

Why Prices Don't Respond Sooner to a Prospective Sovereign Debt Crisis

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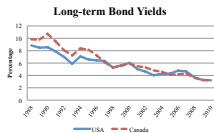
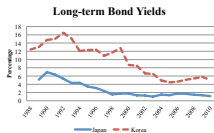
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Preliminary. Comments Welcome. These are our own personal views.

Fiscal problems



Resolutions

- It seems fairly obvious that the current level of deficit spending in Japan and U.S. cannot be sustained for ever.
- Possible resolutions:
 - ① Increase taxes (or reduce spending);
 - ② fail to increase taxes (sovereign debt crisis):
 - inflation?
 - payment suspension?
- Is the second scenario a realistic possibility?
 - If so, how can we explain the current state of Japan or the U.S. with
 - stable or declining prices;
 - high government-bond prices;
 - high yen rate, etc?

Our message

- The fact that inflation and bond yields are low today does **not** mean that the risk of a debt crisis is low.
- The current situations in Japan and the U.S. are perfectly consistent with the view that there is a non-negligible probability of a sovereign debt crisis.

How we make our point

- Consider two types of default (separately):
 - ① Implicit default via inflation (fiscal theory of the price level);
 - ② Explicit default on long-term government debt.
- Compare two specifications:
 - ① Frictionless asset markets (complete markets);
 - ② Financial frictions.
- Agents have heterogenous beliefs about the probability of default.
- Financial frictions are modeled as in Geanakoplos (2003,2010):
 - no contingent claims are traded;
 - agents can borrow to purchase govt debt;
 - govt debt cannot be short sold.

Properties of the model

- Complete markets:
 - Prices respond instantly to news about the possibility of a debt crisis.
 - Inflation smoothing.
- Financial frictions:
 - No response of price to news about the possibility of a debt crisis.
 - Price responses are concentrated in states immediately prior to default state.
- Key in our model with financial frictions:
 - Some individuals want to use leverage to purchase govt debt;
 - Others do not want to purchase govt debt by themselves, but are willing to lend to those who buy it.
- In reality, a large proportion of sovereign debt is held by leveraged financial institutions.

Related literature

- Our model builds on the following two strands of literature:
 - ① Fiscal theory of the price level (FTPL):
 - Leeper (1991); Sims (1994); Woodford (1994); Cochrane (2001); Bassetto (2002); etc.
 - ② Collateral, beliefs, and leverage:
 - Geanakoplos (1997, 2003, 2010); Fostel and Geanakoplos (2008); Geanakoplos and Zame (2009), Simsek (2010), etc.

Fiscal theory of the price level

- “Naive” assumptions made in the standard FTPL:
 - The government commits to a fixed sequence of real tax revenues, $\{T_t\}$.
 - Such a commitment is made both in and out of the equilibrium path.
 - The price level “adjusts” so that the govt budget constraint holds.
- Criticism against the FTPL by Bassetto (2002):
 - It is impossible to consider “out of the equilibrium path” in the Walrasian framework assumed in the FTPL.
 - Bassetto (2002) considers a market game and finds that a version of the FTPL holds.
- For simplicity, here we follow the naive version of the FTPL, but it is straightforward to build a market game similar to Bassetto’s for our model.

Theory of leverage by Geanakoplos

- Conditions of loans:
 - interest rate;
 - collateral;
 - collateral rate.
- How can the interest rate and the collateral rate be determined in markets simultaneously?
 - That is, how can **one** demand-equals-supply equation for a loan determine **two** variables — the interest rate and the collateral rate?
- Geanakoplos has developed a competitive-equilibrium framework determining the interest rate and the collateral rate for loans simultaneously.
 - The key is to consider loans with different collateral rates as different assets.
- In our model only one type of loans are traded in equilibrium, whose collateral rate is given by the ‘no-default constraint.’

Some evidence

- Reinhart and Rogoff (2010) find that the probability of sovereign debt crises goes up following:
 - banking crises;
 - sharp increases in government and external debt.
- Nieto Parra (2008):
 - Investment banks demand higher underwriting fees 1 to 3 years before debt crisis.
 - bond spreads do not respond to the news and remain stable up to the crisis.
- Lau (2003): Argentine CDS only increase about 2 months prior to IMF package and 15 months prior to default.
- Greek sovereign debt spread first increased to 2 percent in December 2008 about 16 months before their request for funds from EU/IMF.

Plan of the talk

- 1 Introduction
- 2 Implicit default: 2 period model
- 3 Implicit default: T -period model
- 4 Explicit default
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Model

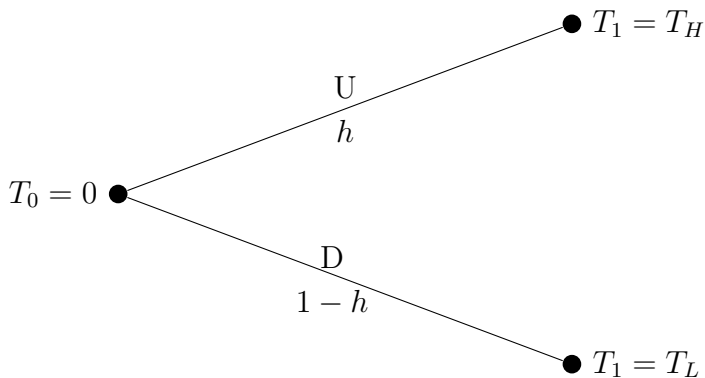
- Two periods: $t = 0, 1$.
- Two states at date 1: $s_1 \in \{U, D\}$.
 - Notation: $s^0 \in S^0 = \{0\}$ and $s^1 = s_1 \in S^1 = \{U, D\}$.
- More generally, s_t denotes the shock realized in period t and s^t denotes the history of shocks.
- States are distinguished by the amount of taxes:

$$T_1 = \begin{cases} T_H, & \text{if } s_1 = U, \\ T_L, & \text{if } s_1 = D \text{ (debt crisis)}. \end{cases}$$

where $T_L \ll T_H$.

- A continuum of agents $h \in [0, 1]$.
 - At date 0, agent h believes that $s_1 = U$ with probability h .
 - Agents are identical except for their beliefs.

Event tree



Individuals

- Preferences:

$$c_0 + \sum_{s^1 \in S^1} \gamma^h(s^1) c(s^1)$$

where $\gamma^h(s^1)$ = subjective probabilities given by

$$\gamma^h(s^1) = \begin{cases} h, & \text{for } s_1 = U, \\ 1 - h, & \text{for } s_1 = D. \end{cases}$$

- Endowments:
 - y_0 at date 0, and y_1 at date 1 (for all $s^1 \in S^1$).
- Storage technology:
 - Gross real rate of return = R (riskfree).

Government

- Flow budget constraint:

$$\bar{B} = P_0 T_0 + q_0 B_0,$$

$$B_0 = P(s^1) T(s^1), \quad \text{for } s^1 \in S^1.$$

where \bar{B} = initial amount of govt debt (nominal); B_0 = amount of govt bonds issued at date 0; $P(s^t)$ = price level at date-event s^t ; q_0 = nominal price of govt bonds at date 0.

- Monetary policy: the nominal interest rate, $\frac{1}{q_0}$.
- Fiscal policy: real amount of taxes collected in each period:

$$T_0 = 0,$$

$$T(s^1) = \begin{cases} T_H, & \text{if } s_1 = U, \\ T_L, & \text{if } s_1 = D. \end{cases}$$

Three market structures

- 1 “complete markets”
 - asset markets without frictions.
 - complete set of contingent claims with a “natural debt limit.”
- 2 “no borrowing”
 - two assets:
 - govt bonds, and storage.
 - frictions:
 - no borrowing;
 - no short sales of govt bonds.
- 3 “leverage”
 - three assets:
 - govt bonds, storage, and loans.
 - frictions:
 - agents can borrow to purchase govt debt;
 - borrowing is limited by the “no-default constraint.”
 - no short sales of govt debt.

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Complete Markets

- frictionless asset markets:
 - complete set of contingent claims (Arrow securities) are traded under the mildest possible debt limit (natural debt limit).
- Arrow security ($s^1|s^0$):
 - traded at s^0 and pays off one unit of account in period 1 iff s^1 occurs.
 - $b^h(s^1|s^0)$ = quantity of Arrow security ($s^1|s^0$) purchased by individual h .
 - $q(s^1|s^0)$ = price of Arrow security ($s^1|s^0$).
- Govt bonds pay one unit of account at every state in period 1.
 - No arbitrage condition:

$$q_0 = \sum_{s^1 \in S^1} q(s^1|s^0).$$

Individual h

- Utility maximization problem of agent h :

$$\max c_0 + \sum_{s^1 \in S^1} \gamma^h(s^1) c(s^1)$$

subject to

$$c_0 + k_0 + \sum_{s^1 \in S^1} q(s^1|s^0) \frac{b(s^1|s^0)}{P_0} + q_0 \frac{b_0}{P_0} \leq \frac{\bar{B}}{P_0} + y_0,$$

$$c(s^1) \leq y_1 - T(s^1) + \frac{b(s^1|s^0)}{P(s^1)} + \frac{b_0}{P(s^1)} + Rk_0, \quad s^1 \in S^1,$$

$$c_0, k_0, b_0, c(s^1) \geq 0.$$

Equilibrium with complete markets

- $c_0^h = 0$ for all $h \in [0, 1]$.
- “Marginal agent”: $h_0 = \frac{1}{2}$.
 - Pessimistic agents, $h \leq h_0$,

$$c^h(s^1) = \begin{cases} 0, & \text{for } s^1 = U, \\ \frac{1}{h_0}(Ry_0 + y_1), & \text{for } s^1 = D. \end{cases}$$

- Optimistic agents, $h > h_0$,

$$c^h(s^1) = \begin{cases} \frac{1}{1-h_0}(Ry_0 + y_1), & \text{for } s^1 = U, \\ 0, & \text{for } s^1 = D. \end{cases}$$

- Equilibrium prices:

$$\frac{\bar{B}}{P_0} = \frac{1}{R} \left\{ h_0 T_H + (1 - h_0) T_L \right\},$$

$$\frac{B_0}{P(s^1)} = T(s^1), \quad \text{for } s^1 \in S^1.$$

Equilibrium Trading Strategies

- Optimistic and pessimistic agents hold 'symmetric' portfolios:
 - Optimistic agents purchase Arrow security U , and sell Arrow security D .
 - Pessimistic agents do the opposite.
- The equilibrium price level will equally reflect the views of optimists and pessimists.

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Market structure

- Two assets:
 - govt bonds and storage.
- Frictions:
 - no short sales of govt bonds;
 - no borrowing.
- Budget set for each individual:

$$c_0 + k_0 + q_0 \frac{b_0}{P_0} \leq \frac{\bar{B}}{P_0} + y_0,$$

$$c(s^1) \leq Rk_0 + \frac{b_0}{P(s^1)} - T(s^1) + y_1, \quad \text{for } s^1 \in S^1,$$

$$c_0, k_0, b_0, c(s^1) \geq 0.$$

Utility maximization

- h_0 = marginal buyer of govt bonds.
- Pessimistic agents only invest in storage: For $h \leq h_0$,

$$c_0^h = b_0^h = 0,$$

$$k_0^h = \frac{\bar{B}}{P_0} + y_0,$$

$$c^h(s^1) = Rk_0^h + y_1 - T(s^1)$$

- Optimistic agents only invest in govt bonds: For $h > h_0$,

$$c_0^h = k_0^h = 0,$$

$$b_0^h = \frac{P_0}{q_0} \left(\frac{\bar{B}}{P_0} + y_0 \right),$$

$$c^h(s^1) = \frac{1}{P(s^1)} b_0^h + y_1 - T(s^1)$$

Equilibrium without borrowing

- h_0 = marginal buyer of govt bonds:

$$\frac{q_0}{P_0} = \frac{1}{R} \left\{ \frac{1}{P(U)} h_0 + \frac{1}{P(D)} (1 - h_0) \right\},$$

- Market clearing condition for govt bonds:

$$\frac{q_0}{P_0} B_0 = (1 - h_0) \left(\frac{\bar{B}}{P_0} + y_0 \right),$$

- Equilibrium prices at date 1:

$$P(s^1) = \frac{B_0}{T(s^1)}, \quad \text{for } s^1 \in S^1,$$

- Evolution of the govt debt:

$$\bar{B} = q_0 B_0.$$

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Market structure

- Three kinds of assets:
 - govt bonds, storage and loans.
- Frictions:
 - govt bonds cannot be short sold;
 - agents can borrow to purchase govt bonds using those bonds as collateral;
 - loans are limited by collateral requirements.
- Use Geanakoplos's (2003, 2010) theory of collateral contracts.
 - Loans with different collateral rates are traded.
- In this model, only one type of loans are traded in equilibrium.
 - It is the loan contract with the lowest collateral rate sufficient to avoid default.
 - Thus, we need to consider only one type of loans, which is characterized by the risk-free interest rate R , and the 'no-default constraint.'

Budget constraint with loans

- The budget set for each agent can be defined by

$$c_0 + k_0 + q_0 \frac{b_0}{P_0} \leq \frac{\bar{B}}{P_0} + y_0 + \phi_0,$$

$$c(s^1) \leq y_1 - T(s^1) + \frac{b_0}{P(s^1)} + Rk_0 - R\phi_0, \quad \text{for } s^1 \in S^1,$$

$$R\phi_0 \leq \frac{b_0}{P(D)}, \quad (\text{no-default condition}),$$

$$c_0, k_0, b_0, c(s^1) \geq 0.$$

- Loan contract with one unit of govt bond b_0 as collateral:

- $\frac{1}{RP(D)}$ = (real) amount of borrowing;
- $\frac{q_0}{P_0}$ = (real) value of the bond (collateral) at date 0;
- collateral rate = value of the bond/ amount of borrowing = $q_0 \frac{P(D)}{P_0} R$.

Utility maximization

- Equilibrium leverage:
 - Optimistic agents borrow as much as they can and use the proceeds to purchase government debt.
 - Pessimistic agents lend to optimistic agents.
- Asymmetry between optimists and pessimists:
 - Optimistic agents can bet on their beliefs.
 - Indeed, borrowing to purchase govt debt is effectively equivalent to purchasing Arrow security U .
 - Pessimistic agent cannot bet on their beliefs.
 - Short selling of government debt is ruled out so that there is no trading strategy mimicking Arrow security D .
- The price level will reflect the optimists' view more than the pessimists'.
 - This generates deflationary pressure.

Equilibrium with leverage

- Utility maximization:

$$c_0^h = 0, \quad h \in [0, 1],$$

$$b_0^h = \begin{cases} \left(\frac{q_0}{P_0} - \frac{1}{RP(D)} \right)^{-1} \left(\frac{\bar{B}}{P_0} + y_0 \right), & h > h_0, \\ 0, & h \leq h_0 \end{cases}$$

$$k_0^h - \phi_0^h = \begin{cases} -\frac{1}{R} \frac{b_0^h}{P(D)}, & h > h_0, \\ \frac{\bar{B}}{P_0} + y_0, & h \leq h_0 \end{cases}$$

$$c^h(s^1) = y_1 - T(s^1) + \frac{b_0^h}{P(s^1)} + R(k_0^h - \phi_0^h), \quad h \in [0, 1]$$

Equilibrium with leverage

- h_0 = marginal buyer of govt bonds:

$$h_0 \frac{\frac{1}{P(U)} - \frac{1}{P(D)}}{\frac{q_0}{P_0} - \frac{1}{RP(D)}} = R,$$

- Market clearing condition for govt bonds:

$$\left(\frac{q_0}{P_0} - \frac{1}{RP(D)} \right) B_0 = (1 - h_0) \left(\frac{\bar{B}}{P_0} + y_0 \right),$$

- Equilibrium prices at date 1:

$$P(s^1) = \frac{B_0}{T(s^1)}, \quad \text{for } s^1 \in S^1,$$

- Evolution of the govt debt:

$$\bar{B} = q_0 B_0.$$

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Two Period model: numerical example

Inflation rates (%) at $t = 0$ and $s^1 = D$, and marginal buyers

	π_{-1}	π_0	$\pi(D)$	h_0
(1) complete markets	-1.96	30.72	47.06	0.5
(2) no borrowing	-1.96	25.57	53.09	0.56
(3) leverage	-1.96	9.46	75.62	0.79

- At $t = -1$, everyone believes $\Pr(s_1 = U) = 1$.
- At $t = 0$, news arrives so that agents start to hold different views.
- Parameters: $\bar{B} = 1$, $y_0 = 1$, $q_0 = 1$, $R = 1.02$, $T_H = 1$, $T_L = 0.5$.
- Because $q_0 = 1$ and $R > 1$, there is deflation in period -1 ($\pi_{-1} < 0$).

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Environment

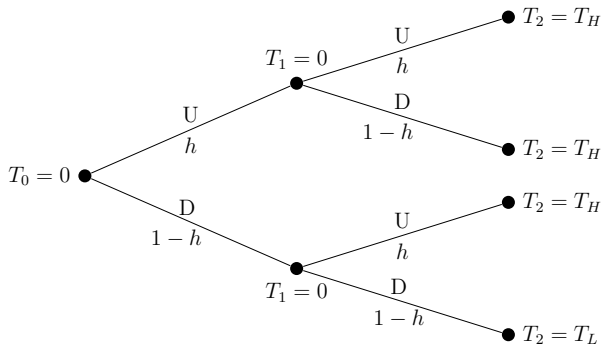
- Shocks: $s_t \in \{U, D\}$, $t = 1, \dots, T$.
- Endowments:

$$y(s^t) = \begin{cases} y_0, & \text{for } t = 0, \\ 0, & \text{for all } s^t \text{ with } t = 1, \dots, T - 1, \\ y_T, & \text{for all } s^T. \end{cases}$$

- Taxes:

$$T(s^t) = \begin{cases} 0, & \text{for all } s^t \text{ with } t = 0, \dots, T - 1, \\ T_L, & \text{for } s^T = D^T, \\ T_H, & \text{for all } s^T \neq D^T. \end{cases}$$

Example: Event tree in three period model



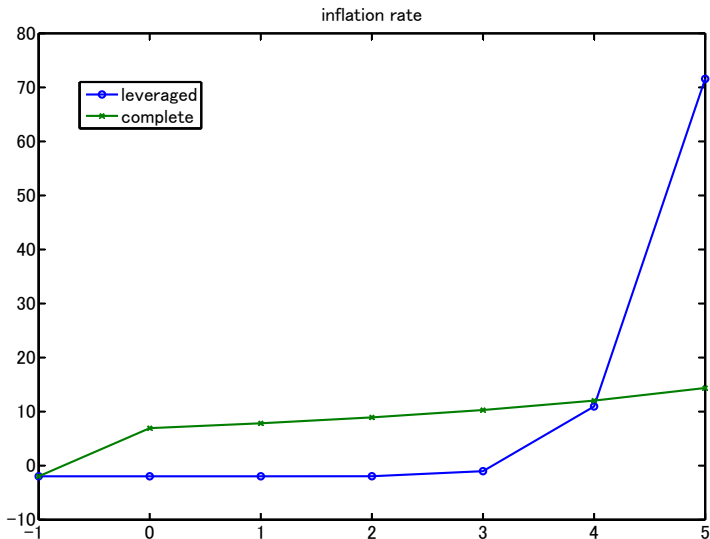
Numerical example: Three period model

Inflation rates and marginal buyers

	π_{-1}	π_0	$\pi(D)$	$\pi(D^2)$	h_0	$h(D)$
(1) complete markets	-1.96	17.65	22.55	30.72	0.50	0.33
(2) with leverage	-1.96	-1.09	10.86	71.89	0.94	0.75

- At $t = -1$, everyone believes $\Pr(s_t = U) = 1$.
- At $t = 0$, news arrives so that agents start to hold different views.
- $\bar{B} = 1$, $y_0 = 1$, $q_0 = q(D) = q(U) = 1$, $R = 1.02$, $T_H = 1$, $T_L = 0.5$.

Inflation rates at $s^t = D^t$ when $T = 5$



Properties of the equilibrium

- Complete Markets
 - Inflation rate jumps on the news in period 0
 - Smoothing. Inflation is smooth along the path to a debt crisis.
 - Along path to crisis, marginal buyer is falling at the rate $1/(t+2)$
- Financial frictions
 - No response of inflation rate to news.
 - Concentration: Inflation rate is low except in states near and during the debt crisis.
 - Marginal buyer is much higher (above 0.74 in all periods)

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Setup with long-term bonds and explicit default

- Shock U, D is realized in each period.
- Price levels, $\{P(s^t)\}$, are exogenously given.
- Govt debt:
 - \bar{B} = amount of government debt in period 0.
 - No new debt issued in any other period.
 - All debt is long-term and matures in period T .
- Sovereign debt crisis:
 - Govt defaults in period T only if $S^T = D^T$.
 - When the govt defaults, it repays only a fraction $\alpha \in (0, 1)$ of \bar{B} .
 - Govt only collects taxes in final period.

$$T(s^T) = \begin{cases} \frac{\bar{B}}{P(s^T)}, & \text{if } s^T \neq D^T, \\ \alpha \frac{\bar{B}}{P(s^T)}, & \text{if } s^T = D^T. \text{ (debt crisis)} \end{cases}$$

Numerical example

- Look at the evolution of the log yield of the govt debt in s^t :

$$\rho(s^t) \equiv \frac{1}{T-t} \ln \left[\frac{1}{q(s^t)} \right],$$

where $q(s^t) =$ price of govt debt in s^t , which matures in period T .

- parameter values:
 - Constant price levels: $P(s^t) = P$ for all s^t and t .
 - Real interest rate: $R = 1.02$.
 - Default rate: $\alpha = 0.2$.
- Prior to period 0, everyone believes that there is not govt default, i.e., $\alpha(s^T) = 1$ with probability one.
 - Under this assumption, the log yield of govt debt in period -1 is

$$\rho_{-1} = \ln(1.02) = 1.98\%.$$

Yields on long-term bond at $s^t = D^t$ when $T = 5$

Log yields in the five-period model (%)

	ρ_{-1}	ρ_0	$\rho(D)$	$\rho(D^2)$	$\rho(D^3)$
(1) complete markets	1.98	6.34	14.84	34.68	104.15
(2) financial frictions	1.98	1.98	2.05	5.04	38.70

- Complete markets
 - bond yield responds to news
 - yield rises along the path towards default
- Financial frictions
 - bond yield does not respond to news
 - Bond yield response is delayed.
 - Magnitude of the increase in bond yield is smaller along path to default.

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Who holds Japanese govt debt?

Holdings of Japanese Government Debt
End of fiscal year 2008

	Amount (trillion yen)	Fraction (%, net of govt)
Total	936.63	
Government	114.04	
Individuals and non-financial companies	75.88	9.2
Domestic Financial Institutions	687.45	83.6
Private	354.27	43.1
Public	268.06	32.6
Central Bank	65.12	7.9
Foreign sector	59.26	7.2

- financial sector: accepts deposits and holds government debt
- individual holdings of government debt are small.

Who holds US debt?

Holdings of U.S. Government Debt
End of Calendar year 2010

	Amount (trillion \$)	Fraction (%, net of govt)
Total	14.03	
Government	6.17	
Individuals and non-financial companies	1.41	12.4
Domestic Financial Institutions	5.38	47.2
Private	1.82	16.0
Public	2.44	21.4
Central Bank	1.11	9.8
Foreign sector	4.44	38.9

- Financial sector also holds a lot of government debt.
- Foreign sector is also important.
- Our result is robust to the introduction of a foreign sector.

Discussion: Restrictions on short selling govt debt.

- A cheap way to finance a mortgage: short government debt.
 - ① Borrow government debt today.
 - ② Sell it. Use proceeds to purchase a home.
 - ③ Repay at the interest rate on government debt.
- Mortgage rates are higher than the yield on government debt.

Spreads on Mortgage Rates over
Government Debt in U.S. and Japan
Data collected on October 24, 2011*

	U.S. (Percentage)	Japan (Percentage)
1- Year ARM	2.84	0.86
5/1-year ARM	1.92	1.22
15-Year Fixed	0.5125	n.a.
20-Year Fixed	n.a.	0.47
30-Year Fixed	1.06	0.46

*Government debt yields and
U.S. Mortgage rates are from Bloomberg.
Japan Mortgage rates are from Shinsei Bank.

Government restrictions

- Basel I and II induce banks to take long leveraged positions in government debt.
- Governments take actions to restrict short selling in states where the risk of default is high.
- Our model suggests that banning short sales of government debt is effective in reducing price pressure!

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Summary

- We have presented a model in which deflation/low yields can persist even when people recognize that a debt fiscal crisis is not a negligible possibility.
- Crucial features of our model:
 - asset markets are imperfect:
 - borrowing is limited by the no-default constraint;
 - individuals hold heterogeneous portfolios:
 - some agents want to purchase govt debt;
 - others do not want to purchase it by themselves, but are willing to lend to those who buy it.
- Compared to the case with frictionless asset markets, our model implies:
 - the inflation rate is much lower before the crisis, but it gets much higher once the crisis occurs.
 - In the model with long-term govt debt, the yield on govt debt behaves similarly.

Robustness

- Here we have assumed that individuals have different beliefs on how likely the debt crisis occurs.
 - Any other assumption that leads to the same type of heterogeneity in portfolios would work too.
 - Example: different degrees of risk aversion.
- We have also assumed that loans are risk-free.
 - If we interpret “loans” in our model as “demand deposits” in banks, this may sound odd, because they are also subject to the risk of inflation.
 - In reality, govt bonds offer higher interest rates than demand deposits.
 - Any other assumption that generates this type of rate-of-return differentials would work too.
 - Example: difference in maturity.

Some directions for future research

- other forms of heterogeneity (e.g., degrees of risk aversion).
- allowing some agents to short-sell government debt (arbitrageurs) as in e.g. Vayanos and Gromb (2010), Chen et al. (2001), but impose limits on arbitrage.
- Endogenous default
- normative analysis.
- more traditional DSGE framework.
- open economy.