The Natural Rate of Interest in a Nonlinear DSGE Model

Yasuo Hirose[1] Takeki Sunakawa[2]

[1]Keio University

[2]Kobe University

Fourth Annual CIGS End of Year Macroeconomics Conference

December 27, 2017

Outline

Background

- Since Wicksell (1898), the natural rate of interest has been one of the key concepts of monetary policy analyses.
- Modern New Keynesian (NK) framework makes the concept of the natural rate relevant for welfare (Woodford, 2003; Galí, 2008).
  - Provides a useful benchmark for policymakers to measure the stance of monetary policy.

Background, cont'd

- The natural rate is unobservable and must be estimated.

Short-run estimates based on DSGE models:

  - Andres, Lopez-Salido, and Nelson (2009), Barsky, Justiniano, and Melosi (2014), Curdia (2015), Curdia, Ferrero, Ng, and Tambalotti (2015), Del Negro, Giannone, Giannoni, and Tambalotti (2017), Justiniano and Primiceri (2010), Edge, Kiley, and Laforte (2008), Neiss and Nelson (2003), Chung, Fuentes-Albero, Paustian and Pfajfar (2016)
  - Employ linearized DSGE models that abstract from the ZLB

Long-run estimates based on semi-structural/reduced-form models:

  - Holston, Laubach, and Williams (2016), Johannsen and Mertens (2016), Kiley (2015), Laubach and Williams (2003, 2016), Lubik and Matthes (2015), Pescatori and Turunen (2015), Williams (2015), Lewis and Vazquez-Grande (2017)

What we do

- Estimate the natural rate of interest in the U.S. using a nonlinear New Keynesian DSGE model with the ZLB.

- Examine how and to what extent nonlinearities affect the estimates of the natural rate.
  - The recent experience of the low interest rates has led researchers to conduct an empirical analysis that takes account of the ZLB:

    Gust, Herbst, Lopez-Salido and Smith (2017), Reichter and Throckmorton (2016a; 2016b), Aruoba, Cuba-borda and Shorfheide (2017), Hills, Nakata and Schmitt (2016), Iiboshi, Shintani and Ueda (2017)

What we do: Estimation strategy

- Estimating the nonlinear model is very costly.
- The two-step approach as in Aruoba, Cuba-Borda, and Schorfheide (2017):
  1. Estimate a linearized model using data prior to the date when the nominal interest rate is bounded at zero.
  2. Given the estimated parameters, we solve a fully nonlinear model with the ZLB and apply a nonlinear filter to extract the sequence of the natural rate for a full sample.
- For comparison, the natural rate based on its linear counterpart is also computed.

What we found

- The estimated natural rate based on the nonlinear model is higher than that based on the linearized model in the periods when the nominal interest rate is close to or bounded at zero.
- This difference is mainly ascribed to a contractionary effect of the ZLB which is considered only in the nonlinear model.
- Other nonlinearities, such as price and wage dispersion, have a limited effect on the estimate of the natural rate.

Outline

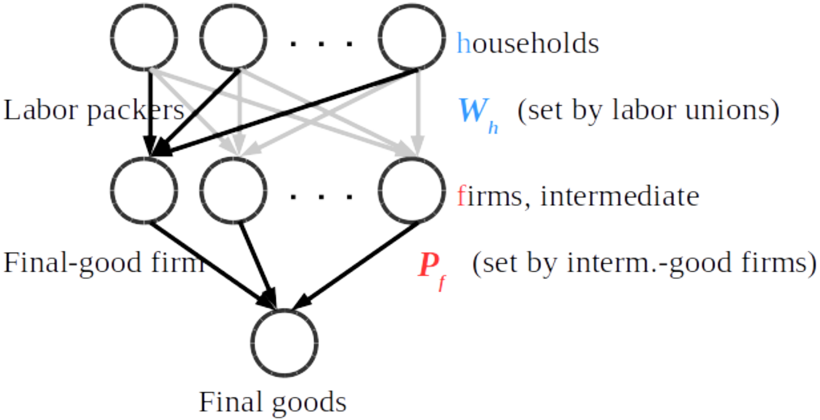
Overview

- A standard empirical NK model with sticky wages and no capital
  1. Households (labor unions, labor packers)
  2. Intermediate-good firms
  3. Final-good firm
  4. Central bank

  - Calvo (1983)-type stickiness in prices and nominal wages.
  - Stochastic trend in TFP.
  - Exogenous shocks to: Productivity, Discount factor, Price markup, and Monetary Policy.

- The natural rate of interest is defined as the real interest rate that would prevail if prices and wages were fully flexible with no price markup shocks.

Overview, cont'd



Households: make consumption & saving decisions
Central bank: set the nominal interest rate

Source of nonlinearity: Price and wage dispersions

- The final-good market clearing condition is

$$Y_t = C_t.$$

- The labor market clearing condition is

$$l_t = \frac{\Delta_{w,t} \Delta_{p,t} Y_t}{A_t},$$

where $l_t = \int_0^1 \int_0^1 l_{f,h,t} df dh$ is the aggregate labor input.

  - $\Delta_{w,t} = \int_0^1 \left(W_{h,t}/W_t\right)^{-\theta_w} dh > 1$: wage dispersion
  - $\Delta_{p,t} = \int_0^1 \left(P_{f,t}/P_t\right)^{-\theta_p} df > 1$: price dispersion

Source of nonlinearity: ZLB

- Monetary policy rule is given by:

$$R_t^n = \max\left[\widehat{R}_t^n, 1\right],$$

$$\widehat{R}_t^n = \left(\widehat{R}_{t-1}^n\right)^{\phi_r}\left[\bar{R}\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{\phi_y}\right]^{1-\phi_r}\exp(\varepsilon_{r,t}).$$

  - The output gap is defined as the ratio of the actual output to the natural output.
  - The ZLB has a contractionary effect, not only when it is binding but also when there is uncertainty about whether it will bind or not in the future.

The natural rate of interest

- The natural output and the natural rate are defined as the ones that would prevail if both prices and wages are perfectly flexible with no price markup shocks:

$$\left(Y_t^* - \gamma Y_{t-1}^*\right)\left(\frac{Y_t^*}{A_t}\right)^{\eta} = \mu A_t,$$

$$R_t^* = \beta^{-1} d_t \left(\mathbb{E}_t \left\{\frac{Y_t^* - \gamma Y_{t-1}^*}{Y_{t+1}^* - \gamma Y_t^*}\right\}\right)^{-1}.$$

  - $Y_t^*$is determined by the sequence of A_t.
  - R_t^* is determined by the sequences of Y_t^* and d_t.
  - The sequences of (A_t, d_t) is indentified by the whole model including other equilibrium conditions.

Outline

Strategy for estimation

- The two-step approach as in Aruoba, Cuba-Borda, and Schorfheide (2017)
  1. Estimate a linearized model using data prior to the date when the nominal interest rate is bounded at zero.
  2. Given the estimated parameters, we solve a fully nonlinear model with the ZLB and apply a nonlinear filter to extract the sequence of the natural rate for a full sample.
- For comparison, the natural rate based on the linearized model is also computed.

Estimation of parameters

- Data: the growth rate of per capita real GDP ($\Delta \log GDP_t$); the inflation rate of GDP deflator ($\Delta \log PGDP_t$); the federal funds rate (FF_t); the log of hours worked ($\log H_t$).

- Sample: 1983:I–2007:IV, up to the date when the nominal interest rate is bounded at zero.

- Observation equations ($y_t = Y_t/A_t$ is the detrended output):

$$\begin{bmatrix} \Delta \log GDP_t \\ \Delta \log PGDP_t \\ FF_t \\ \log H_t \end{bmatrix} = 100 \begin{bmatrix} \bar{a} \\ \bar{\pi} \\ \bar{\pi} + \bar{r} \\ \bar{l} \end{bmatrix} + 100 \begin{bmatrix} \log(y_t/y_{t-1}) + a_t \\ \log(\Pi_t/\Pi) \\ \log(R_t^n/R^n) \\ \log(l_t/l) \end{bmatrix}.$$

- Estimated by Bayesian methods.

- In the subsequent analysis, parameters are fixed at the posterior mode.

Nonlinear solution

- The policy functions:

$$\mathbb{S} = h(\mathbb{S}_{-1}, \tau),$$

where $\mathbb{S}_{-1} = (y_{-1}, \Pi_{-1}, w_{-1}, \widehat{R}^{n}_{-1}, \Delta_{p,-1}, \Delta_{w,-1}, y^{*}_{-1})$; $\tau = (d, a, z, \varepsilon_r)$.

- Computed using the time-iteration method (c.f., Christiano and Fisher 2000; Gust et al. 2017) with a Smolyak approximation of sparse grids.
  - Apply more efficient algorithm developed by Judd, Maliar, Maliar, and Valero (2014).
  - Very accurate, albeit with the reduced number of grid points.

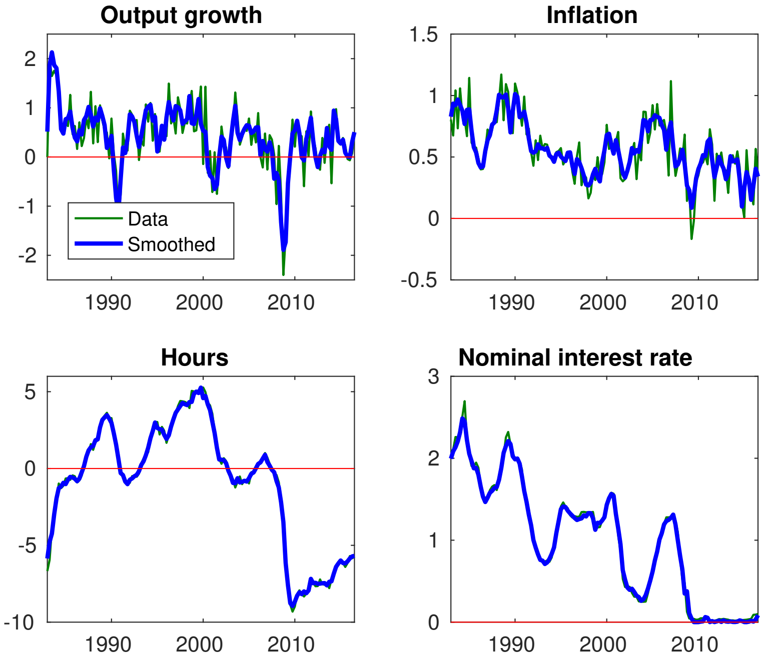
Nonlinear solution

- According to samples simulated from the nonlinear solution, the economy is at the ZLB for 11.8% of quarters, and the average duration of the ZLB spells is 4.3 quarters.
  - Fernández-Villaverde et al. (2015): ZLB prob. = 5.5%; ZLB duration = 2.1 qtrs.
  - Gust et al. (2017): ZLB prob. = 4%; ZLB duration = 3.5 qtrs.

Filtering

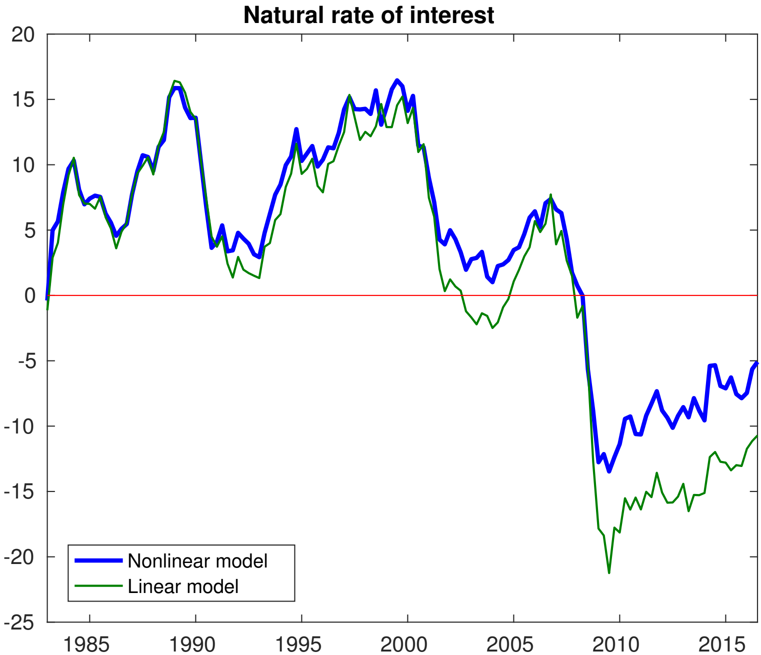
- Apply a boot-strap particle filter as in Fernández-Villaverde and Rubio-Ramirez (2007) and Herbst and Schorfheide (2015).
- The data is the same as those used for the parameter estimation but the sample period is extended to 2016:III.
- The observation equations are the same but with measurement errors. Measurement errors of output growth, inflation, the nominal interest rate, and hours worked are respectively set to 20%, 10%, 5% and 5% of their standard deviations in the data.
- The number of particles is set $N = 20000$.

Data vs. Smoothed estimates



Outline

Natural rate of interest



Some notes on the estimates

- Our estimate of the natural rate ranges from -12% to 15% in the nonlinear model (from -20% to 15% in the linear model).
- Even though the timing of peaks and troughs is similar, the magnitude is a bit larger than the ones in previous studies.
  - Barsky, Justiniano and Melosi (2014): -6% to 12%
  - Del Negro, Giannone, Ginannoni and Tambalotti (2017): -6% to 12%

Identification of the natural rate of interest

- The natural output and the natural rate are determined by:

$$\left(Y_t^* - \gamma Y_{t-1}^*\right)\left(\frac{Y_t^*}{A_t}\right)^{\eta} = \mu A_t,$$

$$R_t^* = \beta^{-1} d_t \left(\mathbb{E}_t\left\{\frac{Y_t^* - \gamma Y_{t-1}^*}{Y_{t+1}^* - \gamma Y_t^*}\right\}\right)^{-1}.$$

  - Y_t^* is determined by the sequence of A_t.
  - R_t^* is determined by the sequences of Y_t^* and d_t.

- Therefore, the natural rate is pinned down by identifying A_t and d_t.

Identification of productivity shock a_t

- In the linearized model, $a_t = \log(A_t/A)$ is identified by the data of output growth and hours worked.
  - Labor market clearing condition: $\tilde{l}_t = \tilde{y}_t$.
  - Observation equations: $\frac{\Delta \log GDP_t}{100} = \bar{a} + \tilde{y}_t - \tilde{y}_{t-1} + a_t$; $\frac{\log H_t}{100} = \bar{l} + \tilde{l}_t$.
- In the nonlinear model, the labor market clearing condition is

$$l_t = \Delta_{p,t} \Delta_{w,t} y_t,$$

where $\Delta_{p,t} \geq 1$ and $\Delta_{w,t} \geq 1$.

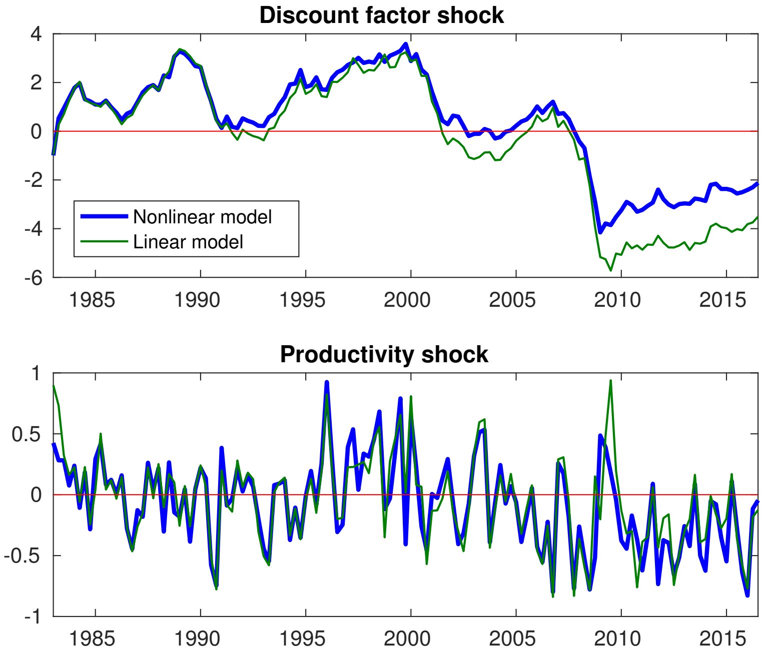
  - y_t becomes smaller and a_t is identified to be larger, which results in the higher natural rate.

Identification of discount factor shock d_t

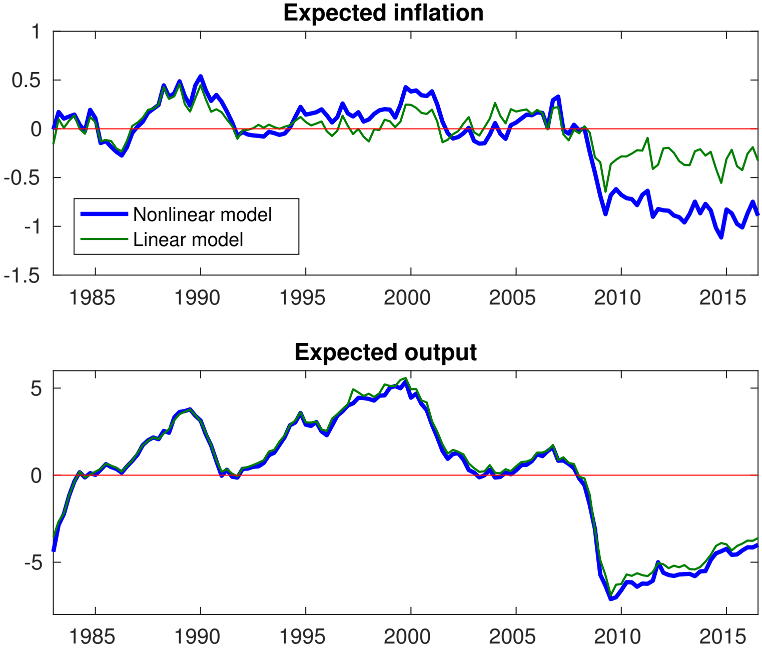
- In the nonlinear model, the ZLB has a contractionary effect:
  - not only when the ZLB is binding,
  - but also when there is uncertainty about whether the ZLB will bind or not in the future (c.f., Hills, Nakata and Schmidt, 2016).
- While such an effect lowers expected output and inflation, actual output and inflation are pegged to the data in the filtering process.
  - d_t must increase, or expectation terms must decrease, to satisfy the consumption Euler equation (as well as the other optimality conditions):

$$\tilde{d}_t \approx \tilde{R}_t^n - \mathbb{E}_t \tilde{\Pi}_{t+1} + \tilde{y}_t - \mathbb{E}_t \tilde{y}_{t+1}$$
$$+ \text{(other terms pegged to the data \& at higher order).}$$

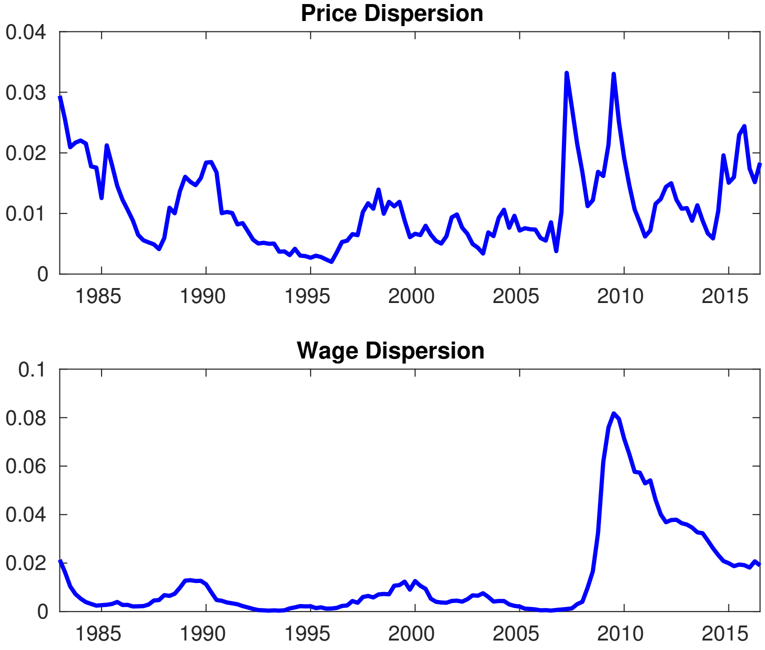
Estimated Shocks



Expected inflation and output



Price and wage dispersions

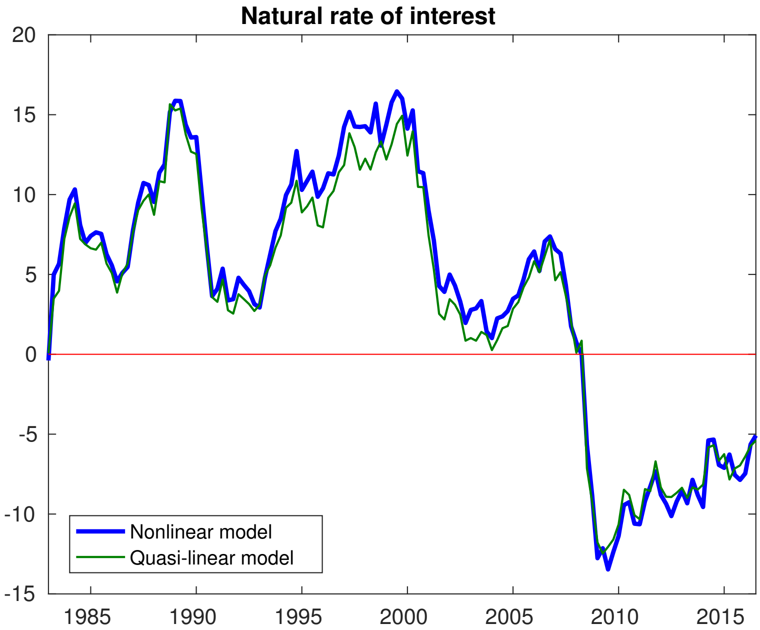


Quasi-linear model

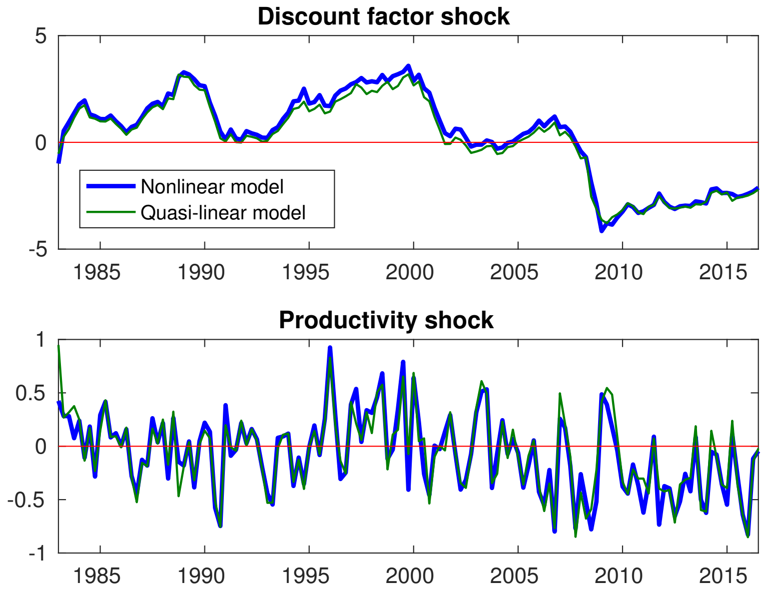
- Consider a quasi-linear model, in which the ZLB constraint is imposed but all the equilibrium conditions are linearized
- Monetary policy rule is replaced with ($\tilde{\cdot}$ stands for log-deviation from the deterministic steady state)

$$\tilde{R}_t^n = \max\left[\tilde{\hat{R}}_t^n, -\log R^n\right],$$
$$\tilde{\hat{R}}_t^n = \phi_r \tilde{\hat{R}}_{t-1}^n + (1-\phi_r)\left(\phi_\pi \tilde{\Pi}_t + \phi_y\left(\tilde{y}_t - \tilde{y}_t^*\right)\right) + \varepsilon_{r,t}.$$

The natural rate of interest: Nonlinear vs. Quasi-linear



Estimated Shocks: Nonlinear vs. Quasi-linear



Outline

Concluding Remarks

- The estimated natural rate is very different from the one estimated with its linear counterpart.
- This difference is mainly explained by a contractionary effect of the ZLB.
- Other nonlinearities than the ZLB, such as price and wage dispersions, have a limited effect.
- Our analysis could be extended by exploiting a medium-scale model and/or estimating the original nonlinear model.

Household

- Each household $h \in [0,1]$ maximizes the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} D_{0,t} \left\{ \log\left(C_{h,t} - \gamma C_{t-1}\right) - \frac{l_{h,t}^{1+\eta}}{1+\eta} \right\},$$

subject to the budget constraint

$$P_t C_{h,t} + B_{h,t} \leq W_{h,t}^n l_{h,t} + R_{t-1}^n B_{h,t-1} + T_{h,t},$$

where $D_{t,t+j} = \prod_{k=t+1}^{t+j} \beta / d_k$ is the cumulative discount factor.

- Discount factor shock:

$$\log d_t = \rho_d \log d_{t-1} + \varepsilon_{d,t},$$

where $\varepsilon_{d,t} \sim N(0, \sigma_d^2)$.

Labor packer

- A labor packer resells the labor package $l_{f,t}$ to intermediate-good firm $f \in [0,1]$ by collecting each differentiated labor $\{l_{f,h,t}\}$ from households so as to maximize profit

$$W_t^n l_{f,t} - \int_0^1 W_{h,t}^n l_{f,h,t} dh,$$

subject to a CES aggregator

$$l_{f,t} \equiv \left(\int_0^1 l_{f,h,t}^{\frac{\theta_w - 1}{\theta_w}} dh \right)^{\frac{\theta_w}{\theta_w - 1}}.$$

Labor union

- Labor unions set nominal wages on a staggered basis à la Calvo (1983) (c.f., Erceg, Henderson and Levin, 2000).
  - In each period, a fraction $1-\xi_w \in (0,1)$ labor unions reoptimize nominal wages, while the remaining fraction ξ_w index nominal wages (adjusted by the economy's deterministic trend $\gamma_a \equiv e^{\bar{a}}$) to a weighted average of Π_{t-1} and $\bar{\Pi}$.
- The unions that reoptimize their nominal wages then maximize expected utility

$$E_t \sum_{j=0}^{\infty} \xi_w^j D_{t,t+j} \left(\frac{\gamma_a^j W_{h,t}^n}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\iota_w} \bar{\Pi}^{1-\iota_w} \right) \Lambda_{h,t+j} l_{h,t+j} - \frac{l_{h,t+j}^{1+\eta}}{1+\eta} \right),$$

where $l_{h,t} \equiv \int_0^1 l_{f,h,t} df$, subject to the labor demand

$$l_{f,h,t+j} = \left[\frac{\gamma_a^j W_{h,t}^n}{W_{t+j}^n} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\iota_w} \bar{\Pi}^{1-\iota_w} \right) \right]^{-\theta_w} l_{f,t+j}.$$

Final-Good Firm

- The representative final-good firm produces output Y_t by choosing intermediate inputs $\{Y_{f,t}\}$ so as to maximize profit

$$P_t Y_t - \int_0^1 P_{f,t} Y_{f,t} df,$$

subject to a CES production technology

$$Y_t = \left(\int_0^1 Y_{f,t}^{\frac{\theta_p - 1}{\theta_p}} df \right)^{\frac{\theta_p}{\theta_p - 1}}.$$

Intermediate-Good Firm

- Each intermediate-good firm f produces differentiated good $Y_{f,t}$ by cost-minimizing labor input $l_{f,t}$ (purchased from labor packer f) subject to the production function

$$Y_{f,t} = A_t l_{f,t}.$$

- A_t is represents total factor productivity which follows

$$\log A_t = \log \gamma_a + \log A_{t-1} + a_t,$$

where $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$ and $\varepsilon_{a,t} \sim N(0, \sigma_a^2)$.

Intermediate-Good Firm, cont'd

- Intermediate-good firms set prices à la Calvo (1983).

- The firms that reoptimize their prices then maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \xi_p^j D_{t,t+j} \frac{\Lambda_{t+j}}{\Lambda_t} \left[\frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} \right) - \frac{W_{t+j}}{A_{t+j}} z_{t+j} \right] Y_{f,t+j},$$

subject to the final-good firm's demand

$$Y_{f,t+j} = \left[\frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} \right) \right]^{-\theta_p} Y_{t+j}.$$

- Cost-push shock:

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_{z,t},$$

where $\varepsilon_{z,t} \sim N(0, \sigma_z^2)$.

Market clearing conditions

- The final-good market clearing condition is

$$Y_t = C_t.$$

- The labor market clearing condition is

$$l_t = \frac{\Delta_{p,t}\Delta_{w,t}Y_t}{A_t},$$

where $l_t = \int_0^1 \int_0^1 l_{f,h,t} df dh$ is aggregate labor input,

  - $\Delta_{p,t} = \int_0^1 \left(P_{f,t}/P_t\right)^{-\theta_p} df > 1$ represents price dispersion, and
  - $\Delta_{w,t} = \int_0^1 \left(W_{h,t}/W_t\right)^{-\theta_w} dh > 1$ represents wage dispersion.

Closing the model

- Monetary policy rule is given by:

$$R_t^n = \max\left[\widehat{R}_t^n, 1\right],$$

$$\widehat{R}_t^n = \left(\widehat{R}_{t-1}^n\right)^{\phi_r}\left[\bar{R}\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{\phi_y}\right]^{1-\phi_r}\exp(\varepsilon_{r,t}),$$

where monetary policy shock is $\varepsilon_{r,t} \sim N(0, \sigma_r^2)$.

- Since A is nonstationary, some variables are detrended such as $y = Y/A$, $w = W/A$ and $y^* = Y^*/A$.

Linearized model

$$
\begin{aligned}
\tilde{y}_t &= \frac{\gamma_a}{\gamma_a+\gamma}\left(\mathbb{E}_t\tilde{y}_{t+1}+a_{t+1}\right)+\frac{\gamma}{\gamma_a+\gamma}\left(\tilde{y}_{t-1}-a_t\right) \\
&\quad -\frac{\gamma_a-\gamma}{\gamma_a+\gamma}\left(\tilde{R}_t-\mathbb{E}_t\tilde{\Pi}_{t+1}-\tilde{d}_t\right), \\
\tilde{\Pi}_t &= \frac{\beta}{1+\beta\iota_p}\mathbb{E}_t\tilde{\Pi}_{t+1}+\frac{\iota_p}{1+\beta\iota_p}\tilde{\Pi}_{t-1}+\kappa_p\left(\tilde{w}_t+\tilde{z}_t\right), \\
\tilde{\Pi}_{w,t} &= \beta\left(E_t\tilde{\Pi}_{w,t+1}+a_{t+1}-\iota_w\tilde{\Pi}_t\right)+\iota_w\tilde{\Pi}_{t-1}-a_t \\
&\quad +\kappa_w\left[\left(\eta+\frac{\gamma_a}{\gamma_a-\gamma}\right)\tilde{y}_t-\frac{\gamma}{\gamma_a-\gamma}\tilde{y}_{t-1}+\frac{\gamma}{\gamma_a-\gamma}a_t-\tilde{w}_t\right], \\
\tilde{w}_t &= \tilde{\Pi}_{w,t}-\tilde{\Pi}_t+\tilde{w}_{t-1}, \\
\tilde{R}^n_t &= \phi_r\tilde{R}^n_{t-1}+(1-\phi_r)\left(\phi_\pi\tilde{\Pi}_t+\phi_y\left(\tilde{y}_t-\tilde{y}^*_t\right)\right)+\varepsilon_{r,t},
\end{aligned}
$$

where $\kappa_p \equiv \frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p(1+\beta\iota_p)}$ and $\kappa_w \equiv \frac{(1-\xi_w)(1-\beta d^{-1}\xi_w)}{\xi_w(1+\eta\theta_w)}$.

Priors and posteriors

	Priors			Posteriors		
Param.	Dist.	Mean	S.D.	Mode	Mean	90%
η	G	2.000	0.250	1.782	1.796	[1.381, 2.194]
γ	B	0.500	0.050	0.684	0.685	[0.633, 0.736]
ξ_p	B	0.500	0.050	0.737	0.740	[0.698, 0.790]
ι_p	B	0.500	0.050	0.514	0.515	[0.429, 0.596]
ξ_w	B	0.500	0.050	0.618	0.603	[0.509, 0.701]
ι_w	B	0.500	0.050	0.526	0.525	[0.440, 0.610]
ϕ_π	G	1.500	0.250	2.075	2.053	[1.732, 2.376]
ϕ_y	G	0.125	0.050	0.096	0.097	[0.058, 0.137]
ϕ_r	B	0.500	0.050	0.784	0.770	[0.750, 0.790]
$\bar{a}$	N	0.540	0.100	0.484	0.487	[0.384, 0.587]
$\bar{\pi}$	N	0.624	0.100	0.634	0.634	[0.550, 0.717]
$\bar{r}r$	G	0.699	0.100	0.675	0.677	[0.581, 0.772]
$\bar{h}$	N	0.000	0.100	-0.001	-0.001	[-0.163, 0.149]

Priors and posteriors, cont'd

	Priors			Posteriors		
Param.	Dist.	Mean	S.D.	Mode	Mean	90%
ρ_d	B	0.500	0.050	0.691	0.681	[0.627, 0.740]
ρ_a	B	0.500	0.050	0.402	0.406	[0.339, 0.475]
ρ_p	B	0.500	0.050	0.664	0.659	[0.577, 0.743]
$100\sigma_d$	IG	0.500	2.000	0.635	0.673	[0.499, 0.848]
$100\sigma_a$	IG	0.500	2.000	0.520	0.531	[0.463, 0.595]
$100\sigma_r$	IG	0.500	2.000	0.145	0.152	[0.134, 0.172]
$100\sigma_p$	IG	0.500	2.000	1.901	2.050	[1.380, 2.753]