A Regime-Switching SVAR Analysis of ZIRP

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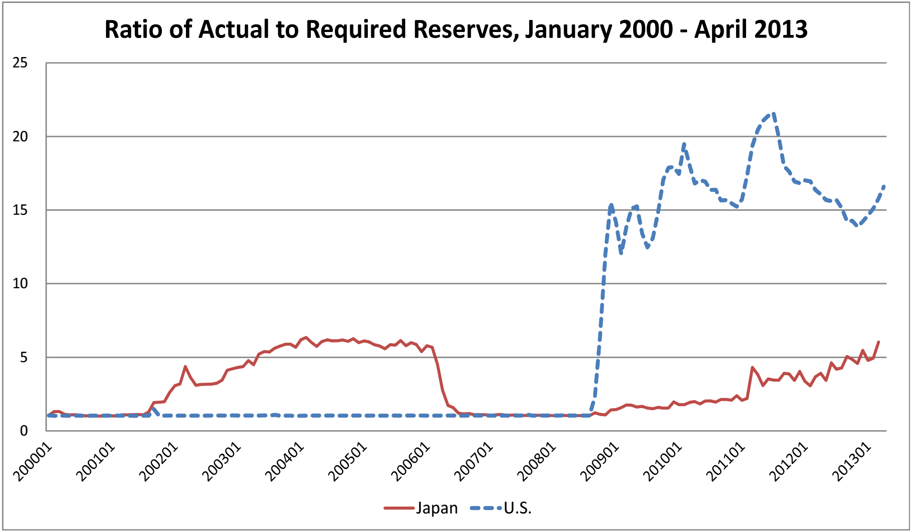
To get started...



What This Paper Does

- VAR/SVAR analysis of the effect of ZIRP/QE on macro variables (inflation and output)
  - Japan has, by our count, 130 months of ZIRP (as of Dec. 2012), maybe enough for time-series analysis
- Unique in two respects:
  - The regime is observable and endogenous (unlike in the hidden-stage Markov switching model)
  - can study IR to regime changes.

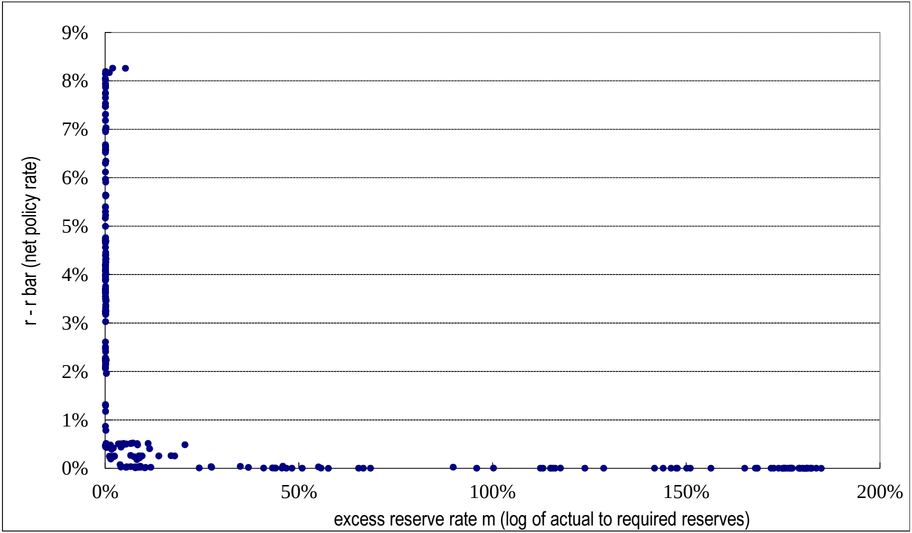
Relation to the Lit

- Previous SVAR-IR papers using Japanese data
  - Iwata-Wu (JME 2004): VAR with nonnegativity constraint on the interest rate.
  - Honda *et. al.* (2007): straight SVAR-IR (prices, output, money) on the 2nd QE period (2001-2006).
  - Fujiwara (JJIE 2006), Inoue-Okimoto (JJIE 2008): Markov-switching SVAR-IR (prices, output, policy rate, money).

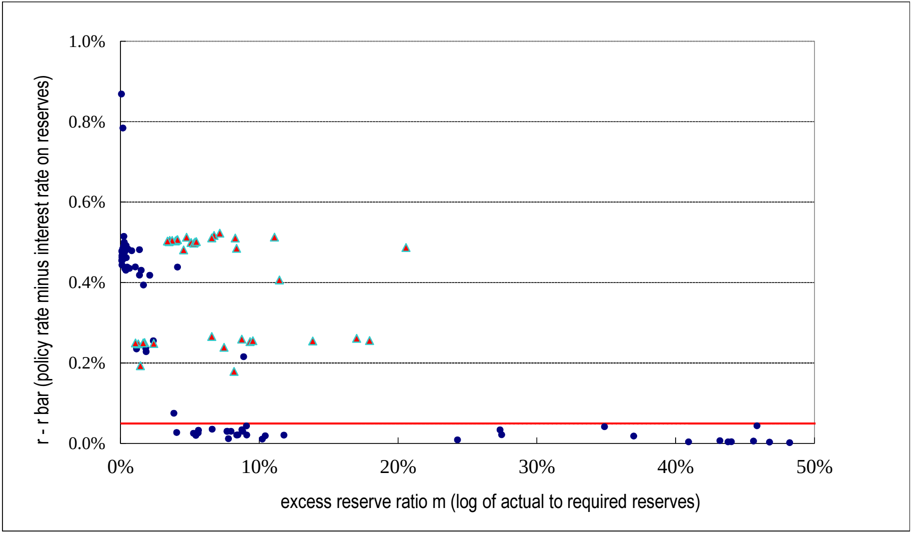
Plan of Talk

- Identifying the Zero Regime
- The Model
- Estimation Results
- IRs
- Conclusions about BOJ's ZIRP

Identify Zero Regime: the "L", $r - \bar{r}$ against m, 1985-2012



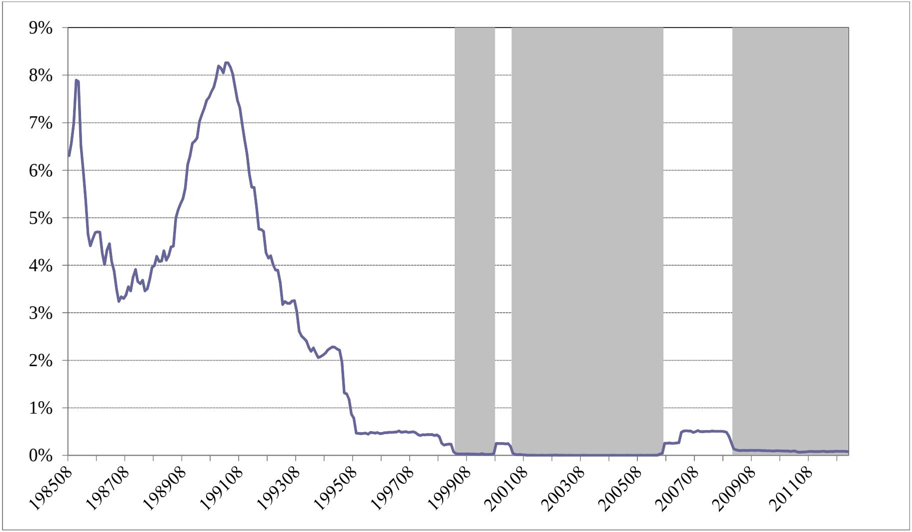
Identify Zero Regime: the "L", $r - \bar{r}$ against m, magnified



Identifying Zero Regime: Three Spells of Z

- Periods of Zero Regime
  - (i) March 1999 - July 2000
  - (ii) March 2001 - June 2006
  - (iii) December 2008 to date

Policy Rate (r) in Japan, August 1985-December 2012



The Model, 1 of 4: textbook SVAR

- Point of departure: textbook 3-variable SVAR
  - (p, x, r), $p =$ monthly inflation rate, $x =$ output gap, $r =$ policy rate, in that order.
  - The first two equations are reduced forms in (p, x).
  - The third equation is the Taylor rule.
  - The error in the Taylor rule is independent of (p, x) reduced-form shocks.
- Taylor rule: with $\pi_t \equiv \frac{1}{12}(p_t + \cdots + p_{t-11})$,

$$r_t = \underbrace{\rho_r r_t^* + (1-\rho_r) r_{t-1}}_{\equiv r_t^e,\text{“Taylor rate”}} + \sigma_r v_{rt}, \quad r_t^* \equiv \underbrace{\alpha_r^* + \underset{(1\times 2)}{\boldsymbol{\beta}_r^{*\prime}} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}}_{\text{“desired Taylor rate”}}, \quad v_{rt} \sim \mathcal{N}(0,1).$$

- See, e.g., Stock-Watson's review article in *J. of Economic Perspectives*, 2001.

The Model, 2 of 4: Add Zero Lower Bound

- Impose the lower bound

$$\text{(censored Taylor rule)} \quad r_t = \max\left[r_t^e + \sigma_r v_{rt}, \bar{r}_t\right], \quad v_{rt} \sim \mathcal{N}(0,1).$$

Equivalently,

$$r_t = \begin{cases} r_t^e + \sigma_r v_{rt}, \quad v_{rt} \sim \mathcal{N}(0,1) & \text{if } s_t = \mathrm{P}, \\ \bar{r}_t & \text{if } s_t = \mathrm{Z}, \end{cases}$$

$$\text{where} \quad s_t = \begin{cases} \mathrm{P} & \text{if } \; r_t^e + \sigma_r v_{rt} \geq \bar{r}_t, \\ \mathrm{Z} & \text{otherwise.} \end{cases}$$

- Note: the regime s_t is endogenous.

The Model, 3 of 4: Add m

- Add m to (p, x, r).

$$m_t = \begin{cases} 0 & \text{if } s_t = \text{P}, \\ \max\left[m_t^e + \sigma_m v_{mt}, 0\right], \quad v_{mt} \sim \mathcal{N}(0,1) & \text{if } s_t = \text{Z}, \end{cases}$$

where

$$m_t^e \equiv \rho_m \left(\alpha_m^* + \boldsymbol{\beta}_m^{*\,\prime} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} \right) + (1 - \rho_m) m_{t-1},$$

- (reminder)
  - $p \equiv$ monthly inflation rate, $\pi_t \equiv \frac{1}{12}(p_t + \cdots + p_{t-11})$,
  - $x \equiv$ output gap,
  - $r \equiv$ policy rate,
  - $m \equiv \log\left(\frac{\text{actual reserves}}{\text{required reserves}}\right)$.

The Model, 4 of 4: Introduce Exit Condition/Forward Guidance

- If $s_{t-1} = \mathrm{Z}$,

$$s_t = \begin{cases} \mathrm{P} & \text{if} \quad r_t^e + \sigma_r v_{rt} \geq \overline{r}_t \quad \text{and} \quad \pi_t \geq \underbrace{\overline{\pi} + \sigma_\pi v_{\pi t}}_{\text{period } t \text{ target inflation rate}}, \\ \mathrm{Z} & \text{otherwise.} \end{cases}$$

- If $s_{t-1} = \mathrm{P}$, as before, i.e.,

$$s_t = \begin{cases} \mathrm{P} & \text{if} \quad r_t^e + \sigma_r v_{rt} \geq \overline{r}_t, \\ \mathrm{Z} & \text{otherwise.} \end{cases}$$

- (reminder) Taylor rule: with $\pi_t \equiv \frac{1}{12}(p_t + \cdots + p_{t-11})$,

$$r_t = \underbrace{\rho_r r_t^* + (1 - \rho_r) r_{t-1}}_{\equiv r_t^e, \text{“Taylor rate”}} + \sigma_r v_{rt}, \quad r_t^* \equiv \underbrace{\alpha_r^* + \underset{(1\times 2)}{\beta_r^{*\prime}} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}}_{\text{“desired Taylor rate”}}.$$

Model Recap: Sequence of Events from t to $r+1$

(1) (p_t, x_t) (inflation and output) determined by the reduced form equations.

(2) CB draws $(v_{rt}, v_{\overline{\pi}t})$ and determines s_t.

- s_t is a Markov chain. Transition probabilities depend on (p_t, x_t).

(3) CB draws v_{mt}.

- If $s_t = \mathrm{P}$, then $r_t = r_t^e + \sigma_r v_{rt}$, $m_t = 0$.
- If $s_t = \mathrm{Z}$, then $r_t = \overline{r}_t$, $m_t = \max[m_t^e + \sigma_m v_{mt}, 0]$.

(1) (p_{t+1}, x_{t+1}) determined given s_t.

- (reminder)

$$m_t^e \equiv \rho_m \left(\alpha_m^* + \boldsymbol{\beta}_m^{*\prime} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} \right) + (1 - \rho_m) m_{t-1},$$

Implications for Estimation

- Reduced-form equations: will take Lucas critique seriously.
  - Given estimation done conditional on regime, no need for selectivity correction.
- Taylor rule:
  - if no exit condition/forward guidance, Tobit
  - with exit condition/forward guidance, only slightly more complicated ML.
- Reserve supply function: No selectivity bias.

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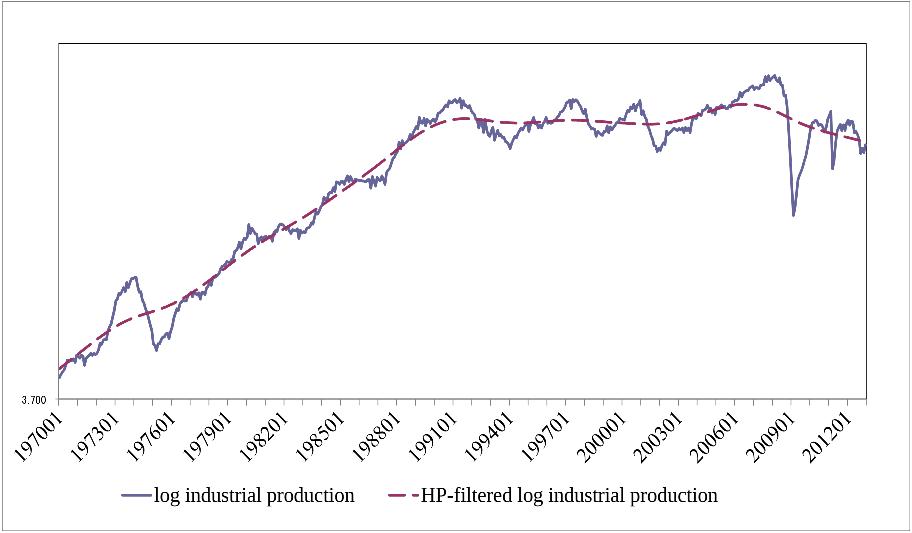
Estimation: Data

- p (monthly inflation rate): $p_t \equiv 1200 \times [\log(CPI_t) - \log(CPI_{t-1})]$.
- π (12-month inflation rate): $\pi_t = \frac{1}{12}(p_t + \cdots + p_{t-11})$.
- m (excess reserve rate): $m_t \equiv 100 \times \log\left(\frac{\text{actual reserves}_t}{\text{required reserves}_t}\right)$.
- r (policy rate): collateralized overnight interbank rate.
  - r_t =average of daily values over the reserve maitenance period (16th day of month t to 15th day of month $t+1$).
- x (output gap): See next picture.

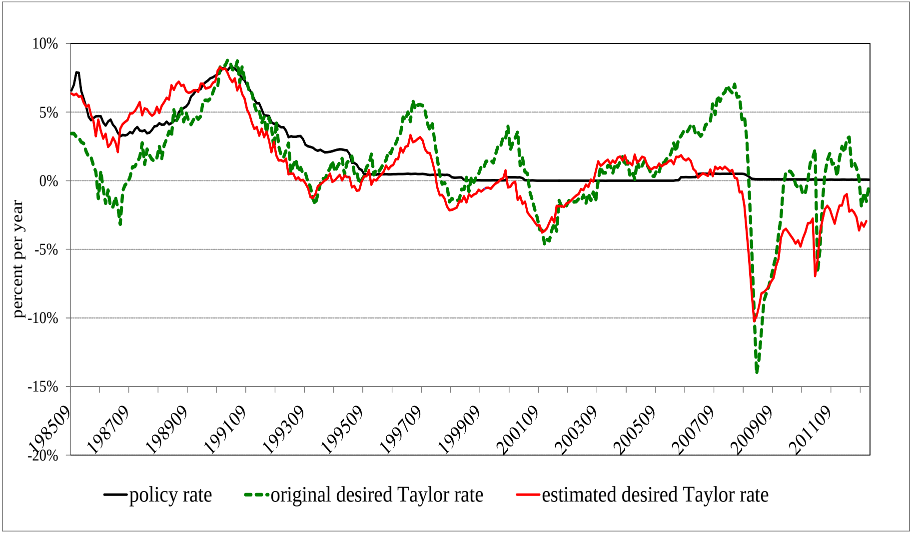
Japanese log IP, 1970-2012



Estimation Results: Executive Summary

- (reminder) $r_t = \underbrace{\rho_r r_t^* + (1-\rho_r) r_{t-1}}_{\equiv r_t^e,\text{ "Taylor rate"}} + \sigma_r v_{rt}, \quad r_t^* \equiv \underbrace{\alpha_r^* + \underset{(1\times 2)}{\beta_r^{*\prime}} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}}_{\text{"desired Taylor rate"}}.$
- (reminder) $m_t = \max[m_t^e + \sigma_m v_{mt}, 0], \quad m_t^e \equiv \rho_m \left(\alpha_m^* + \beta_m^{*\prime} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} \right) + (1-\rho_m) m_{t-1}.$
- Reserve supply on sample Z: has right sign, but not sharply estimated.
- Taylor rule (more in next picture):
  - Need to allow α_r^* to decline after the bubble period of 1991.
  - Parameters sharply estimated.
  - Satisfies Taylor principle.
- Reduced forms:
  - Lucas should apply with full force.
  - not sharply estimated.

Policy Rate and Desired Taylor Rates (r_t^*), 1985 - 2012



Policy Rate and Desired Taylor Rates (r_t^*), 1985 - 2012

- Explain why α_r^* has to be lower post-bubble
- Show the exit condition/forward guidance in action.
- (reminder) $r_t = \underbrace{\rho_r r_t^* + (1-\rho_r) r_{t-1}}_{\equiv r_t^e, \text{“Taylor rate”}} + \sigma_r v_{rt}, \quad r_t^* \equiv \underbrace{\alpha_r^* + \underset{(1\times 2)}{\boldsymbol{\beta}_r^{*\prime}} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}}_{\text{“desired Taylor rate”}}.$

Estimation: (p, x) Reduced Form

sample period is January 1992 - December 2012

subsample P (s_t = P in the prev. month, sample size = 123)						
dep. var.	const.	p_t	x_t	r_t	m_t	R^2
p_{t+1}	-0.23	0.10	0.02	0.40		0.16
	[-1.1]	[1.1]	[0.6]	[3.6]		
x_{t+1}	-0.003	0.064	0.95	-0.11		0.87
	[-0.01]	[0.6]	[24]	[-0.8]		
subsample Z (s_t = Z in the prev. month, sample size = 129)						
dep. var.	const.	p_t	x_t	r_t	m_t	R^2
p_{t+1}	-0.63	0.05	0.03		0.0018	0.03
	[-2.5]	[0.6]	[1.2]		[0.9]	
x_{t+1}	-0.21	0.18	0.90		0.0027	0.84
	[-0.5]	[1.1]	[25]		[0.8]	

Things to Note about Reduced Forms

- Structural change in 1991. Inflation persistence (much higher before 1992).
- lagged m coeffficient under P and lagged r coefficient under Z set to 0.
- lagged m coefficient in p_{t+1} equation under P positive and significant.
- Intercepts lower under Z.

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GRT's IR of i to j

- Gallant-Rossi-Tauchen (*Econometrica*, 1993): For a possibly nonlinear stationary process $\underset{(n\times 1)}{\mathbf{y}_t}$ in general,

$$\mathrm{E}\big(y_{i,t+k} \mid \underbrace{y_{jt}+\delta, y_{j-1,t}, \ldots, y_{1t}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots}_{\text{alternative history}}\big) - \mathrm{E}\big(y_{i,t+k} \mid \underbrace{y_{jt}, y_{j-1,t}, \ldots, y_{1t}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots}_{\text{baseline history}}\big).$$

- Generally history-dependent and not proportional to δ in general.
- Baseline history doesn't have to be the actual history.
- Calculation by Monte Carlo integration.
- Natural extension to nonlinear case. Reduces to the familiar orthogonalized IR in the linear case (see Hamilton's time series text).

Application to our Model: m-IR and r-IR

- m-IR (IR to a change in m):

$$\mathrm{E}\big(y_{t+k}\,|\,s_t = \mathrm{Z}, \underbrace{(m_t + \delta_m, \bar{r}_t, p_t, x_t),}_{\mathbf{y}_t \text{ in the alternative history}} \mathbf{y}_{t-1}, \ldots\big)$$

$$- \mathrm{E}\big(y_{t+k}\,|\,s_t = \mathrm{Z}, \underbrace{(m_t, \bar{r}_t, p_t, x_t),}_{\mathbf{y}_t \text{ in the baseline history}} \mathbf{y}_{t-1}, \ldots\big), \; y = p, x, r, m.$$

- r-IR (IR to a change in r):

$$\mathrm{E}_t\big(y_{t+k}\,|\,s_t = \mathrm{P}, \underbrace{(0, r_t - \delta_r, p_t, x_t)}_{\mathbf{y}_t \text{ in the alternative history}}, \mathbf{y}_{t-1}, \ldots\big)$$

$$- \mathrm{E}_t\big(y_{t+k}\,|\,s_t = \mathrm{P}, \underbrace{(0, r_t, p_t, x_t)}_{\mathbf{y}_t \text{ in the baseline history}}, \mathbf{y}_{t-1}, \ldots\big), \; y = p, x, r, m.$$

Estimated m-IR and r-IR

- Reserve increases are expansionary: $m \uparrow \Rightarrow p \uparrow$, $x \uparrow$. Consistent with previous studies (Inoue-Okimoto, Honda *et. al.*).
- Price puzzle observed: $r \downarrow \Rightarrow p \downarrow$, $x \uparrow$.

ZP-IR: IR to Regime Change from P to Z

- ZP-IR:

$$\mathrm{E}\big(y_{t+k} \,|\, s_t = \mathrm{P}, \underbrace{(0, \overline{r}_t, p_t, x_t),}_{\mathbf{y}_t \text{ in the alternative history}} \mathbf{y}_{t-1}, \ldots\big)$$

$$- \mathrm{E}\big(y_{t+k} \,|\, s_t = \mathrm{Z}, \underbrace{(m_t, \overline{r}_t, p_t, x_t),}_{\mathbf{y}_t \text{ in the baseline history}} \mathbf{y}_{t-1}, \ldots\big), \; y = p, x, r, m.$$

- Can be rewritten as

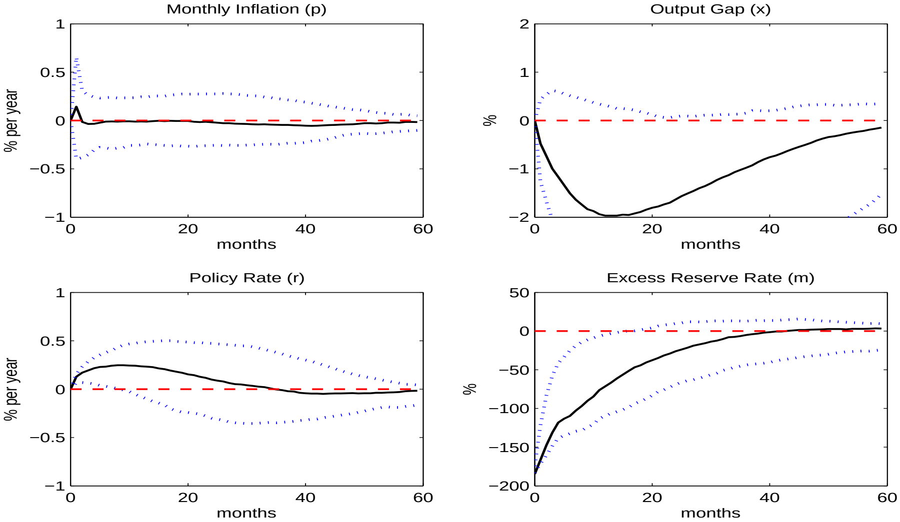
$$\underbrace{\Big[\mathrm{E}\big(y_{t+k} \,|\, s_t = \mathrm{P}, (0, \overline{r}_t, p_t, x_t), \ldots\big) - \mathrm{E}\big(y_{t+k} \,|\, s_t = \mathrm{Z}, (0, \overline{r}_t, p_t, x_t), \ldots\big)\Big]}_{\text{pure regime change effect}}$$

$$- \underbrace{\Big[\mathrm{E}\big(y_{t+k} \,|\, s_t = \mathrm{Z}, (m_t, \overline{r}_t, p_t, x_t), \ldots\big) - \mathrm{E}\big(y_{t+k} \,|\, s_t = \mathrm{Z}, (0, \overline{r}_t, p_t, x_t), \ldots\big)\Big]}_{m\text{-IR}}$$

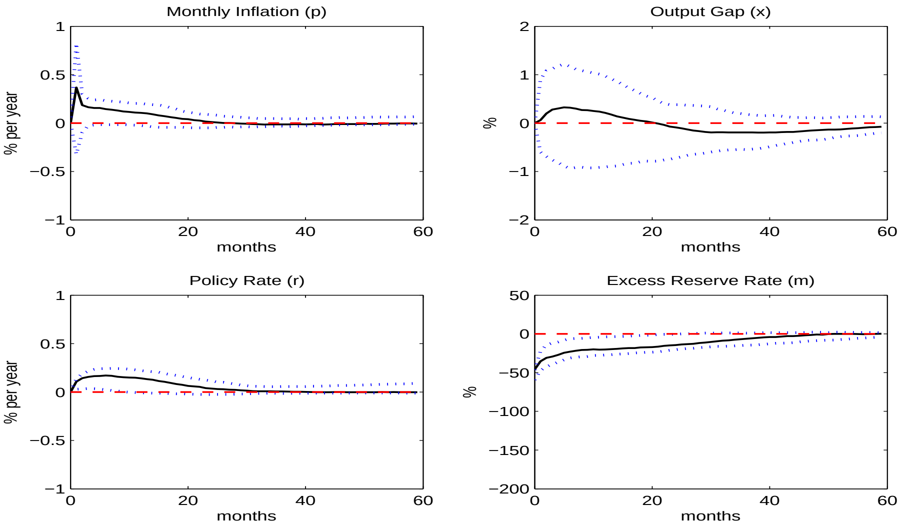
Estimated ZP-IR

- The pure ZP-IR tend to be *expansionary*: Z $\rightarrow$ P $\Rightarrow$ $p \uparrow$, $x \uparrow$.
- Does the effect of *m*-IR more than offset it? Depends on t.
- See IR plots for:
  - $t =$ February 2004 (peak of QE),
  - $t =$ June 2006 (one month before the BOJ terminated Z).

ZP-IR, February 2004



ZP-IR, June 2006



Conclusions about BOJ's ZIRP

- Increases in reserves under ZIRP (the Zero regime) raise both inflation and output.
- Terminating ZIRP is not necessarily deflationary.
- In particular, the termination in July 2006 may have been *inflationary*.
- However, note the wide error bands.