Endogenously Procyclical Liquidity, Capital Reallocation, and q

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Capital reallocation in business cycles

Eisfeldt and Rampini (06, JME):

1. Capital reallocation is large:
   - 1/4 of total investment;
   - 1.4% $\sim$ 5.5% of the capital stock
2. capital reallocation is procyclical:
   - mean(reallocation rate | GDP $>$ trend)
     $=$ 1.59 $\times$ mean(reallocation rate | GDP $<$ trend)
3. benefit of reallocation is acyclical or counter-cyclical:

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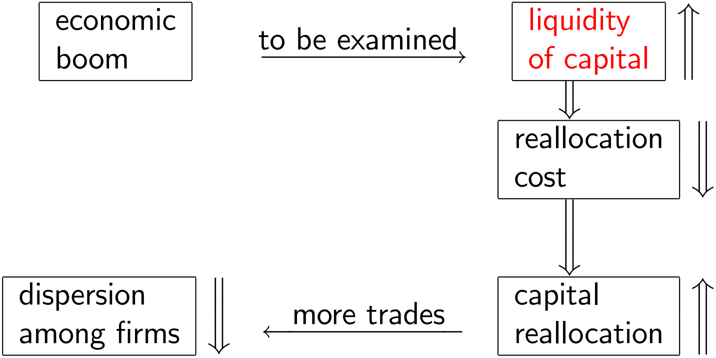
3. benefit of reallocation is acyclical or counter-cyclical:
   - $corr$(GDP, dispersion in z) $=0$ or <0.
   - $z=$ Tobin's q, capital utilization rate, etc.

A possible explanation

Liquidity of capital rises in booms and falls in recessions.



What we do in this paper

1. build a stochastic equilibrium model to capture this mechanism of endogenously procyclical liquidity
2. prove that the mechanism can help
   - explain facts 1 - 3;
   - generate new results on cyclicality of q across firms.

fshocks

Why use a search model?

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Specifically, search

- captures extensive margin and trading probabilities, which are important for the proposed mechanism;
- is a simple way to model trading frictions, which has provided insights in money, finance, and other fields

Related literature

- endogenous liquidity of assets:
  - Eisfeldt (14) uses adverse selection to emphasize cyclical variation in the supply of assets.
    (We emphasize cyclical variation in the demand)
  - Yang (14), Cui and Radde (14):
    focus on financial shocks, not capital reallocation
- Tobin's q: Tobin (69), Gould (68), Sargent (80) ......
- q vs. financing constraints:
  Gilchrist and Himmelberg (95), Gomes (01),
  Hennessy and Whited (07), Cao, et al. (13) ......
  (We focus on liquidity frictions, not financing constraints, and on cyclical reallocation and dispersion in q)

Baseline model environment

- Time is discrete and lasts forever
- all firms are risk neutral, with discount rate r

- firms with capital: each has one unit;
  endogenously classified into
  - producing firms ($j = 1$): common output y (stochastic)
  - displaced firms ($j = 0$): want to sell capital

- firms without capital:
  - buyers: competitive entry; participation cost, ψ

  - capital makers: fixed measure μ; draw cost $K \sim F$.

Search and matching

- Market tightness $\theta \equiv \frac{\text{measure of buyers}}{\text{measure of sellers}}$
- matching prob: $p(\theta)$ for a buyer, $\theta p(\theta)$ for a seller:
  - monotonicity: $p'(\theta) < 0$, $[\theta p(\theta)]' > 0$
    market is more liquid if θ is higher
  - diminishing marginal productivity: $[\theta p(\theta)]'' < 0$
  - usual boundary conditions
- matching prob's incorporate capital specificity
- Nash bargaining: a seller's weight is $\sigma \in (0, 1)$.

Shocks

Aggregate shocks:

- value of output is stochastic, $y_{+1} \sim \Phi(y_{+1}, y)$: transition function $\Phi(y_{+1}, y)$ increases in y

Individual shocks:

- displacement: (business idea no longer works) capital used in production is displaced with prob δ
- depreciation: old capital disappears with prob d.

Timing in a period

- distribution of firms, $n = (n_1, n_0)$, is measured;
  y is realized and but output is not produced yet

- buyers' participation choice;
  capital makers' draws of K and decision

- trading, production, capital making

- displacement shock to capital just used in production
- depreciation shock to capital

- firm values, (V_1, V_0), are measured. flow graph 1

Value functions and decisions

- value functions at the end of a period: $V_j(n, y)$
  $j = 1$ (producing), $j = 0$ (displaced)
  $n = (n_1, n_0)$: distribution of firms
  y: realization of current period output

- capital maker makes capital iff $K < \bar{K} = V_1$

- a producing firm:

$$(1 + r)\, V_1 = \mathbb{E}\left[y_{+1} + \delta V_{0,+1} + (1 - \delta - d)\, V_{1,+1}\right]$$

Value functions and decisions

- a displaced firm (seller in the next period):

$$(1+r)\,V_0 = \mathbb{E}\{\theta_{+1} p_{+1} [m_{+1} - (1-d)\,V_{0,+1}] + (1-d)\,V_{0,+1}\}$$

m: price of capital (not the price of equity)

- Nash bargaining:

$$\max_m [m - (1-d)\,V_0]^{\sigma} [(1-d)\,V_1 - m]^{1-\sigma}$$

$$\Longrightarrow m = (1-d)\,[\sigma V_1 + (1-\sigma)\,V_0]$$

a seller's surplus $= \sigma \times (1-d)\,\Delta,\quad \Delta \equiv V_1 - V_0$.

Bargaining

- Competitive entry of buyers:

$$p\left[(1-d)\,V_1 - m\right] = \psi \text{ (participation cost)}$$

$$\Longrightarrow \theta = \theta(\Delta) \equiv p^{-1}\left(\frac{\psi}{(1-d)(1-\sigma)\Delta}\right)$$

- subtracting Bellman equations for V_1 and V_0 $\Longrightarrow$

$$\Delta(n,y) = \mathbf{T}\Delta(n,y) \qquad \text{where}$$

$$\mathbf{T}\Delta \equiv \frac{1}{1+r}\mathbb{E}\left\{y_{+1} + (1-\delta-d)\,\Delta_{+1} - \frac{\sigma\psi}{1-\sigma}\theta(\Delta_{+1})\right\}$$

Existence of equilibrium

▸ equilibrium definition

Assume: ▸ existence details

- (i) $\mathbb{E}y_{+1}$ is high enough to induce buyers to enter;
- (ii) $p^{-1}(.)$ is not too elastic.

Then,

- **T** is a monotone contraction and, hence, has a unique fixed point $\Delta \in \mathcal{C}$ (recall $\Delta = V_1 - V_0$)
- Δ is strictly increasing in y: joint surplus is procyclical
- $\Delta(y)$ is independent of distribution, n block recursivity (Shi 09, ECMA)

Proposition (continued)

All of the following objects are independent of n and strictly increasing functions of y:

- values, (V_0, V_1): all firms benefit from higher y
- market tightness, θ: liquidity is procyclical
- price of capital, m
- cutoff cost for making capital, $\bar{K}$: capital creation is procyclical

Reiterate

Endogenously procyclical liquidity:

- high realization of $y \Longrightarrow$ high future $y_{+1} \Longrightarrow$
  value of production $\Uparrow$;
  surplus of a match $\Uparrow$
- more buyers participate in the market $\Longrightarrow$
  market tightness $\Uparrow$; selling probability $\Uparrow$
- increased value of production increases capital creation

What about q and its dispersion?

Terms used

- Tobin's q:

$$\text{original:} \quad \frac{\text{market value of a firm}}{\text{replacement cost of capital}}$$

$$\text{in practice:} \quad \frac{\text{market value of a firm}}{\text{book value of a firm}}$$

- our use:

$$q_j = \frac{V_j\text{: market value of a firm } j}{m\text{: market price of capital}}, \; j \in \{1, 0\}$$

- dispersion in q:
  spread: $q_1 - q_0$; standard deviation: S_q

A representative example

Consider $p(\theta) = \frac{1}{1+\theta}$ (telephone matching) and

$$\mathbb{E}y_{+1} = (1-\rho)\,y^* + \rho y, \quad \rho \in (0,1].$$

- The equilibrium has the form:

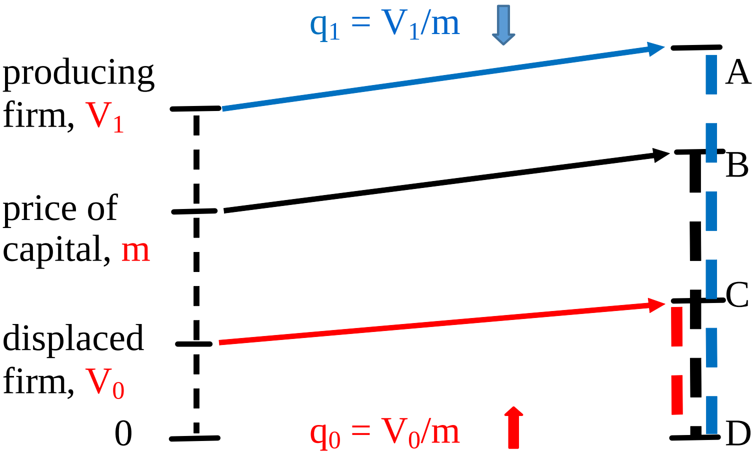
$$\Delta(y) = b_0 + b_1 y, \quad V_0(y) = c_0 + c_1 y.$$

- There exists $\underline{\rho} \in (0,1)$ such that

$$q_0'(y) > 0 \text{ (i.e., } q_1'(y) < 0) \iff \rho > \underline{\rho}$$

- $S_q'(y)$ is ambiguous.

Illustration for a positive shock to y



Extend to have heterogeneous productivity

- A productive firm's output flow is now $y + z$:
  - $z \sim G\left[z_L, z_H\right]$: firm specific and permanent
  - drawn for a new firm, i.e., immediately after a firm buys or makes a unit of capital
- in bargaining, the two sides do not know z that the buyer of capital will draw
- assume that z_L is sufficiently high so that

$$\Delta\left(y, z\right) \geq 0 \text{ for all } \left(y, z\right)$$

i.e., a firm sells capital only when hit by δ. flow graph 2

Previous results extend to this setup

- $(\theta, m, V_0, V_e, \bar{K})(y)$ and $V_1(y, z)$ are strictly increasing; in particular, liquidity is procyclical.
  - e indicates expected value of a new firm (or average value of producing firms)

- Previous results for (q_1, q_0) are valid for (q_e, q_0):
  - $q_e'(y) < 0, \quad q_0'(y) > 0$

New results: among producing firms

- an increase in y:
  - reduces $\left[q_1\left(y, z_H\right) - q_e\left(y\right)\right]$ and $\left[q_e\left(y\right) - q_1\left(y, z_L\right)\right]$,
  - reduces the standard deviation in $q_1\left(y, z\right)$ across z.

- If $q_e'\left(y\right)$ is close to zero, $\exists\ z_c \in \left(z_L, z_H\right)$ such that

$$\frac{\partial}{\partial y} q_1\left(y, z\right) > 0 \iff z < z_c.$$

q is more likely to be procyclical if firm value is lower.

Conclusion

- We formulated a stochastic equilibrium model with endogenously procyclical liquidity
- proven that the model generates:
  - procyclical reallocation of capital;
  - procyclical creation of new capital;
  - acyclical or countercyclical dispersion in q
- plan to test the new prediction with data:
  q is procyclical for low value firms;
  and countercyclical for high value firms.
- another use of the mechanism: role of financial shocks

Another motivation/use

Endogenously procyclical liquidity can help account for the role of financial frictions in business cycles.

- Recent literature emphasizes the role of financial shocks in business cycles:
  - negative financial shocks reduce collateral
  - reduce firms' borrowing and investment

- exogenous liquidity generates counterfactual result: negative financial shock $\Longrightarrow$ equity price $\Uparrow$ (Shi 15, JME)

- but if asset liquidity is endogenous and falls after a negative financial shock, equity price can fall. Return

Together, these facts are puzzling

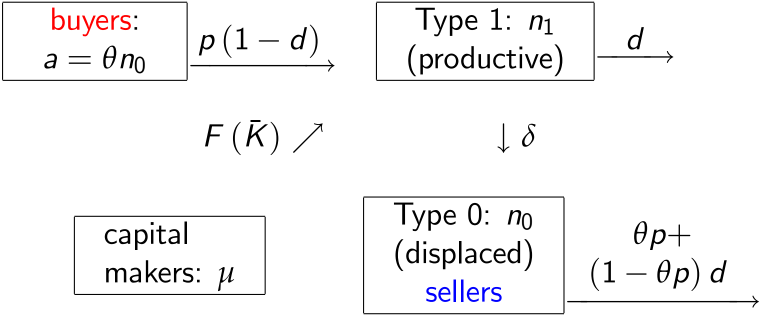
- capital reallocation requires firm heterogeneity:
  - capital moves from low-value to high-value firms
  - heterogeneity measures benefit of reallocation
- procyclical reallocation suggests:
  - heterogeneity increases more in booms;
    e.g., dispersion in q increases more in booms
- but this is not the case in the data

Frictions and liquidity in capital reallocation

- frictions in the trading of capital make market liquidity a determinant of reallocation:
  - difficulty in matching;
  - specificity of capital;
  - information asymmetry

- why is liquidity endogenous and procyclical?
  - economic boom increases return on capital
  - more buyers participate in the market
  - selling probability (liquidity) increases

Flows of firms (and capital)



Return

Definition of equilibrium

distribution of firms $n = (n_1, n_0)$, with $a = \theta n_0$,
market tightness $\theta(n, y)$, price of capital: $m(n, y)$,
value functions $V_j(n, y)$, capital making: $\bar{K}(n, y)$.

- optimality of capital making and buyers' participation
- value functions satisfy Bellman equations
- price m is the result of bargaining
- distribution of firms is stationary.

Return

Existence conditions

- (i) $\mathbb{E}y_{+1}$ is high enough to induce buyers to enter:

$$\mathbb{E}y_{+1} > \frac{(r+\delta+d)\,\psi}{(1-d)\,(1-\sigma)}$$

- (ii) $p^{-1}(.)$ is not too elastic:
  $\exists$ a constant $A \in \left(0, \frac{1-\delta-d}{(1-d)\sigma}\right]$ such that

$$\left|p^{-1}(x_1) - p^{-1}(x_2)\right| \leq A\left|\frac{1}{x_1} - \frac{1}{x_2}\right| \; \forall x_1, x_2 \in [0,1].$$

(The response in θ does not make **T** non-monotone.)

Return

q1 and q0 respond to y in opposite directions

$$\text{bargaining} \implies (1-d)\left[\sigma V_1 + (1-\sigma) V_0\right] = m$$

$$\implies \sigma q_1 + (1-\sigma) q_0 = \frac{1}{1-d}$$

But this alone does not say whether

- q_1 increases in y (i.e., the spread is procyclical)
- or q_1 decreases in y (i.e., the spread is countercyclical)

steady state q

Properties of q in deterministic steady state

If y is constant over time at y^*, then

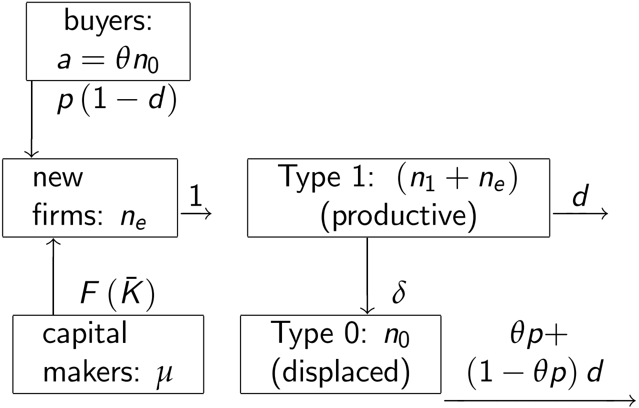
- $\frac{\mathrm{d}q_0^*}{\mathrm{d}y^*} > 0, \quad \frac{\mathrm{d}q_1^*}{\mathrm{d}y^*} < 0$
- $\frac{\mathrm{d}(q_1^* - q_0^*)}{\mathrm{d}y^*} < 0$: spread in q decreases in y^*
- the signs of $\frac{\mathrm{d}(Eq^*)}{\mathrm{d}y^*}$ and $\frac{\mathrm{d}S_q^*}{\mathrm{d}y^*}$ are ambiguous:
  - $\frac{\mathrm{d}(Eq^*)}{\mathrm{d}y^*} > 0$ if $\frac{n_1^*}{n_1^* + n_0^*} < \sigma$;
  - $\frac{\mathrm{d}S_q^*}{\mathrm{d}y^*} < 0$ if $n_1^* \geq n_0^*$.

Return

Need for heterogeneous productivity

- General interpretation of previous results on q:
    - q is procyclical for low-value firms, countercyclical for high-value firms
    - spread in q is countercyclical.
- To map these predictions into the data:
    - need to show that they hold for productive firms, not just between productive and displaced firms
    - this requires productive firms to be heterogeneous.

Flows of firms



Return

Value functions

- new firm (also the average value of productive firms):

$$V_e(y) = \int V_1(y,z)\,\mathrm{d}G(z)$$

- cutoff cost for making capital: $\bar{K}(y) = V_e(y)$
- competitive entry of buyers:

$$p(\theta)\left[(1-d)V_e - m\right] = \psi$$

- Bellman equations for $V_1(y,z)$ and $V_0(y)$ are modified similarly, and gains are

$$\begin{aligned}\Delta(y,z) &= V_1(y,z) - V_0(y),\\ \Delta_e(y) &= V_e(y) - V_0(y).\end{aligned}$$

Proposition

Assume $\Delta(y,z) \geq 0$ for all (y,z).

- There is a unique fixed point for $\Delta_e(y)$, and

$$\Delta(y,z) = \Delta_e(y) + \frac{z}{r+\delta+d}$$

- $(\Delta_e, \theta, m, V_0, V_e, \bar{K})(y)$ and $V_1(y,z)$ are strictly increasing; in particular, liquidity is procyclical.

- Previous results for (q_1, q_0) are valid for (q_e, q_0), where $q_e = V_e / m$ is average q of productive firms:
  - $q_e'(y) < 0, \quad q_0'(y) > 0$