

# Revisiting the Fiscal Theory of Sovereign Risk from a DSGE Viewpoint \*

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## Abstract

In this paper, we revisit Uribe's[24] 'Fiscal Theory of Sovereign Risk' suggesting a trade-off between stabilizing inflation and suppressing default. We develop a class of dynamic stochastic general equilibrium models with nominal rigidities and compare two de facto inflation-stabilization policies; namely, optimal monetary policy and optimal monetary and fiscal policy with an interest rate spread-minimizing policy that completely suppresses default. Under the optimal monetary and fiscal policy, not only the nominal interest rate but also the tax rate works to minimize welfare costs through stabilizing inflation. Under the optimal monetary and the interest rate spread-minimizing policies, only the nominal interest rate is available as a policy instrument. Under the optimal monetary and fiscal policy, both inflation and the output gap completely stabilize, although these fluctuate under the optimal monetary policy. In addition, the volatility of the default rate under the optimal monetary and fiscal policy is considerably lower than under the optimal monetary policy. Thus, there is not necessarily a trade-off between stabilizing inflation and suppressing default. While the optimal monetary and fiscal policy stabilizes both inflation and default, the interest rate spread-minimizing policy makes the inflation rate volatile. However, inflation is not especially volatile when prices are sufficiently sticky. Thus, the trade-off between stabilizing inflation and suppressing default is then not as severe as that suggested by Uribe[24], even when there is a trade-off. In sum, our results are: 1) there is not necessarily a trade-off between stabilizing inflation and suppressing default, and 2) the trade-off between stabilizing inflation and suppressing default is not as severe as Uribe[24] suggests. As policy implications, we argue: 1) we can practically solve the trade-off between stabilizing inflation and suppressing default by adopting optimal monetary and fiscal policy, and 2) the interest rate spread-minimizing policy does not represent an inferior policy from the viewpoint of dissolving the trade-off between stabilizing inflation and suppressing default if price stickiness is sufficiently high.

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# 1 Introduction

Uribe[24] argues that if the central bank's policy is to peg the price level, government surrenders its ability to inflate away the real value of nominal public liabilities, and so public debt default is inevitable. Alternatively, if the central bank's policy is to peg the nominal interest rate, government preserves its ability to suppress public debt default, but it no longer stabilizes the price level. This argument may be consistent with readers' intuition. Through a series of recent default scares stemming from the Greek debt crisis, the stabilizing of inflation and the suppressing of the default trade-off (SI–SD trade-off) observed by Uribe[24] appears to be increasingly emphasized, especially in the Euro area.

However, after revisiting Uribe's[24] fiscal theory of sovereign risk (FTSR) from the viewpoint of a dynamic stochastic general equilibrium (DSGE) model with nominal rigidities as per Woodford[25], we find that there is not necessarily an SI–SD trade-off, and even if there is, it is not as severe as that suggested by Uribe[24]. That is, we develop a class of DSGE models with nominal rigidities and find that inflation stabilization is not inconsistent with default suppression. Default risk could be mitigated through stabilizing inflation, and this result differs markedly from that in Uribe[24]. We can then solve practically the SI–SD trade-off by adopting an optimal monetary and fiscal (OMF) policy where both the nominal interest and tax rates are available as policy instruments to minimize welfare costs through mostly stabilizing inflation. This is our most important policy contribution.

Uribe's[24] FTSR appears to affect strongly the European Central Bank (ECB) in conducting monetary policy. Amid the sovereign debt crisis, conducting monetary policy appears to be extremely difficult. Even if Greece did not default when its huge fiscal deficit was revealed in October 2009, when the Greek 10-Year credit default swap premium began to soar and reached USD 20,404 on April 2012, the ECB faced increased difficulty in conducting monetary policy. Subsequently, the harmonized consumer price index (HCPI) inflation rate started to increase from -0.6% in July 2009, and the ECB's policy interest rate (the short-run buying operation rate) remained at 1% until April 2011, when HCPI inflation was 2.8%. The ECB thus seemed reluctant to stabilize inflation because of sovereign debt problems in Greece and being fully aware of Uribe's[24] FTSR.

In this paper, we double-check the work in Uribe[24] by developing a class of DSGE models with nominal rigidities. We use this to compare the optimal monetary (OM) policy and the OMF policy with the interest rate spread-minimizing (MIS) policy, being that policy which minimizes the interest rate spread, namely, the difference between the nominal interest rate for safe assets and the government debt yield excluding default risk in an economy with sovereign risk. Note that both the OM and OMF policies correspond to the Taylor rule and the price level targeting in Uribe[24] because they are both de facto inflation stabilization policies, and the MIS policy corresponds to the interest rate peg in Uribe[24] because these policies either set the expected default rate to zero or minimize it.

In our model, while we adopt Uribe's[24] default rule, we refocus our attention on the fiscal balance, which is an exogenous shock in Uribe[24], and note that this exogenous setting generates Uribe's[24] result that there is an SI–SD trade-off. The most important mechanism in our model is endogenized production, which is a commonplace setting in the literature on optimal monetary policy in the DSGE established by Woodford[25], making the fiscal balance endogenous and generating a policy implication quite unlike that in Uribe[24]. Thus, the difference in results and/or policy implications between our study and Uribe[24] depends on the assumption of exogenous or

endogenous production.

At this point, we review Uribe's[24] FTSR. By iterating the government budget constraint forward and imposing an appropriate transversality condition, Uribe[24] shows that the default rate depends on the ratio of the net present value of the real fiscal surplus to real government debt with interest payment. That is, the default rate depends on government solvency. Thus, a decrease in the fiscal surplus, which is exogenous in his setting, decreases government solvency. Facing such a case, if the central bank stabilizes inflation, the burden of government debt redemption cannot be mitigated, and the default rate increases. If the central bank gives up trying to stabilize inflation, the burden of government debt redemption can be mitigated by inflation, which decreases real government debt, and the default is mitigated. This is Uribe's[24] FTSR as hinted at by 'fiscal theory of price level' in Cochrane[10], Leeper[20] and Woodford[26], and the FTSR shows that there is indeed an SI-SD trade-off.

How then does endogenized production derive quite different results? First, recall that the fiscal surplus is the difference between tax revenue and government expenditure, and suppose that a tax, which is one of the policy instruments in the OMF policy in our analysis, is levied on output and government expenditure is exogenous. Here, the most important thing is that the fiscal surplus not only acutely involves the default rate but also involves inflation through the output gap. That is, stabilizing the fiscal surplus stabilizes not only the default rate but also both inflation and the output gap. Note that the OM policy and the OMF policy are de facto inflation stabilization policies because inflation volatility determines welfare costs stemming from household utility.

Then, suppose that there is an increase in government expenditure, which is exogenous, and the policy authorities, the government and the central bank, adopt the OMF policy, where the nominal interest and tax rates are policy instruments. Here, production is endogenous; therefore, the fiscal surplus is also endogenous. Facing an increase in government expenditure, which applies pressure to increasing inflation because government expenditure increases the GDP gap through an increase in the marginal cost, the government hikes the tax rate to decrease the GDP gap by lowering consumption. As a result, the inflation–output gap trade-off is completely dissolved by coping with the central bank, whose policy instrument is the nominal interest rate (because the basic mechanism for stabilizing inflation in DSGE models with Calvo pricing will be familiar to most readers, we skip to explaining why stabilizing the output gap stabilizes inflation). Although an increase in government expenditure applies pressure to worsening the fiscal deficit, the increased taxation cancels out any such pressure, so the fiscal deficit improves. Because the fiscal deficit is almost zero as a result and the fiscal balance is more stabilized than under the OM policy where the tax rate is constant over time, the default rate is roughly zero. In short, the more stabilized is inflation, the more stabilized is the default rate, and vice versa, under the OMF policy. Thus, there is not a necessarily an SI–SD trade-off.

We do not necessarily deny Uribe[24] because we can replicate the SI–SD trade-off clearly under the OM policy, which corresponds to the Taylor rule in Uribe[24]. Under the OM policy, facing an increase in government expenditure, inflation is stabilized (it fluctuates more than under the OMF policy because only the nominal interest rate is available to stabilize inflation). Because the tax rate is constant over time, the tax rate is not increased, the fiscal deficit worsens, and the default rate soars. Thus, there is the SI–SD trade-off. What about the MIS policy corresponding to the interest rate peg in Uribe[24]? Under the MIS policy, and as in Uribe[24], the interest rate spread is zero. Because the nominal interest rate for safe assets definitely falls in line with the nominal

interest rate for risky assets—namely, the government debt yield—the expected default rate is stabilized. In addition, because we assume that the policy authorities commit to their policies, the actual default rate over time is zero. Although the default rate is completely stabilized, inflation rises through an increase in the output gap when government expenditure increases. Thus, there is an SI–SD trade-off similar to that in Uribe[24].

However, the SI–SD trade-off that we find is not as severe as that suggested in Uribe[24]. We calculate the volatilities on inflation and the default rate under the OM, the OMF, and the MIS policies for various and plausible levels of price stickiness. Under the OMF policy, both volatilities are quite low and do not depend on price stickiness (in particular, the volatility on inflation is definitely zero). Under the OM policy, the volatility on the default rate is quite high for any plausible price stickiness, although inflation is well stabilized, unlike the MIS policy. Under the MIS policy, the volatility on the default rate is definitely zero, while the inflation volatility depends on price stickiness, such that the greater the price stickiness, the less the inflation volatility, and vice versa. In addition, if price stickiness is quite high, such as 0.95, which implies that the duration of price revision is five years, the volatility on inflation is close to zero. Because the volatility on the default rate is definitely zero, the SI–SD trade-off that we find is not as severe as that suggested in Uribe[24]. Summing up, our results are: 1) there is not necessarily an SI–SD trade-off, and 2) the trade-off is not as severe as that suggested in Uribe[24]. As policy implications, we argue: 1) we can practically solve the SI–SD trade-off by adopting the OMF policy, and 2) the MIS policy does not represent inferior policy from the viewpoint of dissolving the SI–SD trade-off if the price stickiness is sufficiently high.

Finally, we discuss the relationship between our work and previous work addressing sovereign risk or crises in the field of macroeconomics. First, Arellano[2] develops a model in which the default probability depends on some stochastic process and shows that default is more likely in recessions. He succeeds in matching his model with Argentinian data, and his assumption concerning the default mechanism is subsequently applied by Mendoza and Yue[21] and Corsetti, Kuester, Meier and Mueller[12]. In their analysis, Mendoza and Yue[21] attempt to explain the negative relationship between output and default observed in the data. That is, they clarify the reason why deep recession often accompanies sovereign default. Corsetti, Kuester, Meier and Mueller[12] develop a model including financial intermediaries showing that sovereign risk may give rise to indeterminacy and imply that fiscal retrenchment via government spending cuts can help to curtail the risk of macroeconomic instability and, in extreme cases, even stimulate economic activity. Their model stems from Curdia and Woodford[13] and is inclusive of the zero lower bound of nominal interest rates.

Subsequently, Corsetti and Dedola[11] develop a model for a sovereign debt crisis driven by either self-fulfilling expectations or weak fundamentals and analyze the mechanism through which either conventional or unconventional monetary policy can rule out the former. Their finding that swapping government debt for monetary liabilities can preclude self-fulfilling debt crises one of several unconventional monetary policies. Elsewhere, and similar to our analysis, Bacchetta, Perazzi and Wincoop[3] develop a class of DSGE models and analyze both conventional and unconventional monetary policy. They find that the central bank cannot credibly avoid a self-fulfilling debt crisis.

Here, we make it clear where our analysis differs from this earlier body of work. With the exception of Corsetti and Dedola[11] and Bacchetta, Perazzi and Wincoop[3], the main concerns

in these analyses are how sovereign default affects the macroeconomic dynamics, especially the output dynamics, whereas we focus on how the OMF policy affects default. Although Corsetti and Dedola[11] and Bacchetta, Perazzi and Wincoop[3] analyze monetary policy, they do not consider fiscal policy or how to use it as a stabilization or welfare cost-minimization tool. Thus, our purposes are not identical, and we can say that we propose monetary and fiscal policies to both stabilize inflation and suppress default risk, whereas they propose monetary policy only to suppress default risk.<sup>1</sup>

Repeatedly, we emphasize that while previous work in the area obtains important implications, none of these examines the SI–SD trade-off in detail. While Uribe[24] certainly discusses this trade-off, we emphasize that there is not necessarily a trade-off. Needless to say, neither Uribe[24] nor Corsetti, Kuester, Meier and Mueller[12] nor Mendoza and Yue[21] derives this result. Examining this trade-off and deriving the policy implications from the viewpoint of solving the SI–SD trade-off in this paper is novel.

The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 defines the policy target under the three policies mentioned. Section 4 solves the linear-quadratic (LQ) problem, shows the first-order necessary conditions (FONCs) for the policy authorities. Section 5 calibrates the model under the three policies. Section 6 clarifies the SI–SD trade-off under the three policies. Section 7 concludes the paper. The appendices provide some additional analysis with Appendix A examining the steady state and Appendices B to E providing some empirical evidence.

## 2 The Model

We introduce firms into Uribe’s[24] model and develop a class of DSGE models with nominal rigidities following Gali and Monacelli[17], although we do assume a closed economy.<sup>2</sup> Thus, the default mechanism is quite similar to Uribe[24]. We follow Benigno[4] (an earlier working paper version of Benigno[6]) to clarify the households’ choice of risky assets. The household  $i$  on the interval  $i \in [0, 1]$  supplies labor and owns firms. We adopt Calvo pricing and assume that a tax is levied on output and is distorted. Thus, monopolistic power remains, and the steady state is distorted, unlike Gali and Monacelli[17].

### 2.1 Households

A representative household’s preference is given by:

$$U \equiv \mathbf{E}_0 \left( \sum_{t=0}^{\infty} \beta^t U_t \right), \quad (1)$$

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<sup>1</sup>Furthermore, they do not focus on fiscal policy, and their model is unsuitable for analyzing fiscal policy regardless, whereas our model can analyze and evaluate the effect of fiscal policy. In terms of other differences, the government in Arellano[2] does not levy tax on any economic agents, while Mendoza and Yue[21], Corsetti, Kuester, Meier and Mueller[12] and Bacchetta, Perazzi and Wincoop[3] assume either lump-sum taxes or transfers. Thus, under their settings, it is not possible to analyze fiscal policy. In contrast, in our work, government changes the tax rate to minimize welfare costs, and fiscal policy can be analyzed specifically, and we can then easily observe the effects of the OMF policy on default. This is the advantage of our analysis over these existing studies from the viewpoint of model building.

<sup>2</sup>Following Ferrero[14], we introduce government into Gali and Monacelli[17]. In other words, the model is a closed economy version of Okano[22].

where  $U_t \equiv \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}$  denotes the period utility,  $E_t$  is the expectation conditional on the information set at period  $t$ ,  $\beta \in (0, 1)$  is the subjective discount factor,  $C_t$  is the consumption index,  $N_t \equiv \int_0^1 N_t(i) dh$  is the hours of labor, and  $\varphi$  is the inverse of the elasticity of labor supply.

The consumption index of the continuum of differentiated goods is defined as follows:

$$C_t \equiv \left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (2)$$

where  $\varepsilon > 1$  is the elasticity of substitution across goods.

The price level is defined as follows:

$$P_t \equiv \left[ \int_0^1 P_t(i)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}. \quad (3)$$

The maximization of Eq.(1) is subject to a sequence of intertemporal budget constraint of the form:

$$R_{t-1}D_{t-1}^n + R_{t-1}^G B_{t-1}^n (1 - \delta_t) + W_t N_t + PR_t \geq \int_0^1 P_t(i) C_t(i) di + D_t^n + B_t^n, \quad (4)$$

where  $R_t \equiv 1 + r_t$  denotes the gross (risk-free) nominal interest rate,  $R_t^G \equiv R_t \Gamma(-sp_t)$  denotes the government debt coupon rate,  $r_t$  is the net interest rate,  $D_t^n$  denotes the nominal state contingent claim,  $B_t^n$  is the nominal government debt,  $W_t$  is the nominal wage,  $PR_t$  denotes profits from the ownership of the firms,  $\delta_t$  is the default rate,  $sp_t \equiv \frac{SP_t}{SP} - 1$  is the percentage deviation of the (real) fiscal surplus from its steady-state value,  $SP_t \equiv \tau_t Y_t - G_t$  denotes the (real) fiscal surplus,  $\tau_t$  denotes the tax rate,  $Y_t \equiv \left[ \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$  denotes (aggregated) output, and  $G_t \equiv \left( \int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$  denotes (aggregated) government expenditure. Furthermore, we define  $V$  as the steady-state value of any variables  $V_t$  and  $v_t$  as the percentage deviation of  $V_t$  from its steady-state value. Thus,  $SP$  is the steady-state value of the fiscal surplus.

Now we discuss the government debt coupon rate  $R_t^G \equiv R_t \Gamma(-sp_t)$ , where  $\Gamma'(-sp_t) > 0$  by assumption. Our assumption implies that government decides the government debt coupon rate depending on its fiscal situation, such that if the fiscal situation worsens, the government increases the coupon rate. Note that the government debt coupon rate  $R_t^G$  is not government debt yield, which is fully endogenized. In our setting, the government debt yield is decided by households' intertemporal optimal condition; namely, the Euler equation. Thus, the government debt yield is decided endogenously, although the government debt coupon rate depends on our assumption.

As mentioned, the function  $\Gamma(-sp_t)$  is hinted at by Benigno[4], who develops a two-country model with imperfect financial integration, although the details are somewhat different from Benigno[4]. Benigno[4] assumes that households in the home country face a burden in international financial markets. As borrowers, households in the home country will be charged a premium on the foreign interest rate; as lenders, they will receive remuneration less than the foreign interest rate. Following his setting, Benigno[4] assumes  $\Gamma'(\cdot) < 0$ , which implies that the higher the foreign country's government debt, the lower the remuneration for holding the foreign country's government debt.<sup>3</sup> However, on the contrary, our setting implies that the lower the fiscal surplus, the less the remuneration for holding government debt owing to default, which in turn harms capital and

<sup>3</sup>Benigno[4] observes that this function, which depends only on the level of real government bonds in his setting, captures the costs of undertaking positions in the international asset market or the existence of intermediaries in the foreign asset market.

makes households hesitate to hold government debt. The government has to pay additional remuneration for holding government debt, which provides households with a motivation for doing so. Thus, we assume that  $\Gamma'(\cdot) > 0$ . That is, the lower the fiscal surplus, the higher the interest rate multiplier. Another assumption that differs from Benigno[4] is that  $\Gamma(\cdot)$  is a function of the fiscal surplus, while Benigno[4] assumes that it is a function of current government debt with an interest payment; that is,  $R_t B_t$ . Our setting for  $\Gamma(\cdot)$  follows Corsetti, Kuester, Meier and Mueller[12] indirectly. Corsetti, Kuester, Meier and Mueller[12] assume that the higher the fiscal deficit, the greater the probability of default, and vice versa. If we are given that the higher the probability of default, the higher the government debt coupon rate, our assumption that  $\Gamma(\cdot)$  is a decreasing function of the fiscal surplus is consistent with their analysis because the assumption implies that the higher the fiscal surplus, the higher the government debt coupon rate. That is, if we are given that the higher the probability of default, the higher the government debt coupon rate, it can be said that we indirectly assume that the lower the fiscal surplus, the higher the default rate, and this is similar to Corsetti, Kuester, Meier and Mueller[12]. Furthermore, our setting on  $\Gamma(\cdot)$  is supported by some empirical evidence. We analyze whether a fiscal deficit or government debt with interest payment increases the interest rate multiplier  $\Gamma(\cdot)$  using Greek data. These data imply that the fiscal deficit but not government debt with interest payment increases  $\Gamma(\cdot)$ .<sup>4</sup> Thus, our assumption regarding  $\Gamma(\cdot)$  is consistent with some previous work and the available data.

By solving cost-minimization problems for households, we have the optimal allocation of expenditures as follows:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t. \quad (5)$$

Once we account for Eq.(5), the intertemporal budget constraint can be rewritten as:

$$R_{t-1} D_{t-1}^n + R_{t-1}^G B_{t-1}^n (1 - \delta_t) + W_t N_t + P R_t \geq P_t C_t + D_t^n + B_t^n.$$

The remaining optimality conditions for the household's problem are given by:

$$\beta \mathbb{E}_t \left( \frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \frac{1}{R_t}, \quad (6)$$

which is the intertemporal optimality condition—namely, the Euler equation—and:

$$C_t N_t^\varphi = \frac{W_t}{P_t}, \quad (7)$$

which is the standard intratemporal optimality condition.

There is another intertemporal optimality condition depicting the households' motivation to hold government debt with default risk. This is obtained by differentiating the Lagrangian by government nominal debt and is given by:

$$\beta \mathbb{E}_t \left( \frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \frac{1}{R_t^H \mathbb{E}_t (1 - \delta_{t+1})}. \quad (8)$$

with  $R_t^H \equiv R_t \left\{ \Gamma(-sp_t) + B_t \Gamma'(-sp_t) [B(R-1)]^{-1} \right\}$ , and  $R_t^H$  can be interpreted as the government debt yield (excluding the default risk).

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<sup>4</sup>See Appendix C for details.

Combining Eqs.(6) and (8), we have:

$$R_t = R_t^H \mathbf{E}_t (1 - \delta_{t+1}), \quad (9)$$

which shows that the marginal rate of substitution for consumption is the same for households holding either (real) state-contingent claims  $D_t$  or (real) government debt  $B_t$  because both  $R_t$  and  $R_t^H \mathbf{E}_t (1 - \delta_{t+1})$  equal the marginal rate of substitution  $\beta \mathbf{E}_t \left( \frac{P_t C_t}{P_{t+1} C_{t+1}} \right)$ . That is, the consumption schedule is the same whether households hold state-contingent claims  $D_t$  or government debt  $B_t$ . Log-linearizing Eq.(9) yields:

$$\hat{r}_t = \hat{r}_t^H - \mathbf{E}_t (\delta_{t+1}), \quad (10)$$

with  $\hat{r}_t \equiv \frac{dR_t}{R_t}$  and  $\hat{r}_t^H \equiv \frac{dR_t^H}{R_t^H}$ .

Log-linearizing the definition of government debt yield  $R_t^H$ , we have:

$$\hat{r}_t^H = \frac{\omega_\phi}{1 - \beta} \hat{r}_t - \frac{\phi \omega_\gamma}{1 - \beta} sp_t + \frac{\phi \beta}{1 - \beta} b_t, \quad (11)$$

with  $\omega_\phi \equiv 1 - \beta(1 - \phi)$ , and  $\omega_\gamma \equiv 1 + \beta(\gamma - 1)$ , where  $\phi \equiv \Gamma'(0)$  denotes the interest rate spread in the steady state and  $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)}$  denotes the elasticity of the interest rate spread to a one percent change in the fiscal deficit in the steady state. Following Benigno[4], we define the interest rate spread for government debt  $\phi$  and assume  $\Gamma(0) = 1$ . The elasticity  $\gamma$  is an unfamiliar parameter, and we assume  $|\Gamma'(\cdot)| < |\Gamma''(\cdot)|$ ; thus,  $\gamma > 1$ . Our assumption implies that a decrease in the fiscal surplus increases the government debt coupon rate via an increase in the interest rate multiplier, and vice versa, and that changes in the government debt coupon rate are larger than the changes in the fiscal surplus in absolute value. Note that our assumption is supported by the data, which we discuss in Appendix B, estimating the elasticity of the interest rate spread given a one percent change in the fiscal deficit  $\gamma$ .

Given our assumption, Eq.(11) implies that an increase in the fiscal surplus decreases the government debt yield, and vice versa. This is intuitively consistent because an increase in fiscal surplus decreases the interest rate multiplier and decreases the government debt yield. In addition, in the third term on the right-hand side (RHS), the sign is positive. This shows that the government debt yield is an increasing function of government debt. An increase in government debt coincides with a decrease in the fiscal surplus, and vice versa. Thus, this positive sign is consistent with the negative sign in the second term. That is, an increase in government debt increases the government debt yield through an increase in the interest rate multiplier  $\Gamma(\cdot)$ , which is brought about by a decrease in the fiscal surplus.

## 2.2 Government

### 2.2.1 Government Budget Constraint and the FTPL

Fiscal policy consists of choosing the mix between taxes and the one-period nominal debt with sovereign risk to finance the exogenous process of government expenditure. The flow government budget constraint is given by:

$$B_t^n = R_{t-1}^G (1 - \delta_t) B_{t-1}^n - \int_0^1 P_t(i) [\tau_t Y_t(i) - G_t(i)] di.$$



Because the optimal allocation of generic goods is given by  $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$  and  $G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t$ , this equality can be rewritten as:

$$B_t^n = R_{t-1}^G (1 - \delta_t) B_{t-1}^n - P_t S P_t.$$

Dividing both sides of the equality by  $P_t$  yields:

$$B_t = R_{t-1} \Gamma (-s p_{t-1}) (1 - \delta_t) B_{t-1} \Pi_t^{-1} - S P_t. \quad (12)$$

with  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  being the gross inflation rate. The first term on the RHS corresponds to the amount of redemption with the nominal interest payment and shows that the lower the past fiscal surplus, the higher the interest payments, and the higher the default rate, the lower the redemption, and vice versa.

Log-linearizing Eq.(12) yields:

$$b_t = \frac{1}{\beta} \hat{r}_{t-1} - \frac{1}{\beta} \delta_t - \frac{1}{\beta} \pi_t + \frac{1}{\beta} b_{t-1} - \frac{1-\beta}{\beta} s p_t - \frac{\phi}{\beta} s p_{t-1}, \quad (13)$$

where we use the log-linearized definition of the government debt coupon rate  $\hat{r}_t^G = \hat{r}_t - \phi s p_t$  with  $\hat{r}_t^G \equiv \frac{dR_t^G}{R_t^G}$  and  $\pi_t \equiv \log \Pi_t$ . Eq.(13) implies that not only the higher the current fiscal surplus but also the higher the past fiscal surplus, the lower the current government debt because an increase in the fiscal surplus decreases the interest payment via a decrease in the interest rate multiplier.

The appropriate transversality condition for government debt is given by:

$$\lim_{j \rightarrow \infty} \beta^{t+j+1} \mathbf{E}_t \left[ R_{t+j}^G (1 - \delta_{t+j+1}) \frac{P_{t+j} B_{t+j}}{P_{t+j+1}} \right] = 0.$$

By iterating the second equality in Eq.(12) forward, plugging Eq.(6) into this iterated equality, and imposing the appropriate transversality condition for government debt, we have:

$$C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1} (1 - \delta_t) = C_t^{-1} S P_t + \beta \frac{R_t^H}{R_t^G} \mathbf{E}_t (C_{t+1}^{-1} S P_{t+1}) + \beta^2 \mathbf{E}_t \left( \frac{R_t^H}{R_t^G} \frac{R_{t+1}^H}{R_{t+1}^G} C_{t+1}^{-1} S P_{t+1} \right) + \dots, \quad (14)$$

which roughly shows that the burden of government debt redemption with interest payment in terms of consumption, or the left-hand side (LHS), corresponds to the expected sum of the discounted value of the fiscal surplus in terms of consumption, or the RHS, because of the transversality condition. Here,  $\frac{R_t^H}{R_t^G}$  and so forth appear on the RHS. An increase in the government debt coupon rate  $R_t^G$  then worsens the fiscal situation through the increase in the interest payment. Thus,  $R_t^G$  is the denominator. An increase in the government debt yield facilitates the purchase of government debt even though consumption decreases. A decrease in the consumption then improves the fiscal situation because the decrease in the consumption increases the fiscal surplus in terms of consumption. Thus,  $R_t^H$  appears as the numerator. If  $R_t^G = R_t^H$  is applied for all  $t$ , which implies that the government debt coupon rate corresponds to the government debt yield, Eq.(14) reduces to:

$$C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1} (1 - \delta_t) = C_t^{-1} S P_t + \beta \mathbf{E}_t (C_{t+1}^{-1} S P_{t+1}) + \beta^2 \mathbf{E}_t (C_{t+1}^{-1} S P_{t+1}) + \dots$$

In this case, the burden of government debt redemption with interest payment in terms of consumption simply corresponds to the expected sum of the discounted value of the fiscal surplus in terms of consumption.

Eq.(14) can be rewritten as:

$$\delta_t = 1 - \frac{\frac{R_{t-1}^G}{R_{t-1}^H} \sum_{k=0}^{\infty} \prod_{h=0}^k \beta^k \mathbf{E}_t \left( \frac{R_{t+h-1}^H}{R_{t+h-1}^G} C_{t+k}^{-1} S P_{t+k} \right)}{C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1}}. \quad (15)$$

Eq.(15) is our FTSR and implies that an increase in inflation does not necessarily occur even if the government's solvency is lost, and vice versa, similar to Uribe[24]. Not only inflation, but also default, can mitigate the burden of government debt redemption with interest payment. Suppose that the price level is constant and there is no inflation. In this situation, if the net present value of the fiscal surplus in terms of consumption (the numerator) is about to fall below the burden of government debt redemption with interest payment in terms of consumption (the denominator), the second term on the RHS is less than unity. Simultaneously, the LHS exceeds zero; that is, default occurs. In other words, if the government falls insolvent while the price level is strictly stable, default is inevitable. Uribe[24] shows the SI–SD trade-off by introducing default—namely, sovereign risk—into the central equation of the fiscal theory of the price level (FTPL). Similar to Uribe[24], at first glance, Eq.(15) also implies that there is an SI–SD trade-off. Furthermore, he calibrates his model and compares the Taylor rule that stabilizes inflation with the interest rate peg. Under the interest rate peg, the interest rate on risky assets corresponds to the risk-free asset interest rate pegged to the steady-state rate. This calibration shows that default ceases just one period after the shock decreasing the fiscal surplus, even though default continues under the Taylor rule after the shock. This implies that a Taylor rule to stabilize inflation includes the unwelcome possibility of magnifying sovereign risk, and this calls for an interest rate peg to counter default. Although Uribe[24] ignores the welfare perspective of these actions, his policy implications are persuasive. Paying attention to just Eq.(15), which is similar to that in Uribe's[24] model, we seem to obtain policy implications quite similar to those in Uribe[24].

We now present the relationship between our FTSR; namely, Eq.(15) and the FTPL. If there is neither default risk nor an interest rate multiplier in Eq.(15), Eq.(15) reduces to the following because of  $R_t^G = R_t^H = R_t$ :

$$1 = \frac{\sum_{k=0}^{\infty} \beta^k \mathbf{E}_t (C_{t+k}^{-1} S P_{t+k})}{C_t^{-1} R_{t-1} B_{t-1} \Pi_t^{-1}},$$

which is our version of the FTPL. On the RHS in this equality, the numerator is the net present value of the sum of the fiscal surplus in terms of consumption, and the denominator is the burden of the government debt redemption with interest payment in terms of consumption divided by inflation. The LHS is unity. If solvency worsens, the price level increases; that is, inflation occurs, such that the burden of government debt redemption is mitigated. For now, we introduce sovereign risk, and this mechanism is no longer fully applicable, as Eq.(15) implies.

### 2.2.2 Default Rule

Because the default rate is decided endogenously, it may not be said that the government chooses the default rate following a certain rule. However, although the default rule is endogenous in Uribe[24], Uribe[24] considers the default rule that the government does not default unless the tax-to-debt-ratio falls below a certain threshold.<sup>5</sup> Following this idea, we say that the default rate

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<sup>5</sup>The tax-to-debt ratio in Uribe[24] measures government solvency and corresponds to the second term in Eq.(15).

is decided by the following rule. Let us define  $\Psi \equiv \frac{\frac{R_{t-1}^G}{R_{t-1}^H} \sum_{k=0}^{\infty} \prod_{h=0}^k \frac{R_{t+h-1}^H}{R_{t+h-1}^G} \beta^k \mathbf{E}_t (C_{t+k}^{-1} SP_{t+k})}{C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1}}$ , where  $\Psi$  denotes the threshold chosen arbitrarily by the government. Around the steady state,  $\Psi = 1$ , and we set our threshold to one. The government chooses  $\delta_t > 0$  if  $\Psi < 1$ ; that is, the government defaults if solvency worsens. The government chooses  $\delta_t < 0$  if  $\Psi > 1$ ; that is, the government can afford not to default. The government chooses  $\delta_t = 0$  if  $\Psi = 1$ .

### 2.2.3 Relationship between Default Rate and Fiscal Surplus

By leading Eq.(15) one period and plugging this into Eq.(15) itself, we can rewrite Eq.(15) as a second-order differential equation as follows:

$$\delta_t = 1 - \frac{1}{R_{t-1}^G \Pi_t^{-1} B_{t-1}} \left\{ SP_t + \beta \mathbf{E}_t \left[ \left( \frac{C_t}{C_{t+1}} \Pi_{t+1}^{-1} \right) R_t^H (1 - \delta_{t+1}) B_t \right] \right\}. \quad (16)$$

In Eq.(16), the current government debt  $B_t$  appears in the second term on the RHS and the sign is negative. That is, a decrease in current government debt increases the default rate, and vice versa. Why is the sign of government debt  $B_t$  in the second term on the RHS negative? This stems from the transversality condition for government debt. Because of the transversality condition, Eq.(15) and its second-order differential version Eq.(16) are strictly applicable. That is, once issued, government debt must be redeemed. Otherwise, the burden of redemption is mitigated by default or inflation. To keep Eq.(15), once government debt is issued, the fiscal surplus must be improved while newly issued government debt is about to reduce the fiscal surplus. Because the fiscal surplus must improve to redeem debt, the default rate declines as a result of an improvement in the fiscal surplus when government debt increases. Thus, the sign is negative.

In addition, we can easily imagine that the fiscal surplus is a function of the output gap. In fact, the log-linearized fiscal surplus is given by:

$$sp_t = \frac{\beta\tau}{(1-\beta)\varsigma_B} \hat{\tau}_t + \frac{\beta\tau}{(1-\beta)\varsigma_B} y_t - \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B} g_t. \quad (17)$$

with  $\varsigma_B \equiv \frac{B}{Y}$  and  $\varsigma_G \equiv \frac{G}{Y}$  being the steady-state ratio of government debt to output and the steady-state ratio of government expenditure to output, respectively where  $\hat{\tau}_t \equiv \frac{d\tau_t}{\tau}$  denotes the percentage deviation of the tax rate from its steady-state value. We simply refer to the percentage deviation of the tax rate from its steady-state value  $\hat{\tau}_t$  as the tax gap. By using Gali and Monacelli's[17] definition of the output gap—namely,  $\tilde{y}_t \equiv y_t - \bar{y}_t$ , where  $\tilde{y}_t$  and  $\bar{y}_t$  denote the output gap and the natural rate of output, respectively—we can recognize that stabilizing the fiscal surplus leads to the stabilization of the output gap.<sup>6</sup> As mentioned, we need to stabilize the default rate to stabilize the fiscal surplus. In addition, stabilizing the fiscal surplus leads to the stabilization of the output gap, which positively links with inflation under Calvo pricing. Thus, we can imagine that there is not necessarily an SI–SD trade-off.

### 2.2.4 Log-linearizing the Government Budget Constraint

Log-linearizing Eq.(16) yields:

$$c_t = \mathbf{E}_t (c_{t+1}) - \beta \hat{r}_t + \mathbf{E}_t (\pi_{t+1}) - \frac{\omega_\phi}{1-\beta} b_t + \mathbf{E}_t (\delta_{t+1}) - \frac{\omega_{sp}}{\beta(1-\beta)} sp_t + \frac{1}{\beta} \hat{r}_{t-1} - \frac{1}{\beta} \pi_t$$

<sup>6</sup>In our model, the steady state is not efficient because friction stemming from the monopolistically competitive market cannot be dissolved by taxation. Thus, the target level of the output gap (or efficient output gap) is not zero, even though the target level is zero in Gali and Monacelli[17], because the steady state is efficient.

$$+ \frac{1}{\beta} b_{t-1} - \frac{1}{\beta} \delta_t - \frac{\phi}{\beta} s p_{t-1}, \quad (18)$$

with  $\omega_{sp} \equiv (1 - \beta)^2 - \phi \omega_\gamma \beta$ , where we use the log-linearized definition of the government debt coupon rate. Eq.(18) is our log-linearized Euler equation.

### 2.3 Firms

This subsection outlines the production, price setting, marginal cost, and features of the firms, and these are quite similar to Gali and Monacelli[17], although here the tax is levied on firm sales and is not constant.<sup>7</sup>

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(i) = A_t N_t(i),$$

where  $A_t$  denotes the productivity.

By combining the production function and the optimal allocation for goods, we have an aggregate production function relating to aggregate employment as follows:

$$N_t = \frac{Y_t Z_t}{A_t}, \quad (19)$$

where  $Z_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} di$  denotes the price dispersion.

Log-linearizing Eq.(19) yields:

$$n_t = y_t - a_t. \quad (20)$$

We assume that productivity follows an AR(1) process; namely,  $E_t(a_{t+1}) = \rho_A a_t$ , similar to government expenditure.  $Z_t$  disappears in Eq.(15) because of  $o(\|\xi\|^2)$ .

Each firm is a monopolistic producer of one of the differentiated goods. Each firm sets its prices  $P_t(i)$  taking as given  $P_t$  and  $C_t$ . We assume that firms set prices in a staggered fashion, Calvo pricing, according to which each seller has the opportunity to change its price with a given probability  $1 - \theta$ , where an individual firm's probability of reoptimizing in any given period is independent of the time elapsed since it last reset its price. When a firm has the opportunity to set a new price in period  $t$ , it does so in order to maximize the expected discounted value of its net profits. The FONCs for firms are given by:

$$\tilde{P}_t = \frac{E_t \left( \sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k} \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k} \right)}{E_t \left( \sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k} \right)}, \quad (21)$$

where  $MC_t \equiv \frac{W_t}{(1-\tau_t)P_t A_t}$  denotes the real marginal cost,  $\tilde{Y}_{t+k} \equiv \left( \frac{\tilde{P}_t}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}$  denotes the demand for goods when firms choose a new price, and  $\tilde{P}_t$  denotes the newly set prices. Note that we assume that government levies a tax on firm sales.

By log-linearizing Eq.(21), we have:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa mc_t, \quad (22)$$

<sup>7</sup>Unlike our setting, Gali and Monacelli[17] assume that under constant employment subsidies, monopolistic power completely disappears.

with  $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$  being the slope of the New Keynesian Phillips Curve (NKPC). Eq.(22) is the fundamental equality of our NKPC.

Substituting Eq.(7) into the definition of the real marginal cost yields:

$$MC_t = \frac{C_t N_t^\varphi}{(1-\tau_t) A_t}. \quad (23)$$

Note that the marginal cost in the steady state, which is the inverse of a constant markup, is smaller than one, while the gross tax rate  $1-\tau$  is definitely smaller than one. In such a case, the steady-state wedge between the marginal product of labor and the marginal utility of consumption is not unity. That is, monopolistic power remains because it is unable to be completely absorbed through taxation. As we discuss later, we need to derive our welfare criteria following Benigno and Woodford[8] because monopolistic power is no longer removed completely, and the steady state is distorted.

Log-linearizing Eq.(23) yields:

$$mc_t = c_t + \varphi y_t + \frac{\tau}{1-\tau} \hat{r}_t - (1+\varphi) a_t. \quad (24)$$

## 2.4 Equilibrium

### 2.4.1 Market-Clearing Conditions

The market-clearing condition requires:

$$Y_t(i) = C_t(i) + G_t(i),$$

for all  $i \in [0, 1]$  and all  $t$ . By plugging the optimal allocation for generic goods including Eq.(5) into this market-clearing condition, we have:

$$Y_t = C_t + G_t. \quad (25)$$

By log-linearizing Eq.(25), we obtain:

$$y_t = \varsigma_C c_t + \varsigma_G g_t, \quad (26)$$

where  $\varsigma_C \equiv 1 - \varsigma_G$  denotes the steady-state ratio of consumption to output.

### 2.4.2 Output, Nominal Interest Rate and Inflation Dynamics

Plugging Eq.(26) into Eq.(18) yields:

$$\begin{aligned} y_t = & \mathbf{E}_t(y_{t+1}) - \varsigma_C \hat{r}_t + \varsigma_C \mathbf{E}_t(\pi_{t+1}) - \frac{\varsigma_C \omega_\phi}{1-\beta} b_t + \varsigma_C \mathbf{E}_t(\delta_{t+1}) + \frac{\varsigma_C}{\beta} \hat{r}_{t-1} - \frac{\varsigma_C}{\beta} \pi_t + \frac{\varsigma_C}{\beta} b_{t-1} \\ & + \frac{\varsigma_C}{\beta} \delta_t - \frac{\varsigma_C \omega_{sp}}{\beta(1-\beta)} sp_t - \frac{\phi \varsigma_C}{\beta} sp_{t-1} + \varsigma_G (1-\rho_G) g_t, \end{aligned} \quad (27)$$

where we assume that the government expenditure follows an AR(1) process and  $\mathbf{E}_t(g_{t+1}) = \rho_G g_t$ .

Plugging Eqs.(24) and (26) into Eq.(22), we have:

$$\pi_t = \beta \mathbf{E}_t(\pi_{t+1}) + \frac{\kappa [1 + \varphi \varsigma_C]}{1 - \varsigma_G} y_t + \frac{\kappa \tau}{1 - \tau} \hat{r}_t - \frac{\kappa \varsigma_G}{1 - \varsigma_G} g_t - \kappa (1 + \varphi) a_t. \quad (28)$$

Eq.(28) stemming from the firms' FONC Eq.(16) does not have any notable features.

### 3 Policy Target

We analyze three policies, the OM, the OMF, and the MIS policies to contrast with Uribe[24], who analyzes inflation stabilization policy including the Taylor rule and price level targeting, and an interest rate peg that pegs both the nominal interest rate for safe assets and the nominal interest rate for risky assets. Because the OM and the OMF policies are both de facto inflation stabilization policies, these clearly correspond to the Taylor rule and the price level targeting in Uribe[24]. At first glance, there is some difference between the interest rate peg in Uribe[24] and the MIS policy, which minimizes the difference between the nominal interest rate  $\hat{r}_t$ , which is the interest rate for safe assets, and the government debt yield  $\hat{r}_t^H$ . However, both policies are intrinsically the same thing. The expected default rate converges to zero under the interest rate peg in Uribe[24], and the MIS policy makes the expected default rate zero *ex ante*. In fact, as shown in Eqs.(9) and (10), the expected default rate  $\mathbb{E}_t(\delta_{t+1})$  should be zero if the nominal interest rate completely corresponds to the government debt yield; that is,  $R_t = R_t^H$ . Thus, the MIS policy imitates the interest rate peg in Uribe[24] in this regard. <sup>8</sup>

We now discuss the details of each policy. Under the MIS policy, the policy authorities minimize the interest rate spread between the nominal interest rate and the government debt yield  $\hat{r}_t^S \equiv \hat{r}_t^H - \hat{r}_t$  over time. That is, they minimize the following:

$$\mathcal{L}^R \equiv \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0(L_t^R) \quad (29)$$

with:

$$L_t^R \equiv \frac{1}{2} (\hat{r}_t^S)^2.$$

Because of Eq.(10), the expected default rate will be zero under the MIS policy. As mentioned, from the viewpoint of minimizing the expected default rate, this policy corresponds to the interest rate peg in Uribe[24]. Note that Uribe[24] shows that the default is settled just one period after an exogenous negative fiscal surplus shock under the interest rate peg. Because of the zero expected default rate, default no longer occurs after the second period.

Under the OM and the OMF policies, the policy authorities minimize the welfare cost function over time. We derive the period welfare cost function from the welfare criterion following Gali[15]. However, because of the distorted steady state, we need to eliminate the linear term, which generates the ‘‘Welfare Reversal’’.<sup>9</sup> To eliminate the linear term, we need to derive the second-order approximated aggregate supply equation and the second-order approximated intertemporal government solvency condition. Here, the aggregate supply equation and the intertemporal government solvency condition correspond to Eqs.(21) and (15), respectively. Thus, we follow not only Gali[15] but also Benigno and Woodford[8] and Benigno and Woodford[25] to derive the welfare criterion similar to Ferrero[14], analyzing monetary and fiscal policy rules in a currency union with a distorted steady state.

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<sup>8</sup>Policy objectives in Uribe[24] such as the price level targeting and the interest rate peg are given exogenously. However, unlike Uribe[24], we do not give policy objective exogenously because this generates indeterminacy. See Gali and Monacelli[17].

<sup>9</sup>The presence of linear terms generally leads to the incorrect evaluation of welfare, with a simple example of this result proposed by Kim and Kim[19]. Tesar[23] used the log-linearization method and derived the paradoxical result that the incomplete-markets economy produces a higher level of welfare than the complete-markets economy. Kim and Kim[19] point out that the reversal of welfare ordering implies approximation errors owing to the linearization.

Note that we impose  $R_t^G = R_t^H$  when we derive the second-order approximated intertemporal government solvency condition because of the limits of our abilities. However, this restriction has no impact on our analyzing the SI-SD trade-off because our welfare cost function implies that stabilizing inflation is almost the only policy target, and this implies that the OM and OMF policies are de facto inflation stabilization policies.<sup>10</sup> In addition, as shown in Appendix E, our empirical analysis shows that the hypothesis that the government bond yield is consistent with the coupon rate on the benchmark 10-year government bond cannot be rejected for actual data from Italy, Spain, Germany, and the US. This result implies that  $R_t^G = R_t^H$  cannot be denied in even those countries facing significant sovereign risk, such as Italy and Spain. Thus, it cannot necessarily be said that our welfare cost function is derived under a strong assumption.

Following Gali[15], the second-order approximated utility function is given by:

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 \left( \frac{U_t - U}{UCC} \right) &= \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 \left[ \frac{\Phi}{1 - \varsigma_G} y_t - \frac{(1 - \Phi)(1 + \varphi)}{\varsigma_C 2} y_t^2 + \frac{(1 - \Phi)(1 + \varphi)}{1 - \varsigma_G} y_t a_t \right. \\ &\quad \left. - \frac{(1 - \Phi)\varepsilon}{\varsigma_C 2\kappa} \pi_t^2 \right] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \quad (30)$$

where t.i.p. denotes the terms independent of policy,  $o(\|\xi\|^3)$  are the terms of order three or higher, and  $\Phi \equiv 1 - \frac{1-\tau}{\varepsilon-1}$  denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. On the RHS, there are linear terms  $\sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 \left( \frac{\Phi}{1 - \varsigma_G} y_t \right)$  generating the welfare reversal. To avoid welfare reversal, we need to eliminate the linear terms on the RHS in Eq.(30). Following Benigno and Woodford[8] and Benigno and Woodford[25], those linear terms can be rewritten as follows:

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 \left( \frac{\Phi}{1 - \varsigma_G} y_t \right) &= - \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 \left[ \frac{\Phi [(1 - \tau)(1 + \omega_g)\omega_{\nu 1} - \tau\omega_{\omega 1}]}{2\Gamma\varsigma_C^2} y_t^2 \right. \\ &\quad \left. - \frac{\Phi [\omega_{\omega 2}\tau - (1 - \tau)(1 + \omega_g)\omega_{\nu 3}]}{\Gamma\varsigma_C^2} y_t g_t - \frac{\Phi (1 - \tau)(1 + \omega_g)\omega_{\nu 4}}{\Theta\varsigma_C} y_t a_t \right. \\ &\quad \left. + \frac{\Phi (1 - \tau)(1 + \omega_g)\varepsilon(1 + \varphi)}{2\Theta\kappa} \pi_t^2 \right] + \Upsilon_0 + o(\|\xi\|^3), \end{aligned}$$

with  $\omega_g \equiv \frac{G}{SP} = \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B}$ ,  $\Theta \equiv (1 + \omega_g)(1 - \tau)[1 + \varsigma_C\varphi] + \tau[1 - \varsigma_C(1 + \omega_g)]$ ,  $\omega_{\nu 1} \equiv \varsigma_C\varphi[\varsigma_C(1 + 2\varphi) + 2(2 - \varsigma_G)]$ ,  $\omega_{\omega 1} \equiv (1 + \varsigma_G)[1 - \varsigma_C(1 + \omega_g)]$ ,  $\omega_{\omega 2} \equiv \varsigma_C[\varsigma_G(1 + \omega_g) + \omega_g] - 2\varsigma_G$ ,  $\omega_{\nu 3} \equiv 1 - \varsigma_C\{\varsigma_G(1 - 2\varsigma_G) - \varphi[\varsigma_G(2 - \varsigma_G) - 2]\}$ , and  $\omega_{\nu 4} \equiv \varphi\varsigma_C[1 + 2(1 + \varphi)] + (1 + \varphi)(2 - \varsigma_G)$ , where  $\Upsilon_0 \equiv -\frac{\tau\Phi}{\Gamma(1-\beta)}\omega + \frac{(1-\tau)(1+\omega_g)\Phi}{\Theta\kappa}\nu$  denotes a transitory component and  $\omega$  and  $\nu$  are the second-order approximated FONC for firms and the second-order approximated solvency condition for government. Plugging the previous equality into Eq.(30) yields:

$$\sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 \left( \frac{U_t - U}{UCC} \right) \simeq -\mathcal{L} + \Upsilon_0 + \text{t.i.p.} + o(\|\xi\|^3),$$

where:

$$\mathcal{L} \equiv \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 (L_t) \quad (31)$$

<sup>10</sup>In this model, and similar to other DSGE models assuming nominal rigidities, the only practical friction is price stickiness. Thus, our welfare cost function implies that stabilizing inflation is almost the only policy target. In fact, our welfare cost function consists of just the quadratic term for inflation and the output gap from its target level which is defined later. The value of the coefficient on the quadratic term of inflation is approximately 120.2, although the value of the coefficient on the quadratic term of the output gap from its target level is only approximately 2.4 under our parameterization introduced in Section 5.1.

denotes the expected welfare costs:

$$L_t \equiv \frac{\Lambda_x}{2} x_t^2 + \frac{\Lambda_\pi}{2} \pi_t^2,$$

with  $\Lambda_x \equiv \frac{\omega_{u1}}{\Theta \varsigma_C^2}$  and  $\Lambda_\pi \equiv \frac{\varepsilon[\Phi(1-\tau)(1+\omega_g)(1+\varphi)\varsigma_C + \Theta(1-\Phi)]}{\Theta \kappa \varsigma_C}$ ,  $\omega_{u1} \equiv \Phi[(1-\tau)(1+\omega_g)\omega_{\nu1} - \tau\omega_{\omega1}] + (1-\Phi)(1+\varphi)\varsigma_C\Theta$ ,  $\omega_{u2} \equiv \varsigma_C[\Phi(1-\tau)(1+\omega_g)\omega_{\nu4} + (1-\Phi)(1+\varphi)\Theta]$ , and  $\omega_{u3} \equiv \Phi[\omega_{\omega2}\tau - (1-\tau)(1+\omega_g)\omega_{\nu3}]$  where  $L_t$  denotes the period welfare costs,  $x_t \equiv y_t - y_t^*$  denotes the output gap from the target level (OGTL), and  $y_t^* \equiv \frac{\omega_{u2}}{\omega_{u1}} a_t + \frac{\omega_{u3}}{\omega_{u1}} g_t$  denotes the target level of output.

## 4 The LQ Problem

### 4.1 NKIS, NKPC, Government Budget Constraint and the Fiscal Surplus

Plugging the definition of the OGTL into Eq.(27) yields:

$$\begin{aligned} x_t = & \mathbf{E}_t(x_{t+1}) - \varsigma_C \hat{r}_t + \varsigma_C \mathbf{E}_t(\pi_{t+1}) - \frac{\varsigma_C(1-\beta)}{\beta\phi} \mathbf{E}_t(\delta_{t+1}) + \frac{\varsigma_C}{\beta} \hat{r}_{t-1} - \frac{\varsigma_C}{\beta} \pi_t + \frac{\varsigma_C \omega_o}{\beta^2 \phi} \delta_t \\ & - \frac{\varsigma_C \varpi}{\beta} s p_t + \frac{\varsigma_C(\omega_\gamma - \phi\beta)}{\beta^2} s p_{t-1} + \epsilon_{x,t}, \end{aligned} \quad (32)$$

with  $\omega_o \equiv \beta(1+\phi) - 1$  and  $\varpi \equiv 2(1-\beta) + \beta\gamma$ , where  $\epsilon_{x,t} \equiv -\frac{\omega_{u2}(1-\rho_A)}{\omega_{u1}} a_t - \frac{\omega_{u3} - \varsigma_C \omega_{u1}(1-\rho_A)}{\omega_{u1}} g_t$  denotes the demand shock. Note that we use Eqs.(10) and (11) to derive Eq.(32). Eq.(32) is our version of the New Keynesian IS (NKIS) curve. Because of using Eqs.(10) and (11), the terms for the government debt disappear in Eq.(32).

Plugging the definition of the OGTL into Eq.(28) yields:

$$\pi_t = \beta \mathbf{E}_t(\pi_{t+1}) + \frac{\kappa(1+\varphi\varsigma_C)}{\varsigma_C} x_t + \frac{\kappa\tau}{1-\tau} \hat{r}_t + \epsilon_{\pi,t}, \quad (33)$$

where  $\epsilon_{\pi,t} \equiv \frac{\kappa[(1+\varphi\varsigma_C)\omega_{u3} - \varsigma_C \omega_{u1}]}{\varsigma_C \omega_{u1}} g_t - \frac{\kappa[(1+\varphi\varsigma_C)\omega_{u2} - (1+\varphi)\omega_{u1}]}{\varsigma_C \omega_{u1}} a_t$  denotes the cost-push shock. Eq.(33) is our version of the NKPC.

Plugging Eqs.(10) and (11) into Eq.(13) yields:

$$s p_t = \frac{1}{\varpi} \hat{r}_{t-1} - \frac{\omega_o}{\phi\beta\varpi} \delta_t - \frac{1}{\varpi} \pi_t + \frac{\omega_\gamma - \phi\beta}{\varpi\beta} s p_{t-1} + \frac{1-\beta}{\phi\varpi} \mathbf{E}_t(\delta_{t+1}). \quad (34)$$

Plugging the definition of the OGTL into Eq.(17) yields:

$$s p_t = \frac{\beta\tau}{(1-\beta)\varsigma_B} \hat{r}_t + \frac{\beta\tau}{(1-\beta)\varsigma_B} x_t + \epsilon_{sp,t}, \quad (35)$$

where  $\epsilon_{sp,t} \equiv \frac{\beta\tau}{(1-\beta)\varsigma_B} a_t - \frac{\beta(\varsigma_C \omega_{u1} - \tau\omega_{u3})}{(1-\beta)\varsigma_B \omega_{u1}} g_t$  is the fiscal surplus shock.

### 4.2 FONCs for the Policy Authorities

The policy authorities minimize Eq.(31) under the OM and OMF policies, while they minimize Eq.(29) under the MIS policy, subject to Eqs.(32)–(35). We assume that there are two policy instruments, the tax gap and the nominal interest rate. Under the OM and MIS policies, the policy instrument is just the nominal interest rate  $\hat{r}_t$ , and the tax gap  $\hat{r}_t$  is zero over time; that is, the tax rate



is fixed at its steady-state level. The policy authorities choose the sequence  $\{x_t, \pi_t, \hat{r}_t, \delta_t, sp_t\}_{t=0}^\infty$ .<sup>11</sup> Under the OMF policy, the policy instruments are not only the nominal interest rate  $\hat{r}_t$  but also the tax gap  $\hat{\tau}_t$ . The policy authorities select the sequence  $\{x_t, \pi_t, \hat{r}_t, \hat{\tau}_t, \delta_t, sp_t\}_{t=0}^\infty$ .

The OM and the OMF policies are synonyms for an inflation stabilization policy because the weight on the quadratic term of inflation in Eq.(31) is extremely high. Thus, analyzing the effects on the default rate under the OM or OMF policy is analogous to analyzing the effects on the default rate under an inflation stabilization policy. Furthermore, there is one policy instrument under the OM policy, while there are two policy instruments under the OMF policy. This means that the OM policy regime lacks one of the available policy instruments to conduct policy or to stabilize inflation, while the OMF policy regime is more aggressive in stabilizing inflation than the OM policy regime. Thus, we can find how stabilizing inflation affects the default rate through comparing the dynamics on both inflation and the default rate under both policies.

#### 4.2.1 FONCs under the OM Policy

Now, we show the FONCs under the OM policy. The FONCs for the OGTL and for inflation are given by:

$$\Lambda_x x_t = -\mu_{1,t} + \frac{\kappa(1 + \varphi_{SC})}{\varsigma_C} \mu_{2,t} + \frac{\beta\tau}{(1 - \beta)\varsigma_B} \mu_{4,t} + \frac{1}{\beta} \mu_{1,t-1}, \quad (36)$$

$$\Lambda_\pi \pi_t = -\frac{\varsigma_C}{\beta} \mu_{1,t} - \mu_{2,t} + \frac{1}{\varpi} \mu_{3,t} + \frac{\varsigma_C}{\beta} \mu_{1,t-1} + \mu_{2,t-1}, \quad (37)$$

where  $\mu_{1,t}$ ,  $\mu_{2,t}$ ,  $\mu_{3,t}$ , and  $\mu_{4,t}$  are the Lagrange multipliers on Eqs.(32), (33), (34), and (35), respectively. By following Benigno and Benigno[5], we can interpret Eqs.(36) and (37) as the targeting rule. Because of default risk, these FONCs are somewhat different from the familiar ones. However, by ignoring the Lagrange multipliers  $\mu_{3,t}$  and  $\mu_{4,t}$ , we can understand that inflation is stabilized via stabilizing the OGTL because the Lagrange multipliers  $\mu_{1,t}$  and  $\mu_{2,t}$  are multiplied on the NKIS Eq.(32) and NKPC Eq.(33). The mechanism for stabilizing inflation is similar to that in the New Keynesian literature, including Benigno and Benigno[5].

The FONCs for the nominal interest rate and the default rate are given by:

$$\varsigma_C \mu_{1,t} = \varsigma_C \mathbf{E}_t(\mu_{1,t+1}) + \frac{\beta}{\varpi} \mathbf{E}_t(\mu_{3,t+1}), \quad (38)$$

$$\frac{\varsigma_C \omega_o}{\phi\beta} \mu_{1,t} = -\frac{\omega_o}{\varpi} \mu_{3,t} - \frac{\varsigma_C(1 - \beta)}{\beta} \mu_{1,t-1} - \frac{1 - \beta}{\varpi} \mu_{3,t-1}, \quad (39)$$

where Eqs.(38) and (39) are the FONCs for the nominal interest rate and the default rate, respectively. These show that there is a close relationship between the NKIS in Eq.(32) and the government budget constraint in Eq.(34). The FONC for the fiscal surplus is given by:

$$\frac{\varsigma_C \varpi}{\beta} \mu_{1,t} = -\mu_{3,t} - \mu_{4,t} + \frac{\varsigma_C(\omega_\gamma - \phi\beta)}{\beta} \mathbf{E}_t(\mu_{1,t+1}) + \frac{\omega_\gamma - \phi\beta}{\varpi} \mathbf{E}_t(\mu_{3,t+1}), \quad (40)$$

which shows that changes in the fiscal surplus affect the NKIS Eq.(32) and the government budget constraint Eq.(34). In addition, Eqs.(39) and (40) imply that changes in the fiscal surplus affect the default rate because of the Lagrange multipliers on the government budget constraint  $\mu_{3,t}$  and the definition of the fiscal surplus  $\mu_{4,t}$ .

<sup>11</sup>Because government debt disappears in our model, at first glance the policy authorities' instrument is merely the nominal interest rate. However, government debt is indirectly chosen by choosing the fiscal surplus and the default rate.

#### 4.2.2 FONCs under the MIS Policy

The FONCs for the OGTL and for the inflation are given by:

$$\mu_{1,t} = \frac{\kappa(1 + \varphi\varsigma_C)}{\varsigma_C} \mu_{2,t} + \frac{\beta\tau}{(1 - \beta)\varsigma_B} \mu_{4,t} + \frac{1}{\beta} \mu_{1,t-1}, \quad (41)$$

$$\mu_{1,t} = \mu_{1,t-1} - \frac{\beta}{\varsigma_C} (\mu_{2,t} - \mu_{2,t-1}) - \frac{\beta}{\varsigma_C \varpi} \mu_{3,t}. \quad (42)$$

Eqs.(41) and (42) are equivalent to Eqs.(36) and (37) although inflation and the OGTL disappear in Eqs.(41) and (42) because the period loss function  $L_t^R$  does not include the quadratic terms for inflation and the OGTL. That is, both inflation and the OGTL are not intended to be stabilized by the policy authorities.

The FONCs for the nominal interest rate and the fiscal surplus are given by Eqs.(38) and (40), respectively, even under the MIS policy. The FONC for the default rate has distinctive features and is given by:

$$\delta_t = -\frac{\varsigma_C \omega_o}{\beta \phi} \mu_{1,t} - \frac{\omega_o}{\phi \varpi} \mu_{3,t} - \frac{\varsigma_C (1 - \beta)}{\beta \phi} \mu_{1,t-1} - \frac{1 - \beta}{\phi \varpi} \mu_{3,t-1}, \quad (43)$$

which can be interpreted as a targeting rule under the MIS policy. As Eq.(10) implies, stabilizing the default rate is essential to minimizing the interest rate spread. Recall that  $\mu_{1,t}$  and  $\mu_{3,t}$  are Lagrange multipliers on Eqs.(32) and (34); namely, the NKIS and the log-linearized government budget constraint. Thus, to stabilize the default rate, both the NKIS (its LHS is the output gap) and the government budget constraint (its LHS is the fiscal surplus) must shift downward when the default rate is about to increase, and vice versa, because the signs on the first and second terms on the RHS are negative. The negative sign on the first term on the RHS implies that when the default rate is about to increase, the output gap must decrease, and vice versa. As long as government expenditure does not change, the decrease in the output gap coincides with the decrease in consumption. The decrease in the consumption increases the discounted value of the sum of the fiscal surplus in terms of consumption, or solvency. This improvement in solvency applies pressure to decreasing the default rate and the default rate stabilizes.

The negative sign for the second term on the RHS implies that when the default rate is about to increase, the fiscal surplus decreases. This is not inconsistent with our intuition. As long as government expenditure is constant, the fiscal surplus decreases when output decreases. A decrease in output coincides with a decrease in consumption. As mentioned, a decrease in consumption increases the fiscal surplus in terms of consumption or solvency and removes the pressure to increase the default rate. Thus, the negative sign for the second term on the RHS is plausible.

#### 4.2.3 FONCs under the OMF Policy

Under the OMF policy, the FONCs are given not only by Eqs.(36)–(40), but also by the FONC for the tax gap as follows:

$$\mu_{2,t} = -\frac{(1 - \tau)\beta}{(1 - \beta)\varsigma_B \kappa} \mu_{4,t}.$$

As mentioned,  $\mu_{2,t}$  and  $\mu_{4,t}$  are Lagrange multipliers on NKPC Eq.(33) and the definition of the fiscal surplus Eq.(35), respectively. This equality shows that changes in the fiscal surplus affect the NKPC via changes in the tax gap under the OMF policy. In the FONC for inflation Eq.(37),

$\mu_{2,t}$  appears with a negative sign. By plugging this FONC into Eq.(37), we can understand that the definition of the fiscal surplus Eq.(35) must shift upward to stabilize inflation when inflation is about to increase. Because the LHS of Eq.(35) is the fiscal surplus, this means that the fiscal surplus has to increase to stabilize inflation. This mechanism to stabilize inflation has another effect. As shown in Eq.(15) (that is, our FTSR), an increase in the fiscal surplus decreases the default rate, and vice versa. Thus, stabilizing the fiscal surplus not only stabilizes inflation but also suppresses the default under the OMF through manipulating the tax gap. Uribe[24] highlights the SI–SD trade-off. However, by endogenizing production, which also endogenizes the fiscal balance, we find that there is not necessarily an SI–SD trade-off. Under the OMF policy, the tax gap works not only to stabilize inflation, but also to suppress the default through stabilizing the fiscal balance.

## 5 Numerical Analysis

### 5.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. The calibrated parameters mainly follow Ferrero[14] who analyzes optimal monetary and fiscal policy, except for the unfamiliar parameters, which are estimated, including the interest rate spread for risky assets  $\phi$  and the elasticity of the interest rate spread to a one percent change in the fiscal deficit  $\gamma$  and an important parameter for analyzing monetary policy, being the price stickiness  $\theta$ . In addition, we assume that productivity and government expenditure follow AR(1) processes, and we estimate the persistence and standard errors of the innovations from the data.

Following Ferrero[14], the values for the subjective discount factor  $\beta$ , the elasticity of substitution across goods  $\varepsilon$ , the inverse of the labor supply elasticity  $\varphi$ , the steady-state ratio of government debt to output  $\varsigma_B$ , the steady-state ratio of government expenditure to output  $\varsigma_G$ , and the steady-state tax rate  $\tau$ , are set to 0.99, 11, 0.47, 2.4, 0.276, and 0.3, respectively.<sup>12</sup> Using our empirical results for the Greek data reported in Appendices B and D, the spread of the nominal interest rate  $\phi$ , the elasticity of the interest rate spread to the fiscal deficit  $\gamma$ , the price stickiness  $\theta$ , the persistence of productivity  $\rho_A$ , the persistence of government expenditure  $\rho_G$ , and the standard errors of the innovations on productivity and government expenditure are set to 0.138, 1.145, 0.705, 0.976, 0.927, 0.0316, and 0.0728, respectively.

### 5.2 Macroeconomic Dynamics

#### 5.2.1 Macroeconomic Volatility and Correlation

We first discuss macroeconomic volatility (Tab. 1). The inflation volatility under the MIS policy is 1.0977 and is higher than under the OM policy, where it is 0.0012, even though the default rate volatility under the OM policy is 1.0554 and higher than that under the MIS policy, which is definitely zero. This implies that there is an SI–SD trade-off. If policy authorities choose stabilizing inflation, they have to give up suppressing default, and vice versa. This result is consistent with Uribe[24]. However, by comparing the OM policy with the OMF policy, we recognize that there is not necessarily an SI–SD trade-off. Note that both the OM and the OMF policies focus on stabilizing inflation. While the OM policy has just one policy instrument, the OMF policy has

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<sup>12</sup> $\varsigma_B = 2.4$  is consistent with quarterly time periods in the model and implies that the annual steady-state debt–output ratio is 0.6.

two policy instruments. Thus, the volatilities for the OGTL and inflation are definitely zero, which means that the inflation–output gap trade-off is completely dissolved and that these are smaller than the volatilities on the OGTL and inflation under the OM policy (0.0526 and 0.0012, respectively). Notable results are the volatilities on default rate. The volatility under the OMF policy is 0.1884, which is 82% smaller than under the OM policy. Because the volatility on inflation under the OMF policy is definitely zero and smaller than under the OM policy, we can say that there is not necessarily an SI–SD trade-off. This result is quite different from Uribe[24].

We now discuss the correlation between selected variables (Tab. 2). The correlation between inflation and default is -0.8770 under the OM policy. This implies that there is an SI–SD trade-off as long as inflation is stabilized without operating the tax gap. This result is consistent with Uribe[24]. That is, the lower inflation, the higher the default, and vice versa. How does the OMF policy dissolve or mitigate the SI–SD trade-off? The correlation between the default rate and the fiscal surplus under the OMF policy is -0.4537, and the sign is negative. That is, the higher the fiscal surplus, the lower the default rate, and vice versa. In addition, the correlation between the fiscal surplus and the tax gap under the OMF policy is 0.7191, and the sign is positive. This implies that the tax gap increases facing shocks that increase inflation and that an increase in the tax gap contributes to an increase in the fiscal surplus. As shown in Eqs.(32) and (33)—namely, the NKIS and NKPC—an increase in the fiscal surplus decreases inflation through a decrease in the OGTL, and vice versa. Thus, inflation is stabilized through an increase in the tax gap. In addition, an increase in the tax gap contributes to decreasing the default rate through an increase in the fiscal surplus, as mentioned in Section 4.2.3. In fact, as shown in Eq.(35), an increase in the tax gap increases the fiscal surplus and Eq.(34) shows that the higher the fiscal surplus, the lower the default rate. Thus, the default rate is stabilized through an increase in the tax gap. An increase in the tax gap then stabilizes both inflation and the default rate when facing pressure to inflation. Stabilizing inflation is then consistent with suppressing default. There is not necessarily representative of the SI–SD trade-off.

### 5.2.2 Impulse Response Functions

We discuss the impulse response functions (IRFs) and focus on a one standard deviation positive change in government expenditure (Fig. 1). An increase in government expenditure applies pressure to decrease the fiscal surplus and to increase the OGTL. Under the MIS policy, the default rate is completely stabilized, while inflation severely rises (Panels 2 and 7). That is, there is clearly an SI–SD trade-off. Similar to the MIS policy, the OM policy generates the SI–SD trade-off. While inflation is more stable than under the MIS policy, the default rate severely rises (Panels 3 and 7). Under the OMF policy, however, while the default rate is not completely stabilized, it is more stable than under the OM policy (Panel 7). In addition, inflation is completely stabilized under the OMF policy (Panel 3). Thus, there is not necessarily an SI–SD trade-off.

## 6 The Trade-off between Stabilizing Inflation and Suppressing the Default Rate

Is the SI–SD trade-off as severe as that highlighted by Uribe[24]? To respond, we calculate both volatilities on inflation and the default rate under various levels of price stickiness  $\theta$  from 0.6 to 0.95 every 0.05 (Fig. 2). Note that we just focus on a one standard deviation positive change in

government expenditure. Under the OM policy, there is clearly an SI–SD trade-off (Panel 1). The higher the price stickiness, the higher the volatility on the default rate and the lower the volatility on inflation, and vice versa. The higher the price stickiness, the higher the weight on inflation in the period welfare costs  $\Lambda_\pi$ . Thus, the higher the price stickiness, the lower the volatility on inflation. However, as mentioned, aggressively stabilized inflation under the OM policy induces high volatility on the default rate. Thus, there is clearly an SI–SD trade-off. The volatility on inflation depends on price stickiness under the MIS policy, similar to the OM policy (Panel 2). However, unlike the OM policy, the default volatility does not depend on the price stickiness and is definitely zero. In addition, the standard deviation on inflation is just 0.0084 when the price stickiness is 0.95. While the standard deviation on the inflation is nearly zero ( $3.4 \times 10^{-4}$ ), the standard deviation on the default rate is 0.9670 under the OM policy when the price stickiness is 0.95. Policy authorities may then choose the MIS policy rather than the OM policy because the default rate volatility is quite high under the OM policy. Uribe[24] then shows not only the SI–SD trade-off, but also the suggestion of suppressing default by giving up on stabilizing inflation. It seems that Uribe’s[24] suggestion is then not irrelevant but may be realistic if price stickiness is high.

What about the SI–SD trade-off under the OMF policy? The inflation volatility is definitely zero, and on the default rate, it is 0.0076, which is constant and does not depend on the price stickiness (Panel 2). Of course, while inflation is completely stabilized, the volatility on the default rate is quite low, but not zero. However, both inflation and the default are well and aggressively stabilized, rather than the OM policy. Thus, it can be said that there is not necessarily an SI–SD trade-off. Or if there is an SI–SD trade-off, the SI–SD trade-off is not as severe as that suggested by Uribe[24]. If price stickiness is sufficiently high, and the MIS policy is adopted instead of the OMF policy, both inflation and default are well stabilized, although the volatility on the former is not zero.

Which policy should be adopted? This cannot be judged unconditionally because the volatility on the default rate is not definitely zero, even under the OMF, and we assume  $R_t^H = R_t^G$ , which means that the government debt yield equals the government debt coupon rate when we derive the welfare cost function Eq.(29). Thus, we cannot strongly recommend the adoption of the OMF from the viewpoint of minimizing welfare costs. However, if  $R_t^H = R_t^G$  is applied, the policy target in our analysis corresponds to the welfare costs. Actually, as also mentioned, we cannot reject hypothesis  $R_t^H = R_t^G$  either in Germany and the US or in Italy and Spain, where the latter face significant sovereign risk. Thus, even in countries such as Italy and Spain facing sovereign risk, we cannot deny that the government debt yield equals the government debt coupon rate. In that case, countries should adopt the OMF policy, not the MIS policy, from the viewpoint of minimizing welfare cost.

## 7 Conclusion

We develop a class of DSGE models with nominal rigidities and find that: 1) there is not necessarily an SI–SD trade-off, and 2) the trade-off is not as severe as what Uribe[24] described. As policy implications, we argue: 1) we can practically solve the SI–SD trade-off by adopting the OMF policy, and 2) the MIS policy is not an inferior policy from the viewpoint of dissolving the SI–SD trade-off if the price stickiness is sufficiently high.

While the ECB appears to be reluctant to stabilize inflation because of smoldering sovereign risk, our results imply that there is another choice for policy authorities without becoming too concerned about the SI–SD trade-off. That is, the OMF policy may be the first option, and the policy authorities should focus on stabilizing inflation through fiscal policy without hesitation even if there is default risk. At the very least, we can surely maintain that the SI–SD trade-off is not as severe as that suggested by Uribe[24], and therefore we cannot support the assertion that simultaneously stabilizing inflation and suppressing default is impossible.

In terms of future research directions, in this paper, the welfare criteria and thus the welfare cost function is not completely consistent with the household utility function. Deriving welfare criteria that is completely consistent with the households' utility function is then a possible avenue of future work.

## Appendices

### A Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which  $\Pi_t = 1$  and  $\frac{\tilde{P}_t}{P_t} = 1$ . Because this steady state is nonstochastic, the productivity has unit values; i.e.,  $A = 1$ . We assume that the default rate in the steady state is zero; i.e.,  $\delta = 0$  is applied.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = \beta^{-1}.$$

Eq.(21) can be rewritten as:

$$\tilde{P}_t = E_t \left( \frac{K_t}{P^{-1}F_t} \right) \quad (\text{A.1})$$

with:

$$K_t \equiv \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} (P_{t+k}C_{t+k})^{-1} \tilde{Y}_{t+k} MC_{t+k}^n \quad ; \quad F_t \equiv P_t \sum_{k=0}^{\infty} (P_{t+k}C_{t+k})^{-1} \tilde{Y}_{t+k},$$

which implies that:

$$K = \frac{\frac{\varepsilon}{\varepsilon-1} Y MC^n}{(1 - \alpha\beta)(PC)} \quad ; \quad F = \frac{PY}{(1 - \alpha\beta)(PC)}.$$

These equalities imply that:

$$P = \frac{\varepsilon}{\varepsilon - 1} MC^n.$$

Thus, we have:

$$MC = \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{-1}. \quad (\text{A.2})$$

Furthermore, Eqs.23) and (A.2) imply the following:

$$CN^\varphi = \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}. \quad (\text{A.3})$$

Eq.(A.3) implies the familiar expression:

$$(1-\tau)U_C = \frac{\varepsilon}{\varepsilon-1}U_N.$$

Note that because  $\tau \in (0,1)$  and  $\varepsilon > 1$ , this steady state is distorted.

Eq.(12) yields the following:

$$B \left( \frac{1-\beta}{\beta} \right) = SP, \quad (\text{A.4})$$

with  $B \equiv \frac{B^n}{P}$ .

Note that  $R = R^H$  because of  $\delta = 0$  and  $R^G = R\Gamma(0)$ . Plugging this into Eq.(14) yields:

$$C^{-1}R\Gamma(0)B = C^{-1}SP + \frac{\beta}{\Gamma(0)}C^{-1}SP + \left( \frac{\beta}{\Gamma(0)} \right)^2 C^{-1}SP + \dots = \frac{1}{1-\beta[\Gamma(0)]^{-1}}C^{-1}SP,$$

which implies:

$$\Gamma(0)B\beta^{-1} = \frac{1}{1-\beta[\Gamma(0)]^{-1}}SP. \quad (\text{A.5})$$

Plugging Eq.(5) into this equality yields:

$$\Gamma(0) = \frac{1-\beta}{1-\beta[\Gamma(0)]^{-1}},$$

which implies that  $\Gamma(0) = 1$ . Thus, our assumption that  $\delta = 0$  is consistent with  $\Gamma(0) = 1$ .

Because of  $\Gamma(0) = 1$ ,  $R^G = R$ . Thus,

$$R^G = R^H. \quad (\text{A.6})$$

In the steady state, Eq.(15) reduces to:

$$1 = \frac{\frac{1}{(1-\beta\frac{R^H}{R^G})} (C^{-1}SP)}{C^{-1}RB}. \quad (\text{A.7})$$

Note that the RHS in Eq.(A.7) corresponds to the steady-state value of  $\Psi$ . That is,  $\Psi = 1$  is applied in the steady state. This implies that the default rate is zero in the steady state.

## B Empirical Evidence on Calibrated Unfamiliar Parameters and AR(1) Processes

One of our calibrated parameters, the elasticity of the interest rate spread to the fiscal deficit  $\gamma$ , is based on the following regression:

$$\ln \left( R_t^{risky} - R_t \right) = \alpha_0 + \alpha_1 df_t + \alpha_2 DUM_t + \alpha_3 df_t DUM_t, \quad (\text{B.1})$$

where  $R_t^{risky}$  corresponding to  $R_t^G$  denotes the nominal interest rate for risky assets, where  $DUM_t$  is a dummy.  $\alpha_1$  and  $\alpha_3$  measure how changes in the percentage deviation of the fiscal deficit  $df_t \equiv -sp_t$  widen or narrow the interest rate spread  $R_t^{risky} - R_t$ . Thus,  $\alpha_1$  and  $\alpha_3$  correspond to  $\gamma$ . Data are monthly and are retrieved from Datastream, and we use the yields on 10-year government bonds and the real government budget balance in Greece.<sup>13</sup> In the steady state, as shown in Eq.(A.6), the government debt coupon rate equals the government debt yield and both  $\phi$  and  $\gamma$  are the steady-state values. Thus, we adopt the yields on government bonds as our data. The sample period is from January 2005 to April 2015. Note that the Athens Olympics were held in January 2005, at the beginning of the period when the unhealthy fiscal deficit started. The real government budget balance is seasonally adjusted and Hodrick–Prescott (HP) filtered. The data frequency is monthly. We assign  $DUM_t = 1$  during May 2010 to June 2012, otherwise  $DUM_t = 0$ . Note that Greece requested fiscal support from both the International Monetary Fund (IMF) and the ECB in April 2010, May 2010 was the following month, and Greece decided to adopt a reduced budget following the results of the poll in June 2012. That is,  $DUM_t = 1$  is assigned during the severe debt crisis in Greece. The estimators on  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  are -4.302, 0.170, 2.196, and 1.145, respectively. Those of the standard errors are 0.896, 0.099, 0.852, and 0.077, respectively. The results of the estimators on  $\alpha_0$  and  $\alpha_3$  are significant at the 1% level, and those on  $\alpha_2$  are significant at the 5% level. The result that  $\alpha_1$  is not significant and  $\alpha_3$  is significant implies that the elasticity of the interest rate spread to the fiscal deficit  $\gamma$  is significant during the severe debt crisis when the nominal interest rate rose rapidly, and its elasticity is 1.145. Thus, we set  $\gamma$  to 1.145. Because  $\gamma$  is significant during May 2010 to June 2012, we regard the average of the interest rate spread  $R_t^{risky} - R_t$  as the risk premium, and we find that the interest rate spread for risky assets  $\phi$  is 0.138.

The AR(1) processes are also estimated from data on real GDP, the GDP deflator, nominal government expenditure and employment in Greece retrieved from IMF World Economic Outlook, and the sample period is from January 2005 to April 2015. Productivity is GDP divided by employment and real government expenditure is nominal government expenditure divided by the GDP deflator. The generated data are HP filtered. Our results for the persistence of productivity  $\rho_A$  and the persistence of government expenditure are 0.976 and 0.927, respectively, and the innovations for productivity and government expenditure are 0.0316 and 0.0728, respectively, as mentioned in Section 5.1.

As we mentioned in Section 2.1, our assumption concerning the elasticity of the interest rate spread to the fiscal deficit  $\gamma > 1$  is supported by the data. This is true because the t-statistic for the null hypothesis  $\alpha_3 = 1$  against the alternative hypothesis  $\alpha_3 > 1$  is 1.88, which is larger than the 5% critical value of 1.7, and thus  $\alpha_3 > 1$  is supported statistically. Note that as mentioned,  $\alpha_3$  corresponds to  $\gamma$ .

## C Empirical Evidence on Government Debt with Interest Payment as an Argument for $\Gamma(\cdot)$

Similar to Eq.(B.1), we estimate the following:

$$\ln\left(R_t^{risky} - R_t\right) = \tilde{\alpha}_0 + \tilde{\alpha}_1 r b_t + \tilde{\alpha}_2 DUM_t + \tilde{\alpha}_3 r b_t DUM_t,$$

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<sup>13</sup>The original data include the nominal government budget balance, which we deflate using the CPI.



where  $\tilde{\alpha}_1$  and  $\tilde{\alpha}_3$  measure how changes in the percentage deviation of government debt with interest payment from its steady-state value  $rb_t \equiv \frac{R_t B_t}{RB} - 1$  widen or narrow the interest rate spread  $R_t^{risky} - R_t$ . Thus,  $\tilde{\alpha}_1$  and  $\tilde{\alpha}_3$  correspond to  $\gamma$ . Data are quarterly and are retrieved from Datastream, and we use government debt in Greece and government interest payment in Greece. We sum government debt in Greece and government interest payment in Greece and divide it by CPI in Greece. The generated data are HP filtered. The sample period runs from Q1, 2005 to Q1, 2015. We assign  $DUM_t = 1$  during Q2, 2010 to Q2, 2012, otherwise  $DUM_t = 0$ . The estimators on  $\tilde{\alpha}_0$ ,  $\tilde{\alpha}_1$ ,  $\tilde{\alpha}_2$ , and  $\tilde{\alpha}_3$  are -4.316, 0.385, 2.366, and -4.120, respectively. Those of the standard errors are 0.890, 10.307, 0.841, and 10.367, respectively. The results for the estimators on  $\tilde{\alpha}_0$  and  $\tilde{\alpha}_2$  are significant at the 1% level. The fact that  $\tilde{\alpha}_1$  and  $\tilde{\alpha}_3$  are not significant means that  $\gamma$  cannot be estimated if we assume that the argument for  $\Gamma(\cdot)$  is government debt with interest payment in Greece. This estimation result and the result on Appendix B imply that the (negative) fiscal surplus as an argument for  $\Gamma(\cdot)$  is plausible, although government debt with interest payment as an argument for  $\Gamma(\cdot)$  is not plausible.

## D Empirical Evidences on Price Stickiness

Following Gali and Gertler[16] and Benigno and Lopez-Salido[7], we estimate an equation as follows:

$$E_t [\theta\pi_t - \theta 0.99\pi_{t+1} - (1 - \theta)(1 - \theta 0.99)mc_t] = 0. \quad (\text{D.1})$$

The estimation method is the generalized method of moments developed by Hansen[18]. We use quarterly data in Greece for the GDP deflator and nominal unit labor cost retrieved from Datastream, and these are seasonally adjusted. The sample period runs from Q1, 2005 to Q3, 2015. The rate of change in the GDP deflator is regarded as the data series for inflation  $\pi_t$ . We deflate the nominal unit labor cost by the GDP deflator to generate the real unit labor cost. Finally, we calculate the percentage deviation of the marginal cost from its steady-state value following  $mc_t = \frac{MC_t - MC_t^{HP}}{MC_t^{HP}}$ , where  $MC_t^{HP}$  is the HP-filtered real marginal cost.

To estimate,  $\pi_{t-1}$ ,  $\pi_{t-2}$ ,  $mc_{t-1}$ , and  $mc_{t-2}$  are designated as instrumental variables. We use heteroscedasticity and autocorrelation-consistent standard errors. The spectral estimation method is the quadratic spectral kernel, and the bandwidth parameter is selected using the Andrews[1] procedure. The J-statistic for the validity of overidentifying restrictions is 2.03, and the associated p-value is 0.56. This suggests that the above equation is successfully estimated.

As estimation results, we obtain the estimator 0.705 and standard error 0.206. Because the p-value is 0.001, our estimator is significant at the 1% level.

## E Empirical Evidence on the Relationship between the Redemption Yield and the Coupon Rate

We estimate an equation as follows:

$$r_t^H = \beta_0 + \beta_1 r_t^G,$$

where  $r_t^H$  and  $r_t^G$  denote the yield and the coupon rate on benchmark 10-year government bonds, respectively. Here, the coupon rate is the monthly average. We use monthly data on the PIIGS—namely, Portugal, Italy, Ireland, Greece and Spain—and Germany and the US, and retrieve the

data from Datastream. The sample period runs from January 2005 to September 2015. We verify  $\beta_0 = 0$  and  $\beta_1 = 1$ , which implies that the yield equals the coupon rate on average. Our results for  $\beta_0$  in Portugal, Italy, Ireland, Greece, Spain, Germany, and the US are 9.501, 0.353, -5.419, 7.939, 0.353, -0.176, and 0.129, respectively, and those of the standard errors are 4.349, 0.542, 2.718, 3.898, 0.542, 0.131, and 0.089, respectively. The estimator on  $\beta_0$  in Portugal, Ireland, and Greece is significant at the 5% level, while the remainder are not significant. Our results on  $\beta_1$  in Portugal, Italy, Ireland, Greece, Spain, Germany, and the US are -0.919, 0.893, 2.204, 0.350, 0.893, 1.020, and 0.960, respectively, and those of the standard errors are 0.852, 0.126, 0.659, 1.0418, 0.126, 1.020, and 0.960, respectively. We cannot reject that  $\beta_1 = 1$  in Italy, Ireland, Spain, Germany, and the US because the estimators are significant at the 1% level, while the estimator on  $\beta_1$  in Portugal and Greece is not significant.

We also conduct F-tests, and we obtain F-statistics of 2.670, 0.567, 3.036, 5.187, 0.567, 2.584, and 1.082 for Portugal, Italy, Ireland, Greece, Spain, Germany, and the US, respectively. The p-values are 0.073, 0.568, 0.052, 0.007, 0.569, 0.079, and 0.342 for Portugal, Italy, Ireland, Greece, Spain, Germany, and the US, respectively. Our null hypothesis is  $\beta_0 = 0$  and  $\beta_1 = 1$ . Because the F-statistics in Greece are significant at the 1% level, we cannot accept our hypothesis  $r_t^H = r_t^G$  in Greece.

Summarizing our results, the hypothesis  $\beta_0 = 0$  and  $\beta_1 = 1$  is supported in Italy, Spain, Germany, and the US. That is, roughly speaking, the yield is consistent with the coupon rate on benchmark 10-year government bonds in these countries. However, in Portugal, Ireland, and Greece, the yield is not consistent with the coupon rate on the benchmark 10-year government bond.

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Table 1: Macroeconomic Volatility

Variable	OMP	OMFP	MIS
$x_t$	0.0526	0.0000	0.2347
$\pi_t$	0.0012	0.0000	1.0977
$\hat{r}_t$	2.7636	0.0085	1.0707
$\hat{\tau}_t$	NA	0.2336	NA
$\delta_t$	1.0554	0.1884	0.0000
$sp_t$	2.6391	0.6411	0.4677
$\hat{r}_t^S$	0.2271	0.0761	0.0000

Table 2: Correlation between Selected Variables

Variable	Policy	$x_t$	$\pi_t$	$\hat{r}_t$	$\hat{\tau}_t$	$\delta_t$	$sp_t$	$\hat{r}_t^S$
$x_t$	OM	1.0000						
	OMF	1.0000						
	MIS	1.0000						
$\pi_t$	OM	-0.1098	1.0000					
	OMF	NA	1.0000					
	MIS	-0.0577	1.0000					
$\hat{r}_t$	OM	0.0835	-0.7653	1.0000				
	OMF	NA	NA	1.0000				
	MIS	-0.1318	0.9972	1.0000				
$\hat{\tau}_t$	OM	NA	NA	NA	1.0000			
	OMF	NA	NA	0.9164	1.0000			
	MIS	NA	NA	NA	1.0000			
$\delta_t$	OM	0.2879	-0.8770	0.8405	NA	1.0000		
	OMF	NA	NA	-0.4620	-0.5566	1.0000		
	MIS	NA	NA	-0.2082	NA	1.0000		
$sp_t$	OM	-0.9102	0.2644	-0.4343	NA	-0.4754	1.0000	
	OMF	NA	NA	0.4104	0.7191	-0.4537	1.0000	
	MIS	0.9360	-0.0541	-0.1241	NA	NA	1.0000	
$\hat{r}_t^S$	OM	-0.3444	0.2108	0.3720	NA	-0.1270	-0.0022	1.0000
	OMF	NA	NA	-0.3571	-0.6769	0.4331	-0.9982	1.0000
	MIS	NA	NA	NA	NA	NA	NA	1.0000

Figure 1: IRFs to Government Expenditure

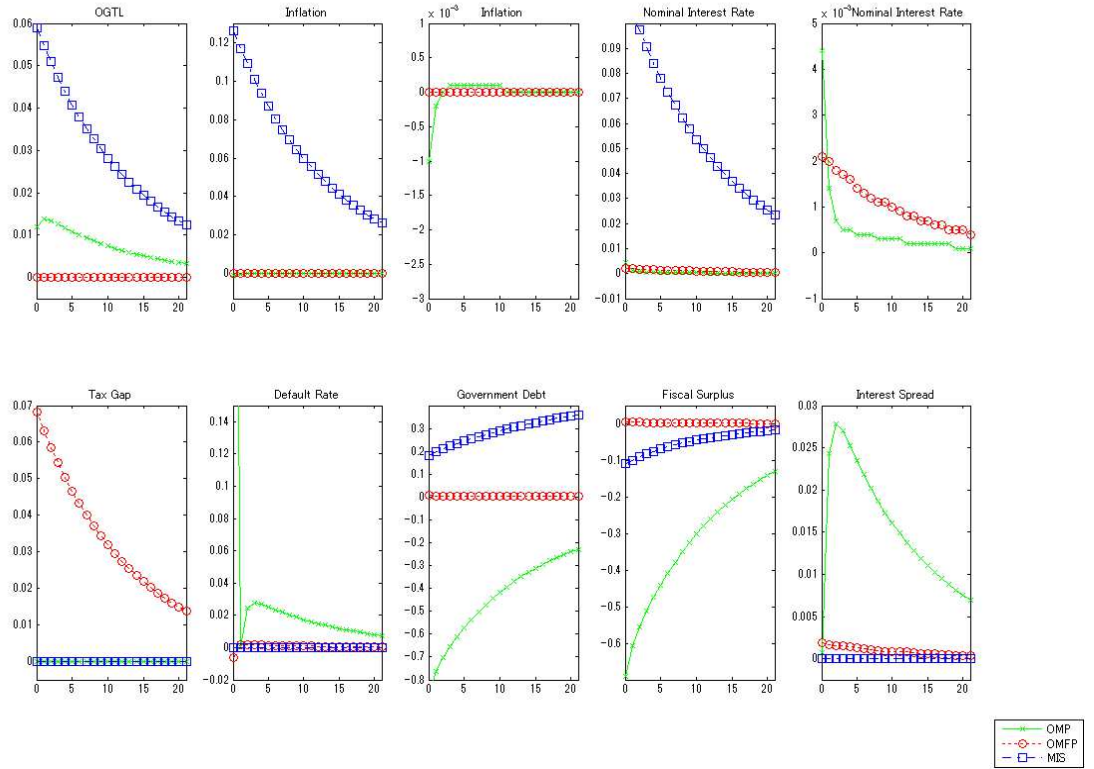


Figure 2: The Trade-off between Stabilizing Inflation and the Default Rate Volatilities

