The Intertemporal Keynesian Cross

Adrien Auclert[*] Matthew Rognlie[†] Ludwig Straub[‡]

March 20, 2018

Abstract

We demonstrate the importance of intertemporal marginal propensities to consume (IMPCs)—the impulse response of consumption to income—in the transmission mechanism of shocks in macroeconomic models. By representing general equilibrium as an intertemporal Keynesian cross, we show that IMPCs are sufficient statistics for the determinacy of equilibrium outcomes and for the impulse response of output. Models with precautionary savings, borrowing constraints, and low liquidity can capture the shape of IMPCs in the data, while alternative models with complete markets or hand-to-mouth agents cannot. Calibrating our model to existing evidence on IMPCs, we find that the economy is determinate for a Taylor rule coefficient below 1, and that deficit-financed fiscal multipliers are well above 1 under most plausible degrees of monetary policy responsiveness—in sharp contrast with both representative agent and saver-spender models.

1 Introduction

Recent empirical work on consumption has revolutionized the study of monetary and fiscal policy by macroeconomists. In the data, marginal propensities to consume (MPCs) out of one-time income shocks are much larger than what standard representative agent models imply, yet most known results on the effects on monetary and fiscal policy are based on a representative agent construct. This has prompted a recent literature to revisit the standard answers using models that match the magnitude and the heterogeneity of MPCs in the data.

There exists several competing models of household behavior that can all be made consistent with the large average MPCs observed empirically. Two well-known examples are models built around the saver-spender allegory of Campbell and Mankiw (1989), where a fraction of the population lives hand to mouth, and models with precautionary savings and borrowing constraints

[*]Stanford University and NBER. Email: aauclert@stanford.edu.
[†]Northwestern University. Email: matthew.rognlie@northwestern.edu.
[‡]MIT. Email: straub@mit.edu.

with liquid assets as in Deaton (1991), Carroll (1997), and Zeldes (1989), or with illiquid assets as in Kaplan and Violante (2014). As a result, both classes of models have been widely used to study fiscal and monetary policy in recent years.[1]

While these models all reflect the existing MPC evidence, in this paper we argue that they are not all consistent with the evidence on *intertemporal marginal propensities to consume* (IMPCs), which describes the *impulse response* of consumption to income at various horizons. We show that under certain conditions, IMPCs are *sufficient statistics* for answers to general equilibrium macroeconomic questions. Therefore, the model delivers correct answers to these questions if *and only if* its IMPCs are aligned with those in the data. While previous work has emphasized the importance of MPCs as sufficient statistics for certain partial equilibrium questions (Auclert 2017, Auclert and Rognlie 2018), to the best of our knowledge, this is the first result establishing the precise sense in which MPCs matter for general equilibrium. This result has significant implications for empirical work on consumption, and for theoretical work developing business cycle models with heterogeneous agents.

We make the case for the importance of IMPCs in the context of a simple heterogeneous agent New Keynesian model. We focus on fiscal policy, and consider changes in the time paths of government spending or taxes, which we write as vectors $d\mathbf{G}$ and $d\mathbf{T}$. We postulate a homothetic retention function and labor demand schedule, and assume that monetary policy follows a rule that maintains a constant real interest rate. Under these natural benchmark assumptions, we show that a general equilibrium impulse response of output $d\mathbf{Y}$ to any fiscal shock $(d\mathbf{G}, d\mathbf{T})$ that satisfies the government's intertemporal budget constraint is a solution to the recursive equation

$$d\mathbf{Y} = -\mathbf{M}d\mathbf{T} + d\mathbf{G} + \mathbf{M}d\mathbf{Y} \tag{1}$$

where $\mathbf{M}$ is a matrix whose entries are average IMPCs. Hence, $\mathbf{M}$ is all we need to know to solve for this impulse response. Other aspects of the data, such as household preferences, the nature of constraints they face, the distributions of income and wealth, or the comovement between consumption, output and government spending, are irrelevant conditional on $\mathbf{M}$.

Equation (1) is reminiscent of a staple of macroeconomics known as the Keynesian cross, which played an important role in macroeconomics before the advent of modern microfounded models, and is still widely used in policy circles today. The Keynesian cross is a fixed point equation that postulates that the output effect in a period dY of a change in fiscal policy dG or taxes dT in the same period solves the equation

$$dY = -MPC \cdot dT + dG + MPC \cdot dY \tag{2}$$

where MPC is "the" marginal propensity to consume. The intuition stems from the feedback

[1] Iacoviello (2005), Galí, López-Salido and Vallés (2007), Bilbiie (2008) and Eggertsson and Krugman (2012) study monetary or fiscal policy with saver-spender models. Guerrieri and Lorenzoni (2017), McKay and Reis (2016), McKay, Nakamura and Steinsson (2016) and Kaplan, Moll and Violante (2018) study the same issues in models of precautionary savings with or without illiquidity.

between income and consumption. In a closed economy, assuming no investment response to the fiscal change, the change in GDP must equal the sum of changes in consumption and government spending $dY = dC + dG$, and this is also the change in aggregate income. But consumption responds to changes in income and taxes according to the MPC, $dC = MPC\,(-dT + dY)$. Hence the fixed point equation (2), from which one can solve for the output effect of the fiscal shock using only information on the MPC.

Equation (1) is a generalization of the Keynesian cross (1) that involves impulse responses to output rather than static effects, and allows for an evaluation of the effect of complex government spending plans and tax plans that can be financed with goverment borrowing, as they often are in practice. Its sufficient statistics are a *set* of IMPCs rather than a static MPC. We therefore call it the *intertemporal Keynesian cross*, or IKC for short. While we focus most of the paper on the analysis of fiscal policy for concreteness, we show in section 8.2 that a similar equation involving $\mathbf{M}$ allows to solve for the general equilibrium effect of many other shocks. In all of these cases, the *same* set of IMPCs enter as sufficient statistics for the output impulse response.

Motivated by this result, we examine the existing IMPC evidence and confront it with predictions from a battery of dynamic consumption models. There exists some limited evidence on the dynamics of consumption responses to increases in income. The best evidence we are aware of—which uses a well-identified empirical strategy as well as comprehensive information on household balance sheets, allowing researchers to back out a precise measure of consumption—comes from the Norwegian lottery data analyzed in Fagereng, Holm and Natvik (2016). Their evidence suggests that the annual MPC out of an unexpected lottery gain is around 0.5 in the first year, dropping off sharply to about 0.16 in the second year, and decaying slowly thereafter. As stressed by previous literature, a representative agent model cannot match the large impact MPC of 0.5. We show that the data also rejects saver-spender models, since they imply negligible effects on consumption after year 1. A representative agent model with bonds in the utility can come close, but implies a pattern of decay in IMPCs that is too slow relative to the data. Models with precautionary savings and borrowing constraints calibrated to standard aggregate macroeconomic targets such as the liquidity-to-GDP ratio also fall short of the overall level of IMPCs. However, when we calibrate the aggregate liquidity level so as to minimize the distance between model and data IMPCs, we obtain a good fit to the full path of IMPCs in the data. Since IMPCs are all that matter for the question we posed, we use this low-liquidity model as our benchmark calibration in the rest of the paper.[2]

Before turning to the evaluation of fiscal policy, we discuss how to solve the IKC (1). This is not straightforward, because of the potential for multiple equilibria that plagues all dynamic general equilibrium macroeconomic models with nominal rigidities. In our context, multiplicity corre-

[2] Hence, in the "TANK vs HANK" debate in the literature over whether saver-spender models or models with more complex heterogeneity should be used for monetary-fiscal analysis (see Debortoli and Galí 2017, Bilbiie 2017, or Kaplan and Violante 2018), our results favor HANK models, but only those with a calibration to low levels of liquidity. This is close to, but not quite as extreme as the zero-liquidity limits of Ravn and Sterk (2017a), Werning (2015), or Ravn and Sterk (2017b), which the IMPC data also rejects.

sponds to output impulse response vectors $\mathbf{v}$ that satisfy the equation $\mathbf{M}\mathbf{v} = \mathbf{v}$. This multiplicity has a natural interpretation as a self-fulfilling change in economic activity. By interpreting $\mathbf{M}$ as the transition matrix of an infinite-dimensional Markov chain, we provide a simple condition for determinacy based on the limit behavior of IMPCs. Hence IMPCs are sufficient statistics for equilibrium determinacy, as well as for impulse responses to shocks. To the best of our knowledge, our paper provides the first general determinacy condition for a model with an infinite dimensional state space. We then provide the solution(s) to equation (1), with or without determinacy. Applying our results, we find that our benchmark calibration is determinate under constant real interest rates, in contrast to the representative agent model.

We next turn to our main application, which is the evaluation of fiscal policy shocks. We start by providing a useful benchmark result: when the government changes its spending plan by raising taxes at the same time ($d\mathbf{G} = d\mathbf{T}$), then the fiscal multiplier is exactly 1 ($d\mathbf{Y} = d\mathbf{G} = d\mathbf{T}$). This provides a significant generalization of Woodford (2011)'s results on the government expenditure multiplier for a model with heterogeneous agents where Ricardian equivalence fails. This result implies that IMPCs matter for fiscal policy because they determine the output effect of changes in fiscal policy that involve primary deficits—such as increases in government spending paid for by taxes in the future, or transfers to households offset by higher future taxes. For these changes, we find constant-r fiscal multipliers that increase rapidly in the degree to which taxes are delayed, with multipliers that can be above 3 if the half-life of government debt is above 10 years. The key are IMPCs: in the data, agents react little to changes in income taking place more than 5 years in the future. When government spending increases but taxes are delayed for a long time, higher government spending increases consumption for a number of years, which in turn increases incomes and therefore consumption, and so on. While our results on large multipliers are consistent with those on the theoretical literature focusing on the zero lower bound (see for example Christiano, Eichenbaum and Rebelo 2011 or Farhi and Werning 2016), our findings on the importance of fiscal deficits for the multipliers, and the relevance of IMPCs to study this effect, is new to the literature.[3]

We study a number of extensions of our framework. First, we consider endogenous monetary policy responses, deviating from our benchmark of constant real interest rates. The most substantive economic change is that our IKC equation (1) now involves consumption responses to real interest rates in addition to consumption responses to income. Adapting our determinacy results, we provide a new Taylor principle ensuring determinacy for heterogeneous agent economies. In our benchmark calibration, it is enough that the coefficient of response of nominal interest rates to inflation ϕ_π be greater than 0.7 for the economy to be determinate—a significant constrast to the well-known representative agent threshold of 1 (Bullard and Mitra 2002, Woodford 2003). While our fiscal multipliers are dampened by the presence of endogenous Taylor rule responses, our main result regarding the importance of deficit finance survives, and our multipliers remain

[3] In parallel and independent work, Hagedorn, Manovskii and Mitman (2017) also study fiscal multipliers in a heterogeneous agent model and find larger multipliers for deficit-financed fiscal policy.

above 1 even with Taylor rule response coefficients well above 3 under a realistic degree of price rigidity. In other extensions, we study cases where the economy's income incidence departs from our benchmark assumption, consider alternative shocks beyond fiscal shocks, and add investment to the model.

Our results have significant implications for empirical work on consumption, and for theoretical work developing business-cycle models with heterogeneous agents. On the empirical side, it suggests an important role for measuring the impulse response of consumption to income rather than MPCs alone; both for unexpected income shocks (for which some empirical evidence exists) as well as expected transfers in the future (for which there is essentially none). Our results on endogenous monetary policy also call for empirical work measuring dynamic consumption responses to real interest rate changes. Just as the literature has used evidence on MPCs as a model selection device to move away from the representative agent model, we expect IMPC evidence to help discriminate among existing models going forward. On the theoretical side, our results show the importance of developing macroeconomic models that match the IMPC evidence while maintaining consistency with other macroeconomic targets. While our benchmark calibration does achieve a good fit to the IMPC, it comes at the cost of missing on the wealth to GDP ratio. While this may not be a worry within the class of questions for which IMPCs are strict sufficient statistics, such as fiscal multipliers under constant real interest rates that we focus on for much of the paper, it may become more problematic as we extrapolate the model out to consider situations in which the real interest rate adjusts, or the incidence of income is more complex than in our benchmark model.

The rest of the paper is organized as follows. Section 2 presents our baseline model, for which we derive the intertemporal Keynesian cross equation (1) in section 3. Section 4 confronts our model as well as a battery of other ones to existing IMPC evidence and presents our benchmark calibraiton. Section 5 presents model determinacy conditions and the solution to the IKC. Section 6 contains our main results on fiscal multipliers. We cover endogenous monetary policy in section 7 and other extensions in section 8. Section 9 concludes.

2 Baseline model

In this section, we introduce our baseline model for the study of fiscal policy. We keep the model as simple and close to the Heterogeneous-Agent New Keynesian literature as possible. Our assumptions on income incidence and fiscal-monetary rules help us derive a very simple intertemporal Keynesian cross representation of equilibrium; we relax these assumptions and consider more general shocks in later sections.

Model setup. Time is discrete and runs from $t = 0$ to ∞. The economy is populated by a continuum measure 1 of ex-ante identical agents who face no aggregate uncertainty, but may face idiosyncratic uncertainty. Individuals can be in various idiosyncratic ability states e_i, and transition between those states according to a Markov process with fixed transition matrix Π. We assume that the mass of worker type i in idiosycratic state e_i is always equal to $\pi(e_i)$, the probability of e_i in the stationary distribution of Π. Ability levels are normalized to be one on average, so that $\mathbb{E}_e[e] = \sum_{e_i} \pi(e_i) e_i = 1$.

Agents. Agents have time-0 utility over consumption and labor supply plans given by separable preferences

$$\mathbb{E}\left[\sum_{t\geq 0} \beta^t \left\{u(c_{it}) - v(n_{it})\right\}\right] \tag{3}$$

Each period t, an agent with incoming real wealth a_{it-1} and realized earnings ability e_{it} enjoys the consumption of a generic consumption good c_{it} and gets disutility from working n_{it} hours. Consumption goods have price p_t and the nominal wage per unit of ability is w_t. The agent faces a loglinear retention function on pretax labor income as in Bénabou (2000), can trade in real bonds that deliver a net real interest rate r_t, and faces a borrowing constraint. Specifically, his budget constraint in period t in units of date-t consumption goods is

$$c_{it} + a_{it} = (1 + r_t) a_{it-1} + \tau_t \left(\frac{w_t}{p_t} e_{it} n_{it}\right)^{1-\lambda} \quad \forall t, i \tag{4}$$

$$a_{it} \geq \underline{a} \tag{5}$$

The agent maximizes (3) by choice of c_{it} and a_{it}, subject to (4) and the borrowing constraint (5). By contrast, due to frictions in the labor market, the agent is restricted to supply n_{it} hours. Hence the agent takes total net of tax income $z_{it} \equiv \tau_t \left(\frac{w_t}{p_t} e_{it} n_{it}\right)^{1-\lambda}$ as given. From the household's perspective, therefore, equation (4) simplifies to

$$c_{it} + a_{it} = (1 + r_t) a_{it-1} + z_{it} \quad \forall t, i \tag{4'}$$

Maximization of (3) subject to (4') and (5) constitutes the core of the standard incomplete market model of consumption and savings.

Labor market. Following standard practice in the New Keynesian sticky-wage literature, labor hours n_{it} are determined by aggregate union labor demand. The union allocates hours among heterogeneous households as a function of aggregate employment. In this section, we assume that this allocation is uniform: when labor demand is l_t, households must supply

$$n_{it} = l_t \tag{6}$$

Hence skill-weighted hours worked $\mathbb{E}_I[e_{it} n_{it}]$ are always equal to aggregate labor demand l_t. Combined with our assumption on the retention function, this *equal incidence* assumption implies that all macroeconomic changes affect households net-of-tax incomes z_{it} in proportion, irrespective of their skill level e_{it}. This is a natural benchmark case to study. We explore how our results change when we relax it in section 8.1.

Households are off their labor supply curves because the nominal wage w_t is partially sticky. Optimization by unions given previous-period wages w_{t-1} implies a value for current-period wages w_t. Under our monetary policy rule, the exact wage adjustment rule is irrelevant for aggregate outcomes, so we defer discussion of it to section 7.

Production of final goods. Firms operate a simple, time-invariant technology

$$y_t = l_t \tag{7}$$

They are perfectly competitive and set prices flexibly, so that the final goods price is given by

$$p_t = w_t \tag{8}$$

and profits are 0, justifying why no dividends enter households' budget constraints in (4). Hence the real wage per efficient hour is constant and equal to $\frac{w_t}{p_t} = 1$, and goods price inflation and wage inflation are equal at all times, $\frac{p_{t+1}}{p_t} = \frac{w_{t+1}}{w_t}$.

Government. The government sets an exogenous plan for spending g_t and tax revenue t_t. Assuming initial government debt b_{-1}, the sequences $\{g_t, t_t\}$ must satisfy the intertemporal budget constraint

$$(1+r_{-1})b_{-1} + \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1+r_s} \right) g_t = \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1+r_s} \right) t_t \tag{9}$$

In each period $t \geq 0$, the government implements this plan by issuing or retiring debt as needed. Its outstanding debt at the end of period t is

$$b_t = (1+r_{t-1})\, b_{t-1} + g_t - t_t \tag{10}$$

The government also adjusts the coefficient τ_t on the labor income retention function such that aggregate tax revenue equals t_t, ie

$$\mathbb{E}_I \left[e_{it} n_{it} - \tau_t \left(e_{it} n_{it} \right)^{1-\lambda} \right] = t_t \tag{11}$$

Monetary policy sets the nominal interest rate i_t by following a simple rule. Given the path for goods prices p_t, the real interest rate at t (the price of date-$t+1$ goods in units of date-t goods) is equal to

$$1 + r_t \equiv (1+i_t)\, \frac{p_t}{p_{t+1}} \tag{12}$$

In this section, we consider a rule in which the real interest rate is a strictly positive constant

$$r_t = \overline{r} > 0 \tag{Constant-r}$$

Such a rule has proved useful in the equilibrium analysis of both representative- and heterogeneous-agent New Keynesian models,[4] and we will show that, in our context, it facilitates a simple Intertemporal Keynesian Cross representation. In section 7 we replace (Constant-r) by a Taylor rule. The key difference will be that household consumption responses to interest rates will matter alongside consumption responses to income.

Definition 1. Given an initial nominal wage w_{-1}, initial government debt b_{-1}, an initial distribution $\Psi_{-1}(a, e)$ over assets a and idiosyncratic states e, and exogenous sequences for fiscal policy $\{\overline{g_t}, t_t\}$ that satisfy the intertemporal budget constraint (9), a *general equilibrium* in this economy is a path for prices $\{p_t, w_t, r_t, i_t\}$, aggregates $\{y_t, l_t, c_t, b_t, g_t, t_t\}$, individual allocations rules $\{c_t(a, e), n_{it}\}$, and joint distributions over assets and productivity levels $\{\Psi_t(a, e)\}$, such that households optimize, unions optimize, firms optimize, monetary and fiscal policy follow their rules, and the goods and bond markets clear:

$$g_t + \int c_t(a, e)\, d\Psi_t(a, e) = y_t \tag{13}$$

$$\int a d\Psi_t(a, e) = b_t \tag{14}$$

This completes the description of our environment. Except in very simple cases, the model cannot be solved analytically and one has to resort to numerical simulations for solutions—the usual practice in the literature. However, in the next section, we show that we can characterize impulse responses from steady state in terms of simple averages of intertemporal marginal propensities to consume.

Calibration. For our numerical simulations, we stay as close as possible to a standard calibration for a Huggett model. We assume the economy is initially at a steady state. Households have constant CES utility over consumption $u(c) = \frac{c^{1-\nu^{-1}}}{1-\nu^{-1}}$ with a standard value of $\nu = \frac{1}{2}$. We set a steady-state value for output of $y = 1$ and a steady-state inflation rate of $\pi = 0$. The specific form of labor disutility $v(n)$ and the parameters governing union maximization are irrelevant for steady state outcomes.

Following the standard practice in the literature, we assume that gross income follows an AR(1) process, which we discretize using an 11 point Markov chain using the Rouwenhorst method. We use Floden and Lindé (2001)'s estimates of the persistence of US wage process, equal to 0.9136 yearly, and set variance of innovations such that the standard deviation of log gross earnings is equal to its US value of 0.92 as in Auclert and Rognlie (2018).[5] We calibrate the curvature pa-

[4] See for example Woodford (2011) or McKay et al. (2016),

[5] This is the same value as in Kaplan et al. (2018), and is higher than the value of 0.51 implied by Floden and Lindé

	Targets	IMPCs	Liquidity
Parameters	Description	Value	Value
ν	Elasticity of intertemporal substitution	0.5	—
β	Discount factor	0.75	0.89
r	Real interest rate	2%	—
π	Inflation rate	0%	—
b/y	Government debt to GDP	11%	140%
$\underline{a}/y$	Borrowing constraint	0	—
g/y	Government spending to GDP	18.9%	—
λ	Retention function curvature	0.181	—

Table 1: Calibrated parameters (external calibration)

rameter of the retention function to $\lambda = 0.181$ and the ratio of government spending to GDP to $g/y = 18.9\%$ following Heathcote, Storesletten and Violante (2017). Following McKay et al. (2016), we assume that households cannot borrow ($\underline{a} = 0$) and target an equilibrium real interest rate of $r = 2\%$.

We pursue two different strategies to calibrate the remaining parameters. In our *IMPC target* calibration, we calibrate the household discount factor β using the IMPC moment matching procedure described in section 4. Given β, we finally find the level of government liquidity b and the intercept of the retention function τ such that the asset market clears and the government budget constraint is satisfied. This delivers a liquidity to GDP ratio of b/y =11%. In our *liquidity calibration*, by contrast, we follow the literature and calibrate to an observed level of government debt, which we choose to be b/y =140% of output as in McKay et al. (2016). We then obtain τ residually from the government budget constraint, and finally calibrate β to hit our target for the steady state interest rate r. Table 1 summarizes our two calibrations.

3 Deriving the Intertemporal Keynesian Cross

3.1 Aggregate income and consumption function.

We start with a useful simplification of aggregate household behavior. Given (6), each agent has net of tax income $z_{it} = \tau_t (e_{it} l_t)^{1-\lambda}$. Aggregate net of tax income can then be written as

$$z_t \equiv \tau_t l_t^{1-\lambda} \underbrace{\mathbb{E}_I[e_{it}^{1-\lambda}]}_{\equiv E^{1-\lambda}} \tag{15}$$

and each z_{it} can be written in terms of the aggregate as $z_{it} = \frac{e_{it}^{1-\lambda}}{E^{1-\lambda}} z_t$.

If we substitute this into (4′), then conditional on the individual state (a_{it}, e_{it}), the only variables affecting the household problem are the aggregate sequences $\{z_t\}$ and $\{r_t\}$. Households'

(2001) to capture the "new view" of household idiosyncratic risk. Since our tax system is progressive, it effectively dampens the amount of idiosyncratic risk by a factor $1 - \lambda$.

optimal policy rules $c_t(a,e)$ are determined by these sequences, and aggregate consumption $c_s = \int c_s(a,e)\, d\Psi_s(a,e)$ can therefore be written as a function of them:

$$c_s = c_s(\{z_t\}, \{r_t\}) \tag{16}$$

which we call the **aggregate consumption function**. Substituting (15) into (11) and using $\mathbb{E}_I[e_{it}n_{it}] = l_t = y_t$, in equilibrium we also have

$$z_t = y_t - t_t \tag{17}$$

i.e. that aggregate net income z_t equals output minus tax revenue.

3.2 Exogenous-y partial equilibrium

Although the aggregate consumption function must be calculated numerically, (16) and (17) together offer a straightforward characterization of aggregate household behavior, which depends only on real interest rates and aggregate output net of taxes.

General equilibrium as described in definition 1, by contrast, is much more complicated, since it imposes market clearing in goods and assets each period. This motivates us to define a simpler partial equilibrium notion, which we call exogenous-y partial equilibrium, that drops these market clearing conditions and instead takes the path y as given.

Definition 2. Given exogenous sequences for fiscal policy $\{\overline{g_t}, t_t\}$ that satisfy the intertemporal budget constraint (9), and a path for output supply $\{y_t\}$, an ***exogenous-y partial equilibrium*** is a path for prices $\{p_t, w_t, r_t, i_t\}$, aggregates $\{y_t, l_t, c_t, b_t, g_t, t_t\}$, individual rules $\{c_t(a,e)\}$, and joint distributions over assets and productivity levels $\{\Psi_t(a,e)\}$, such that households optimize, unions optimize, firms optimize, and monetary and fiscal policy follow their rules. We define $y_t^{PE} \equiv c_t + g_t$ as the resulting path of output demand.

This shuts off many of the complex intertemporal feedbacks in the general equilibrium model, which operate through market clearing. Indeed, we can immediately derive the following.

Lemma 1. *There is a unique exogenous-y partial equilibrium corresponding to a given path for output supply $\{y_t\}$ and fiscal policy $\{g_t, t_t\}$. This defines a mapping $y_s^{PE}(\{y_t\};\{g_t, t_t\})$, which can be written in terms of the consumption function as*

$$y_s^{PE}(\{y_t\};\{g_t, t_t\}) = c_s(\{y_t - t_t\}, \{\bar{r}\}) + g_s \tag{18}$$

Partial equilibrium in definition 2 only differs from general equilibrium in definition 1 by omitting goods and asset market clearing, replacing them with exogenous y. By Walras' law, goods market clearing in each period is sufficient for asset market clearing, and we can therefore recover general equilibrium as a fixed point of partial equilibrium, in which output demand equals output supply.

Lemma 2. *A general equilibrium is an exogenous-y partial equilibrium that satisfies the additional requirement that goods markets clear at every point in time, that is, a path* $\{y_t\}$ *that solves the system of equations*

$$y_s = y_s^{PE}\left(\{y_t\}\,;\{g_t, t_t\}\right) \quad \forall s \tag{19}$$

Partial equilibrium effects of shocks. We will primarily study the effects of shocks to policy. Since a given shock may impact policy variables in many periods, it is convenient to define a one-dimensional policy shifter ϵ, and to write spending and tax revenue $g_t(\epsilon)$ and $t_t(\epsilon)$ as functions of this shifter. To simplify, we can then write partial equilibrium output demand as a function of only $\{y_t\}$ and ϵ:

$$y_s^{PE}(\{y_t\}, \epsilon) \equiv y_s^{PE}\left(\{y_t\}\,;\{g_t(\epsilon), t_t(\epsilon)\}\right)$$

We will be particularly interested in the effects of a policy shock starting from a steady state. Letting y denote steady-state output, and assuming that $\epsilon = 0$ corresponds to steady-state policy, we write

$$\partial y_t \equiv \frac{\partial y_t^{PE}\left(\{y\}, 0\right)}{\partial \epsilon} d\epsilon \tag{20}$$

for each t. We call $\{\partial y_t\}$ the **partial equilibrium effect** on output demand of the policy shifter $d\epsilon$.

Lemma 3. *The partial equilibrium effect of a fiscal policy shock starting from steady state is given by*

$$\partial y_s = -\left(\sum_{t=0}^{\infty} \frac{\partial c_s}{\partial z_t} dt_t\right) + dg_s \tag{21}$$

The first term in (21) is the effect of the changing tax sequence on consumption, filtered through the impact on net incomes entering into the consumption function. The second term is simply the direct effect of increasing spending.

Since fiscal policy sequences must obey (9), the net present value of the perturbation to taxes $\{dt_t\}$ must equal the net present value of the perturbation to government spending $\{dg_t\}$. Together with household budget constraints, which require that household incomes and consumption equal in net present value, this implies the following lemma.

Lemma 4. *The partial equilibrium effect* $\{\partial y_t\}$ *of any shock has zero present value, that is,*

$$\sum_{t=0}^{\infty} \frac{\partial y_t}{(1+r)^t} = 0 \tag{22}$$

This is an important feature of our policy shocks in partial equilibrium: they cannot lead to a change in the present value of aggregate spending, even though they can affect its path over time. An increase in government spending in one period will be offset, in present value terms, by a decrease in consumption demand in other periods resulting from the taxes that finance the spending.

3.3 General equilibrium and the intertemporal Keynesian cross

Define $y_t(\epsilon)$ to be a function mapping each shock ϵ to a general equilibrium output response. (If there are multiple equilibria for a given ϵ, this can be any differentiable selection.) Write $dy_t \equiv \frac{dy_t}{d\epsilon} d\epsilon$. We can now relate $\{dy_t\}$ to the partial equilibrium $\{\partial y_t\}$ from the previous section.

Totally differentiating (19), we obtain

$$dy_s = \partial y_s + \sum_{t=0}^{\infty} m_{s,t} dy_t \quad \forall s \tag{23}$$

where

$$m_{s,t} \equiv \frac{\partial y_s^{PE}(\{y\}, \epsilon)}{\partial y_t} = \frac{\partial c_s(\{y-t\}, r)}{\partial z_t} \tag{24}$$

For any given $\{\partial y_s\}$, a general equilibrium $\{dy_t\}$ is a solution to the infinite-dimensional linear system in (23).

We can prove the following analogue of lemma 4:

Lemma 5. *For any t, the present value of $m_{s,t}$ discounted to date t equals one:*

$$\sum_{s=0}^{\infty} \frac{m_{s,t}}{(1+r)^{s-t}} = 1 \tag{25}$$

As with lemma 23, this comes from the observation that all income earned in the economy is spent at some point in time. Incrementing y_t by one therefore also increases the date-t present value of goods demand across all periods by one.

The intertemporal Keynesian cross. Our main result, the intertemporal Keynesian cross, is an expression of equation (23) in vector form. Since lemmas 4 and 5 will prove to be important, it is convenient to rescale quantities so that they have especially simple vector interpretations. We therefore write $dY_t \equiv \frac{1}{(1+r)^t} dy_t$, $\partial Y_t \equiv \frac{1}{(1+r)^t} \partial y_t$, and $M_{s,t} \equiv \frac{1}{(1+r)^{t-s}} m_{s,t}$. All other capital letters will refer to present value concepts as well.

Define the vector $\partial \mathbf{Y} \equiv (\partial Y_0, \partial Y_1, \cdots)$ and the infinite-dimensional matrix $\mathbf{M} \equiv (M_{s,t})$. The system of equations (23) delivers our main proposition.

Proposition 1 (The intertemporal Keynesian cross.)**.** *To first order, for any shock, impulses to partial equilibrium output $\partial \mathbf{Y}$ and general equilibrium output $d\mathbf{Y}$ satisfy*

$$d\mathbf{Y} = \partial \mathbf{Y} + \mathbf{M} d\mathbf{Y} \tag{26}$$

The matrix $\mathbf{M}$ has entries

$$M_{s,t} = \frac{1}{(1+r)^{s-t}} \frac{\partial c_s}{\partial z_t} \tag{27}$$

where c_s is the aggregate consumption function (16), and $\mathbf{M}$ is column stochastic, $\mathbf{1}'\mathbf{M} = \mathbf{1}'$.

The vector $\partial\mathbf{Y}$ *is given by*

$$\partial\mathbf{Y} = -\mathbf{M}d\mathbf{T} + d\mathbf{G} \tag{28}$$

where $d\mathbf{T}$ *and* $d\mathbf{G}$ *are the shocks to tax revenue and government spending, respectively, and has mean zero,* $\mathbf{1}'\partial\mathbf{Y} = 0$.

This proposition separates the determination of general equilibrium output into two steps. First, there is the exogenous-y partial equilibrium effect $\partial\mathbf{Y}$, which is easy to obtain from (28). Then, given this $\partial\mathbf{Y}$, general equilibrium output $d\mathbf{Y}$ solves the fixed point in equation (26), the **intertemporal Keynesian cross** (**IKC**), whose structure is given by a matrix $\mathbf{M}$ that is independent of the shock. All the complexity of general equilibrium—which arises from the need to clear markets in every period—appears only in this second step.

$\mathbf{M}$ embodies a transmission mechanism from partial to general equilibrium that is common to all shocks. When (26) has a unique solution—as it does in our main calibration, and for which we will provide sufficient conditions on $\mathbf{M}$ in section 5—$\partial\mathbf{Y}$ is *sufficient* for $d\mathbf{Y}$: shocks that have the same partial equilibrium effect also have the same general equilibrium effect. Conceivably, an observer interested in $d\mathbf{Y}$ could ignore the details of a shock and retain only information on $\partial\mathbf{Y}$.[6]

We will refer to entries in $\mathbf{M}$ as **intertemporal marginal propensities to consume**, or **IMPC**s. They tell us, in present value terms, how much households will consume in period s out of a shock to aggregate net income in period t. The intertemporal Keynesian cross (26) tells us that an impulse to general equilibrium output $d\mathbf{Y}$ must equal the partial equilibrium impulse $\partial\mathbf{Y}$, plus the marginal consumption $\mathbf{M}d\mathbf{Y}$ out of itself. The name is inspired by the presence of a similar fixed point in the traditional Keynesian cross, which we will discuss further in the next section.

The intertemporal Keynesian cross (26) turns out to be quite general: we will show in section 8.2 that the same relationship between $d\mathbf{Y}$ and $\partial\mathbf{Y}$ holds for a variety of shocks, including shocks to household preferences, borrowing constraints, income distribution, and monetary policy. We continue to have $\mathbf{1}'\partial\mathbf{Y} = 0$ in all cases—in partial equilibrium, these shocks shift output demand intertemporally, but leave its present value unchanged. Crucially, the $\mathbf{M}$ matrix remains independent of the *shock*.

By contrast, the $\mathbf{M}$ matrix will change as we explore more complex specifications of *policy*, becoming a more complex function of IMPCs when we consider more general fiscal rules in section 6.3, and including household responses to real interest rates when we consider more general monetary rules in section 7. In all cases, $\mathbf{M}$ continues to be a stochastic matrix, and the intertemporal Keynesian cross (26) holds, allowing us to treat partial equilibrium $\partial\mathbf{Y}$ as sufficient for general equilibrium $d\mathbf{Y}$.

Finally, note that we did we not immediately solve (26) to find $d\mathbf{Y}$, even though it would be natural to try writing $d\mathbf{Y} = (\mathbf{I} - \mathbf{M})^{-1}\partial\mathbf{Y}$. This is because the inverse $(\mathbf{I} - \mathbf{M})^{-1}$, where $\mathbf{I}$ is the identity matrix, often does not exist: $\mathbf{1}'(\mathbf{I} - \mathbf{M}) = \mathbf{0}$ by the stochasticity of $\mathbf{M}$ in theorem 1, implying immediately that $\mathbf{I} - \mathbf{M}$ is non-invertible if the horizon was finite, and possibly that

[6] When (26) does not have a unique solution, this reflects an underlying multiplicity of general equilibria. $\partial\mathbf{Y}$ is still sufficient for the *set* of possible $d\mathbf{Y}$, but the specific $d\mathbf{Y}$ depends on the selection rule. We discuss more in section 5.

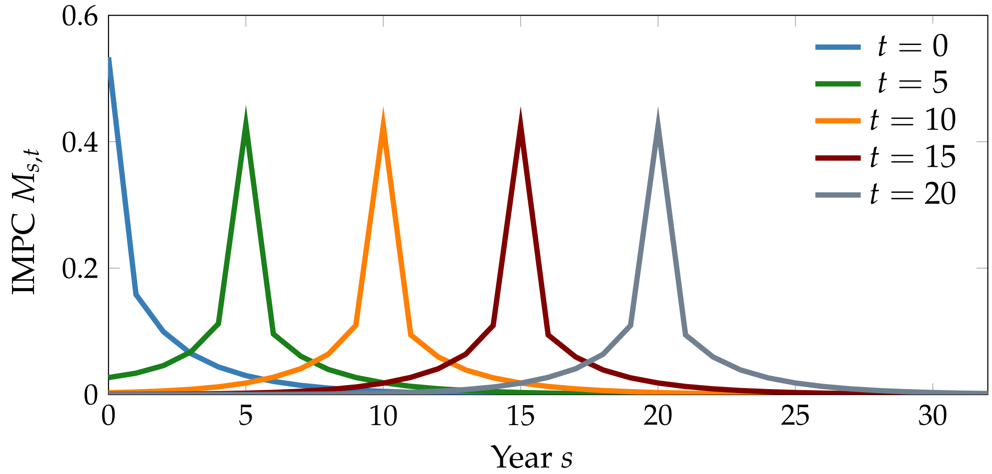


Figure 1: IMPCs in calibrated model

it is non-invertible in our infinite horizon case as well.[7] Fortunately, even when $\mathbf{I} - \mathbf{M}$ is non-invertible, there generally still exist solutions to (26), because $\mathbf{1}'\partial\mathbf{Y} = 0$ means that $\partial\mathbf{Y}$ can still be in the column space of $\mathbf{I} - \mathbf{M}$.

3.4 Properties of IMPCs

Figure 1 plots selected columns of the $\mathbf{M}$ matrix for our calibrated model: the IMPCs $M_{s,t}$ for $t = 0, 5, 10, 15, 20$. For each t, the plotted column corresponds to the impulse response of consumption to an increase in aggregate income at t, when both are in present value units.

Thanks to the incomplete markets and borrowing constraints facing households, the strongest consumption response is contemporaneous with the receipt of income, and then diminishes in both directions—less is spent from an impulse to income when that income is expected further in the future, or was received in the more distant past. In short, elements of $\mathbf{M}$ tend to be larger the closer they are to the main diagonal.

Two key features of the IMPCs in figure 1 turn out to be provably general to our model, not specific to this particular calibration. First, the IMPCs are all strictly positive: at the margin, an increase in aggregate income in one period leads to an increase in consumption in all periods. Second, the IMPCs $M_{s,t}$ become translation invariant as we take $s, t \to \infty$: they depend only on $s - t$. When t is close to 0, households do not have many periods to consume in anticipation of the income they will eventually receive; but when $t \to \infty$, households in the aggregate increase their consumption both before and after t in the same pattern. In matrix terms, this means that IMPCs $M_{s,t}$ for high s, t depend in the limit only on the distance from the main diagonal. We call

[7]This is much simpler in the finite-horizon case, because the usual results of finite-dimensional linear algebra apply: $\mathbf{1}'(\mathbf{I} - \mathbf{M}) = \mathbf{0}$ means that $\mathbf{M}$ is both rank-deficient and has a nontrivial null-space of vectors $\mathbf{v}$ such that $\mathbf{M}\mathbf{v} = \mathbf{v}$, which also gives us multiplicity. With infinite dimensions, this need not be the case (since, for instance, left and right eigenvalues need not coincide), and indeed we explore these issues at length in section 5, making use of the stochasticity of $\mathbf{M}$ to apply results on Markov chains.

this feature of $\mathbf{M}$ *self-similarity*.

Proposition 2. *The* $\mathbf{M}$ *matrix of IMPCs is* positive, *that is,* $M_{s,t} > 0$ *for all* $s, t \in \mathbb{N}$*, and* asymptotically self-similar, *that is, for any given* $t_0 \in \mathbb{N}$*, the sequence of IMPCs* $(M_{t+s,t})_{s \geq -t_0}$ *converges uniformly in* $t \in \{t_0, t_0 + 1, \ldots\}$ *as* $t \to \infty$*.*

Both features in proposition 2 are shared by many models with heterogeneity and incomplete markets, and will be convenient for our analytical results in section 5. Self-similarity is *not* present, however, in representative-agent models with unconstrained households. Indeed, in these models, all columns of $\mathbf{M}$ are identical, as the consumption response to income shocks of a certain present value is the same whether these shocks occur at $t = 0$ or $t = 1000$.

3.5 Relation to the static Keynesian cross and network interpretation

The traditional, static Keynesian cross is frequently used in discussions of policy. In contrast to the intertemporal Keynesian cross, it is a *scalar* fixed point equation relating the one-period impulse to output to itself. It can be derived by starting with the market-clearing condition $dY = dC + dG$, and then substituting $dC = MPC \cdot (dY - dT)$, where MPC is the one-period aggregate marginal propensity to consume out of income, and $dY - dT$ is the rise in net income after taxes. This can be written as

$$dY = (-MPC \cdot dT + dG) + MPC \cdot dY \tag{29}$$

(29) is identical to a scalar version of (26) and (28), with the $\mathbf{M}$ matrix of intertemporal MPCs replaced by a one-period MPC.[8] Since this is only a scalar equation, it is easy to solve (29) for dY and write

$$dY = \frac{1}{1 - MPC} \cdot (-MPC \cdot dT + dG) \tag{30}$$

In (30), the impact of a fiscal shock on output equals the direct effect on consumption and government output demand, $-MPC \cdot dT + dG$, times the "multiplier" $\frac{1}{1-MPC}$, which can equivalently be written as the infinite sum $1 + MPC + MPC^2 + \cdots$. This multiplier reflects the accumulation of feedback from output to consumption demand.

Although they have a similar structure, there are profound economic differences between the static Keynesian cross (29) and the intertemporal Keynesian cross (26). The static Keynesian cross is static: it ignores the impact of future taxes on demand, and it also ignores feedback from endogenous changes in output that occur in later periods. For instance, higher output today might lead to higher consumption demand and output tomorrow, leading in turn to a rise in anticipated income that boosts consumption demand today. The intertemporal Keynesian cross captures this feedback, while the static version does not.

Network interpretation. These feedbacks across time periods, missing in the static Keynesian cross, can be interpreted as taking place in an intertemporal *network*. Formally, we can view the in-

[8] Versions of (29) appear in most undergraduate macro textbooks, e.g. in chapter 10 of Mankiw (2010) or chapter 23 of Taylor and Weerapana (2011) .

tertemporal Keynesian cross (26) as an equation for flow equilibrium in a network; in the network, nodes are time periods t, and each node has some level dY_t of output demand.

The network exogenously sends a flow of ∂Y_t to each node t, with the sum of these flows across all t being zero. The network also endogenously sends a flow $M_{s,t}dY_t$ from each node t to each node s, and the sum of all these flows away from t is $\sum_{s=0}^{\infty} M_{s,t}dY_t = dY_t$. The network is in equilibrium when the net flow in and out of each node s is zero, i.e. when

$$\partial Y_s + \sum_{t=0}^{\infty} M_{s,t}dY_t - dY_s = 0$$

which is just a restatement of (26).

This interpretation of general equilibrium as the flow equilibrium of an intertemporal network, where flows are given by IMPCs, will prove convenient when deriving analytical properties of the solution in section 5.

Special case with the static Keynesian cross: the saver-spender model. The intertemporal Keynesian cross gives a more comprehensive picture of macroeconomic feedbacks because it is derived from a formal model, with optimizing agents that respect budget constraints. Nevertheless, it is worthwhile to note that a version of the static Keynesian cross *is* correct, period-by-period, for a popular simple model: the *saver-spender* model, where a share λ of households is hand-to-mouth and a share $1-\lambda$ is unconstrained, with the households being otherwise identical and facing no idiosyncratic shocks. For this model, described in more detail in appendix B.1, we have the following.

Lemma 6. *If the equilibrium with* $\lim_{t\to\infty} dY_t = 0$ *is selected, then in the saver-spender model the general equilibrium response to a fiscal shock is*

$$dY_t = (-\lambda \cdot dT_t + dG_t) + \lambda \cdot dY_t \tag{31}$$

for each t.

For each t, (31) is identical to the static Keynesian cross (29), except that MPC is replaced by the fraction λ of hand-to-mouth households. λ can be interpreted as the aggregate MPC in the saver-spender model out of an income increase in the current period that is offset by a decrease of the same present value in a later period.[9]

Hence the static Keynesian cross, despite its analytical shortcomings, is a valid depiction of equilibrium in the saver-spender model, where IMPCs have a special structure and the complexity of the intertemporal network disappears. As we will demonstrate in the next section, however, the IMPCs predicted by the saver-spender model have a very poor fit to the data—a mismatch we will show in section 6 to have large practical consequences for the analysis of fiscal policy.

[9] One example of such a shock to the path of incomes would be a deficit-financed tax cut, paid for by later tax increases. Note that λ is slightly lower than the aggregate MPC out of a one-time income shock, since the $1-\lambda$ unconstrained households have a small but strictly positive MPC.

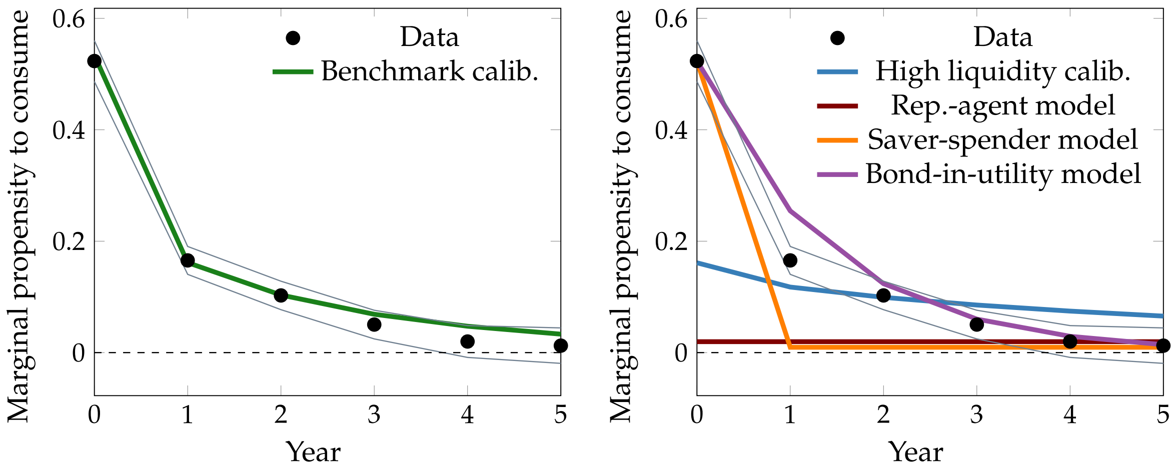


Figure 2: IMPCs in the data and several models

4 Evidence on IMPCs and model validation

A large empirical literature has sought to estimate marginal propensities to consume. Broadly speaking, authors have pursued two types of empirical strategies. One class of studies, follow the "reported preference" approach of surveying households and asking them directly how how they would spend unexpected one-time transfers (see Shapiro and Slemrod (2003), Jappelli and Pistaferri (2014), Fuster, Kaplan and Zafar (2018)). Another class of studies follow the "revealed preference" approach of estimating MPCs out transitory shocks from actual behavior, either by fitting a model that extracts the transitory component of income (Blundell, Pistaferri and Preston (2008)) or by using quasi-random variation in income induced by tax rebates (Johnson, Parker and Souleles (2006)) or in lotteries (Fagereng et al. (2016)).

Our results in section 3 show the importance of IMPCs as sufficient statistics for a range of macroeconomic questions, including the aggregate effects of fiscal policy. While there has so far been little direct work on IMPCs, existing MPC studies deliver some useful information. Our preferred evidence comes from the Norwegian data analyzed by Fagereng et al. (2016). The data includes comprehensive information on consumption and uses the random winnings of lotteries to identify the dynamic consumption responses to income shocks. The author's main estimating equation is:

$$C_{it} = \alpha_i + \tau_t + \sum_{t=0}^{5} \gamma_s \text{lottery}_{i,t-s} + \theta X_{it} + \epsilon_{it} \tag{32}$$

where C_{it} is consumption of individual i in year t, α_i an individual fixed effect, τ_t a time effect, X_{it} are household characteristics, and $\text{lottery}_{i,t}$ is the amount household i wins in year t. Since lottery winnings are not forecastable and are disbursed at the time they are announced, the estimated $\widehat{\gamma}_s$ correspond to our concept of date-0 IMPCs, as we discuss further below.

The black dots in figure 2 represent the point estimates for $\widehat{\gamma_0}$ through $\widehat{\gamma_5}$, together with 99% confidence intervals. Consistent with a large empirical literature, the annual MPC out of a one-time transfer is large, at about 0.52. What the literature has not stressed as much, but clearly appears in the Norwegian data, is that the MPC in year following the transfer is also fairly large, at around 0.17. The IMPCs then slowly decay and become statistically insignificant around year 4.

MPCs out of invididual vs aggregate income. The empirical moments from the data are informative about our general equilibrium sufficient statistics. Recall from section 3 that the first column of the M matrix is the response of aggregate consumption to an increase in aggregate income. In particular, we can write that column in current value terms as

$$m_{s,0} = \mathbb{E}\left[\frac{\partial c_{is}}{\partial z_{i0}} \frac{z_{i0}}{\mathbb{E}\left[z_{i0}\right]}\right]$$

which is a sum of individual IMPCs $\partial c_{is}/\partial z_{i0}$ out of an unanticipated income shock at date 0, weighted by net incomes z_{i0}. This is precisely what regression (32) estimates, when weighted with time t net incomes z_{i0}.[10] Thus, $\gamma_s = m_{s,0}$. Since our results show it is crucial to match the IMPCs $\widehat{\gamma}_s$, we calibrate the remaining parameter of our model, the household discount factor β, so as to minimize the least-square distance between model and data, in other words

$$\beta = \arg\min_{\widetilde{\beta}} \sum_{s=0}^{5} \left(m_{s,0}\left(\widetilde{\beta}\right) - \widehat{\gamma}_s\right)^2$$

This procedure allows us to recover the only missing parameter β from our calibration.

Comparison to alternative models. We now discuss the fit of our model relative to other models of consumption behavior that might be used as alternatives to evaluate fiscal policy. Our results show that the key distinguishing features of these models is the pattern of IMPCs they generate. We therefore organize our discussion around the fit of each model to the pattern of γ_s's in the data.

The first model we consider is the liquidity calibration of our baseline model, as summarized in the last column of table 1. The blue line in the right panel of figure 2 shows the pattern of IMPCs for this model. As is well-known, a standard calibration of a Huggett model cannot match the high MPCs in the data: here, the on-impact annual MPC is only 0.18. By contrast, our model matches both the MPC on impact as well as the pattern of IMPCs for later dates, but it is inconsistent with typical targets for aggregate liquidity, as table 1 illustrates. While our results show that matching the pattern of IMPCs is sufficient for studying fiscal policy under a policy of constant real interest rates, there are two downsides to our benchmark calibration. First, in the absence of IMPC data for future income, we have to rely on the extrapolations to later dates generated by the model (see figure 1). Second, when we consider interest rate adjustment in section 7, the relevant sufficient

[10] The current results for $\hat{\gamma}_s$ are unweighted but will be weighted in future versions of this paper.

statistics will include intertemporal consumption responses to real interest rates, for which even less empirical evidence is currently available.

Next, we turn to two standard models with no heterogeneity. The IMPCs of an infinitely-lived representative agent are those of the permanent income hypothesis—flat and low, and are therefore rejected by the data.

The so-called 'saver-spender models', popularized by Campbell and Mankiw (1989) and used for fiscal policy analysis by Galí et al. (2007) among many others, can be calibrated to fit the high levels of MPCs observed in the data. These models (sometimes also called TANK, or Two-Agent New Keynesian models) feature an exogenous fraction λ of agents that are hand to mouth and therefore have a marginal propensity to consume of 1.[11] We can then calibrate λ to hit a given MPC $\widehat{\gamma_0}$. As show in the orange line of figure 2, this model is inconsisent with the high empirical $\widehat{\gamma_1}$ and $\widehat{\gamma_2}$.

Finally, a recent branch of the New Keynesian literature has considered models with bonds in the utility (see Michaillat and Saez 2018). As we explain in appendix B.2, it turns out that these models generate a pattern of IMPCs that looks much like our model's dynamic response to income shocks. It is possible to calibrate the curvature of the wealth in the utility function to match a given level of initial MPCs. As we show in the appendix, such a model delivers an exponential for MPCs, contrary to what the data suggests.

Hence, we conclude that our benchmark calibration delivers the best fit to empirical IMPCs among this wide set of candidate models, although a bond in the utility model may come close, and we acknowledge that the model is used for extrapolation where the data is currently missing (consumption responses to future income shocks and consumption responses to real interest rates).[12] An important empirical agenda is to provide more of these empirical moments, so that we can afford to rely less on model extrapolations going forward.

5 IMPCs and determinacy

Before proceeding further with our model, it is useful to ask about determinacy. For monetary models with no or little household heterogeneity, there is a large theoretical and numerical literature studying the conditions under which there is local determinacy. These conditions can be stated in terms of the recursive state-space formulation of a problem—as, for instance, in Blanchard and Kahn (1980). In our model, by contrast, household-level idiosyncratic shocks imply an infinite-dimensional state space, and these results do not readily apply.[13]

[11] In the interest of space, we defer a detailed discussion of the TANK and the BIU model to appendices B.1 and B.2, respectively.

[12] Another candidate model that may be consistent with both IMPCs and standard macroeconomic targets is the illuquid asset model of Kaplan and Violante (2014). We leave an exploration of this model to future research.

[13] One possible approach is to discretize the infinite-dimensional state space and apply the usual tools on the resulting recursive system. With a fine discretization, however, the state-space formulation is very high-dimensional, and this approach only gives numerical answers without indicating the economic mechanisms, unlike our IMPC-based approach. It also requires that equilibrium is solved recursively in the first place—rather than by iterating over time paths,

Instead, we offer a simple test for local determinacy—defined as the uniqueness of equilibrium paths in the neighborhood of the steady state—based on IMPCs. The key measure is

$$\mu \equiv \lim_{t\to\infty} \sum_s m_{t+s,t} = \lim_{t\to\infty} \sum_s \left(\frac{1}{1+r}\right)^s M_{t+s,t} \tag{33}$$

the sum of the *current*-value IMPCs $m_{t+s,t}$ with respect to income at date t, in the limit of $t \to \infty$. In other words, if an aggregate income shock has been anticipated for an arbitrarily long period, μ tells us the cumulative consumption response to that shock. For our calibration, it is the area under the red impulse response in the left panel of figure 3.

Indeterminacy of equilibrium is equivalent to the existence of $\mathbf{v}$ that is bounded in current values and is a fixed point $\mathbf{M}\mathbf{v} = \mathbf{v}$ of $\mathbf{M}$. This is because the intertemporal Keynesian cross (26) reduces to $d\mathbf{Y} = \mathbf{M}d\mathbf{Y}$ in the absence of shocks, and if there is some nondegenerate solution $\mathbf{v} = d\mathbf{Y}$ with no shocks, it must correspond to another equilibrium. We have:

Proposition 3 (Local determinacy)**.** *If $\mu > 1$, general equilibrium is locally determinate around the steady state. If $\mu < 1$, there is local multiplicity of general equilibrium around the steady state, corresponding to a one-dimensional space of solutions $\mathbf{v}$ to $\mathbf{M}\mathbf{v} = \mathbf{v}$.*

Why does determinacy hinge on whether μ is greater or less than 1? Recall that $\mathbf{M}$ is stochastic, so that every column sums to 1: $\sum_s M_{t+s,t} = 1$. In *present*-value terms, the cumulative consumption response to an income shock is always equal to the shock. μ, by contrast, measures the cumulative *current*-value consumption response, which can be less or more than the income shock depending on the timing of consumption. μ will be greater than 1 when more of the consumption response takes place after the income is received, rather than in anticipation of it.

Multiplicity exists when there is some self-sustaining shock to output $d\mathbf{Y}$: when the rise in incomes associated with $d\mathbf{Y}$ causes an equal rise in consumption spending $d\mathbf{C} = d\mathbf{Y}$. It turns out that this cannot happen if consumption takes place predominantly after income is received for high t: if so, income received at t will be spent more at $t+1$, $t+2$, etc., which will be spent more at $t+2$, $t+3$, etc., and no self-sustaining income pattern is possible. Proposition 3 finds that $\mu > 1$ is the precise measure of this notion—spending after an income shock is received—that matters for determinacy.

This is visualized for our calibration in figure 3. In the left panel, the area under the black present-value impulse is exactly 1, while the area under the red current-value impulse is μ, in this case slightly greater than 1. It follows from proposition 3 that equilibrium is locally determinate for our benchmark. In the right panel, we vary the level of the liquidity b/y in the calibration (adjusting the discount factor β to maintain the same equilibrium real interest rate $r = 2\%$). We find local determinacy for the model at all levels of liquidity. In principle, adding liquidity could go in either direction—since it increases the scope for consuming with either a lead or a lag relative to an income shock—but in practice more liquidity means more of a lag, and μ rises.

for which we have an efficient algorithm, which we describe in our online appendix.

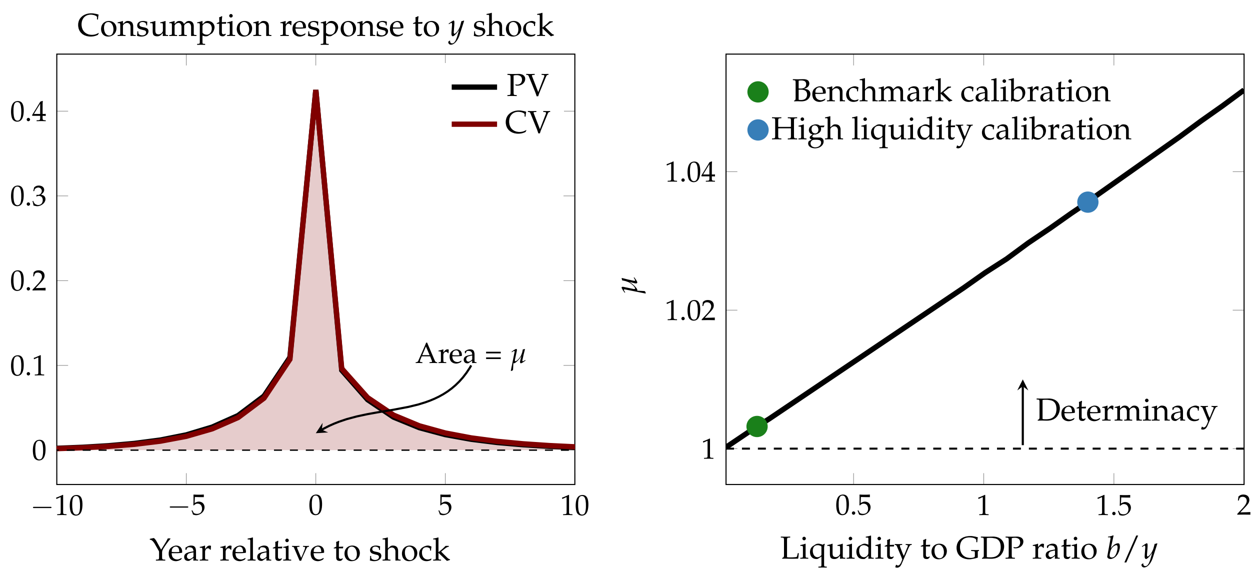


Figure 3: Determinacy in benchmark calibration and as we vary liquidity

Proposition 3 can be stated entirely in terms of the $\mathbf{M}$ matrix, and generalizes beyond the model in section 2. The key features of the $\mathbf{M}$ matrix for the proposition to apply are positivity and asymptotic self-similarity, proven for our case in proposition 2. The advantage of this approach is that it is clear how different model elements contribute to determinacy, based solely on how they alter the consumption impulse to a far-out shock to y.

Later in this paper, we will explore various ways in which perturbations to the model alter determinacy via this framework. For instance, in section 6.3 we show that with a more sophisticated fiscal rule, the consumption response to income is delayed and determinacy becomes even stronger; in section 7 we show that with endogenous inflation, we can get determinacy or indeterminacy depending on whether the interest rate response is strong enough to shift consumption into the future.

Solutions to the intertemporal Keynesian cross. The IKC characterizes the impulse response $d\mathbf{Y}$ to shocks with partial equilibrium effect $\partial\mathbf{Y}$. Where there is local indeterminacy around the steady state, corresponding to some current value bounded $\mathbf{v}$ such that $\mathbf{Mv} = \mathbf{v}$, there are also multiple solutions to the IKC (26). Each of these solutions corresponds to the impulse response under a particular (differentiable) equilibrium selection. The following theorem provides the structure of these solutions.

Theorem 1 (Solving the Intertemporal Keynesian Cross)**.** *In response to a shock with a partial equilibrium output effect of $\partial\mathbf{Y}$, the (local) general equilibrium output effect $d\mathbf{Y}$ solves the IKC (26) and is given by*

$$d\mathbf{Y} = \sum_{k=0}^{\infty} \mathbf{M}^k \partial\mathbf{Y} + \mathbf{v}, \tag{34}$$

where $\mathbf{v}$ *satisfies* $\mathbf{M}\mathbf{v} = \mathbf{v}$*.*

Conversely, any current-value bounded $d\mathbf{Y}$ *of the form in* (34) *solves the IKC and is the general equilibrium output response (for some selection, if there is multiplicity).*

If $\mu < 1$*, there is a multiplicity of* $d\mathbf{Y}$ *corresponding to a one-dimensional subspace of* $\mathbf{v}$*. If* $\mu > 1$*, there is a unique* $d\mathbf{Y}$*, corresponding to a unique* $\mathbf{v}$*. If additionally* $\lim_{t\to\infty}\sum_s sM_{t+s,t} > 0$*, this* $\mathbf{v}$ *is* $\mathbf{0}$*.*

Theorem 1 shows that general equilibrium output response consists of two summands, an infinite sum $\sum_{k=0}^{\infty}\mathbf{M}^k\partial\mathbf{Y}$ and a vector $\mathbf{v}$. The first is strikingly reminiscent of the solution (29) to the static Keynesian cross, which can be written as a multiplier $(1 + mpc + mpc^2 + \cdots)$ times the partial equilibrium effect of a shock. Replacing mpc with $\mathbf{M}$, however, brings in a much richer array of intertemporal feedbacks.

One intuition for this sum is that it can be obtained by attempting to solve (26) with fixed-point iteration, starting with $\partial\mathbf{Y}$ as an initial guess on the right, and iteratively evaluating the right side of (26) to obtain each new guess. Each iteration adds a new layer of intertemporal responses, giving us another term $\mathbf{M}^n\partial\mathbf{Y}$ in the partial sum $\sum_{k=0}^{n}\mathbf{M}^k\partial\mathbf{Y}$, until in the limit we obtain the infinite sum in (34). Higher-order terms relate to $\partial\mathbf{Y}$ in potentially quite complicated ways: for instance, the $t = 7$ entry in $\mathbf{M}^2\partial\mathbf{Y}$ includes the impact of ∂Y_{10} on output demand dY_{12} at $t = 12$, multiplied by the impact of dY_{12} on dY_7—and the same is true if we replace $t = 10, 12$ by any other two periods. As section 3.5 suggests, we can think of the final sum as the accumulation of flows in an intertemporal network.

The first role of the $\mathbf{v}$ term in (34) is to index multiplicity when $\mu < 1$. In this case, we can think of the space of possible equilibrium responses as consisting of $\sum_{k=0}^{\infty}\mathbf{M}^k\partial\mathbf{Y}$ plus a one-dimensional subspace of $\mathbf{v}$ satisfiying $\mathbf{M}\mathbf{v} = \mathbf{v}$. An equilibrium selection rule can then choose, among other things, the relationship between $\mathbf{v}$ and $\partial\mathbf{Y}$. Alternatively, when there is a unique solution $\mu > 1$, there are two possible cases. If $\lim_{t\to\infty}\sum_s sM_{t+s,t} > 0$, meaning that the consumption response to far-out income shocks in present-value terms takes place on average after the shock (a stronger condition than $\mu > 1$), $\mathbf{v} = \mathbf{0}$, and the sum $\sum_{k=0}^{\infty}\mathbf{M}^k\partial\mathbf{Y}$ directly gives the unique equilibrium.[14] When this limit condition does not hold, there is a more subtle case: there is a unique $\mathbf{v}$ such that the expression (34) is bounded in current value—but this $\mathbf{v}$ is not necessarily $\mathbf{0}$. Instead, $\sum_{k=0}^{\infty}\mathbf{M}^k\partial\mathbf{Y}$ may be unbounded in current values, and there is a unique also-unbounded $\mathbf{v}$ that can be combined with it to obtain a finite equilibrium response.

With unique equilibrium $\mu > 1$, in both cases above it is possible to write $d\mathbf{Y}$ as a function of $\partial\mathbf{Y}$, as the following corollary to theorem 1 reveals.

Corollary 1. *If* $\mu > 1$*, equilibrium is locally unique and there exists a linear mapping* $\mathcal{G}$ *such that* $d\mathbf{Y} = \mathcal{G}\partial\mathbf{Y}$ *for all shocks. This mapping is given by*

$$d\mathbf{Y} = \mathcal{G}\partial\mathbf{Y} \equiv \sum_{k=0}^{\infty}\mathbf{M}^k\partial\mathbf{Y}$$

[14]Formally, the $\lim_{t\to\infty}\sum_s sM_{t+s,t} > 0$ condition turns out to be decisive because it implies that the Markov chain whose transition matrix is given by $\mathbf{M}$ is transient.

if $\lim_{t\to\infty}\sum_s sM_{t+s,t} > 0$, *and*

$$d\mathbf{Y} = \mathcal{G}\partial\mathbf{Y} \equiv (1+\mathbf{v}\mathbf{a}')\sum_{k=0}^{\infty}\mathbf{M}^k\partial\mathbf{Y}$$

otherwise, where $\mathbf{v}$ *is the unique vector (up to scale) satisfying* $\mathbf{M}\mathbf{v} = \mathbf{v}$, *and* $\mathbf{a}'$ *is a unique element of the dual of* $\mathbb{R}^{\mathbb{N}}$.

In short, the linear mapping $\mathcal{G}$ from partial equilibrium to the unique general equilibrium takes an especially simple form when $\lim_{t\to\infty}\sum_s sM_{t+s,t} > 0$, but there is an additional term corresponding to $\mathbf{v}$ in (34) otherwise.

The most important aspect of corollary 1 for our purposes will be simply the existence of $\mathcal{G}$.

6 IMPCs and fiscal stimulus

We can now turn to our analysis of fiscal policy using IMPCs as sufficient statistics. Since our benchmark model is locally determinate under the constant r monetary policy, Proposition 1 shows that we can analyze the effect of *any* perturbation to the path of spending or taxes statisfying the intertemporal budget constraint $\mathbf{1}'d\mathbf{G} = \mathbf{1}'d\mathbf{T}$ using the unique locally determinate solution to

$$d\mathbf{Y} = -\mathbf{M}d\mathbf{T} + d\mathbf{G} + \mathbf{M}d\mathbf{Y} \tag{35}$$

This only requires information on $\mathbf{M}$. We proceed in two steps: first, we consider balanced-budget spending shocks, where spending is financed by contemponaneous increases in taxes ($d\mathbf{G} = d\mathbf{T}$). In this case, $\mathbf{M}$ turns out not to matter at all. We next turn to the case where the fiscal shock involves deficits.

6.1 Balanced-budget fiscal policy

Even though we have so far stressed the importance of IMPCs for understanding the effects of government spending, there exists a special case in which they are *irrelevant*, as the next proposition illustrates.

Proposition 4. *Assume that the government changes its path for spending and raises taxes contemporaneously, so that* $d\mathbf{G} = d\mathbf{T}$. *Then, the fiscal multiplier is 1 for every date:* $d\mathbf{Y} = d\mathbf{G}$.

While this result follows immediately from equation (35) and the assumption that the economy is determinate so that the solution $d\mathbf{Y} = d\mathbf{G}$ is unique, it is nevertheless very striking. Economically, it reflects the cancellation of two forces. In partial equilibrium, an increase in spending that contemporaneously raises taxes has a less than one-for-one effect on economic activity $(I-\mathbf{M})\,d\mathbf{G}$ because of the fall in private agents' consumption in response to the taxes. However, the increase in pretax incomes raises consumption, and this turns out to exactly offset the increase in taxes at every date and for every agent. Hence, under our assumptions, every agent's consumption decision is unchanged in the new equilibrium, so that aggregate consumption is unchanged as well. The multiplier is exactly 1 at every single date, irrespective of the timing of spending.

Proposition 4 provides a very clean heterogeneous-agent counterpart to Woodford (2011)'s representative-agent result under constant real interest rates. It can serve as a useful benchmark as the literature on fiscal multipliers in heterogeneous-agent models develops (see for example Ferriere and Navarro (2017) and Hagedorn et al. (2017)).[15]

6.2 Deficit-financed fiscal policy

For any shock that involves primary deficits, IMPCs are crucial for the fiscal multiplier, as the following proposition shows.

Proposition 5 (IMPCs and deficit financing)**.** *The general equilibrium consumption effect of any fiscal shock only depends on the path of primary deficits $d\mathbf{G} - d\mathbf{T}$, not on their composition in terms of spending versus taxes. The general equilibrium output effect is the sum of the direct spending effect and the consumption effect of deficits:*

$$d\mathbf{Y} = d\mathbf{G} + \mathcal{G} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T}) \tag{36}$$

Recall from theorem 1 that $\mathcal{G}$ is a matrix sum of IMPCs. Hence, proposition 5 shows exactly why IMPCs matter for the evaluation of fiscal shocks: they determine the consumption effect of primary deficits. It is for these shocks that getting IMPCs aligned with the data matters the most.

Figure 4 considers the output effect of a hypothetical path for government spending g_t that requires no taxes ($t_t = 0$) and satisfies the government intertermporal budget constraint. Spending initially increases, generating an increase in government debt, which is later stabilized by a reduction in spending.[16] The output effect of this shock in our model is plotted in the solid blue line of the right panel. The shock is very expansionary, with an impact multiplier above 3. Consumption is plotted in the dashed blue line. The large effect reflects equilibrium feedback from IMPCs. Since the higher government spending in the first few periods is not offset by taxes, it raises incomes in these periods and leads agents to increase their consumption. Moreover, as figure 3 shows, agents are essentially unresponsive to income shocks more than a few years in advance, and therefore they do not initially lower consumption in response to the future decline in income. By the time

[15] The results of Hagedorn et al. (2017) are consistent with this benchmark. Their monetary policy specification is extremely close to maintainting constant-r at all times. When they study contemporaneous taxation, they find a fiscal multiplier slightly below 1. This is because all taxes are raised by a lump sum at the margin, which is regressive and therefore raises consumption by less than in our benchmark. We study the effect of unequal incidence in section 8.1. When the government raises taxes at a later date, they find a multiplier higher than 1, consistent with the large expansionary effects of deficits in our model.

[16] Specifically, this is the path generated by the following fiscal rule. Write (g_{ss}, b_{ss}) for the steady-state levels of government spending and debt. Assume the fiscal rule for government spending is $g_t - g_{ss} = -\psi\,(b_{t-1} - b_{ss}) + \epsilon_t^g$, where ϵ_t^g is an innovation to spending with AR coefficient ρ. This rule stipulates a reduction in spending when debt is above steady state level and ensures stability. The figure plots the effect of an innovation to ϵ_t^g when $\rho = 0.7$ and $\psi = 0.3$.

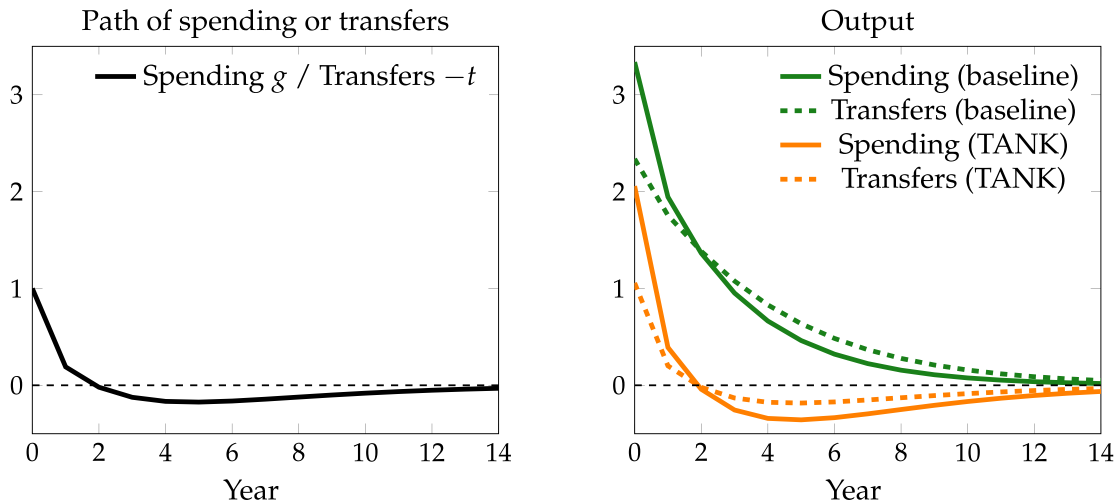


Figure 4: Output effect of a plan for spending or transfers

lower government spending comes on stream, higher past income has spilled over to raise consumption in the current period, explaining why consumption stays positive throughout.

The dashed line also shows the effect on output and consumption of a direct change in transfers $(-t)$ that has the path shown in the left panel. This is sometimes known as the transfer multiplier. Since these have the same effect on primary deficits, by proposition 5, they must also have the same consumption effect. In other words, the transfer and the spending multiplier differ by the direct effect of government spending.

Contrast these results with the saver-spender or 'TANK' model. Lemma 6 established that for this model, all that matters is current deficits. Indeed, a trivial application of the lemma yields that the consumption response is precisely $\lambda/(1-\lambda)$ times the current primary deficit $dG_t - dT_t$. This explains why the output response to a transfer shock (dashed orange line) in the right panel of figure 4 follows the exact same path as the primary deficits themselves in the left panel, and is scaled up slightly since $\lambda/(1-\lambda)$ is just above 1 in our calibration. Following the same logic, the output response to a spending shock (solid orange line) is $1+\lambda/(1-\lambda)$ times the primary deficit.

It is worth stressing that neither the on-impact fiscal multiplier in a saver-spender model, nor the time path of the output responses are close to the one of our baseline model. This illustrates that matching regular MPCs, as our saver-spender model does, is not sufficient to ensure that GE amplification is in line with the data.

The difference between our baseline model and the saver-spender model becomes even more pronounced as we vary the speed of government debt reduction in response to a deficit-financed spending shock. The top left panel in figure 5 shows a benchmark spending shock that is initially debt-financed and the debt is paid down at various speeds (top right panel). We parametrize this speed as ψ (see footnote 16). The consumption responses in the two economies are strikingly different. While the speed of debt repayment is irrelevant for the on-impact multiplier according

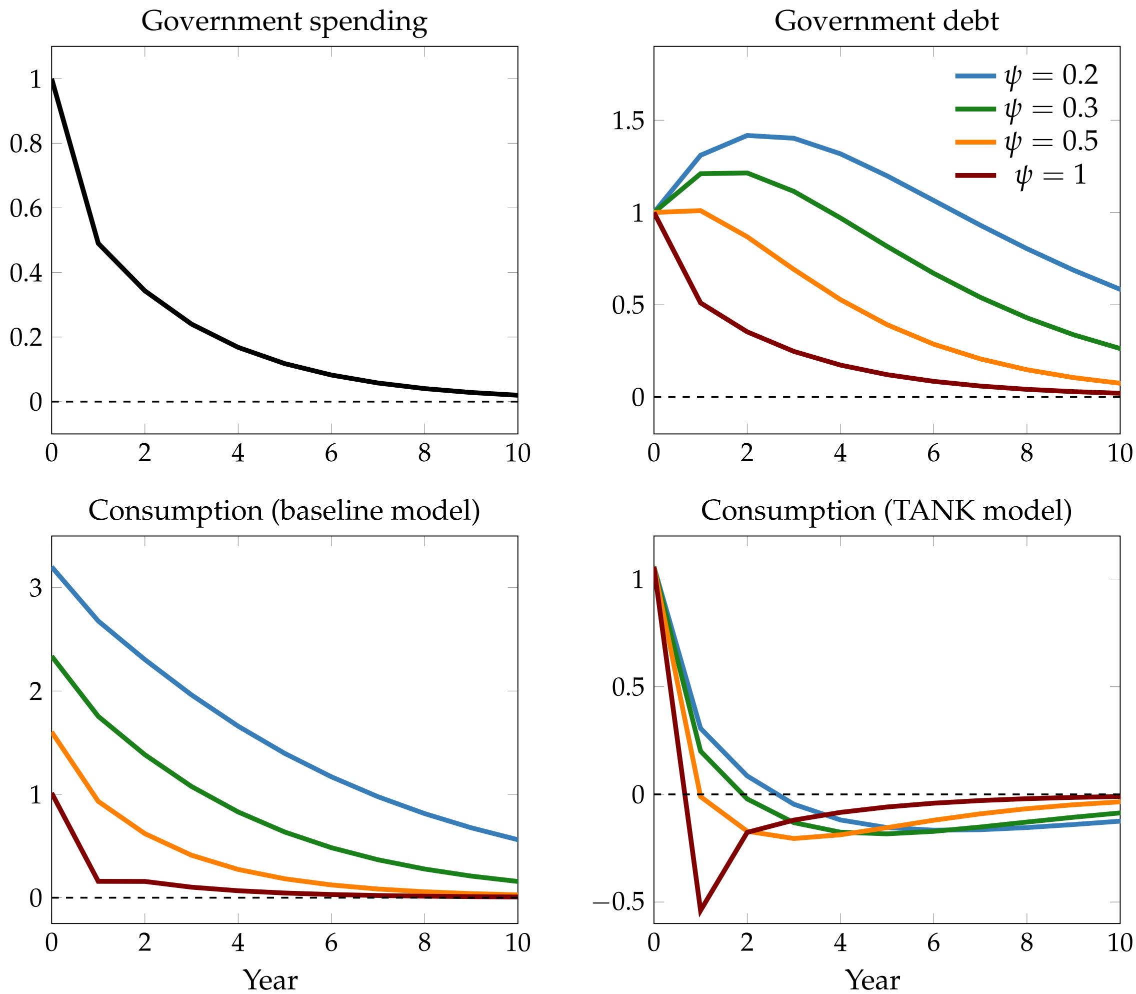


Figure 5: Fiscal policy under various speeds of debt reduction ψ

to the saver-spender model, our baseline model suggests that fiscal policy is a lot more powerful on impact when debt is reduced more slowly. In addition, our baseline model finds a positive impact on consumption throughout as agents smooth out the spending of income gains over the first few years—in stark contrast with the very sharp reversal in consumption predicted by the saver-spender economy.

6.3 Tax rate cuts vs tax revenue cuts

The results so far treat tax revenues dT_t as a direct policy instrument, with the understanding that there is always a path of average retention rates τ_t that can implement any given path of tax revenues dT_t. Yet, the mapping between τ_t and dT_t is highly nonlinear and deserves further attention. For instance, even (or especially) in the absence of labor supply changes, it is an interesting question whether tax *rate* cuts, modeled as an increase in the average retention rate τ_t, can be self-financing, and more generally how tax revenues are affected by tax rate changes. [to be completed]

7 Endogenous monetary policy

So far we have focused on the simple yet powerful benchmark case where monetary policy follows a constant real interest rate rule. We now explore the consequences of a Taylor rule for nominal interest rates. This is especially interesting in our setting as it can shed light on precisely how endogenous policy rules can shape general equilibrium amplification, determinacy and the size of the fiscal multiplier.

We assume in this section that the central bank follows a Taylor rule,

$$i_t = \bar{i} + \phi_\pi \pi_t + \phi_y \hat{y}_t \tag{37}$$

where $\hat{y}_t \equiv \log(y_t/y)$. The real rate is then pinned down by the Fisher equation,

$$r_t = i_t - \pi_{t+1} \tag{38}$$

Inflation follows a standard Phillips curve

$$\pi_t = \kappa \hat{y}_t + \beta \pi_{t+1} \tag{39}$$

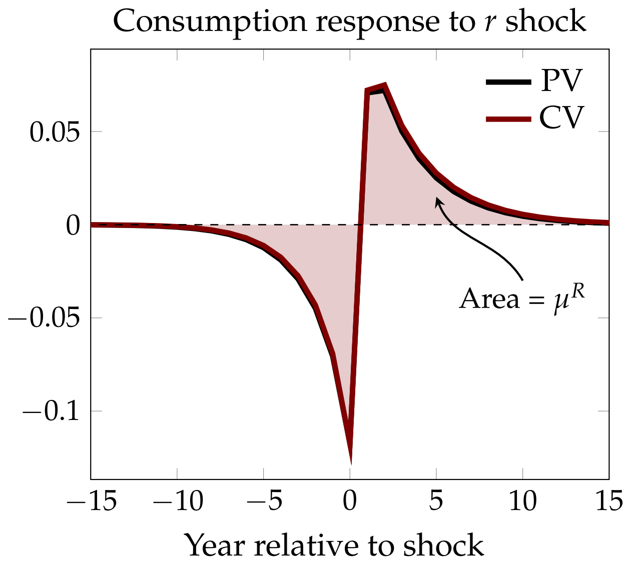
which we microfound using staggered wage setting by unions, in the spirit of Erceg, Henderson and Levin (2000) (see appendix). Since the real rate is no longer constant, it is important to specify how the government deals with changes in interest expenses. We assume that, in each period, the government adjusts its aggregate tax revenue t_t so as to ensure that its budget constraint holds,

$$t_t = (1 + r_t) b_{t-1} - b_t + g_t \tag{40}$$

It takes as given an exogenous schedule $\{g_t, b_t\}$ of government expenses and debt levels, as well as real rates $\{r_t\}$. We define general and exogenous-y equilibria as in definitions 1 and 2, with the exception that the exogenous government policy is now stated as a schedule $\{g_t, b_t\}$ and no longer involves t_t, and real rates are determined according to (37)–(39). This still defines a mapping from output supply $\{y_t\}$ to exogenous-y partial equilibrium demand $\{y_t^{PE}\}$ as in (18), only that t_t is determined by (40) and real interest rates are determined by (37)–(39). Thus,

$$y_s^{PE}(\{y_t\}; \{g_t, b_t\}) = c_s(\{y_t - t_t\}, \{r_t\}) + g_s$$

As before, a general equilibrium satisfies $y_s = y_s^{PE}(\{y_t\}; \{g_t, b_t\})$.



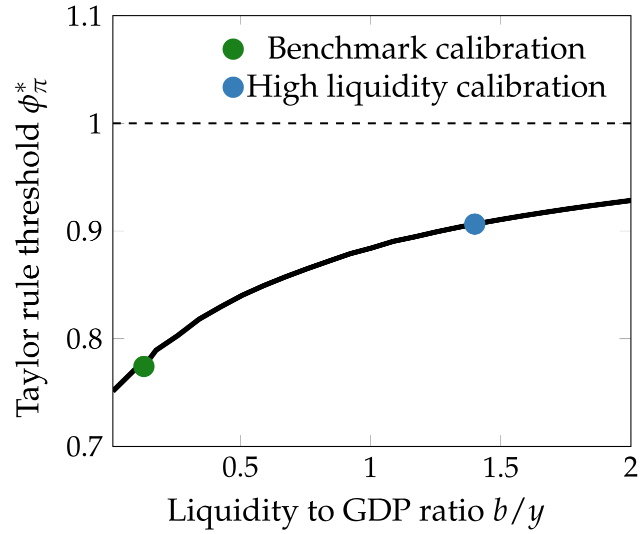


Figure 6: Intertemporal interest rate responses and Taylor rule thresholds for determinacy.

Intertemporal interest rate responses and the IKC. An important aspect of understanding the general equilibrium response to shocks in this economy is the response of consumption to interest rate changes at various horizons. Similar to our definition of IMPCs $m_{s,t}$ in (24), we define the *intertemporal interest rate responses* as

$$m^R_{s,t} \equiv \frac{dc_s}{dr_t} = y^{-1}\frac{\partial c_s}{\partial r_t} - by^{-1}\frac{\partial c_s}{\partial z_t}$$

For given s, t, $m^R_{s,t}$ captures the response of date s aggregate consumption to a small change in the real interest rate at date t, adjusted for the income change induced by the government's tax response. Ideally, we would bring direct empirical evidence to bear on $m^R_{s,t}$. Since we are not aware of any such evidence, we are using our model to compute the model-implied interest rate responses. Figure 6 shows $m^R_{s,t}$ as a function of s for a far-out shock (panel (a)). As before, we summarize the responsiveness of consumption to far-out interest shocks as the integral of the current-value response,

$$\mu^R \equiv \lim_{t\to\infty}\sum_s m^R_{s,t} \tag{41}$$

Below, we summarize the interest responses in a matrix $\mathbf{M}^R$, whose entries, similar to $\mathbf{M}$, are defined in present values,

$$M^R_{s,t} \equiv \frac{1}{(1+r)^{s-t}} m^R_{s,t} \tag{42}$$

We assume $\mathbf{M}^R$ to be self-similar, in line with our simulations and much like $\mathbf{M}$ (proposition 2).

The IKC with a Taylor rule. We can linearize the equation for the general equilibrium output response, $y_s = y_s^{PE}(\{y_t\}; \{g_t, b_t\})$, around a steady state to get the Taylor-rule version of the IKC.

Proposition 6 (IKC with Taylor rule). *Suppose monetary policy follows the Taylor rule (37) and inflation is given by the Phillips curve (39). To first order, impulses to partial equilibrium demand $\partial\mathbf{Y}$ and general equilibrium output $d\mathbf{Y}$ satisfy*

$$d\mathbf{Y} = \partial\mathbf{Y} + \left(\mathbf{M} + \mathbf{M}^R\Phi\right) d\mathbf{Y}.$$

Here, $\mathbf{M}$ is as before in (27); $\mathbf{M}^R$ is as in (42); Φ has entries

$$\Phi_{s,t} \equiv \frac{\partial r_s}{\partial \widehat{y_t}} = \begin{cases} 0 & s > t \\ (\phi_y + \kappa\phi_\pi) & s = t \\ \left(\phi_\pi - \beta^{-1}\right)\beta^{t-s}\kappa & s < t \end{cases}.$$

Note that $\mathbf{1}'\mathbf{M}^R = 0$.

Proposition 6 generalizes the constant-r IKC in proposition 1 to the case where the real rate responds to output, that is, $\Phi \neq 0$. This introduces a new term in the IKC, $\mathbf{M}^R\Phi d\mathbf{Y}$: The vector of interest rates adjusts according to $\Phi d\mathbf{Y}$, inducing a consumption response $\mathbf{M}^R\Phi d\mathbf{Y}$.

Determinacy with Taylor rules. It is known since Woodford (2000) that Taylor rules can ensure local determinacy of a textbook 3-equation New-Keynesian model if and only if interest rates are sufficiently responsive to inflation and output, that is, if ϕ_π and ϕ_y satisfy the Taylor principle,

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1 \tag{43}$$

There is no comparable result for models with idiosyncratic risk and incomplete markets, such as the one in section (2).

We present such a result in the following proposition.

Proposition 7. *(Modified Taylor Principle) Suppose monetary policy follows the Taylor rule (37) and inflation is given by the Phillips curve (39). Suppose $\mathbf{M}, \mathbf{M}^R, \partial\mathbf{Y}$ are as in proposition 6 with $\mathbf{M} + \mathbf{M}^R\Phi$ positive, $\mathbf{M}^R$ self-similar, and $\mu^R > 0$. Then,*

(a) if ϕ_y, ϕ_π are such that $\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1 + \frac{1-\beta}{\kappa}\frac{1-\mu}{\mu^R}$ the model is locally determinate;

(b) if ϕ_y, ϕ_π are such that $\phi_\pi + \frac{1-\beta}{\kappa}\phi_y < 1 + \frac{1-\beta}{\kappa}\frac{1-\mu}{\mu^R}$ the model is locally indeterminate.

In particular, when the Taylor rule is only based on inflation, $\phi_y = 0$, the threshold responsiveness ϕ_π^* for determinacy is equal to $1 - \frac{1-\beta}{\kappa}\frac{1-\mu}{\mu^R}$. The model is locally determinate under a fixed nominal interest rate, $\phi_\pi = \phi_y = 0$, if $\frac{1-\mu}{\mu^R} > \frac{\kappa}{1-\beta}$.

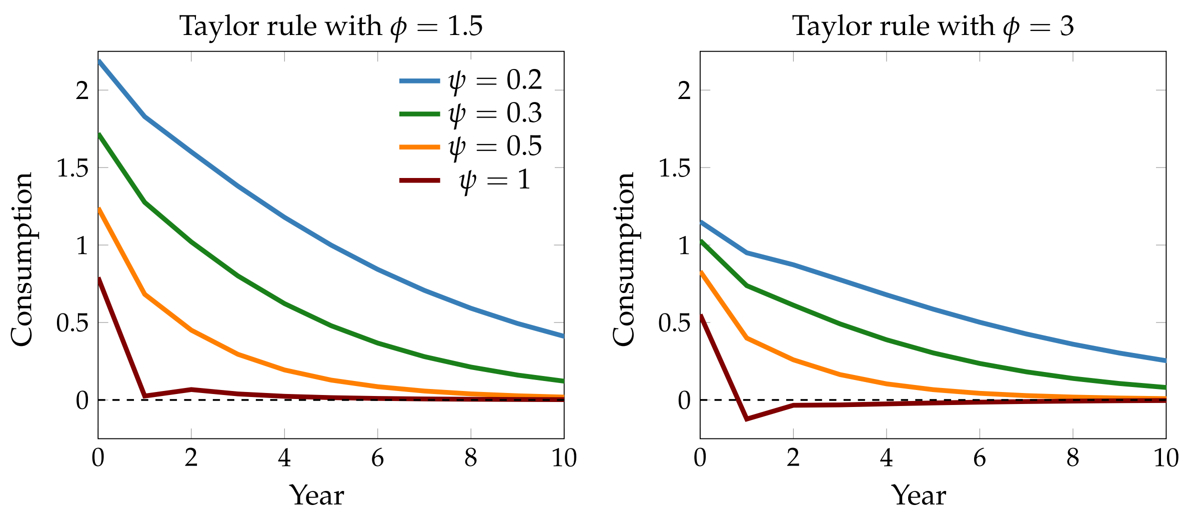


Figure 7: Consumption effect of the shock in figure 5 under a Taylor rule response

Our result in proposition 7 succinctly summarizes how strong a Taylor rule is required to be as a function of IMPCs and intertemporal interest rate responses. Specifically, it stresses the role of μ and μ^R. We explain the intuition behind both in the simple case where $\phi_y = 0$.

The position of μ relative to 1 is a measure of how much demand is shifted into the future. In section 6, we saw that under a constant real interest rate, 1 is precisely the threshold μ has to exceed for determinacy. Theorem 7 adds to this result that where μ lies relative to 1 determines whether the Taylor rule has to respond to inflation ϕ_π more or less than one-for-one. If μ is large compared to 1 and prices are not too flexible, that is, κ is small, it may even be that the economy is determinate under a fixed nominal interest rate.

To illustrate why μ^R is important, imagine an economy with $\mu < 1$. How responsive does the Taylor rule have to be to shift enough demand out into the future and generate determinacy? This is exactly where μ^R matters: if the economy's responsiveness to real interest rates μ^R were large (say infinite), even a small increase in ϕ_π relative to 1 will be enough to ensure determinacy. Thus, the threshold ϕ_π^* declines in μ^R when $\mu < 1$, and vice versa if $\mu > 1$.

We graphically illustrate this relationship on the right panel of figure 6.

Relationship with the regular Taylor principle. Proposition 7 is not readily applicable to a representative-agent economy as $\mathbf{M}$ and $\mathbf{M}^R$ are not self-similar. Still, there is a sense in which the regular Taylor principle (43) can be regarded as a limit case. First, note that both of the infinite sums in the definitions for μ and μ^R diverge to infinity for large t. It turns out that the "right" way to resolve this ∞/∞ problem is to modify both sums by introducing a dampening term $e^{-\epsilon s}$ for some small $\epsilon > 0$, and then to consider the limit of the ratio as $T \to \infty$. This procedure yields a limit of 0 and thus reconciles our modified Taylor principle in proposition 7 with the regular one in (43). The details behind this procedure can be found in the appendix.

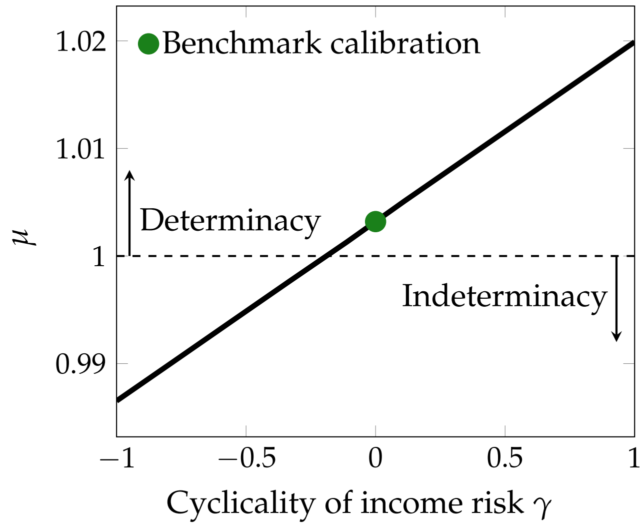


Figure 8: Determinacy as a function of cyclicality of income risk

Fiscal stimulus with Taylor rule: determinacy vs. multiplier. We are ready to investigate the way in which a Taylor rule affects the output effects of government spending in our model. Figure 7 repeats the exercise of figure 5 and plots the consumption responses for various speeds ψ of debt repayment. The left and right panels in figure 7 do so for two different assumptions on ϕ_π. In the left panel, ϕ_π is set to be equal to 1.5, a standard value. In the right panel, ϕ_π is set to 3, to simulate an aggressive Taylor rule.

The difference is noticeable. While the more responsive Taylor rule puts the model safely into the determiancy region, away from any self-fulfilling fluctuations, it also significantly dampens the effects of fiscal stimulus and reduces the on-impact consumption response by around half. This illustrates a policy trade-off between achieving determinacy on the one hand and not dampening fiscal multipliers.

8 Extensions

8.1 Cyclicality of income risk

In this section we extend our model to allow for departures from our benchmark assumption of equal income incidence. This can be achieved in two ways: by assuming an income retention function of a different form, or by assuming that the labor demand function depends on individual type. Here we follow the latter route and replace (6) with a more general rationing function (see Werning (2015) and Auclert and Rognlie (2018))

$$n_{it} = \Gamma\left(e_{it}, l_t\right) \tag{44}$$

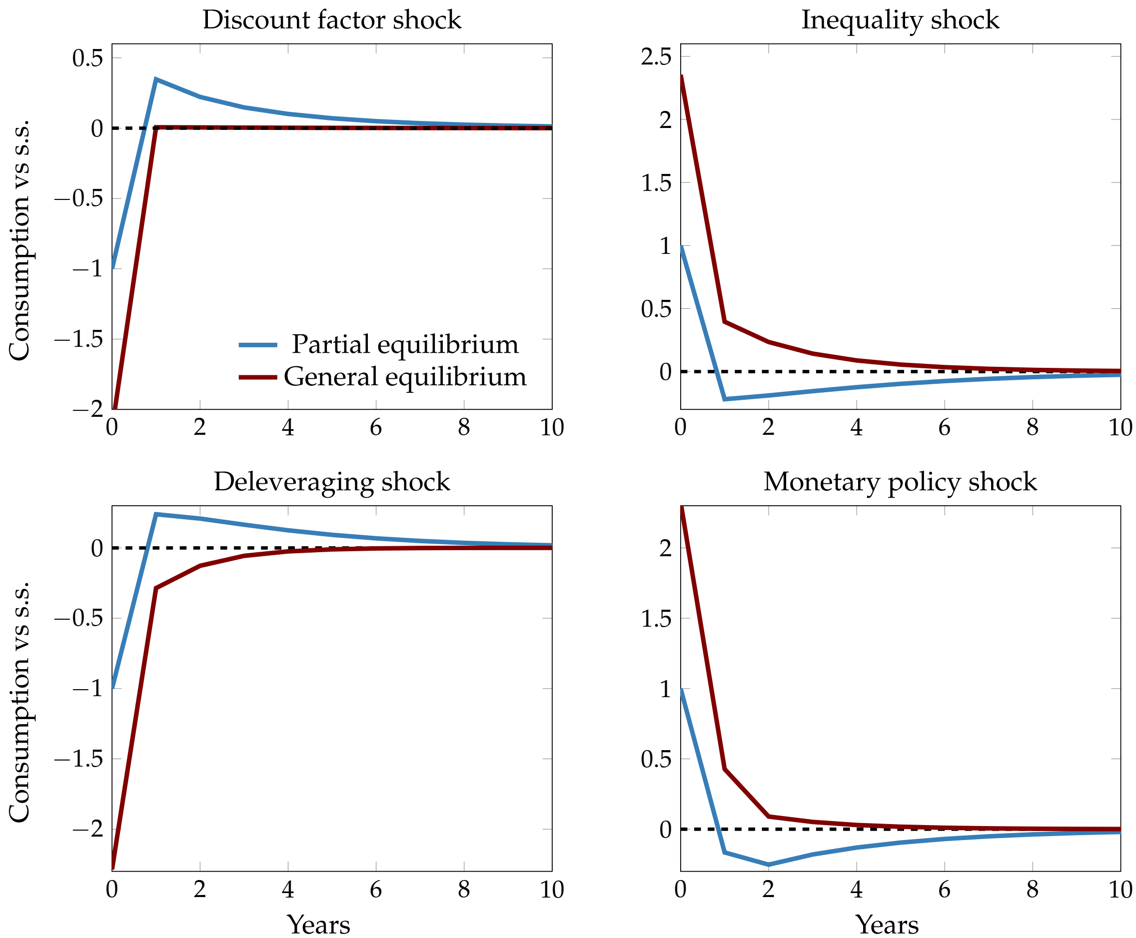


Figure 9: Other shocks

We use the parameterization proposed by Auclert and Rognlie (2018), which makes the standard deviation of log earnings take a constant elasticity γ with respect to aggregate employment l, and revisit our results depending on the value of $\gamma = \frac{\partial^2 \log \Gamma}{\partial \log e \partial \log l}$. We find two main reasons why γ, the cyclicality of income risk parameter, matters.

The cyclicality of income risk changes the determinacy properties of the model. This is illustrated in figure 8, which repeats the exercise in figure 3 by changing γ from its benchmark value of 0. More countercyclical income risk makes the model more indeterminate because it leads income shocks to be spent in earlier periods, making μ more negative and therefore pushing the model towards indeterminacy, as per proposition 3.

8.2 Other shocks

In this section, we show that a version of the intertemporal Keynesian cross (26) obtains for many shocks other than fiscal policy shocks. We now allow for the possibility of preference shocks, shocks to the income distribution (inequality shocks), deleveraging shocks and monetary policy shocks. As in Section 2, we assume that monetary policy implements a path of real interest rates $\{\bar{r}_t\}$, which only for monetary policy shocks is allowed to be time-varying. The only difference of these shocks to fiscal policy shocks lies in their partial equilibrium response $\partial\mathbf{Y}$. In the case of fiscal policy, $\partial\mathbf{Y}$ is equal to $\partial\mathbf{G} - \mathbf{M}d\mathbf{T}$. In this section, $\partial\mathbf{Y}$ is determined as follows.

Proposition 8. *For each of the following shocks, starting from the steady state, we define ∂y_t to be the first order response of partial equilibrium consumption to the following changes:*

a) For preference shocks, to discount factors $\{\beta_t\}$,

b) For inequality shocks, to a one-time increase in the standard deviation of date 0 log productivities $\log e_{i0}$, subject to the normalization $\mathbb{E}e_{i0} = 1$,

c) For deleveraging shocks, to borrowing constraints $\{\underline{a}_t\}$,

d) For monetary policy, to real interest rates $\{\bar{r}_t\}$.

In all cases, $\sum(1+r)^{-t}\partial y_t = 0$, that is, ∂y_t has zero net present value, and the IKC (26) holds.

For these other shocks, the same IKC still applies, linking zero-NPV partial equilibrium shocks to general equilibrium responses. We illustrate the workings of the GE amplification captured by the IKC in Figure 9. It shows partial and general equilibrium responses for these four shocks.

8.3 Investment

In this section, we add investment to our model. We show that a variant of the IKC still holds and examine the implications of investment for determinacy and fiscal multipliers. [to be completed]

9 Conclusion

In this paper, we established IMPCs as sufficient statistics for a broad class of general equilibrium questions. We showed that a simple linear recursive equation encapsulating a dynamic income-consumption feedback—the intertemporal Keynesian cross—characterizes the impulse response of output to macroeconomic shocks. Under homogeneous income incidence and constant real interest rates, the adjustment matrix in this equation consists of an income-weighted average of IMPCs. Calibrating our model to existing evidence, we found that deficit financed fiscal stimulus had large multipliers for plausible degrees of monetary responsiveness, and that an inflation Taylor rule coefficient well below 1 was sufficient to ensure equilibrium determinacy.

Existing evidence on IMPCs speaks in favor of models with precautionary savings and borrowing constraints, but with a calibration that cannot match the wealth to GDP ratio—a standard and important macroeconomic target. This calls for more empirical evidence on the impulse response of consumption at various horizons, and for the development of new models that can match these impulse responses while being consistent with these alternative targets. Just like empirical evidence on MPCs has moved the literature away from standard representative agent models, empirical evidence on IMPCs can advance the development of macroeconomic models that provide a more accurate depiction of household behavior, ultimately making us more confident about our predictions regarding the aggregate effects of macroeconomic shocks.

References

Auclert, Adrien, "Monetary Policy and the Redistribution Channel," *Manuscript*, May 2017.

_ **and Matthew Rognlie**, "Inequality and Aggregate Demand," Working Paper 24280, National Bureau of Economic Research February 2018.

Bénabou, Roland, "Unequal Societies: Income Distribution and the Social Contract," *The American Economic Review*, 2000, *90* (1), 96–129.

Bilbiie, Florin O., "Limited Asset Markets Participation, Monetary Policy and (inverted) Aggregate Demand Logic," *Journal of Economic Theory*, May 2008, *140* (1), 162–196.

_ , "The New Keynesian Cross: Understanding Monetary Policy with Hand-to-Mouth Households," 2017.

Blanchard, Olivier Jean and Charles M. Kahn, "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, July 1980, *48* (5), 1305–1311.

Blundell, Richard, Luigi Pistaferri, and Ian Preston, "Consumption Inequality and Partial Insurance," *American Economic Review*, December 2008, *98* (5), 1887–1921.

Bullard, James and Kaushik Mitra, "Learning About Monetary Policy Rules," *Journal of Monetary Economics*, September 2002, *49* (6), 1105–1129.

Campbell, John Y. and N. Gregory Mankiw, "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence," *NBER Macroeconomics Annual*, January 1989, *4*, 185–216.

Carroll, Christopher D., "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics*, February 1997, *112* (1), 1–55.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo, "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, February 2011, *119* (1), 78–121.

Deaton, Angus, "Saving and Liquidity Constraints," *Econometrica*, 1991, *59* (5), 1221–1248.

Debortoli, Davide and Jordi Galí, "Monetary Policy with Heterogeneous Agents: Insights from TANK models," *Manuscript*, September 2017.

Eggertsson, Gauti B. and Paul Krugman, "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *Quarterly Journal of Economics*, August 2012, *127* (3), 1469–1513.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin, "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, October 2000, *46* (2), 281–313.

Fagereng, Andreas, Martib B. Holm, and Gisle J. Natvik, "MPC Heterogeneity and Household Balance Sheets," Working Paper 2861053, SSRN, October 2016.

Farhi, Emmanuel and Iván Werning, "Chapter 31 - Fiscal Multipliers: Liquidity Traps and Currency Unions," in Harald Uhlig and John B. Taylor, eds., *Handbook of Macroeconomics*, Vol. 2, Elsevier, 2016, pp. 2417–2492.

Ferriere, Axelle and Gaston Navarro, "The Heterogeneous Effects of Government Spending: It's All About Taxes," *Manuscript*, November 2017.

Floden, Martin and Jesper Lindé, "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?," *Review of Economic Dynamics*, April 2001, *4* (2), 406–437.

Fuster, Andreas, Greg Kaplan, and Basit Zafar, "What Would You Do With $500? Spending Responses to Gains, Losses, News and Loans," Working Paper 24386, National Bureau of Economic Research March 2018.

Galí, Jordi, J. David López-Salido, and Javier Vallés, "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, 2007, *5* (1), 227–270.

Guerrieri, Veronica and Guido Lorenzoni, "Credit Crises, Precautionary Savings, and the Liquidity Trap," *Quarterly Journal of Economics*, August 2017, *132* (3), 1427–1467.

Hagedorn, Marcus, Iourii Manovskii, and Kurt Mitman, "The Fiscal Multiplier," *Manuscript*, 2017.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante, "Optimal Tax Progressivity: An Analytical Framework," *The Quarterly Journal of Economics*, 2017, *132* (4), 1693–1754.

Iacoviello, Matteo, "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, June 2005, *95* (3), 739–764.

Jappelli, Tullio and Luigi Pistaferri, "Fiscal Policy and MPC Heterogeneity," *American Economic Journal: Macroeconomics*, October 2014, *6* (4), 107–136.

Johnson, David S., Jonathan A. Parker, and Nicholas S. Souleles, "Household Expenditure and the Income Tax Rebates of 2001," *American Economic Review*, December 2006, *96* (5), 1589–1610.

Kaplan, Greg and Giovanni L. Violante, "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, July 2014, *82* (4), 1199–1239.

— **and Giovanni L Violante**, "Microeconomic Heterogeneity and Macroeconomic Shocks," *Journal of Economic Perspectives*, 2018, *forthcoming.*

— **, Benjamin Moll, and Giovanni L. Violante**, "Monetary Policy According to HANK," *American Economic Review*, March 2018, *108* (3), 697–743.

Mankiw, N. Gregory, *Macroeconomics*, 7 ed., New York, N.Y: Worth Publishers, January 2010.

McKay, Alisdair and Ricardo Reis, "The Role of Automatic Stabilizers in the U.S. Business Cycle," *Econometrica*, January 2016, *84* (1), 141–194.

— **, Emi Nakamura, and Jón Steinsson**, "The Power of Forward Guidance Revisited," *American Economic Review*, October 2016, *106* (10), 3133–3158.

Michaillat, Pascal and Emmanuel Saez, "A New Keynesian Model with Wealth in the Utility Function," *Manuscript*, February 2018, p. 34.

Ravn, Morten O. and Vincent Sterk, "Job Uncertainty and Deep Recessions," *Journal of Monetary Economics*, October 2017, *90* (Supplement C), 125–141.

— **and** — , "Macroeconomic Fluctuations with HANK & SAM: An Analytical Approach," January 2017.

Shapiro, Matthew D. and Joel Slemrod, "Did the 2001 Tax Rebate Stimulate Spending? Evidence from Taxpayer Surveys," in "Tax Policy and the Economy, Volume 17," MIT Press, 2003, pp. 83–110.

Taylor, John B. and Akila Weerapana, *Principles of Economics*, 7 ed., Mason, OH: Cengage Learning, October 2011.

Werning, Iván, "Incomplete Markets and Aggregate Demand," Working Paper 21448, National Bureau of Economic Research, August 2015.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, August 2003.

— , "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, 2011, *3* (1), 1–35.

Zeldes, Stephen P., "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy*, April 1989, *97* (2), 305–346.

Appendix [to be completed]

A Union wage setting

A.1 Details of the labor market

We assume that the worker provides n_{ijt} hours of work to each of a continuum of unions indexed by $j \in [0,1]$, so that his total labor effort is

$$n_{it} \equiv \int_0^1 n^i_{jt} dj$$

The agent gets paid a nominal market wage of w_{jt} per efficient unit of work in union j. Hence his total nominal gross earnings are

$$w_t e_{it} n_{it} \equiv \int_0^1 w_{jt} e_{it} n_{ijt} dj$$

Each union j aggregates efficient units of work into a union-specific task l_{jt}, such that

$$l_{jt} = \sum_{e_i} \pi\left(e_{it}\right) e_{it} n_{ijt} \tag{45}$$

Aggregate employment l_t, in turn, is an aggregate of union-specific tasks that are imperfect substitutes for one another,

$$l_t \equiv \left(\int_0^1 l_{jt}^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

Each union sets its own wage w_{jt} per unit of task provided. We assume that nominal wages w_{jt} are partially rigid, and only reset occasionally in a Calvo fashion: each period, any given union j keeps its wage constant with probability θ. At every point in time, the wage index

$$w_t = \left(\int_0^1 w_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$

serves as a reference for final goods firms's demand for tasks. Conditional on employment l_t, this demand is

$$l_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\epsilon} l_t$$

In turn, union j, when producing l_{jt} tasks, pays all of its workers the same wage w_{jt} and demands that those with skill e_{it} work n_{ijt} hours according to a particular rule Γ, so that

$$n_{ijt} = \Gamma\left(e_{it}, l_{jt}\right) \tag{46}$$

The Γ function must be consistent with aggregation (45) for any l_{jt}. A simple example is $\Gamma\left(e,l\right) = l$. In this case, each union member works $n_{ijt} = l_{jt}$ hours irrespective of their skill or their consumption level.

A wage-resetting union maximizes an average of the utilities (3) of each of its workers, with each worker receiving weight λ^i. In appendix A, we write down the wage setting problem of the union conditional on aggregate employment l_t and a real wage $\frac{w}{p_t}$. We show that in a steady state in which employment is constant at l^*, prices are constant at p^* and all resetting unions set wages at w^*, the consumption distribution $\Psi(c)$ statisfies

$$\frac{w^*}{p^*} = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_{e,\Psi(c)}\left[\lambda(e,c)\,\Gamma_l(e,l^*)\,l^*\frac{\frac{dv^i}{dn}(\Gamma(e,l^*))}{e}\right]}{\mathbb{E}\left[\int \lambda(e,c)\,\gamma_l(e,l^*)\,l^*\frac{\epsilon - \frac{\Gamma(e,l^*)}{\Gamma_l(e,l^*)l^*}}{\epsilon - 1}\frac{du}{dc}(c)\right]} \tag{47}$$

Intuitively, union wage-setting ensures that the real wage $\frac{w^*}{p^*}$ is at a markup over some average across union members of the marginal rate of susbtitution between consumption and hours.

The wage setting rule (47) requires knowledge of the consumption distribution in the population c^{*i}. Away from steady-state, we assume that unions reset wages under a behavioral approximation that this distribution remains constant. The underlying assumption is that it is too complicated for the union to keep track of each member's asset position over time when that distribution is changing. Practically, this implies that the union ignores income effects on labor supply when resetting wages away from steady-state. Below we show that this problem leads to a simple New Keynesian Phillips curve for aggregate wage growth, $\pi_t^w \equiv \log\left(\frac{W_t}{W_{t-1}}\right)$, given to first order by

$$\pi_t^w = \lambda_t\left(\frac{1}{\psi}\widehat{l}_t - (w_t - p_t)\right) + \beta_t \pi_{t+1}^w \tag{48}$$

where $l_t \equiv \log(l_t/l^*)$, ψ is a constant, and κ_t, β_t are deterministic functions of time, with $\beta_t = \beta$ when $\mathcal{T} = \infty$.

A.2 Derivation of the general case

Consider the problem of a union j. The union supplies l_{jt} units of efficient work by aggregating labor from each of its members according to (45), by allocating work across its living members acording to the rule in (46), that is $e_t^i n_t^i = \gamma\left(l_{jt}, e_t^i\right)$.

Consider first the general case. Given a path for agregates $\{l_t, P_t, W_t\}$, wages at every union $\{W_{jt}\}$, and an assumed distribution for the consumption of every agent $\{c_t^i\}$, the problem that the union k solves when it gets a change to reset its wage W_{kt} is

$$\max_{W_{kt}} \sum_{\tau \geq 0} (\beta\theta)^\tau \int \lambda^i \left(u^i\left(c_{t+\tau}^i;\theta\right) - v^i\left(\int n_{jt+\tau}^i dj;\theta\right)\right)$$

where λ^i is the weight assigned to member i in social utility.

The first-order condition for this problem is

$$\sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \frac{\frac{du^i}{dc}\left(c^i_{t+\tau}\right)}{P_{t+\tau}} \left(\gamma\left(l_{kt+\tau}, e^i_{t+\tau}\right) - \epsilon\gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau}\right) di$$
$$+ \sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \epsilon \frac{\frac{dv^i}{dn}\left(\int n^i_{jt+\tau} dj\right)}{e^i_{t+\tau} W_{kt}} \gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau} di = 0$$

which we can rewrite as

$$\sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \frac{du^i}{dc}\left(c^i_{t+\tau}\right) \gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau}$$
$$\left[\left(\epsilon - \frac{\gamma\left(l_{kt+\tau}, e^i_{t+\tau}\right)}{\gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau}}\right)\frac{W_{kt}}{P_{t+\tau}} - \epsilon \frac{\frac{dv^i}{dn}\left(\int n^i_{jt+\tau} dj\right)}{\frac{du^i}{dc}\left(c^i_{t+\tau}\right) e^i_{t+\tau}}\right] di \quad = \quad 0$$

Hence all resetting unions choose the same wage $W_{kt} = \overline{W_t}$, satisfying the implicit equation

$$\overline{W_t} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau} \frac{\frac{dv^i}{dn}\left(\int n^i_{jt+\tau} dj\right)}{e^i_{t+\tau}} di}{\sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau} \left(\frac{\epsilon - \frac{\gamma\left(l_{kt+\tau}, e^i_{t+\tau}\right)}{\gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau}}}{\epsilon - 1}\right) \frac{\frac{du^i}{dc}\left(c^i_{t+\tau}\right)}{P_{t+k}} di} \tag{49}$$

In the special case where γ is linear in l, so that $\gamma_l l = \gamma$, (49) simplifies to

$$\overline{W_t} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \gamma\left(l_{kt+\tau}, e^i_{t+\tau}\right) \frac{\frac{dv^i}{dn}\left(\int n^i_{jt+\tau} dj\right)}{e^i_{t+\tau}} di}{\sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \gamma\left(l_{kt+\tau}, e^i_{t+\tau}\right) \frac{\frac{du^i}{dc}\left(c^i_{t+\tau}\right)}{P_{t+k}} di}$$

In a steady-state in which the price level is P^*, the nominal reset wage is equal to the constant $\overline{W} = W^*$, labor is the constant l^*, and the consumption distribution is c^{*i}, (49) yields

$$\epsilon \int \lambda^i \gamma_l\left(l^*, e^i\right) l^* \frac{\frac{dv^i}{dn}\left(\frac{1}{e^i}\gamma\left(l^*, e^i\right)\right)}{e^i} di = \frac{W^*}{P^*} \int \lambda^i \left(\epsilon\gamma_l\left(l^*, e^i\right) l^* - \gamma\left(l^*, e^i\right)\right) \frac{du^i}{dc}\left(c^{*i}\right) di \tag{50}$$

which is equation (47). In general equilibrium, equation (50) implicitly defines the level of steady-state employment l^*, where the consumption distribution is consistent with a real earnings process given by $\frac{W^*}{P^*}\gamma\left(l^*, e^i\right)$. We linearize around such a steady state.

From (49), we can write the reset wage relative to the current wage W_t as

$$\frac{\overline{W_t}}{W_t} = \frac{\epsilon \sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau} \frac{\frac{dv^i}{dn}\left(\int n^i_{jt+\tau} dj\right)}{e^i_{t+\tau} W_t} di}{\sum_{\tau\geq 0} (\beta\theta)^{\tau} \int \lambda^i \left(\epsilon\gamma_l\left(l_{kt+\tau}, e^i_{t+\tau}\right) l_{kt+\tau} - \gamma\left(l_{kt+\tau}, e^i_{t+\tau}\right)\right) \frac{\frac{du^i}{dc}\left(c^i_{t+\tau}\right)}{P_{t+k}} di} = \frac{G_t}{H_t} \tag{51}$$

where G_t, H_t satisfy the recursions

$$G_t = \epsilon \int \lambda^i \gamma_l \left(l_{kt}, e_t^i\right) l_{kt} \frac{\frac{dv^i}{dn}\left(\int n_{jt}^i dj\right)}{e_t^i W_t} di + \beta\theta \frac{W_{t+1}}{W_t} G_{t+1} \tag{52}$$

$$H_t = \int \lambda^i \left(\epsilon \gamma_l \left(l_{kt}, e_t^i\right) l_{kt} - \gamma\left(l_{kt}, e_t^i\right)\right) \frac{\frac{du^i}{dc}\left(c_t^i\right)}{P_t} di + \beta\theta H_{t+1} \tag{53}$$

The wage index W_t at time t satisfies the recursion

$$W_t^{1-\epsilon} = \theta W_{t-1}^{1-\epsilon} + (1-\theta)\left(W_t^*\right)^{1-\epsilon} \tag{54}$$

Take logs on both sides of (54) and (51) and rearrange to get

$$\pi_t^w \equiv w_t - w_{t-1} = \frac{1-\theta}{\theta}\left(w_t^* - w_t\right) = \frac{1-\theta}{\theta}\left(g_t - h_t\right) \tag{55}$$

Consider the steady state where $l_t = l^*$, $P_t = P^*$, $W_t = W_{jt} = W^*$ for all j and the distribution of consumption is at its steady state c^{*i}. From (46), all workers with productivity e_t^i work equally at each firm,

$$n_{jt}^i = \frac{\gamma\left(l^*, e_t^i\right)}{e_t^i} \quad \forall j$$

Then (52)–(53) deliver

$$G_t^* = \Phi_t \epsilon \int \lambda^i \gamma_l \left(l^*, e^i\right) l^* \frac{\frac{dv^i}{dn}\left(\frac{\gamma\left(l^*, e^i\right)}{e^i}\right)}{e_t^i W^*}$$

$$H_t^* = \Phi_t \int \lambda^i \left(\epsilon \gamma_l \left(l^*, e^i\right) l^* - \gamma\left(l^*, e^i\right)\right) \frac{\frac{du^i}{dc}\left(c^{*i}\right)}{P^*} di$$

where $\Phi_t = \sum_{\tau=0}^{T-t} (\beta\theta)^\tau$. Using (50), we obtain

$$G_t^* = C\Phi_t$$

$$H_t^* = C\Phi_t$$

with $C = \epsilon \int \lambda^i \gamma_l \left(l^*, e^i\right) l^* \frac{\frac{dv^i}{dn}\left(\frac{1}{e^i}\gamma\left(l^*, e^i\right)\right)}{e^i W^*} di = \int \lambda^i \left(\epsilon \gamma_l \left(l^*, e^i\right) l^* - \gamma\left(l^*, e^i\right)\right) \frac{\frac{du^i}{dc}\left(c^{*i}\right)}{P^*} di.$

Approximating (52)–(53) around G_t^*, H_t^*, and defining $\widehat{g}_t \equiv \log\left(\frac{G_t}{G_t^*}\right)$ and $\widehat{g}_t \equiv \log\left(\frac{H_t}{H_t^*}\right)$, we obtain

$$\widehat{g}_t = \frac{1}{\Phi_t} a_t + \left(1 - \frac{1}{\Phi_t}\right)\left(w_{t+1} - w_t + \widehat{g_{t+1}}\right) \tag{56}$$

$$\widehat{h}_t = \frac{1}{\Phi_t} b_t + \left(1 - \frac{1}{\Phi_t}\right)\widehat{h_{t+1}} \tag{57}$$

Where we have defined

$$a_t \equiv \int \omega^{Gi} \left\{ \left(\frac{\gamma_{ll}\left(l^*, e^i\right) l^*}{\gamma_l\left(l^*, e^i\right)} + 1 \right) \left(-\epsilon\left(w_{kt} - w_t\right) + \widehat{l_t} \right) + \frac{1}{\psi^i\left(e^i\right)} \frac{\gamma_l\left(l^*, e^i\right) l^*}{\gamma\left(l^*, e^i\right)} \widehat{l_t} - w_t \right\} di$$

In this expression, ψ^i is the Frisch elasticity of labor supply of individual i in state e^i, and individual weights are the following transformation of social weights:

$$\omega^{Gi} = \frac{\lambda^i \gamma_l\left(l^*, e^i\right) l^* \frac{\frac{dv^i}{dn}\left(\frac{\gamma\left(l^*, e^i\right)}{e^i}\right)}{e^i}}{\int \lambda^i \gamma_l\left(l^*, e^i\right) l^* \frac{\frac{dv^i}{dn}\left(\frac{1}{e^i}\gamma\left(l^*, e^i\right)\right)}{e^i} di}$$

Similarly,

$$b_t = \int \omega^{Hi} \left(\left\{ \frac{\epsilon}{\epsilon - 1} \left(\frac{\gamma_{ll}\left(l^*, e^i\right) l^*}{\gamma_l\left(l^*, e^i\right)} + 1 \right) - \frac{1}{\epsilon - 1} \left(\frac{\gamma_l\left(l^*, e^i\right) l^*}{\gamma\left(l^*, e^i\right)} \right) \right\} \left(-\epsilon\left(w_{kt} - w_t\right) + \widehat{l_t} \right) - \frac{1}{\sigma\left(e^i\right)} \widehat{c_t^i} - p_t \right) di$$

with

$$\omega^{Hi} = \frac{\lambda^i \left(\epsilon \gamma_l\left(l^*, e^i\right) l^* - \gamma\left(l^*, e^i\right)\right) \frac{du^i}{dc}\left(c^{*i}\right)}{\int \lambda^i \left(\epsilon \gamma_l\left(l^*, e^i\right) l^* - \gamma\left(l^*, e^i\right)\right) \frac{du^i}{dc}\left(c^{*i}\right) di}$$

Our behavioral assumption for the union consists in assuming that $\int \omega^{Hi} \frac{1}{\sigma^i\left(e^i\right)} \widehat{c_t^i} di = 0$, that is, the consumption distribution remains at its steady-state. Using this assumption to simplify b_t, we can solve for the difference

$$\begin{aligned} a_t - b_t &= \left(\int \omega^{Gi} \frac{1}{\psi^i\left(e^i\right)} \frac{\gamma_l\left(l^*, e^i\right) l^*}{\gamma\left(l^*, e^i\right)} di \right) \widehat{l_t} - \left(w_t - p_t\right) \\ &\quad + \left(\int \left\{ \omega^{Gi} \frac{1}{\epsilon - 1} \left(\frac{-\gamma_{ll}\left(l^*, e^i\right) l^*}{\gamma_l\left(l^*, e^i\right)} \right) \right\} di \right) \left(\widehat{l_t} - \epsilon\left(\overline{w_t} - w_t\right) \right) \\ a_t - b_t &= \frac{1}{\psi} l_t - \left(w_t - p_t\right) - \epsilon \nu \left(\overline{w_t} - w_t\right) \end{aligned} \tag{58}$$

where

$$\begin{aligned} \frac{1}{\psi} &\equiv \int \omega^{Gi} \frac{1}{\psi^i\left(e^i\right)} \frac{\gamma_l\left(l^*, e^i\right) l^*}{\gamma\left(l^*, e^i\right)} di + \nu \\ \nu &\equiv \int \omega^{Gi} \frac{1}{\epsilon - 1} \left(\frac{-\gamma_{ll}\left(l^*, e^i\right) l^*}{\gamma_l\left(l^*, e^i\right)} \right) di \end{aligned}$$

We finally obtain the wage Phillips Curve as follows. Start with equation (55)

$$\pi_t^w = w_t - w_{t-1} = \frac{1-\theta}{\theta}\left(\overline{w_t} - w_t\right) = \frac{1-\theta}{\theta}\left(g_t - h_t\right) = \frac{1-\theta}{\theta}\left(\widehat{g_t} - \widehat{h_t}\right) \tag{59}$$

where the last line follows $G_t^* = H_t^*$. Next, combine (59), (56), (57) and (58) into

$$
\begin{aligned}
\overline{w_t} - w_t &= \widehat{g_t} - \widehat{h_t} \\
&= \frac{1}{\Phi_t}\left(a_t - b_t\right) + \left(1 - \frac{1}{\Phi_t}\right)\left(w_{t+1} - w_t + \widehat{g_{t+1}} - \widehat{h_{t+1}}\right) \\
&= \frac{1}{\Phi_t}\left(\frac{1}{\psi} l_t - \left(w_t - p_t\right) - \epsilon\nu\left(\overline{w_t} - w_t\right)\right) + \left(1 - \frac{1}{\Phi_t}\right)\left(w_{t+1} - w_t + \widehat{g_{t+1}} - \widehat{h_{t+1}}\right)
\end{aligned}
$$

solve out for $\overline{w_t} - w_t$

$$
\overline{w_t} - w_t = \left(1 + \frac{\epsilon\nu}{\Phi_t}\right)^{-1}\left\{\frac{1}{\Phi_t}\left(\frac{1}{\psi} l_t - \left(w_t - p_t\right)\right) + \left(1 - \frac{1}{\Phi_t}\right)\left(w_{t+1} - w_t + \widehat{g_{t+1}} - \widehat{h_{t+1}}\right)\right\}
$$

Wage inflation π_t^w is then

$$
\begin{aligned}
w_t - w_{t-1} &= \frac{1-\theta}{\theta}\left(1 + \frac{\epsilon\nu}{\Phi_t}\right)^{-1}\left\{\frac{1}{\Phi_t}\left(\frac{1}{\psi} l_t - \left(w_t - p_t\right)\right) + \left(1 - \frac{1}{\Phi_t}\right)\left(\left(\frac{1}{\theta} - 1\right)\left(w_{t+1} - w_t\right) + \frac{1-\theta}{\theta}\left(\widehat{g_{t+1}} - \right.\right.\right. \\
&= \left(1 + \frac{\epsilon\nu}{\Phi_t}\right)^{-1}\left\{\frac{1}{\Phi_t}\frac{1-\theta}{\theta}\left(\frac{1}{\psi} l_t - \left(w_t - p_t\right)\right)\right\} + \left(1 - \frac{1}{\Phi_t}\right)\frac{1}{\theta}\left(w_{t+1} - w_t\right)
\end{aligned}
$$

This gives us our final expression for the wage Phillips curve

$$
\pi_t^w = \lambda_t\left(\frac{1}{\psi}\widehat{l_t} - \left(w_t - p_t\right)\right) + \beta_t \pi_{t+1}^w \tag{60}
$$

where

$$
\begin{aligned}
\lambda_t &\equiv \left(1 + \frac{\epsilon\nu}{\Phi_t}\right)^{-1}\left(\frac{1}{\Phi_t}\frac{1-\theta}{\theta}\right) \\
\beta_t &\equiv \left(1 - \frac{1}{\Phi_t}\right)\frac{1}{\theta} \\
\Phi_t &\equiv \sum_{\tau=0}^{T-t}\left(\beta\theta\right)^{\tau}
\end{aligned}
$$

As $T \to \infty$, $\Phi_t \to \frac{1}{1-\beta\theta}$ and therefore $\beta_t \to \beta$, so that (60) becomes

$$
\pi_t^w = \frac{1-\beta\theta}{1+\epsilon\nu\left(1-\beta\theta\right)}\frac{1-\theta}{\theta}\left(\frac{1}{\psi}\widehat{l_t} - \left(w_t - p_t\right)\right) + \beta\pi_{t+1}^w
$$

In general equilibrium, $w_t = p_t$, $\pi_t^w = \pi_t$ and $\widehat{l_t} = \widehat{y_t}$, so we recover (39) with $\kappa = \frac{1-\beta\theta}{1+\epsilon\nu(1-\beta\theta)}\frac{1-\theta}{\theta}\frac{1}{\psi}$.

B Alternative economies

B.1 Saver-spender (TANK) economy

- Discrete time $t = 0, \ldots, \infty$
- Two kinds of agents $i \in \{c, u\}$
  - fraction μ of permanently hand-to-mouth agents (c)
  - fraction $1 - \mu$ of infinitely-lived unconstrained agents (u)
- At time t, agent type $i \in \{c, u\}$
  - works n_t^c hours at a nominal market wage w_t,
  - pays real taxes t_t^c,
  - can buy consumption goods with price p_t.
- Unconstrained agents can save in nominal bonds with return r_t. Constrained agents cannot save or borrow.

Constrained agents. Constrained agents just consume their income in each period

$$c_t^c = \frac{w_t}{p_t} n_t^c - t_t^c \tag{61}$$

Unconstrained agents. Each unconstrained agent solves the maximization problem

$$\begin{aligned} \max \quad & \sum \beta^t u\left(c_t^u\right) \\ & c_t^u + a_t^u = \left(1 + r_{t-1}\right) a_{t-1}^u + \frac{w_t}{p_t} n_t^u - t_t^u \end{aligned}$$

where u has constant elasticity of substitution σ, $u\left(c\right) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$.

Technology. All agents supply as many hours n_t^i as firms demand given the real wage $\frac{w_t}{p_t}$. These firms operate the linear technology

$$y_t = n_t = \mu n_t^c + \left(1 - \mu\right) n_t^u.$$

They are perfectly competitive and set prices flexibly, so that $p_t = w_t$ and profits are zero.

Wage determination. To focus on aggregate demand considerations, wages are perfectly rigid at a constant level, which we normalize to 1 for convenience:

$$w_t = 1$$

Hence firm optimization also implies that the nominal price level is constant at $p_t = 1$. The Γ function rations in the following way:

$$\begin{aligned} n_t^u &= C_y \times n_t^{\widehat{\gamma_y}} \\ \mu n_t^c + (1-\mu)\, n_t^u &= n_t = y_t \end{aligned} \tag{62}$$

Fiscal policy. In each period, the government has an outstanding amount of debt b_t. It pays for spending g_t, sets taxes t_t, and issues new debt b_t, so as to satisfy its nominal budget constraint

$$b_t + t_t = (1 + r_{t-1})\, b_{t-1} + g_t \tag{63}$$

The government has initially issued $b_{-1} = \overline{b}$ bonds. Taxes t_t^u, t_t^c on each agent are set such that

$$\begin{aligned} t_t^u &= C_{tax} \times t_t^{\widehat{\gamma_{tax}}} \\ \mu t_t^c + (1-\mu)\, t_t^u &= t_t \end{aligned} \tag{64}$$

Monetary policy. Monetary policy sets a path for the nominal interest rate i_t. Combined with $p_t = 1$, this implies that the real interest rate at t and the nominal interest rate are equal

$$1 + r_t \equiv (1 + i_t)\, \frac{p_t}{p_{t+1}} = 1 + i_t$$

We further assume a constant-r rule, so that

$$i_t = r = \beta^{-1} = 1$$

which is exogenous.

Equilibrium. Given a path for government policy $\{b_t, g_t, t_t, i_t\}$ satisfying (63), an equilibrium is a path for quantities $\{y_t, n_t^c, n_t^u, c_t^c, c_t^u\}$, such that households optimize, firms optimize, and the goods and bond markets clear:

$$g_t + \mu c_t^c + (1-\mu)\, c_t^u = y_t \tag{65}$$

$$(1-\mu)\, a_t^u = b_t \tag{66}$$

Solving for government spending multipliers. Given a feasible path for policy $\{b_t, g_t, t_t, r_t\}$, equilibrium is determined as follows. Firm price-setting implies $p_t = 1$ at every t. Combined with the central bank policy, this determines the path for $r_t = i_t$. Constrained households' consumption given the real wage $\frac{w_t}{p_t} = 1$ is

$$c_t^c = n_t^c - t_t^c$$

Unconstrained households' optimal intertemporal choice implies the familiar permanent-income solution

$$c_t^u = c_0^u = (1-\beta)\left\{\sum_{s=0}^{\infty}\beta^s\left(n_s^u - t_s^u\right) + \beta^{-1}a_{-1}^u\right\} \quad \forall t \tag{67}$$

We are interested in the effects of perturbations to the path of government spending dg_t. The government budget constraint implies that these must be paid by taxes at some point, ie

$$\sum_{t=0}^{\infty}\beta^t dg_t = \sum_{t=0}^{\infty}\beta^t dt_t$$

The model features a steady state level of output $\overline{y}$. Suppose that government policy returns to steady state past date T. Then we select the standard equilibrium[17] by imposing that output also returns to steady state by T, $y_T = \overline{y}$.

This implies in particular that $dc_T^u = 0$, but then from (67) this must imply that

$$dc_t^u = 0 \quad \forall t \leq T$$

as well. The present value of net of tax income for the unconstrained agent must be unchanged, else he would change her behavior past the point at which fiscal policy changes, which is incompatible with equilibrium selection.

Now, given (63), we have

$$d\left(b_t - \beta^{-1}b_{t-1}\right) = dg_t - dt_t$$

But given (66) and $dc_t^u = 0$, this is also

$$\begin{aligned} d\left(b_t - \beta^{-1}b_{t-1}\right) &= (1-\mu)\, d\left(a_t^u - \beta^{-1}a_{t-1}^u\right) \\ &= (1-\mu)\, d\left(n_t^u - t_t^u\right) \end{aligned}$$

While the rationing rules (62) and (64) imply

$$d\left(n_t^u - t_t^u\right) = \gamma_y dy_t - \gamma_{tax} dt_t$$

with $\gamma_y = \widehat{\gamma_y}\frac{n^u}{y}$, $\gamma_{tax} = \widehat{\gamma_{tax}}\frac{t^u}{t}$. Combining these expressions we obtain the TANK government spending multiplier *purely* as a function of current g_t and current t_t:

$$dy_t = \frac{dg_t}{(1-\mu)\gamma_y} + \frac{(1-\mu)\gamma_{tax} - 1}{(1-\mu)\gamma_y}dt_t \tag{68}$$

This formula is likely to extend to alternative environments. For example, in a model with flexible labor supply, if the unconstrained also own the firms with countercyclical profits then this will

[17] It is easy to show directly that this model has multiple equilibria at constant-r. Alternatively, our determinacy condition in ARS explains why.

tend to generate a $\gamma_y < 1$.

Interpretation of multiplier formula (68).

a) Consider first the case of contemporaneous financing $dg_t = dt_t$ and equal incidence $\gamma_{tax} = \gamma_y = 1$. Then
$$dy_t = \frac{1-\mu}{1-\mu} dg_t = dg_t$$
This is our benchmark government spending multiplier of 1.

b) Maintain equal incidence but assume taxes are delayed. Then
$$dy_t = \frac{1}{1-\mu} dg_t - \frac{\mu}{1-\mu} dt_t \tag{69}$$
The effects of government spending are amplified by $\frac{1}{1-\mu}$ when they take place due to the income-consumption feedback of the constrained agents. Taxation has contemporaneous impact $\frac{\mu}{1-\mu}$ since the constrained agents react to it, which then gets amplified.

c) With unequal incidence but $\gamma_{tax} = \gamma_y = \gamma$, the formula is
$$dy_t = \frac{dg_t}{(1-\mu)\gamma} + \frac{(1-\mu)\gamma - 1}{(1-\mu)\gamma} dt_t$$
ie it remains the same as in (69) provided we relabel the effective fraction of constrained agents as $1 - \widetilde{\mu} = (1-\mu)\gamma$

d) Even in the most general case of (68) we see that the effects of government spending and taxation are entirely static. In particular there is no effect from government spending "spilling over to closeby periods". This is a key difference with our HANK model that stems from the structure of the M matrix.

Calibrating the model. A natural calibration for a TANK model sets the fraction of constrained agents μ to hit a given population average MPC,
$$\overline{mpc} = \mu \times 1 + (1-\mu) \times (1-\beta)$$
in other words $\mu = 1 - \beta^{-1}(1 - \overline{mpc})$. With $\beta = 0.96$ to replicate our steady state real interest rate of 4%, we can therefore 'mimic' our HANK model that has $\overline{mpc} = 0.225$ by setting $\mu = 0.194$.

B.2 Bond-in-utility (BIU) economy

We now show that the representative agent-BIU model is better able to match a HANK model. Consider one representative agent who values bonds in the utility. His problem is

$$\begin{aligned} \max \quad & \sum \beta^t \{u(c_t) + v(a_t)\} \\ \text{s.t.} \quad & c_t + a_t = (1 + r_{t-1}) a_{t-1} + \frac{w_t}{p_t} n_t - t_t \end{aligned} \tag{70}$$

Production is linear,

$$y_t = n_t$$

Prices are flexible, so $\frac{w_t}{p_t} = 1$, and wages are rigid so employment is demand-determined. The government budget constraint is still given by

$$b_t + t_t = (1 + r_t) b_{t-1} + g_t \tag{71}$$

In equilibrium,

$$\begin{aligned} c_t + g_t &= y_t = n_t \\ a_t &= b_t \end{aligned}$$

The agent's first order condition is

$$u'(c_t) = \beta (1 + r_t) u'(c_{t+1}) + v'(a_t) \tag{72}$$

Partial equilibrium. Given a_{-1}, and exogenous income $z_t = y_t - t_t$, the perfect foresight solution to the agent's problem is given by (70)–(72). In steady state (72) implies

$$u'(z + ra)(1 - \beta(1 + r)) = v'(a) \tag{73}$$

For income $z_t \to z$, the agent's asset choice will converge back to $a_\infty = a_{-1}$. We can linearize the dynamics in (72) around the steady state

$$\begin{aligned} u''(c) dc_t &= \beta(1 + r) u''(c) dc_{t+1} + v''(a) da_t \\ dc_t + da_t &= (1 + r) da_{t-1} + dz_t \end{aligned}$$

and combine those to get the second-order difference equation

$$(1 + r) da_{t-1} - \left(1 + \frac{v''(a)}{u''(c)} + \beta(1 + r)^2\right) da_t + \beta(1 + r) da_{t+1} = -dz_t + \beta(1 + r) dz_{t+1}$$

Consider the roots of

$$P(X) = 1 + r - \left(1 + \frac{v''(a)}{u''(c)} + \beta(1+r)^2\right)X + \beta(1+r)X^2$$

We have $P(0) = 1 + r > 0$, and

$$P(1) = r(1 - \beta(1+r)) - \frac{v''(a)}{u''(c)} = r\frac{v'(a)}{u'(c)} - \frac{v''(a)}{u''(c)}$$

where we have used (73). Since $\frac{v''(a)}{u''(c)} = \frac{\gamma}{\theta}\frac{c}{b}\frac{v'(a)}{u'(c)}$ where γ, θ are the local curvatures in v and u, assuming the stability condition $c > \frac{\theta}{\gamma}rb$ we see that $P(1) < 0$, and therefore P has two roots $0 < \lambda_1 < 1 < \lambda_2$. The local policy function for assets is given by

$$da_t = \lambda_1 da_{t-1} + \sum_{s=0}^{\infty}(\beta\lambda_1)^{s+1}\left\{\frac{1}{\beta(1+r)}dz_{t+s} - dz_{t+s+1}\right\}$$

from which we find the local consumption policy function

$$dc_t = (1 + r - \lambda_1)da_{t-1} + dz_t + \sum_{s=0}^{\infty}(\beta\lambda_1)^{s+1}\left\{dz_{t+s+1} - \frac{1}{\beta(1+r)}dz_{t+s}\right\}$$

Hence, the consumption function in response to impulses $dz_t = 1_{\{t=T\}}$ has teh following shape:

- For immediate shocks $T = 0$, the date-0 MPC is

$$\overline{m} \equiv 1 - \frac{\lambda_1}{1+r}$$

  consumption then declines at rate λ_1, ie[18]

$$dc_t = \overline{m}\lambda_1^t$$

- For future shocks, immediate MPC is $\left(\frac{1}{\lambda_2}\right)^T\overline{m}$. Solving for assets,

$$da_t = \begin{cases} \lambda_1 da_{t-1} - \overline{m}(\beta\lambda_1)^{T-t} & t < T \\ \lambda_1 da_{t-1} + 1 - \overline{m} & t = T \\ \lambda_1 da_{t-1} & t > T \end{cases}$$

  and consumption follows as residual. Hence assets fall in anticipation, bounce back upon

[18]One way to check this is to use the fact that the PV of MPCs must be 1, which relates $\overline{m}$ and λ_1 through

$$\overline{m}\sum_{t=0}^{\infty}\left(\frac{\lambda_1}{1+r}\right)^t = \frac{\overline{m}}{1 - \frac{\lambda_1}{1+r}} = 1$$

receipt, and then decline at rate λ_1.

General equilibrium BIU-NK model. Consider first the steady state with output y, bonds b and spending g. Then (71) reads $t = rb + g$, and $z + ra = y - t = y - g$. This implies that the steady state real rate r must solve

$$u'(y-g)(1-\beta(1+r)) = v'(b)$$

Next consider transition dynamics. Given a path for g_t and b_t compatible with (71), equilibrium in the BIU-NK model is a sequence $\{y_t\}$ solving

$$u'(y_t - g_t) = \beta(1+r)\, u'(y_{t+1} - g_{t+1}) + v'(b_t) \tag{74}$$

when linearized this delivers

$$\begin{aligned} dy_t - dg_t &= \beta(1+r)(dy_{t+1} - dg_{t+1}) + \frac{v''(b)}{u''(c)} db_t \\ dy_t - dg_t &= \frac{v''(b)}{u''(c)} \sum_{k=0}^{\infty} [\beta(1+r)]^k\, db_{t+k} \end{aligned}$$

For example with our mean-reversion-in-debt rule in (**??**) we obtain

$$dy_t = dg_t + \frac{v''(b)}{u''(c)} \frac{1}{\beta(1+r) - \rho} db_t \tag{75}$$

This is very different from the TANK fiscal multiplier (68) and much closer to what we saw in HANK (or in the OLG model), namely that the level of debt matters for the fiscal multiplier rather than the flow of taxes at a point in time.

Calibration. Assume $v'(b) = \varphi b^{-\gamma}$, $u'(c) = c^{-\theta}$. Then we have

$$\frac{v''(b)}{u''(c)} = \frac{\gamma}{\theta} \frac{c}{b} \frac{v'(b)}{u'(c)} = \frac{\gamma}{\theta} \frac{c}{b} (1 - \beta(1+r))$$

and

$$P(1) = (1 - \beta(1+r)) \left(r - \frac{\gamma}{\theta} \frac{c}{b} \right)$$

The stablity condition is $c > \frac{\theta}{\gamma} rb$, which is easily satistifed. We fix θ, β and R as in our model calibration. Then, given a target $\overline{m}$ we find γ that delivers the desired $\lambda_1 = (1+r)(1-\overline{m})$. This delivers a BIU calibration that matches the impact MPC of our HANK.

C Main proofs

C.1 Section 3 proofs

C.1.1 Proof of lemma 4

According to our partial equibrium definition **??**, each household takes z_{it} and maximizes (3) subject to

$$c_{it} + a_{it} = (1 + r_t)\, a_{it-1} + z_{it} \quad \forall t, i \tag{4'}$$

Let $q_t \equiv \prod_{s=0}^{t} \frac{1}{1+r_s}$. Along any realized path, households satisfy

$$(1 + r_0)\, a_{i,-1} + \left(\sum_{t=0}^{\mathcal{T}} q_t z_{it} \right) = \left(\sum_{t=0}^{\mathcal{T}} q_t c_{it} \right) + q_{\mathcal{T}} a_{i\mathcal{T}}$$

Taking expectations at date 0 for a given household, we have,

$$(1 + r_0)\, a_{i,-1} + \left(\sum_{t=0}^{\mathcal{T}} q_t \mathbb{E}_0[z_{it}] \right) = \left(\sum_{t=0}^{\mathcal{T}} \mathbb{E}_0[c_{it}] \right) + q_{\mathcal{T}} \mathbb{E}_0[a_{i\mathcal{T}}]$$

Next, taking the population mean $\mathbb{E}_I$ of both sides gives, using iterated expectations $\mathbb{E}_I[\mathbb{E}_0[\cdot]] = \mathbb{E}_I[\cdot]$, and noting that asset market clearing implies $\mathbb{E}_I\left[a_{i,-1}\right] = b_{-1}$ as well as $\mathbb{E}\left[a_{i,T}\right] = b_{\mathcal{T}}$ (the level of government debt outstanding), we have

$$(1 + r_0)\, b_{-1} + \left(\sum_{t=0}^{\mathcal{T}} q_t \mathbb{E}_I[z_{it}] \right) = \left(\sum_{t=0}^{\mathcal{T}} q_t \mathbb{E}_I[c_{it}] \right) + q_{\mathcal{T}} b_{\mathcal{T}} \tag{76}$$

But, from the definition of labor incomes and constant-y,

$$\begin{aligned} \mathbb{E}_I[z_{it}] &= t_t + (1 - \tau_t)\, \mathbb{E}_I\left[e_{it} n_{it}\right] \\ &= t_t + (1 - \tau_t)\, y \\ &= y - rev_t \\ &= y - g_t + b_t - (1 + r_t)\, b_{t-1} \end{aligned}$$

Now, telescoping the sum

$$\sum_{t=0}^{\mathcal{T}} q_t \left(b_t - (1 + r_t)\, b_{t-1} \right) = -\left(1 + r_0\right) b_{-1} + q_{\mathcal{T}} b_{\mathcal{T}}$$

and therefore (76) is simply

$$\sum_{t=0}^{\mathcal{T}} q_t \left(c_t + g_t - y \right) = 0$$

Hence, in our definition of an EYE $\partial y_t = dc_t^{PE} + \partial g_t$, any shock must satisfy

$$\sum_{t=0}^{\mathcal{T}} q_t d\left(c_t + g_t\right) + \sum_{t=0}^{\mathcal{T}} dq_t \times \left(c_t + g_t - y\right) \quad = \quad 0$$

Hence

$$\sum_{t=0}^{\mathcal{T}} q_t \partial y_t + \sum_{t=0}^{\mathcal{T}} dq_t \times 0 = 0$$

where the first term follows from our definition of ∂y_t and the second term from goods market clearing at every date before the shock.