Paralyzed by Fear: Rigid and Discrete Pricing Under Demand Uncertainty

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Motivation

- Why do we care?
  - Price rigidity: crucial to understanding propagation mechanism of monetary policy and business cycle fluctuations
- How to model?
  - Taylor, Calvo, menu costs, sticky information, rational inattention, etc.
- How to choose between models?
  - to guide us, large empirical literature on documenting price stickiness
  - rich set of 'overidentifying' restrictions on the theory
- This paper: a new model of rigid prices
  - intuitive and parsimonious

Key Mechanism: uncertainty about competition

1. Uncertainty about demand function
   - Not confident about potentially complex shape of demand curve
   - Learn through noisy demand signals at posted price
   - Reduction in uncertainty: stronger locally, not confident to extrapolate
   - Uncertainty aversion $\longrightarrow$ kinks in *as if* expected demand at past prices
     - ★ If increase price $\Rightarrow$ worry demand is very elastic
     - ★ If decrease price $\Rightarrow$ worry demand is very inelastic

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2. Uncertainty about relevant relative price (the argument of demand)
   - Relevant price index of competition is unknown; review it infrequently
   - Short run: unknown relation b/w price index and observed aggr. price
   - Firm takes action robust to worst-case demand schedule
     - ★ action: relative price against last observed competition price index
     - ★ worst case: agg prices are uninformative about competition price index

Key Implications

- Kinks from lower uncertainty at previously posted prices $\Rightarrow$ prices that are endogenously:
  1. sticky : do not want to move and face higher uncertainty
  2. discrete : conditional on price change, move to 'safer' prices
  3. increasingly attractive: larger kinks if posted more often
  4. both flexible and sticky: endogenous cost of adjustment
- Novel empirical implications: prices with unusually high demand realizations are stickier

Literature

1. Sticky prices
   1. Empirical
      - ★ Micro data: Bils & Klenow (2004), Klenow & Kryvtsov (2008), Nakamura & Steinsson (2008), Eichenbaum et al. (2011), Vavra (2014)
   2. Theory: pricing rigidities
      - ★ Real: Ball & Romer (1990), Kimball (1995), kinked demand curves (Stigler 1947, Stiglitz 1979)
      - ★ Nominal: Calvo, Taylor, menu costs (eg. Kehoe & Midrigan, 2010), rational inattention (eg. Matejka (2014), Stevens (2014))
2. Pricing under demand uncertainty
   - Parametric Bayesian learning: Rothschild (1974), Willems (2011), Bachmann & Moscarini (2011)
3. Knightian uncertainty
   - Decision theory: Gilboa & Schmeidler (1989), Pires (2002), Epstein & Schneider (2007)

Outline

1. **Analytical Model**
   - Learning under ambiguity
   - Optimal pricing
     - static and dynamic tradeoffs
     - policy functions
2. Quantitative Model
   - Nominal Rigidity
   - Quantitative Results
   - Novel Empirical Implications
   - Monetary Policy

Information structure

- The firm faces log marginal cost c_t, sells single good for price p_t
- Time t profit:

$$\upsilon(p_t, q_t, c_t) = (e^{p_t} - e^{c_t})e^{q(p_t)}$$

  - demand:

$$q_t = x(p_t) + z_t$$

- Information:
  - not observe $x(p_t)$ and z_t separately
  - z_t is purely risky - i.e. know that

$$z_t \sim iidN(0, \sigma_z^2)$$

  - $x(.)$ is ambiguous – not know its probability distribution
  - the firm learns about $x(p_t)$ through past sales data $\{q^{t-1}, p^{t-1}\}$

Learning Framework

- Prior is a Gaussian Process distr: for any price vector $\mathbf{p} = [p_1, ..., p_N]'$

$$x(\mathbf{p}) \sim N\left(\begin{bmatrix} m(p_1) \\ \vdots \\ m(p_N) \end{bmatrix}, \begin{bmatrix} K(p_1, p_1) & \dots & K(p_1, p_N) \\ \vdots & \ddots & \vdots \\ K(p_N, p_1) & \dots & K(p_N, p_N) \end{bmatrix} \right)$$

1. Ambiguity – the firm entertains a **set** of priors Υ
   - Priors have different mean function $m(p)$
   - Same covariance function (infinitely differentiable):

$$K(p, p') = \sigma_x^2 \exp(-\psi(p - p')^2)$$

2. Non-parametric – not restricted to a parametric family, just:
   - Lay inside some bounds

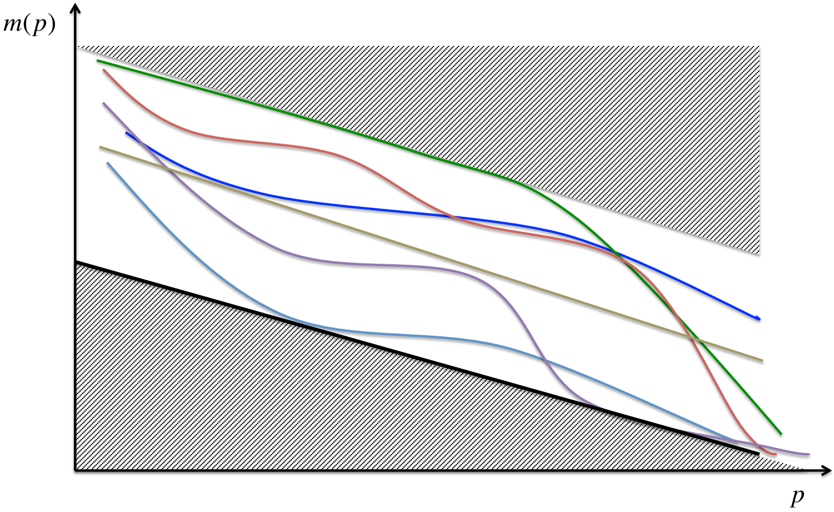
$$m(p) \in [\gamma_L - bp, \gamma_H - bp]$$

   - Non-increasing, i.e. is a demand curve

$$m(p') \leq m(p), \text{ for } \forall p' > p$$

   - Maximum derivative (ensures continuity): $|m'(p)| \leq b_{\max}$

Admissible Prior Mean Functions



Learning: Prior-by-prior Bayesian updating

- The firm uses data $\varepsilon^{t-1} = (p^{t-1}, q^{t-1})$ to update each prior
- Recursive multiple priors utility (Epstein-Schneider (2007))

$$V\left(\varepsilon^{t-1}, c_t\right) = \max_{p_t} \min_{m(p)} E^{\widehat{x}_{t-1}(p_t; m(p))} \left[\upsilon(\varepsilon_t, c_t) + \beta V\left(\varepsilon^{t-1}, \varepsilon_t, c_{t+1}\right)\right]$$

  - Min operator is conditional on price choice
    - ★ The firm looks for the p_t most robust to the set of possibilities it faces
  - Price choice – affects profits today and information set tomorrow

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- Worst-case $m(p)$ – lowest expected demand $\hat{x}_{t-1}(p_t; m(p))$:

$$m^*(p; p_t) = \underset{m(p)\in\Upsilon}{\text{argmin}}\ \widehat{x}_{t-1}(p_t; m(p))$$

Illustration

- Imagine firm has observed p_0 for N_0 times, with avg demand

$$q_0 = x(p_0) + \frac{1}{N_0} \sum_i z_i$$

- Then signal-to-noise ratio for a given p' is

$$\alpha(p', p_0) = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2 / N_0} \exp(-\psi(p' - p_0)^2)$$

Kinks in expected demand

- **Set** of conditional expectations, indexed by priors

$$\widehat{x}_0(p'; m(p)) = \underbrace{(1-\alpha(p', p_0))\, m(p')}_{\text{Prior demand at } p'} + \underbrace{\alpha(p', p_0)\left[q_0 + m(p') - m(p_0)\right]}_{\text{Signal } + \, \Delta \text{ in Demand between } p' \text{ and } p_0}$$

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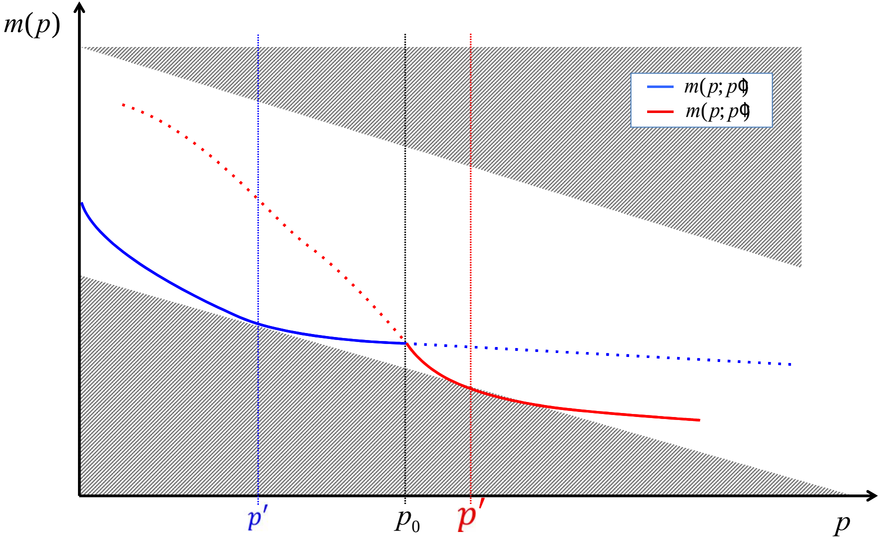
- Worst-case priors: minimize

1. Prior demand at p': $m^*(p') = \gamma_L - bp'$
2. Change in demand from p' to p_0: worst-case is conditional on price p'

- For $p' > p_0$: worry demand is elastic between p' and p_0
  $m^*(p') - m^*(p_0) = -b_{max}(p' - p_0)$
- For $p' < p_0$: worry demand is inelastic between p' and p_0
  $m^*(p') - m^*(p_0) = 0$

Worst-case is conditional on price



Worst-case expected demand

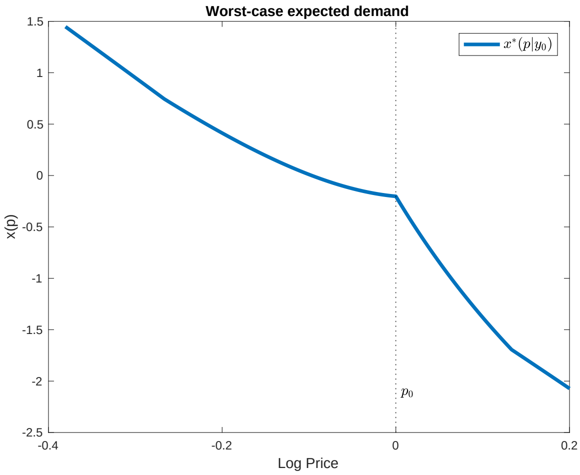
- Kink in worst-case expected demand at p_0: from endogenous switch in worst-case prior
  - Demand elasticity to the left ($p' \to p_0^-$):

$$-(1 - \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2/N_0})b$$

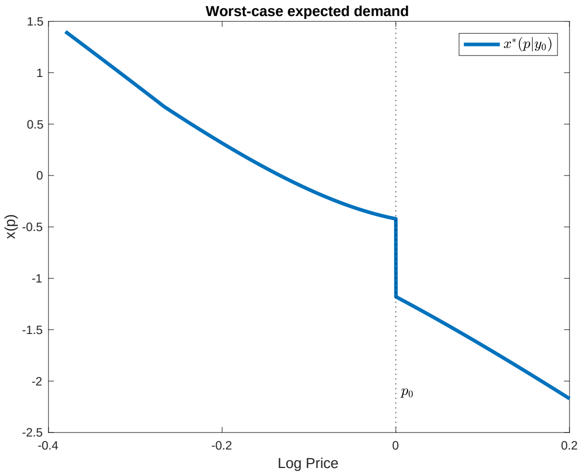
  - Demand elasticity to the right ($p' \to p_0^+$)

$$-(1 - \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2/N_0})b - \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2/N_0} b_{max}$$

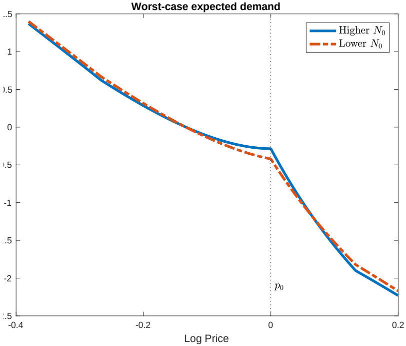
As if kinked expected demand

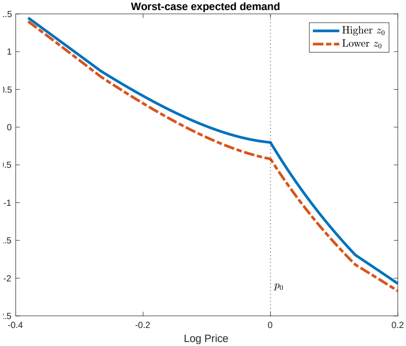


As if kinked expected demand: $b_{max} \to \infty$

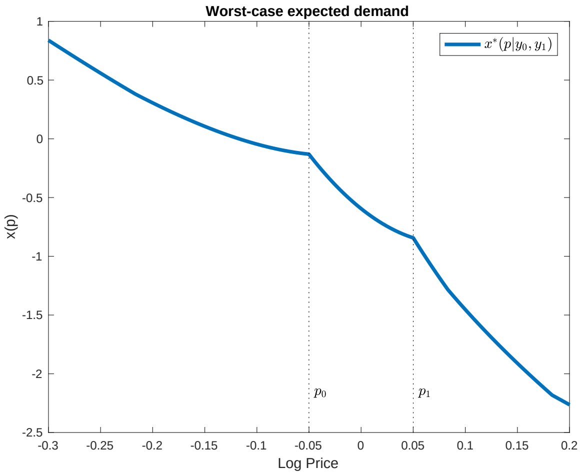


As if kinked expected demand: more exercises





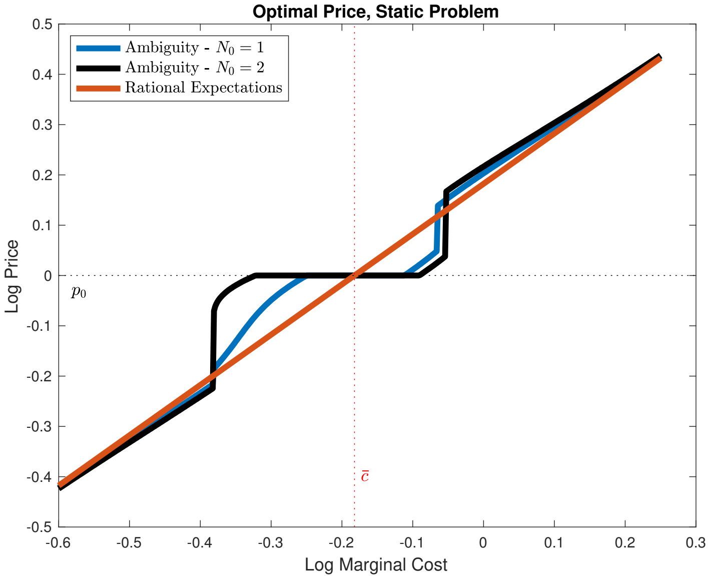
As if kinked expected demand: 2 past prices



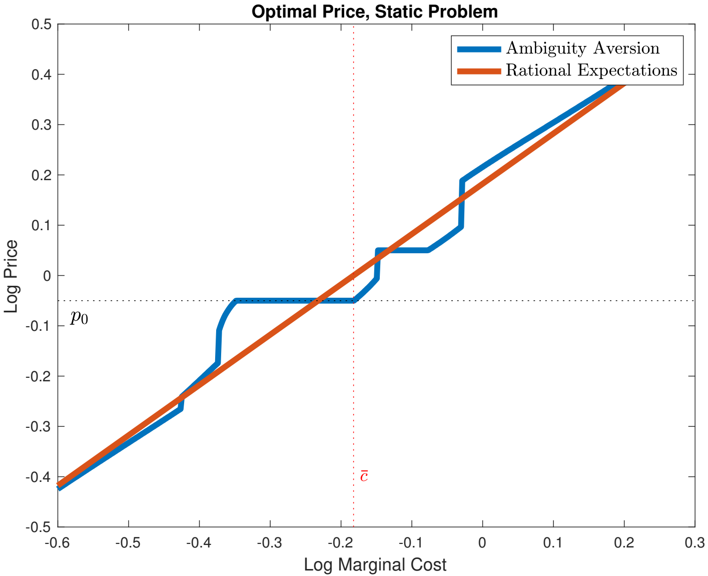
Optimal pricing: Myopic (static) maximization

- Perceived kinks lead to price stickiness
  - Intuition: higher uncertainty at new prices $\Rightarrow$ kink at p_0 $\Rightarrow$ stickiness
- Inaction regions (stickiness) are price and history specific
  - Increase with information precision (N) and level of past demand (q)
- Past price not only 'sticky' but also attractive – i.e. 'reference' prices
  1. Memory / discreteness (positive probability of revisiting past prices)
  2. Declining hazard – prob. of revisit increases with N
  3. Flexibility and stickiness – small price changes could be optimal
- Theory of **endogenous, time-varying** cost of price change

Myiopic Optimal Price: kinked expected demand



Myiopic Optimal Price: kinked expected demand



Dynamics: Experimentation Motive

- Full model infeasible: infinite state space
  - Whole history of prices and demand observations
- Consider instead
  - Firm understands how action at $t = 1$ affects information set at $t \geq 2$

$$\max_{p_1} E(\pi(p_1, c_1) + \beta V(c_2, \mathcal{I}_1) | \mathcal{I}_0)$$

  s.t.

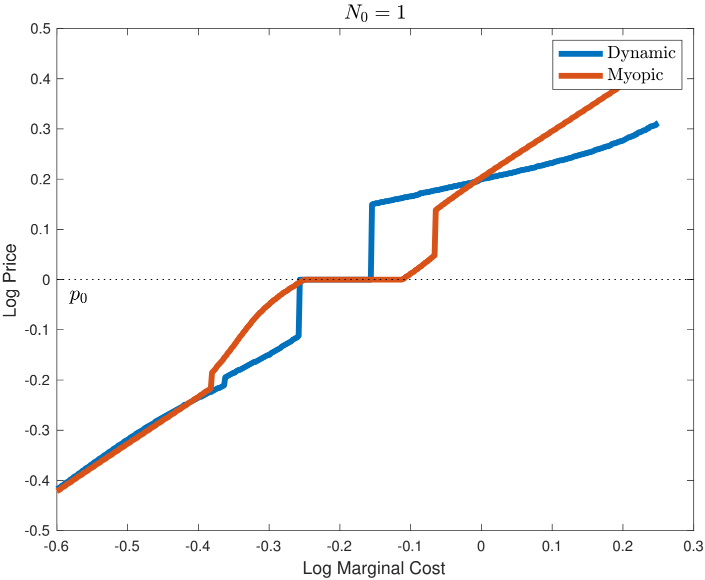
$$\mathcal{I}_1 = \mathcal{I}_0 \cup \{p_1, q_1\}$$

  - But thinks there are no updates to information for $t \geq 2$, so

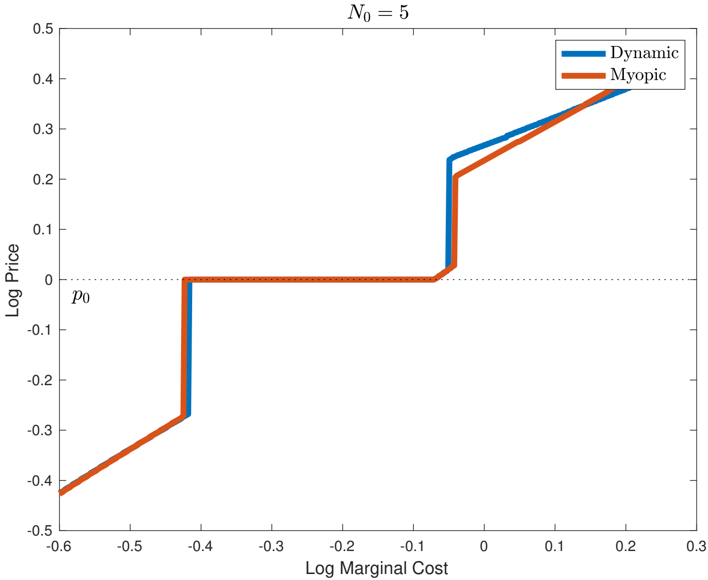
$$V(c_t, \mathcal{I}_1) = \max_{p_t} E_t(\pi(p_t, c_t)) + \beta E_t(V(c_{t+1}, \mathcal{I}_1),$$

- Puts an upper bound on experimentation motive
  - Today is last period in which you can acquire new information

Forward looking policy



Experimentation Motive: existing information matters

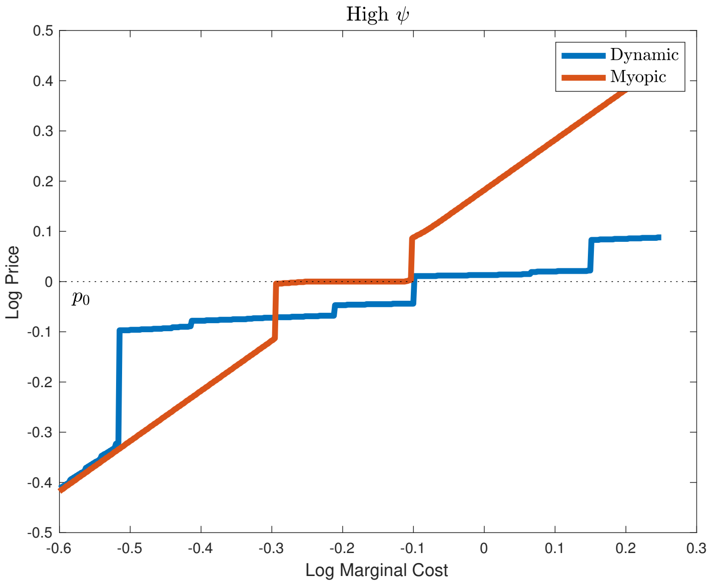


Reduced benefits of experimentation

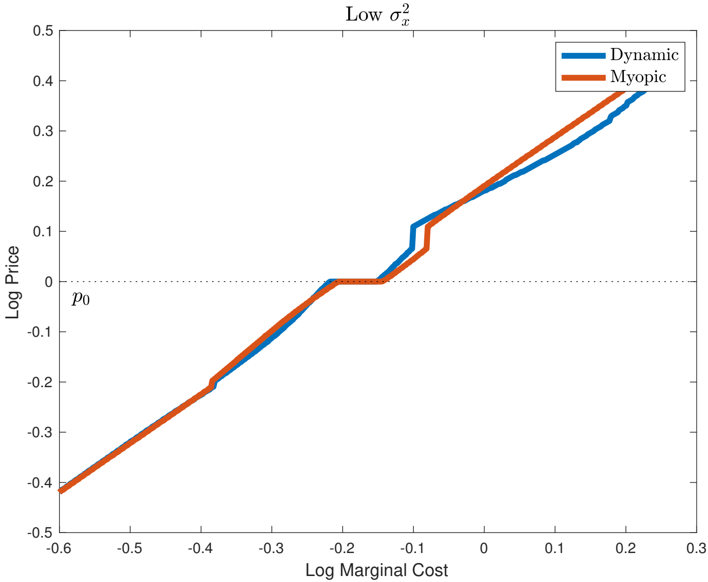
1. Better information about $x(p_t) \Rightarrow$ get closer to true optimal price
   - More useful if you set price further away from p_0 (influential point)
   - **Here**: information is local, reducing effect of influential points
2. Option value of new information: if bad signal, go back to "safe" p_0
   - Higher value if close to p_0 (marginal cost is persistent)
   - **Here** (unlike independent arm bandit models): $x(p)$ and $x(p')$ are *correlated* $\Rightarrow p_1 \approx p_0$ carries little new information
   - Likely to set p_0 again (sticky price) $\Rightarrow$ best to draw new signal there

- Why does higher N_0 reduce experimentation motive?
  1. Cost of forgone profit of large experimentation is large
  2. New signal at $p_1 \approx p_0$ will have little effect on beliefs

Move a little when $\text{corr}(x(p), x(p'))$ is low



Low experimentation motive with low signal-to-noise ratio



Outline

1. Analytical Model

2. **Quantitative Model**
   - Nominal Rigidity
   - Calibration and Quantitative Results
   - Novel Empirical Implications
   - Monetary Policy Effects

A monopolistically competitive model with nominal prices

- Household: CES aggregator over goods produced by industries j

$$P_t = \left(\int P_{j,t}^{1-b} dj \right)^{\frac{1}{1-b}}$$

- Industry j: aggregates over interm. goods $\Rightarrow$ demand for good i

$$q_{i,j,t} = h(p_{i,t} - p_{j,t}) \underbrace{- b(p_{j,t} - p_t) + c_t}_{=\text{demand for industry } j} + z_{i,t}$$

1. Firm i observes aggregate and own realizations: $\{p_t, c_t, p_{i,t}, q_{i,j,t}\}$
2. Firm i observes relevant prices $p_{j,t}$ infrequently, with prob. λ_T
3. Firms exit with exogenous probability λ_ϕ

- Ambiguity about competition: two layers
  1. **demand function**: functional form of industry demand $h(.)$
  2. **argument of demand function**: ambiguity about $p_{j,t}$

Ambiguous demand $y_{i,j,t} = h(p_{i,t} - p_{j,t}) - bp_{j,t} + bp_t + c_t + z_{i,t}$

- Relation between $p_{j,t}$ and p_t. If $p_{j,s}$ last observed ind price

$$p_{jt} - p_{js} = \phi(p_t - p_{js}) + \nu_{jt}$$

  - Long-run cointegrated but in short-run ambiguous relationship:

$$\phi(p_t - p_{js}) \in [-\gamma_p, \gamma_p], \text{ for } |p_t - p_{js}| \leq K.$$

  - We empirically document imprecise industry - aggregate inflation link
    ▸ Inflation Evidence

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- Identification problem: nature draws jointly $h(.)$ and $\phi(.)$

$$h(p_{it} - p_{jt}) = h(\underbrace{p_{it} - p_{js}}_{=\hat{r}_{it}} - \phi(p_t - p_{js}) + \nu_{jt})$$

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$$h(p_{it} - p_{jt}) = h(\underbrace{p_{it} - p_{js}}_{=\hat{r}_{it}} - \phi(p_t - p_{js}) + \nu_{jt})$$

- Firm's action is robust against worst-case demand schedule:

$$h^*(\hat{r}_{it}, \nu_{jt}) \approx x(\hat{r}_{it}) + \varepsilon_{it}; \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$$

  - no p_t because nature chooses some (unidentifiable) $\phi^*(p_t - p_{js}) = \bar{\phi}$

Nominal rigidity from learning the worst-case demand

- Demand signals

$$y_{i,j,t} = x(\hat{r}_{it}) + c_t + b(p_t - p_{js}) + \varepsilon_{it} + z_{it}$$

- Ambiguity about competition

1. **demand function**: kinks formed in relative prices $\hat{r}_{it} = p_{it} - p_{js}$
2. **argument of demand function**: p_{jt} beliefs constant in the short-run $\Rightarrow$ nominal rigidity

- Potential for 'pricing regimes': sticky nominal prices with memory

Quantitative model

- GE model with measure zero of ambiguity-averse firms
  - Aggregate shocks: money supply and TFP
  - endogenous aggregates evolve as with flex prices
- Ergodic distribution: beliefs of firms converge to a stable distribution
  - Learning friction still present at aggregate & individual firm level
    - Endogenous reference prices means firms select from coarse set
    - Never learns demand at all possible prices, friction remains in long-term
- Parameters:
  - macro: calibrate to standard moments on inflation and aggregate TFP
  - micro: use micro-data pricing and quantity moments (IRI dataset)
  - take out sales (V-shape filter)
  - some direct evidence:

Parameter	Value	Source/Target
λ_ϕ	0.0075	mean lifespan of a product 2.5 yrs (Argente-Yeh 2017)
σ_z	0.61	median demand forecast error

Calibration micro parameters: SMM

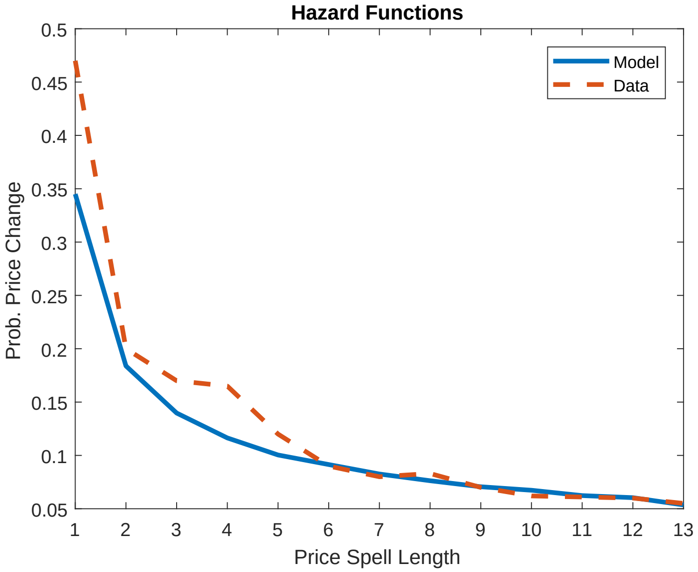
Parameter	Value	Description
ρ_w	0.784	Persistence of idiosyncratic productivity
σ_w	0.047	St. dev. of idiosyncratic productivity shock
ν	1.15	Ambiguity parameter
ψ	4	Prior covariance function smoothing parameter
σ_x	0.51	Prior variance of $x(.)$
b_{max}	3.4*b	Maximum derivative
λ_T	0.015	Frequency of price reviews

Target Moments	Data	Model
Frequency of price changes	0.11	0.11
Fraction of price increases	0.54	0.55
Mean size of abs price changes	0.19	0.20
Lower quartile of abs price change distribution	0.069	0.069
Upper quartile of abs price change distribution	0.27	0.28
Frequency of modal price change (13 week window)	0.027	0.029
Mean duration of pricing regimes (weeks - Stevens, 2017)	29.1	32.1

Additional Implications: discrete prices with memory

Moment	Data	Model
Probability of revisiting a price (last 26 weeks)	0.62	0.68
Avg # uniq. prices (26 weeks) / (# price changes + 1)	0.77	0.73
Fraction of time at modal price	0.83	0.85
Prob. price change goes to modal price	0.43	0.51

Price change hazard



Additional Implications: demand signals matter for pricing

- Model predicts that stickiness is stronger for a price:

1. posted more often ('high N')
   - already some evidence to this: e.g. declining hazard
2. with unusually positive demand realizations ('high $\bar{z}$')
   - intuitive: more likely to remain at prices that appear 'profitable'
   - stronger effect at young prices: kink mostly driven by $\bar{z}$
   - at older prices: $\bar{z}$ changes little the large kink that comes from N
   - novel empirical implications: link quantity data to stickiness

Demand signals matter for pricing: data and model

- Regression that tests those predictions

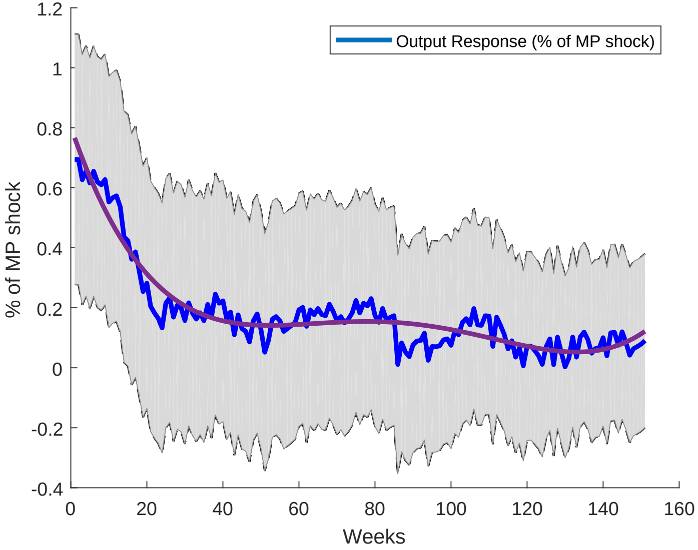
$$I(p_{i,t} \neq p_{i,t-1}) = \alpha_i + \xi_t + \beta_Z \bar{z}_{i,t-1} + \beta_N N_{i,t-1} + \varepsilon_t$$

  - $\beta_Z < 0$ ($\beta_N < 0$): less likely to change a price $p_{i,t-1}$ with high $\bar{z}$ (N)
  - subsample with young prices: effects stronger for Z

	Young ($N_{i,t-1} \leq 8$)			All ($N_{i,t-1} \leq 26$)		
	$\Pr(\Delta > 0)$	Z effect	N effect	$\Pr(\Delta > 0)$	Z effect	N effect
Data	0.14	- 7.9%	- 6.5%	0.1	- 5%	- 23%
Model	0.15	- 6.1%	- 9.7%	0.1	- 3.9%	- 15.6%

Regression

Monetary Policy IRF



Conclusions

- Firm exploits demand curve under ambiguity
  - learning about non-parametric demand
  - firm acts *as if* kinked expected demand at previously observed prices
  - generates 'price memory' and makes them endogenously:

    sticky, discrete, increasingly attractive

- With imperfect info on competitors' prices: nominal rigidity

- Endogenous cost of price change: rigidity is history and state dependent
  - implications for policy

Evidence on weak aggregate - industry prices link

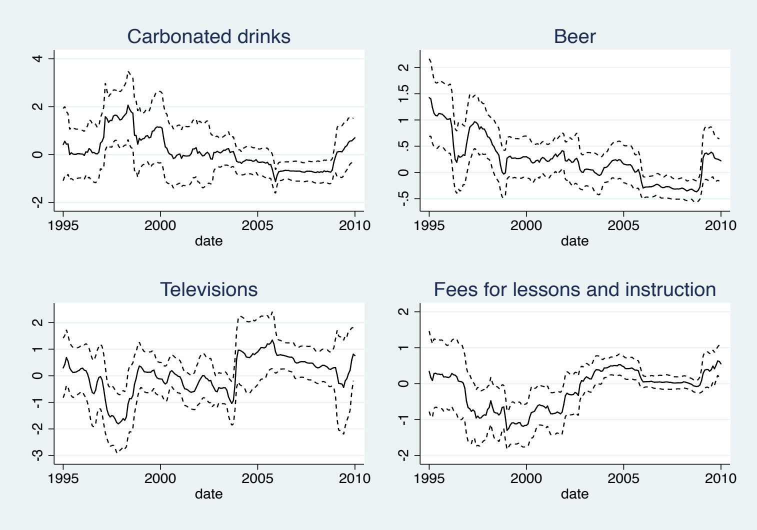


Figure: 3-year rolling regressions of 3-month industry inflation on 3-month aggregate inflation. ▸ Back

Demand regression

- Regression to recover $z_{ijt} > 0$ realizations

$$q_{ijt} = \beta_0 + \beta_1 q_{i,j,t-1} + \beta_2 p_{ijt} + \beta_3 p_{ijt}^2 + \beta_4 cpi_t + \\ + week_t' \theta_1 + store_j' \theta_2 + item_i' \theta_3 + z_{ijt}$$

where

- q_{ijt}, p_{ijt} are quantities and prices in logs
- cpi_t is the consumer price index for food and beverages
- $week_t$ is a vector of week dummies
- $store_j$ is a vector of store dummies
- $item_i$ is a vector of item dummies

Back