Cross-Sectional and Aggregate Labor Supply

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Modern Business Cycle Analysis

- Early representative agent models
  - Kydland-Prescott, ...
  - Largely ignore cross-sectional moments
- Recent advances in hetero agent models
  - Aiyagari (1994), ...
  - Heterogeneity: idiosyncratic productivity shocks
  - Testable **cross-sectional** implications

Labor Supply in Macro Models

- Representative agent models
  - Offsetting income and substitution effects
  - So-called "Balanced Growth Path" preferences
- Aiyagari-type Hetero agent models
  - Idiosyncratic productivity shocks
  - Inherits "Balanced Growth Path" preferences
  - Cross-sectional $cor(w, h) \approx$ **0.7** $\leftrightarrow$ **0** in data

Standard Macro Models with Idiosyncratic Productivity Shocks

- Standard heterogeneous agent macro models that highlight idiosyncratic productivity shocks do not generate the near **zero cross-sectional correlation between hours and wages**.
- Ask whether matching this moment matters for business cycle properties of these models

Two Extensions

We consider two extensions from standard model

- Departure from **balanced growth path preferences**.
- Introduction of idiosyncratic **shocks to the opportunity cost of working**.

Preview

- Both extensions can match the empirical correlation.
- Large and opposing effects on the cyclical volatility of the labor market.
- Cross-sectional moments are important for business cycle analysis.

Illustrative Example

Consider a household with utility:

$$\frac{c^{1-\sigma}}{1-\sigma} - \frac{h^{1+1/\gamma}}{1+1/\gamma}$$

F.O.C. for labor supply:

$$h = \left\{ \frac{w}{c^{\sigma}} \right\}^{\gamma}$$

This Paper

- Examine this issue using Chang-Kim-Kwon-Rogerson (2019)
- A heterogeneous agent model that features both intensive and extensive margins of labor.
- Chang & Kim (2007) + Rogerson & Wallenius (2009)

Benchmark: "One-Shock" Model

$$\max_{\{c_t, h_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - B \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right\} \right]$$

$$c_t + a_{t+1} = (1 + r_t) a_t + w_t z_t g(h_t)$$

$$g(h_t) = \max\{0, h_t - \hat{h}\}, \quad h_t \in [0, 1]$$

$$a_{t+1} \geq \bar{a}$$

$$\ln z_{t+1} = \rho_z \ln z_t + \epsilon_{zt}, \qquad \epsilon_{zt} \sim N(0, \sigma_z^2)$$

- z: idiosyncratic productivity
- minimum hours $\hat{h}$ reflects setup costs, commuting, etc.
- Both margins are chosen optimally.

Technology: Representative Firm

$$\max_{L_t, K_t} \; Y_t = Z_t L_t^{\alpha} K_t^{1-\alpha}.$$

$$\ln Z_{t+1} = \rho_Z \ln Z_t + \varepsilon_{Zt}, \;\; \varepsilon_{Zt} \sim N(0, \sigma_Z^2).$$

$K_t = \int a_t \, d\mu$: Aggregate Capital
$L_t = \int h_t z_t \, d\mu$: Aggregate Efficiency unit of labor

$\mu(a, z)$ Cross-sectional distribution of workers

Calibration: One-Shock Model

α	β	γ	δ	B	ρ_z	σ_z	$\hat{h}$
0.36	0.976	1.00	0.025	18.9	0.975	0.165	0.151

- ρ_z, σ_z from panel data on wages (e.g., Floden-Linde, 2001)
- γ: Frisch Elast of labor supply
- B: chosen to match employment rate (70%)
- $\hat{h}$: to match the average hours (0.33)
- β: to match 4% annual rate of return

Cross-Sectional Dispersion

	Model	Data
S.D. of Annual Hours	0.32	0.45 (CPS)
Earnings Gini	0.59	0.63 (SCF)
Wealth Gini	0.71	0.78 (SCF)
$Corr(w, h)$	0.78	0 (PSID, SIPP)

- Does well on earnings and wealth dist.
- Not enough dispersion in hours
- Too high correlation b/w wages and hours

Two-Shock Model

The only change:

$$\ln c - B\mathbf{x}\frac{h^{1+1/\gamma}}{1+1/\gamma}$$

$$\log x_{it+1} = \rho_x \log x_{it} + \varepsilon_{xit+1},\ \ \varepsilon_{xit} \sim N(0, \sigma_x^2).$$

- **x** : opportunity cost of working (preference for leisure, home productivity)
- $cor(\epsilon_z, \epsilon_x) = \rho_{zx}$

Household's Labor Supply Decision

Intensive Margin:

$$h = \left\{ B \frac{w}{c} \frac{z}{x} \right\}^{\gamma}$$

Extensive Margin (Work if):

$$w(h - \hat{h}) \frac{z}{x} \geq B\, h^{\frac{1}{\gamma}} c$$

Comparative Advantage

- What matters is comparative advantage (z/x).
- Shape of cross-sectional distribution of (z,x) crucial for aggregate labor supply.

Calibration: Two-Shock Model

β	B	ρ_z	σ_z	ρ_x	σ_x	$\hat{h}$
0.977	20.6	0.975	0.165	0.975	0.103	0.133

- $\rho_x = \rho_z = 0.975$
- $\rho_{zx} = corr(x, z) = 0$
- σ_x to match the disperson of hours
- B: chosen to match employment rate (70%)
- $\hat{h}$: to match the average hours (0.33)
- β: to match 4% annual rate of return

Annual Hours Transition: PSID

		$t+1$					
		Not Work	1st	2nd	3rd	4th	5th
	Not Work	83.57	12.25	1.69	0.91	0.99	0.60
	1st	21.08	49.45	14.91	6.15	5.29	3.12
	2nd	4.77	15.40	45.77	18.27	11.15	4.63
t	3rd	2.81	6.75	19.77	46.24	17.88	6.54
	4th	2.26	5.14	10.63	19.91	42.42	19.64
	5th	1.81	3.41	4.69	6.80	19.77	63.52

Annual Hours Transition: Model

		$t+1$					
		Not Work	1st	2nd	3rd	4th	5th
	Not Work	72.49	17.64	6.53	2.39	0.81	0.14
	1st	21.12	34.43	24.53	12.60	5.61	1.72
	2nd	8.46	20.54	37.54	23.19	7.88	2.39
t	3rd	3.24	11.03	17.67	37.28	24.68	6.09
	4th	1.69	6.99	7.57	19.31	39.95	24.48
	5th	0.49	3.57	3.83	5.37	20.49	66.25

Two-Shock Model

- Does well on transition of hours
- More importantly, $corr(w, h) = 0.53$.

Two Extensions from Benchmark Model

To achieve $corr(w, h) \approx 0$,
consider two extensions:

- Departure from **balanced growth path preferences**: $\sigma \neq 1$.
- Departure from $\rho_{zx} = 0$

Extension I: $\sigma \neq 1$

$$h = \left\{ \frac{w}{c^{\sigma}} \frac{z}{x} \right\}^{\gamma}$$

- $\sigma \uparrow$ making the wealth effect in labor supply stronger $corr(w, h) \downarrow$.
- With $\sigma = 2$, we achieve $corr(w, h) \approx 0$
- Pijoan-Mas (2006), Heathcote et al. (2016), ...

Calibration of Extension I Model

β	B	ρ_z	σ_z	ρ_x	σ_x	$\hat{h}$	σ
0.96946	77.0	0.975	0.165	0.975	0.144	0.144	2

Extension II: $\rho_{zx} \neq 0$

$$h = \left\{ \frac{w}{c} \frac{z}{x} \right\}^{\gamma}$$

- $corr(z, x) = \rho_{zx} \uparrow \rightarrow corr(w, h) \downarrow$.
- With $\rho_{zx} = 0.9$, we achieve $corr(w, h) \approx 0$
- Weak cross-sectional comparative advantage

ρ_{zx}	-0.9	-0.5	0	0.5	0.9
$corr(w, h)$	0.78	0.66	0.53	0.34	-0.004

Calibration of Extension II Model

ρ_{zx}	β	B	$\hat{h}$	σ_x
-0.9	0.97537	19.0	0.128	0.0825
-0.5	0.976	19.2	0.130	0.09
0.5	0.97818	22.6	0.135	0.124
0.9	0.9814	26.2	0.139	0.147

Wealth and Earnings

Gini Coefficient: Wealth					
PSID=0.76, SCF=0.78					
$\sigma = 2,\ \rho_{zx} = 0$			0.63		
$\rho_{zx} =$	-0.9	-0.5	0.0	0.5	0.9
	0.63	0.64	0.65	0.67	0.68
Gini Coefficient: Earnings					
PSID=0.53, SCF=0.63					
$\sigma = 2,\ \rho_{zx} = 0$			0.53		
$\rho_{zx} =$	-0.9	-0.5	0.0	0.5	0.9
	0.60	0.59	0.59	0.57	0.53

Wealth Share by Quintile

	I	II	III	IV	V
PSID	-0.52	0.50	5.06	18.74	76.22
SCF	-0.39	1.74	5.72	13.43	79.49
$\sigma = 2,\ \rho_{zx} = 0$	0.07	2.19	9.80	24.77	63.17
$\rho_{zx} = -0.9$	0.16	2.66	9.51	23.80	63.88
$\rho_{zx} = 0.0$	0.08	2.10	8.75	23.20	65.87
$\rho_{zx} = 0.9$	0.04	1.16	7.54	22.51	68.76

Earnings Share by Wealth Quintile

	I	II	III	IV	V
PSID	7.51	11.31	18.72	24.21	38.23
SCF	7.05	14.50	16.48	20.76	41.21
$\sigma = 2,\ \rho_{zx} = 0$	10.05	14.97	18.79	23.38	32.81
$\rho_{zx} = -0.9$	5.56	11.14	16.91	24.87	41.52
$\rho_{zx} = 0.0$	6.26	11.95	17.06	24.43	40.31
$\rho_{zx} = 0.9$	10.27	13.09	18.65	23.21	34.78

Cross-Sectional & Aggregate Fluctuations

- Both extensions can match the empirical correlation ($corr(w, h) \approx 0$).
- Large and opposing effects on the cyclical volatility of the labor market.

Business Cycle Implications

	σ_Y	σ_H	$\frac{\sigma_H}{\sigma_Y}$	σ_E	σ_h	σ_L	σ_w
Data (BLS)	2.01	1.80	0.89	1.51	0.48	–	0.98
Chang et al(2019)	1.74	0.79	0.45	0.69	0.12	1.09	0.83
$\sigma = 1,\ \rho_{zx} = 0$	1.65	0.72	0.44	0.58	0.17	0.93	0.85
$\sigma = 2,\ \rho_{zx} = 0$	1.46	0.39	0.28	0.44	0.09	0.72	0.66
$\sigma = 1,\ \rho_{zx} = 0.9$	1.92	0.93	0.48	0.95	0.11	1.47	0.84

- Cross-sectional moments are important for business cycle analysis.
- $H = E \times h$, E: Employment, h: Hours per worker
  L: Efficiency units

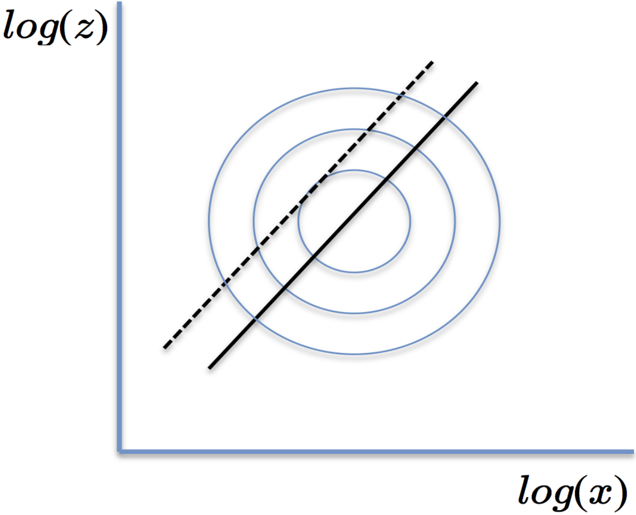
Effect of ρ_{zx} on Business Cycle Statistics

	σ_Y	σ_H	$\frac{\sigma_H}{\sigma_Y}$	σ_E	σ_h	σ_L	σ_w
$\rho_{zx} = -0.9$	1.59	0.66	0.41	0.47	0.22	0.83	0.87
$\rho_{zx} = -0.5$	1.62	0.68	0.42	0.51	0.19	0.87	0.86
$\rho_{zx} = 0$	1.65	0.72	0.44	0.58	0.17	0.93	0.85
$\rho_{zx} = 0.5$	1.72	0.77	0.45	0.68	0.14	1.04	0.83
$\rho_{zx} = 0.9$	1.92	0.93	0.48	0.95	0.11	1.47	0.84

Comparative Advantage

- What matters is comparative advantage (z/x).
- Shape of cross-sectional distribution of (z,x) crucial for aggregate labor supply.

Cross-Sectional Comparative Advantage and Aggregate Employment Response



Special Case: No Heterogeneity

No heterogeneity in z and x.

- Hansen-Rogerson Lottery Economy
- Infinitely elastic aggregate labor supply

Special Case: $x = \psi * z$

z and x are perfectly correlated.

- No Comparative Advantage (z/x is a constant)
- Reservation wage distribution is degenerate
- Infinitely elastic aggregate labor supply

Special Case: z only

Heterogeneity in z only: e.g., Aiyagari (1994).

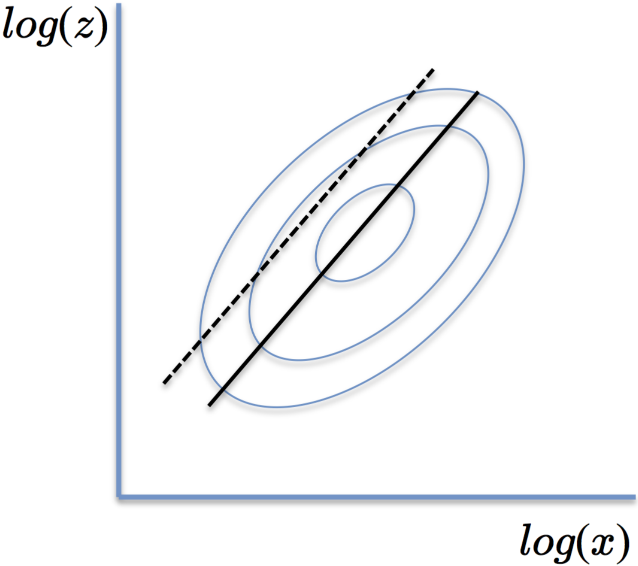
- Reservation wage depends on market productivity and wealth.

Cross-Sectional Comparative Advantage

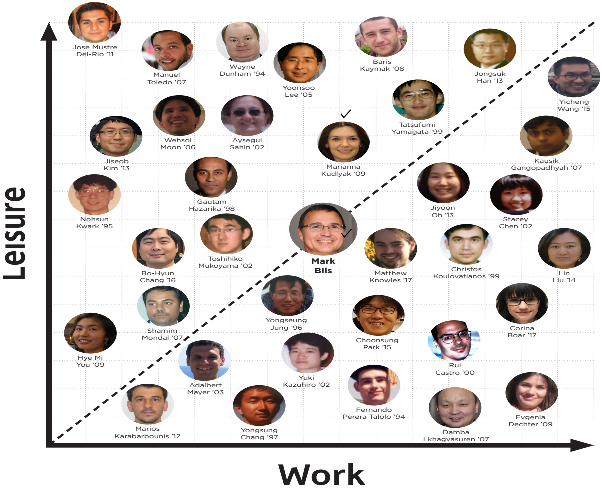
z and x are **positively** correlated: $cor(z, x) > 0$.

- "Weak" Comparative Advantage
- Workers with high productivity in the market are also good at home production
- Sensitive to the change in relative return
- Elastic aggregate labor supply

$cor(z, x) > 0$



Economy made with Mark, Marianna, Toshi,..



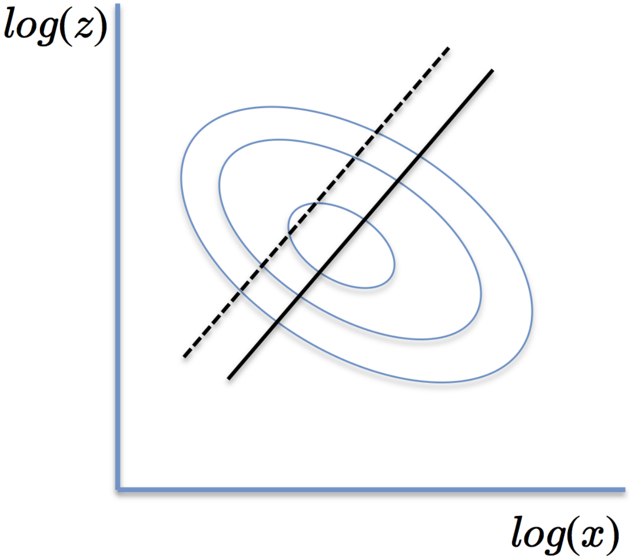
Celebrating 30 Years at the University of Rochester and a Happy 60th Birthday for Mark Bils

Cross-Sectional Comparative Advantage

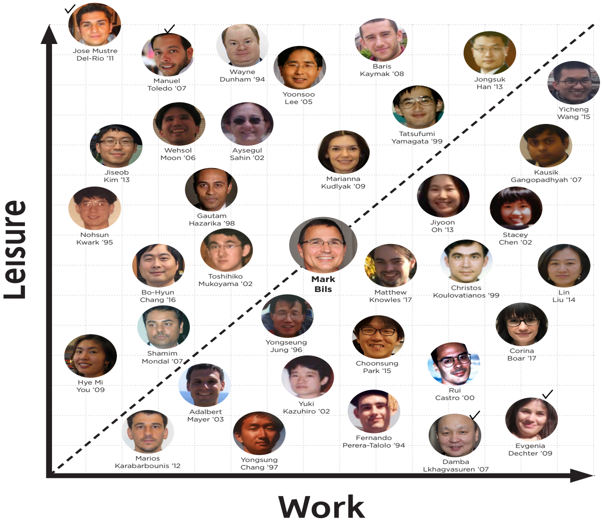
z and x are **negatively** correlated: $cor(z, x) < 0$.

- "Strong" Comparative Advantage
- Workers with high productivity in the market are bad at home production
- Not willing to move between activities
- z/x distribution is dispersed.
- Inelastic aggregate labor supply

$cor(z, x) < 0$



Economy w/ Jose, Geni, Corina, Damba,...



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Conclusion

- Standard heterogeneous agent model fail to match the near zero cross-sectional corr(w,h).
- Economy with weak comparative advantage.
  - Match cross-sectional corr(w,h).
  - Exhibits an elastic aggregate labor supply.
- Cross-sectional moments are important for business cycle analysis.