The Global Diffusion of Ideas

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The Global Diffusion of Ideas

- Long held belief that openness affects the diffusion of technologies/ideas
  - Pirenne (1936), Diamond (1997)
- Empirical debate
  - Sachs & Warner (95), Coe & Helpman (95), Frankel & Romer (99), Rodriguez & Rodrik (00), Keller (09), Feyrer (09a,b), Pascali (2014)
- Growth Miracles: Openness and protracted periods of growth
- But standard mechanisms imply relatively small effects
  - e.g., Connolly & Yi (14)

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The Global Diffusion of Ideas

- Provide explicit model of diffusion process based on local interactions
  - Kortum (1997), Eaton & Kortum (1999), Alvarez, Buera, & Lucas (2008), Lucas (2009) Lucas & Moll (2014), Perla & Tonetti (2014) Luttmer (2012, 2014), Jovanovic & Rob (1989)
- How does openness shape ideas to which individuals are exposed?
  - Alvarez, Buera, & Lucas (2014), Perla, Tonetti & Waugh (2014), Sampson (2014), Monge-Naranjo (2012)
- Combine new ideas with insights from others $\Rightarrow$ "general" Frechet limit
  - related to model of random networks in Oberfield (2013)
- Interface with static models of trade, multinational production (MP)
  - Eaton & Kortum (2002), Bernard, Eaton, Jensen, & Kortum (2003), Alvarez & Lucas (2007), Ramondo & Rodriguez-Clare (2014)

The Global Diffusion of Ideas

- How does openness affect development? Potential for growth miracles?

- Which interactions facilitate exchange of ideas? Does it matter?

- Role of policy, international barriers in shaping interactions?

- Rich and tractable enough to take to cross-country data

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  - (Potentially) Large dynamic gains, protracted transition after openness
  - Especially for countries close to autarky
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  - Accounting for cross-sectional TFP-trade relationship...
  - Accounting for changes in TFP, growth miracles...

Roadmap

- Learning from an arbitrary source distribution, Frechet Limit
- Trade
  - Illustrate implications of alternative learning channels
  - Static and dynamic gains from trade
  - Long-run and short-run (liberalization)
- Quantitative exploration
  - Cross-sectional TFP-trade relationship in 1960
  - South Korea: trade and development in the postwar period
- (probably not today) Incentives for Innovation
- (probably not today) Trade and Multinational Production

Learning from an Arbitrary Source

Innovation and Diffusion

- Continuum of goods $s \in [0,1]$
  - For each good m managers (m is large)
  - Bertrand Competition
- Manager with productivity q
  - Ideas arrive stochastically at rate α_t
  - New idea has productivity zq'^{β}
    - Insight from someone with productivity $q' \sim \tilde{G}_t(q')$
    - Original component $z \sim H(z)$
  - Adopts if $zq'^{\beta} > q$
- β measures strength of diffusion
  - Pure innovation: $\beta = 0$ (Kortum (1997))
  - Pure diffusion: $\beta = 1$, H degenerate (ABL (2008, 2014), with Poisson arrivals)

Productivity Distribution

- Distribution of productivity among managers $M_t(q)$

- Frontier of knowledge $\tilde{F}_t(q) = M_t(q)^m$

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$$\frac{1}{m}\frac{d}{dt}\log \tilde{F}_t(q) = \frac{d}{dt}\log M_t(q) = -\alpha_t \Pr(zq'^{\beta} > q)$$

$$= -\alpha_t \int_0^{\infty} \left[1 - \tilde{G}_t\left((q/z)^{1/\beta}\right)\right] dH(z)$$

Frechet Limit

Assumptions

- Distr. of original component of ideas has Pareto tail: $\lim_{z\to\infty} \frac{1-H(z)}{z^{-\theta}} = 1$
- For now: $\tilde{G}_t$ has sufficiently thin right tail: $\lim_{q\to\infty} q^{\beta\theta}[1-\tilde{G}_t(q)] = 0$
  - Later: initial distribution $M_0(q)$ has sufficiently thin tail
- $\beta < 1$

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Convenient to study productivity scaled by number of managers

$$F_t(q) = \tilde{F}_t\left(m^{\frac{1}{(1-\beta)\theta}} q\right) \qquad G_t(q) = \tilde{G}_t\left(m^{\frac{1}{(1-\beta)\theta}} q\right)$$

Proposition ▸ Formal Statement

As $m \to \infty, t \to \infty$, $\quad F_t(q) = e^{-\lambda_t q^{-\theta}}, \quad \dot{\lambda}_t = \alpha_t \int_0^\infty x^{\beta\theta} dG_t(x)$

- λ_t: stock of knowledge

Simple Example

- Individuals learn from managers at frontier

$$G_t(q) = F_t(q)$$

- Then stock of knowledge evolves as

$$\dot{\lambda}_t = \Gamma(1-\beta)\alpha_t \lambda_t^{\beta}$$

- Long-run growth requires the arrival rate grows, $\frac{\dot{\alpha}_t}{\alpha_t} = \gamma$
- Implies growth in stock of knowledge at rate

$$\frac{\dot{\lambda}}{\lambda} = \frac{\gamma}{1-\beta}$$

- Compounding: New ideas lead to even better insights

Trade

World Economy (BEJK, 2003)

- n countries, defined by
  - Labor, L_i
  - Stock of knowledge, λ_i
  - Iceberg trade costs, κ_{ij}

- Household in i has Dixit-Stiglitz preferences $C_i = \left[\int_0^1 c_i(s)^{\frac{\varepsilon-1}{\varepsilon}} ds\right]^{\frac{\varepsilon}{\varepsilon-1}}$

- Production is linear, uses only labor

- For manager in j, unit cost of providing good to country i is

$$\frac{w_j \kappa_{ij}}{q}$$

- Bertrand Competition:

$$p_i(s) = \min\left\{\frac{\varepsilon}{\varepsilon-1}\begin{array}{c}\text{lowest}\\\text{unit cost}\end{array}, \begin{array}{c}\text{second lowest}\\\text{unit cost}\end{array}\right\}$$

Static Trade Equilibrium

- Price index

$$P_i^{-\theta} \propto \sum_j \lambda_j (w_j \kappa_{ij})^{-\theta}$$

- Trade Shares

$$\pi_{ij} = \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}}$$

- Labor market clearing (under balanced trade)

$$w_i L_i = \sum_j \pi_{ji} w_j L_j$$

The Global Diffusion of Ideas

Diffusion of ideas

1. Learn from Sellers
   - Equally exposed to goods consumed (Alvarez-Buera-Lucas)
   - Learn in proportion to quantity consumed (or expenditure)
2. Learn from Producers
   - Equal exposure to active domestic producers (Perla-Tonetti-Waugh, Sampson)
   - Exposed in proportion to labor used (Monge-Naranjo)

Source distributions

- Let S_{ij} be set of goods for which j is lowest-cost provider for i

- Learning from sellers
  - in proportion to expenditure on good

$$G_i^S(q) \equiv \sum_j \int_{s \in S_{ij} | q_j(s) < q} \frac{p_i(s)c_i(s)}{P_i C_i} ds$$

- Learning from producers
  - in proportion to labor used to produce good

$$G_i^P(q) \equiv \sum_j \int_{s \in S_{ji} | q_i(s) \leq q} \frac{1}{L_i} \frac{\kappa_{ji}}{q_i(s)} c_j(s) ds$$

Learning From Sellers

$$\dot{\lambda}_i = \alpha_i \int_0^\infty q^{\beta\theta} dG_i(q) \qquad \propto \qquad \alpha_i \sum_j \pi_{ij} \left(\frac{\lambda_j}{\pi_{ij}} \right)^\beta$$

- Expenditure-weighted average
- Selection: hold fixed λ_j
  - lower π_{ij} $\Rightarrow$ import goods with higher q
- To maximize growth:

$$\frac{\lambda_j}{\lambda_{j'}} = \frac{\pi_{ij}}{\pi_{ij'}}$$

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$$\frac{\lambda_j}{\lambda_{j'}} = \frac{\pi_{ij}}{\pi_{ij'}} \left(= \frac{\lambda_j \left(w_j \kappa_{ij}\right)^{-\theta}}{\lambda_{j'} \left(w_{j'} \kappa_{ij'}\right)^{-\theta}} \right)$$

  - Import more from high wage countries
  - Conflicts with maximizing current welfare

Learning from Producers

- Stock of knowledge

$$\dot{\lambda}_i = \alpha_i \int_0^\infty q^{\beta\theta} dG_i(q) \qquad \propto \qquad \alpha_i \sum_j r_{ji} \left(\frac{\lambda_i}{\pi_{ji}} \right)^\beta$$

- Revenue-weighted average: $r_{ji} = \frac{\pi_{ji} P_j C_j}{\sum_k \pi_{ki} P_k C_k}$ is i's revenue share
- Impact of trade: Selection
  - High productivity producers likely to expand
  - Low productivity producers likely to drop out

Gains from Trade

Static and Dynamic Gains from Trade

Real income is

$$y_i \propto \frac{w_i}{P_i} \propto \left(\frac{\lambda_i}{\pi_{ii}}\right)^{1/\theta}$$

- Static gains from trade: hold λ fixed
- Dynamic gains from trade: operate through idea flows

A Symmetric World

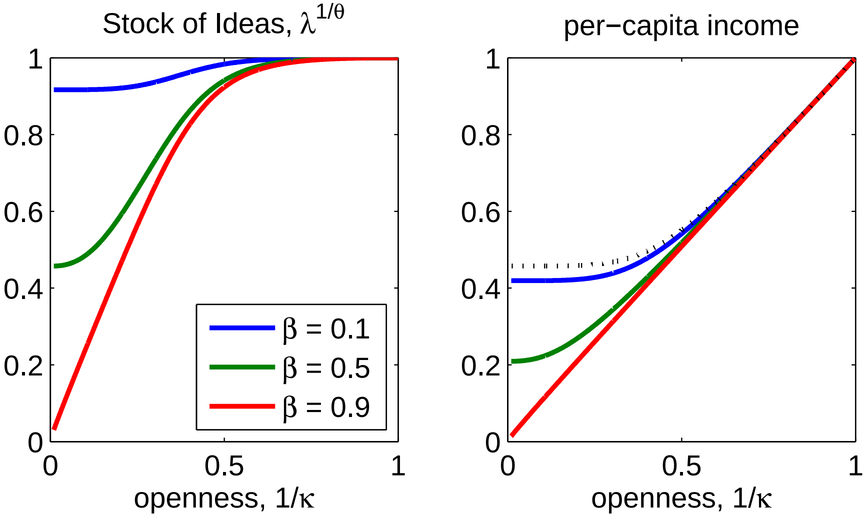
- Consider world with n symmetric countries
- Long-run gains from trade

$$\frac{y^{FT}}{y^{AUT}} = \underbrace{n^{\frac{1}{\theta}}}_{static} \underbrace{n^{\frac{\beta}{(1-\beta)\theta}}}_{dynamic} = n^{\frac{1}{\theta}\frac{1}{1-\beta}}$$

- Dynamic gains from trade
  - Increase with β
  - Similar to input-output multiplier

Note: For special case of symmetric world, specifications of learning are identical

Long-Run Gains from Trade: Reduction in common κ

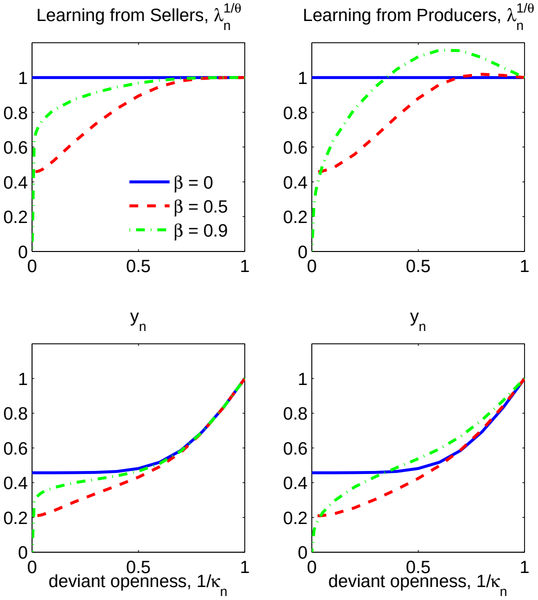


Long-Run Gains from Trade: Single Deviant

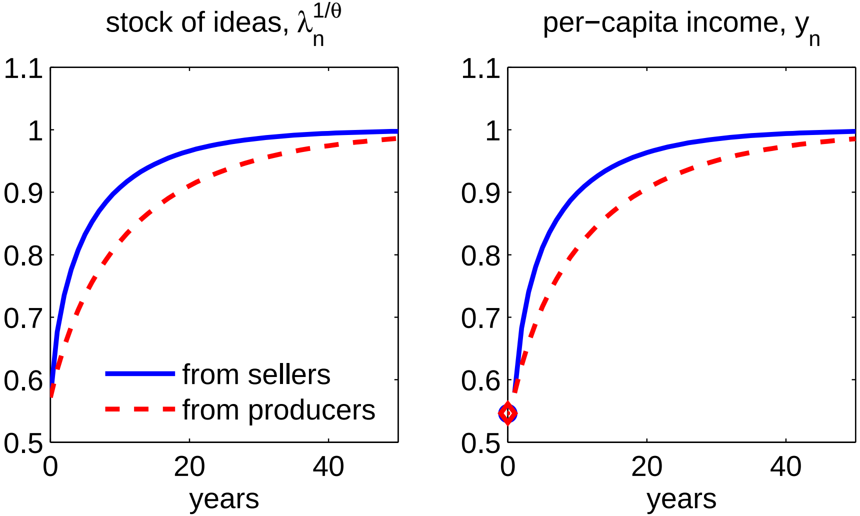
What is the fate of a single country that is isolated?

- Trade among $n-1$ countries is costless
- Trade to and from “deviant” economy incurs iceberg cost κ_n

Long-Run Gains from Trade: Single Deviant



Trade Liberalization, Isolation → 20% Import Share



$\beta = 0.5,\ \theta = 5,$ TFP Growth rate on BGP $= 0.01$

Gains from Trade: Takeaways

- Static gains relevant when economy relatively open
  Dynamic gains relevant when economy relatively closed
- For moderately open economy, dynamic gains non-monotonic in β
- Learning from producers: open economy can get better insights if more isolated
- Small open economy, (relatively) simple expressions for speed of convergence
  expressions
  - Faster with high β
  - α plays no role
  - Slower with learning from domestic producers

Quantitative Exploration

Quantitative Exploration

- Generalized trade model: intermediate inputs, capital, non-traded goods
  details
- Let L_{it} be equipped labor ($= K_{it}^{1/3} \left(pop_{it} \cdot h_{it}\right)^{2/3}$, from the PWT)
- Questions:
  - Can model account for the cross-section relationship between TFP and trade?
  - Can openness account for a significant part of the evolution of TFP of growth miracles?

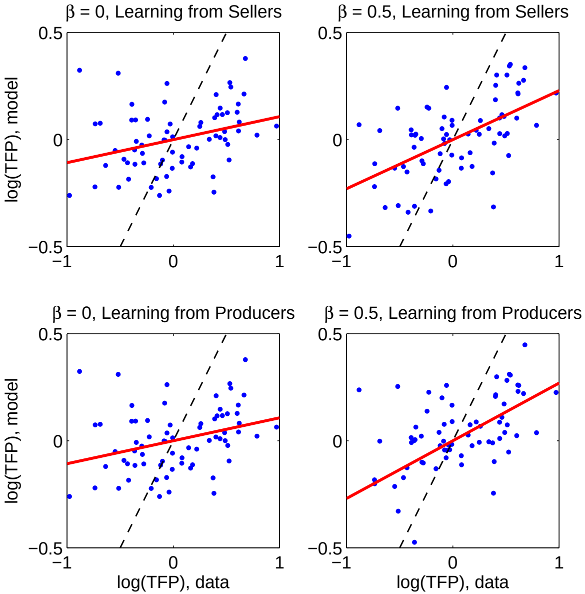
Calibration

- Calibrate the evolution of trade costs, κ_{ijt}, to match bilateral trade flows
  details

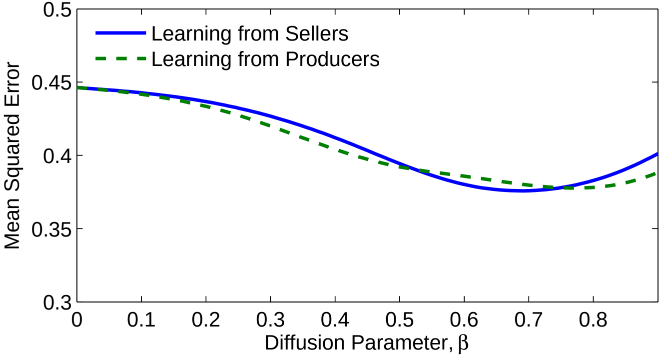
Parameter	Value
θ	5
Share of Non-Traded Goods	0.5
Intermediate Good Share of Cost	0.5
Capital Share of VA	$1/3$
TFP Growth on BGP	1% per year

- α_{it}, β?
  - Homogenous $\alpha_{it} = \alpha L_{it}^{\Upsilon}$. Cross-sectional TFP-trade relationship?
  - Heterogenous α_{it}. Match TFP in 1962. Allow α_i to change?
  - Explore the effects for various β.

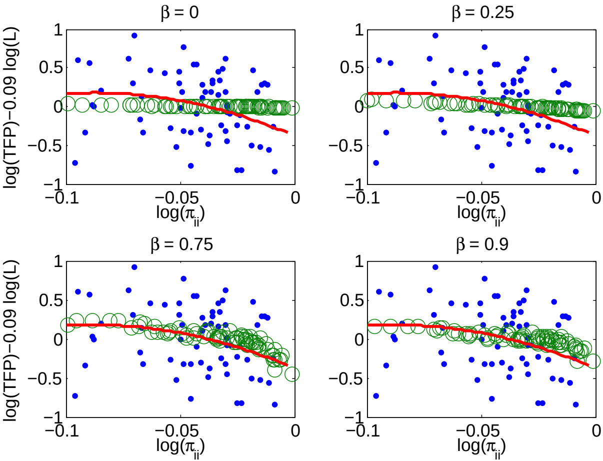
Distribution of TFP in 1962



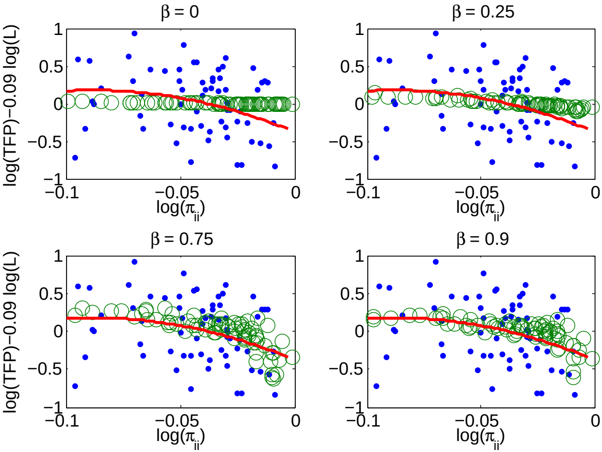
Distribution of TFP in 1962



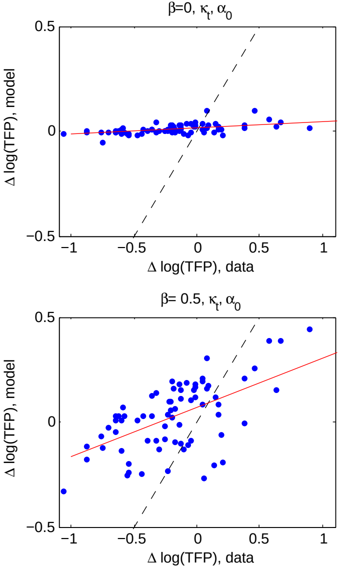
TFP and Trade in 1962, Learning from Sellers



TFP and Trade in 1962, Learning from Producers



Transitions, Learning from Sellers



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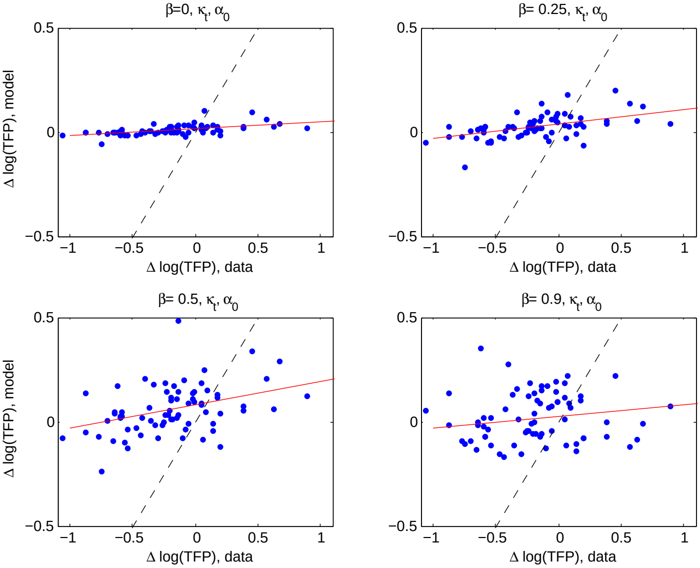

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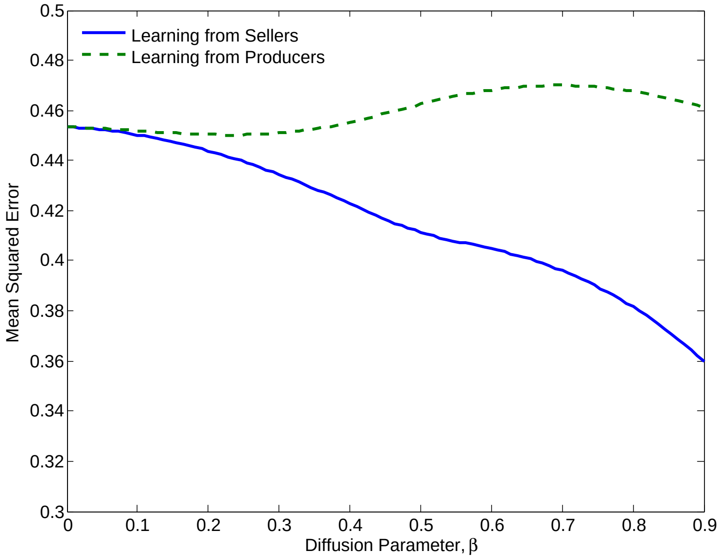
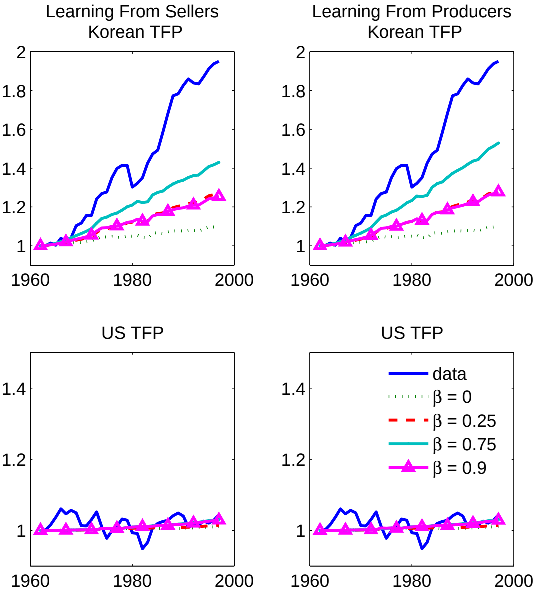
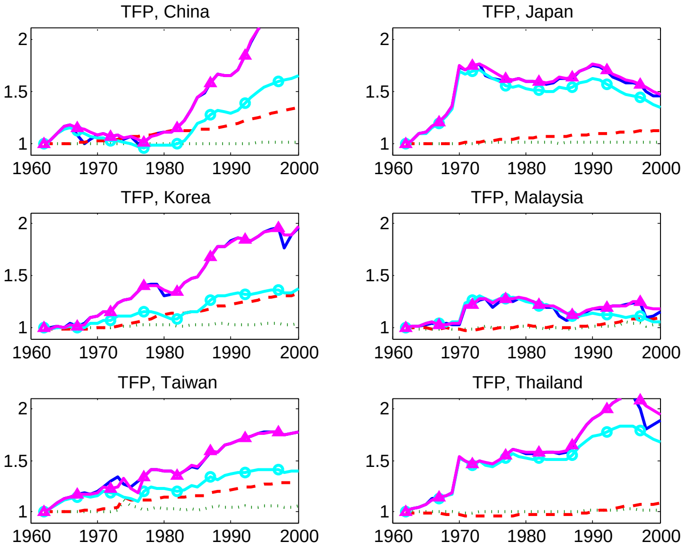


Development Dynamics, South Korea (vs. US)



Development Dynamics, Growth Miracles



Other Applications/Extensions

- Incentives for Innovation: endogenizing α
- Trade and Multinational Production

Incentives to Innovate

$$L_{jt} = L_{jt}^{Production} + L_{jt}^{R\&D}$$

- Across BGPs, $\frac{L_{jt}^{R\&D}}{L_{jt}}$ independent of trade barriers
  - Market size ↑, but competition ↑
  - Like Eaton & Kortum (2001), Atkeson & Burstein (2010)
- But, openness ⇒ same R&D effort leads to better insights
  - Related to Baldwin & Robert-Nicoud (2008)

Multinational Production (MP)

- Multinational Production (build on Ramondo & Rodriguez-Clare (2013))
- Manager associated with
  - Home country i
  - Profile of productivities, $\{q_1, ..., q_n\}$
- Iceberg MP costs δ_{ij}
- Trade equilibrium: Eaton-Kortum

Multinationals and Learning

- Manager with $\{q_1, ..., q_n\}$ draws insight from good with q'
- Location-specific $\{z_1, ..., z_n\}$, drawn from $H(z_1, ..., z_n)$
- New Profile
$$\left\{\max\{q_1, z_1^{1-\beta} q'^{\beta}\}, ..., \max\{q_n, z_n^{1-\beta} q'^{\beta}\}\right\}$$
- $\{z_1, ..., z_n\}$ drawn from multivariate Pareto, correlation ρ Details
- $F_{it}(q_1, ..., q_n)$ is multivariate Frechet
$$F_{it} = e^{-\lambda_{it}\left(\sum_j q_j^{-\frac{\theta}{1-\rho}}\right)^{1-\rho}} \text{ and } \dot{\lambda}_{it} = \alpha \int_0^{\infty} q^{\beta\theta} dG_{it}(q)$$

Multinational Production

- Learning from Sellers & Producers

$$\text{Sellers:} \qquad \dot{\lambda}_i \quad \propto \quad \alpha \sum_j \sum_k \pi_{ijk} \left(\frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[\sum_l \pi_{ilk}\right]^\rho} \right)^\beta$$

$$\text{Producers:} \qquad \dot{\lambda}_i \quad \propto \quad \alpha \sum_j \sum_k r_{jik} \left(\frac{\lambda_k}{\pi_{jik}^{1-\rho} \left[\sum_l \pi_{jlk}\right]^\rho} \right)^\beta$$

where $r_{jik} = \frac{w_j \pi_{jik}}{w_i}$

- Autarky vs Free Trade, Free MP

$$\frac{y^{FT}}{y^{AUT}} = \underbrace{n^{\frac{2-\rho}{\theta}}}_{\text{static}} \times \underbrace{n^{\frac{(2-\rho)\beta}{1-\beta}}}_{\text{dynamic}}$$

Trade and FDI

Are trade and FDI complements or substitutes?

- Let $y(\kappa, \delta)$ be real income for symmetric world with
  - trade costs κ
  - FDI costs δ

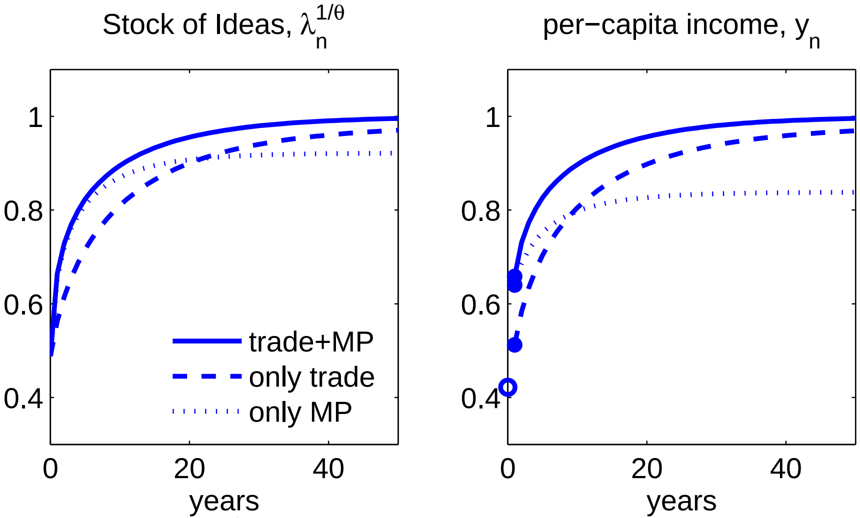
- Depends on ρ. Two polar cases:

$$\lim_{\rho \to 0} \frac{y(\kappa, \delta)}{y(1,1)} = \left[\left(\frac{1 + (n-1)\kappa^{-\theta(1-\beta)}}{n} \right) \left(\frac{1 + (n-1)\delta^{-\theta(1-\beta)}}{n} \right) \right]^{\frac{1}{\theta(1-\beta)}}$$

and

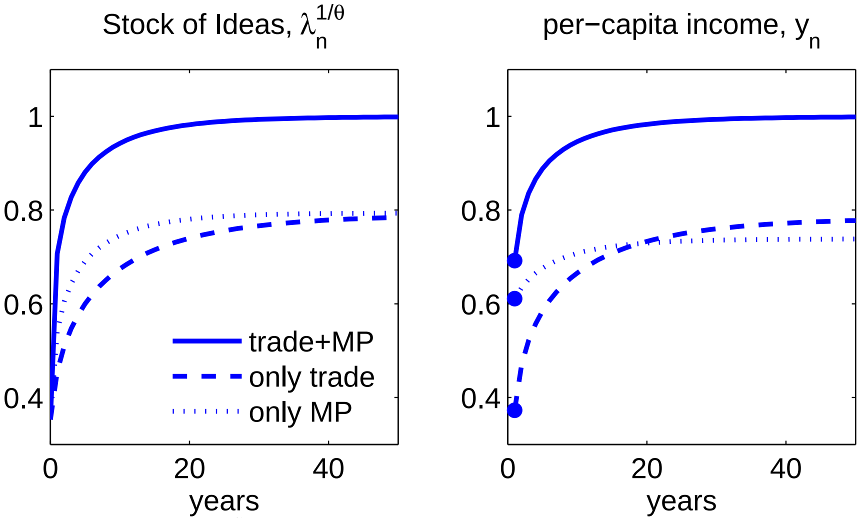
$$\lim_{\rho \to 1} \frac{y(\kappa, \delta)}{y(1,1)} = \max \left\{ \left(\frac{1 + (n-1)\kappa^{-\theta(1-\beta)}}{n} \right), \left(\frac{1 + (n-1)\delta^{-\theta(1-\beta)}}{n} \right) \right\}^{\frac{1}{\theta(1-\beta)}}$$

Opening to Trade and/or MP, $\rho = 0.5$



$$\beta = 0.5, \ \frac{\alpha^S}{\alpha^S + \alpha^P} = 0.1, \rho = 0.5, \kappa = 100 \rightarrow 2.15, \ \delta = 100 \rightarrow 3$$

Opening to Trade and/or MP, $\rho = 0.1$



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Conclusions/Future Research

- Present tractable model that incorporates large class of diffusion mechanisms, based on local interactions
- Common message:
  - Large dynamics gains from trade, specially for intermediate values of β
  - able to account for the cross-sectional TFP-trade relationship
  - ... generate growth miracles with a significant role for trade
- Future research:
  - Infer value for β: aggregate TFP-trade dynamics, e.g., Feyrer (2009a,b), Hanson & Muendler (2013), Levchenko & Zhang(2014), Pascali (2014); micro evidence, e.g., Aitken & Harrison (1999), Javorcik (2004.
  - Endogenizing α, role for human capital

Frechet Limit

Proposition

Given assumptions, the frontier of knowledge evolves as:

$$\lim_{m\to\infty} \frac{d \ln F_t(q)}{dt} = -\alpha_t q^{-\theta} \int_0^\infty x^{\beta\theta} dG_t(x)$$

Define $\lambda_t = \int_{-\infty}^{t} \alpha_\tau \int_0^\infty x^{\beta\theta} dG_\tau(x)$

Corollary

Suppose that $\lim_{t\to\infty} \lambda_t = \infty$. Then $\lim_{t\to\infty} F_t(\lambda_t^{1/\theta} q) = e^{-q^{-\theta}}$.

Back

Learning from Producers

in proportion to employment

$$G_i(q) = \sum_{j=1}^{n} \int_0^q \underbrace{\frac{L_j w_j}{L_i w_i} \left(\frac{w_i \kappa_{ji}}{P_j} \right)^{1-\varepsilon} x^{\varepsilon-1}}_{fraction\ of\ employment\ in\ x} \underbrace{\prod_{k \neq j} F_k \left(\frac{w_k \kappa_{ik}}{w_i \kappa_{ii}} x \right)}_{prob.\ j\ buys\ x\ from\ i} dF_i(x)$$

back

Learning from Producers

uniformly

$$G_i(q) = \sum_{j=1}^{n} \int_0^q \frac{1}{\pi_{ii}} \prod_{k \neq j} F_k \left(\frac{w_k \kappa_{jk}}{w_i \kappa_{ji}} x \right) dF_i(x)$$

The evolution of the stock of knowledge

$$\dot{\lambda}_i \propto \left(\frac{\lambda_i}{\pi_{ii}} \right)^{\beta}$$

back

Multivariate Pareto

$$H(z_1, ..., z_n) = \max \left\{ 1 - \left(\sum_j \left(\frac{z_i}{z_0} \right)^{-\frac{\Theta}{1-\rho}} \right)^{1-\rho}, 0 \right\}$$

- Each marginal is distribution is Pareto
- $\rho \in [0, 1]$ like a correlation

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Endogenous Growth Case, $\beta = 1$

Alvarez, Buera & Lucas (2013)

- Learning from sellers
- Trade only
- Evolution of the distribution of productivities

$$\frac{\partial \log(F_{it}(q))}{\partial t} = -\alpha \left[1 - \sum_{j=1}^{n} \int_0^q \prod_{k \neq j} F_{kt} \left(\frac{w_k \kappa_{ik}}{w_j \kappa_{ij}} x \right) dF_{jt}(x) \right]$$

Endogenous Growth Case, $\beta = 1$
Alvarez, Buera & Lucas (2013)

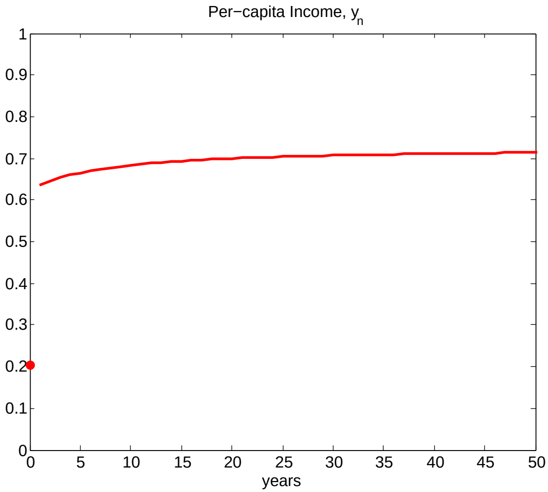
- Growth rate in a BGP, $\nu = n\alpha/\theta$

- Tails converge if $\kappa_{ij} < \infty$

$$\lim_{q\to\infty}\lim_{t\to\infty}\frac{1 - F_{it}\left(qe^{\nu t}\right)}{\lambda q^{-\theta}} = 1$$

- Distribution not Frechet (log-logistic if $\kappa_{ij} = w_i = 1$)

Single Deviant: Stock of Knowledge



Generalized Trade Model

- Technology requiring an intermediate aggregate and labor

$$y_i(\mathbf{q}) = \frac{1}{\eta^\eta \zeta^\zeta (1-\eta-\zeta)^{1-\eta-\zeta}} q_i x_i(\mathbf{q})^\eta k_i(\mathbf{q})^\zeta l_i(\mathbf{q})^{1-\eta-\zeta}$$

- Intermediate (investment) aggregate technology

$$X_i = \left[\int c_{xi}(\mathbf{q})^{1-1/\epsilon} dF_i(\mathbf{q}) \right]^{\epsilon/(\epsilon-1)}$$

- Fraction μ of the goods are tradable, i.e.,

$$p_i^{1-\varepsilon} = (1-\mu) \int_0^\infty \left(\frac{p_i^\eta R_i^\zeta w_i^{1-\eta-\zeta}}{q} \right)^{1-\varepsilon} dF_j(q)$$
$$+ \mu \sum_{j=1}^n \int_0^\infty \left(\frac{p_j^\eta R_i^\zeta w_j^{1-\eta} \kappa_{ij}}{q} \right)^{1-\varepsilon} \prod_{k \neq j} F_k \left(\frac{p_k^\eta R_i^\zeta w_k^{1-\eta-\zeta} \kappa_{ik}}{p_j^\eta R_i^\zeta w_j^{1-\eta-\zeta} \kappa_{ij}} q \right) dF_j(q)$$

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Speed of Convergence: Small Open Economy

For small open economy, speed of convergence is

- If agents learn from sellers

$$\gamma \left\{ 1 - \frac{\Omega^S_{ii} - \pi_{ii}}{1 + \theta \left(1 + \pi_{ii}\right)} + \frac{\beta}{1 - \beta} \left(1 - \Omega^S_{ii}\right) \right\}$$

- If agents learn from producers

$$\gamma \left\{ 1 - \frac{\Omega^P_{ii} - \pi_{ii}}{1 + \theta \left(1 + \pi_{ii}\right)} + \frac{\beta}{1 - \beta} \frac{\left(1 - \Omega^P_{ii}\right)\left(1 + \pi_{ii}\right)}{1 + \theta \left(1 + \pi_{ii}\right)} \right\}$$

where $\Omega^S_{ii} \equiv \frac{\pi_{ii}(\lambda_i/\pi_{ii})^\beta}{\sum_j \pi_{ij}(\lambda_j/\pi_{ij})^\beta}$ and $\Omega^P_{ii} \equiv \frac{r_{ii}(\lambda_i/\pi_{ii})^\beta}{\sum_j r_{ji}(\lambda_i/\pi_{ji})^\beta}$.

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Calibrating Trade Costs

Use trade data from Feenstra et al. (2005), GDP from PWT 8.0 and the equilibrium relations

$$\kappa_{ijt} = \kappa_{jit} = \left[\frac{1-\pi_{iit}}{\pi_{ijt}} \frac{1-\pi_{jjt}}{\pi_{jit}} \left(\frac{Z_{it}}{1-Z_{it}} \right) \left(\frac{1-Z_{jt}}{Z_{jt}} \right) \right]^{\frac{1}{2\theta}}$$

where Z_{it} solves

$$\pi_{iit} = \frac{(1-\mu) + \mu Z_{it}^{1-\frac{\varepsilon-1}{\theta}}}{(1-\mu) + \mu Z_{it}^{-\frac{\varepsilon-1}{\theta}}}.$$

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