Asset Bubbles and Bailout

Tomohiro Hirano (University of Tokyo)
Noriyuki Yanagawa (University of Tokyo)

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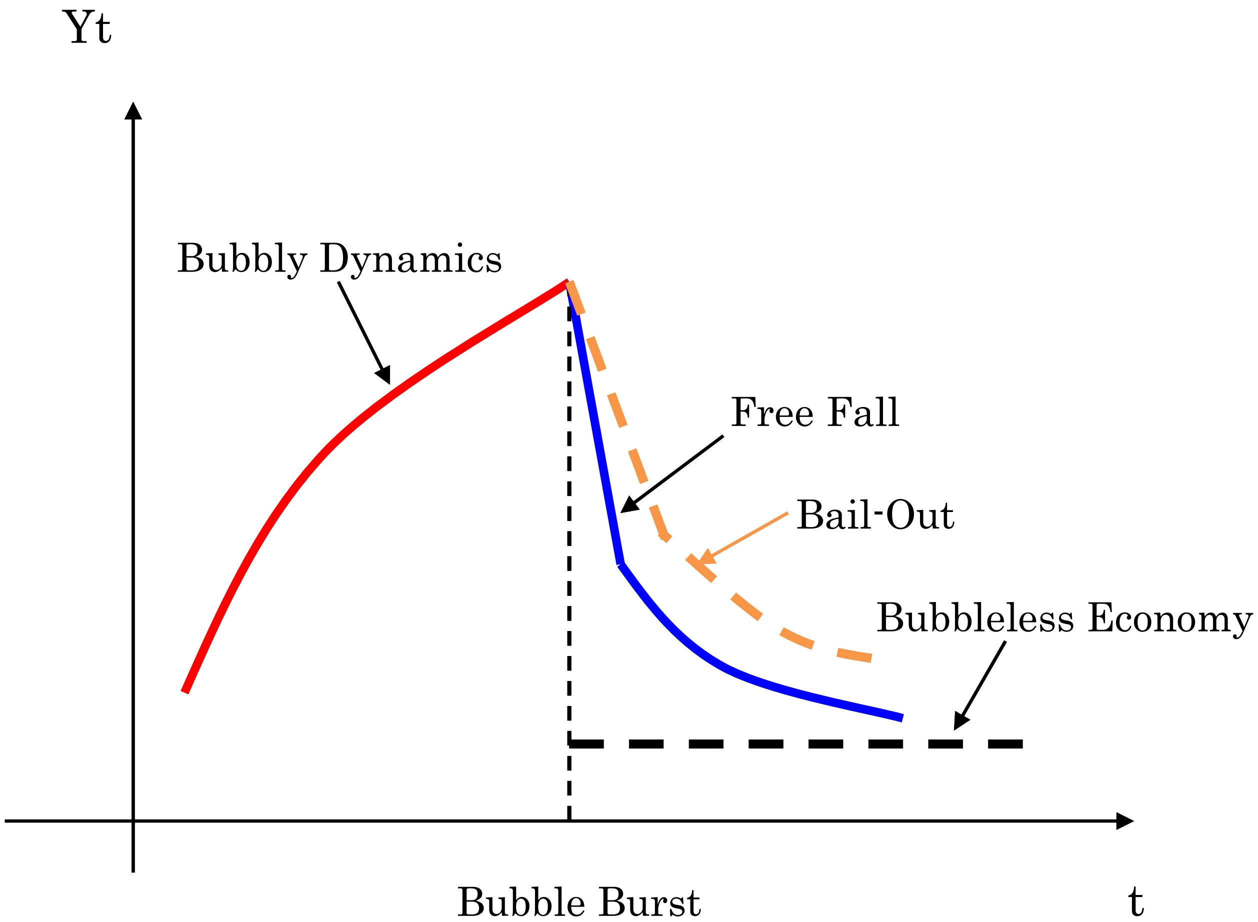
Motivations

- Bubble bursts cause a free-fall in real economic activity.
- To mitigate it, government takes various types of bailouts:

  Capital injection$\rightarrow$recapitalizations of the net worth

Although bailouts may mitigate adverse effects of the bubble bursts, what happens if bailout policies are anticipated?

Boom and Bust of Asset Bubbles



Questions

- How does the anticipated bailout affect the emergence of bubbles?
- To what extent is ex-post bailout efficient from ex-ante perspective?
- How does the anticipated bailout affect boom-bust cycles?

Finally, we discuss the effects of bailout on tax payers (households)?

Bailout we consider:

Government levies a lump sum tax on households when bubbles burst.

Direct transfer to entrepreneurs with bubble assets

→Households pay direct costs for the bubble bursts.

Bailout(Conti):

A fraction $\lambda \in [0, 1]$ of entrepreneurs with bubble assets is rescued.

- $\lambda = 1$ means all entrepreneurs are rescued.
- $\lambda = 0$ means no entrepreneurs are rescued.

An increase in λ means an expansion of bailout, i.e., more entrepreneurs are rescued.

From ex-ante perspective, each entrepreneur anticipates government bailout with probability λ .

Related Literature on bailout (monetary policy)

- Three period model:

Farhi and Tirole(2009, 2011), Diamond and Rajan(2011)

- Dynamic macro model:

Gertler, Kiyotaki, and Queralto(2011)

We analyze the effect of bailout on *stochastic bubbles* in a dynamic macro model.

Related Literature on rational bubbles

Recent developments:

Woodford(1990), Caballero/Krishnamurthy(2006), Kiyotaki/Moore(2008), Farhi and Tirole (2009), Kocherlakota(2009), Hirano/Yanagawa(2010), Martin/Ventura(2010), Aoki/Nikolov(2011)

We consider bailout in a rational bubbles model.

Model: A extension of Kiyotaki (1998)

- Infinitely-lived agents model
- Two types of goods: consumption and capital goods
- Agents: entrepreneurs and households (workers)
  - ·entrepreneurs with H-projects (H-entrepreneurs)
  - ·entrepreneurs with L-projects (L-entrepreneurs)
- Investment technologies follow

$$k^i_{t+1} = \alpha^i_t z^i_t \tag{1}$$

$z^i_t (\geq 0)$ is the investment at date t, and k^i_{t+1} is the capital at date $t+1$. α^i_t is the marginal productivity of investment at date t. $\alpha^H > \alpha^L$.

In this economy, there are competitive firms which produce consumption goods using capital and labor.

- Production function

$$y_t = k_t^{\sigma} n_t^{1-\sigma}, \tag{2}$$

  k_t and n_t are capital goods and labor input at date t. y_t is output at date t.

For simplicity, we assume that capital fully depreciates in one period.

Factors of production are paid their marginal product:

$$q_t = \sigma K_t^{\sigma-1} \quad \text{and} \quad w_t = (1-\sigma) K_t^{\sigma}. \tag{3}$$

K_t is aggregate capital goods. w_t is wage rate.

Each household (worker) supplies one unit of labor inelastically in each period, and earns wage income.

In equilibrium,

$$c_t^u = w_t - T_t$$

$T_t > 0$ if bubbles collapse.

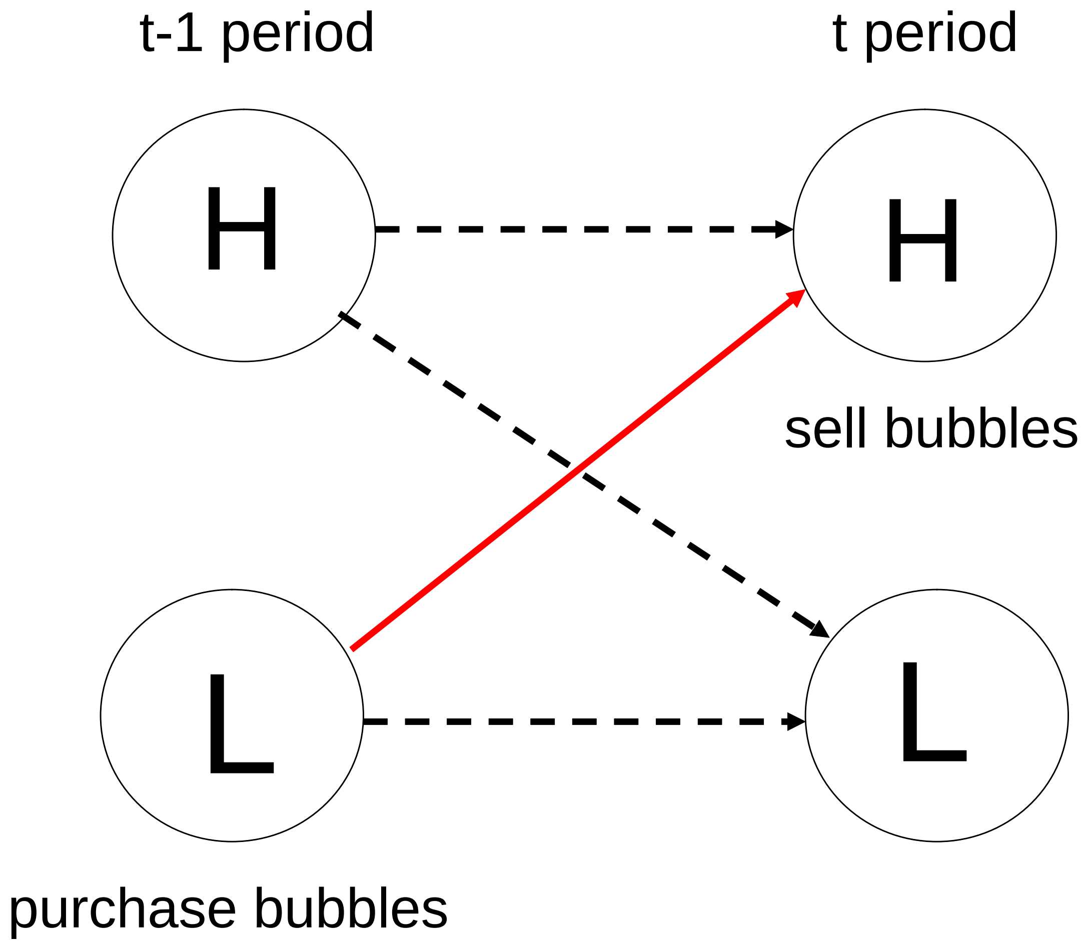
Two Technical assumptions

- At each date, each entrepreneur meets H-projects with probability p, and L-projects with probability $1-p$. The entrepreneur knows his own type of date t.

  →Type of entrepreneurs changes over time.

- Borrowing constraint

  →The entrepreneur cannot borrow as much as they want.


t-1 period
t period
H
H
sell bubbles
L
L
purchase bubbles

Entrepreneur's Maximization Problem

$$\underset{\{c_t^i,\, z_t^i,\, b_t^i,\, x_t^i\}_{t=0}^{\infty}}{Max} \qquad E_0\left[\sum_{t=0}^{\infty} \beta^t \log c_t^i\right]$$

$$c_t^i + z_t^i + P_t x_t^i = q_t k_t^i - r_{t-1} b_{t-1}^i + b_t^i + P_t x_{t-1}^i, \tag{4}$$

$$r_t b_t^i \leq \theta q_{t+1} \alpha_t^i z_t^i, \tag{5}$$

$$x_t^i \geq 0, \tag{6}$$

x_t^i is the level of bubble assets purchased by a type i entrepreneur at date t, and (5) is the borrowing constraint, and (6) is the short sale constraint. P_t is the per unit price of bubble assets in terms of consumption goods on survive at date t.

- Stochastic Bubbles (Weil 1989)

Here we assume the following Markov chain:

$$\Pr(P_t > 0 \mid P_{t-1} > 0) = \pi,$$

Once bubbles burst, they never arise again.

- Bailout Scheme

Each entrepreneur anticipates government bailout with probability λ.

For H-types:

- If $q_{t+1}\alpha^H > r_t$, BC and SSC bind.

$$z_t^i = \frac{1}{1 - \frac{\theta q_{t+1}\alpha^H}{r_t}} \beta(q_t k_t^i - r_{t-1} b_{t-1}^i + P_t x_{t-1}^i), \qquad (7)$$

$$e_t^i \equiv y_t^i - r_{t-1} b_{t-1}^i + P_t x_{t-1}^i.$$

- Thanks to log-utility,

$$c_t^i = (1-\beta)(q_t k_t^i - r_{t-1} b_{t-1}^i + P_t x_{t-1}^i) \qquad (8)$$

Effects of bubbles on investment

Two competing effects

- Crowd-in Effect
  Bubbles→increase net worth→finance more investment
  ⇒Bubbles generate "balance sheet effects".

- Crowd-out Effect(Traditional Tirole Effect)

Which one of these dominates depends on α^H.

For L-types: BC and SSC do not bind.

They decide optimal portfolio allocations.

Risk averse L-types allocate their savings between L-projects(safe), lending to H-types(safe), and bubble assets(risky).

$$z_t^i + P_t x_t^i - b_t^i = \beta e_t^i \tag{9}$$

They allocate their savings, so that marginal expected utility is equalized between three assets.

F.O.Cs with respect to z_t^i, b_t^i, and x_t^i,

$$(z_t^i): \frac{1}{c_t^i} = \pi\beta\frac{q_{t+1}\alpha^L}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{q_{t+1}\alpha^L}{c_{t+1}^{i,(1-\pi)\lambda}} + (1-\pi)(1-\lambda)\beta\frac{q_{t+1}\alpha^L}{c_{t+1}^{i,(1-\pi)(1-\lambda)}} + \mu_t^i, \tag{10}$$

$$(b_t^i): \frac{1}{c_t^i} = \pi\beta\frac{r_t}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{r_t}{c_{t+1}^{i,(1-\pi)\lambda}} + (1-\pi)(1-\lambda)\beta\frac{r_t}{c_{t+1}^{i,(1-\pi)(1-\lambda)}}, \tag{11}$$

$$(x_t^i): \frac{P_t}{c_t^i} = \pi\beta\frac{P_{t+1}}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{1}{c_{t+1}^{i,(1-\pi)\lambda}}\frac{m_{t+1}^i}{x_t^i}, \tag{12}$$

- $c_{t+1}^{i,\pi} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i + P_{t+1}x_t^i)$
- $c_{t+1}^{i,(1-\pi)\lambda} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i + m_{t+1}^i)$
- $c_{t+1}^{i,(1-\pi)(1-\lambda)} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i)$

We analyze a bailout that fully recovers the net worth, $m_{t+1}^i = P_{t+1}x_t^i$.

From this portfolio decisions,

$$P_t x_t^i = \frac{\delta \frac{P_{t+1}}{P_t} - r_t}{\frac{P_{t+1}}{P_t} - r_t} \beta e_t^i \qquad (13)$$

$$z_t^i - b_t^i = \frac{(1-\delta)\frac{P_{t+1}}{P_t}}{\frac{P_{t+1}}{P_t} - r_t} \beta e_t^i . \qquad (14)$$

$\delta \equiv \pi + (1-\pi)\lambda.$

A rise in λ induces L-types to take on more risks, i.e., they are willing to buy more bubble assets.

In equilibrium,

- If $\alpha_1^H < \alpha^H < \alpha_2^H$, collateral value is low.

$$z_t^L > 0.$$

- If $\alpha^H \geq \alpha_2^H$, collateral value is high.

$$z_t^L = 0.$$

L-types invest in L-projects for risk-hedge when collateral value is low.

Maximization Problem of Household

$$\underset{\{c_t^u, b_t^u, x_t^u\}_{t=0}^{\infty}}{Max} \quad E_0\left[\sum_{t=0}^{\infty} \beta^t \log c_t^u\right] \tag{15}$$

$$c_t^u + P_t x_t^u = w_t - r_{t-1} b_{t-1}^u - T_t + b_t^u + P_t x_{t-1}^u, \tag{16}$$

$$r_t b_t^u \leq 0, \tag{17}$$

$$x_t^u \geq 0. \tag{18}$$

Each worker is endowed with one unit labor endowment at each period, which is supplied inelastically in the labor market and earns wage rate.

The Definition of Competitive Equilibrium

Competitive equilibrium is a set of prices $\{r_t, q_t, w_t\}_{t=0}^{\infty}$ and quantities $\{C_t^H, C_t^L, C_t^u, B_t^H, B_t^L, B_t^u, Z_t^H, Z_t^L, G_t, X_t, K_{t+1}, Y_t\}_{t=0}^{\infty}$ such that

(i) Each entrepreneur (as well as each household) maximizes his/her expected discounted utility under the constraint, choosing $\{c_t^i, b_t^i, z_t^i, x_t^i\}_{t=0}^{\infty}$.

(ii) Each firm maximizes its profit.

(iii) All markets clear.

- Credit Market Clearing Condition:

$$B_t^H + B_t^L + B_t^u = 0 \tag{19}$$

$\sum_{i \in H_t} b_t^i \equiv B_t^H$, $\sum_{i \in L_t} b_t^i \equiv B_t^L$,

- Goods Market Clearing Condition:

$$C_t^H + C_t^L + C_t^u + Z_t^H + Z_t^L = Y_t, \tag{20}$$

$\sum_{i \in H_t} z_t^i \equiv Z_t^H$, $\sum_{i \in L_t} z_t^i \equiv Z_t^L$, $\sum_{i \in H_t} c_t^i \equiv C_t^H$, $\sum_{i \in L_t} c_t^i \equiv C_t^L$.

- Labor Market Clearing Condition:

$$N_t = 1. \tag{21}$$

- Capital Market Clearing Condition:

$$K_t^H + K_t^L = K_t. \tag{22}$$

- Bubble Market Clearing Condition:

$$X_t = X. \tag{23}$$

Evolution of Capital

- If $r_t = q_{t+1}\alpha^L$,

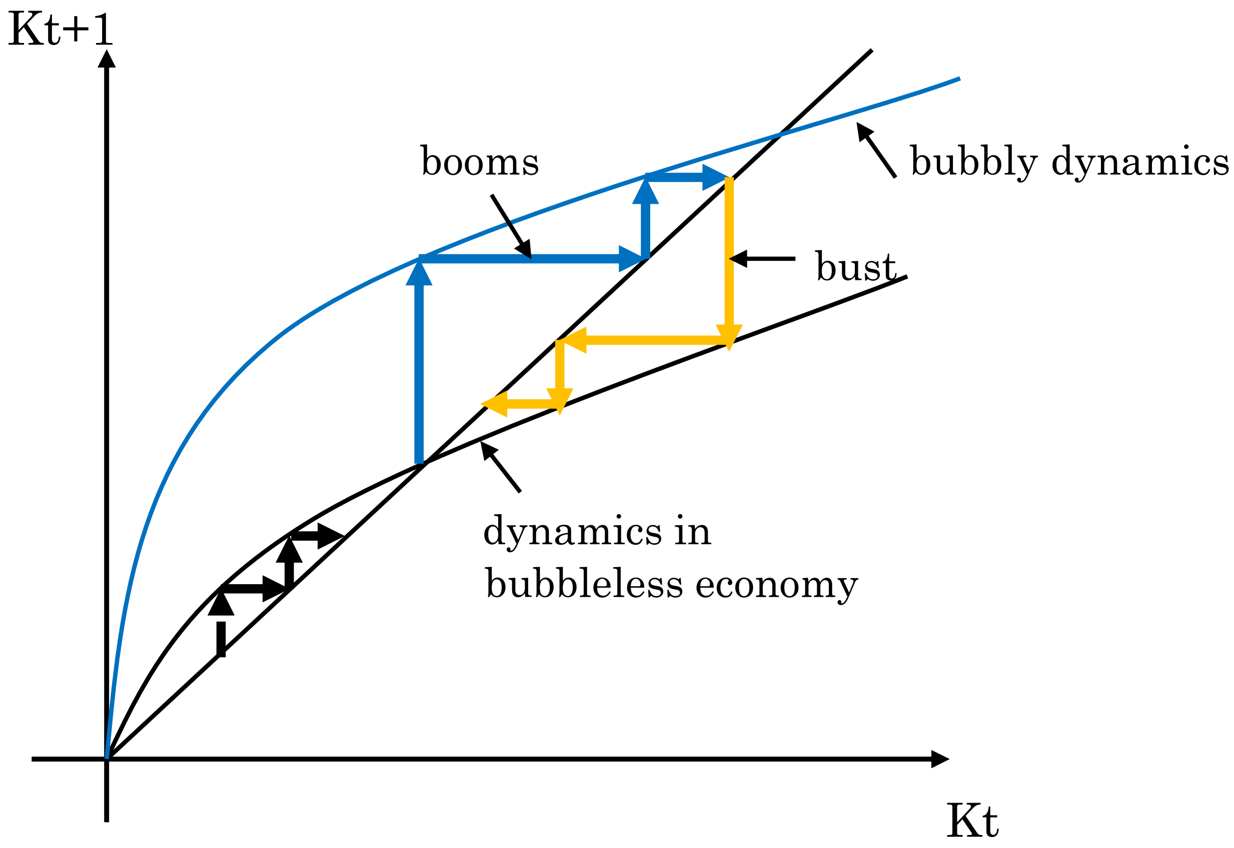
$$K_{t+1} = \alpha^H \underbrace{\frac{\beta p A_t}{1 - \frac{\theta\alpha^H}{\alpha^L}}}_{Z_t^H} + \alpha^L \underbrace{\left(\beta A_t - \frac{\beta p A_t}{1 - \frac{\theta\alpha^H}{\alpha^L}} - P_t X\right)}_{Z_t^L}.$$

- If $q_{t+1}\alpha^L < r_t < q_{t+1}\alpha^H$,

$$K_{t+1} = \alpha^H \underbrace{[\beta A_t - P_t X]}_{Z_t^H}.$$

$A_t \equiv \sigma K_t^\sigma + P_t X$ is aggregate wealth of entrepreneurs.

When α^{H} is relatively low



Dynamic Effect of Stochastic Bubbles on the saddle path

Sustainability Condition:

In order that stochastic bubbles can exist,

$$\frac{A_{t+1}}{A_t} \geq \frac{P_{t+1}}{P_t}. \tag{24}$$

$\frac{A_{t+1}}{A_t}$: wealth's growth rate
$\frac{P_{t+1}}{P_t}$: bubbles' growth rate

Otherwise, the size of bubbles explodes and the economy eventually cannot sustain bubbles.

A change in λ affects both $\frac{A_{t+1}}{A_t}$ and $\frac{P_{t+1}}{P_t}$.

Main Result 1: The existence condition of stochastic bubbles when government bailout is anticipated with probability λ is

$$\theta < \delta\beta(1-p).$$

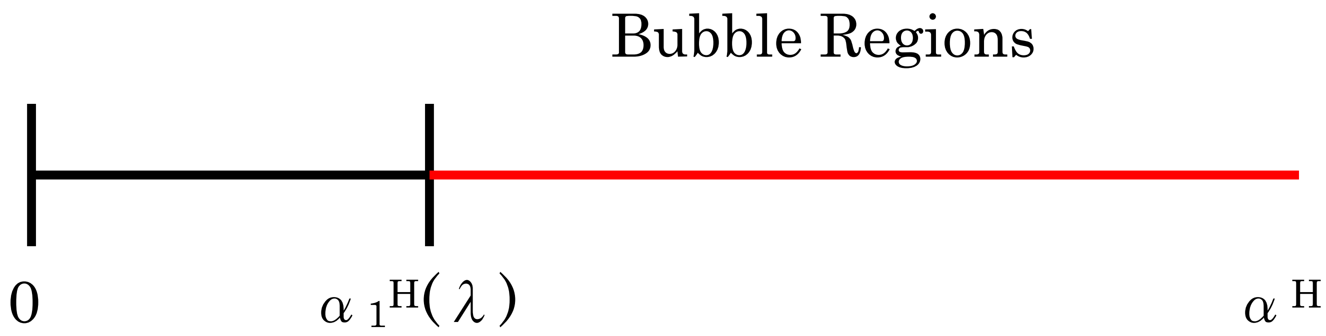
and

$$\alpha^H > \alpha^L \frac{1-\delta(1-p)\beta}{(1-\delta\beta)\theta + p\beta\delta} \equiv \alpha_1^H,$$

$\delta \equiv \pi + (1-\pi)\lambda.$

$\rightarrow$ Bubbles cannot arise in low α^H economies.

Given θ and π,



α_1^H is a decreasing function of λ.

Bubbles cannot occur in low α^H economies,

because wealth's growth rate is too low.

Result 1 suggests

The more government bailout is expected, bubble regions become wider and wider.

Bubbles are more likely to occur, the more government bailout is expected.

Note:

We can also characterize bubble regions with π or θ.

Why?

$$\frac{A_{t+1}}{A_t} \geq \frac{P_{t+1}}{P_t}. \tag{25}$$

- Required Rate of Return:

$$\frac{P_{t+1}}{P_t} = \frac{r_t(1-p-\phi_t)}{\delta(1-p)-\phi_t}. \tag{26}$$

where $\delta \equiv \pi + (1-\pi)\lambda$ and $\phi_t \equiv P_t X/\beta A_t$

When λ rises, P_{t+1}/P_t declines. Even low α^H economies can sustain bubbles.

Allen and Gale (2007):

> ⋯ In the 1930's the market was the problem and government intervention through regulation or direct ownership of banks was the solution. Today many argue that inconsistent government macroeconomic policies or moral hazard in the financial system caused by government guarantees is at the root of recent crises. ⋯

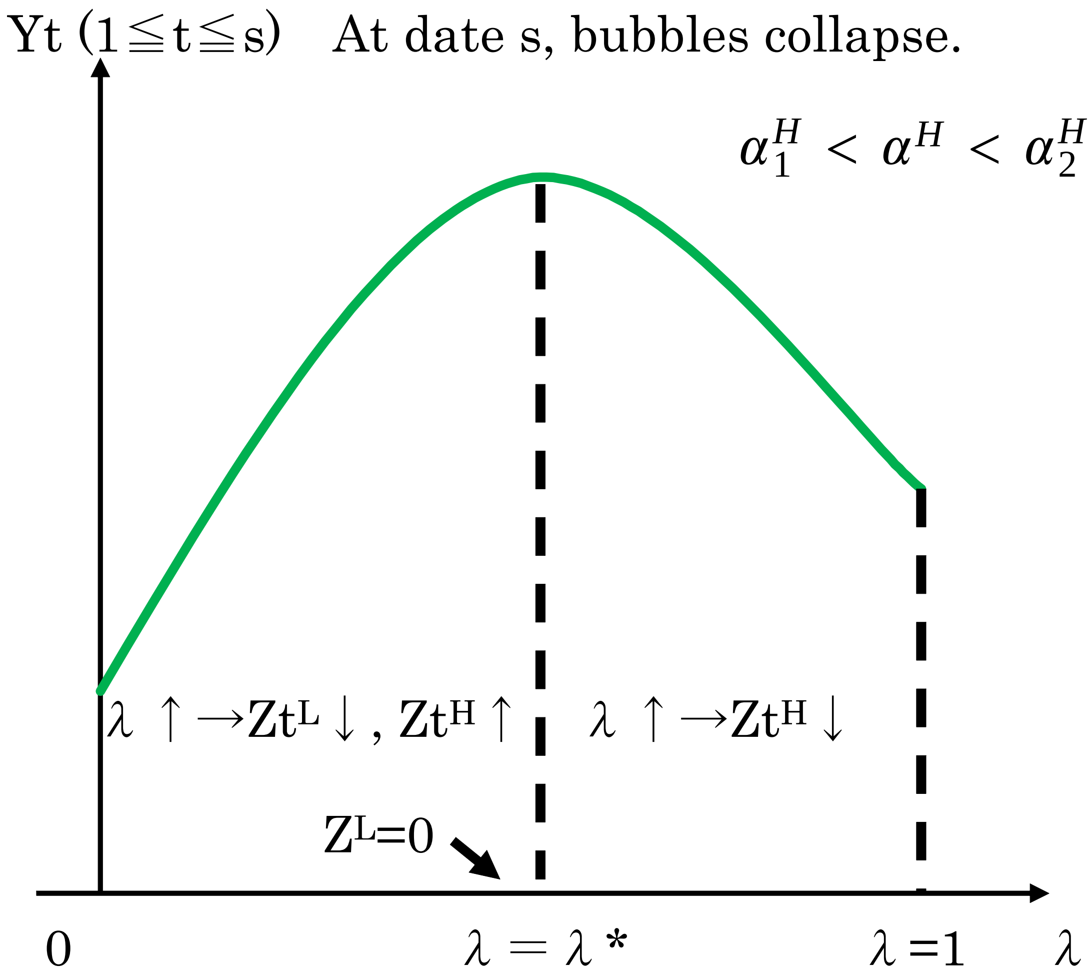
Quote from their book titled "Understanding Financial Crisis"

Question 2:

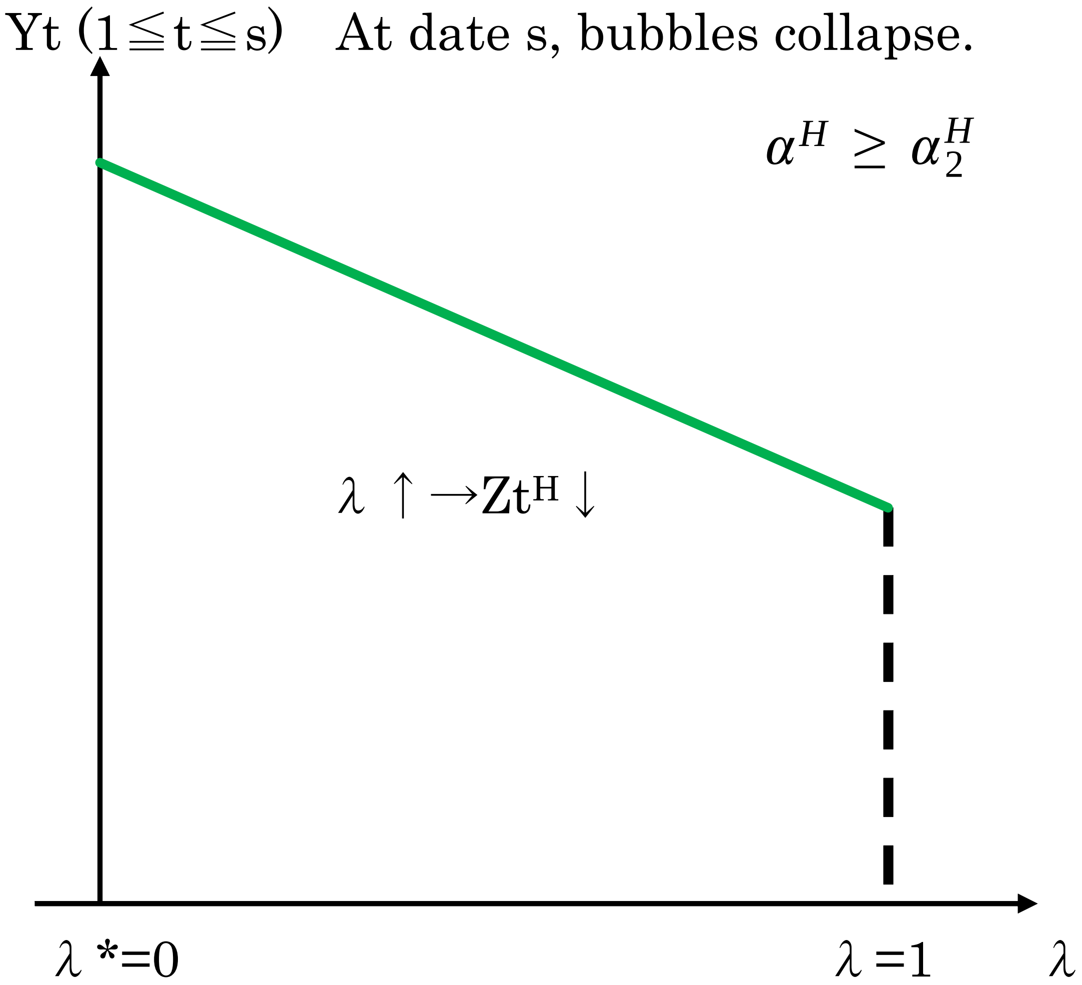
To what extent is ex-post bailout efficient from ex-ante perspective?

We consider ex-ante efficiency in terms of output.

Main result 2 is the following Figure.



Value of λ where ex-ante efficiency is maximized.



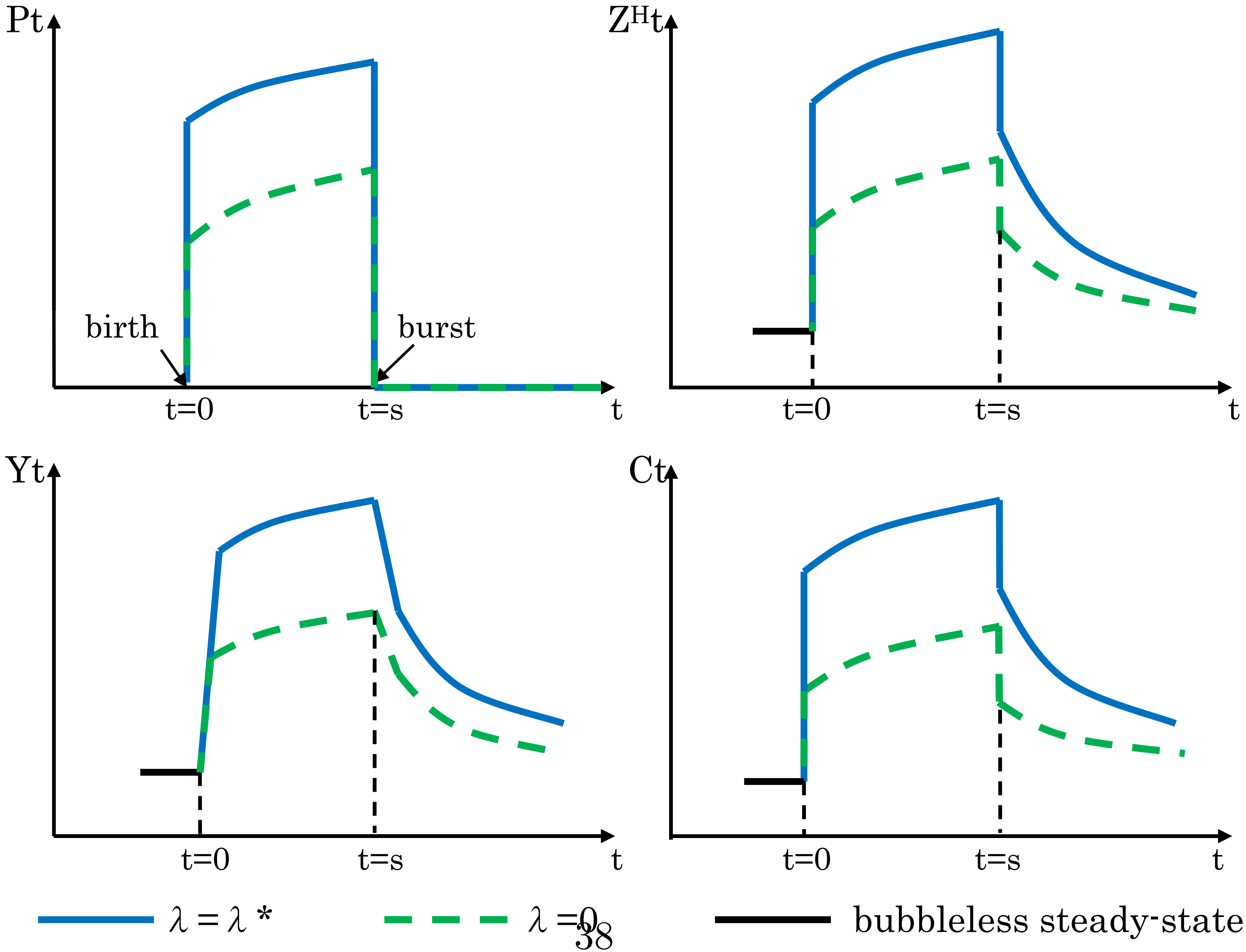
Value of λ where ex-ante efficiency is maximized.

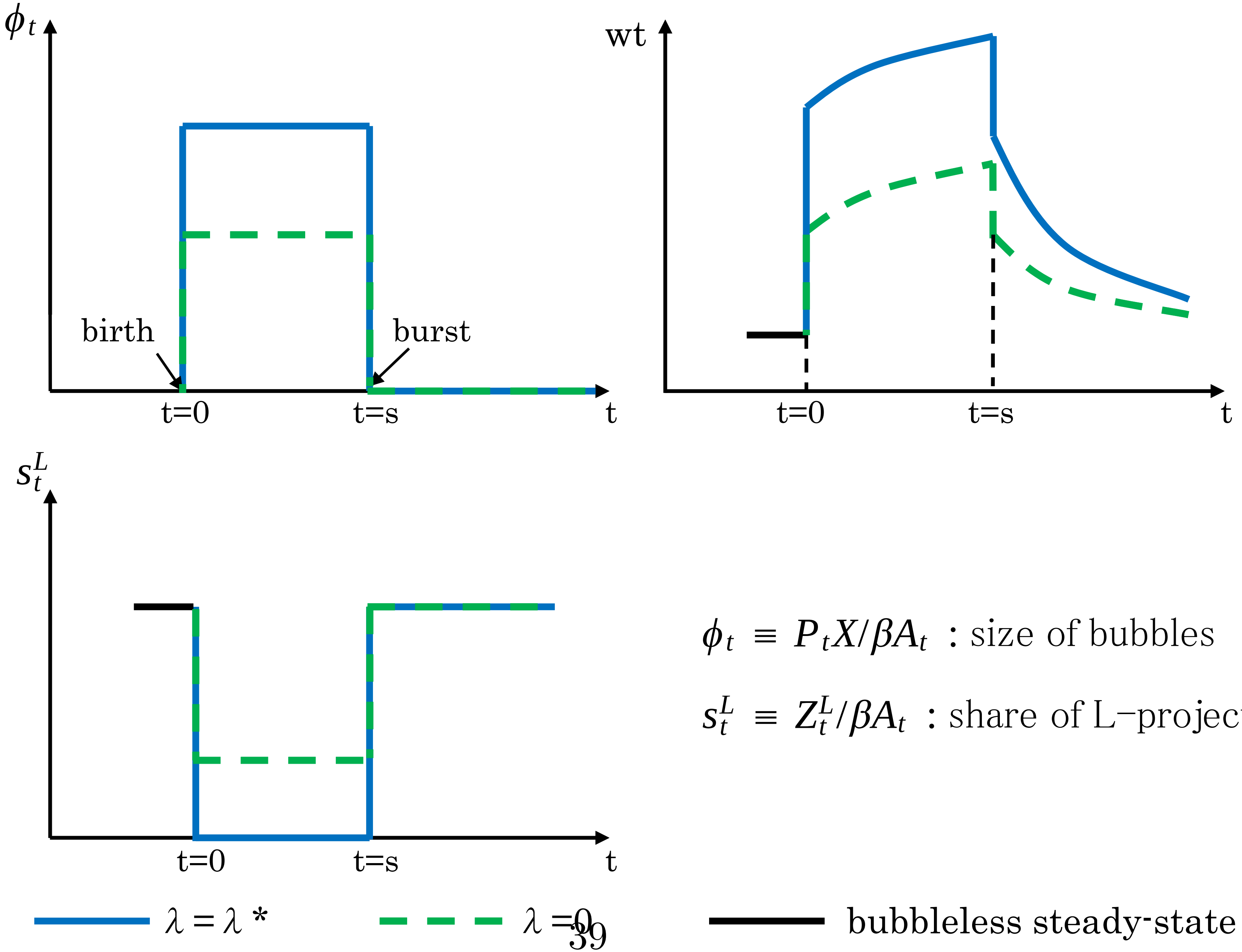
Question 3:

Suppose that government chooses λ^*, and entrepreneurs believe it.

How does the anticipated bailout affect macrodynamics?

Main Result 3: Impulse Response in the Figure.


Pt
birth
burst
t=0
t=s
t
Z^H t
t=0
t=s
t
Yt
t=0
t=s
t
Ct
t=0
t=s
t
λ = λ *
λ =0
bubbleless steady-state



$\phi_t \equiv P_t X / \beta A_t$: size of bubbles

$s_t^L \equiv Z_t^L / \beta A_t$: share of L−projects

Finally, we discuss how does bailout affect tax payers (households)?

- Negative effect:
  They pay direct costs of the bubble bursts.

- Positive effect:
  1. Wage rate after the bursts increases thanks to the bailout.

  2. Wage rate before the bursts increases when the bailout is anticipated.