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The Liquidity Coverage Ratio, Mortgage-Backed Securities, and Mortgage-Market Instability

Chris Mitchell*

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Abstract

Under Basel III, banks are subject to a new constraint known as the Liquidity Coverage Ratio (LCR). The LCR is meant to ensure that banks have a sufficient quantity of high quality liquid assets (HQLA) that can be sold during periods of funding stress to meet outflows. In many jurisdictions, mortgage-backed securities (MBS) are an important asset for meeting the LCR. This may result in mortgage-market instability, however, if banks subsidize mortgage credit during periods of high HQLA demand – to acquire additional MBS – and subsequently remove this subsidy once HQLA demand wanes. This paper develops a model of banking to explore this mechanism. The primary result is that intertemporal variation in HQLA requirements, stemming from intertermporal variation in corporate-loan demand, may lead to mortgage and housing market instability.

Keywords: Banking, Mortgage Market, Regulation

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*Institute of Social and Economic Research, Osaka University, 6-1, Mihogaoka, Ibaraki, Osaka 567-0047, Japan. Email: mitchell@iser.osaka-u.ac.jp.

1 Introduction

During the recent financial crisis there was a systematic run on wholesale funding (McCabe 2010 and Covitz et al. 2013). In response to this, the Basel Committee on Banking Supervision (BCBS) introduced a new global liquidity standard as part of Basel III. This standard, known as the Liquidity Coverage Ratio (LCR), is meant to ensure that banks have a sufficient quantity of high quality liquid assets (HQLA) that can be sold during periods of funding stress to meet outflows over a period of 30 calendar days, thereby preempting future runs, or mitigating their adverse consequences (see Liebmann and Peek 2015 for a discussion). However, as with any new regulation, the LCR will have unanticipated consequences: this paper develops a model of banking to illustrate how mortgage-market instability may arise from the preferential treatment of mortgage-backed securities (MBS) under the LCR.

The treatment of an HQLA-eligible asset has two dimensions: its haircut, and its inclusion limit. Both of which are determined in accordance with the asset's expected liquidity during periods of funding stress. In general, public-sector debt and central-bank liabilities have the lowest haircuts (zero in most cases), and can be included without limit, while qualifying corporate debt (rated BBB- or higher) and common equity shares receive a 50% haircut, and are subject to a 15% aggregate inclusion limit.[1] Under the original LCR proposal (BCBS 2010), MBS (including covered bonds) received generous haircuts of between 15% - 25%, and inclusion limits of between 15% - 40%. However, and crucial for this paper, many G7 countries are granting MBS even more favourable treatment; haircuts have been reduced to 0% - 7%, and inclusion limits have been increased to 70% - 100%. Taken together, MBS is the most favourable source of private-sector HQLA for meeting the LCR.

The mechanism formalized in this paper accounts for this preferential treatment, and can be described as follows. Banks in the model have a menu of assets for meeting the LCR, and profit maximization ensures that the most cost-effective set is chosen. Due to the

[1] An exception is made for qualifying corporate debt rated AA- or higher. These securities receive a 15% haircut, and 40% inclusion limit.

favourable treatment of MBS under Basel III, this security is preferred. When banks experience intertemporal variation in HQLA requirements - stemming from loan-demand-driven funding shocks, possibly through the business cycle - the marginal value of MBS adjusts in the same direction. This, in turn, has a bearing on the marginal value of mortgage credit, which is a composite-good under the LCR; its value derives from both principle and interest payments, and its use as HQLA once securitized. When banks experience favourable loan-demand conditions, their funding and HQLA requirements are high. This leads to a high marginal value of MBS, and a low equilibrium mortgage interest rate (discussed in Section 3). Households respond to this by acquiring large mortgage balances, making them susceptible to future interest-rate shocks (discussed in Section 4). This build-up of risk materializes once loan-demand dissipates, and the marginal value of MBS becomes zero for LCR adherence. As a consequence, the equilibrium mortgage interest rate increases, and some households default in equilibrium. This depresses real-estate prices, and results in financial-sector losses (discussed in Section 5).

The aforementioned mechanism has a similar propagation channel to the one operating during the recent sub-prime crisis. In that crisis, innovations to mortgage securitization during the early 2000's resulted in lower interest rates and looser underwriting standards (Brunnermeier 2009). This culminated in widespread mortgage default once sub-prime rates rose, and house prices fell (Mayer et al. 2009). Like the sub-prime crisis, the mechanism discussed herein has a similar potential for mortgage-market instability. Furthermore, the fall in house prices predicted by the model will likely have a first-order impact on consumption spending through a housing-wealth effect (see Case et al. 2013, Campbell and Cocco 2007, and Bhatia and Mitchell 2016 for US, UK, and Canadian evidence, respectively).

This paper is related to a small theoretical literature analyzing liquidity regulation. Within which is Bech and Keister (2013), who argue that monetary policy is less effective under an LCR. In their model, banks have a preference for settling interbank short positions with term funding, rather than overnight funding, as the former relaxes the LCR constraint. In this environment, central banks have difficulty influencing policy (overnight) rates through open-market operations. Balasubramanyan and VanHoose (2013), who study deposit stability under the LCR, provide mixed results. In certain cases analyzed, the LCR promotes stability, while in others, it does the opposite. In the model of Malherbe (2014), liquidity regulation has the paradoxical effect of reducing market liquidity during periods of funding stress. In his model, higher ex-ante liquidity increases the likelihood that asset-sales are used to offload under-performing securities, and the resulting adverse-selection problem reduces ex-post liquidity. Contrary to these papers, the conclusions drawn in Adrian and Boyarchenko (2013) are more supportive of the LCR. In their model, capital and liquidity regulation are substitutes for promoting financial stability. While the former involves a trade-off between risk reduction and consumption growth, the latter does not, and is preferred as a result. The current paper offers a new perspective on liquidity regulation by analyzing individual HQLA assets, and the inefficiencies that arises from their unique treatment.

There is also a small empirical literature analyzing the balance-sheet consequences of liquidity regulation. These studies have primarily relied on English and Dutch data, given each country's preexisting liquidity regulation.[2] Among these is Banerjee and Mio (2015), who find that English banks respond to liquidity regulation by substituting HQLA-eligible assets for short-term intra-financial loans, and also substitute "stable" non-financial deposits for "less stable" short-term wholesale funding. Bonner (2014) finds similar results for Holland, specifically, Dutch banks substitute highly-liquid sovereign debt for less-liquid private-sector debt. These results are consistent with the current model's prediction that banks adjust asset portfolios - in favor of HQLA-eligible securities - in response to the LCR.

The remainder of this paper is organized as follows. Section 2 discusses the LCR. Section 3 presents the banking model. Section 4 develops a model of households. Section 5 discusses the LCR's destabilizing property, and Section 6 concludes.

[2]England: Individual Liquidity Guidance, and Holland: Dutch Liquidity Requirement.

2 The Liquidity Coverage Ratio

Once the LCR is fully implemented in 2019, the stock of HQLA held by a bank must be weakly greater than its expected net cash outflows during a 30-day funding-stress scenario prescribed by the BCBS (BCBS 2010 and 2013a). To minimize undue economic disruption, there is a 5-year phase-in period; the LCR's statutory minimum is set to 60% in 2015, and increases by 10% in each subsequent year until reaching 100% in 2019. A number of jurisdictions have accelerated this process, however. Canada implemented a 100% LCR in 2015, while the US and EU are expected to do likewise by 2017 and 2018, respectively. In addition, banks are likely to hold excess HQLA to reduce the chance of non-compliance. Evidence of this is provided by de Haan and van den End (2013), and the results of Schmaltz et al. (2014) predict LCR buffers of between 50% - 80%.

The LCR is commonly written as:

$$\frac{\text{Stock of HQLA}}{\text{Total Net Cash Outflows Over the Next 30 Calender Days}} \geq 100\%.$$

When calculating a bank's stock of HQLA, haircuts are applied to each permissible asset at rates commiserate with its expected liquidity. In general, public-sector debt and central-bank liabilities have the lowest haircuts (zero in most cases), followed by MBS, highly-rated corporate debt, and common equity shares. These haircuts are reported in Table 1 of Appendix A. Assets are also categorized according to their inclusion limits. Level 1 assets (such as government debt) generally have no limit, while Level 2A and 2B assets have inclusion limits of 40% and 15%, respectively (and cannot exceed 40% combined). These limits are also reported in Table 1 of Appendix A.

Among private-sector HQLA, MBS receives the most favorable treatment under the LCR. In addition to being the only asset-backed security with global HQLA eligibility, many G7 countries are treating MBS in a fashion similar to that of government debt and central bank liabilities. Key examples include the full Level 1 status of most Canadian

MBS,[3] and the EU's reclassification of AA- rated covered bonds as Level 1 HQLA (with a 93% inclusion rate and 70% aggregate cap).[4] As argued below, the preferential treatment of MBS creates a market distortion with negative consequences for mortgage-market stability.

The second component of the LCR is its denominator, which depends on expected net cash outflows (i.e., gross outflows minus gross inflows) over a 30-day funding stress event. Gross outflows are determined by multiplying a bank's outstanding liabilities by "run-off rates" prescribed by the BCBS. These are reported in Table 2 of Appendix A. Retail deposits have the lowest run-off rates, generally ranging from 3% to 10%, while those of wholesale funding are higher, and generally range from 10% to 100%.[5] Banks may partially offset gross outflows by subtracting a part of contractual inflows (within the same 30-day period), up to a maximum of 75% of outflows. However, in practice, these offsets are likely to be small for most banks.[6]

To simplify the model presented below, there are only two HQLA-eligible assets: government debt; and MBS, and both may be included without limit. In addition, a bank's HQLA requirement depends on its gross outflows only (i.e., contractual inflows are not explicitly accounted for). However, as discussed below, each liability-type funds a specific asset-type, and as such, the reader may interpret each liability-specific run-off rate as incorporating an asset-specific inflow rate.

[3] The Canadian MBS market is highly concentrated in National Housing Act Mortgage Backed Securities and Canada Mortgage Bonds. Both are fully-backed by the Canadian government.

[4] Alternatively, the US has not granted government-sponsored-enterprise debt (i.e., Fannie Mae and Freddie Mac) Level 1 status.

[5] The exceptions are: secured funding with central banks (0% run-off); funding backed by Level 1 assets (0%); small business deposits deemed "stable" (5%); and insured operational deposits generated by clearing, custody and cash management activities (5%).

[6] The "inflow rate" on contractual cash inflows (i.e., the portion of inflows that may be subtracted from outflows) is relatively small for most inflow types.

3 Banks and the Supply of Mortgage Credit

This section presents the banking model. It is used to derive mortgage supply curves, which are subsequently used in Section 5 to derive mortgage-market equilibria.

3.1 Banking Model Set-Up

There is a measure B of identical banks indexed by i, which operate competitively in the mortgage market. Banks acquire four types of one-period assets: mortgage loans (I^m), mortgage-backed securities (MBS), government debt (I^g), and corporate loans (I^c). In each period t, the banking sector faces aggregate mortgage demand of $I_t^m(r_t^m)$ (derived in Section 4) where r_t^m is the competitive mortgage interest rate. Banks acquire full mortgage insurance on all loans at the unit cost c^I. This simplifies the analysis by abstracting from mortgage-loan uncertainty - from the bank's perspective - (see BCBS 2013b for a discussion of international mortgage-insurance regimes). In each period t, the banking sector is endowed with a stochastic quantity of corporate loans. These loans are divided equally among the banks, and always generate positive profits, which are characterized by the function: $\Pi^c(I^c) > 0 \ \forall \ I^c > 0$. Government debt is elastically-supplied at the interest rate r^g, and banks create MBS by securitizing mortgage credit at the unit cost c^M.

Banks finance mortgage loans, corporate loans, and government debt in unique funding markets. This assumption reflects the work of Damar et al. (2015), which argues that banks generally fund assets with specific liabilities; typically using retail deposits to fund mortgages, and wholesale funding for corporate loans. All three funding markets have perfectly elastic supply at the interest rates r^{fm}, r^{fc}, and r^{fg}, respectively, and corresponding run-off rates of θ^m, θ^c, θ^g, respectively. Furthermore, governments have a funding advantage over banks (i.e., $r^{fg} > r^g$). In practice, this funding advantage stems from the superior credit quality and liquidity of highly-rated government debt.[7] Finally, banks finance MBS creation with internal resources, which are always sufficient. Note that MBS in this model is

[7] See, for example, Beber et al. 2009 and Krishnamurthy and Vissing-Jorgensen 2012.

not used for funding, rather, it is used exclusively for HQLA.

To reflect the central role that government securities plays in modern financial systems, it is also assumed that banks derive an implicit value from holding some government debt. This value is represented by the function $F(I^g)$, which satisfies the Inada conditions.

3.2 The Bank's Problem and Results

Banks seek to maximize the following "augmented" profit function[8] in each period t (with embedded budget constraints), with respect to the arguments $I^m_{i,t}$, $I^g_{i,t}$, and $MBS_{i,t}$:

$$Max_{\{I^m_{i,t}, I^g_{i,t}, MBS_{i,t}\}} \Pi_{i,t} = (r^m_t - r^{fm} - c^I)I^m_{i,t} + (r^g - r^{fg})I^g_{i,t} + F(I^g_{i,t}) - c^M MBS_{i,t} + \Pi^c(I^c_{i,t}),$$

subject to the following three inequality constraints:[9]

1. LCR constraint:

$$\frac{(\psi^g I^g_{i,t} + \psi^m MBS_{i,t})}{(\theta^m I^m_{i,t} + \theta^c I^c_{i,t} + \theta^g I^g_{i,t})} \geq L,$$

2. Mortgage loan constraint:

$$MBS_{i,t} \leq \delta I^m_{i,t},$$

3. Non-negative MBS constraint:

$$MBS_{i,t} \geq 0,$$

where $\delta \; \epsilon \; (0, 1]$ is the fraction of mortgage credit that can be securitized.

[8] It is "augmented" due to the presence of $F(I^g)$.

[9] Implicitly, the constraints $I^m_{i,t} \geq 0$ and $I^g_{i,t} \geq 0$ are also imposed. Both will hold in equilibrium.

The following two propositions characterize the set of equilibria in the limit as the cost of insurance (c^I) and the cost of securitization (c^M) go to zero.[10]

Proposition 1 *In the absence of an LCR constraint (i.e., $L = 0$), the competitive equilibrium interest rate is r^{fm}, and banks extend:*

$$I^m_{i,t} = \frac{I^m_t(r^{fm})}{B}$$

units of mortgage credit, while acquiring:

$$I^g_{i,t} = F'^{-1}(\Delta r^g)$$

units of government debt (where $\Delta r^g = r^{fg} - r^g > 0$). Furthermore, banks create no MBS. For a proof of these results see Appendix B.

Proposition 2 *In the presence of an LCR constraint (i.e., $L > 0$), the competitive equilibrium interest rate is:*

$$r^m_t = \begin{cases} r^{fm} & \text{if } I^c_t < \Omega, \\ r^{fm} - \frac{\delta\psi^{MBS} - L\theta^m}{\psi^G - L\theta^g}\left[\Delta r^g - F'(I^g_{i,t})\right] & \text{if } I^c_t \geq \Omega \end{cases}$$

where:

$$\Omega = \frac{B(\psi^g - L\theta^g)F'^{-1}(\Delta r^g) + (\delta\psi^m - L\theta^m)I^m_t(r^m_t)}{L\theta^c}.$$

Banks extend:

$$\frac{I^m_t(r^m_t)}{B}$$

units of mortgage credit, acquire:

[10] In addition to simplifying the exposition, assuming that c^M tends to zero is needed to guarantee the existence of a competitive equilibrium. The same is not needed of c^I.

$$I_{i,t}^{g} = \begin{cases} F^{'-1}(\Delta r^{g}) & \textit{if } I_t^c < \Omega \\ \frac{L\theta^c I_t^c - (\delta\psi^M - L\theta^m) I_t^m(r_t^m)}{B(\psi^g - L\theta^g)} & \textit{if } I_t^c \geq \Omega \end{cases}$$

units of government debt, and create:

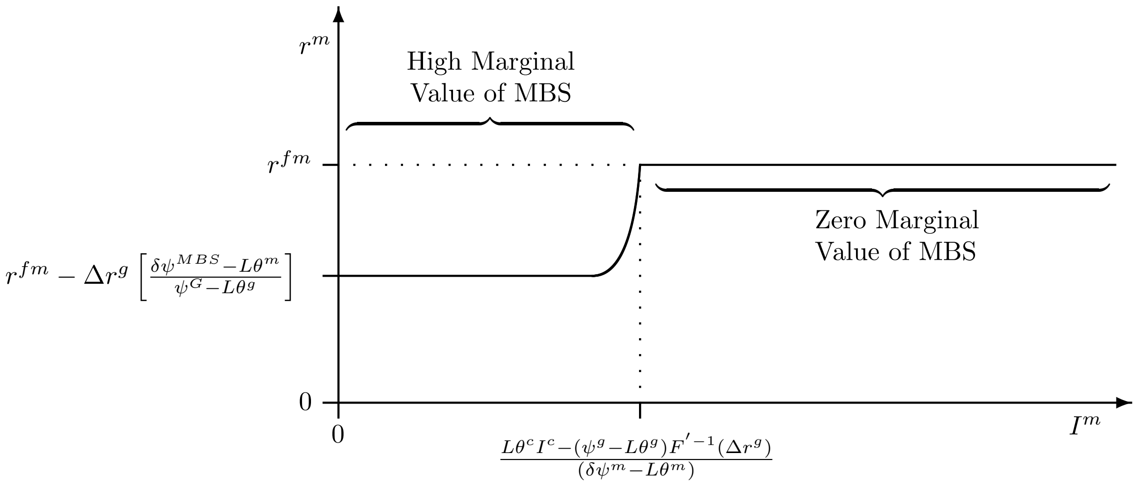
$$MBS_{i,t} = \begin{cases} \frac{\frac{L}{B}[\theta^c I_t^c + \theta^m I_t^m(r_t^m)] - (\psi^g - L\theta^g) F^{'-1}(\Delta r^g)}{\psi^m} & \textit{if } I_t^c < \Omega \\ \delta \frac{I_t^m(r_t^m)}{B} & \textit{if } I_t^c \geq \Omega \end{cases}$$

units of MBS. For a proof of these results see Appendix B.

According to Proposition 1, in the absence of an LCR constraint, mortgage credit is always supplied at a bank's marginal cost of funding, and banks hold "minimum quantities" of government debt (i.e., $I_{i,t}^g = F^{'-1}(\Delta r^g)$). That is, government debt is acquired until its marginal value equals the interest rate differential (Δr^g). Conversely, according to Proposition 2, in the presence of an LCR constraint, mortgage interest rates depend on the realized value of I_t^c. When I_t^c is relatively small (first case), HQLA requirements are satisfied through minimum government debt and MBS. Consequently, the marginal value of MBS is zero, and mortgage credit is supplied at its marginal cost (i.e., $r_t^m = r^{fm}$). When I_t^c is relatively large, marginal MBS is used to reduce costly government debt, which was acquired to meet elevated HLQA requirements stemming from high realizations of I_t^c (i.e., $I_t^g > F^{'-1}(\Delta r^g)$). Consequently, MBS has a strictly positive marginal value, and banks subsidize mortgage credit (i.e., $r_t^m < r^{fm}$) to acquire additional quantities.

Figure 1 depicts the mortgage supply curve arising from Proposition 2 (for a particular realization of I_t^c). The left-hand-side corresponds to the second case of Proposition 2, where marginal MBS is used to reduce costly government debt. In this case, mortgage credit is subsidized. The right-hand-side of Figure 1 corresponds to the first case of Proposition 2, where the marginal value of MBS is zero. In this case, mortgage credit is unsubsidized. For this particular illustration, the implicit marginal value of government debt approaches zero quickly. This assumption is maintained throughout the paper, and implies a near-horizontal lower line, and a steep curve connecting the lower and upper horizontal line segments. Conversely, the mortgage supply curve arising from Proposition 1 is everywhere horizontal.

Figure 1: Mortgage Supply Curve when $L > 0$



This figure plots the mortgage supply curve arising from Proposition 2. The left-hand-side corresponds to the second case of Proposition 2, in which mortgage credit is subsidized. The subsidy approaches $\Delta r^g[(\delta\psi^{MBS} - L\theta^m)/(\psi^G - L\theta^g)]$ as $I^m \to 0$. The right-hand-side corresponds to the first case of Proposition 2, in which mortgage credit is unsubsidized.

In practice, the mortgage subsidy depends on a number of factors, including a bank's regulatory jurisdiction and prevailing market conditions. However, Proposition 2 can be used to derive a first-order approximation to this subsidy, once the relevant parameter values are known.

4 Households and the Demand for Mortgage Credit

To complete the analysis, we require mortgage-demand curves, and housing-market equilibrium. Towards this end, Subsection 4.1 develops a simple model of households, which is used in Subsection 4.2 to derive aggregate mortgage demand. Subsection 4.3 goes on to characterize the housing-market equilibrium, while Subsection 4.4 derives conditions under which households default in equilibrium.

4.1 Households

Consider an overlapping-generations model, where ex-ante homogeneous households live for T periods. Each cohort has a unit measure of households indexed by j, where j is their period of birth. Each household has N income-generating periods, where $T = N + 1$ is the age of retirement. Households are endowed with Y units of income in each income-generating period, and of this, they spend the fraction $\alpha \in (0, 1)$ on non-discretionary consumption, and the remaining $1 - \alpha$ on housing. Households purchase homes in the first period of their lives using mortgages with initial loan-to-value ratios of 1.[11] In subsequent periods, households refinance mortgage credit to borrow the maximum amount possible subject to their intertemporal budget constraints. These funds are use to purchase additional housing. Upon retirement, households liquidate their real-estate, and consume the proceeds.

Notice that households do not have explicit intertemporal preferences over consumption and housing services. Rather, their demand for each is summarized by the parameters Y and α. This simplifies the analysis while maintaining the fundamental result that mortgage-interest-rate shocks may lead to mortgage default (see Campbell and Cocco 2015 for a theoretical underpinning of this result).

4.2 Mortgage Contracts and the Demand for Mortgage Credit

All mortgages in the model have fixed-interest one-period terms. These are similar to variable-rate mortgages with one period lock-ins. Using the standard mortgage-payment calculator, we can determine each cohort $j^{'s}$ period-t mortgage payment ($M^P_{j,t}$) as a function of its mortgage principle ($I^m_{j,t}$), remaining amortization period ($A_{j,t}$), and interest rate (r^m_t), as follows:

$$M^P_{j,t} = I^m_{j,t} \frac{r^m_t (1 + r^m_t)^{A_{j,t}}}{(1 + r^m_t)^{A_{j,t}} - 1}. \tag{1}$$

Taking the inverse of Equation 1, and substituting $(1 - \alpha)Y$ for $M^P_{j,t}$, produces cohort $j^{'s}$

[11] It is assumed that households enter the world with no capital.

period-t mortgage demand:

$$I^m_{j,t} = (1-\alpha)Y\frac{[(1+r^m_t)^{A_{j,t}}-1]}{r^m_t(1+r^m_t)^{A_{j,t}}}. \tag{2}$$

It was assumed that households borrow the maximum amount possible subject to their intertemporal budget constraints. This implies that a household's amortization period equals its number of remaining income-generating periods (i.e., $A_{j,t} = N + j - t$). From this, Equation 2 can be rewritten as:

$$I^m_{j,t} = (1-\alpha)Y\frac{[(1+r^m_t)^{N-t+j}-1]}{r^m_t(1+r^m_t)^{N-t+j}}, \tag{2.1}$$

$$s.t.\ j > t - N,$$

where the condition $j > t - N$ ensures that only households with future income-generating periods obtain mortgage credit. Equation 2.1 implicitly assumes that households condition mortgage demand on current interest rates only, i.e., they do not factor-in possible interest-rate changes. This stems from the fixed-housing-expenditure assumption, and is consistent with recent empirical studies on mortgage demand (see, for example, Miles 2004 and Bucks and Pence 2008).

From Equation 2.1, we can derive the aggregate mortgage-demand curve in period t (I^m_t), by summing across all income-generating cohorts. This produces:

$$I^m_t = \sum_{j=t-N+1}^{t}\frac{[(1+r^m_t)^{(N-t+j)}-1](1-\alpha)Y}{r^m_t(1+r^m_t)^{(N-t+j)}}. \tag{3}$$

Aggregate demand for mortgage credit is a decreasing function of r^m_t, and an increasing function of Y.

4.3 Housing Market

Aggregate expenditure on housing in period t (E_t^H) is the summation of new mortgage credit (Equation 3), and the accumulated housing equity of non-retired homeowners:[12]

$$E_t^H = I_t^m + \sum_{j=t-N+1}^{t\text{-}1} \left[P_t^H h_{j,t} - MB_{j,t}\right], \tag{4}$$

where P_t^H is the unit price of housing, $h_{j,t}$ is cohort $j^{'s}$ stock of housing at the beginning of period t, and $MB_{j,t}$ is cohort $j^{'s}$ beginning-of-period mortgage balance, which equals:

$$MB_{j,t} = (1 + r_{t-1}^m) I_{j,t-1}^m - (1 - \alpha) Y. \tag{5}$$

It should be noted that $I_{j,t}^m$ and $MB_{j,t}$ are not the same quantity. The former is cohort $j^{'s}$ new mortgage balance - based on prevailing interest rates - while $MB_{j,t}$ is cohort $j^{'s}$ remaining principle on last period's mortgage.

The housing demand curve can be derived by rearranging Equation 4 as follows:

$$H_t^D = \frac{1}{P_t^H}\left[I_t^m - \sum_{j=t-N+1}^{t-1} MB_{j,t}\right] + \sum_{j=t-N+1}^{t-1} h_{j,t}. \tag{6}$$

H_t^D is a decreasing function of price, and an increasing function of both aggregate mortgage credit and housing equity, where the latter quantity is captured by the share of housing owned by non-retired homeowners (i.e., the $h_{j,t}$ terms), and their outstanding mortgage balances ($MB_{j,t}$).

To derive housing-market equilibrium, it is assumed that total housing is fixed at $\overline{H}$ units, and perfectly divisible. From this and Equation 6, if follows that Equation 7 characterizes the market-clearing price of housing in period t:

[12] Recall that newly-retired homeowners liquidate their real-estate, and do not contribute to housing demand. Also, cohort $j = t$ does not own a house at the beginning of period t, and thus, has no equity.

$$P_t^H = \frac{1}{\overline{H} - \sum_{j=t-N+1}^{t-1} h_{j,t}} \left[I_t^m - \sum_{j=t-N+1}^{t-1} MB_{j,t} \right]. \tag{7}$$

P_t^H is an increasing function of aggregate mortgage credit and the fraction of housing owned by non-retired homeowners. Conversely, it is a decreasing function of aggregate mortgage balances.

4.4 Mortgage Default

Households in the model can default in equilibrium. This occurs when housing equity is negative *and* households cannot roll-over mortgage credit (discussed below).[13] This necessitates an assumption regarding access to mortgage credit post-bankruptcy. Empirical evidence on this issue is somewhat mixed. However, recent studies by Han and Li (2011) and Cohen-Cole et al. (2013) provide evidence that households are able to regain access to secured-credit markets shortly after bankruptcy. In light of this, defaulting households reenter the mortgage market immediately after bankruptcy with a clean slate. This softens the the model's primary result by adding additional house-price support.

Households never default in a steady state, that is, when interest rates are constant, since $I_{j,t}^m = MB_{j,t} \; \forall \; j$ (from Equations 2.1 and 5). However, households may default when interest rates increase, owing to two reinforcing effects. First, outstanding mortgage balances cannot be refinanced with new mortgage credit, owing to each household's reduced borrowing capacity (Equations 2.1 and 5), and second, the lower quantity of aggregate mortgage credit places downward pressure on house prices (Equation 7). This makes negative housing equity and default more likely.[14] The interest-rate change necessary for mortgage default is cohort-specific, owing to differences in housing equity and amortization lengths. Proposition 3 provides a convenient approximation to the interest-rate change necessary for

[13]The second condition rules-out strategic default (see Kau and Keenan 1995 for a discussion). That is, a necessary condition for mortgage default is the presence of liquidity constraints.

[14]The inability to refinance mortgage credit for any interest rate increase follows from the fixed-housing-expenditure assumption (i.e., $(1-\alpha)Y$). In practice, most households have some financial slack. This will alleviate the problem somewhat.

default when the mortgage and housing markets are in a steady state:

Proposition 3 *Households in cohort j default in equilibrium whenever interest rates increase by (approximately):*

$$\Delta r(j) = \frac{\sum\limits_{\{K:N\geq K>N-t+j+1\}} \eta(j)K + (\eta(j)-1)(N-t+j) + \eta(j)}{\sum\limits_{K=1}^{N} \eta(j)\left[\frac{K^2+K}{2}\right]}, \tag{8}$$

where:

$$\eta(j) = \frac{h_{j,t}}{\overline{H} - \sum\limits_{k=t-N+1}^{j} h_{k,t}}$$

is cohort $j^{'s}$ share of housing, divided by the number of unoccupied housing units (i.e., homes previously owned by newly-retired households, and those of defaulting households). For a proof of this result, see Appendix D.

From Proposition 3, higher interest rates do not necessarily result in mortgage default, even though households are unable to roll-over outstanding mortgage balances. When the interest-rate change is relatively small, households down-size real-estate, and reduce mortgage payments accordingly. However, each cohort has a threshold interest-rate change that causes default, due to negative housing equity.

Consistent with a number of empirical studies (see Campbell and Dietrich 1983 and Schwartz and Torous 1993), younger homeowners in the model are the most susceptible to default, owing to larger mortgage balances and limited housing equity. This can be seen from Equation 8, which is a decreasing function of j.

5 The LCR and Mortgage-Market Instability

With the modelling environment now characterized, the main result is presented next: the mortgage-market instability arising from intertemporal variation in HQLA requirements.

To illustrate the model's underlying mechanism, an example is provided for two consecutive periods. In the first period, corporate-loan demand is relatively high, which induces banks to subsidize mortgage credit (the second case of Proposition 2). In the second period there is a negative corporate-loan demand shock. This reduces aggregate HQLA requirements, which increases the equilibrium mortgage interest rate, and may cause mortgage default.

5.1 First Period

Suppose the mortgage and housing markets are in a steady-state, and that MBS and minimum government debt are *insufficient* to meet HQLA requirements due to a relatively high level of corporate loans. I.e.:

$$I_1^c > \frac{B(\psi^g - L\theta^g)F^{'-1}(\Delta r^g) + (\delta\psi^m - L\theta^m)I_1^m(r_1^m)}{L\theta^c}.$$

This corresponds to the second case of Proposition 2. As a result, banks acquire excess quantities of government debt to meet their high HQLA requirements (i.e., $I_1^g > F^{'-1}(\Delta r^g)$), and subsidize mortgage credit to create additional MBS. In this setting, the equilibrium mortgage interest rate approaches:

$$r_1^m = r^{fm} - \Delta r^g \left[\frac{\delta\psi^{MBS} - L\theta^m}{\psi^G - L\theta^g}\right],$$

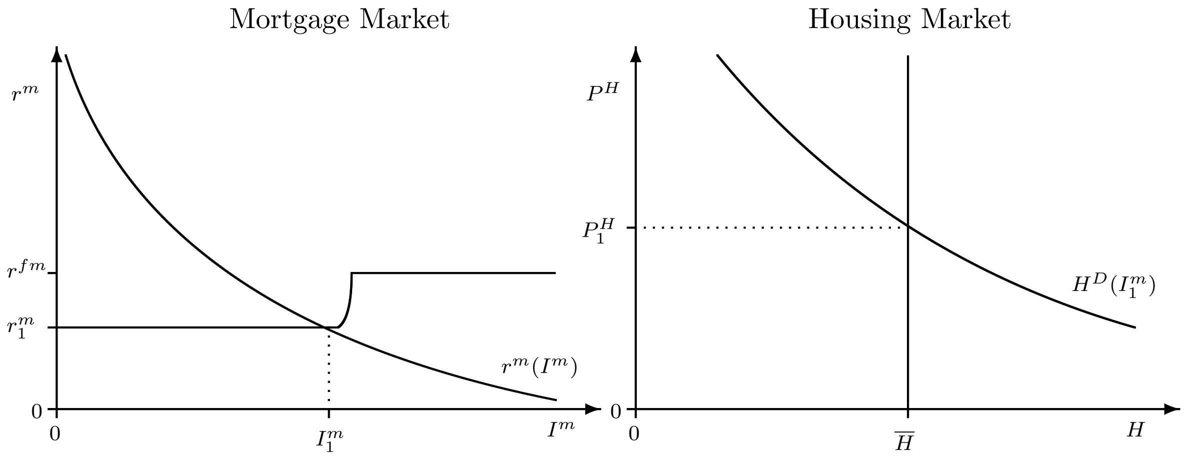
while the equilibrium quantity of mortgage credit supplied is:

$$I_1^m = \sum_{j=t-N+1}^{t} \frac{[(1+r_1^m)^{(N-t+j)} - 1](1-\alpha)Y}{r_1^m(1+r_1^m)^{(N-t+j)}}. \tag{9}$$

This scenario is depicted in the left panel of Figure 2.

In a steady state, no default occurs given interest rates are constant (from Proposition 3). Furthermore, all non-retired cohorts own an equivalent fraction of housing:

Figure 2: Equilibrium in the Mortgage and Housing Markets: First Period



This figure depicts the mortgage and housing markets in period 1. With a relatively high value of I_1^c, mortgage-market equilibrium occurs at the interest rate $r_1^m \approx r^{fm} - \Delta r^g(\delta\psi^{MBS} - L\theta^m)/(\psi^G - L\theta^g)$, and the equilibrium quantity of mortgage credit supplied is I_1^m. This generates the housing demand curve $H^D(I_1^m)$, which leads to the equilibrium price of housing P_1^H.

$$h_{j,1} = \frac{\overline{H}}{N} \;\forall\; j \in \{t-N+1,..,t-1\}, \tag{10}$$

and aggregate beginning-of-period mortgage balances are:

$$MB_1 = \sum_{j=t-N+1}^{t-1} \frac{[(1+r_1^m)^{(N-t+j)} - 1](1-\alpha)Y}{r_1^m(1+r_1^m)^{(N-t+j)}}. \tag{11}$$

From Equations 9, 10, and 11, the equilibrium price of housing in period 1 is thus:

$$P_1^H = \frac{N(1-\alpha)Y}{\overline{H}}\left[\frac{(1+r_1^m)^N - 1}{r_1^m(1+r_1^m)^N}\right]. \tag{12}$$

This is depicted in the right panel of Figure 2.

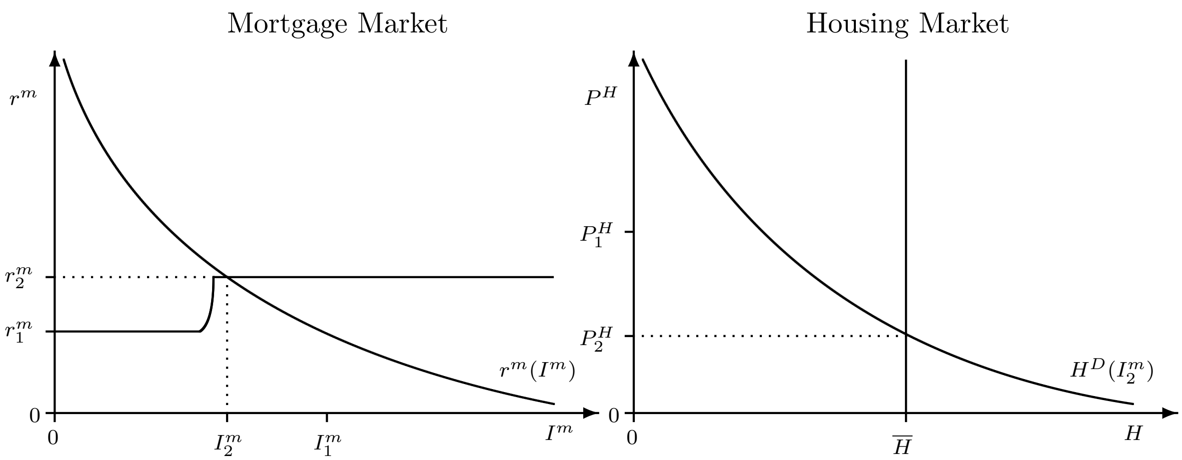
5.2 Second Period

Now suppose the banking sector is hit with an unfavorable corporate-loan demand shock in period 2 (i.e., $I_2^c < I_1^c$). This shifts the mortgage supply curve to the left, and provided I_2^c is sufficiently small, the equilibrium mortgage interest rate approaches the cost of funding (i.e., $r_2^m = r^{fm}$) from Proposition 2. As a result, the quantity of mortgage credit supplied decreases to (approximately):

$$I_2^m \approx I_1^m - (1-\alpha)Y\Delta r^g \left[\frac{\delta\psi^{MBS} - L\theta^m}{\psi^G - L\theta^g}\right] \sum_{\mathrm{K}=1}^{\mathrm{N}} \left[\frac{K^2+K}{2}\right].$$

This is depicted in the left panel of Figure 3.

Figure 3: Equilibrium in the Mortgage and Housing Markets: Second Period



This figure depicts the mortgage and housing markets in period 2. When the banking sector experiences a negative corporate-loan demand shock (i.e., $I_2^c < I_1^c$), the mortgage supply curve shifts to the left. This results in a higher equilibrium interest rate (i.e., $r_2^m = r^{fm} > r_1^m$), and a lower equilibrium quantity of mortgage credit supplied ($I_2^m(r_2^m) < I_1^m(r_1^m)$). This, in turn, shifts the housing demand curve ($H^D(\cdot)$) downwards, resulting in a lower equilibrium price of housing (i.e., $P_2^H < P_1^H$).

Since borrowing constraints are tighter in period 2, households are unable to roll-over mortgage credit (from Equations 2.1 and 5), and cohort j defaults in equilibrium whenever:

$$r_2^m - r_1^m \approx \Delta r^g \left[\frac{\delta\psi^{MBS} - L\theta^m}{\psi^G - L\theta^g}\right] > \frac{\sum\limits_{\{K:N\geq K>t-(j+1)\}} \eta(j)K + (\eta(j)-1)(N-t+j) + \eta(j)}{\sum\limits_{\mathrm{K}=1}^{\mathrm{N}} \eta(j)\left[\frac{K^2+K}{2}\right]}$$

from Proposition 3. Furthermore, the fall in mortgage credit puts downward pressure on house prices (the right panel of Figure 3). The exact magnitude of this effect depends on the interest-rate change, the number of defaulting cohorts, and the share of housing owned by each cohort. Proposition 4 provides an approximate value for period 2 house prices, relative to those in period 1:

Proposition 4 *In the absence of mortgage default, period 2 house prices are (approximately):*

$$P_2^H \approx P_1^H - \frac{N(1-\alpha)Y}{\overline{H}} \Delta r^g \left[\frac{\delta \psi^{MBS} - L\theta^m}{\psi^G - L\theta^g} \right] \sum_{K=1}^{N} \left[\frac{K^2+K}{2} \right].$$

while, when at least one household defaults, period 2 house prices are at most (approximately):

$$\overline{P_2^H} \approx P_1^H - \frac{\Upsilon N}{\overline{H}}(1-\alpha)Y.$$

Where Υ is the number of defaulting households. For a proof of these results see Appendix D.

5.3 Discussion

In Section 3 it was assumed that banks acquire mortgage insurance. As such, insurers are the financial institutions directly affected by the type of default described herein. However, as illustrated by the recent financial crisis, losses in one part of the financial system can quickly spread to others, causing systemic events. Additionally, the destruction of housing equity generates additional losses on all defaulting mortgage credit, whether insured or uninsured. This, in turn, has a first-order effect on all banks.

6 Conclusion

This paper developed a model of banking to highlight a potentially-destabilizing property of the Liquidity Coverage Ratio (LCR). This results from the preferential treatment that mortgage-backed securities (MBS) receive as high quality liquid assets (HQLA) under the LCR: in many jurisdictions MBS are considered Level 1 HQLA. When banks experience positive HQLA-demand shocks, the marginal value of MBS increases. This places downward pressure on mortgage interest rates, and households acquire larger mortgage balances as a result. Once the HQLA-demand shock dissipates, the marginal value of MBS becomes zero, and mortgage interest rates increase. This, in turn, may cause mortgage default, and will depress house prices.

Moderate use of MBS for LCR adherence is likely beneficial. It diversifies the pool of HQLA, and may alleviate the LCR's regulatory burden in jurisdictions with limited alternative assets. However, policy-makers should ensure that the LCR does not contribute to another mortgage-market-induced systemic event.

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A Initial LCR Parameters

Table 1: Inclusion Rates for HQLA-Eligible Assets (BCBS 2010)

Level 1 Assets (No Limit)	Inclusion Rate
• Coins and Bank Notes • Qualifying Marketable Securities from Sovereigns, Central Banks, PSEs, and Multilateral Development Banks • Qualifying Central Bank Reserves • Domestic Sovereign or Central bank debt for non-0% risk-weighted sovereigns	100%
Level 2A Assets (Maximum of 40% of HQLA)	
• Sovereign, Central Bank, Multilateral Development Banks, and PSE Assets qualifying for 20% Risk Weight • Qualifying Corporate Debt Securities Rated AA- or Higher • Qualifying Covered Bonds Rated AA- or Higher	85%
Level 2B Assets (Maximum of 15% of HQLA)	
• Qualifying Residential Mortgage Backed Securities	75%
• Qualifying Corporate Debt Securities Rated Between A+ and BBB-	50%
• Qualifying Common Equity Shares	50%

Table 2: Outflow Rates for Liabilities (BCBS 2010)

Retail Deposits:	Outflow Rate
Demand Deposits and Term Deposits (Less than 30 Days)	
• Insured Stable Deposits	3%
• Stable Deposits	5%
• Less Stable Retail Deposits	10%
Term Deposits with Residual Maturity Greater than 30 Days	0%
Unsecured Wholesale Funding:	

Demand and Term Deposits (Less than 30 Days Maturity) Provided by Small Business Customers	
• Stable Deposits	5%
• Less Stable Deposits	10%
• Operational Deposits Generated by Clearing, Custody and Cash Management Activities	25%
• Portion Covered by Deposit Insurance	5%
• Cooperative Banks in an Institutional Network (Qualifying Deposits with the Centralized Institution)	25%
• Non-Financial Corporates, Sovereigns, Central Banks, Multilateral Development Banks, and PSEs	40%
• If the Entire Amount is Fully Covered by Deposit Insurance Scheme	20%
Other Legal Entity Customers	100%
Secured Funding:	
• Secured Funding Transactions with a Central Bank Counterparty or Backed by Level 1 Assets with any Counterparty	0%
• Secured Funding Transactions Backed by Level 2A Assets, with any Counterparty	15%
• Secured Funding Transactions Backed by Non-Level 1 or Non-Level 2A Assets, with Domestic Sovereigns, Multilateral Development Banks, or Domestic PSEs as a Counterparty	25%
• Backed by RMBS Eligible for Inclusion in Level 2B	25%
• Backed by Other Level 2B Assets	50%
• All Other Secured Funding Transactions	100%

B Proof of Propositions 1 & 2

Banks seek to maximize the following objective function:

$$Max_{\{I^m_{i,t}, I^g_{i,t}, MBS_{i,t}\}} \Pi_t = (r^m_t - r^{fm} - c^I) I^m_{i,t} + (r^g - r^{fg}) I^g_{i,t} + F(I^g_{i,t}) + c^M MBS_{i,t} + \Pi^c(I^c_{i,t}),$$

subject to:

$$C^1 = L(\theta^m I^m_{i,t} + \theta^c I^c_{i,t} + \theta^g I^g_{i,t}) - (\psi^g I^g_{i,t} + \psi^m MBS_{i,t}) \leq 0,$$

$$C^2 = MBS_{i,t} - \delta I^m_{i,t} \leq 0$$

$$C^3 = -MBS_{i,t} \leq 0.$$

The first-order necessary conditions are:

$$I^m_{i,t}: \ r^m_t - r^{fm} - c^I - \lambda^1 L \theta^m - \lambda^2 \delta = 0, \tag{13}$$

$$I^g_{i,t}: \ r^g - r^{fg} + F'(I^g_{i,t}) + \lambda^1 [\psi^g - L\theta^g] = 0, \tag{14}$$

$$MBS: \ -c^M + \lambda^1 \psi^m + \lambda^2 + \lambda^3 = 0, \tag{15}$$

$$\lambda^i \geq 0 \ \forall \ i = 1, 2, 3, \tag{16}$$

$$\lambda^i C^i = 0 \ \forall \ i = 1, 2, 3, \tag{17}$$

where λ^1, λ^2, and λ^3 are constraint multipliers on the first, second, and third constraints, respectively.

There are two cases of interest. In the first, $L = 0$ (Proposition 1). Here, the LCR constraint is slack, and from Equation 17, $\lambda^1 = 0$. It follows that $I^g_{i,t} = F'^{-1}(\Delta r^g)$ (where

$\Delta r^g = r^{fg} - r^g > 0$) from Equation 14. For any positive level of mortgage credit, either $\lambda^2 > \lambda^3 = 0$, $\lambda^3 > \lambda^2 = 0$, or $\lambda^2 = \lambda^3 = 0$ (as $c^M \to 0$). Given, $\lambda^1 = 0$, it follows that $\lambda^2 = \lambda^3 = 0$ from Equation 15. Furthermore, given $\lambda^2 = 0$, it follows that $r_t^m = r^{fm} + c^I$ from Equation 13, and in the limit as $c^I \to 0$, $r_t^m = r^{fm}$. Finally, since $c^M > 0$, it follows that $MBS = 0$.

In the second case $L > 0$ (Proposition 2). Here, there are two sub-cases. In the first:

$$I_t^c \leq \frac{B(\psi^g - L\theta^g)F'^{-1}(\Delta r^g) + (\delta\psi^m - L\theta^m)I_t^m(r_t^m)}{L\theta^c}.$$

That is, minimum government debt and MBS are sufficient to meet each bank's HQLA requirement. To see this, note that each bank's LCR constraint is:

$$\psi^g F'^{-1}(\Delta r^g) + \psi^m \delta \frac{I^m(r_t^m)}{B} \geq L(\theta^m \frac{I^m(r_t^m)}{B} + \theta^g F'^{-1}(\Delta r^g) + \theta^c \frac{I_t^c}{B})$$

when they hold minimum government debt ($F'^{-1}(\Delta r^g)$). Solving for I_t^c produces the inequality above. From Equation 14, we have:

$$\lambda^1 = \frac{\Delta r^g - F'(F'^{-1}(\Delta r^g))}{[\psi^g - L\theta^g]} = 0.$$

As with the first case above ($L = 0$), $\lambda^1 = 0$ implies that $\lambda^2 = 0$ (Equation 15 and $c^M \to 0$). Given $\lambda^1 = \lambda^2 = 0$, it follows that $r_t^m = r^{fm} + c^I$ from Equation 13, and in the limit as $c^I \to 0$, $r_t^m = r^{fm}$. Finally, to satisfy C^1, MBS must equal:

$$\frac{\frac{L}{B}[\theta^c I_t^c + \theta^m I_t^m(r_t^m)] - (\psi^g - L\theta^g)F'^{-1}(\Delta r^g)}{\psi^m}.$$

In the second sub-case of Proposition 2, minimum government debt and MBS are not sufficient to meet HQLA requirements. This happens whenever:

$$I_t^c > \frac{B(\psi^g - L\theta^g)F'^{-1}(\Delta r^g) + (\delta\psi^m - L\theta^m)I_t^m(r_t^m)}{L\theta^c},$$

i.e., whenever I_t^c is sufficiently high. In this case, banks acquire additional government debt to satisfy C^1, i.e., $I_t^g > F^{'-1}(\Delta r^g)$. From Equation 14, we have:

$$\lambda^1 = \frac{\Delta r^g - F^{'}(I_t^g)}{\psi^g - L\theta^g} > 0.$$

That is, given government debt has diminishing returns, and $I_t^g > F^{'-1}(\Delta r^g)$, it follows that $\Delta r^g > F^{'}(I_t^g)$, and $\lambda^1 > 0$. Whenever $I_t^g > F^{'-1}(\Delta r^g)$, it must be that $MBS > 0$, as banks reduce I_t^g to its lowest amount possible. This implies that:

$$MBS = \delta I_t^m(r_t^m)$$

from C^2. Since $MBS > 0$, it follows that C^3 is slack, and $\lambda^3 = 0$ (Equation 17). Given $\lambda^3 = 0$, it follows that:

$$\lambda^2 = -\psi^m \left[\frac{\Delta r^g - F^{'}(I_t^g)}{\psi^g - L\theta^g}\right] + c^M$$

from Equation 15. Substituting the expressions for λ^1 and λ^2 into Equation 13, produces the following equilibrium mortgage interest rate (as $c^I \rightarrow 0$ and $c^M \rightarrow 0$):

$$r_t^m = r^{fm} - \frac{\delta\psi^{MBS} - L\theta^m}{\psi^G - L\theta^g}\left[\Delta r^g - F'(I_{i,t}^c)\right].$$

Finally, each bank acquires:

$$I_t^g = \frac{L\theta^c I_t^c - (\delta\psi^M - L\theta^m) I_t^m(r_t^m)}{B(\psi^g - L\theta^g)}$$

units of government debt to meet C^1.

C Proof of Proposition 3

Cohort j^* is forced to default on mortgage credit when the interest rate increases by approximately:

$$\Delta r(j^*) = \frac{\sum\limits_{\{K:N\geq K>t-(j^*+1)\}} \eta(j^*)K + (\eta(j^*)-1)(N-t+j^*) + \eta(j^*)}{\sum\limits_{\mathrm{K}=1}^{\mathrm{N}} \eta(j^*)\left[\frac{K^2+K}{2}\right]},$$

where

$$\eta(j^*) = \frac{h_{j^*,t}}{\overline{H} - \sum\limits_{j=t-N+1}^{j^*} h_{j,t}}.$$

Households default in equilibrium when they have negative housing equity, and cannot roll-over mortgage credit due to a higher mortgage interest rate. The second condition is always met when interest rates increase, while the first is met when:

$$MB_{j^*,t} > P_t^H h_{j^*,t}. \tag{18}$$

Denote last period's mortgage interest rate by r, and the current-period's interest rate by r'. Further, denote cohort $j's$ outstanding mortgage balance by:

$$MB(r,j) = \frac{(1-\alpha)Y[(1+r)^{N-t+j} - 1]}{r(1+r)^{N-t+j}},$$

and their available mortgage credit by:

$$I^m(r',j) = \frac{(1-\alpha)Y[(1+r')^{N-t+j} - 1]}{r'(1+r')^{N-t+j}}.$$

These definitions explicitly account for the relevant interest rates (r, and r'), and the amortization periods of each cohort j (via $N-t+j$). Furthermore, note that the beginning-of-period mortgage balance is zero for both newly-born households ($j=t$) and retired households ($j=t-N$), and that retired households cannot acquire mortgage credit ($I^m(r',t-N)=0$). From Equation 7 of Subsection 4.3, the price of housing in period t is:

$$P_t^H = \left[\sum_{j=t-N+1}^{t} I^m(r',r) - \sum_{j=t-N+1}^{t-1} MB(r,r)\right]\theta(j^*),$$

where:

$$\theta(j^*) = \frac{1}{\overline{H} - \sum\limits_{j=t-N+1}^{j^*} h_{j,t}},$$

and j^* is the oldest defaulting household (note: households default in order from youngest to oldest). The following equality (from Equation 5 of Section 4):

$$MB(r,j) = (1+r)I^m(r,j+1) - (1-\alpha)Y$$

allows us to rewrite Equation 7 as:

$$P_t^H = \left[\sum_{j=t-N+1}^{t} I^m(r',j) - \sum_{j=t-N+1}^{j^*+1} (1+r)I^m(r,j) + \sum_{j=t-N+1}^{j^*+1} (1-\alpha)Y \right] \theta(j^*). \tag{19}$$

$I^m(r',f)$ can be approximated using a first-order Taylor-series expansion of $I^m(\cdot,j)$ around r, resulting in:

$$I^m(r',j) \approx I^m(r,j) + \Delta r[I^{m'}(r,j)], \tag{20}$$

where $\Delta r = r' - r$. Equation 20 allows us to approximate Equation 19 by:

$$\begin{aligned} P_t^H \approx \Bigg[\sum_{\{j:t \geq j > j^*+1\}} I^m(r,j) + \Delta r \sum_{j=t-N+1}^{t} [I^{m'}(r,j)] - \sum_{j=t-N+1}^{j^*+1} r[I^m(r,j)] \\ + \sum_{j=t-N+1}^{j^*+1} (1-\alpha)Y \Bigg] \theta(j^*), \end{aligned} \tag{21}$$

and for small interest rates, Equation 21 can be further approximated by:

$$\begin{aligned} \lim_{r \to 0} P_t^H \approx \Bigg[\sum_{\{K:N \geq K > N-t+j^*+1\}} (1-\alpha)YK - \Delta r \sum_{K=1}^{N} \left[\frac{(1-\alpha)YK^2 + (1-\alpha)YK}{2} \right] \\ +(1-\alpha)Y(N-t+j^*+1) \Bigg] \theta(j^*), \end{aligned} \tag{22}$$

since:

$$\lim_{r \to 0} I^m(r,j) = (1-\alpha)Y(N-t+j),$$

$$\lim_{r\to 0} I^{m'}(r,j) = -\left[\frac{(1-\alpha)Y(N-t+j)^2 + (1-\alpha)Y(N-t+j)}{2}\right],$$

and

$$\sum_{j=t-N+1}^{j^*+1} (1-\alpha)Y = (1-\alpha)(N-t+j^*+1).$$

Finally, substituting Equation 22 into Equation 18, taking the limit of MB_{t,j^*} as $r \to 0$, canceling the $(1-\alpha)Y$ terms, and solving for Δr, provides the result in Proposition 3.

D Proof of Proposition 4

In the absence of mortgage default, aggregate mortgage balances in period 2 are equal to those in period 1. Furthermore, the fraction of housing owned by non-retired cohorts in period 2 is the same as in period 1. This allows us to write the difference between period 2 and period 1 house prices as follows:

$$P_2^H - P_1^H = \frac{N}{\overline{\overline{H}}}\left[\sum_{K=1}^{N} I^m(r_2^m, K) - \sum_{K=1}^{N} I^m(r_1^m, K)\right], \tag{23}$$

where:

$$I^m(r,K) = (1-\alpha)Y\frac{[(1+r)^K - 1]}{r(1+r)^K}.$$

If we approximate each $I^m(r_2^m, K)$ term using a first-order Taylor-series expansion of $I^m(\cdot, K)$ around r_1^m, we can approximate Equation 23 as:

$$P_2^H - P_1^H \approx \frac{N}{\overline{\overline{H}}}\left[\Delta r \sum_{K=1}^{N} I^{m'}(r_1^m, K)\right]. \tag{24}$$

Furthermore, noting that:

$$\lim_{r\to 0} I^{m'}(r,K) = -\left[\frac{(1-\alpha)YK^2 + (1-\alpha)YK}{2}\right],$$

and:

$$\Delta r \approx \Delta r^g \left[\frac{\delta\psi^{MBS} - L\theta^m}{\psi^G - L\theta^g}\right],$$

allows us to rewrite Equation 24 as:

$$P_2^H \approx P_1^H - \frac{N(1-\alpha)Y}{\overline{H}}\Delta r^g \left[\frac{\delta\psi^{MBS} - L\theta^m}{\psi^G - L\theta^g}\right] \sum_{\mathrm{K}=1}^{\mathrm{N}} \left[\frac{K^2 + K}{2}\right].$$

Providing the first part of the proof.

For cohort j to default, it must be that

$$P_2^H \frac{\overline{H}}{N} < MB_{j,2}, \tag{25}$$

from the equal-housing result, and the fact that households only default when they have negative housing equity. Isolating P_2^H, and noting that:

$$\lim_{r \to 0} MB_{j,t} = (1-\alpha)Y(N - t + j)$$

allows us to approximate the above inequality as follows:

$$P_2^H < \frac{N}{\overline{H}}(1-\alpha)Y(N - t + j).$$

The youngest home-owning cohort in period 2 is $j = 1$. Thus, cohort 1 defaults when:

$$P_2^H < \frac{N}{\overline{H}}(1-\alpha)Y(N - 1). \tag{26}$$

Furthermore, note that period 1 house prices are:

$$P_1^H = \frac{N(1-\alpha)Y}{\overline{H}} \left[\frac{(1 + r_1^m)^N - 1}{r_1^m(1 + r_1^m)^N}\right],$$

from Equation 12, which can be approximated as:

$$P_1^H \approx \frac{N}{\overline{H}}(1-\alpha)YN, \tag{27}$$

in the limit as $r \to 0$. Combining 26 and 27 allows us to provide an approximate upper-bound on period 2 house prices - relative to period 1 house prices - when the youngest home-owning cohort defaults:

$$P_2^H < P_1^H - \frac{N}{\overline{\overline{H}}}(1-\alpha)Y.$$

Continuing this exercise for each cohort, and noting that cohorts default from youngest to oldest, produces the general result that period 2 house prices have an approximate upper bound of:

$$\overline{P_2^H} \approx P_1^H - \frac{\Upsilon N}{\overline{\overline{H}}}(1-\alpha)Y,$$

where Υ is the number of defaulting cohorts.