Knowledge, Diffusion and Reallocation

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- Quantitatively important
- Contributes to productivity and diffusion of new ideas
- Recent contributions: barriers to reallocation very costly
- Results are very sensitive to assumptions about returns to scale or demand elasticity.

Introduction

Returns to scale and knowledge transmission

- Fixed factors.vs. replication
- Links to knowledge transmission – in general costly to replicate.
- What is fixed or not may depend on incentives for knowledge transmission
- Develop a deeper theory of replication/knowledge transmission

Roadmap

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2. Theory of learning and replication of knowledge
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4. Links to firm dynamics.

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- $\alpha < 1$, then smaller gains.

Example: productivity gains

	$z = 1.5$	$z = 2$	$z = 4$
$\alpha = 0.5$	2%	5%	17%
$\alpha = 0.8$	7%	17%	39%
$\alpha = 1$	20%	33%	60%

- Gains depend on returns to scale and speed of reallocation.
- Maximum with CRS (case $\alpha = 1$)
- Models explicitly or implicitly make assumptions about this.

Predicted gains from lowered barriers:

Hopenhayn and Rogerson	5%	eliminate layoff costs
Eaton and Kortum	3.5%	loss going to autharky
Burstein Monge	8% to 15%	zero cost to FDI
Ramondo	50%	zero costs to FDI
McGrattan and Prescott	30%	form union 20 countries

A model of learning and diffusion

- GE economy, fixed labor endowment L, representative agent – preferences: balanced growth

$$\begin{aligned} U &= \int e^{-rt} \frac{c\left(t\right)^{1-\theta}}{1-\theta} dt \\ r &= \rho + \theta g \end{aligned}$$

- Technology: Solow (vintage model) meets Lucas (adjustment cost)
- Basic component: knowledge capital pair (z, k) : z is knowledge embodied in this k
- production technology $zf(k, n)$, CRS
- $f(k, n) = \min(k, n)$ and $\theta = 1$ for this talk.

Technology for replication

- $\dot{k}(z)$ has cost $C\left(\frac{\dot{k}}{k}\right) zk$, depreciation δk.
- $C(\cdot)$ increasing, convex.
- CRS in $k, \dot{k}$
- More costly to replicate better knowledge

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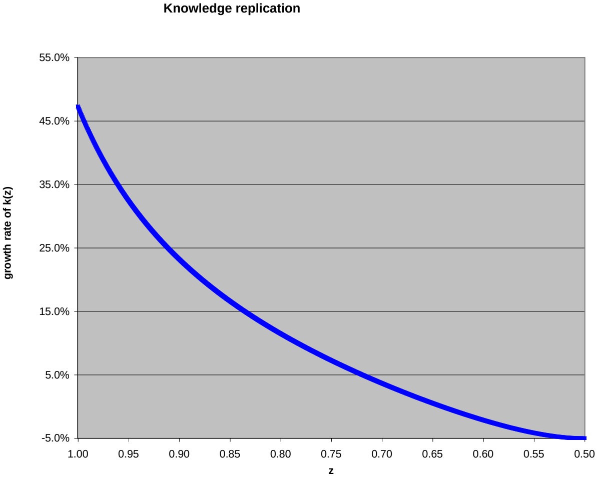
  - $\dot{k}(z) / k(z)$ increasing in z

Rates of replication



Entry of new vintages

- Technological frontier $\gamma(t) = e^{gt}$
- Technology for entry: one worker $\longrightarrow$ k_0 units of knowledge capital of type $x\gamma(t)$
- $x \in [0, 1] \sim F(dx)$
- Heterogeneity in productivity within a cohort
- Coexistence of several cohorts

Equilibrium with no new arrivals of z

- $g = 0$
- Fixed labor endowment L
- Initial distribution $k(z)$ with highest $\bar{z}$
- Converge to steady state: $k(\bar{z}) = L$ (if $C'(0) = 0$)
- Complete reallocation: all resources flow to most productive

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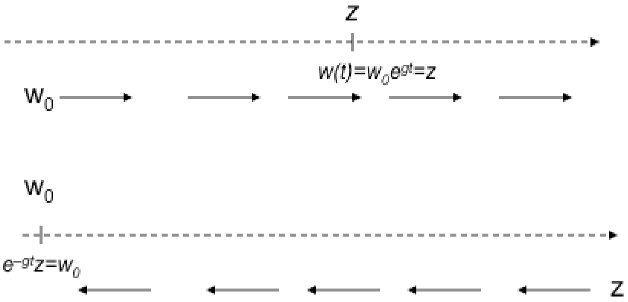

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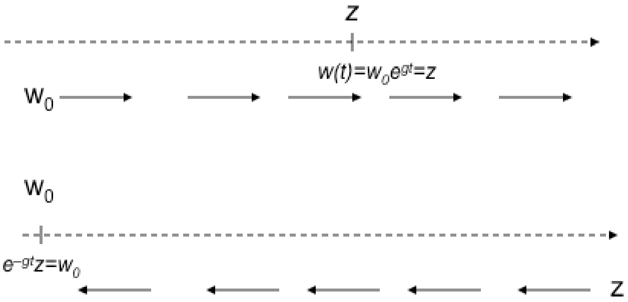

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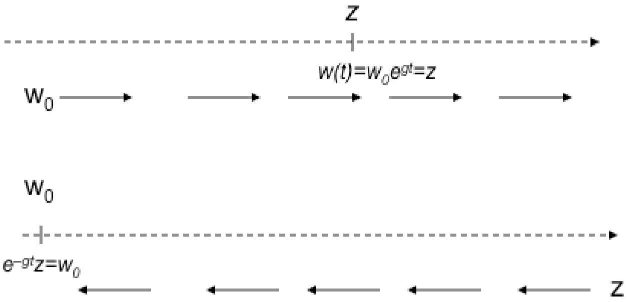

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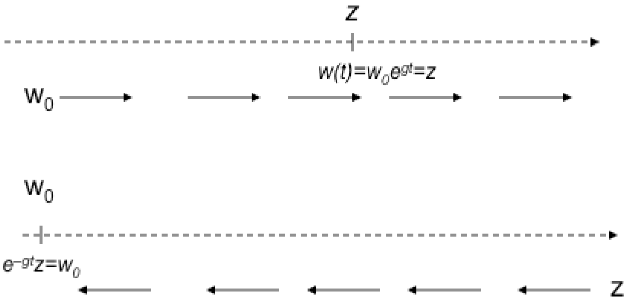

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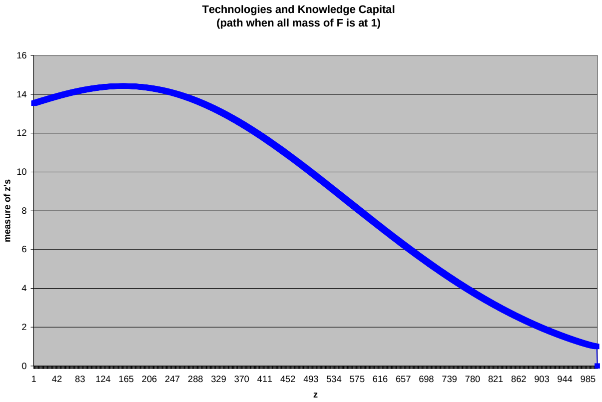
- Vintage is active and producing while $z(t) > w_0$. When $z(t) = w_0$ it is discontinued and the stock $k(t)$ lost.

Life cycle of new innovation

- Innovator starts with k_0 units of knowledge capital
  $z_0 \in [0,1]\,, \gamma(t) = 1$
- Active only if $z_0 > w\,(t)$.
- Replicates at declining rate
- Shut down after s periods when $z_0 = e^{gs} w\,(t)$

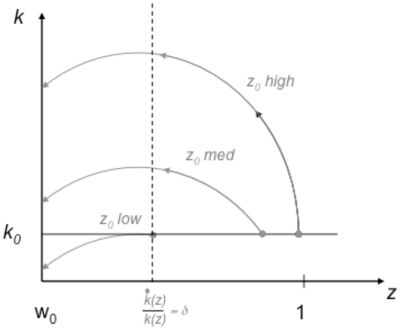
Life cycle of innovation



Stationary distribution of knowledge capital

- Normalize $\gamma(t) = 1$
- constant flow of entry m
- Steady state – stationary distribution $k(z)$

Path of knowledge capital for different initial z_0' s



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- $L_R + L_P = L$
- flow of entry $m = k_0 L_R$
- Total labor demand linear in L_R.

Reallocation and the incentives for replication

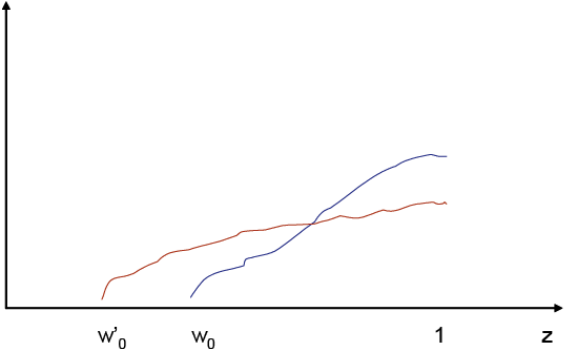
Productivity of R&D

- An increase in k_0 (productivity of frontier research)
  - Increases w_0
  - Lowers $v(z, w_0)$ and thus $\dot{k}(z)/k$ for all z
  - Decreases lifetime of vintages and slows down diffusion
- Similar effect for improvement in F

Reallocation and the incentives for replication

Higher rate of technological advance

- increase in rate of obsolescence
- Decreases w_0 (ratio $w(t)/\gamma(t)$)
- Increases $v(z, w_0)$ for low $z\prime s$ but reduces it for high $z's$.



- Intuition: Low z discounts more the future (when wage will be higher)
- Flattens replication profile and lowers gain of drawing better $z's$

Reallocation and the incentives for replication

Higher cost of replication

- $\gamma C\left(\dot{k}\right)$, increase in γ
- Bigger direct impact on higher $z's$ (envelope argument)
- w_0 decreases
- Again, flatten $v\left(z, w_0\right)$ profile.
- Lower replication.

Connection to decreasing returns

- Static allocation: $\ln n = A + \frac{1}{1-\alpha} \ln z$
  Higher elasticity of n with respect to z iff higher α
- Consider $n(z,a) = k(z,a)$ employment of one original unit of type z after a periods
- $\partial \ln k(z,a) / \partial \ln z = \frac{\dot{k}(z) - \dot{k}(ze^{-ga})}{g}$
- For case $C(\dot{k}) = c\dot{k}^2/2$ equals $\left[\frac{v(z)}{z} - \frac{v(ze^{-ga})}{ze^{-ga}}\right] / gc$
- Elasticity falls with c.

Policy experiment

- Tax on investment $tC\left(\dot{k}\left(z\right)\right)z$
- Tax decreases investment and reallocation from less to more productive
- Impact: depends on importance of reallocation
- 3 scenarios: baseline, high adjustment cost, high g

Scenarios

- Baseline, high adjustment cost, high g
- $C\left(\dot{k}\right)$ quadratic;
- $r = 5\%,\ \delta = 5\%,\ g = 3\%$; baseline $w_0 = 0.5$ (Bartelsman and Domes)
- $F(z) = \frac{1-\exp(-\lambda z)}{1-\exp(-\lambda)}$, $\lambda = 2$

Impact of tax

- Higher taxes reduce the incentive to invest.
- lowers equilibrium wage
- Less turnover of knowledge chains
- Lower average productivity

Impact of tax

	Base case		high adj. cost		high g	
	w_0	prod	w_0	prod	w_0	prod
$t = 0$	100	100	93.7	93.7	83.9	80.7
$t = 0.5$	-6.5%	-2.1%	-3.5%	-0.9%	-3.9%	-0.8%
$t = 1.0$	-9.8%	-4.1%	-6.1%	-2%	-6.4%	-1.7%

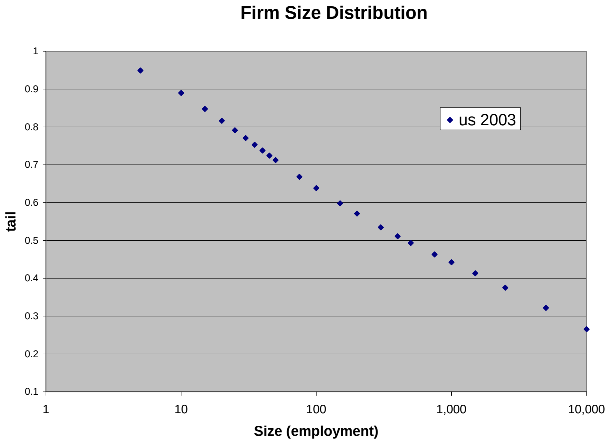
Firms and Knowledge Capital

- Above considers only allocations. Data is based on firms/plants.
- Quantitative discipline
- Help explain some facts?
- Recent paper Luttmer - motivation: explaining the rapid growth of large firms.
- How can we take this model to the data?

Some motivating facts

- Large degree of reallocation: 10% yearly job creation and destruction (Davis, Haltiwanger, Schuh)
- Fairly large changes in firm size over 10 year horizon period.
- Growth rate independent of sizes (Gibrat's law)
- Growth rate decreasing in age
- Productivity differences are persistent (Bartelsman and Doms)
- Low productivity helps predict exit.
- Entry and exit play an important role (15% of yearly job creation and 20% of job destruction.)

Size distribution - Zipf's law



Firm as one Knowledge chain and no obsolescence (g=0)

Counterfactual implications:

1. Firms grow at different rates
2. No heterogeneity in the long run

Obsolescence - g>0

- If firms homogeneous (standard vintage model)
  - strong firm life-cycle
  - death increases with age
- Sources of heterogeneity:
  1. Initial draws
  2. random success in staying at the frontier/upgrading

Heterogeneity in initial draws

- Distribution $F(s)$
- Depreciates relative to the frontier at rate g
- Exit rates and age: more flexibility but increases at some point
- Older firms tend to be larger
- Productivity decreases with age in some range.
- Still some strong life cycle effects

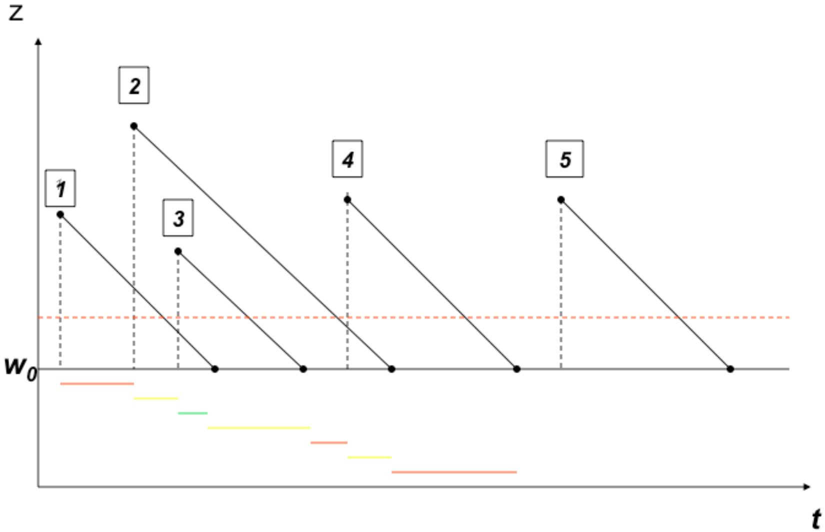
2. Random success in staying at the frontier

- Potential way of getting firms to grow fast for longer time
- Indivisibility? What is a firm?
  - A firm as specialized knowledge capital.
  - Frontier could move randomly, perhaps drastic
  - Or smoother: pieces of knowledge capital may fail to learn
- Promising road

Firms as portfolios of knowledge capital.

- Lots of R&D done in existing firms
- State of the firm $(z_1, k_1, z_2, k_2..., z_n, k_n)$, new draws arrival rate m.
- Firm grows or contracts. When number of $z's$ in operation goes to zero, consider an exit. Substituted by a draw from an outsider.
- Simple aggregation procedure - no change in behavior.

Firm's life-cycle



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  - Productivity of lower end $= 0.5$
  - Growth rate $g = 0.03$
  - $\Longrightarrow$ 24 years to go from frontier to 0.5
  - Expected turnover $= 1/24 = 4\%$ is too small *even without resampling*.

Stochastic draws (distribution F)

- Very few *free* parameters: λ, w_0, g, m where w_0 and g are pinned down.
- Growth and size: Gibrat's law (sort of)

Table 1. growth vs size and age

Variable	Estimate	Standard error	t-value
constant	0.07	0.002384	29.1
size	0.002	0.001098	1.6
age	-0.002	0.000088	-27.5

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- *Survival:* Firms running more vintages are less likely to exit - also tend to be larger.

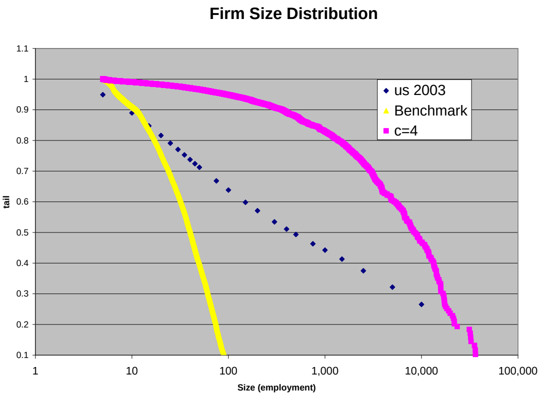
	model	US
size of entrants/incumbents	45%	35%
Rate of entry/exit (annual)	5%	7%
job creation/destruction rate	7%	10%
Share of entry/exit	30%	20%

Age, size, productivity and growth

age	size/avg	average z	growth	B&D
less than 5	0.3	0.71	8.6%	7.7%
5 to 10	0.6	0.69	5.7%	3.7%
10 to 15	0.9	0.65	2.6%	2.9%
15 to 25	1.1	0.60	0.1%	
25 to 50	1.0	0.61	0.4%	
more than 50	1.1	0.60	-0.2%	
total	1.0	0.61	2.0%	

Size distribution

- Zipf's law: $(1 - F(size)) = size^{-1}$



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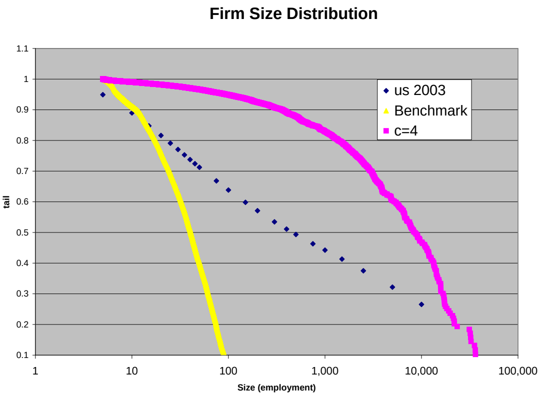

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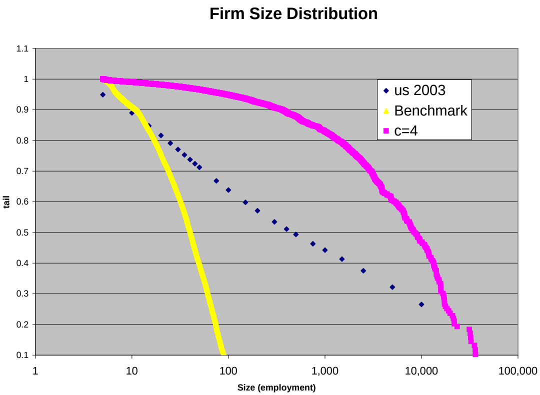

- Fails in our benchmark (too few large firms)
- With lower $C0$, "missing middle".

Final Remarks

- The study of diffusion through replication of knowledge important area.
- Important to understand the gains from reallocation and overall productivity.
- Reduced-form returns to scale have implicit assumptions about replication.
- Incentives to replicate may vary significantly across economies, time and space
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