

Debt-Ridden Borrowers and Productivity Slowdown

(Incomplete and preliminary)

Keiichiro Kobayashi* and Daichi Shirai†

May 2012 (First version: March, 2012)

Abstract

Many authors argue that financial constraints have been tightened since the great recession in 2007–2009. To explain this, we construct a model in which borrowing constraints for firms are tightened as a result of mass default due to a bubble collapse. In Jermann and Quadrini (2012)’s model, a defaulted firm either goes back to a normal firm by (partial) repayment of the debt or is liquidated. We assume that there is an intermediate status: a “debt-ridden” firm, which is defined as a firm whose lender retains the right to liquidate them. The lender allows the debt-ridden firm to continue if it pays continuation fee. In our model debt forgiveness is infeasible: once a firm defaults on the debt, it is either liquidated or kept as a debt-ridden firm. The defaulter can never go back to a normal firm unless it repays all the unpaid debt. Prohibition of debt forgiveness can be justified as a collective choice of the society to expand the borrowing limit of normal firms.

It is shown that borrowing constraints are tighter for debt-ridden firms than for normal firms. This implies that emergence of a large mass of debt-ridden borrowers may be a cause of the “financial shocks” in the recent macroeconomic literature. Tightened borrowing constraints due to emergence of debt-ridden firms lowers the aggregate productivity. The negative effect on productivity can be permanent. In a version of the model with endogenous growth the growth rate of aggregate productivity becomes zero if the number of debt-ridden firms exceeds a certain threshold.

Keywords: Borrowing constraint, working capital, financial shocks, productivity slowdown.

JEL Classification numbers: E30, G01, O40.

*Institute of Economic Research, Hitotsubashi University; CIGS; and RIETI. Email: kerkbys@ier.hit-u.ac.jp

†The Canon Institute for Global Studies. Email: shirai.daichi@gmail.com

1 Introduction

The decade after a financial crisis tends to be associated with low economic growth (Reinhart and Rogoff 2009, Reinhart and Reinhart 2010). It is also known that the growth of total factor productivity slows down or even becomes negative for a decade (Kehoe and Prescott 2007). Why productivity growth slows down persistently after a financial crisis? Many authors argue that financial constraints have been tightened during/after the great recession in 2007–2009. Why can financial constraints be tightened due to a financial crisis or a collapse of asset-price bubble? Can tightening of financial constraints cause a persistent slowdown of economic growth? These are the question we consider in this paper. In this paper we propose a theoretical model in which the emergence of debt-ridden borrowers lowers the aggregate productivity persistently through tightening financial constraints.

We construct a general equilibrium model, in which an exogenous shock makes substantial number of firms default on their debt. The model is based on Jermann and Quadrini’s (2006, 2012) models, which explicitly consider the bargaining after default to derive a borrowing constraint à la Kiyotaki and Moore (1997). In Jermann and Quadrini, firms’ ability to borrow is bounded by the limited enforceability of debt contract. The borrowing firm can default on the debt obligation and renegotiate on the repayment. The borrowing is limited such that the amount borrowed is renegotiation-proof. In the imaginary renegotiation in Jermann and Quadrini, the defaulted firm can go back to a normal firm if the lender and the firm agree on repayment, or the firm is liquidated if the renegotiation breaks down. Thus there are two status for a firm: being a normal firm or being liquidated.

A novel feature of our model is that we assume that there is an intermediate status of a firm: being a “debt-ridden” firm. We define a debt-ridden firm as a firm whose lender retains the right to liquidate it. The lender allows the debt-ridden firm to continue its operation if she agrees the amount of continuation fee to be paid by the firm. We assume that once a firm defaults on its debt, it can never go back to normal unless it pays all the original debt. The defaulted firm is either liquidated or kept as a debt-ridden firm.

We analyze the borrowing constraint for the debt-ridden firms and show that they face tighter borrowing constraint when they borrow working capital loans than normal firms do. This result seems counterintuitive since debt-ridden firms are under the control of their lenders, while the normal firms are not. The reason for this counterintuitive result is that a normal firm loses more when it defaults on its debt than a debt-ridden firm do. In our model, if a normal firm defaults on its debt, it inevitably becomes a debt-ridden firm unless it repays all the original debt. If a normal firm defaults it loses the status of a normal firm, while if a debt-ridden firm defaults it continues as a debt-ridden firm after

renegotiation. Thus a normal firm loses more by defaulting than a debt-ridden firm do. Because of this fact the incentive-compatibility condition (or no-default condition) implies that a normal firm can borrow more than a debt-ridden firm can.

Tighter borrowing constraint for working capital loans of debt-ridden firms makes their production activity inefficient. If a substantial number of firms become debt-ridden, both the aggregate borrowing capacity and the aggregate productivity decline. This implies that emergence of debt-ridden borrowers may be a cause of the “financial shocks” in the recent macroeconomic literature. After the great recession in 2007–2009, many authors argue that a shock in the financial sector can cause a severe recession (e.g., a risk shock in Christiano, Motto, and Rostagno 2009, and a financial shock in Jermann and Quadrini 2012). In our model, emergence of a substantial number of debt-ridden firms manifests itself as tightening of aggregate borrowing constraint, which can be interpreted as a financial shock.

We also show that emergence of debt-ridden firms has a persistent negative effect on productivity. The higher inefficiency due to tighter borrowing constraints lowers the value of new entry to the market for a potential entrant, and discourages the R&D activity. The decrease in R&D activity leads the economy into a steady state with low productivity. We consider a modified version of the model where the economy grows endogenously, and show in a numerical example that the growth rate of aggregate productivity may become zero permanently if the number of debt-ridden firms exceeds a certain threshold. This result seems consistent with the facts that we observe in persistent recessions after financial crises (see Section 2).

Related literature: Our theory is related to the literature on debt overhang. Myers (1977) pointed out the suboptimality of debt in the corporate finance literature and Lamont (1995) applied the notion of debt overhang in macroeconomics.¹ Debt overhang problem is typically that a firm cannot borrow new money for a productive projects when it has a too large amount of existing debt. The debt overhang arises if the existing debt holder is different from the potential lender who would lend new money. In this paper we take a small step forward by proposing a new theory that an inefficiency can arise even if the lender of new money *is* the existing debt holder. This paper is also close to Caballero, Hoshi, and Kashyap (2008). They analyze the “zombie lending” which is defined as a de facto subsidy to unproductive firms from the banks. They argue that congesting zombie firms hinder entry of highly productive firms and lower the aggregate productivity. In this paper, we make a complementary point to their argument: even an intrinsically productive firm can become unproductive when it becomes debt-ridden. This is because a debt-ridden firm is subject to tighter borrowing constraint for working capital loans.

¹See also Krugman (1988) on debt overhang in the international finance. See Moyen (2007) and Chen and Manso (2010), for example.

This paper is organized as follows. In the next section we review the facts on persistent recessions after financial crises. Section 3 proposes and analyzes the model. We modify the model to an endogenous growth model in Section 3.4. Section 4 concludes.

2 Facts on persistent recessions after financial crises

There observed numerous examples of productivity slowdown after a financial crisis. The most notable is the Great Depression in the 1930s in the US and the similar depressions in that period in the major nations. Ohanian (2001) shows that there was a large productivity decline during the US Great Depression, which is unexplained by capital utilization or labor hoarding. Kehoe and Prescott (2007) drew our attention to the fact that many countries experienced a decade-long severe recessions, which they call the “great depressions” in the twentieth century. Papers in their book unanimously emphasize that the declines in the growth rate of total factor productivity was the primal cause of the great depressions.

Another example of a decade-long recession after a financial crisis is the 1990s in Japan. The growth rates of the gross domestic product (GDP) and the total factor productivity (TFP) in the 1990s are both lower than in the 1980s. Figure 1 shows the GDP along with the potential capacity, which has an apparent kink at the beginning of the 1990s, when huge asset-price bubbles burst in the stock market and the real estate market. See Figure 2 for asset prices in Japan in the 1990s. Table 1 shows various estimates of TFP growth rate in Japan. Hayashi and Prescott (2002) emphasize that the growth of TFP slowed down in the 1990s in Japan.² One notable feature in the 1990s is significant decrease in entries of new firms and increase in exits. See Figure 3 for comparison of entry and exit of firms between Japan and the US. In the literature, the procyclicality of net entry is well known (Bilbiie, Ghironi, and Melitz 2007). Net entry also contributes significantly to TFP growth for US manufacturing establishments (Bartelsman and Doms 2000). Nishimura, Nakajima and Kiyota (2005) argue that malfunctioning of entry and exit contributes substantially to a fall in Japan’s TFP in the late 1990s. Another characteristic in the 1990s in Japan was the persistently lingering nonperforming loans (NPLs) in the banking sector. The NPLs were the excess debt for nonfinancial firms in mainly real-estate, wholesale, retail, and construction sectors. Figure 4 shows development in the amount of the NPLs in Japan in the 1990s and the 2000s. The delayed disposal of the huge NPLs was seen as de facto subsidy to nonviable firms (“zombie lending”). The zombie lending has been named as

²There are substantial debate on whether the TFP slowdown in Japan is truly a slowdown of technical progress or just a measurement error (see Kawamoto 2005, Fukao and Miyagawa 2008). Tentative conclusion on this issue in the literature is that there was a slowdown in technical progress in Japan, though it may not be so severe as Hayashi and Prescott originally measure.

the cause of Japan’s persistent recession (Peek and Rosengren 2005, and Caballero, Hoshi, and Kashyap 2008).

3 Model

We consider a closed economy in which the final good is produced competitively from labor input and varieties of intermediate goods. The firms are monopolistic competitors and they produce respective varieties of intermediate goods from material input, which is the final good, and capital input. In our model, when a firm defaults on the debt, the lender cannot forgive debt. Instead, the lender can choose whether to liquidate the firm or to allow it to continue operation as a “debt-ridden firm.” In this paper a debt-ridden firm is a firm whose lender retains a unilateral discretion to liquidate the firm. Later we clarify the difference between normal firms and debt-ridden firms by formally defining respective optimization problems they solve. The model is a version of expanding variety model in which new entry of firms increases the aggregate productivity. The expanding variety model is proposed by Rivera-Batiz and Romer (1991) and simplified by Acemoglu (2009). We basically follow Acemoglu’s exposition. In Sections 3.1–3.3, we analyze the model in which the economy converges to a steady state. In Section 3.4, we introduce a positive externality from the variety of goods to the aggregate productivity, which enables endogenous growth. We show in Section 3.4 that the growth rate falls to zero if there emerge sufficiently many debt-ridden firms.

3.1 Basic setup

There are fixed supplies of capital K and labor L . The labor is used for production of the final good, and the capital is used for production of the intermediate goods. There is a mass of firms indexed by $i \in [0, N_t]$, which produce intermediate goods, where N_t is the measure of varieties of the intermediate goods in period t . The firms are either normal firms or debt-ridden firms, while the status of being normal and debt-ridden are clarified later. A representative household owns these firms and solve the following program:

$$\begin{aligned} \max_{C_t, I_t, b_{t+1}} \quad & E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln C_t \right], \\ \text{subject to} \quad & C_t + \frac{b_{t+1}}{1+r_t} + I_t \leq w_t L + \int_0^{N_t} \pi_{it} di + \xi_t b_t + d_t, \\ & N_{t+1} = (1-\delta)N_t + \chi I_t, \\ & I_t \geq 0. \end{aligned}$$

where β is the subjective discount factor, C_t is consumption, L is the fixed amount of labor supply, I_t is the R&D investment, w_t is wage rate, r_t is the market interest rate, b_t

is bond issued by the normal firms, d_t is payment by the debt-ridden firms, and π_{it} is the profit from firm i , where $i \in [0, N_t]$. The bond is risky debt and ξ_t is the recovery rate of the bond, where $\xi_t = 1$ if no firm defaults and $\xi_t < 1$ if some firms default. The cost of R&D investment is measured in the unit of the final good and one unit of the R&D investment creates χ units of new variety of the intermediate goods. The δ fraction of the varieties depreciates every period. The first-order conditions (FOCs) with respect to I_t and N_{t+1} imply that the R&D investment takes place if $\chi V_{nt} = 1$ and it does not take place if $\chi V_{nt} < 1$, where V_{nt} is the value of a normal firm. The cost of the R&D investment is unity and the expected gain from the R&D is χV_{nt} . The amount of the R&D investment is determined such that the cost and the gain are equalized: $\chi V_{nt} = 1$.

The final good is produced competitively from the intermediate goods x_{it} , $i \in [0, N_t]$ and the labor by the following production function:

$$Y_t = \frac{1}{\eta} \left(\int_0^{N_t} x_{it}^\eta di \right) L^{1-\eta},$$

where $0 < \eta < 1$. Since the final good producer maximizes $Y_t - \int_0^{N_t} p_{it} x_{it} di - w_t L_t$, where p_{it} is the real price of the intermediate good i , perfect competition in the final good market implies that

$$\begin{aligned} p_{it} &= p(x_{it}) = L^{1-\eta} x_{it}^{\eta-1}, \\ w_t &= \frac{1-\eta}{\eta} \frac{Y_t}{L}. \end{aligned}$$

Firm i produces the intermediate good i from capital input k_{it} and the material input m_{it} , where m_{it} is the final good, by the following production function:

$$x_{it} = A_{it} k_{it}^\alpha m_{it}^{1-\alpha},$$

where A_{it} is the productivity parameter. For simplicity of the analysis, we assume that the productivity is identical across firms and constant over time:

$$A_{it} = A \quad \text{for all } i \text{ and } t.$$

Firm i needs to buy k_{it} at the price of q_{t-1} in period $t-1$ and buys m_{it} at the price of unity in period t . We assume for simplicity that on one hand there is no financial friction in buying and selling the physical capital k_{it} in the market. Thus firm i can pay any amount of $q_{t-1} k_{it}$. On the other hand, firm i needs to borrow working capital m_{it} from the representative household to buy the material input and this debt is subject to the financial constraint that we specify below. Firm i borrows both inter-temporal debt b'_{it} and intra-temporal debt (working capital) m_{it} . The inter-temporal debt contract is such that firm i borrows $\frac{b'_{it}}{1+r_{t-1}}$ at the end of period $t-1$ where r_{t-1} is the market rate. The

sole shock in this economy is the idiosyncratic redistribution shock Δ_{it} that changes the amount of the inter-temporal debt to be repaid in period t . We define b_{it} by

$$b_{it} \equiv b'_{it} + \Delta_{it},$$

which is the amount that firm i must repay in period t to the lender of the inter-temporal debt. All agents know the probability distribution function of Δ_{it} and they take the expectation over the distribution of Δ_{it} when they decide their actions.

Timing of actions of firm i is as follows. At the end of period $t - 1$ firm i borrows $\frac{b'_{it}}{1+r_{t-1}}$ and purchases capital stock k_{it} at price q_{t-1} . At the beginning of period t the redistribution shock Δ_{it} is revealed and the inter-temporal debt becomes $b_{it} = b'_{it} + \Delta_{it}$, then the firm borrows working capital m_{it} , purchases the material input, and produces x_{it} . After selling x_{it} it repays debt $m_{it} + b_{it}$ and sells k_{it} at price q_t . Note that there is no stochastic shock during the short span when the firm borrows the working capital. So the gross interest rate for the intra-period working capital is 1.

Now we specify the borrowing constraint for firm i . In what follows we omit the subscript i unless there is a risk of confusion. It will be shown that the firm's debt is subject to the borrowing constraint:

$$m_t + b_t \leq \phi q_t k_t + V_{nt} - V_{zt}, \quad (1)$$

where V_{nt} is the value of a normal firm and V_{zt} is the value of a debt-ridden firm, while later we will specify V_{nt} by (2) and V_{zt} by (9). It will be shown that $V_{nt} > V_{zt}$ in equilibrium. The borrowing limit is derived from the limited enforceability of debt contracts as firms can default on their debt obligations. The basic logic is the same as Jermann and Quadrini (2012). The decision of default arises after the realization of revenues but before repaying the inter-period and intra-period loans. At this stage the total liabilities are $m_t + b_t$, that is, the intra-period loan plus the inter-period debt due in period t . As in the Jermann-Quadrini model, the lender and the firm renegotiate on repayment if the firm defaults, and if the renegotiation breaks down and the firm is liquidated at this point in time the lender obtains $(\phi + \psi)q_t k_t$, by confiscating a part of capital stock (k_t), where $0 < \phi + \psi < 1$. A departure from the Jermann-Quadrini model is the following assumption:

Assumption 1. The institutional environment of this economy is such that a lender cannot forgive debt of the borrowing firm if it defaults on the debt. Instead the lender either liquidates the defaulted firm immediately or allows it to continue operation as a debt-ridden firm. The debt-ridden firm is a firm whose lender retains the right to liquidate it and the lender allows it to continue operation if she agrees on the continuation fee that it pays to the lender. The lender and the debt-ridden firm negotiate at most twice in each period. The first negotiation arises if the firm defaults on the debt obligation in the middle of the period. In this negotiation the lender and the firm negotiate on the repayment f ,

while the lender obtains $(\phi + \psi)q_t k_t$ if she liquidates the firm at this stage. The second negotiation, which arises at the end of the period, is over the continuation fee d_{t+1} that the firm should pay in the next period. If the lender liquidates the firm in the second negotiation, she obtains only $\psi q_t k_t$.

The difference of the liquidation value of the firm in the first and the second negotiation can be justified as follows: the first negotiation takes place in the middle of the period and the second negotiation takes place at the end of the period; and the firm can conceal or sell off a part of capital (ϕk_t) during the period between the two negotiations. Given this institutional environment, a normal firm inevitably becomes a debt-ridden firm once it defaults on the debt. The defaulted firm loses $V_{nt} - V_{zt}$ inevitably. Appendix A shows that the defaulted firm and the lender renegotiate on repayment and agree that the defaulted firm repays $\phi q_t k_t$ in period t . The incentive-compatibility constraint for a normal firm not to default is that the original debt ($m_t + b_t$) is no greater than the value it loses by defaulting ($\phi q_t k_t + V_{nt} - V_{zt}$). Thus a normal firm solves the following Bellman equation:

$$V_{nt} = \max_{k, b'} \frac{b'}{1 + r_t} - q_t k + E_t \left[\max_m \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ p(x)x - m - \tilde{b} + q_{t+1}k + \tilde{V}_{t+1} \right\} \right], \quad (2)$$

$$\text{subject to } x = A_{t+1} k^\alpha m^{1-\alpha}, \quad (3)$$

$$b = b' + \Delta, \quad (4)$$

$$m \leq \max\{0, \phi q_{t+1}k + V_{nt+1} - V_{zt+1} - b\}, \quad (5)$$

$$\tilde{b} = b \text{ and } \tilde{V}_{t+1} = V_{nt+1} \text{ if } b \leq \phi q_{t+1}k + V_{nt+1} - V_{zt+1}, \quad (6)$$

$$\tilde{b} = \phi q_t k_t \text{ and } \tilde{V}_{t+1} = V_{zt+1} \text{ if } b > \phi q_{t+1}k + V_{nt+1} - V_{zt+1}, \quad (7)$$

where λ_t is the Lagrange multiplier of the budget constraint in the household's problem and \tilde{b} is the realized repayment of the inter-temporal debt b . Constraints (5) says that if the realization of Δ is such that $\phi q_{t+1}k + V_{nt+1} - V_{zt+1} - b < 0$, the firm sets $m = 0$. Since Appendix A shows that the repayment after default is $\phi q_t k_t$, it is easily shown that the payoff of no default ($q_t k_t + V_{nt+1} - b$) is strictly less than that of defaulting ($(1 - \phi)q_t k_t + V_{zt+1}$). Therefore, the firm defaults on b if $\phi q_{t+1}k + V_{nt+1} - V_{zt+1} - b < 0$.

To exclude the equilibrium in which all firms intentionally defaults after borrowing too much b_t , we assume that the values of the parameters are chosen such that

$$\forall t, \quad V_{nt} - V_{zt} \geq \psi q_t k_t, \quad (8)$$

in the equilibrium path.³

³ Note that the lender is willing to lend as long as $m_t + b_t \leq (\phi + \psi)q_t k_t$, because she can obtain $(\phi + \psi)q_t k_t$ by liquidating the firm if it defaults on the debt. So if $V_{nt} - V_{zt} < \psi q_t k_t$, all firms set the highest possible b_t at the end of period $t - 1$ such that $\phi q_t k_t + V_{nt} - V_{zt} < m_t + b_t \leq (\phi + \psi)q_t k_t$, and all of them intentionally default in period t . In this case, all firms become debt-ridden firms from period t on. This equilibrium path is self-consistent but does not seem to be relevant to the reality. Condition (8) excludes the possibility of emergence of this "all default" equilibrium.

Why debt forgiveness is not allowed? Our institutional setting that debt forgiveness is not allowed is justified as a collective choice of the society to expand the borrowing limit. Given that the parameter values satisfy (8), the borrowing limit for a normal firm in the economy where debt forgiveness is feasible is $(\phi + \psi)q_t k_t$ as we show in Appendix D, while the limit is $\phi q_t k_t + V_{nt} - V_{zt}$ in the economy where debt forgiveness is prohibited. (8) implies that the borrowing limit is higher in the economy where debt forgiveness is prohibited than in the economy it is not. Thus the prohibition of debt forgiveness can be justified as a social choice to expand the borrowing limit.

Now we consider the behavior of a debt-ridden firm. We assume the following for the institutional setting surrounding the debt-ridden firm.

Assumption 2. At the end of period t the lender and the debt-ridden firm bargain over the continuation fee, d_{t+1} , that the debt-ridden firm should pay to the lender in period $t + 1$. If they agree on d_{t+1} , the lender allows the firm to continue operation in period $t + 1$ and the firm maximize its net profit after paying d_{t+1} . The debt-ridden firm cannot accumulate financial assets inter-temporally.

The last sentence of the above assumption implies that the debt-ridden firm cannot make savings. As we show in Appendix C, this assumption is not necessary for our results in the case of the deterministic equilibrium.

The bargaining over d_{t+1} takes place at the end of period t and we assume that if the bargaining breaks down and the firm is liquidated at this point, the lender obtains only $\psi q_t k_t$ by confiscating a part of capital stock. We assume that the lender cannot confiscate $(\phi + \psi)q_t k_t$ because $\phi q_t k_t$ has been concealed from the lender at the end of the period.⁴

Therefore, if the debt-ridden firm continues the lender can obtain $D_t \equiv \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + D_{t+1} \right\} \right]$, where $\tilde{d}_t \leq d_t$ is the realized payment, while the promised amount is d_t , and D_t is the present value of the flow of payments from the firm, while the lender can obtain $\psi q_t k_t$ if she liquidates the firm immediately. As we see in Appendix B, the bargaining outcome d_{t+1} is determined by

$$D_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + D_{t+1} \right\} \right] = \psi q_t k_t.$$

The lender and the firm determine d_{t+1} , taking $\{k_{t+j}\}_{j=1}^{\infty}$ as given. See Appendix B for detailed description of the values of d_{t+1} and \tilde{d}_{t+1} . After d_{t+1} is agreed at the end of period t , the debt-ridden firm is allowed to operate in period $t + 1$. Actions of the debt-ridden firm are as follows. At the end of period t , after d_{t+1} is agreed it sells k_t and buys

⁴The debt-ridden firm and the lender can determine the value of d_{t+1} in the first negotiation in the middle of period t . But since they have the second chance of the renegotiation at the end of period t , the value of d_{t+1} must be identical to the value which is determined in the second renegotiation. Otherwise d_{t+1} is revised in the second renegotiation at the end of period t .

k_{t+1} in the market. At the beginning of period $t + 1$ after realization of A_{t+1} , it borrows working capital m_{t+1} to purchase the material input (the final good). It produces the intermediate good $A_{t+1}k_{t+1}^\alpha m_{t+1}^{1-\alpha}$ and sells it in the monopolistically competitive market. After it receives the proceeds of sales, it repays the debt $m_{zt+1} + d_{t+1}$.

The debt-ridden firm's ability to borrow the working capital m_{zt+1} is bounded by the limited enforceability of debt contracts. After realization of revenues and before repaying m_{zt+1} , the debt-ridden firm has a chance to default on its debt ($m_{zt+1} + d_{t+1}$). If the firm defaults, the firm and the lender renegotiate on the repayment f as described in Appendix B. If they agree on f , the lender obtains $f + D_{t+1}$, while she obtains $(\phi + \psi)q_{t+1}k_{t+1}$ by liquidating the firm if the bargaining breaks down. Therefore, $f + D_{t+1} = (\phi + \psi)q_{t+1}k_{t+1}$ and the no-default condition ($m \leq \max\{0, f - d_{t+1}\}$) implies that

$$m_{zt+1} \leq \max\{0, (\phi + \psi)q_{t+1}k_{t+1} - d_{t+1} - D_{t+1}\}.$$

Since it is shown that $D_{t+1} = \psi q_{t+1}k_{t+1}$ in Appendix B, the borrowing constraint for m_{zt+1} is $m_{zt+1} \leq \max\{0, \phi q_{t+1}k_{t+1} - d_{t+1}\}$. The debt-ridden firm maximizes its own value V_{zt} , which is defined by the following Bellman equation. Given $\{d_{t+j}\}_{j=1}^\infty$, the debt-ridden firm solves

$$V_{zt} = \max_{k_{t+1}} -q_t k_{t+1} + E_t \left[\max_{m_{t+1}} \beta \frac{\lambda_{t+1}}{\lambda_t} \{p(x)x - m_{t+1} - d_{t+1} + q_{t+1}k_{t+1} + V_{zt+1}\} \right], \quad (9)$$

$$\text{subject to } x = A_{t+1}k_{t+1}^\alpha m_{t+1}^{1-\alpha}, \quad (10)$$

$$m_{t+1} \leq \max\{0, \phi q_{t+1}k_{t+1} - d_{t+1}\}. \quad (11)$$

Note that d_{t+1} does not depend on the firm's choice of k_{t+1} , since the lender and the firm agree on d_{t+1} at the end of t , taking the expected value of k_{t+1} as given.

One may suspect that prohibition of corporate savings by the debt-ridden firms in Assumption 2 is crucial in deriving a persistent inefficiency due to the binding borrowing constraint (11). If the firm can accumulate financial assets, it could be possible to relax the borrowing constraint eventually. But we show in Appendix C that at least in the deterministic equilibrium the borrowing constraint is identical to (11) even if the debt-ridden firm can make savings, under the assumption that the lender can confiscate the savings when she liquidates the firm. Thus there is no incentive for the debt-ridden firm to make savings in the deterministic case.

3.2 Equilibrium without debt-ridden firms

In this subsection we characterize the deterministic equilibrium in which $\Delta_{it} = 0$ with probability one. We also assume that $\delta = 0$, that is, a variety of good is not depleted once it is created.

We consider the economy in which all firms are normal firms and there is no debt-ridden firm. The resource constraints of the economy are

$$C_t + I_t + \int_0^{N_t} m_{it} = Y_t,$$

$$K = \int_0^{N_t} k_{it} di.$$

The economy converges to a stationary equilibrium, in which the number of firms $N_t = N$ is time-invariant and determined by

$$V_{nt} = \frac{1}{\chi}.$$

In the symmetric stationary equilibrium, there is no R&D investment ($I_t = 0$) and the resource constraints are $C = Y - Nm$ and $K = Nk$, where k and m are the capital and material inputs for one firm. We focus on the parameter values that makes the borrowing constraint (5) non-binding in the stationary equilibrium. In this case $m = \hat{m}$, where $\hat{m} = \arg \max_m p(A_{it+1} k_{it+1}^\alpha m^{1-\alpha}) A_{it+1} k_{it+1}^\alpha m^{1-\alpha} - m$. That the borrowing constraint is non-binding implies that

$$\hat{m} \leq \phi q_{t+1} k_{t+1} + V_{nt+1} - V_{zt+1}.$$

In this case, b_{t+1} is indeterminate. But we assume that there is infinitesimally small tax benefit for issuing intertemporal debt such that firms are willing to borrow intertemporal debt up to the borrowing limit. In the case of a deterministic equilibrium, the amount of b_{t+1} is determined by $b_{t+1} = \phi q_{t+1} k_{t+1} - \hat{m} + V_{nt+1} - V_{zt+1}$. Note that V_{nt} does not depend on b_{t+1} , because the tax benefit is infinitesimal, and the loan rate and the market rate are equal and satisfy

$$\frac{b_{t+1}}{1+r_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] b_{t+1}.$$

3.3 Equilibrium with debt-ridden firms

We assume that the economy is initially in the symmetric stationary equilibrium where all firms are normal, $I_t = 0$, and $N_t = N$, where N is determined so that $V_n = \frac{1}{\chi}$. We assume that in period τ the economy is hit by redistribution shocks $\{\Delta_{i\tau}\}_{i \in [0, Z]}$, which makes $b_{i\tau} > \phi q_\tau k_\tau + V_{n\tau} - V_{z\tau}$ for $i \in [0, Z]$ and $b_{i\tau} \leq \phi q_\tau k_\tau + V_{n\tau} - V_{z\tau}$ for $i \in (Z, N]$. Thus firm $i \in [0, Z]$ choose to default on $b_{i\tau}$ and pay $\phi q_\tau k_\tau$, and firm $i \in (Z, N]$ do not default on their debt. At the end of period τ , the firms in $[0, Z]$ and their lenders negotiate on the amount of the continuation fee $d_{\tau+1}$. We conjecture and verify later that $V_{nt} < \frac{1}{\chi}$ along with the equilibrium path. Since $V_{nt} < \frac{1}{\chi}$, there takes place no R&D investment and N_t does not change from N . Since N , K , and L are all constant over time, the equilibrium is the steady state. We focus on the equilibrium where the borrowing constraint (5) for

normal firms is not binding and (11) for debt-ridden firms does bind. Denoting variables for debt-ridden firms and normal firms by those with subscript z and n , respectively, the equilibrium is described by the following system of equations:

$$1 = (1 - \alpha)\eta L^{1-\eta} A^\eta k_n^{\alpha\eta} m_n^{(1-\alpha)\eta-1}, \quad (12)$$

$$1 + \mu_z = (1 - \alpha)\eta L^{1-\eta} A^\eta k_z^{\alpha\eta} m_z^{(1-\alpha)\eta-1}, \quad (13)$$

$$q = \beta \left[\alpha\eta L^{1-\eta} \frac{(A k_n^\alpha m_n^{1-\alpha})^\eta}{k_n} + q \right], \quad (14)$$

$$q = \beta \left[\alpha\eta L^{1-\eta} \frac{(A k_z^\alpha m_z^{1-\alpha})^\eta}{k_z} + (1 + \phi\mu_z)q \right], \quad (15)$$

$$Y = \frac{1}{\eta} A^\eta L^{1-\eta} \{ Z(k_z^\alpha m_z^{1-\alpha})^\eta + (N - Z)(k_n^\alpha m_n^{1-\alpha})^\eta \}, \quad (16)$$

$$C = Y - (N - Z)m_n - Zm_z, \quad (17)$$

$$K = Zk_z + (N - Z)k_n, \quad (18)$$

$$m_n < \phi q k_n + V_n - V_z, \quad (19)$$

$$d = (\beta^{-1} - 1)\psi q k_z, \quad (20)$$

$$m_z + d = \phi q k_z, \quad (21)$$

$$V_n = -qk_n + \beta \{ L^{1-\alpha} (A k_n^\alpha m_n^{1-\alpha})^\eta - m_n + qk_n + V_n \}, \quad (22)$$

$$V_z = -qk_z + \beta \{ L^{1-\alpha} (A k_z^\alpha m_z^{1-\alpha})^\eta - m_z - d + qk_z + V_z \}, \quad (23)$$

where μ_z is the Lagrange multiplier for (11). This system of equations is solved as follows. First, (13), (15), (20), and (21) imply that

$$\mu_z = \frac{1 - \beta - [\phi - (\beta^{-1} - 1)\psi] \frac{\alpha\beta}{1-\alpha}}{\phi\beta + [\phi - (\beta^{-1} - 1)\psi] \frac{\alpha\beta}{1-\alpha}} \quad (24)$$

Given μ_z , we define Λ by

$$\Lambda = \left[\frac{1 - \beta}{1 - (1 + \phi\mu_z)\beta} \right]^{\frac{1-(1-\alpha)\eta}{1-\eta}} (1 + \mu_z)^{-\frac{(1-\alpha)\eta}{1-\eta}}. \quad (25)$$

The macroeconomic variables are given by

$$k_n = \frac{K}{N + (\Lambda - 1)Z}, \quad (26)$$

$$k_z = \frac{\Lambda K}{N + (\Lambda - 1)Z}, \quad (27)$$

$$m_n = [(1 - \alpha)\eta L^{1-\eta} A^\eta k_n^{\alpha\eta}]^{\frac{1}{1-(1-\alpha)\eta}}, \quad (28)$$

$$m_z = \left(\frac{\Lambda^{\alpha\eta}}{1 + \mu_z} \right)^{\frac{1}{1-(1-\alpha)\eta}} m_n, \quad (29)$$

$$q = \frac{\alpha\beta}{(1 - \alpha)(1 - \beta)} \frac{m_n}{k_n}, \quad (30)$$

$$d = (\beta^{-1} - 1)\psi q k_z \quad (31)$$

$$V_n = \frac{\beta}{(1 - \alpha)(1 - \beta)} (\eta^{-1} - 1) m_n, \quad (32)$$

$$V_z = \frac{\beta}{1 - \beta} \left\{ \frac{(1 + \mu_z)}{(1 - \alpha)\eta} m_z - m_z - d \right\} - q k_z. \quad (33)$$

These variables must satisfy (8) and (19), which we check numerically in the example of Figure 5.

Figure 5 shows the variables in the steady state equilibrium as functions of Z . The parameter values are given in the caption of Figure 5. We set χ such that the value of N equals unity in the stationary equilibrium where $Z = 0$. The value of χ is therefore given by $V_{SS} = \chi_{SS}^{-1} = 3.4234$, where V_{SS} is the value of the normal firm in the stationary equilibrium where $Z = 0$. The aggregate productivity of the economy is proportional to

$$\frac{Y - (N - Z)m_n - Zm_z}{K^\theta L^{1-\theta}},$$

where $\theta = \frac{\alpha\eta}{1-(1-\alpha)\eta}$. As $C = Y - (N - Z)m_n - Zm_z$ decreases as Z increases in this figure, the aggregate productivity is decreasing in Z . Figure 5 also justifies our conjecture that $V_n < \chi_{SS}^{-1}$ ($= V_{SS}$) in the equilibrium where $Z > 0$. This is because V_n is decreasing in Z as Figure 5 shows. Why does V_n decrease as the debt-ridden firms increase? Since the borrowing constraint is tight for the debt-ridden firms, the value of capital stock k_t as a collateral asset for the debt-ridden firms are higher and they purchase more capital than the normal firms. As a result, the price of capital increases and the increased cost of capital pushes V_n down. This is how the increase of Z decreases V_n . This mechanism is similar to the congestion effect of zombie firms in Caballero, Hoshi, and Kashyap's (2008) model. Because of this congestion effect the decline of productivity is permanent. This result forms a striking contrast to the equilibrium outcome in the case where debt forgiveness is feasible, which is described in Appendix D. We see in Appendix D that the aggregate productivity comes back to the normal level immediately after a mass default if the lenders and the firms can agree on debt forgiveness.

Next we consider the endogenous growth model, and show that the congestion effect can lower the growth rate to zero permanently because the raised cost of capital due to emergence of debt-ridden firms discourages R&D investment.

3.4 Endogenous growth and the zero growth path

To enable endogenous growth in our model we need the following externality:

Assumption 3. There is a positive externality from the number of variety to the productivity: $A_t = \hat{A}N_t^\alpha$, where \hat{A} is an exogenous parameter. All agents take A_t as given, and households do not recognize the effect of their choice of N_t on A_t .

The externality from expanding variety enables endogenous growth in this economy. It is shown below that if Z is small the economy converges to the balanced growth path (BGP), in which the productivity grows at a constant rate, while if Z is large the economy falls into the zero growth path (ZGP), in which the growth rate is zero.

The BGP in this economy is an equilibrium path in which all firms are normal and the economy grows at a constant rate, i.e., $N_{t+1}/N_t = Y_{t+1}/Y_t = C_{t+1}/C_t = 1 + g$, where $g > 0$. Since the capital K and the labor L are fixed supply, the expansion of variety N_t is the only source of economic growth in this model. We assume that the borrowing constraint (5) does not bind in the BGP. The price of capital in the BGP is $N_t q$. The variables (m, q, g) in BGP are specified by

$$\begin{aligned} m &= [(1 - \alpha)\eta L^{1-\eta}(\hat{A}K^\alpha)^\eta]^{-\frac{1}{1-(1-\alpha)\eta}}, \\ q &= \frac{\alpha\beta}{(1-\alpha)(1-\beta)} \frac{m}{K}, \\ \frac{1}{\chi} &= \left[\frac{\beta}{1+g-\beta} \left\{ \frac{1}{(1-\alpha)\eta} - 1 \right\} - \frac{\alpha\beta}{(1-\alpha)(1-\beta)} \right] m. \end{aligned}$$

We consider a numerical example, the parameter values of which are the same as those in the example in the previous subsection, except for χ . We set the value of χ such that the growth rate of the BGP equals 1 %. The value of χ is therefore given by $V_{BGP} = \chi_{BGP}^{-1} = 2.9307$. In this example, the variables are given by $m = 0.6657$, $q = 1.997$, and $g = 0.01$.

We are interested in whether the emergence of the debt-ridden firms make the economy follow the zero growth path (ZGP), in which R&D investment does not take place and the economy does not grow. Suppose that $N_t = 1$ and there emerge debt-ridden firms, whose measure is Z , in period t . Given Z , the ZGP is specified by the system of equations in the previous subsection (12)–(23). We consider the same numerical example as in the previous subsection, except that we change the value of χ from χ_{SS} to χ_{BGP} . If V_n calculated by (22) satisfies $V_n < \chi_{BGP}^{-1}$, then the R&D does not take place and therefore N_{t+j} stays at 1 for $j \geq 1$. In this case the economy stays at a steady state where the productivity stays constant, which is the ZGP. As Figure 5 shows, $V_n > \chi_{BGP}^{-1}$ for $0 \leq Z < 0.5439$

and $V_n < \chi_{BGP}^{-1}$ for $0.5439 < Z \leq 1$. The economy falls into the ZGP if Z is larger than 0.5439.

In this example, V_n calculated by (22) is larger than χ_{BGP}^{-1} for $Z \in [0, 0.4647)$. In this case the equilibrium is *not* given by (12)–(23) since if the equilibrium were the ZGP, the value of new entry, V_{nt} given by (22), would be strictly greater than the cost, χ_{BGP}^{-1} , and the R&D investment would take place, leading to $N_{t+1} > N_t$, a contradiction. The economy grows such that the value of new entry V_{nt} is equal to χ_{BGP}^{-1} , and converges to the BGP eventually. We describe the dynamics of the economy for a small $Z \in [0, 0.5439)$ in Appendix E.

Another numerical example is shown in Figure 6. In this example we set $\phi = 0.13$ instead of 0.15 in the example of Figure 5. Figure 6 shows the variables in the ZGP. With $\phi = 0.13$, the value of a debt-ridden firm along with the BGP is negative: $V_{z,BGP} = -0.5505$, while it is positive along with the ZGP as the panel of V_z in Figure 6 shows. This means that there exist multiple equilibria, the BGP and the ZGP, in this economy for a large initial value of Z . First, for any value of Z the BGP can be an equilibrium. Suppose that macroeconomic expectations prevail in the initial period that the economy follows the BGP. The all Z firms defaulted in the initial period exit immediately anticipating their negative payoff, $V_z = -0.5505$, and only the normal firms remain in the market. Since the debt-ridden firms exit from the market, the capital price q_t becomes sufficiently cheap such that the R&D investment takes place and the economy grows at the BGP rate. Second, for $Z > 0.4651$ the ZGP can be an equilibrium. Suppose that macroeconomic expectations prevail in the initial period that the economy follows the ZGP. All the Z firms defaulted in the initial period stay in the market anticipating their positive payoff, $V_z > 0$. As Figure 6 shows that $\chi_{BGP} V_{n,ZGP} < 1$ for $Z > 0.4651$, where $V_{n,ZGP}$ is the value of a new firm in the ZGP, the gains from the R&D investment is strictly less than its cost along with the ZGP when the initial value of Z is larger than 0.4651. Therefore if $Z > 0.4651$ and the macroeconomic expectations are that the economy follows the ZGP, then no households conduct the R&D investment and the economy stays at the ZGP.⁵

These examples in Figures 5 and 6 show that a mass default may shift the equilibrium path from a growth path to the ZGP. This shift occurs because the debt-ridden firms facing tighter borrowing constraints purchase collateralizable asset, i.e., physical capital in our model, aggressively, leading to a rise of the cost of capital. The higher capital cost reduces the expected gain of the R&D investment and discourages entry of new firms. If there emerges a sufficient number of the debt-ridden firms the expected gain of the

⁵Note that the ZGP cannot exist in this economy if $Z \leq 0.4651$. This is because $\chi_{BGP} V_{n,ZGP} \geq 1$ for $Z \leq 0.4651$, implying that the gains from the R&D investment is greater than or equal to its cost along with the ZGP. Since the R&D is conducted and N_t increases if the initial value of Z is no greater than 0.4651, the economy inevitably grows and the ZGP cannot be an equilibrium.

R&D falls below its cost and no one undertakes the R&D, leading to the zero growth of the aggregate productivity. This negative effect on the firms' entry from higher capital cost is similar to the negative congestion effect on productivity of the zombie lending in Caballero, Hoshi, and Kashyap (2008).

4 Conclusion

Decade-long recessions with low productivity growth are often observed after financial crises. We proposed a hypothesis that emergence of debt-ridden borrowers cause a persistent productivity slowdown. Economic agents become overly indebted as a result of boom and bust of asset-price bubble. If debt reduction or debt forgiveness is not easily implemented due to some rigidities of the market institution, the borrowers become debt-ridden. Analyzing the bargaining after default on the original debt, we show that the debt-ridden borrowers are subject to tighter borrowing constraint than the normal firms although they are under the control of the lenders. Emergence of a substantial number of debt-ridden borrowers lowers the aggregate productivity through tightening the aggregate borrowing constraint. Emergence of debt-ridden firms also has congestion effect as in Caballero, Hoshi, and Kashyap (2008), because they purchase the collateralizable asset aggressively and push its price up, discouraging entry of new firms. We show in a version of our model in which the economy grows endogenously that the congestion effect lowers the growth rate of the aggregate productivity to zero if the measure of debt-ridden firms exceeds a certain threshold level and also pessimistic expectations prevail.

Tightening of the aggregate borrowing constraints due to emergence of many debt-ridden borrowers may manifest itself as a “financial shock” during or after a financial crisis. The mechanism of tightening of the borrowing constraint in this model is simple. We can easily embed this model into a standard dynamic stochastic general equilibrium model and assess qualitatively or quantitatively whether emergence of debt-ridden borrowers is a primal cause of the financial shocks in the recent macroeconomic events. We leave this topic for future research.

Appendix A: Derivation of borrowing constraint for normal firms

The borrowing constraint is derived from the argument about what happens if the firm defaults on its debt. There are the following three options for the firm and the lender to do after default.

- Liquidation: The firm manager refuses to pay and walks away. The lender confiscates a part of the asset of the firm.

- Making the firm debt-ridden: In exchange for a continuation fee, the lender allows the firm to continue operation, while she retains the right to liquidate it.
- Debt forgiveness: In exchange for a partial or zero repayment, the lender releases the firm and waives the right to liquidate it.

We need to specify the details of the institutional setting about bargaining after default. So we make the following detailed assumption instead of Assumptions 1 and 2.

Assumption 4. In period t , firm i can default on the debt obligation of $m_t + b_t$ after the firm obtains the proceeds $p(x_t)x_t$. If firm i defaults then the firm and the lender can renegotiate on the amount of repayment f_t .

1. Once firm i defaults, the lender obtains the unilateral discretion to liquidate it. The legal institution in this economy is such that as long as the repayment f_t is strictly less than the original debt ($m_t + b_t$) the lender can retain the right to liquidate firm i . And the lender can retain and exercise this right without any penalty even after she receives f_t ($< m_t + b_t$) from firm i , implying that the lender never sell the right to liquidate firm i at any price cheaper than the original debt.
2. If the lender liquidates the firm in the middle of the period the lender obtains a part of the physical capital $(\phi + \psi)q_t k_t$, where $0 < \phi + \psi < 1$. When firm i is liquidated, the variety i of the intermediate goods just disappears from this economy.
3. If the lender and the firm agree on f_t , which is equal to or greater than the original debt, then the firm regains the right to choose whether or not to continue its own business. Both the lender and the firm can verify that the lender loses the right to liquidate the firm at the moment she receives f_t ($\geq m_t + b_t$).
4. If the lender and the firm agree on f_t , which is strictly less than the original debt, then the lender retains the right to liquidate the firm. In this case at the end of period t , the lender and the firm negotiate over the continuation fee d_{t+1} , which the firm must pay in period $t + 1$ (in addition to the working capital m_{t+1}).
 - (a) If they agree on d_{t+1} the lender allows the firm to continue operating in the next period.
 - (b) If they do not agree on d_{t+1} , the lender liquidates the firm at the end of period t . Since the bargaining over d_{t+1} takes place at the end of period t when a part of capital $\phi q_t k_t$ has already been concealed from the lender, the lender can confiscate only $\psi q_t k_t$ by liquidating the firm at this stage.
 - (c) The debt-ridden firm cannot make savings inter-temporally.

5. Suppose that a debt-ridden firm and the lender agree on d_{t+1} in period t and the firm continues operating in period $t+1$. Unpaid debt of the firm evolves to $B_{t+j+1} = (1 + r_{t+j})\{m_{t+j} + B_{t+j} - f_{t+j}\}$ in period $t + j + 1$ for $j \geq 0$ with $B_t = b_t$, where f_{t+j} is realized repayment in period $t + j$. As long as $B_{t+j} + m_{t+j} > f_{t+j}$ the lender retains the right to liquidate the firm. If $f_{t+1} \geq m_{t+1} + B_{t+1}$ the firm return to a normal firm and the lender loses the right to liquidate it.

Assumption 4 - 1 implies that debt forgiveness is infeasible because as long as repayment is strictly less than the original debt the lender chooses to retain or exercise the right to liquidate it. Thus what can happen after default is either liquidation or continuation of the firm as a debt-ridden firm.

Suppose that firm i defaults on the debt $m_t + b_t$. After default, they negotiate over repayment f_t . If they do not reach an agreement and the lender liquidates the firm, the lender obtains $(\phi + \psi)q_t k_t$, while the firm obtains $p(x_t)x_t + (1 - \phi - \psi)q_t k_t$. If they agree on repayment f_t ($\leq m_t + b_t$), firm i continues operation as a debt-ridden firm. If firm i becomes a debt-ridden firm by paying f_t , it obtains $p(x_t)x_t + q_t k_t - f_t + V_{zt}$ and the lender obtains $f_t + D_t$, where D_t is the present value of the expected cashflow which the lender can receive from a debt-ridden firm from period $t + 1$ on. The bargaining over f_t after firm i defaults on $m_t + b_t$ is described as a Nash bargaining:

$$\max_{f_t} [(\phi + \psi)q_t k_t + V_{zt} - f_t]^\sigma [f_t + D_t - (\phi + \psi)q_t k_t]^{1-\sigma}.$$

We assume that the firm has all the bargaining power for simplicity of analysis, i.e., $\sigma = 1$. Thus the bargaining outcome is $f_t = (\phi + \psi)q_t k_t - D_t$. Here we use $D_t = \psi q_t k_t$, which is shown in Appendix B, to have $f_t = \phi q_t k_t$. Therefore, if firm i defaults on $m_t + b_t$, the lender and the firm will agree on the repayment $f_t = \phi q_t k_t$ and firm i continues as a debt-ridden firm as a result of the bargaining.

Now we specify the condition for firm i not to default on $m_t + b_t$. After receiving the proceeds, $p(x_t)x_t$, if firm i does not default, it obtains $p(x_t)x_t + q_t k_t - m_t - b_t + V_{nt}$. On the other hand, if it defaults, the firm and the lender bargains over repayment f_t , which leads to the agreement $f_t = \phi q_t k_t$, and firm i obtains $p(x_t)x_t + (1 - \phi)q_t k_t + V_{zt}$. The no default condition for firm i is $p(x_t)x_t + q_t k_t - m_t - b_t + V_{nt} \geq p(x_t)x_t + (1 - \phi)q_t k_t + V_{zt}$, which can be rewritten as

$$m_t + b_t \leq \phi q_t k_t + V_{nt} - V_{zt}.$$

Appendix B: Derivation of borrowing constraint for debt-ridden firms

We consider the bargaining between a debt-ridden firm and the lender over the fee d_{t+1} for continuation in the next period. The bargaining takes place at the end of period t .

If the debt-ridden firm and the lender agree on d_{t+1} , the firm obtains $V_{zt}(d_{t+1})$, which depends on d_{t+1} , and the lender obtains $\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + D_{t+1} \right\} \right]$, where \tilde{d}_{t+1} is the realized payment.⁶ Note that the lender and the firm take D_{t+1} as given in period t because D_{t+1} is determined by the bargaining at the end of period $t + 1$ and they have no ability to precommit to the outcome of the future bargaining. If the firm and the lender does not agree on d_{t+1} , then the firm obtains $(1 - \psi)q_t k_t$ and the lender obtains only $\psi q_t k_t$ by liquidating the firm. This is due to Assumption 4 - 4 - (b). The Nash bargaining between a debt-ridden firm and the lender is therefore,

$$\max_{d_{t+1}} \{V_{zt}(d_{t+1}) - (1 - \psi)q_t k_t\}^\sigma \left\{ \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + D_{t+1} \right\} \right] - \psi q_t k_t \right\}^{1-\sigma}.$$

With $\sigma = 1$, the bargaining outcome is

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + D_{t+1} \right\} \right] = \psi q_t k_t. \quad (34)$$

We will describe how d_{t+1} and \tilde{d}_{t+1} are determined shortly.

After d_{t+1} is agreed at the end of period t , the debt-ridden firm is allowed to operate in period $t + 1$. Actions of the debt-ridden firm are as follows. At the end of period t , it sells k_t and buys k_{t+1} . At the beginning of period $t + 1$, it borrows working capital m_{zt+1} , where m_{zt+1} is the material input (which is the consumer good) for the debt-ridden firm. It produces the intermediate good $A_{t+1}k_{t+1}^\alpha m_{zt+1}^{1-\alpha}$ and sells it in the monopolistically competitive market. After it receives the proceeds of sales, it repays the debt $m_{zt+1} + d_{t+1}$.⁷ The debt-ridden firm can default on the debt ($m_{zt+1} + d_{t+1}$) after production. If the firm defaults on the debt, the lender and the firm renegotiate over repayment f . The renegotiation on f is as follows. If the debt-ridden firm and the lender reach an agreement, the debt-ridden firm obtains $p(x_{t+1})x_{t+1} + q_{t+1}k_{t+1} + V_{zt+1} - f$ and the lender obtains $f + D_{t+1}$. If there is no agreement, the lender liquidates the firm. Thus the debt-ridden firm obtains $p(x_{t+1})x_{t+1} + (1 - \phi - \psi)q_{t+1}k_{t+1}$ and exits the market, and the lender obtains $(\phi + \psi)q_{t+1}k_{t+1}$. As the bargaining power of the debt-ridden firm is 1 and that of the lender is zero, we have

$$f = (\phi + \psi)q_{t+1}k_{t+1} - D_{t+1}.$$

The no renegotiation condition for m_{zt+1} implies that $m_{zt+1} \leq \max\{0, f - d_{t+1}\}$. Therefore, the borrowing constraint that m_{zt+1} must satisfy is

$$m_{zt+1} \leq \max\{0, (\phi + \psi)q_{t+1}k_{t+1} - d_{t+1} - D_{t+1}\}.$$

⁶Note that $V_{zt}(d_{t+1})$ here is not V_{zt} in the Bellman equation (9) but the sum of V_{zt} and the value of the capital stock: $V_{zt}(d_{t+1}) = V_{zt} + q_t k_t$.

⁷We assume for simplicity that the redistribution shock Δ_{it} does not hit the debt-ridden firm.

After the bargaining on d_{t+1} is over at the end of period t , the firm purchases k_{t+1} , knowing that its own choice of k_{t+1} directly changes $D_{t+1} = \psi q_{t+1} k_{t+1}$. Therefore, the borrowing constraint imposed on the working capital loan m_{zt+1} at the beginning of period $t + 1$ is rewritten as

$$m_{zt+1} \leq \max\{\phi q_{t+1} k_{t+1} - d_{t+1}, 0\}. \quad (35)$$

Now we specify how d_{t+1} and \tilde{d}_{t+1} are determined. The borrowing constraint (35) implies that if $\phi q_{t+1} k_{t+1} < d_{t+1}$ the firm cannot borrow working capital and is forced to set $m_{zt+1} = 0$ and $\tilde{d} = \phi q_{t+1} k_{t+1}$. Although the firm defaults the lender allows the firm to continue as a debt-ridden firm from the next period on. On the other hand, if $\phi q_{t+1} k_{t+1} \geq d_{t+1}$, then $\tilde{d}_{t+1} = d_{t+1}$. Thus d_{t+1} is the solution to

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left\{ \tilde{d}_{t+1} + \psi q_{t+1} \bar{k}_{t+1} \right\} \right] = \psi q_t k_t,$$

where $\tilde{d}_{t+1} = q_{t+1} \bar{k}_{t+1}$ if $\phi q_{t+1} \bar{k}_{t+1} < d_{t+1}$, and $\tilde{d}_{t+1} = d_{t+1}$ otherwise, where \bar{k}_{t+1} is the expected amount of capital stock that the debt-ridden firm purchases at the end of period t after the agreement on d_{t+1} is reached. We denote it by \bar{k}_{t+1} , not k_{t+1} , because the firm and the lender take it as given when they bargain over d_{t+1} and neither the firm nor the lender can commit to the amount of capital stock (k_{t+1}) which is to be purchased after the bargaining.

In the deterministic case where $\tilde{d}_{t+1} = d_{t+1}$ the value of d_{t+1} is given by the following equation:

$$d_{t+1} = \frac{\psi q_t k_t}{\beta \frac{\lambda_{t+1}}{\lambda_t}} - \psi q_{t+1} \bar{k}_{t+1}. \quad (36)$$

This is because $D_{t+1} = \psi q_{t+1} \bar{k}_{t+1}$.

Appendix C: Neutrality of corporate savings in the deterministic case

In our basic model, firms cannot make savings (Assumption 2 or Assumption 4 - 4 - (c) in Appendix A). Here we consider the case where the debt-ridden firms can accumulate financial assets. In this appendix we focus on the deterministic case. First, we assume that only if the lender liquidates the firm at the end of period t , she obtains $\psi q_t k_t + s_t$, where s_t is the risk-free bond, which the firm buys in period t . Suppose that the debt-ridden firm chooses a positive amount of savings, $s_t > 0$. The bargaining over d_{t+1} at the end of period t leads to $D_t = \beta \frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} = \psi q_t k_t + s_t$. We define the market interest rate r_t by $(1 + r_t)^{-1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$. Thus we have

$$d_{t+1} = (1 + r_t)(\psi q_t k_t + s_t) - D_{t+1}. \quad (37)$$

At the beginning of period $t + 1$, the firm obtains $(1 + r_t)s_t$ from its asset and it borrows $m_{t+1} - (1 + r_t)s_t$ as the working capital. After production, the firm has to repay $m_{t+1} + d_{t+1} - (1 + r_t)s_t$. Then the firm and the lender renegotiate over the repayment f . Note that at this stage of bargaining the firm does not own financial asset s_{t+1} yet. It buys s_{t+1} after the repayment is done. Now we consider the bargaining. If they agree on f , the lender obtains $f + \hat{D}_{t+1}$, where $\hat{D}_{t+1} = \psi q_{t+1}k_{t+1} + \bar{s}_{t+1}$ and \bar{s}_{t+1} is the expected value of s_{t+1} at the stage of the bargaining, while she obtains $(\phi + \psi)q_{t+1}k_{t+1}$ if they do not agree on f . Since the bargaining power of the firm is one, the bargaining outcome is $f = \phi q_{t+1}k_{t+1} - \bar{s}_{t+1}$, which implies the borrowing constraint for the working capital:

$$m_{t+1} + d_{t+1} - (1 + r_t)s_t \leq \phi q_{t+1}k_{t+1} - \bar{s}_{t+1}, \quad (38)$$

Equations (37) and (38) imply that

$$m_{t+1} + (1 + r_t)\psi q_t k_t + \bar{s}_{t+1} - D_{t+1} \leq \phi q_{t+1}k_{t+1}. \quad (39)$$

Since $(1 + r_t)^{-1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$ and $D_{t+1} = \psi q_{t+1}\bar{k}_{t+1} + \bar{s}_{t+1}$, the borrowing constraint (39) reduces to (11), because d_{t+1} in (11) is rewritten as $(1 + r_t)\psi q_t k_t - \psi q_{t+1}\bar{k}_{t+1}$, as (36) implies. Note that s_t does not appear in this constraint, implying that there is no incentive for the debt-ridden firm to accumulate s_t . Thus we have shown that in the deterministic equilibrium, the corporate savings s_t does not affect the equilibrium prices and allocations.

Appendix D: Equilibrium with debt forgiveness

In this appendix we assume that the lender can commit to debt forgiveness and describe the equilibrium after a mass default under the assumption that debt forgiveness is feasible.

In this Appendix we eliminate Assumption 4 - 1 and assume that the lender can forgive the debt of defaulters. We consider two stage bargaining when the firm default on the debt:

- The lender and the firm negotiate whether or not to forgive the debt. If they agree on repayment f , the firm continues as a normal firm after repaying f .
- If they do not agree on debt forgiveness, they bargain whether to allow the firm to continue as a debt-ridden firm or to liquidate it.

We show in what follows that the borrowing constraint for a normal firm is tighter when debt forgiveness is feasible, and that if a mass default described in Section 3.3 occurs in period t all defaulters return to normal firms in period $t + 1$ by debt forgiveness.

Lemma 1. *If debt forgiveness is feasible, the borrowing constraint for a normal firm is*

$$m_t \leq \max\{0, (\phi + \psi)q_t k_t - b_t\}.$$

(Proof) Suppose that the firm defaults on the debt $m_t + b_t$ in period t after receiving the proceeds $p(x_t)x_t$. The outcome of the second stage bargaining whether to make the firm debt-ridden or to liquidate it is the same as in Appendix B. The firm pays $\phi q_t k_t$ to the lender and becomes a debt-ridden firm, whereas the present value of the expected cashflow that the lender can receive from the debt-ridden firm from period $t + 1$ on is $D_t = \psi q_t k_t$.

Now we consider the first stage bargaining. If the firm and the lender agree on repayment f , the firm continues as a normal firm. Thus if the agreement is reached, the firm obtains $p(x_t)x_t - f + q_t k_t + V_n$ and the lender obtains f . If they do not agree on f , they go to the second stage bargaining, in which the firm obtains $p(x_t)x_t + (1 - \phi)q_t k_t + V_{zt}$ and the lender obtains $\phi q_t k_t + D_t = (\phi + \psi)q_t k_t$. Therefore the first stage bargaining is expressed as the following Nash bargaining:

$$\max_f [\phi q_t k_t + V_{nt} - V_{zt} - f]^\sigma [f - (\phi + \psi)q_t k_t]^{1-\sigma},$$

with $\sigma = 1$, which implies that $f = (\phi + \psi)q_t k_t$. The borrowing constraint for m_t is derived from the no default condition, $m_t \leq \max\{0, f - b_t\}$. (End of proof)

Note that the parameters must be chosen such that (8) holds to avoid “all default” equilibrium. See footnote 3. Thus the borrowing constraint for the normal firms in the case where debt forgiveness is feasible is tighter than in the case where it is not.

Next we briefly describe what happens if a mass default occurs in the economy where debt forgiveness is feasible. Suppose that the redistribution shock Δ_{it} is such that $b_{it} > (\phi + \psi)q_t k_t$ for $i \in [0, Z]$. Firm $i \in [0, Z]$ cannot obtain working capital and cannot produce anything in period t . At the end of period t the firms default on b_t and start bargaining on repayment f , which is the same as above. The bargaining outcome is that the firms pay $f = (\phi + \psi)q_t k_t$ and the lenders forgive the remaining debt $b_t - (\phi + \psi)q_t k_t$ and waiver the right to liquidate the firms. The defaulted firms return to normal in period $t + 1$ and the mass default does not generate inefficiency from period $t + 1$ on.

This result presents a stark contrast to the persistent productivity decline in the case where debt forgiveness is infeasible (Section 3.3).

Appendix E: Dynamics when the number of debt-ridden firms is small

We consider only the deterministic equilibrium. Suppose that $N_0 = 1$ and there emerge debt-ridden firms, whose measure is Z , in period 0. For small Z , the economy grows and converges to the BGP eventually. The equilibrium path in which the R&D investment takes place and N_t grows is described by the following system of equations. Note that

$V_{nt} = \chi^{-1}$ in this equilibrium.

$$1 = (1 - \alpha)\eta L^{1-\eta} \hat{A}^\eta (N_t k_{nt})^{\alpha\eta} m_{nt}^{(1-\alpha)\eta-1},$$

$$1 + \mu_{zt} = (1 - \alpha)\eta L^{1-\eta} \hat{A}^\eta (N_t k_{zt})^{\alpha\eta} m_{zt}^{(1-\alpha)\eta-1},$$

$$q_t = \beta \frac{C_t}{C_{t+1}} \left[\alpha \eta L^{1-\eta} \frac{\hat{A}^\eta (N_{t+1} k_{nt+1})^{\alpha\eta} m_{nt+1}^{(1-\alpha)\eta}}{k_{nt+1}} + q_{t+1} \right],$$

$$q_t = \beta \frac{C_t}{C_{t+1}} \left[\alpha \eta L^{1-\eta} \frac{\hat{A}^\eta (N_{t+1} k_{zt+1})^{\alpha\eta} m_{zt+1}^{(1-\alpha)\eta}}{k_{zt+1}} + (1 + \phi \mu_{zt+1}) q_{t+1} \right],$$

$$Y_t = \frac{\hat{A}^\eta N_t^{\alpha\eta}}{\eta} L^{1-\eta} \left\{ Z (k_{zt}^\alpha m_{zt}^{1-\alpha})^\eta + (N_t - Z) (k_{nt}^\alpha m_{nt}^{1-\alpha})^\eta \right\},$$

$$C_t + I_t = Y_t - (N_t - Z) m_{nt} - Z m_{zt},$$

$$K = Z k_{zt} + (N_t - Z) k_{nt},$$

$$d_t = \frac{\psi}{\beta} \frac{C_{t+1}}{C_t} q_t k_{zt} - \psi q_{t+1} k_{zt+1},$$

$$m_{zt} = \phi q_t k_{zt} - d_t,$$

$$\chi^{-1} = -q_t k_{nt+1} + \beta \frac{C_t}{C_{t+1}} \left\{ L^{1-\eta} (\hat{A} N_{t+1}^\alpha k_{nt+1}^\alpha m_{nt+1}^{1-\alpha})^\eta - m_{nt+1} + q_{t+1} k_{nt+1} + \chi^{-1} \right\},$$

$$V_{zt} = -q_t k_{zt+1} + \beta \frac{C_t}{C_{t+1}} \left\{ L^{1-\eta} (\hat{A} N_{t+1}^\alpha k_{zt+1}^\alpha m_{zt+1}^{1-\alpha})^\eta - m_{zt+1} - d_{t+1} + q_{t+1} k_{zt+1} + V_{zt+1} \right\},$$

$$N_{t+1} = N_t + \chi I_t.$$

The following inequality must be satisfied:

$$m_{nt} < \phi q_t k_t + V_{nt} - V_{zt},$$

$$V_{zt} \geq 0.$$

The first inequality is necessary as we assumed that the borrowing constraint for normal firms is not binding in the equilibrium. The second inequality is necessary since otherwise the debt-ridden firm exits because liquidation is preferable to continuing as a debt-ridden firm. As this equilibrium converges to the balanced growth path, it is convenient to define and analyze the following detrended variables: $c_t = C_t/N_t$, $i_t = I_t/N_t$, $y_t = Y_t/N_t$, $\tilde{q}_t = q_t/N_t$, $K_{nt} = N_t k_{nt}$, $K_{zt} = N_t k_{zt}$, $z_t = Z/N_t$, $G_t = N_{t+1}/N_t$. The above equations

are rewritten as the following system of equations of the detrended variables.

$$\begin{aligned}
1 &= (1 - \alpha)\eta L^{1-\eta} \hat{A}^\eta K_{nt}^{\alpha\eta} m_{nt}^{(1-\alpha)\eta-1}, \\
1 + \mu_{zt} &= (1 - \alpha)\eta L^{1-\eta} \hat{A}^\eta K_{zt}^{\alpha\eta} m_{zt}^{(1-\alpha)\eta-1}, \\
\tilde{q}_t &= \beta \frac{c_t}{c_{t+1}} \left[\alpha\eta L^{1-\eta} \frac{\hat{A}^\eta K_{nt+1}^{\alpha\eta} m_{nt+1}^{(1-\alpha)\eta}}{K_{nt+1}} + \tilde{q}_{t+1} \right], \\
\tilde{q}_t &= \beta \frac{c_t}{c_{t+1}} \left[\alpha\eta L^{1-\eta} \frac{\hat{A}^\eta K_{zt+1}^{\alpha\eta} m_{zt+1}^{(1-\alpha)\eta}}{K_{zt+1}} + (1 + \phi\mu_{zt+1})\tilde{q}_{t+1} \right], \\
y_t &= \frac{\hat{A}^\eta}{\eta} L^{1-\eta} \left\{ z_t (K_{zt}^\alpha m_{zt}^{1-\alpha})^\eta + (1 - z_t) (K_{nt}^\alpha m_{nt}^{1-\alpha})^\eta \right\}, \\
c_t + i_t &= y_t - (1 - z_t)m_{nt} - z_t m_{zt}, \\
K &= z_t K_{zt} + (1 - z_t)K_{nt}, \\
d_t &= \frac{\psi}{\beta} \frac{c_{t+1}}{c_t} G_t \tilde{q}_t K_{zt} - \psi \tilde{q}_{t+1} K_{zt+1}, \\
m_{zt} &= \phi \tilde{q}_t K_{zt} - d_t, \\
\chi^{-1} &= -\tilde{q}_t K_{nt+1} + \frac{\beta}{G_t} \frac{c_t}{c_{t+1}} \left\{ L^{1-\eta} (\hat{A} K_{nt+1}^\alpha m_{nt+1}^{1-\alpha})^\eta - m_{nt+1} + \tilde{q}_{t+1} K_{nt+1} + \chi^{-1} \right\}, \\
V_{zt} &= -\tilde{q}_t K_{zt+1} + \frac{\beta}{G_t} \frac{c_t}{c_{t+1}} \left\{ L^{1-\eta} (\hat{A} K_{zt+1}^\alpha m_{zt+1}^{1-\alpha})^\eta - m_{zt+1} - d_{t+1} + \tilde{q}_{t+1} K_{zt+1} + V_{zt+1} \right\}, \\
G_t &= 1 + \chi i_t, \\
z_{t+1} &= z_t / G_t.
\end{aligned}$$

The detrended variables must satisfy

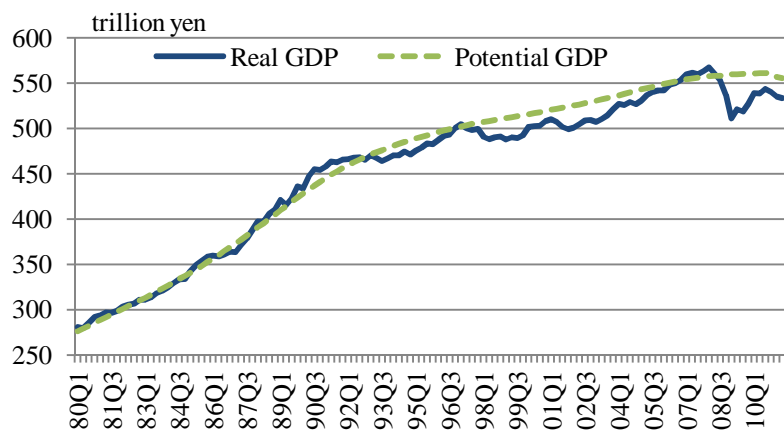
$$\begin{aligned}
m_{nt} &< \phi \tilde{q}_t K_{nt} + V_{nt} - V_{zt}, \\
V_{zt} &\geq 0.
\end{aligned}$$

This system of equation can be solved by the backward shooting method, in which $\{c_t, i_t, y_t, d_t, m_{nt}, m_{zt}, \mu_{zt}, \tilde{q}_t, K_{nt}, K_{zt}, z_t, G_t, V_{zt}\}$ are calculated, given $\{c_{t+1}, i_{t+1}, y_{t+1}, d_{t+1}, m_{nt+1}, m_{zt+1}, z_{t+1}, \mu_{zt+1}, \tilde{q}_{t+1}, K_{nt+1}, K_{zt+1}, G_t, V_{zt+1}\}$. *** The result of numerical simulation is shown in Figure ???. The economy monotonically converges to the BGP. The theory predicts that the economy grows and converges to the BGP if and only if $Z = z_0$ is smaller than a threshold value, which is *** in the numerical example, and it falls into the ZGP otherwise. The numerical result confirms this prediction as Figure ?? shows that there exists the path converging to the BGP only for the initial value z_0 which is smaller than ***. The technical appendix that describes the solution procedures is available from authors upon request. ***

References

- Acemoglu, Daron (2009) *Introduction To Modern Economic Growth*: Princeton University Press.
- Bartelsman, Eric J. and Mark Doms (2000) “Understanding Productivity: Lessons from Longitudinal Microdata,” *Journal of Economic Literature*, Vol. 38, No. 3, pp. 569–594, September.
- Bilbiie, Florin, Fabio Ghironi, and Marc J. Melitz (2007) “Endogenous Entry, Product Variety, and Business Cycles,” *NBER Working Papers*, No. 13646, November.
- Caballero, Ricardo J., Takeo Hoshi, and Anil K. Kashyap (2008) “Zombie Lending and Depressed Restructuring in Japan,” *American Economic Review*, Vol. 98, No. 5, pp. 1943–1977, December.
- Chen, Hui and Gustavo Manso (2010) “Macroeconomic Risk and Debt Overhang.” mimeo.
- Christiano, Lawrence, Roberto Motto, and Massimo Rostagno (2011) “Risk Shocks.” mimeo.
- Fukao, Kyoji and Tsutomu Miyagawa eds. (2008) *Productivity and Japan’s Economic Growth*, Tokyo: University of Tokyo Press. (in Japanese).
- Hayashi, Fumio and Edward C. Prescott (2002) “The 1990s in Japan: A Lost Decade,” *Review of Economic Dynamics*, Vol. 5, No. 1, pp. 206–235, January.
- Jermann, Urban and Vincenzo Quadrini (2006) “Financial Innovations and Macroeconomic Volatility,” *NBER Working Papers*, No. 12308, June.
- (2012) “Macroeconomic Effects of Financial Shocks,” *American Economic Review*, Vol. 102, No. 1, pp. 238–271, February.
- Kawamoto, Takuji (2005) “What Do the Purified Solow Residuals Tell Us about Japan’s Lost Decade?” *Monetary and Economic Studies*, Vol. 23, No. 1, pp. 113–148, February.
- Kehoe, Timothy J. and Edward C. Prescott eds. (2007) *Great Depressions of the Twentieth Century*, Minneapolis: Federal Reserve Bank of Minneapolis.
- Kiyotaki, Nobuhiro and John Moore (1997) “Credit Cycles,” *Journal of Political Economy*, Vol. 105, No. 2, pp. 211–248.
- Kobayashi, Keiichiro and Masaru Inaba (2006) “Business cycle accounting for the Japanese economy,” *Japan and the World Economy*, Vol. 18, No. 4, pp. 418–440, December.

- Krugman, Paul R. (1988) “Financing vs. forgiving a debt overhang,” *Journal of Development Economics*, Vol. 29, No. 3, pp. 253–268, November.
- Lamont, Owen (1995) “Corporate-Debt Overhang and Macroeconomic Expectations,” *American Economic Review*, Vol. 85, No. 5, pp. 1106–1117, December.
- Moyen, Nathalie (2007) “How big is the debt overhang problem?” *Journal of Economic Dynamics and Control*, Vol. 31, No. 2, pp. 433–472, February.
- Myers, Stewart C. (1977) “Determinants of corporate borrowing,” *Journal of Financial Economics*, Vol. 5, No. 2, pp. 147–175, November.
- Nishimura, Kiyohiko G., Takano, Nobu, and Kiyota Kozo (2005) “Does the natural selection mechanism still work in severe recessions?: Examination of the Japanese economy in the 1990s,” *Journal of Economic Behavior & Organization*, Vol. 58, No. 1, pp. 53–78, September.
- Ohanian, Lee E. (2001) “Why Did Productivity Fall So Much during the Great Depression?” *American Economic Review*, Vol. 91, No. 2, pp. 34–38, May.
- Peek, Joe and Eric S. Rosengren (2005) “Unnatural Selection: Perverse Incentives and the Misallocation of Credit in Japan,” *American Economic Review*, Vol. 95, No. 4, pp. 1144–1166, September.
- Reinhart, Carmen M. and Vincent R. Reinhart (2010) “After the Fall,” *NBER Working Papers*, No. 16334, September.
- Reinhart, Carmen M. and Kenneth S. Rogoff (2009) *This Time Is Different: Eight Centuries of Financial Folly*, Princeton: Princeton University Press.
- Rivera-Batiz, Luis A and Paul M Romer (1991) “Economic Integration and Endogenous Growth,” *The Quarterly Journal of Economics*, Vol. 106, No. 2, pp. 531–555, May.



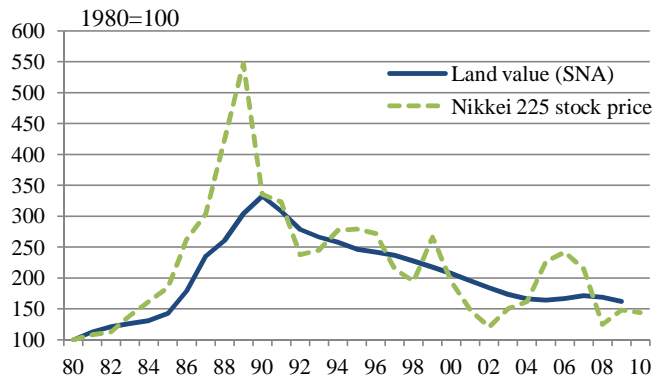
Source: Cabinet Office, Government of Japan, “Annual Report on National Accounts”

Figure 1: Real GDP and Potential GDP in Japan

	HP	KI	JIP2011
1971–80	0.83		1.68
1981–90	1.93	2.06	1.39
1991–2000	0.36	0.35	0.04
2001–2007		0.48	1.13

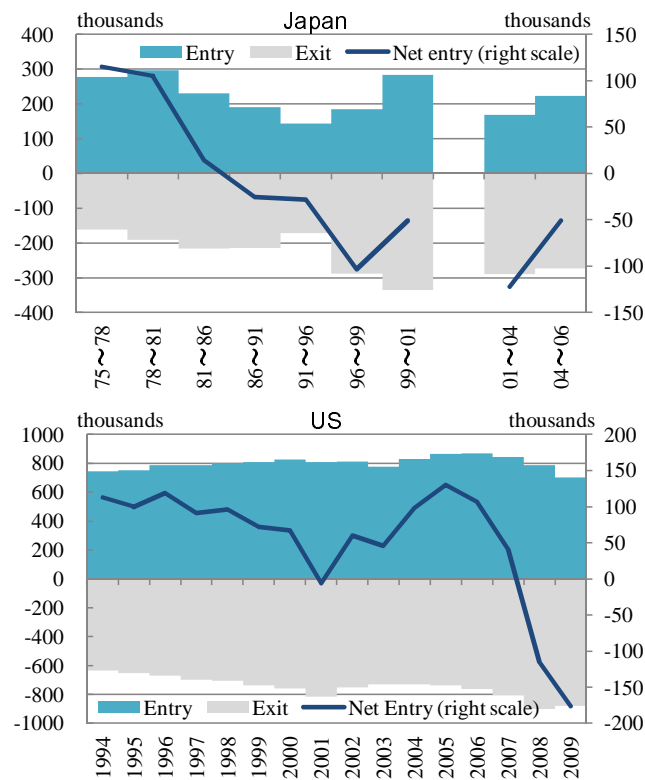
Note: HP, KI, JIP2011 are from updated versions of Hayashi and Prescott(2002), Kobayashi and Inaba (2006), and Fukao and Miyagawa (2008).

Table 1: TFP growth rate in Japan



Sources: Cabinet Office, Government of Japan; Nihon Keizai Shimbun.

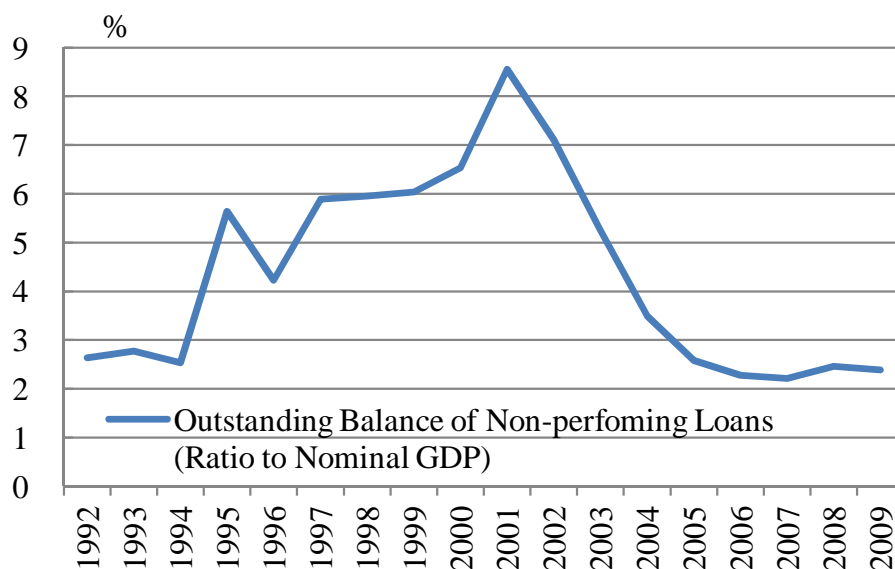
Figure 2: Land value and Stock price in Japan



Note: Japan's figures after 2001 are based on 1993-basis industry classification.

Sources: (Japan) Ministry of Internal Affairs and Communications, "Establishment and Enterprise Census"; (US) Bureau of Labor Statistics, "Business Employment Dynamics".

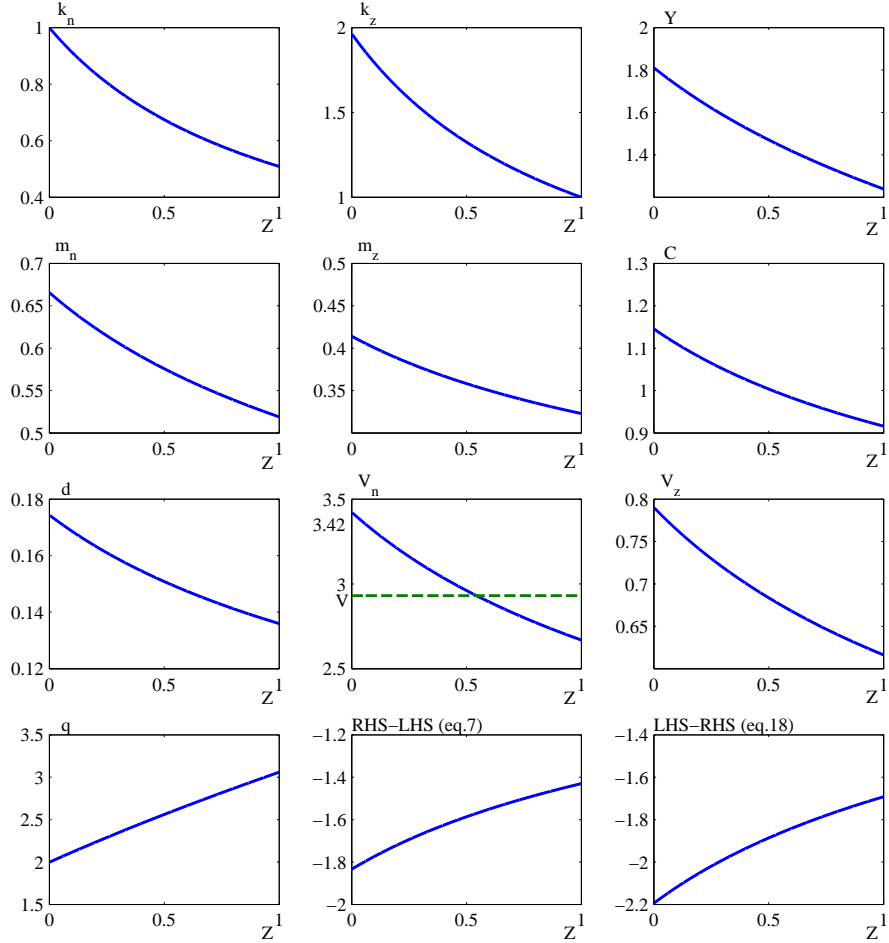
Figure 3: Private sector establishment entry and exits: US and Japan



Note: The non-performing loans is the Risk Management Loans (RMLs) defined in the Banking Act in Japan, which consist of loans to bankrupt borrowers, delayed loans, 3 month overdue loans, and loans with modified terms and conditions. RMLs do not include securitized loans.

Sources: Financial Services Agency, *Status of Non-Performing Loans*; Cabinet Office, Government of Japan, *Annual Report on National Accounts*.

Figure 4: Development of the non-performing loans

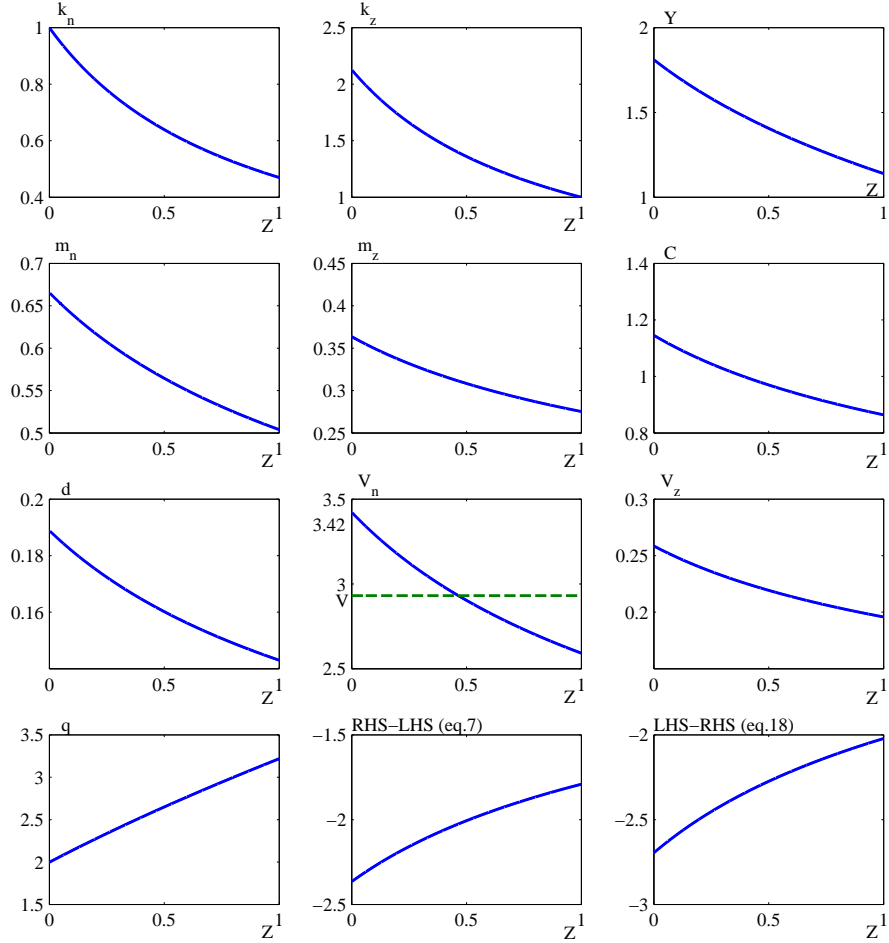


Parameters: $\alpha = 0.25$, $\beta = 0.9$, $\phi = 0.15$, $\eta = 0.7$, $A = 1.9048$,

$K = 1$, $L = 1$, $\psi = 0.4$, $\chi_{SS}^{-1} = 3.4234$, $V = \chi_{BGP}^{-1} = 2.9307$.

Results: $\mu_z = 0.41$, $\Lambda = 1.9648$, $m = 0.6657$, $g = 0.01$. If $z \geq 0.5439$, $Vn < \chi_{BGP}^{-1}$.

Figure 5: Equilibrium with debt-ridden firms



Parameters: $\alpha = 0.25$, $\beta = 0.9$, $\phi = 0.13$, $\eta = 0.7$, $A = 1.9048$,

$K = 1$, $L = 1$, $\psi = 0.4$, $\chi_{SS}^{-1} = 3.4234$, $V = \chi_{BGP}^{-1} = 2.9307$.

Results: $\mu_z = 0.5210$, $\Lambda = 2.1283$, $m = 0.6657$, $g = 0.01$. If $z \geq 0.4651$, $Vn < \chi_{BGP}^{-1}$.

Figure 6: Equilibrium with debt-ridden firms: Multiple equilibria