satisfies the constraint. If $k < \hat{k}$, the solution is not Pareto optimal: by increasing k to $\hat{k}$, S^b reaches its maximum, while S^e increases. In that case, we select $k = \hat{k}$, in accordance with the lexicographic proportional solution. The lending rate when the constraint binds is

$$r_b = \frac{\theta(1-\chi_b)}{\chi_b - \theta} \text{ if } \theta \leq \hat{\theta} \equiv \frac{\chi_b f(\hat{k}) - \hat{k}}{f(\hat{k}) - \hat{k}};\ r_b = \frac{\hat{\theta}(1-\chi_b)}{\chi_b - \hat{\theta}} \text{ otherwise.}$$

Provided θ is not too large, r_b decreasing with χ_b. If $f(k) = zk^\gamma$, e.g., one can check r_b and k are given by:

$$r_b = \begin{matrix} \frac{\theta(1-\gamma)}{\gamma} \\ \frac{\theta(1-\chi_b)}{\chi_b-\theta} \\ \frac{1-\gamma}{\gamma} \end{matrix} \text{ and } k = \begin{matrix} (\gamma z)^{\frac{1}{1-\gamma}} \\ \left[\frac{(\chi_b-\theta)z}{1-\theta}\right]^{\frac{1}{1-\gamma}} \\ (\chi_b z \gamma)^{\frac{1}{1-\gamma}} \end{matrix} \text{ if } \chi_b \begin{matrix} \geq (1-\theta)\gamma + \theta \\ \in \left[\frac{\theta}{1-\gamma(1-\theta)}, (1-\theta)\gamma + \theta\right) \\ < \frac{\theta}{1-\gamma(1-\theta)} \end{matrix}$$

For low χ_b, r_b is maximized and independent of χ_b and θ; in this case the constraint binds and k maximizes S^b. For intermediate χ_b, r_b is decreasing in χ_b and increasing in θ. For high χ_b, the constraint is slack, so k and r_b are independent of χ_b.

A3. Limited commitment

In the text, the entrepreneur's borrowing limit is a fraction χ_b of $f(k)$. This can be motivated by, instead of moral hazard, limited commitment. Assume banks can no longer seize output: entrepreneurs can abscond with it all and default on the loan. However, banks have a record of repayment histories, and can punish defaulters by exclusion from future credit. An endogenous debt constraint ensures entrepreneurs repay debts, which depends on $\bar{W}^e = W^e(0,0) = \beta\{\alpha\lambda[f(k) - k - \phi] + \bar{W}^e\}$. An entrepreneur in stage 2 with no wealth has an investment opportunity in the next period with probability $\alpha\lambda$, in which case he gets surplus $f(k) - k - \phi$. Solving for $\bar{W}^e$, we obtain

$$\bar{W}^e = \frac{\alpha\lambda\left[f(k) - k - \phi\right]}{\rho}. \tag{60}$$

Thus, the value of being an entrepreneur is the discounted sum of profits, net of fees. By defaulting, an entrepreneur is banished to autarky and loses $\bar{W}^e$, making the borrowing constraint $\psi + \phi \le \bar{W}^e$.

Under Nash bargaining the loan contract solves

$$(k, \phi) \in \arg\max \left[f(k) - k - \phi\right]^{1-\theta} \phi^{\theta} \text{ st } k + \phi \le \bar{W}^e. \tag{61}$$

The problem is convex, since $\bar{W}^e$ is independent of k. The frontier is

$$S^e + S^b = f(k^*) - k^* \text{ if } S^b \le \bar{W}^e - k^*;\ \Delta^{-1}(S^e + S^b) + S^b = \bar{W}^e \text{ otherwise,}$$

where $\Delta(k) \equiv f(k) - k$ is the total surplus when the constraint binds. Relative to Figure 12, the frontier now intersects the horizontal axis at $S^e = 0$. Notice $k = k^*$ and $\phi = \theta\left[f(k^*) - k^*\right]$ if $\bar{W}^e \ge k^* + \phi$. Using this, the value of an entrepreneur who is not constrained is $\bar{W}^e = \alpha\lambda\left(1-\theta\right)\left[f(k^*) - k^*\right]/\rho$. Accordingly, entrepreneurs are not constrained if

$$\rho \le \rho^* \equiv \frac{\alpha\lambda\left(1-\theta\right)\left[f(k^*) - k^*\right]}{(1-\theta)k^* + \theta f(k^*)}.$$

Next suppose the constraint binds. The solution to (61) is

$$\bar{W}^e = \frac{\theta f(k) + (1-\theta) f'(k) k}{(1-\theta) f'(k) + \theta}. \tag{62}$$

Now the borrowing limit $\bar{W}^e$ is a weighted average of $f(k)$ and the supplier's cost, k. In this case,

$$\bar{W}^e = \frac{\alpha\lambda}{\rho + \alpha\lambda} f(k). \tag{63}$$

The limit from (63) is analogous to the pledgeability constraint in Section 4 where $\chi_b = \alpha\lambda/(\rho+\alpha\lambda)$. Here pledgeability depends on ρ, λ and α. A difference however is that the RHS of (63) uses future output.

Substituting $\bar{W}^e$ from (63) into (62), k solves

$$\frac{k}{f(k)} = \frac{\alpha\lambda(1-\theta) f'(k) - \rho\theta}{(\rho+\alpha\lambda)(1-\theta) f'(k)}. \tag{64}$$

Notice $k = 0$ always solves (64), as is standard. In addition, there is solution $k > 0$ uniquely determined, since the LHS (64) is increasing in k while the RHS is decreasing for all k such that $\alpha\lambda(1-\theta) f'(k) > \rho\theta$. The positive solution increases with α and λ and decreases with ρ and θ. The lending rate is

$$r_b = \frac{\bar{W}^e - k}{k} = \frac{\alpha\lambda}{\rho+\alpha\lambda}\frac{f(k)}{k} - 1,$$

which increases with θ. Notice r_b depends on ρ, since the debt limit is determined by future surpluses, as well as λ and α.

Given $f(k) = zk^\gamma$, when the borrowing constraint is slack, $k^* = (\gamma z)^{\frac{1}{1-\gamma}}$ and $r_b = \theta(1-\gamma)/\theta$, identical to Section 4. When it binds,

$$k = \left[\frac{\chi_b(1-\theta)z\gamma}{(1-\theta)\gamma + (1-\chi_b)\theta}\right]^{\frac{1}{1-\gamma}} \text{ and } r_b = \frac{(1-\chi_b)\theta}{(1-\theta)\gamma},$$

where $\chi_b \equiv \alpha\lambda/(\rho+\alpha\lambda)$. Now k increases while r_b decreases with pledgeability.

A4. Strategic foundations for bargaining

While the strategic foundations of Nash bargaining are very well known, there are some nuances here, like commitment issues and nonconvexities; therefore, we provide the details. Consider a game with alternating offers between the entrepreneur and bank. There is no discounting, but an exogenous risk of breakdown. At the initial stage, the entrepreneur makes an offer (k^e, d^e, ϕ^e), and the bank can say either yes or no. If it says yes, the offer is implemented. If it say no, the game continues. With probability δ^e negotiations end with no loan; with probability $1-\delta^e$ the bank makes an offer (k^b, d^b, ϕ^b), and the entrepreneur can either say yes or no. If he says yes, the offer is implemented. If he say no, the game continues. With probability δ^b negotiations end; with probability $1-\delta^b$ the games continues as in the initial stage. See the game tree in Figure 13. A node with two players corresponds to a simultaneous move and the risk of breakdown is a move by Nature.

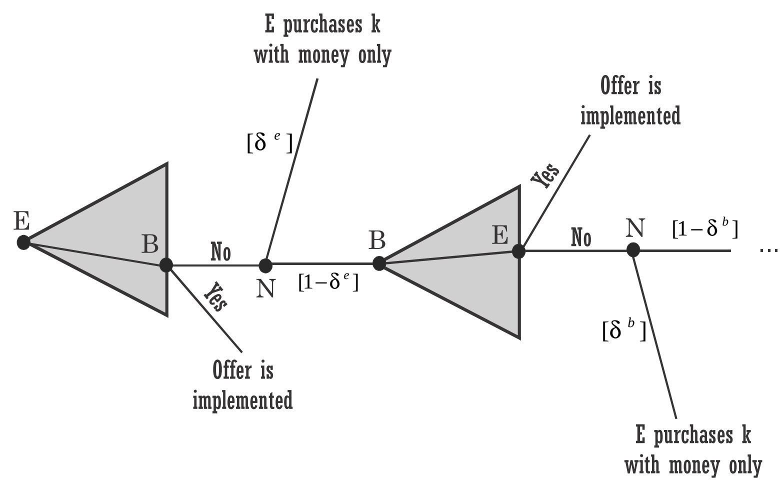

Figure 13: Game tree

Consider stationary equilibria with offers, (k^e, d^e, ϕ^e) and (k^b, d^b, ϕ^b). We restrict attention to acceptance rules in the form of reservation surpluses, R^e and R^b, that specify a minimum surplus required for an agent to accept. Entrepreneurs accept an offer if $f(k) - \psi - \phi \geq R^e$, and banks accept if $\phi \geq R^b$. When it is the entrepreneur turn to make an offer,

$$S^e(R^b) = \max_{k,\phi} \left\{ [f(k) - k - \phi] \, \mathbb{I}_{\{\phi \geq R^b\}} \right\} \text{ st } \; k + \phi \leq \chi_b f(k) + a^e_m,$$

where $\mathbb{I}_{\{\phi \geq R^b\}}$ is an indicator function that equals one if $\phi \geq R^b$ (we ignore the down payment d, because the entrepreneur uses his real balances before requesting a loan). The solution is:

$$S^e(R^b) = f(k^*) - k^* - R^b \text{ if } R^b \le \chi_b f(k^*) - k^* + a^e_m \tag{65}$$

$$= f(k) - k - R^b \text{ if } R^b \in (\chi_b f(k^*) - k^* + a^e_m, \chi_b f(\hat{k}) - \hat{k} + a^e_m] \tag{66}$$

where k is the largest solution to $\chi_b f(k) - k = R^b - a^e_m$. If the reservation surplus of the bank is sufficiently low, the entrepreneur can finance k^* and $\phi = R^b$; if R^b is larger but not too large, the entrepreneur asks for the largest loan satisfying the liquidity constraint; if R^b is too large the entrepreneur cannot satisfy $\phi \ge R^b$ and get a surplus. It can be checked that $S^e(R^b)$ is decreasing and concave with $S^e(0) > 0$.

Similarly, the bank's surplus when it is his turn to make an offer is

$$S^b(R^e) = \max_{k,\phi} \left\{ \phi \mathbb{I}_{\{f(k)-k-\phi \ge R^e\}} \right\} \text{ st } k + \phi \le \chi_b f(k) + a^e_m.$$

The bank maximizes his payoff subject to the acceptance rule and liquidity constraint. The solution is

$$S^b(R^e) = f(k^*) - k^* - R^e \text{ if } R^e \in [(1-\chi_b) f(k^*) - a^e_m, f(k^*) - k^*] \tag{67}$$

$$= \chi_b f(\hat{k}) - \hat{k} + a^e_m \quad \text{if} \quad R^e \le (1-\chi_b) f(\hat{k}) - a^e_m \tag{68}$$

$$= f(k) - k - R^e \quad \text{otherwise,} \tag{69}$$

where k solves $(1-\chi_b) f(k) = R^e + a^e_m$. If the entrepreneur's reservation surplus is large but not so large the bank would not participate, the bank offers to finance k^*; if R^e is low, the bank asks for a payment such that the constraint binds; and below a threshold for R^e, k maximizes $\chi_b f(k) - k$. It can be checked that $S^b(R^e)$ is nondecreasing, concave, and $S^b(R^e) > 0$.

The endogenous reservations surpluses solve

$$R^e = (1-\delta^b) S^e(R^b) + \delta^b \Delta^m(a^e_m) \tag{70}$$

$$R^b = (1-\delta^e) S^b(R^e). \tag{71}$$

Thus, R^e is the surplus that makes the entrepreneur indifferent between accepting or rejecting, and similarly for (71). Note that after a breakdown the bank receives no surplus.

Figure 14 shows (70) in blue and (71) in red; both are downward sloping and concave. To establish existence, let $\bar{R}^e > 0$ be the R^e such that $S^b(R^e) = 0$. By the duality of the entrepreneur and bank problems, $\bar{R}^e = S^e(0)$. Moreover, provided that $a_m^e < k^*$ then $\Delta^m(a_m^e) < S^e(0)$. Hence, the blue curve is below the red curve at $R^b = 0$. The red curve has a maximum $(1 - \delta^e)S^b(0) < \chi_b f(\hat{k}) - \hat{k} + a_m^e$. So at $R^b = \chi_b f(\hat{k}) - \hat{k} + a_m^e$ the blue curve is to the right of the red curve. Hence, they intersect, so a solution exists. Uniqueness follows from concavity of the relationships and the fact that when they are linear, they have different slopes.

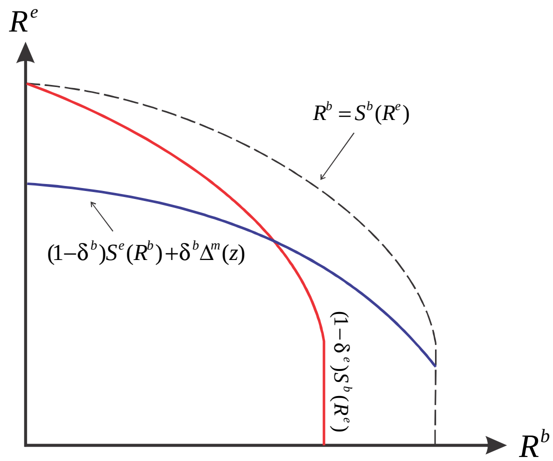

Figure 14: Determination of (R^b, R^e)

A stationary, subgame perfect equilibrium is composed of two offers, (k^e, d^e, ϕ^e) and (k^b, d^b, ϕ^b), and two reservation surpluses, R^e and R^b, solving the above conditions, as is completely standard. Existence and uniqueness here follow from the above discussion. Now consider letting the risk of breakdown get small by rewriting $\delta^e = \varepsilon\bar{\delta}^e$ and $\delta^b = \varepsilon\bar{\delta}^b$. As $\varepsilon \to 0$, $S^e(R^b) - R^e \to 0$ and $S^b(R^e) - R^b \to 0$ (i.e., when the breakdown risk gets small, first-mover advantage vanishes). Graphically, the reservation values at the intersection of the curves in Figure 14 converge to a point on the dashed curve.

Suppose first the borrowing constraint does not bind. Then $S^e(R^b) = f(k^*) - k^* - R^b$ and $S^b(R^e) = f(k^*) - k^* - R^e$. Thus, both the entrepreneur and bank offer

k^*, and use ϕ to satisfy the acceptance rules. Taking the limit as $\varepsilon \to 0$,

$$R^e \;\to\; \frac{\bar{\delta}^e\left[f(k^*) - k^*\right] + \bar{\delta}^b \Delta^m(a^e_m)}{\bar{\delta}^e + \bar{\delta}^b} \tag{72}$$

$$R^b \;\to\; \frac{\bar{\delta}^b}{\bar{\delta}^e + \bar{\delta}^b}\left[f(k^*) - k^* - \Delta^m(a^e_m)\right]. \tag{73}$$

The banks' surplus approaches a fraction $\bar{\delta}^b/(\bar{\delta}^e + \bar{\delta}^b)$ of the total surplus, coinciding with the Nash solution with $\theta = \bar{\delta}^b/\left(\bar{\delta}^e + \bar{\delta}^b\right)$.

Now suppose the liquidity constraint binds. Then $S^e(R^b) = f(k^e) - k^e - R^b$ where k^e is the highest solution to $k^e + R^b = \chi_b f(k^e) + a^e_m$, and $S^b(R^e) = f(k^b) - k^b - R^e$ where k^b is the solution to $(1-\chi_b)f(k^b) = R^e + a^e_m$. Now (k^e, k^b, R^e, R^b) solves

$$R^e = (1 - \bar{\delta}^b\varepsilon)\left[f(k^e) - k^e - R^b\right] + \bar{\delta}^b\varepsilon\Delta^m(a^e_m) \tag{74}$$

$$R^b = (1 - \bar{\delta}^e\varepsilon)\left[f(k^b) - k^b - R^e\right] \tag{75}$$

$$R^b = \chi_b f(k^e) - k^e + a^e_m \tag{76}$$

$$R^e = (1-\chi_b)f(k^b) - a^e_m. \tag{77}$$

Rearranging (74)-(75) we obtain

$$R^e = \frac{(1-\bar{\delta}^b\varepsilon)\left\{f(k^e) - k^e - (1-\bar{\delta}^e\varepsilon)\left[f(k^b) - k^b\right]\right\} + \bar{\delta}^b\varepsilon\Delta^m(a^e_m)}{1 - (1-\bar{\delta}^b\varepsilon)(1-\bar{\delta}^e\varepsilon)}.$$

Letting $\varepsilon \to 0$ and using L'Hopital's rule, we get

$$R^e = \frac{\bar{\delta}^e\left[f(k) - k\right] + \left[f'(k) - 1\right]\left(\frac{dk^e}{d\varepsilon} - \frac{dk^b}{d\varepsilon}\right) + \bar{\delta}^b\Delta^m(a^e_m)}{\bar{\delta}^b + \bar{\delta}^e}. \tag{78}$$

The terms $dk^e/d\varepsilon$ and $dk^b/d\varepsilon$ are obtained by differentiating (74)-(77) in the neighborhood of $\varepsilon = 0$,

$$\frac{dk^e}{d\varepsilon} - \frac{dk^b}{d\varepsilon} = \frac{\bar{\delta}^e\left[f(k) - k - R^e\right]}{1 - \chi_b f'(k)}. \tag{79}$$

Substituting (79) into (78) and replacing R^e by $(1-\chi_b)f(k) - a^e_m$, we get

$$\left(\frac{\bar{\delta}^b}{\bar{\delta}^e}\right)\frac{1 - \chi_b f'(k)}{(1-\chi_b)f'(k)} = \frac{\chi_b f(k) - k + a^e_m}{(1-\chi_b)f(k) - a^e_m - \Delta^m(a^e_m)}. \tag{80}$$

This corresponds to the FOC from Nash bargaining with $\theta = \bar{\delta}^b/(\bar{\delta}^e + \bar{\delta}^b)$. As usual, subgame perfect equilibrium in the game generates the same outcome as Nash bargaining.

A6. Structure of interest rates

We extend our model to characterize the structure of rates of return across different assets. First, we assume that one-period government bonds are partially liquid in the following sense: an investor holding a_g^e units of bonds in stage 1 can trade a fraction $\chi_g \in [0,1]$ in exchange for k. Second, in order to make bank loans comparable to one-period bonds we assume that they are repaid after one period. More precisely, investment opportunities in stage 1 of period t generate output $f(k)$ in stage 2 of period $t+1$ and bank loans offered in t are repaid at the time when investment pays off. Banks who issue IOUs in period t can commit to redeem them in stage 2 of period t. Because there is now a mismatch between the maturity of banks' liabilities and the maturity of the loans we allow banks to produce the numéraire at a unit cost (alternatively, we could assume banks are large entities with a large number of loans and liabilities, or that banks' IOUs are repaid in $t+1$).

The surplus of an unbanked entrepreneur is

$$\Delta^m(a_m^e + \chi_g a_g^e) = \beta f(k^m) - k^m \text{ where } k^m = \min\{a_m^e + \chi_g a_g^e, k^*\},$$

where $f'(k^*) = \beta^{-1} = 1+\rho$. In contrast to the formulation in the main text, output is product with a one-period lag, hence the discounting. Note that the liquid assets of the entrepreneur are composed of real balances, a_m^e, and the pledgeable bonds, $\chi_g a_g^e$. The bargaining problem with the bank is

$$\max_{k,d,\phi} \left[\beta f(k) - d - \beta\Phi - \Delta^m(a_m^e + \chi_g a_g^e)\right]^{1-\theta} \left[-(k-d) + \beta\Phi\right]^{\theta}$$
$$\text{st } \Phi \leq \chi_b f(k) \text{ and } d \leq a_m^e + \chi_g a_g^e,$$

where $\Phi = k - d + \phi$ is the sum of the principal and interest payments paid by the entrepreneur to the bank in $t+1$. The surplus of the bank is the difference between the IOU it must repay in t, $k-d$, and the discounted value of the repayment by the entrepreneur in $t+1$, $\beta\Phi$. The downpayment can now be composed of real balances and bonds and it cannot exceed the liquid wealth $a_m^e + \chi_g a_g^e$. If the liquidity constraint does not bind, $k^c = k^*$ and

$$\Phi = \frac{(k^* - k^m) + \theta\left[\beta f(k^*) - k^* - \Delta^m(a_m^e + \chi_g a_g^e)\right]}{\beta}.$$

The discounted payment to the bank, $\beta\Phi$, is the sum of the repayment of the loan and a fraction θ of the surplus generated by the bank loan. The liquidity constraint does not bind if

$$a_m^e + \chi_g a_g^e + \theta\Delta^m(a_m^e + \chi_g a_g^e) \geq (1-\theta)k^* + (\theta - \chi_b)\beta f(k^*). \tag{81}$$

Using the definition of Φ, interest payments are equal to

$$\phi = \rho(k^* - k^m) + \frac{\theta\left[\beta f(k^*) - k^* - \Delta^m(a_m^e + \chi_g a_g^e)\right]}{\beta}.$$

The first term is the interest payment if banks had no bargaining power. In that case, the real rate of return of the loan would be equal to the rate of time preference. The second term is a rent that the bank can extract given its bargaining power. Dividing by the loan size, $k^* - k^m$, the real lending rate is

$$r_b = \rho + \frac{\theta\left[\beta f(k^*) - k^* - \Delta^m(a_m^e + \chi_g a_g^e)\right]}{\beta(k^* - k^m)}. \tag{82}$$

So the real lending rate is the sum of the rate of time preference and an intermediation premium that depends on banks' bargaining power.

Suppose next that the liquidity constraint does bind. The solution to the bargaining problem is

$$\begin{aligned} \frac{\chi_b\beta f(k) - k + a_m^e + \chi_g a_g^e}{(1-\chi_b)\beta f(k) - (a_m^e + \chi_g a_g^e) - \Delta^m(a_m^e + \chi_g a_g^e)} &= \frac{\theta}{1-\theta}\frac{1-\chi_b\beta f'(k)}{(1-\chi_b)\beta f'(k)} \\ \Phi &= \chi_b f(k). \end{aligned}$$

These equations are analogous to the ones in the main text where the production function is scaled by β and a_m^e is replaced with $a_m^e + \chi_g a_g^e$. In this case the real lending rate is

$$r_b = \frac{\chi_b f(k)}{k - (a_m^e + \chi_g a_g^e)} - 1.$$

From (2) the entrepreneur's choice of money balances and bond holdings solves

$$\max_{a_m^e, a_g^e \geq 0} \left\{ -i a_m^e - \left(\frac{i - i_g}{1 + i_g}\right) a_g^e + \lambda(1-\alpha)\Delta^m(a_m^e + \chi_g a_g^e) + \alpha\lambda\Delta^c(a_m^e + \chi_g a_g^e) \right\}, \tag{83}$$

where

$$\Delta^c(a_m^e) = \begin{cases} (1-\theta)\left[\beta f(k^*) - k^*\right] + \theta \Delta^m(a_m^e + \chi_g a_g^e) & \text{if } \ a_m^e + \chi_g a_g^e \geq a^* \\ (1-\chi_b)\beta f(k^c) - \left(a_m^e + \chi_g a_g^e\right) & \text{if } a_m^e + \chi_g a_g^e < a^*, \end{cases}$$

where a^* is the value of $a_m^e + \chi_g a_g^e$ such that (81) holds at equality.

In order to get closed form expressions, consider the regime where the liquidity constraint does not bind. Using a second-order approximation for $\Delta^m(a_m^e + \chi_g a_g^e)$,

$$\Delta^m(a_m^e + \chi_g a_g^e) \approx \beta f(k^*) - k^* + \beta f''(k^*) \frac{(k^m - k^*)^2}{2}.$$

Plug this expression into (82) to obtain

$$r_b \approx \rho - \frac{\theta f''(k^*)(k^* - k^m)}{2}. \tag{84}$$

Assuming interior solutions the FOC from (83) gives

$$\lambda\left[1 - \alpha(1-\theta)\right]\left[\beta f'(k^m) - 1\right] = i = \frac{i - i_g}{\chi_g\,(1 + i_g)}$$

In order to guarantee that the solution is interior we would have to check that $\chi_g a_g^e < k^m$. It follows that the spread between illiquid and liquid bonds is

$$\frac{i - i_g}{1 + i_g} = \frac{\rho - r_g}{1 + r_g} = \chi_g i.$$

Using a first-order approximation of the LHS,

$$k^m - k^* \approx \frac{i}{\beta f''(k^*)\lambda\left[1 - \alpha(1-\theta)\right]}. \tag{85}$$

Substitute $k^m - k^*$ from (85) into (84), the lending rate can be approximated as:

$$r_b \approx \rho + (1+\rho)\frac{\theta i}{2\lambda\left[1 - \alpha(1-\theta)\right]}. \tag{86}$$

If $\rho = 0$, the expression for r_b corresponds to the one in the text. Alternatively, the yield difference between a bank loan and a risk-free (illiquid) bond is

$$\frac{r_b - \rho}{1+\rho} \approx \frac{\theta i}{2\lambda\left[1 - \alpha(1-\theta)\right]}. \tag{87}$$

A7. Long-lived investment projects

In the main text, investment projects are short-lived: investment opportunities in the first stage have a single pay-off in the second stage. In many macroeconomic applications investment opportunities have long-lasting payoffs, e.g., firms in the Pissarides or Melitz models or Lucas trees. Suppose that entrepreneurs can create long-lived assets (akin to Lucas trees) that generate a payoff $f(k)$ every period that depends on the initial investment, k. (The investment is putty-clay.) Those assets fully depreciate at the end of a period with probability δ. (One can think of it as the death rate of a firm/job.) The discounted sum of the output flows generated by this investment project is:

$$F(k) = \frac{f(k)}{1-(1-\delta)\beta}.$$

The benchmark version of our model corresponds to the case $\delta = 1$.

For simplicity we set $\chi_s = 0$. We consider a lending contract composed of an investment size, k, an initial down payment, d, and a payment to the bank every period, Φ. The per-period payment to the bank is subject to the pledgeability constraint, $\Phi \leq \chi_b f(k)$. Equivalently, we can write the liquidity constraint as:

$$\frac{\Phi}{1-(1-\delta)\beta} \leq \chi_b F(k),$$

where the left side is the entrepreneur's debt expressed as the discounted sum of the payments to the bank and the right side is a fraction χ_b of the value of the investment project. The discounted sum of the banks' profits are:

$$-(k-d) + \frac{\Phi}{1-(1-\delta)\beta}.$$

The first term is the loan size, $k-d$. The second term is the discounted sum of the payments to the bank. If we denote $\phi = \Phi - [1-(1-\delta)\beta](k-d)$ the discounted sum of the bank's profits can be expressed as $\phi/[1-(1-\delta)\beta]$. The surplus of the entrepreneur from a bank loan is

$$F(k) - d - \frac{\Phi}{1-(1-\delta)\beta} - \Delta^m(a_m^e),$$

where $\Delta^m(a_m^e)$ is the surplus if the entrepreneur self-finances the investment. The first term is the value of the investment project, the second term is the down payment, and the third term corresponds to the interest payments to the bank. Using the definitions of ϕ and $F(k)$ we can reexpress this surplus as

$$\frac{f(k)-\phi}{1-(1-\delta)\beta}-k-\Delta^m(a_m^e).$$

The bargaining problem between the bank and entrepreneur becomes:

$$\max_{k,d,\phi}\left[\frac{f(k)-\phi}{1-(1-\delta)\beta}-k-\Delta^m(a_m^e)\right]^{1-\theta}\left[\frac{\phi}{1-(1-\delta)\beta}\right]^{\theta} \tag{88}$$

$$\text{s.t.}\quad \phi+\left[1-(1-\delta)\beta\right](k-d)\leq\chi_b f(k)\quad\text{and } d\leq a_m^e. \tag{89}$$

This bargaining problem coincides with the one in the main text when $\delta=1$. The disagreement point, $\Delta^m(a_m^e)$, is computed as before where $f(k)$ is replaced with $F(k)$, i.e.,

$$\Delta^m(a_m^e)=F(k^m)-k^m\quad\text{where}\quad k^m=\min\{a_m^e,k^*\},$$

where k^* is such that $F'(k^*)=1$. If the liquidity constraint does not bind, then the solution to (88)-(89) is

$$f'(k) = 1-(1-\delta)\beta \tag{90}$$

$$\phi = \theta\left\{f(k)-\left[1-(1-\delta)\beta\right]\left[k+\Delta^m(a_m^e)\right]\right\}. \tag{91}$$

Equation (90) equalizes the marginal product of capital with the rate of time preference adjusted by the depreciation rate. Equation (91) gives the flow payment that splits the match surplus. If the liquidity constraint does bind then the bargaining problem (88)-(89) reduces to:

$$\max_{k\geq 0}\left[\frac{(1-\chi_b)f(k)}{1-(1-\delta)\beta}-a_m^e-\Delta^m(a_m^e)\right]^{1-\theta}\left[\frac{\chi_b f(k)}{1-(1-\delta)\beta}-(k-a_m^e)\right]^{\theta}.$$

The FOC is:

$$\begin{aligned}\frac{\chi_b f(k)-\left[1-(1-\delta)\beta\right](k-a_m^e)}{(1-\chi_b)f(k)-\left[1-(1-\delta)\beta\right]\left[a_m^e+\Delta^m(a_m^e)\right]} &= \frac{\theta}{1-\theta}\frac{1-(1-\delta)\beta-\chi_b f'(k)}{(1-\chi_b)f'(k)}\\ \phi &= \chi_b f(k)-\left[1-(1-\delta)\beta\right](k-a_m^e).\end{aligned}$$

The comparative statics are similar to the ones in the benchmark model when $\delta=1$.

We compute the rate of return on the loan as

$$r_b = \frac{\Phi}{k-d} - \delta.$$

The first term corresponds to the interest payment as a frcation of the loan size, $k-d$. The second term is the probability at which the loan is terminated. It follows that

$$r_b = (1-\delta)(1-\beta) + \frac{\phi}{k-d}.$$

The second term on the RHS is the lending rate as computed in the main text. The first term on the RHS is the return necessary to compensate for the rate of time preference and the termination rate.

Assuming the pledgeability constraint does not bind, the entrepreneur's choice of money balances solves

$$\max_{a_m^e \geq 0} \left\{ -ia_m^e + \lambda(1-\alpha)\Delta^m(a_m^e) + \alpha\lambda\Delta^c(a_m^e) \right\},$$

where $\Delta^c(a_m^e) = (1-\theta)\left[F(k^*) - k^*\right] + \theta\Delta^m(a_m^e)$. The FOC is

$$\frac{i}{\lambda\left[1-\alpha(1-\theta)\right]} = \frac{f'(k)}{1-(1-\delta)\beta} - 1.$$

Using the same approximations as in the main text,

$$r_b \approx \frac{(1-\delta)\rho}{1+\rho} + \frac{(\rho+\delta)\theta i}{2(1+\rho)\lambda\left[1-\alpha(1-\theta)\right]}.$$

The first term is the frictionless rate while the second term is the premium arising from frictions in the credit market.

A8. Money, trade credit, and bank credit

We now let entrepreneurs accumulate real balances and use trade credit. The surplus of an unbanked entrepreneur with a_m^e real balances is now

$$\Delta^m(a_m^e; \chi_s) = \max\left\{f(k^m) - k^m\right\} \text{ st } k^m \leq a_m^e + \chi_s f(k^m).$$

It is easy to check that $k^m = k^*$ if $a_m^e \geq k^* - \chi_s f(k^*)$ and $k^m - \chi_s f(k^m) = a_m^e$ otherwise. Moreover, the marginal value of real balances is

$$\Delta^{m\prime}(a_m^e; \chi_s) = \frac{f'(k^m) - 1}{1 - \chi_s f'(k^m)} \text{ if } a_m^e < k^* - \chi_s f(k^*),$$

where we used that $\partial k^m / \partial a_m^e = 1/\left[1 - \chi_s f'(k^m)\right]$. Money has a multiplier effect on trade credit. An additional unit of real balances allows entrepreneurs to increase investment and hence pledgeable output, which in turn allows suppliers to offer bigger loans. Using a second-order Taylor series expansion for a_m^e close to $a_m^* = k^* - \chi_s f(k^*)$ so that k^m is close to k^*:

$$\begin{aligned}\Delta^m(a_m^e;\chi_s) &= \Delta^m(a_m^*;\chi_s) + \Delta^{m\prime\prime}(a_m^*;\chi_s)\frac{(a_m^* - a_m^e)^2}{2} \\ &= f(k^*) - k^* + \frac{f''(k^*)}{(1-\chi_s)^2}\frac{(a_m^* - a_m^e)^2}{2},\end{aligned}$$

where we used that $\Delta^{m\prime}(a_m^*;\chi_s) = 0$ in the first equality, i.e., a change in real balances only has a second-order effect on the entrepreneur's surplus when k^m is close to k^*. To obtain the second equality we used that

$$\begin{aligned}\Delta^{m\prime\prime}(a_m^e;\chi_s) &= \frac{f''(k^m)\left[1-\chi_s f'(k^m)\right] + \left[f'(k^m)-1\right]\chi_s f''(k^m)}{\left[1-\chi_s f'(k^m)\right]^2}\frac{\partial k^m}{\partial a_m^e} \\ &= \frac{f''(k^*)}{(1-\chi_s)^2}.\end{aligned}$$

Consider next an entrepreneur in contact with a bank, where loan contracts now specify an investment level k, a down payment d, and the bank's fee ϕ. If the loan negotiations are unsuccessful, the entrepreneur can purchase k with cash and trade credit. So his surplus from the bank loan is $f(k) - k - \phi - \Delta^m(a_m^e;\chi_s)$. Then the bargaining problem is

$$\max_{k,d,\phi}\left[f(k) - k - \phi - \Delta^m(a_m^e;\chi_s)\right]^{1-\theta}\phi^\theta \text{ st } k - d + \phi \le \chi_b f(k) \text{ and } d \le a_m^e.$$

This problem is formally equivalent to the one studied earlier where the threat point, $\Delta^m(a_m^e;\chi_s)$, has been generalized. Notice that the contract does not specify if part of the loan, $k - d$, is provided by suppliers since it would not affect payoffs.

We consider equilibria where i is small so that the liquidity constraints, $k-d+\phi \le \chi_b f(k)$ and $d \le a_m^e$, do not bind. Assuming an interior solution, the entrepreneur's money demand is given by:

$$\frac{i}{\lambda\left[1-\alpha(1-\theta)\right]} = \frac{f'(k^m)-1}{1-\chi_s f'(k^m)}.$$

An interior solution exists if χ_s is less than some threshold. A first-order approximation of the RHS gives

$$\frac{i}{\lambda\left[1-\alpha(1-\theta)\right]}=\frac{f''(k^*)}{\left(1-\chi_s\right)^2}(a_m^e-a_m^*). \tag{92}$$

The intermediation payment to the bank is

$$\phi=\theta\left[f(k^*)-k^*-\Delta^m(a_m^e;\chi_s)\right].$$

Using the approximations above,

$$\phi\approx\frac{-\theta\left[\left(1-\chi_s\right)i\right]^2}{2\lambda^2\left[1-\alpha(1-\theta)\right]^2 f''(k^*)}.$$

The availability of trade credit, $\chi_s>0$, reduces the pass through from i to ϕ.

We consider two alternative assumptions for the distribution of loans to the entrepreneur and hence the definition of the lending rate. Suppose first that the bank is offering the full loan of size $k^*-a_m^e$. The lending rate is then defined as:

$$r_b=\frac{\theta\left[f(k^*)-k^*-\Delta^m(a_m^e;\chi_s)\right]}{k^*-a_m^e}.$$

From the approximation for $\Delta^m(a_m^e)$ above and the fact that $k^*-a_m^e\approx k^*-a_m^*$, a change in i only has a second-order effect on r_b.

Alternatively, suppose that the supplier borrows $a_m^*-a_m^e$ from the bank and $k^*-a_m^*=\chi_s f(k^*)$ from the supplier. The lending rate is now defined as

$$r_b'=\frac{\theta\left[f(k^*)-k^*-\Delta^m(a_m^e;\chi_s)\right]}{k^*-\chi_s f(k^*)-a_m^e}$$

Plugging the approximation for $\Delta^m(a_m^e)$ into the expression for r_b' gives

$$\begin{aligned} r_b' &= \frac{-\theta f''(k^*)}{\left(1-\chi_s\right)^2}\frac{\left(a_m^*-a_m^e\right)}{2} \\ &= \frac{\theta i}{2\lambda\left[1-\alpha(1-\theta)\right]}. \end{aligned}$$

Better access to unintermediated credit, through a higher χ_s, does not affect the pass through from the policy rate to the lending rate.

A first-order approximation of $k^m-\chi_s f(k^m)$ in the neighborhood of k^* gives

$$k^m-\chi_s f(k^m)=a_m^*+\left(1-\chi_s\right)\left(k^m-k^*\right).$$

Hence, $a_m^e - a_m^* = (1 - \chi_s)\,(k^m - k^*)$. Substituting this expression into (92),

$$k^m = \frac{(1 - \chi_s)\, i}{\lambda\,[1 - \alpha(1 - \theta)]\, f''(k^*)} + k^*.$$

Investment becomes less responsive to changes in the policy rate as χ_s increases.