

Liquidity Traps and Monetary Policy: Managing a Credit Crunch

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June 21, 2013

Abstract

We present a model of a monetary economy with heterogeneous producers and collateral constraints. We use the model to study the consequences of alternative monetary policies following a tightening in the collateral requirement. Firstly, we show that when policy does not react to the change in the environment, there is a large deflation, and a particularly severe contraction in an economy with nominal private debt. Secondly, we consider a monetary policy that implements a constant inflation target. Price stability imposes a bound on the real interest rate and it requires a sharp increase in the supply of assets by the government, moving the economy into a liquidity trap and crowding out private investment. In this case the credit crunch leads to a less pronounced recession, but slower recovery. Finally, we show that the welfare consequences of alternative monetary policies vary substantially across individuals.

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1 Introduction

The year 2008 will be remembered in the macroeconomics literature for long. This is so, not only because of the massive shock that hit global financial markets, particularly since the bankruptcy of Lehman and the collapse of the interbank market that immediately followed, but also because of the unusual and extraordinary response to it, emanated from all major Central Banks. The reaction of the Fed is a clear example: it doubled its balance sheet in just three months - from 800 billion on September 1st, to 1,6 trillion by December 1st. Then, it kept on increasing it to reach around 3 trillion by the end of 2012.¹ Very similar measures were taken by the European Central Bank and other Central Banks of developed economies. We guess that most macroeconomists would agree with the notion that the 2008-2013 period is, from the point of view of macroeconomic theory and policy, among the most dramatic ones in the past hundred year's history, perhaps second only to the Great Depression period.

Paradoxically, however, none of the models used by Central Banks at the time in developed economies was of any use to study neither the financial shock, nor the reaction of monetary policy. Those models essentially ignore financial markets on one hand, and monetary aggregates on the other. There were good reasons for this: by and large, big financial shocks seemed to belong exclusively to emerging economies since the turbulent 1930's. We are not sure how to define emerging economies, but always had the sense that it meant highly volatile financial markets. According to this narrow definition, it seems that 2008 taught us, among other things, that we live in an emerging world.² In addition, monetary economics developed, in the last two decades, around the Central Bank rhetoric of emphasizing exclusively on the short term nominal interest rate. Measures of liquidity or money, were completely ignored as a stance of monetary policy; one of the reasons being that empirical relationships between monetary aggregates, interest rates and prices, that stood well for most of the twentieth century, broke down in the midst of the banking deregulation that started in the 1980's.³ Consequently, there is a lack of general equilibrium models that can be used to study the effects of monetary policies like the Fed adopted since 2008, during times of financial distress. The purpose of this paper is to provide one such model and study the macroeconomic effects of monetary policy during a credit crunch. We study a model with heterogeneous entrepreneurs that face cash-in-advance constraints on purchases and collateral constraints on borrowing. The collateral constraints give rise to a non-trivial financial market. The cash-in-advance constraints give rise to a money market. We use this model to study the effect of alternative monetary policies in the equilibrium allocation following a shock to financial intermediation.

An essential role of financial markets is to reallocate capital from wealthy individuals with no profitable investment project - savers - to individuals with profitable projects

¹Compare it with 1987, 2000 (Y2K) and 2001 (9-11).

²See Diaz-Alejandro (1985) for a very interesting view on the subject.

³For a detailed discussion of this, and a reinterpretation of the evidence that strongly favors the view of a stable "money demand" relationship, see Lucas and Nicolini (2013).

and no wealth - investors. The efficiency of these markets determines the equilibrium allocation of physical capital across projects and therefore equilibrium intermediation and total output.

The development literature has study models of the financial sector with these properties, the key friction being an exogenous collateral constraint on investors. We borrow the model of financial markets from that literature. The equilibrium allocation critically depends on the nature of the collateral constraints; the tighter the constraints, the less efficient the allocation of capital and the lower are total factor productivity and output. A tightening of the collateral constraint creates disintermediation and a recession. This reduction in the ability of financial markets to properly perform the allocation of capital across projects, we interpret as a negative financial shock. We view this as a sector specific technology shock with, as we will argue, aggregate consequences.

The single modification we introduce to this nowadays standard model in the development literature, is a cash-in-advance constraint on households. While we briefly discuss the case with nominal wage rigidity at several points, we assume prices and wages to be fully flexible, mostly to highlight the effects that are novel in the model. Monetary policy determines equilibrium inflation and nominal interest rates and it has real effects. This is so because of the usual well understood distortionary effects of inflation in a cash-credit world, but, more importantly, because of the zero bound on nominal interest rates restriction that arises from optimizing behavior of individuals in the model. The analysis of the effect of monetary policy at the zero bound is the contribution of this paper.

One attractive feature of the financial sector model we use, is that in the recession that follows a shock to the collateral constraint, the equilibrium real interest rate goes down for several periods.⁴ The reason is that savings must be reallocated to lower productivity entrepreneurs, but they will only be willing to do it for a lower interest rate. To put it differently, the "demand" for loans falls, and this pushes down the real interest rate. If the shock is large enough, the equilibrium interest rate becomes negative. Depending on what monetary policy does, the bound on the nominal interest rate may become a bound on the real interest rate. As an example, imagine a monetary policy that targets, successfully, a constant price level: If inflation is zero, the Fisher equation, which is an equilibrium condition of the model, implies a zero lower bound on the real interest rate. The way the negative real interest rate interacts with the zero bound on nominal interest rates that arises in a monetary economy, under alternative monetary policies, is at the hearth of the mechanism discussed in the paper.

The qualitative properties of the recession generated by a tightening of the collateral constraint, or credit crunch, are in line with some of the events that unfolded since 2008, like the persistent negative real interest rate, the sustained periods with the effective zero bound on nominal interest rates and the substantial drop in investment.

⁴This feature is special of the credit crunch. If the recession is driven by an equivalent, but exogenous, negative productivity shock, the real interest rate remains positive, as we will show.

In addition, the model is consistent with the very large increases in liquidity while the zero bound binds. Thus, we argue, the model in the paper is a good candidate to interpret the way monetary policy is affecting the economy nowadays. Some - but not all! - features of this great contraction make it a - distant - cousin of the great depression of the 30's. The great depression evolved in parallel to a mayor banking crisis and the severity of the depression was unique in US history. The role of monetary policy has also been at the center of the debate: For many, the unresponsive Fed played a key role in the unfolding of events during the great depression.⁵ A strongly held view attributes the reaction of the Fed in September 2008 to the lessons that Friedman and Schwartz draw from the great depression and attributes to the policy reaction the avoidance of an even major recession.

We first study the case in which the monetary authority is irresponsive to the credit crunch and does not change policy. The model implies that the nominal interest rate will be at its zero bound for a finite number of periods and there will be a deflation on impact. To the extent that debt obligations are in nominal terms, this deflation strongly accentuates the recession well beyond the one generated by the credit crunch, due to a debt deflation problem. We then study active inflation targeting policies, for low values of the inflation target. In these case, the deflation and the associated debt deflation problem are avoided by a very large increase in the supply of government liabilities, that must follow the credit crunch. Was the different monetary policy recently adopted, the reason why the great contraction was much less severe than the great depression? Our model suggests this may well be the case.⁶

The number of periods that the economy will be at the zero bound and the amount of liquidity that must be injected depends on the target for the rate of inflation. The evolution of output critically depends on this too. As we mentioned above, the interaction between the inflation target and the zero bound on nominal interest rates is the key to understand the mechanism. Imagine, as before, that the target for inflation is zero. Thus, the Fisher equation plus the zero bound constraint imply that the real interest rate cannot be negative. This imposes a floor on how low can the real interest rate be. But for this to be an equilibrium, private savings must end up somewhere else: This is the role of government liabilities. In this heterogeneous credit-constraint agents model, debt policy does have effects on equilibrium interest rates, even if taxes are lump-sum. Thus, the issuance of government liabilities crowds out private investment.

But there is an additional effect of monetary policy. In the model, a credit crunch generates a recession because total factor productivity falls. The reason, as we mentioned above, is that capital needs to be reallocated from high productivity entrepreneurs

⁵See Friedman and Schwartz (1963)

⁶Friedman and Schwartz argued that the Fed should have increased substantially its balance sheet in order to avoid the deflation during the great depression. In 2002, Bernanke, then a Federal Reserve Board governor, said in a speech in a conference to celebrate Friedman's 90th birthday, "I would like to say to Milton and Anna: Regarding the Great Depression. You're right, we did it. We're very sorry. But thanks to you, we won't do it again." Bernanke's speech has been published in *The Great Contraction, 1929-1933: (New Edition)* (Princeton Classic Editions), 2008.

for which the collateral constraint binds, and therefore must de-leverage, to low productivity entrepreneurs for which the collateral constraint does not bind. As a result of the drop in productivity, output and investment fall, at the same time that financial intermediation shrinks. Therefore, by keeping real interest rates high, an inflation targeting policy leaves the most unproductive entrepreneurs out of production, increasing average productivity. Thus, a target for inflation, if low, implies that the drop in productivity will be lower than in the real economy benchmark - there will be less reallocation of capital to low productivity workers - but the recession will be more prolonged - capital accumulation falls because of the crowding out effect. If the target for inflation is higher, say 1%, then the effective lower bound on the real interest rate is -1%, lower than before. Thus, the amount of government liabilities that must be issued will be smaller, the crowding out will be smaller, but the drop in average productivity will be higher.

The model provides an interpretation of the after 2009 events that is different from the one provided by a branch of the literature that, using New Keynesian models, places a strong emphasis on the interaction between the zero bound constraint on nominal interest rates and price rigidities.⁷ This is also the dominant view of monetary policy at major central banks, including the Fed. According to this view, a shock - often associated to a shock in the efficiency of intermediation⁸ - drove the natural real interest rate to negative values. The optimal monetary policy in those models is to set the nominal interest rate equal to the natural real interest rate. However, due to the zero bound, that is not possible. But it is optimal, unambiguously, to keep the nominal interest rate at the zero bound, as the Fed has been doing for over 4 years now. The model we study stresses a different and novel trade-off between ameliorating the initial recession and delaying the recovery.⁹ When the central bank chooses a lower inflation target, the real interest rate is constrained to be higher, and therefore, there is less reallocation of capital toward less productive, and previously inactive, entrepreneurs. The counterpart of the milder drop in TFP is a drop in investment due to the crowding out, leading to a substantial and persistence decline in the stock of capital.

As in the recent experience, to maintain price stability during a credit crunch the government needs to expand dramatically the size of its liabilities. In a credit crunch the capacity of productive entrepreneurs to supply bonds is reduced, resulting in an excess demand of saving instruments by unproductive entrepreneurs and workers. This is, obviously, a demand for real assets. Without any policy intervention, the adjustment

⁷See Christiano et al. (2011), Correia et al. (forthcoming), Eggertsson (2011), Eggertsson and Woodford (2003), Krugman (1998), Werning (2011) and references therein.

⁸See for example Curdia and Eggertsson (2009), Drautzburg and Uhlig (2011), and Galí et al. (2011).

⁹We consider a model with flexible prices in order to more clearly focus on the novel mechanisms in our paper. However, it is straightforward to extend our analysis and allow for wage rigidity. Nominal wage rigidity would make the recession even stronger in the case of the unresponsive monetary policy due to the ensuing deflation. The effect of nominal wage rigidity will be small in the case of inflation targeting. In any case, the trade-off discussed in the text will remain.

must come about through a deflation. The reason is that a negative real interest rate, together with the zero bound, implies that inflation must be positive. But without any policy intervention, the price level in the steady state must be the same as the initial price level. The initial drop in the price level (initial deflation) allows for a positive inflation along the path, with a zero "long run" inflation. To avoid the deflation, policy must restrict the real interest rate to become negative. Thus, the government must increase the supply of money - or government bonds, which at the zero bound are perfect substitutes, to satisfy the excess demand of mediums to "store value". This increase in the supply of government bonds induces a further increase in the demand of these bonds by unconstrained entrepreneurs, as these agents save in anticipation of the higher taxes that will be raised to pay the interest of this debt.

The paper proceeds as follows. In Section 2, we present the model and solve the individual's problems. In Section 3, define an equilibrium and partially characterize the equilibrium dynamics. In Section 4, we solve numerically the model under alternative monetary policies and discuss the results. We discuss the distribution of welfare consequences of alternative monetary policies in Section 5

2 The Model

In this section we describe the model, that follows closely the framework in Moll (2012). The model's attractive feature for our purposes is that it explicitly deals with heterogeneous agents that are subject to exogenous collateral constraints in a relatively tractable fashion. This model has been used to characterize the aggregate implications of a credit crunch, generated by a drop in the amount of credit that can be obtained from each unit of capital posed as collateral (Buera and Moll, 2012).

We modify the original model by imposing a cash-in-advance constraint on consumer's decision problem, so we can determine the aggregate price level and the nominal interest rate. We can therefore study the effect of alternative monetary policies on equilibrium quantities following a credit crunch.

We analyze a deterministic economy and assume that starting at the steady state, at time zero all agents learn that the collateral will be tightened for a finite number of periods. We then explore the effects of alternative monetary policies. In each case, we assume agents have perfect foresight regarding the evolution of the collateral constraint and of monetary policy.

2.1 Households

All agents have identical preferences, given by

$$\sum_{t=0}^{\infty} \beta^t [\nu \log c_{1t}^i + (1 - \nu) \log c_{2t}^i] \tag{1}$$

where c_{1t}^i and c_{2t}^i are consumption of the cash good and of the credit good, for agent i at time t , and $\beta < 1$. The assumption of logarithmic preferences implies a simple expression for the agents wealth accumulation decision.

Each agent also faces a cash-in-advance constraint of the type

$$c_{1t}^i \leq \frac{m_t^i}{p_t}. \quad (2)$$

where m_t^i is the beginning of period money holdings and p_t is the money price of consumption at time t .

The economy is inhabited by two classes of agents, a mass L of workers and a mass 1 of entrepreneurs, which are heterogeneous with respect to their productivity $z \in Z$. We assume that the productivity is constant through their lifetime. We let $\Psi(z)$ be the measure of entrepreneurs of type z . Every period, each entrepreneur must chose, for the following period, to be an active entrepreneur (to operate a firm as a manager) or to be a passive one (and offer his wealth in the credit market). We proceed now to study the optimal decision problems of agents.

Entrepreneurs There are three state variables for each entrepreneur, her financial wealth - capital plus bonds -, money holdings, and the occupational choice (active or passive) made last period. She must decide the labor demand if active, how much to consume of each good, whether to be active in the following period, and if so, how much capital to invest in her own firm. An entrepreneur's investment is constraint by her financial wealth at the end of the period a and the amount of bonds she can sell $-b$, $k \leq -b + a$, where we assume that decision to be limited by a simple collateral constraint of the form

$$-b^i \leq \theta k^i, \quad (3)$$

for some exogenously given $\theta \in [0, 1)$.

If the entrepreneur decides not to be active (to allocate zero capital to her own firm), then she invest all her non-monetary wealth to purchase bonds.

We assume that the technology available to entrepreneurs of type z is given by the Cobb-Douglas form¹⁰

$$y = (zk)^{1-\alpha} l^\alpha.$$

This technology implies that revenues of an entrepreneur net of labor payments is a linear function of the capital stock, ϱzk , where $\varrho = \alpha ((1 - \alpha)/w)^{(1-\alpha)/\alpha}$ is the return to the effective units of capital zk , and w denotes the real wage.¹¹ Thus, the end

¹⁰The Cobb-Douglas form, together with the log utility imply that aggregate quantities behave as in Solow's model. See Moll (2012).

¹¹The details are in Appendix 1.

of period investment and leverage choice of entrepreneurs with ability z solves the following linear program

$$\begin{aligned} \max_{k,d} \quad & \varrho zk + (1 - \delta)k + (1 + r)b \\ & k \leq a - b, \\ & -b \leq \theta k. \end{aligned}$$

Denoting the maximum leverage by $\lambda = 1/(1 - \theta)$, it is straightforward to show that the optimal capital and leverage choice are given by the following policy rules, with a simple threshold property¹²

$$k(z, a) = \begin{cases} \lambda a, & z \geq \hat{z} \\ 0, & z < \hat{z} \end{cases}$$

and

$$b(z, a) = \begin{cases} -(\lambda - 1)a, & z \geq \hat{z} \\ a, & z < \hat{z} \end{cases}$$

where \hat{z} solve

$$\varrho \hat{z} = r + \delta.$$

Given entrepreneurs' optimal investment and leverage decisions, they would face a linear return to their non-monetary wealth that is a simple function of their productivity

$$R(z) = \begin{cases} \lambda(\varrho z - r - \delta) + 1 + r, & z > \hat{z} \\ 1 + r, & z \leq \hat{z} \end{cases}$$

Given these definitions, the budget constraint of entrepreneur i , with net-worth a_t^i and productivity z^i , will be given by

$$c_{1t}^i + c_{2t}^i + a_{t+1}^i + \frac{m_{t+1}^i}{p_t} = R_t(z^i)a_t^i + \frac{m_t^i}{p_t} - T_t(z^i), \quad (4)$$

where we allow lump-sum taxes (transfers if negative) to be a function of the – exogenous – productivity of entrepreneurs.

Note that these budget constraints imply that agents choose, at t , money balances m_{t+1}^i for next period, as the cash-in-advance constraints (2) make clear. Thus, we are adopting the timing convention of Svensson (1985), in which goods markets open in

¹²Details in Appendix 1.

the morning and asset markets open in the afternoon. Thus, agents buy cash goods at time t with the money holdings they acquired at the end of period $t - 1$. Similarly, production by entrepreneurs at time t is done with capital goods accumulated at the end of period $t - 1$. An advantage of this timing for our purposes is that it treats all asset accumulation decisions symmetrically, using the standard timing from capital theory.¹³

Workers Workers are all identical and are endowed with a unit of time that they inelastically supply to the labor market. Thus their budget constraints are given by

$$c_{1t}^W + c_{2t}^W + a_{t+1}^W + \frac{m_{t+1}^W}{p_t} = (1 + r_t)a_t^W + w_t + \frac{m_t^W}{p_t} - T_t^W \quad (5)$$

where a_{t+1}^W and m_{t+1}^W are real financial assets and nominal money holdings chosen at time t , and T_t^W are lump-sum taxes paid to the government. If $T_t^W < 0$, these represent transfers from the government to workers. We impose on workers a non-borrowing constraint, so $a_t^W \geq 0$ for all t .¹⁴

2.2 Demographics

The decision rules of entrepreneurs imply that the wealth of active entrepreneurs increases over time, while that of inactive entrepreneurs converges to zero. Thus, each active entrepreneur saves away from the collateral constraint asymptotically. In order for the model to have a non-degenerate distribution of wealth across productivity types in a steady state, we assume that a fraction $1 - \gamma$ of entrepreneurs die and are replaced by equal number of new entrepreneurs. The productivity z of the new entrepreneurs is drawn from the same distribution $\Psi(z)$, i.i.d across entrepreneurs and over time. We assume that there are no annuity markets and that each new entrepreneur inherits the assets of a randomly drawn dying entrepreneur. Agents do not care about future generations, so if we let $\hat{\beta}$ be the pure discounting factor, they discount the future with the compound factor $\beta = \hat{\beta}\gamma$, which is the one we used above.

2.3 The Government

In every period the government chooses the money supply M_{t+1} , issues one-period bonds B_{t+1} , and uses type specific lump-sum taxes (subsidies) $T_t(z)$ and T_t^W . Government policies are constrained by a sequence of period by period budget constraints

$$B_{t+1} - (1 + r_t)B_t + \frac{M_{t+1}}{p_t} - \frac{M_t}{p_t} + \int T(z)_t \Psi(dz) + T_t^W = 0. \quad (6)$$

¹³This assumption implies that unexpected changes in the price level have welfare effects, since agents cannot replenish cash balances till next period.

¹⁴This is a natural constraint to impose. It is equivalent to impose on workers the same collateral constraints entrepreneurs face; since workers will never decide to hold capital in equilibrium.

We denote by T_t the total taxes receipts of the government,

$$T_t = \int T_t(z)\Psi(dz) + T_t^W.$$

In representative agent models, monetary policy can be executed via lump-sum taxes and transfers that, because those models satisfy Ricardian equivalence, are neutral. However, in this model, Ricardian equivalence will not hold for three related reasons. First, agents face different rates of return to their wealth. Thus, the present value of a given sequence of taxes and transfers differs across agents. Second, lump-sum taxes and transfers will redistribute wealth in general, and these redistributions do affect aggregate allocations, due to the presence of the collateral constraints. In the numerical sections, we will be explicit regarding the type of transfers we consider and the effect they have on the equilibrium allocation.

2.4 Optimality conditions

The optimal problem of agents is to maximize (1) subject to (2) and (4) for entrepreneurs or (5) for workers. Note that the only difference between the two budget constraints is that entrepreneurs have no labor income. For workers, as for inactive entrepreneurs, the return to their non-monetary wealth equals $1 + r_t$. In what follows, to save on notation, we drop the index for individual entrepreneurs i unless strictly necessary. Since this is a key aspect of the model, we first briefly explain the zero bound equilibrium restriction on the nominal interest rate that arises from the agent's optimization problem.¹⁵ Then, we discuss the other first order conditions.

In this economy, gross savings come from inactive entrepreneurs and, potentially, from workers. Note that the return of holding financial assets for these agents is $R_t(z) = (1 + r_t)$, while the return of holding money - ignoring the liquidity services - is given by p_t/p_{t+1} . Thus, if there is intermediation in equilibrium, the return of holding money cannot be higher than the return of holding financial assets. If we define the nominal return as $(1 + r_t) \frac{p_{t+1}}{p_t}$, then for intermediation to be non-zero in equilibrium, the zero bound constraint

$$(1 + r_t) \frac{p_{t+1}}{p_t} - 1 \geq 0 \tag{7}$$

must hold for all t .

The first order conditions of household's problem imply the standard Euler equation

$$\frac{1}{\beta} \frac{c_{2t+1}}{c_{2t}} = R_{t+1}(z), \quad t \geq 0, \tag{8}$$

and intra-temporal optimality condition between cash and credit goods

$$\frac{\nu}{1 - \nu} \frac{c_{2t+1}}{c_{1t+1}} = R_{t+1}(z) \frac{p_{t+1}}{p_t}, \quad t \geq 1. \tag{9}$$

¹⁵Formal details are provided in Appendix 1.

which requires that the marginal rate of substitution between cash and credit goods at time $t + 1$ equals their relative price, given by the agent specific nominal return of net-worth.

Solving forward the period budget constraint (4), using the optimal conditions (8) and (9) for all periods, and assuming that the cash-in-advance is binding at the beginning of period $t = 0$, we obtain the following solutions for consumption of the credit good and financial assets for agents that face a strictly positive opportunity cost of money in period $t + 1$,¹⁶

$$c_{2t} = \frac{(1 - \nu)(1 - \beta)}{1 - \nu(1 - \beta)} \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \right]$$

and

$$a_{t+1} = \beta \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)}.$$

These equations always characterize the solution for active entrepreneurs. The reason is that for them, the opportunity cost of holding money is given by $R_t(z)p_{t+1}/p_t > (1 + r_t)p_{t+1}/p_t \geq 1$, where the last inequality follows from the zero bound condition (7). Thus, the cash-in-advance constraint is always binding for active entrepreneurs from time $t = 1$ on, even when nominal interest rates are zero.

The solution also characterizes the optimal behavior of inactive entrepreneurs, as long as $(1 + r_t)p_{t+1}/p_t - 1 > 0$. The solution for inactive entrepreneurs in periods in which the nominal interest rate is zero, $(1 + r_t)p_{t+1}/p_t - 1 = 0$, is

$$a_{t+1} + \frac{m_{t+1}}{p_t} - \frac{m_{t+1}^T}{p_t} = \beta \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)}.$$

where

$$\frac{m_{t+1}^T}{p_t} = \frac{\nu(1 - \beta)\beta}{1 - \nu(1 - \beta)} \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] \quad (10)$$

¹⁶Note that it could be possible that initial money holdings are so large for an active entrepreneur, that the cash-in-advance constraint will not be binding the first period. This case will not be relevant provided initial real cash balances satisfy the following condition:

$$\frac{m_0}{p_0} \leq \frac{\nu(1 - \beta)}{1 - \nu(1 - \beta)} \left[R_0(z)a_0 - \sum_{j=0}^{\infty} \frac{T_j(z)}{\prod_{s=1}^j R_s(z)} \right].$$

If this condition is not satisfied, then the optimal policy for period $t = 0$ is to consume a fraction $\nu(1 - \beta)$ and $(1 - \nu)(1 - \beta)$ of the present value of the wealth, inclusive of the initial real money balances, in cash and credit goods. Similarly, the non-monetary wealth and real money holding at the end of the first period are functions of the present value of the wealth, inclusive of the initial real money balances.

are the real money balances that will be used for transaction purposes in period $t + 1$, and $m_{t+1}/p_t - m_{t+1}^T/p_t \geq 0$ are the excess real money balances, hoarded from period t to $t + 1$. Notice that for active entrepreneurs, $R_{t+1}(z)p_{t+1}/p_t > 0$, so $m_{t+1}^T = m_{t+1}$.

The optimal plan for workers is slightly more involved, as their income is non-homogeneous in their net-worth and they will tend to face binding borrowing constraints in finite time. In particular, as long as the $(1 + r_\infty)\beta < 1$, as it will be the case in the equilibria we will discuss, where r_∞ is the real interest rate in the steady state, workers drive their wealth to zero in finite time, and are effectively hand-to-mouth consumers in the long run. That is, for sufficiently large t ,

$$c_{2,t}^W = \frac{1 - \nu}{1 - \nu(1 - \beta)}(w_t - T_t^W)$$

and

$$c_{1,t+1}^W = \frac{m_{t+1}^W}{p_{t+1}} = \frac{\nu\beta}{1 - \nu(1 - \beta)} \frac{p_t}{p_{t+1}}(w_t - T_t^W).$$

Along a transition, workers may accumulate assets for a finite number of periods. This would typically be the case if they expect a future drop in their wages, or they receive a temporarily large transfer, $T_t^W < 0$.

3 Equilibrium

Given sequences of government policies $\{M_t, B_t, T_t\}_{t=0}^\infty$ and collateral constraints $\{\theta_t\}_{t=0}^\infty$, an equilibrium is given by sequences of prices $\{r_t, w_t, p_t\}_{t=0}^\infty$, and corresponding quantities such that:

- Entrepreneurs and workers maximize, taking as given $\{r_t, w_t, p_t\}_{t=0}^\infty$,
- The government budget constraint is satisfied, and
- Bond, labor, and money markets clear

$$\int b_{t+1}^i di + b_t^W + B_{t+1} = 0, \quad \int l_t^i di = 1, \quad \int m_t^i di + m_t^W = M_t, \quad \text{for all } t.$$

To illustrate the mechanics of the model, we first provide a partial characterization of the equilibrium dynamics of the economy for the case in which the zero lower bound is never binding, $1 + r_{t+1} > p_t/p_{t+1}$ for all t , workers are hand-to-mouth, $a_t^W = 0$ for all t , and the share of cash goods is arbitrarily small, $\nu \approx 0$. Then, we discuss some properties of the model when the zero bound constraint binds.

3.1 Equilibrium away from the zero bound.

Let $\Phi_t(z)$ be the measure of wealth held by entrepreneurs of productivity z at time t . Integrating the production function of all active entrepreneurs, equilibrium output is given by a Cobb-Douglas function of aggregate capital K_t , aggregate labor L , and aggregate productivity Z_t ,

$$Y_t = Z_t K_t^\alpha L^{1-\alpha} \quad (11)$$

where aggregate productivity is given by the wealth weighted average of the productivity of active entrepreneurs, $z \geq \hat{z}_t$,

$$Z_t = \left(\frac{\int_{\hat{z}_t}^{\infty} z \Phi_t(dz)}{\int_{\hat{z}_t}^{\infty} \Phi_t(dz)} \right)^\alpha. \quad (12)$$

Note that Z_t is an increasing function the cutoff \hat{z}_t and a function of the wealth measure $\Phi_t(z)$. In turn, given the capital stock at $t + 1$, that we discuss below, the evolution of the wealth measure is given by

$$\begin{aligned} \Phi_{t+1}(z) = & \gamma \left[\beta \left[R_t(z) \Phi_t(z) - \sum_{j=0}^{\infty} \frac{T_{t+j}(z) \Psi(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}(z) \Psi(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] \\ & + (1 - \gamma) \Psi(z) (K_{t+1} + B_{t+1}) \end{aligned} \quad (13)$$

where the first term reflects the decision rules of the γ fraction of entrepreneurs that remain alive, and the second reflects the exogenous allocation of assets of dead entrepreneurs among the new generation.

Then, given the - exogenous - value for λ_{t+1} and the wealth measure $\Phi_{t+1}(z)$ the cutoff for next period is determined by the bond market clearing condition

$$\int_0^{\hat{z}_{t+1}} \Phi_{t+1}(dz) = (\lambda_{t+1} - 1) \int_{\hat{z}_{t+1}}^{\infty} \Phi_{t+1}(dz) + B_{t+1}. \quad (14)$$

To obtain the evolution of aggregate capital, we integrate over the individual decisions and use the market clearing conditions. It results in a linear function of aggregate output, the initial capital stock, and the aggregate of the (individual specific) present value of taxes,

$$\begin{aligned} K_{t+1} + B_{t+1} = & \beta [\alpha Y_t + (1 - \delta) K_t + (1 + r_t) B_t] - \beta \sum_{j=0}^{\infty} \int_0^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) \\ & + \sum_{j=1}^{\infty} \int_0^{\infty} \frac{T_{t+j}(dz)}{\prod_{s=1}^j R_{t+s}(z)}. \end{aligned} \quad (15)$$

Solving forward the government budget constraint (6), using that $\nu \approx 0$, and substituting into (15)

$$\begin{aligned}
K_{t+1} &= \beta [\alpha Y_t + (1 - \delta)K_t] + (1 - \beta) \sum_{j=1}^{\infty} \int_0^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) \\
&\quad - (1 - \beta) \sum_{j=1}^{\infty} \frac{\int T_{t+j}(z) \Psi(dz) + T_{t+j}^W}{\prod_{s=1}^j (1 + r_{t+s})}.
\end{aligned} \tag{16}$$

The first term gives the evolution of aggregate capital in an economy without taxes. In this case, aggregate capital in period $t + 1$ is a linear function of aggregate output and the initial level of aggregate capital. The evolution of aggregate capital in this case is equal to the accumulation decision of a representative entrepreneur (Moll, 2012; Buera and Moll, 2012). The second term captures the effect of alternative paths for taxes, discounted using the type-specific return to their non-monetary wealth, while the last term is the present value of taxes from the perspective of the government. For instance, consider the case in which the government increases lump-sum transfers to entrepreneurs in period t , financing them with an increase in government debt, and therefore, with an increase in the present value of future lump-sum taxes. In this case, future taxes will be discounted more heavily by active entrepreneurs, implying that the last term is bigger than the second. Thus, this policy results in a lower aggregate capital in period $t + 1$.

Finally, we describe the determination of the price level. In the previous derivations, in particular, to obtain (16), we have used that $\nu \approx 0$, and therefore, the money market clearing condition is not necessarily well defined.¹⁷ More generally, given monetary and fiscal policy, the price level is given by the equilibrium condition in the money market

$$\begin{aligned}
\frac{M_{t+1}}{p_t} &= \frac{\nu(1 - \beta)\beta}{1 - \nu(1 - \beta)} \left[\alpha Y_t + (1 - \delta)K_t + (1 + r_t)B_t \right. \\
&\quad \left. - \sum_{j=0}^{\infty} \int_0^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) \right].
\end{aligned} \tag{17}$$

The nominal interest rate is obtained from the intertemporal condition of inactive entrepreneurs

$$\frac{1}{\beta} \frac{c_{2t+1}}{c_{2t}} = \frac{1 + i_t}{\frac{p_{t+1}}{p_t}} = 1 + r_t \tag{18}$$

Note that, except for the well known Sargent-Wallace initial price level indeterminacy result, and as is usual in cash in advance models, we can think of monetary policy as sequences of money supplies $\{M_t\}_{t=0}^{\infty}$, or sequences of nominal interest rates, $\{i_t\}_{t=0}^{\infty}$.

¹⁷To determine the price level in the cash-less limit we need to assume that as $M_{t+1}, \nu \rightarrow 0$, $M_{t+1}/\nu \rightarrow \tilde{M}_{t+1} > 0$.

We will think of policy as one of the two sequences and therefore abstract from the implementability problem.

There are two important margins in this economy. The first, is the allocation of capital across entrepreneurs, that is dictated by the collateral constraints and determines measured TFP (see (12)). The second, is the evolution of aggregate capital over time, which, in the absence of taxes, behaves as in Solow's model (see (16) and set $T_{t+j}(z) = 0$). Clearly, fiscal policy has aggregate implications: the net supply of bonds affects (14) and taxes affect (16). However, monetary policy does not, since none of those equations depend on nominal variables. Monetary policy does have effects, since it distorts the margin between cash and credit goods, but in a fashion that resembles the effects of monetary policy in a representative agent economy.

3.2 Equilibrium at the zero bound

In periods in which the zero bound binds, monetary policy can have a potentially large effect on the equilibrium quantities. The reason is that, by affecting inflation, monetary policy, together with the zero bound on nominal interest rates, can impose a bound on real interest rates. To see this, use (18) and the zero bound to write

$$1 + i_t = \frac{p_{t+1}}{p_t} (1 + r_t) \geq 1$$

which implies

$$r_t \geq \frac{p_t - p_{t+1}}{p_{t+1}} = - \left(\frac{\pi_{t+1}}{1 + \pi_{t+1}} \right) \quad (19)$$

where π_t

Imagine now an economy with zero net supply of bonds that enters a credit crunch, generated by a drop in maximum leverage, λ_t . Equation (14) implies that the threshold \hat{z}_{t+1} has to go down, to reduce the left hand side and increase the right hand side so as to restore the equilibrium. This drop in the supply of bonds will reduce the real interest rate so the marginal entrepreneurs that were lending capital, now start borrowing till market equilibrium is restored. If the credit crunch is large enough, the equilibrium real interest rate may become negative. If inflation is not high enough, the bound (14) may be binding. Imagine, for instance, the case of inflation targeting with a target equal to zero. Then, the real interest rate cannot become negative.

How will an equilibrium look like? In order to support the zero inflation policy the government needs to inject enough liquidity, so the net supply of bonds (or money, since they are perfect substitutes at the zero bound), so that condition (14) is satisfied. This policy will have implications on the equilibrium cutoff \hat{z}_{t+1} . In addition, as it can be seen in (15), the injection of liquidity (increases in B_{t+1}) affects capital accumulation. Thus, at the zero bound, the level of inflation chosen by the central bank, if low enough, can affect the two relevant margins in the economy.

To further explore this implications, we need to numerically solve the model. This, we do next.

4 Numerical Examples

In this section, we numerically solve the model to illustrate the way monetary policy interacts with the credit crunch. For all the experiments we consider, we start the economy at the steady state, and assume that in the first period, agents learn that there will be a deterministic credit crunch. By this, we mean that we assume that the parameter λ_t , that controls the tightness of the collateral constraint, goes down for a number of periods and then slowly goes back to the steady state level. All other parameters are kept constant.

Given this credit crunch, we consider two different scenarios for monetary policy. In the first one, we illustrate the interactions between real and nominal variables as a result purely of the credit crunch, in the absence of a policy response. In the second scenario, we assume that monetary policy is such that inflation is kept low and constant, at target values that are consistent with the typical mandates of Central Banks. To achieve the desired target, policy must be active, and we must be specific with respect to the accompanying debt and transfer policies. Thus, we consider alternative lump-sum tax and subsidy schemes that implement the given inflation target. We also study the effect of alternative inflation targets. We compare, in all cases, the evolution of the equilibrium with a benchmark case in a real economy with no government, i.e., one in which we set the parameter $\nu = 0$ and $T(z)_t = T_t^W = M_{t+1} = 0$. In the real economy there is no money so neither the zero lower bound nor the liquidity trap are relevant considerations.

The model has very few parameters, some of them can be assigned using standard aggregate targets. We set the time period to a quarter. On the production side, we set the capital share in output $\alpha = 1/3$ and the depreciation rate $\delta = 1 - (1 - 0.07)^{1/4}$. For preferences and the demographic structure we set the relative importance of the cash good $\nu = 0.5$, the discount factor $\beta = 0.986$ to match a quarterly interest rate of 0.005, and we set the survival rate $(1 - \gamma) = 0.9^{1/4}$. Finally, we set the leverage parameter $\theta = 0.75$ which implies $\lambda = 4$. The distribution of productivity z is assumed to be lognormal(0, 1). The assumptions that allow us to obtain relatively simple characterizations of individual's problem and aggregation, e.g., log utility and individual technologies with constant returns, make the model less suitable for a full quantitative analysis. For instance, since the model does not have a well-defined size distribution of entrepreneurs, we cannot use moments on the size distribution of establishments or firms to calibrate the distribution of productivities, nor to match the leverage of the economy. Therefore, the numerical examples that follow should be seen as illustrations of the model mechanisms.

4.1 Real Benchmark

As a benchmark, we first present the effects of a credit crunch in an economy without money, the one that obtains when setting the weight of cash goods to zero, $\nu = 0$, and assume that transfers and government liabilities are zero, $B_t = 0, T_t = 0$. The results

in this section follow closely those in Buera and Moll (2012).



Figure 1: Debt to Capital Ratio, $\theta_t = 1 - 1/\lambda_t$.

In Figure 1, we show the evolution of the exogenous driving force of the credit crunch, the debt to capital ratio θ_t , normalized to 1 at the first period. In Figure 2, we show the evolution of output, total factor productivity, Z_t , the capital stock and the real interest rate during a credit crunch (solid line), and compare them with the evolution of these variables following an “equivalent” exogenous TFP shock (dashed line).¹⁸

The immediate effect of the credit crunch is to reduce the amount of bonds that an active entrepreneurs can issue. This means that in the following period they will only be able to manage a lower amount of capital. But as the capital stock is given, some of it will be reallocated to previously inactive - and therefore less productive - entrepreneurs. This immediately lowers total factor productivity (top right panel) and therefore output. But for those entrepreneurs to find optimal to manage capital, the real interest rate has to go down (bottom right panel). The lower output implies that there are fewer resources for investment, and therefore, the capital stock drops below its steady state level (bottom left panel).

As shown in Buera and Moll (2012), the change in aggregate variables, with the exception of the interest rate, is the same in response to a credit crunch or to the corresponding exogenous TFP shock. The dynamics of TFP is identical by construction.

¹⁸In particular, we feed to the model an unanticipated exogenous TFP shock that replicates the evolution of the endogenous TFP during a credit crunch.

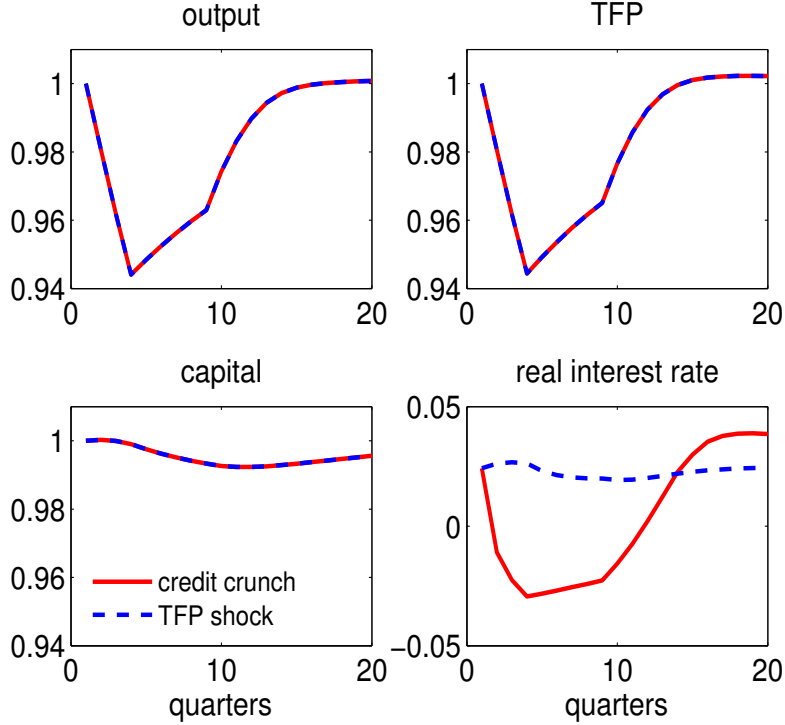


Figure 2: Aggregate Implications of a Credit Crunch: Real Benchmark

As can be seen by specializing equation (16) to the case $T_t = 0$, the evolution of aggregate capital is solely a function of the current level of the capital stock and aggregate output, which is itself only a function aggregate capital and TFP (see equation 11).

As can be seen in the bottom right panel of Figure 2, the drop in the interest rate is substantially more pronounced following a credit crunch, compared to the case of an exogenous TFP shock. As the supply of bond by productive entrepreneurs is further constrained during a credit crunch, the equilibrium interest rate must drop to clear the bond market. This force is not present in a contraction that is driven by an exogenous decline in TFP.

We would like to stress the effect of the credit crunch on the real interest rate. The New-Keynesian literature on the zero bound that represents the dominant view, assumes shocks to the discount factor, or study models where a credit crunch leads to increase in the demand of bonds (Guerrieri and Lorenzoni, 2011), in order to generate a negative “natural” rate of interest. While our model also generates a large drop in the real interest rate, the forces underlying this result are different. As previously discussed, in our framework the drop in the real interest rate is the consequence of a collapse in the ability of productive entrepreneurs to supply bonds, i.e., to borrow from the unproductive entrepreneurs and workers, as oppose to an increase in the demand

for bonds by these agents.

In our numerical exercises we choose a credit crunch - the values for θ_t - such that the equilibrium exhibits negative real interest rate for two years and such that it averages an annualized value of -2% , a value that was suggested in the literature mentioned above.

4.2 Nonresponsive monetary policy

We now show the equilibrium of the monetary model assuming that policy does not respond to the shock, so the quantity of money does not change. Note that while we focus on the case of money rules, in an equilibrium, given a money rule, we obtain a unique sequence of interest rates. One could therefore think of policies as setting those same interest rates.¹⁹ As there is no change in monetary policy, we do not need to change transfers either. We consider an economy with no public debt, and therefore, no taxes or transfers, $B_t = T_t = 0$ all t .

As shown in Figure 2, a credit crunch results in a large decline in the return of real assets. In a monetary economy, the return of real assets cannot be lower than the return of money. If they are the same, the economy is at the zero lower bound. If at the zero bound there is a further tightening of the collateral constraint, there will be an excess demand for “store of value”, leading in equilibrium to the hoarding of real money balances by inactive entrepreneurs, in excess to the ones needed for transaction purposes. As the supply of money is held fixed in this exercise, the price level must drop initially so that, in equilibrium, the supply of *real* balances meets the excess demand of real balances of inactive entrepreneurs. More precisely, for the periods in which agents hoard money, no arbitrage implies that the return of money – the inflation rate – should be equal to the return of bonds, $p_{t+1}/p_t - 1 = -r_{t+1}/(1+r_{t+1})$. Consequently, to be consistent with no arbitrage, in periods in which the return of bonds becomes “sufficiently” negative, the inflation rate must be particularly high. Therefore, the value of money in the first period should be high – the price level low – to compensate for the future low returns.²⁰

The response of the main variables in the nominal economy with a fixed money supply are illustrated in Figure 3. As discussed above and illustrated in the bottom right panel, there is a large deflation on impact and positive inflation afterwards as the supply of bonds by productive entrepreneurs recovers, and the excess demand for

¹⁹If one were to think of policy as setting a sequence of interest rates, the issue of price level determination should be addressed. The literature has adopted two alternative routes, the Taylor principle or the fiscal theory of the price level. We abstract from those implementation issues in this paper.

²⁰More formally, given a real allocation, the price level in the initial period is determined by the inflation rates in the periods in which the zero bound is binding, and the price level of the period in which the economy exits the zero bound (see equation 17). If the credit crunch is sufficiently anticipated, or in the cash-less limit, i.e., if $\nu, M \rightarrow 0$ with $M/\nu \rightarrow \tilde{M}$, the real interest rate, the wage rate, and the aggregate stock of capital are independent of the nominal side of the model, and this intuition is complete.

real balances slowly reverts to zero. The initial deflation increases the value of the money balances at the beginning of the initial period. The increase in the value of the initial money balances leads to an increase in aggregate consumption and, since capital, TFP, and therefore, output, are predetermined, it also leads to a decline in aggregate investment.

For relatively productive entrepreneurs, those with gross return to their net-worth $R_0(z) > p_1/p_0$, the cash-in-advance constraint in the initial period is binding, and the increase in the real value of money balances is solely spent in cash goods. For these individuals the evolution of their net-worth is not affected, but their total consumption increases. For relatively unproductive entrepreneurs and workers, the increase in the value of their initial cash balances is spent in cash and credit goods, and partially saved by hoarding real cash balances. The decline in investment is illustrated in the lower left panel of Figure 3.

Notice that the drop in aggregate investment in the monetary economy (solid line) is larger than the one in the real benchmark (dotted line), while the evolution of TFP (top right panel) is similar to its evolution in the real benchmark. The lower value for investment implies that the recession is deeper, but the overall effect on output is small.²¹

In the context of the model, this unexpected shock has relatively minor consequences. However, it suggests that a potential problem may arise to the extent that debt instruments are nominal obligations.²² If this were the case, a deflation would substantially increase the real value of the debt, making the collateral constraint even tighter.

4.2.1 Nominal Bonds

In order to explore this possibility, we solve the model assuming that entrepreneurs only issue nominal bonds. In particular, we assume that active entrepreneurs finance their investment by issuing one period nominal bonds. As before, the real value of bond issuance are restricted by the collateral constraint in (3).

The results, which are dramatically different, are depicted in Figure 4, which also plots both the benchmark (dotted line) and the case of indexed debt (dashed line). The recession is deeper and more persistent, driven mainly by a sharper decline in TFP. The intuition for the dramatic effect of the debt deflation is simple: The initial deflation implies a large redistribution from high productivity, leveraged entrepreneurs towards bondholders, who are inactive, unproductive entrepreneurs. The ability of productive entrepreneurs to invest is now hampered by both the tightening of collateral constraints and the decline of their net-worth. As a consequence, there needs to be a larger decline in the real interest rate so that in equilibrium more capital is reallocated

²¹Capital will be a third of a percentage point lower, but only 1/3 of the decline in the capital stock translates to output.

²²This “debt deflation” problem has been mentioned as one of the possible costs of deflations before, particularly in reference to the great depression (Fisher, 1933).

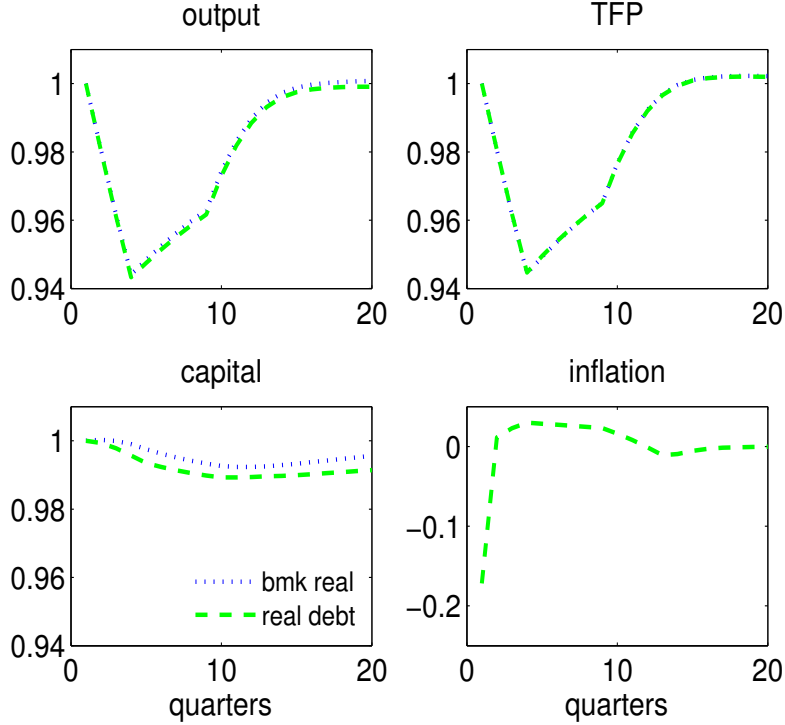


Figure 3: Aggregate Implications of a Credit Crunch: Constant Money

from productive to unproductive entrepreneurs (bottom left panel).

The discussion above suggests that the initial deflation can be very costly in terms of output, in the case in which debt is a nominal obligation. An obvious question is, then, what can monetary policy do, if anything, to stabilize the price level and output. We consider those cases next.

4.3 Inflation targeting

We now consider the case of a Central Bank whose objective is to implement an inflation target of $\pi = p_{t+1}/p_t - 1$ for all t . In order to implement the inflation target π , the government needs to adjust the supply of assets, i.e., real money balances M_{t+1}/p_t and government bonds B_{t+1} , and the associated lump-sum tax sequences $T_t(z)$, T_t^W , to accommodate changes in the desired demand for real money balances, and more importantly, to satisfy the excess demand for assets during a credit crunch. In particular, without loss of generality, we assume that the government sets the quantity of money to be equal to the money required by individuals to finance their purchases of cash good in every period, m_{t+1}^T , given by equation (10),

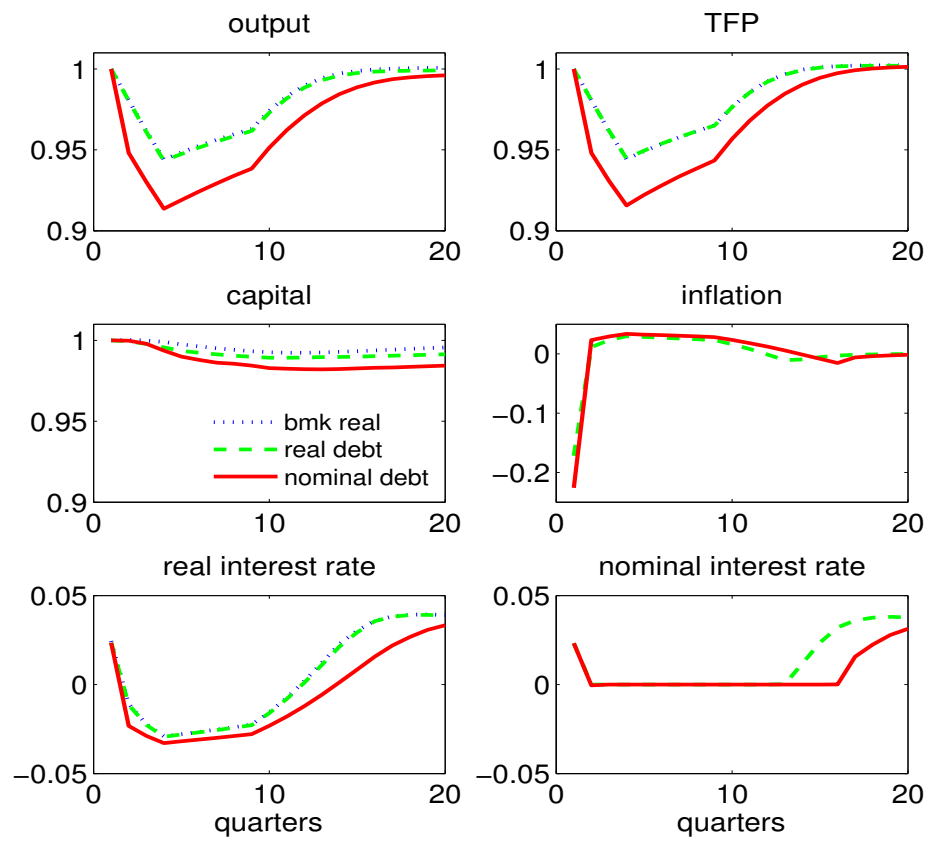


Figure 4: Aggregate Implications of a Credit Crunch: Constant Money

$$M_{t+1} = p_t \frac{\nu(1-\beta)\beta}{1-\nu(1-\beta)} \left[\int R_{t+1}(z) \Phi_{t+1}(dz) - \sum_{j=0}^{\infty} \int_0^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) \right],$$

and that the public debt accommodates the excess demand for bonds in periods where the real interest rate equals the constants return of money $r_{t+1} = -\pi/(1+\pi)$,

$$B_{t+1} = \begin{cases} B_t & \text{if } r_{t+1} > -\frac{\pi}{1+\pi} \\ \int_0^{\hat{z}_{t+1}} \Phi_{t+1}(dz) - (\lambda_{t+1} - 1) \int_{\hat{z}_{t+1}}^{\infty} \Phi_{t+1}(dz) & \text{if } r_{t+1} = -\frac{\pi}{1+\pi} \end{cases}. \quad (20)$$

Obviously, lump-sum taxes (subsidies) must be adjusted accordingly to satisfy the government budget constraint in (6).

These conditions fully determined the evolution of the money supply, government bonds, and the aggregate level of taxes (transfers), but they leave unspecified how taxes (transfers) are distributed across entrepreneurs and workers. We consider two simple cases: First, we present results for the case that taxes (transfers) are purely lump-sum, i.e., $T_t(z) = T_t^W = T_t$ for all t, z .²³ We refer to this as the “lump-sum” case. The second case that we consider is one where taxes (transfers) are purely lump-sum for all period with the exception of those when the government expands the supply of government bonds, i.e., $B_{t+1} > B_t$. In the periods when the government increases the supply of bonds, we assume that the proceeds from the sell bonds, net of interest payments, and the adjustment of the supply of real balances are only rebated to the entrepreneurs, in a lump-sum fashion. The second case captures an scenario in which the government responds to a credit crunch by bailing out productive entrepreneurs and bond holders. We refer to this as the “bailout” case.²⁴

The results for the case in which the government implements a constant inflation of 2%, a value in line with the price stability mandates of major Central Banks, are depicted in Figure 5. The solid line corresponds to the case of pure lump-sum taxes (transfer) while the dashed line shows the results for the case in which the government rebates the proceed of the sell of bonds only to entrepreneurs. For comparison, the dotted line shows again the results for the real benchmark.

Two salient patterns arise. First, to maintain price stability during a credit crunch the government needs to expand dramatically the size of its liabilities (center right panel). Second, when implementing a low inflation, and therefore, constraining the real interest rate to be higher, the government attains a less pronounce recession at the cost of a slower recovery.

²³In this section we need to specify the relative number of workers and entrepreneurs in the economy. We assume that workers are 25% of the population, $L/(1+L) = 1/4$. We choose a low share of workers, who in our model choose to be against their borrowing constraint in a steady state, to limit the non-Ricardian elements in the model.

²⁴The transfer to bond holders is consistent with the evidence presented by Veronesi and Zingales (2010) for the bail-out of the financial sector in 2008.

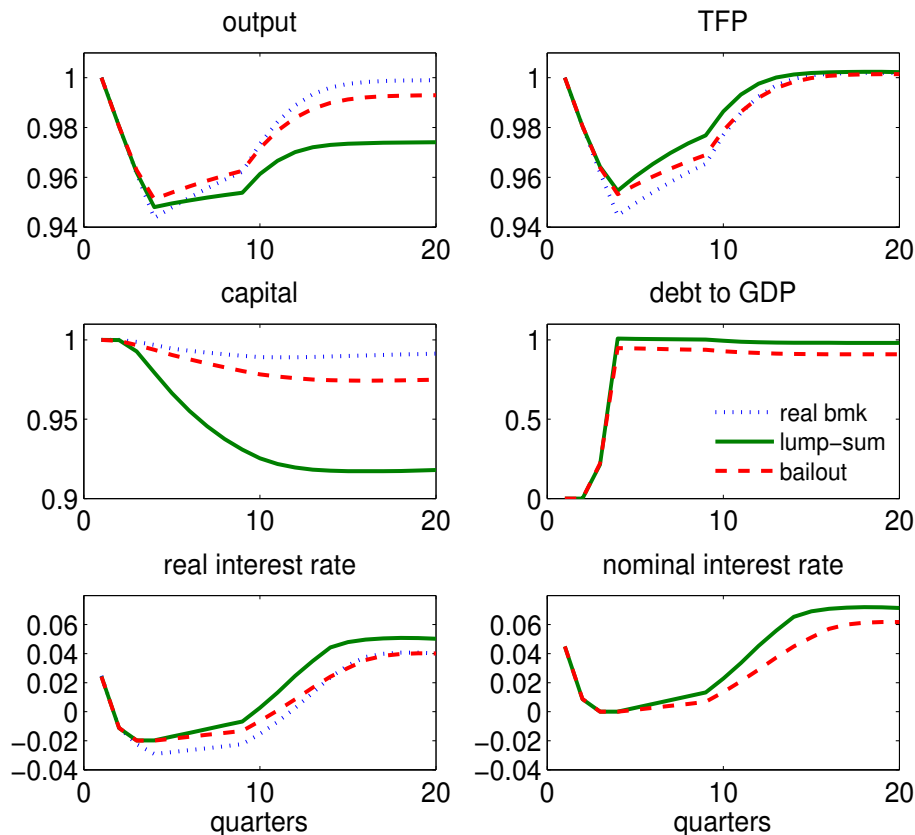


Figure 5: Aggregate Implications of a Credit Crunch with an Inflation Target: Alternative Subsidy Schemes, Inflation Target $\pi = 0.02$.

In a credit crunch the capacity of productive entrepreneurs to supply bonds is reduced, resulting in an excess demand of saving instruments by unproductive entrepreneurs and workers.²⁵ To avoid the deflation induced by the excess demand of mediums to serve as “store of value”, the government must increase the supply of government bonds or money, which at the zero bound are perfect substitute. Furthermore, the increase in the supply of government bonds induces a further increase in the demand of these bonds by unconstrained entrepreneurs, as these agents save in anticipation of the higher taxes that will be raised to pay the interest of this debt.

As the top left panel of Figure 5 shows, with this policy the government accomplishes a slightly less pronounced recession at the cost of significantly more protracted

²⁵In the real benchmark, at the beginning of the credit crunch workers accumulate assets as the credit crunch it anticipated one period in advance, and the lowest value of the collateral constraint is attained in the fourth quarter.

recovery. The milder recession is explained by the smaller drop in TFP. When the government maintains the inflation low, the real interest rate is constrained to be higher, and therefore, there is less reallocation of capital toward less productive, and previously inactive, entrepreneurs. The counterpart of the milder drop in TFP is a collapse in investment, leading to a substantial and persistence decline in the stock of capital.

In our framework Ricardian equivalence does not hold, and increases in government debt crowds out private investment. This is particularly true for the case in which the government uses pure lump-sum taxes (solid line). In this case, part of the transfers go to workers, who in equilibrium have a large marginal propensity to consume as they will be against their borrowing constraint in finite time.²⁶ Thus, when the government increases the supply of bonds, and transfers the proceeds of the sell of these bonds to households, aggregate consumptions increases, and investment decreases, relative to the real benchmark economy.

In comparison, the recovery is faster when the government rebates the proceeds from the increase in the debt solely to entrepreneurs (dashed line). Nevertheless, the drop in investment is still more pronounced than in the real benchmark. There are two reasons why Ricardian equivalence does not hold in this case. Firstly, productive entrepreneurs choose to consume part of the higher government transfers. Productive entrepreneurs discount future taxes at a rate that is higher than the interest rate, i.e., for infra-marginal entrepreneurs, $z > \hat{z}_{t+1}$, $R_{t+1}(z) > (1 + r_{t+1})$. Secondly, even inactive entrepreneurs, who discount the future at the same rate as the government, face initially a sequence of transfers and taxes that have a strictly positive net present value. This is because in this case entrepreneurs receive a disproportionately large share of transfers, while taxes are uniformly distributed among entrepreneurs and workers.

Can the government mitigate the consequences of a credit crunch by choosing alternative inflation targets? In particular, is it desirable that the government chooses a sufficiently high inflation target in order to avoid the zero lower bound? We explore this question in Figure 6. There we present the evolution of four economies differing in the level of the inflation target, $\pi = 0, 0.01, 0.02$, and 0.03 . In all these cases we assume that the government rebates the proceeds from the increase in the debt solely to entrepreneurs (bailout case).

The two main features of the previous examples are reinforced for the economies with a lower inflation target. The lower the inflation target is, the less pronounced the recession in the short run is. At the same time, the recovery is slower. Furthermore, the government will need a larger increase in the supply of bonds to implement a lower inflation target. The larger increase in the government debt will imply a larger crowd-out of investment. On the contrary, for a sufficiently large inflation target, $\pi = 0.03$ in our example, the government reproduces closely the equilibrium in the real benchmark economy. The dynamics of the nominal interest rate is common across these examples.

²⁶In a steady state the interest rate is strictly lower than the rate of time preferences, $(1 + r_\infty)\beta < 1$. Therefore, workers, who earn a flow of labor income each period, will choose to be against their borrowing constraint in finite time.

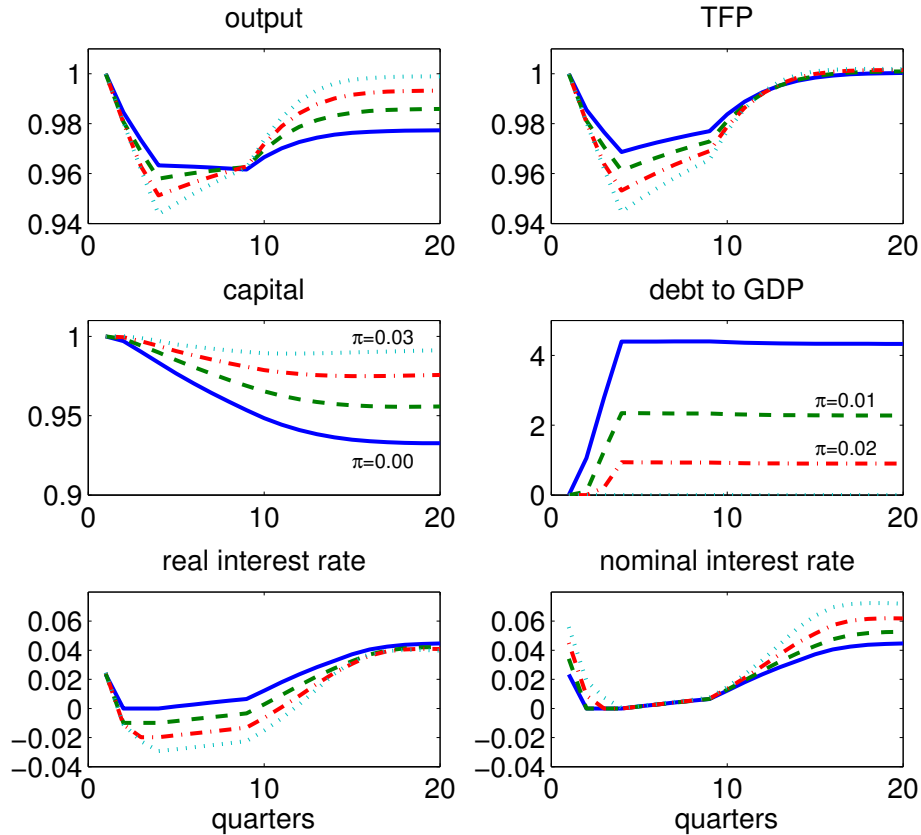


Figure 6: Aggregate Implication of a Credit Crunch with an Inflation Target: Alternative Inflation Targets, Bailout Case.

The nominal interest rate are at zero, or close to zero, for various quarters.

The case of a government implementing a low inflation seems attractive to interpret the Great Recession in the US. Following the 2008 crisis, the economy has been for several quarters at the zero bound, while the Fed has increased substantially its balance sheet. The Fed policy has been directed explicitly to provide the US economy with safe zero nominal interest rate money-like-assets, while inflation has been under total control. All these features are reproduced by this example. Moreover, there is a presumption that these policies avoided a more severe recession, although the recovery is seen as unusually slow. Again, a feature of the aggregate economy in this example.

5 Distribution of Welfare Impacts

In the previous section we focused on the impact of policies on aggregate outcomes and factor prices. The aggregate figures suggest a relatively simple trade-off at the aggregate level. These dynamics, though, hide very disparate effects of a credit crunch, and alternative monetary policies, among different agents. While workers are hurt by the drop in wages, the profitability of active entrepreneurs, and their welfare, increases as a result of lower factor prices. Similarly, unproductive entrepreneurs are bondholders in equilibrium, and therefore, are hurt by a decline in the real interest rate.

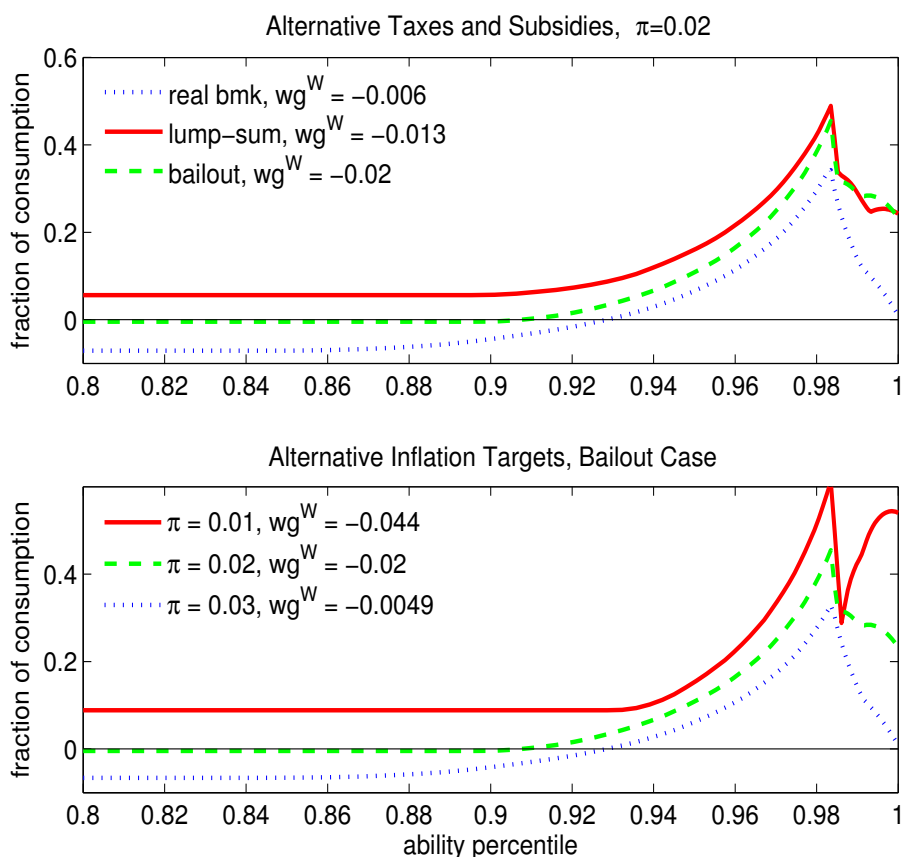


Figure 7: Distribution of Welfare Gains.

Figure 7 presents the impact in the welfare of entrepreneurs of different ability, and workers, of a credit crunch under alternative policy responses. We measure the welfare impact of a credit crunch in terms of the fraction of consumption that an individual is willing to permanently forgo to experience a credit crunch.²⁷ If positive (negative) we

²⁷For entrepreneurs, we consider the welfare of individuals that at the time of the shock have wealth

refer to this measure as the welfare gains (loses) from a credit crunch, and alternative policy responses.

The different curves show the welfare consequences of alternative policy responses. The difference between two curves gives the gains from alternative policies. The curves in the top panel correspond to the alternative tax schemes discussed in Figure 5, while the bottom panel correspond to the alternative inflation targets in Figure 6.

The dotted line in the top panel shows the welfare gains for entrepreneurs from experiencing a credit crunch in the real benchmark, as a function of the percentile of their ability distribution. Unproductive entrepreneurs are clearly hurt by a credit crunch, as the return of the bonds they hold becomes negative for over 10 quarters, and only gradually returns to the original steady state. Their loses amount to over 5% of permanent consumption. On the contrary, entrepreneurs who become active as the credit crunch lowers factor prices, and increase their profitability, benefit the most. Finally, the effect on the welfare of the most productive entrepreneurs is ambiguous, as they are favor by the lower factor prices, but are hurt by the tightening of collateral constraints, which limit their ability to leverage their high productivity.

The welfare loses for workers are shown by the legend of each curve. Clearly workers are hurt by experiencing a credit crunch, as the wages drop for a number of periods. The credit crunch amount to a permanent drop of over half a percentage point in their consumption, $wg^W = -0.006$.

The top panel of Figure 7 also shows the welfare gains of a credit crunch when the government implements an inflation target of 2%, under two alternative tax schemes. The solid line corresponds to the case in which the monetary policy is implemented with pure lump-sum taxes. The dashed line is the case in which the proceed of the sell of government bonds is rebated lump-sum only to entrepreneurs (bailout case).

A policy that implements a relatively low inflation target, leads to a higher real interest rate, resulting in a relatively larger welfare gains for relatively unproductive entrepreneurs who are bondholders. The welfare gains of unproductive entrepreneurs are at expense of workers, who do not hold bonds in the steady state, but end up paying higher taxes to finance the interest payment of the government debt. Intuitively, the welfare loses of workers are highest when the proceed of the sell of bonds is rebated solely to entrepreneurs, $wg^W = 0.02$, compare to the pure lump-sum case, $wg^W = 0.0013$.

The bottom panel shows the welfare consequences of alternative inflation targets, for the case in which the government rebates the proceed of the sell of bonds solely to entrepreneurs. The lowest the inflation target the highest the real interest rate, both during the credit crunch, and in the new steady state.²⁸ Unproductive entrepreneurs benefit from the highest interest rate. Similarly, productive entrepreneurs benefit from

equal to the average wealth of the economy. For workers, their welfare is calculated assuming, as is true in the steady state of the model, that they own no wealth when the credit crunch is announced.

²⁸Given the debt policy equation (20), the government debt in the new steady state will be highest the lowest the inflation target is. In the model, a higher level of government debt implies a lower level of capital in the new steady state.

the lowest wages associated with the lowest capital during the transition, and in the new steady state. Although individual entrepreneurs do not internalize it, collectively they benefit from the lower wages associated with a lower aggregate stock of capital.

6 Conclusions

TO BE WRITTEN.

7 Appendix

TO BE WRITTEN.

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