

Optimal Income Taxation: Mirrlees Meets Ramsey

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System

How should we tax income?

- What **structure of income taxation** offers best trade-off between benefits of public insurance and costs of distortionary taxes?
- Proposals for a flat tax system with universal transfers
 - Friedman (1962)
 - Mirrlees (1971)
- Others have argued for U-shaped marginal tax schedule
 - Saez (2001)

This Paper

We compare 3 tax and transfer systems:

1. **Affine tax system:** $T(y) = \tau_0 + \tau_1 y$
 - constant marginal rates with lump-sum transfers
2. **HSV tax system:** $T(y) = y - \lambda y^{1-\tau}$
 - function introduced by Feldstein (1969), Persson (1983), and Benabou (2000)
 - increasing marginal rates without transfers
 - τ indexes progressivity: $1 - \tau = \frac{1-T'(y)}{1-T(y)/y}$
3. **Optimal tax system**
 - fully non-linear

Main Findings

- Marginal tax rates should be increasing in income, NOT flat or U-shaped
- Best tax and transfer system in the HSV class typically better than the best affine tax system
 - More valuable to have marginal tax rates increase with income than to have lump-sum transfers
- Welfare gains from tax reform sensitive to planner's taste for redistribution
 - May be tiny

Mirrlees Approach to Tax Design: Mirrlees (1971), Diamond (1988), Saez (2001)

- Agents differ wrt unobservable log productivity α
- Planner only observes earnings $x = \exp(\alpha) \times h$
- Think of planner choosing (c, x) for each α type
- Include incentive constraints, s.t. each type prefers the earnings level intended for their type
- Allocations are constrained efficient
- Trace out tax decentralization $T(x(\alpha)) = x(\alpha) - c(\alpha)$

Novel Elements of Our Analysis

1. We explore a range of Social Welfare Functions

- Utilitarian SWF as a benchmark
⇒ Strong desire for redistribution
- Alternative SWF that rationalizes amount of redistribution embedded in observed tax system

2. Our model has a distinct role for private insurance

- Standard decentralization of efficient allocations delivers all insurance through tax system ⇒ Very progressive taxes

Environment 1

- Standard static Mirrlees plus partial private insurance (quantitatively important)
- Heterogeneous individual labor productivity with two stochastic components

$$\log w = \alpha + \varepsilon$$

- ε is privately-insurable, α is not
 - Agents belong to large families
 - α common across all members of a family \Rightarrow cannot be pooled within family
 - ε purely idiosyncratic & orthogonal to $\alpha \Rightarrow$ can be pooled within family
- Planner sees neither component of productivity

Environment 2

- Common preferences

$$u(c, h) = \log(c) - \frac{h^{1+\sigma}}{1+\sigma}$$

- Production linear in aggregate effective hours

$$\int \int \exp(\alpha + \varepsilon) h(\alpha, \varepsilon) dF_\alpha dF_\varepsilon = \int \int c(\alpha, \varepsilon) dF_\alpha dF_\varepsilon + G$$

Planner's Problems

- Seeks to maximize SWF denoted $W(\alpha)$
- Only sees total family income $y(\alpha) = \int \exp(\alpha + \varepsilon)h(\alpha, \varepsilon)dF_\varepsilon$
- First Stage
 - Planner offers menu of contracts $\{c(\tilde{\alpha}), y(\tilde{\alpha})\}$
 - Family heads draw idiosyncratic α and report $\tilde{\alpha}$
- Second Stage
 - Family members draw idiosyncratic ε
 - Family head tells each member how much to work
 - Total earnings must deliver $y(\tilde{\alpha})$ to the planner
 - Must divide consumption $c(\tilde{\alpha})$ between family members

Nature of the Solution

- Planner cannot condition individual allocations on ε , given free within-family transfers
 - equally cheap for any family member to deliver income to the planner, and equally valuable to receive consumption
- Thus, planner cannot take over private insurance
 - ⇒ Distinct roles for public and private insurance
- Note: Extent of private risk-sharing is exogenous with respect the tax system

Planner's Problem: Second Best

$$\begin{aligned} \max_{c(\alpha), y(\alpha)} \quad & \int W(\alpha) U(\alpha, \alpha) dF_\alpha \\ \text{s.t.} \quad & \int y(\alpha) dF_\alpha \geq \int c(\alpha) dF_\alpha + G \\ & U(\alpha, \alpha) \geq U(\alpha, \tilde{\alpha}) \quad \forall \alpha, \forall \tilde{\alpha} \end{aligned}$$

where $U(\alpha, \tilde{\alpha}) \equiv$

$$\left\{ \begin{array}{l} \max_{\{c(\alpha, \tilde{\alpha}, \varepsilon), h(\alpha, \tilde{\alpha}, \varepsilon)\}} \int \left\{ \log(c(\alpha, \tilde{\alpha}, \varepsilon)) - \frac{h(\alpha, \tilde{\alpha}, \varepsilon)^{1+\sigma}}{1+\sigma} \right\} dF_\varepsilon \\ \text{s.t.} \quad \int c(\alpha, \tilde{\alpha}, \varepsilon) dF_\varepsilon = c(\tilde{\alpha}) \\ \int \exp(\alpha + \varepsilon) h(\alpha, \tilde{\alpha}, \varepsilon) dF_\varepsilon = y(\tilde{\alpha}) \end{array} \right.$$

$$U(\alpha, \tilde{\alpha}) = \log(c(\tilde{\alpha})) - \frac{\Omega}{1+\sigma} \left(\frac{y(\tilde{\alpha})}{\exp(\alpha)} \right)^{1+\sigma}$$

$$\text{where } \Omega = \left(\int \exp(\varepsilon)^{\frac{1+\sigma}{\sigma}} dF_\varepsilon(\varepsilon) \right)^{-\sigma}$$

Planner's Problem: Ramsey

$$\begin{aligned} \max_{\tau} \quad & \int W(\alpha) \left\{ \int u(c(\alpha, \varepsilon), h(\alpha, \varepsilon)) dF_{\varepsilon} \right\} dF_{\alpha} \\ \text{s.t.} \quad & \int \int c(\alpha, \varepsilon) dF_{\alpha} dF_{\varepsilon} + G = \int \int \exp(\alpha + \varepsilon) h(\alpha, \varepsilon) dF_{\alpha} dF_{\varepsilon} \end{aligned}$$

where $c(\alpha, \varepsilon)$ and $h(\alpha, \varepsilon)$ are the solutions to

$$\left\{ \begin{array}{l} \max_{\{c(\alpha, \varepsilon), h(\alpha, \varepsilon)\}} \int \left\{ \log c(\alpha, \varepsilon) - \frac{h(\alpha, \varepsilon)^{1+\sigma}}{1+\sigma} \right\} dF_{\varepsilon} \\ \text{s.t.} \quad \int c(\alpha, \varepsilon) dF_{\varepsilon} = y(\alpha) - T(y(\alpha); \tau) \\ y(\alpha) = \int \exp(\alpha + \varepsilon) h(\alpha, \varepsilon) dF_{\varepsilon} \end{array} \right.$$

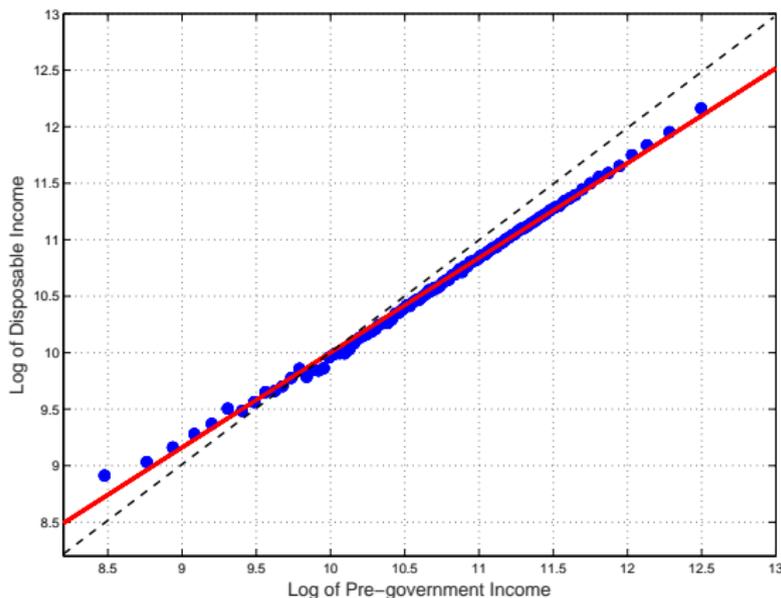
Social Preferences

- Assume SWF takes the form $W(\alpha; \theta) = \exp(-\theta\alpha)$
 - θ controls taste for redistribution
 - $W(\alpha; \theta)$ function could be micro-founded as a probabilistic voting model
- Nests standard SWFs used in the literature:
 - $\theta = 0$: Utilitarian [our benchmark]
 - $\theta = -1$: Laissez-Faire Planner
 - $\theta \rightarrow \infty$: Rawlsian

Empirically Motivated SWF

- Progressivity built into current tax system informative about politico-economic demand for redistribution
- Assume planner (political system) choosing tax system in HSV class: $T(y) = y - \lambda y^{1-\tau}$
- Assume planner has SWF in class $W(\alpha; \theta) = \exp(-\theta\alpha)$
- What value for θ gives observed τ as solution to Ramsey problem?
 - Let $\tau^*(\theta)$ denote welfare-maximizing choice for τ given θ
 - **Empirically Motivated SWF** $W(\alpha; \theta^*)$ s.t. $\tau^*(\theta^*) = \tau^{US}$
 - related to inverse optimum problem
- Ramsey planner with $\theta = \theta^*$ choosing a tax and transfer scheme in the HSV class would choose exactly τ^{US}

Baseline HSV Tax System: $T(y; \lambda, \tau) = y - \lambda y^{1-\tau}$

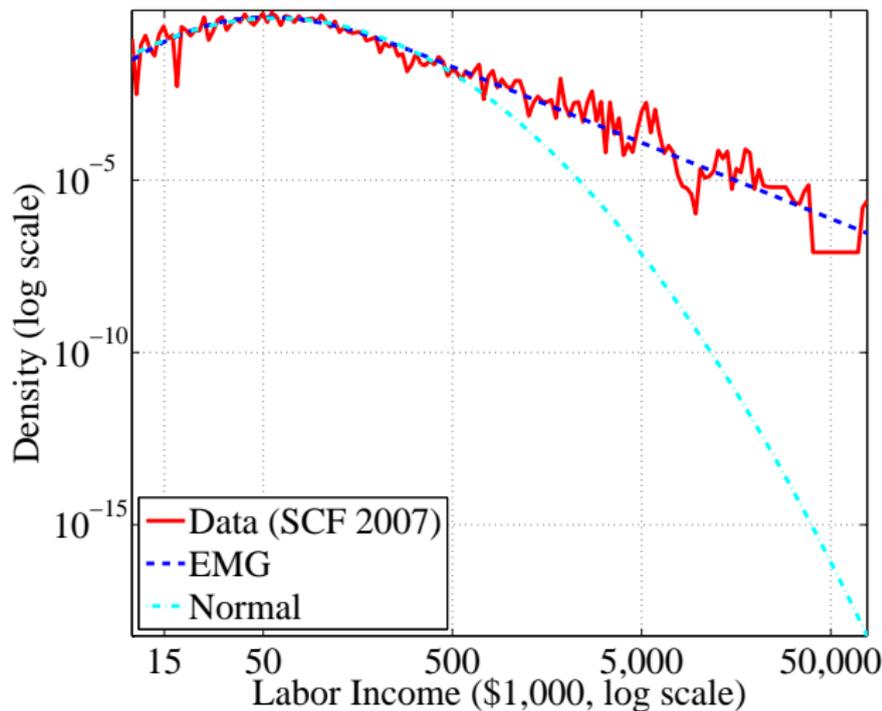


- Estimated on PSID data for 2000-2006
- Households with head / spouse hours ≥ 260 per year
- Estimated value for $\tau = 0.161$, $R^2 = 0.96$

Calibration: Wage Distribution

- Heavy Pareto-like right tail of labor earnings distribution (Saez, 2001)
- Assume Pareto tail reflects uninsurable wage dispersion
- F_α : Exponentially Modified Gaussian $EMG(\mu_\alpha, \sigma_\alpha^2, \lambda_\alpha)$
- F_ε : Normal $N(\frac{-\sigma_\varepsilon^2}{2}, \sigma_\varepsilon^2)$
- $\log(w) = \alpha + \varepsilon$ is itself EMG $\Rightarrow w$ is Pareto log-normal
- $\log(wh)$ is also EMG, given our utility function, private insurance model, and HSV tax system
- Normal variance coefficient in the EMG distribution for log earnings: $\sigma_y^2 = \left(\frac{1+\sigma}{\sigma+\tau}\right)^2 \sigma_\varepsilon^2 + \sigma_\alpha^2$.

Distribution for Labor Income



Use micro data from the 2007 SCF to estimate α by maximum likelihood $\Rightarrow \lambda_\alpha = 2.2$ and $\sigma_y^2 = 0.4117$

Calibration

- Frisch elasticity = 0.5 $\Rightarrow \sigma = 2$
- Progressivity parameter $\tau = 0.161$ (HSV 2014)
- Govt spending G s.t. $G/Y = 0.188$ (US, 2005)
- Variance of normal component of SCF earnings + external evidence on importance of insurable shocks
 $\Rightarrow \sigma_{\varepsilon}^2 = \sigma_{\alpha}^2 = 0.1407$
 - Variance of insurable shocks consistent with HSV 2014
 - Total variance of log wages (0.488) and variance of log consumption (0.246) consistent with empirical counter parts

Bottom of Wage Distribution

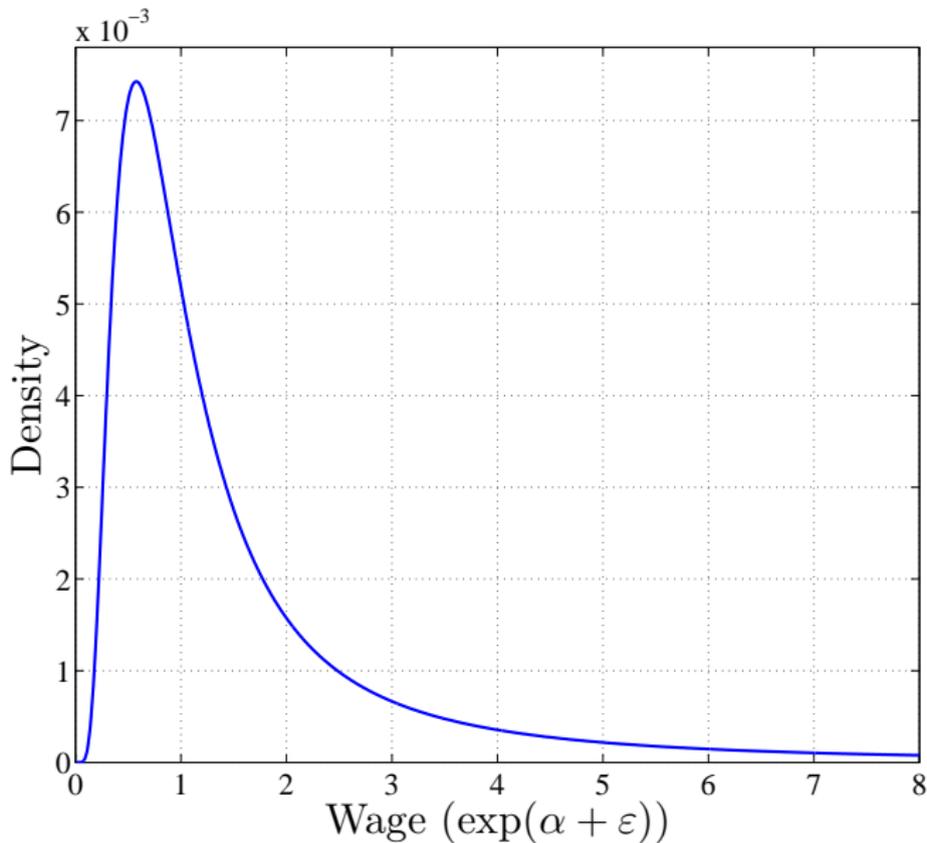
- Difficult to measure distribution of offered wages at the bottom, given selection into participation
- Low and Pistaferri (2015) estimate distribution of latent offered wages within a structural model in which workers face disability risk and choose participation

Percentile Ratios	Model	LP
P5/P1	1.48	1.48
P10/P5	1.24	1.20
P25/P10	1.44	1.40

Numerical Implementation

- Maintain continuous distribution for ε
- Assume a discrete distribution for α
- Baseline: 10,000 evenly-spaced grid points
- α_{\min} : \$2 per hour (5% of the average = \$41.56)
- α_{\max} : \$3,075 per hour (\$6.17m assuming 2,000 hours = 99.99th percentile of SCF earnings distn.)
- Set μ_{α} and σ_{α}^2 to match $E[e^{\alpha}] = 1$ and target for $var(\alpha)$ given $\lambda_{\alpha} = 2.2$

Wage Distribution



Quantitative Analysis

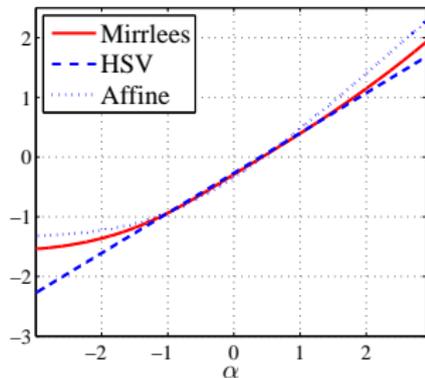
- U.S. tax system approximated by HSV with $\tau = 0.161$
- Focus on three optimal systems:
 1. HSV tax function: $T(y) = y - \lambda y^{1-\tau}$
 2. Affine tax function: $T(y) = \tau_0 + \tau_1 y$
 3. Mirrless tax function (second best allocation)

Quantitative Analysis: Benchmark

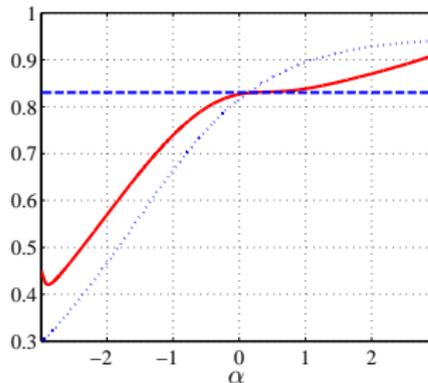
Tax System	Tax Parameters		Outcomes			
			welfare	Y	$T'(y)$	TR/Y
HSV ^{US}	$\lambda : 0.839$	$\tau : 0.161$	–	–	0.319	0.018
HSV	$\lambda : 0.817$	$\tau : 0.330$	2.08	–7.22	0.466	0.063
Affine	$\tau_0 : -0.259$	$\tau_1 : 0.492$	1.77	–8.00	0.492	0.279
Mirrlees			2.48	–7.99	0.491	0.213

Benchmark: Mirrlees vs Ramsey

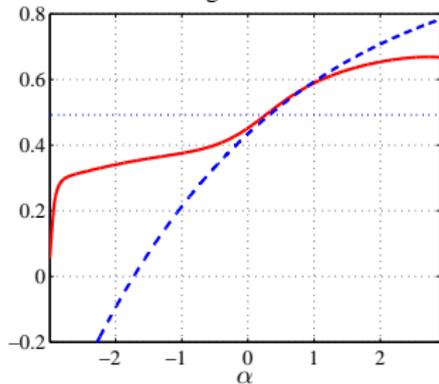
A. Log Consumption



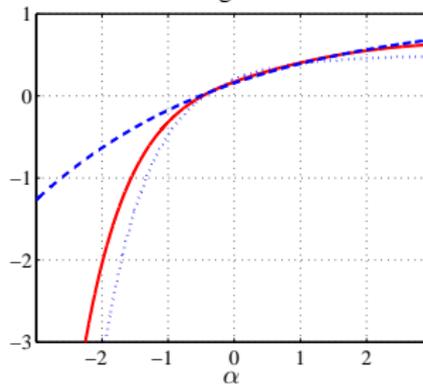
B. Hours Worked



C. Marginal Tax Rate



D. Average Tax Rate



Quantitative Analysis: Benchmark

- Optimal HSV better than optimal affine
 - ⇒ Increasing marginal rates more important than lump-sum transfers
- Moving to fully optimal system generates **substantial gains** (2.5%)
- The optimal **marginal tax rate is around 50%**

Quantitative Analysis: Sensitivity

What drives the results?

1. **Eliminate insurable shocks**: $\tilde{v}_\alpha = v_\alpha + v_\varepsilon$ and $\tilde{v}_\varepsilon = 0$
2. Utilitarian SWF $\theta = 0$
 \Rightarrow Various SWFs including **Empirically motivated SWF**
3. **Increase desire to raise revenue**
4. Wage distribution has thin **Log-Normal** right tail: $\alpha \sim N$

Sensitivity: No Insurable Shocks

Tax System	Tax Parameters		Outcomes			
			welfare	Y	$T'(y)$	TR/Y
HSV ^{US}	$\lambda : 0.842$	$\tau : 0.161$	—	—	0.319	0.019
HSV	$\lambda : 0.804$	$\tau : 0.383$	4.17	-9.72	0.511	0.084
Affine	$\tau_0 : -0.283$	$\tau_1 : 0.545$	5.34	-10.45	0.545	0.326
Mirrlees			5.74	-10.64	0.550	0.284

- No insurable shocks \Rightarrow larger role for public redistribution
 - Want higher tax rates and larger transfers
 - Optimal HSV worse than optimal affine
- \Rightarrow Distinguishing insurable shocks from uninsurable shocks is important

Social Welfare

- Consider alternative SWFs:

- $\theta = -1$: Laissez-Faire Planner
- $\theta \rightarrow \infty$: Rawlsian

- Empirically motivated SWF: $W(\alpha; \theta^*)$ s.t. $\tau^*(\theta^*) = \tau^{US}$

- Closed form expression for θ^* !

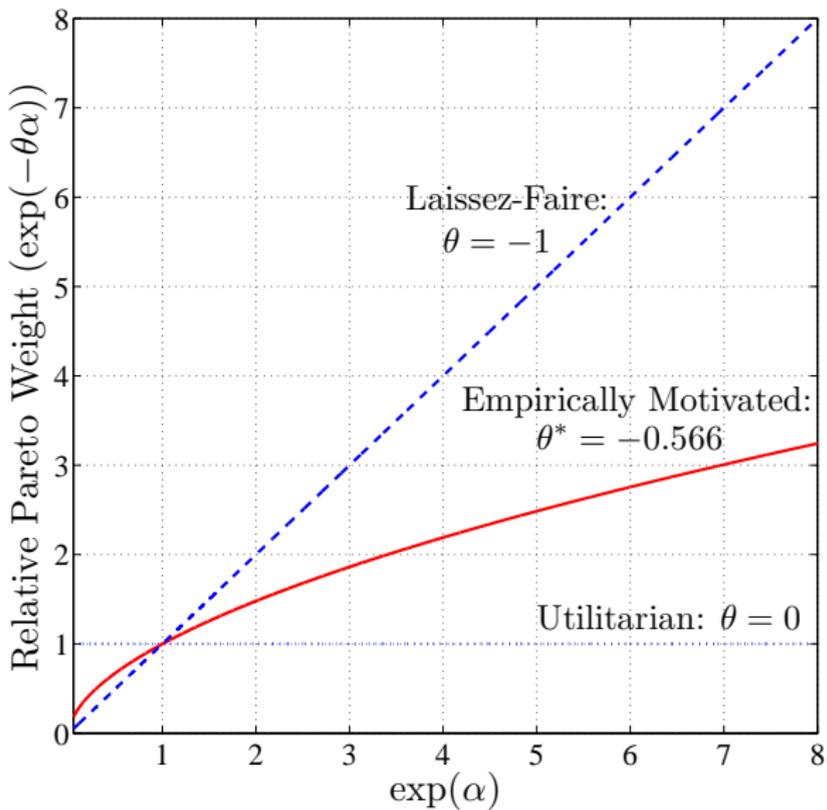
$$\sigma_\alpha^2 \theta^* - \frac{1}{\lambda_\alpha + \theta^*} = -\frac{1}{\lambda_\alpha - 1 + \tau} - \sigma_\alpha^2 (1 - \tau) + \frac{1}{1 + \sigma} \left\{ \frac{1}{(1-g)(1-\tau)} - 1 \right\}$$

- Simple in Normal case ($\lambda_\alpha \rightarrow \infty$)

$$\theta^* = -(1 - \tau) + \frac{1}{\sigma_\alpha^2} \frac{1}{1 + \sigma} \left\{ \frac{1}{(1-g)(1-\tau)} - 1 \right\}$$

- θ^* increasing in τ and g
- θ^* declining in σ and σ_α^2
- θ^* increasing in λ_α (holding fixed $var(\alpha) = \sigma_\alpha^2 + \frac{1}{\lambda_\alpha^2}$)

Social Welfare Functions

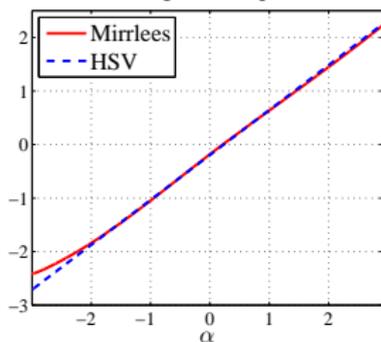


Sensitivity: Alternative SWFs

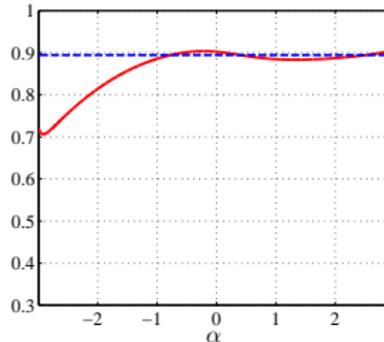
SWF	Mirrlees Allocations				Welfare Change		
	θ	$T'(y)$	TR/Y	ΔY	Mirrlees	Affine	HSV
Laissez-Faire	-1	0.083	-0.082	9.72	3.15	3.14	2.98
Emp. Motivated	-0.57	0.314	0.051	0.16	0.05	-0.48	-
Utilitarian	0	0.491	0.213	-7.99	2.48	1.77	2.08
Rawlsian	∞	0.711	0.538	-22.55	708.28	649.14	354.90

Empirically-Motivated SWF

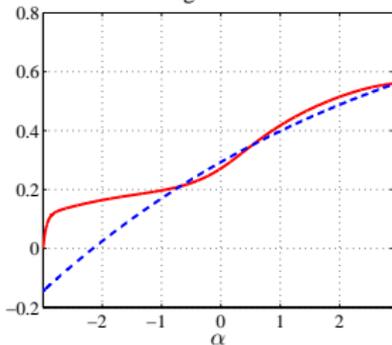
A. Log Consumption



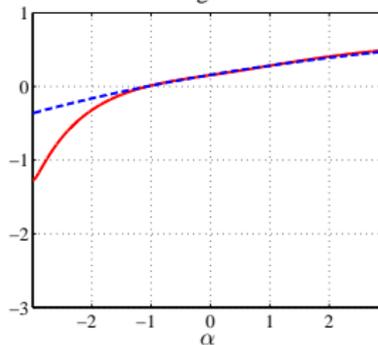
B. Hours Worked



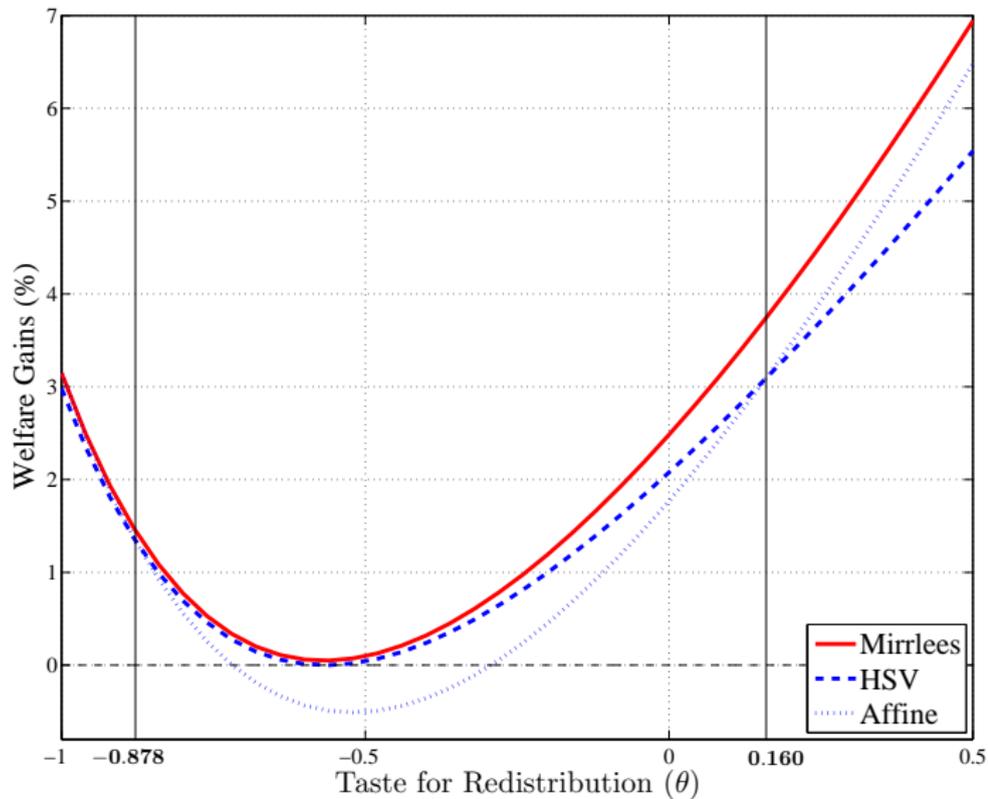
C. Marginal Tax Rate



D. Average Tax Rate



HSV vs Affine with Various SWFs



SWF Sensitivity: Summary

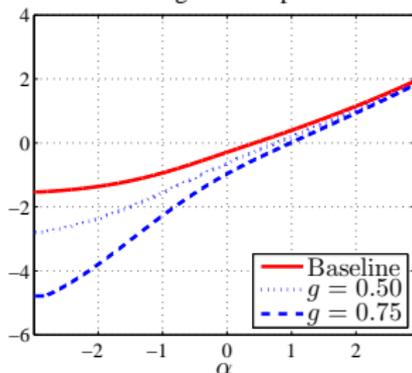
- Optimal tax system very sensitive to assumed SWF
- Welfare gains moving from the current tax system to the optimal one can be tiny
- Affine system works well when preference for redistribution is either very strong or very weak:
 - In the first case, want large lump-sum transfers
 - In the second, want lump-sum taxes
- For intermediate tastes for redistribution ($\theta \in [-0.88, 0.16]$), HSV is better than affine

Sensitivity: Need to Raise Revenue

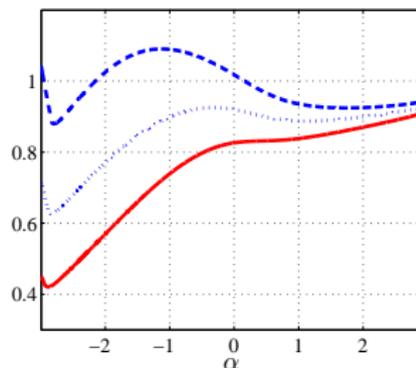
- Saez (2001) found a U-shaped marginal schedule to be optimal
- His intuition: Want to make sure welfare is targeted only to the very poor
- We don't find this. Why?
- Key is **degree of revenue requirement**: to finance
 - exogenous public expenditure G
 - endogenous universal lump-sum transfers Tr

U-shaped Tax Rates with High G

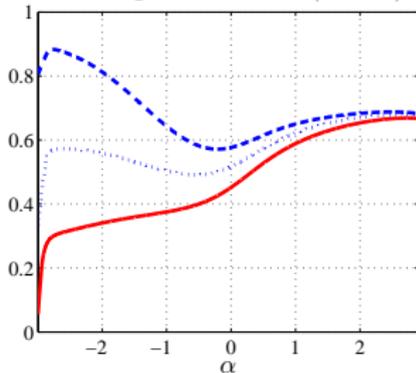
A. Log Consumption



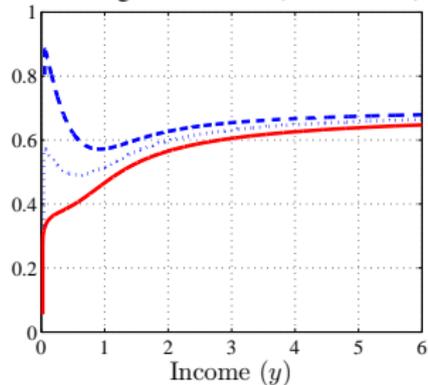
B. Hours Worked



C. Marginal Tax Rate (with α)



D. Marginal Tax Rate (with income)



Intuition: U-shaped Tax Rates with High G

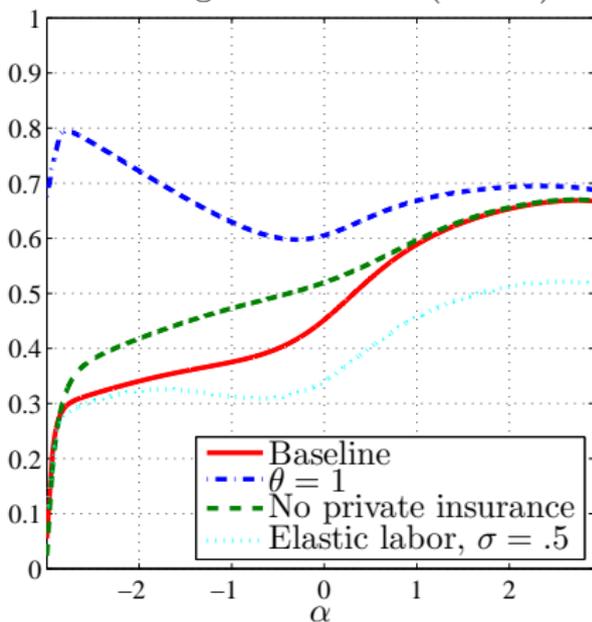
- Tax rates at the top relatively insensitive to the level of G
 - Already close to the top of the Laffer curve
 - Asymptotic rates indicated by Saez (2001): $\frac{1+\sigma}{\sigma+\lambda_\alpha} \approx 71\%$
- Tax rates at low income levels increase in G
 - Little room at the top \Rightarrow instead raise marginal rates at low income levels
- U-shaped rather than monotonically declining
 - Dip in the middle to keep labor supply distortions low where the heaviest population mass is located

Alternative Ways to Increase Fiscal Pressure

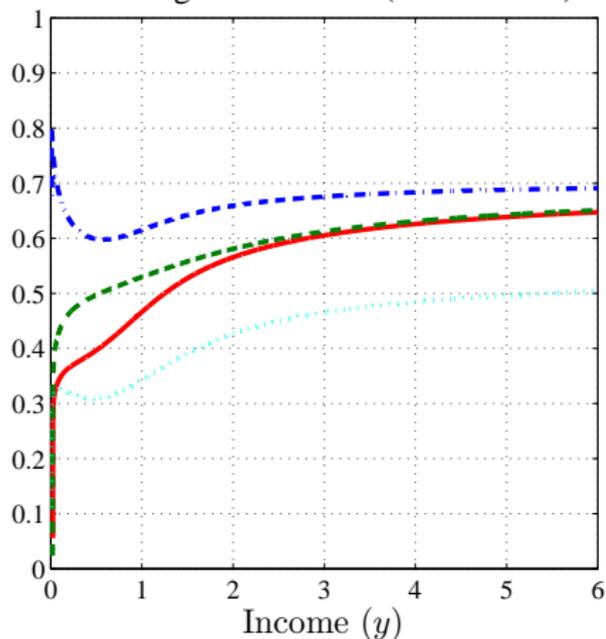
- Increase optimal lump-sum transfers by
 - Increasing the planner's taste for redistribution $\theta = 1$
 - Shutting off private insurance
- Reduce the government's ability to satisfy revenue demands by
 - Increasing the labor supply elasticity $\sigma = 0.5$

Alternative Ways to Increase Fiscal Pressure

A. Marginal Tax Rate (with α)



B. Marginal Tax Rate (with income)



Why does Saez (2001) find U-shaped rates?

- Various assumptions that imply high fiscal pressure:
 - Higher value for government purchases (25% of GDP)
 - Rule out private insurance
 - Use utility functions that limit the government's ability to extract revenue from the rich
- U-shaped profile for marginal rates is not a general feature of an optimal tax system

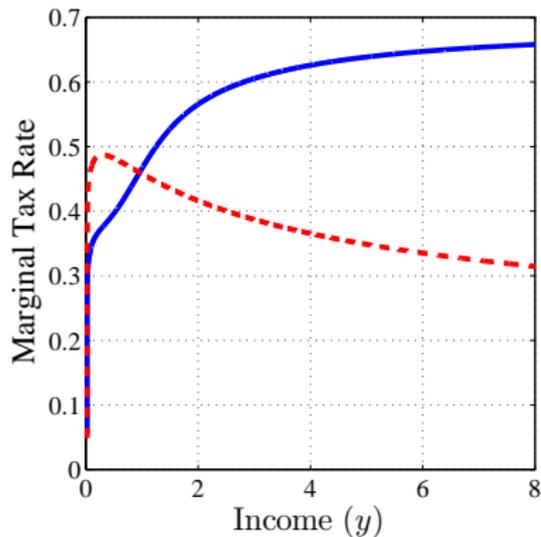
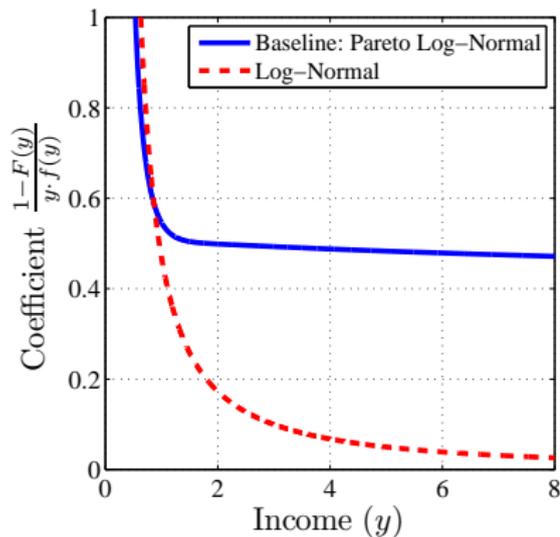
Sensitivity: Log-Normal Wage

Tax System	Tax Parameters		Outcomes			
			welfare	Y	$T'(y)$	TR/Y
HSV ^{US}	$\lambda : 0.828$	$\tau : 0.161$	–	–	0.319	0.017
HSV	$\lambda : 0.813$	$\tau : 0.285$	0.88	–5.20	0.427	0.048
Affine	$\tau_0 : -0.230$	$\tau_1 : 0.451$	2.19	–6.01	0.451	0.242
Mirrlees			2.28	–5.74	0.443	0.254

- Log-normal distribution \Rightarrow thin right tail
- Optimal HSV worse than optimal affine
- Optimal affine nearly efficient

Why Distribution Shape Matters

- Want high top marginal rates when (i) few agents face those marginal rates, but (ii) can capture lots of revenue from higher-income households

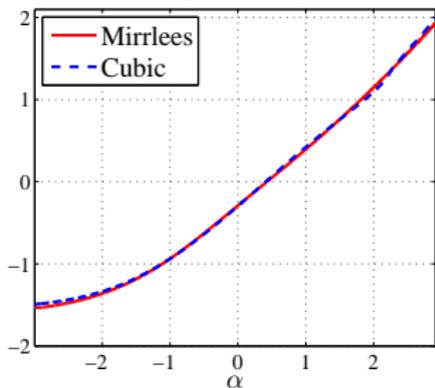


Extension: Polynomial Tax Functions

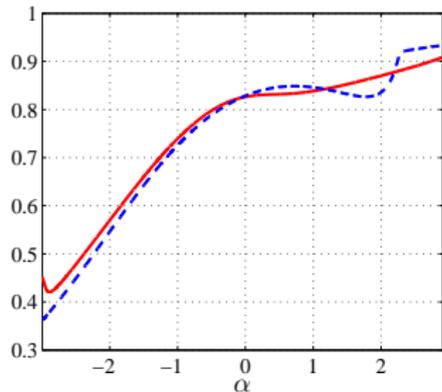
Tax System	Tax Parameters				Outcomes			
					welfare	Y	$T'(y)$	TR/Y
HSV ^{US}	λ 0.839	τ 0.161			—	—	0.319	0.018
Affine	τ_0 -0.259	τ_1 0.492			1.77	-8.00	0.492	0.279
Cubic	τ_0 -0.212	τ_1 0.370	τ_2 0.049	τ_3 -0.002	2.40	-8.01	0.491	0.228
Mirrlees					2.48	-7.99	0.491	0.213

Cubic Tax Function

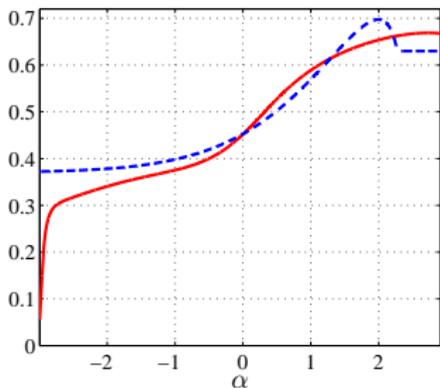
A. Log Consumption



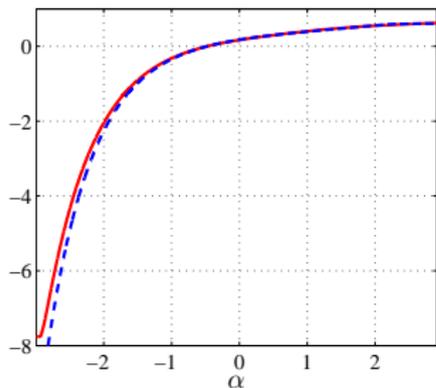
B. Hours Worked



C. Marginal Tax Rate



D. Average Tax Rate



Extension: Type-Contingent Taxes

- Productivity partially reflects observable characteristics (e.g. education, age, gender)
- Some fraction of uninsurable shocks are observable:
 $\alpha \rightarrow \alpha + \kappa$
- Heathcote, Perri & Violante (2010) estimate variance of cross-sectional wage dispersion attributable to observables, $v_{\kappa} = 0.108$
- Planner should condition taxes on observables: $T(y; \kappa)$
- Consider two-point distribution for κ (college vs high school)

Extension: Type-Contingent Taxes

- **Significant welfare gains** relative to non-contingent tax
- Conditioning on observables \Rightarrow marginal tax rates of 42%

System		Outcomes			
		wel.	Y	$T'(y)$	TR/Y
HSV ^{US}	$\lambda : 0.834, \tau : 0.161$	–	–	0.319	0.015 0.020
HSV	$\lambda^L : 1.069, \tau^L : 0.480$	6.21	–2.80	0.416	0.147
	$\lambda^H : 0.595, \tau^H : 0.073$				–0.019
Affine	$\tau_0^L : -0.403, \tau_1^L : 0.345$	6.15	–2.53	0.421	0.420
	$\tau_0^H : -0.032, \tau_1^H : 0.452$				0.008
Mirrlees		6.54	–2.53	0.418	0.368 0.007

Conclusions

- Optimal marginal tax schedule increasing in income, and neither flat nor U-shaped
- Welfare gains moving from the current tax system to the optimal one hinge on the choice of SWF, may be tiny
- Ramsey and Mirrlees tax schemes not far apart: can approximately decentralize Mirrlees with a simple tax scheme