Managing a Polarized Structural Change

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Abstract

We propose a novel framework that integrates an economy's distribution of skills with its occupation and industrial structure. Individuals with heterogeneous skills choose to become a manager or worker by comparative advantage, and those who become workers are again sorted into occupations that differ in terms of skill contents. Industrial sectors differ in how they combine occupational outputs (or tasks) for production. In equilibrium, occupation-specific TFPs and individuals' occupational choices endogenously determine sector-level and aggregate TFPs. Using this model, we show faster TFP growth among middle-skill worker occupations relative to the rest leads not only to job and wage polarization across worker occupations (*horizontal polarization*), but also to a higher employment share and relative wage for managers over workers (*vertical polarization*). Moreover, the speed of both types of polarization is faster in sectors that use more middle-skill worker tasks and less managerial tasks, which also leads to endogenously higher TFP growth for those sectors. We document that such predictions are supported empirically. If sectoral outputs are complementary in producing final output, the faster TFP growth among middle-skill occupations endogenously shifts capital and labor toward sectors that are less dependent on the middle-skill task, resulting in structural change. In the limiting balanced growth path, the middle-skill occupations vanish, but all sectors coexist and grow at the same rate. A quantitative analysis shows that even in the presence of sector-specific TFP growth, occupation-specific TFP growth alone can account for more than half of structural change.

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1 Introduction

We develop a novel model that integrates an economy's occupation and industrial structure with the individual skill distribution. The model jointly determines how heterogeneous individuals sort into different occupations and how much of each occupation is employed by different industrial sectors. In the model, the changes in wage and employment shares across occupations, and across industrial sectors, are interrelated and reinforce each other.

We use this model to connect labor market polarization and structural change. Structural change—the shift of economic output and employment across broadly-defined sectors—is a well-established economic fact. Job and wage polarization—the employment shares and wages of low- and high-skill occupations increasing relative to middle-skill occupations—is a distinct feature of advanced economies (Acemoglu and Autor, 2011; Goos et al., 2014). In addition, we establish two new facts in the U.S. data: (i) the divergence in the relative employment shares and wages between managers and workers, which we call *vertical polarization*, and (ii) the faster speed of both polarization across workers, and between managers and workers, in sectors that use middle-skill occupations more intensively. We then show in our model that all these phenomena can be generated by one common cause: relatively faster growth of the TFP of middle-skill worker occupations, which is not sector-specific.

In our model, individuals are heterogeneous in two dimensions, managerial talent and worker human capital, which are not sector-specific. Individuals decide whether to become a manager or a worker based on comparative advantage, and if they become a worker, also whether to work in a low-, middle- or high-skill occupations based on their *occupation-neutral* worker human capital. Notably, they only choose an occupation (or task) and are indifferent across sectors. The only difference across sectors is how intensively different occupational outputs or tasks are used in production, and sector-level TFP is endogenously determined by the equilibrium distribution of occupation choices.

An exogenous improvement in the productivity of the middle-skill worker occupation, relative to the managerial and other worker tasks, leads to job and wage polarization across workers, i.e., the employment shares and wages of the low- and high-skill occupations increase relative to the middle-skill occupation, assuming complementarity across worker tasks. Part of such an productivity improvement can be viewed as routinization, or technological advancement among routine jobs (which tend to be middle-skill jobs in the data), reducing the need for workers working in such jobs, and recent empirical studies have established that routinization has played a dominant role (Autor and Dorn, 2013; Goos et al., 2014) in polarization. When distinction is necessary, we refer to this as *horizontal* polarization.

A novel feature of our model is that TFP growth among middle-skill jobs leads to an increase in the employment share and wages of managers relative to all workers (vertical polarization), if managerial and worker tasks are also complementary. We also show that the speed of both horizontal and vertical polarization is faster in sectors that uses more middle-skill tasks and less managers. In the U.S. data, this sector is manufacturing, and we document that horizontal and vertical polarization has indeed progressed faster in manufacturing than in services. This implies faster endogenous TFP growth for the manufacturing sector, which is also supported by the data. Then, if manufacturing and service outputs are complementary, it leads to structural change from manufacturing to services. We also show that structural change, although originating from occupation-specific TFP growth, reinforces polarization.

An implication of structural change driven by occupation-level TFP growth is that, in the limiting balanced growth path of our model, some tasks may vanish but both sectors must coexist, unlike many existing theories of structural change. The intuition is straightforward: because most theories rely on sector-specific forces to explain structural change, the shift of resources from one sector to another must continue as long as the same forces exist. In contrast, in our model of endogenous sector-level TFP, occupation-level TFP growth is sector-neutral and only affects sectors indirectly through how they combine task outputs. Once certain occupation shares become negligible, structural change must cease even as TFP continues to grow at the occupation level.

Our model is a first attempt to provide a framework that links the occupational structure of an economy to sectoral aggregates. In particular, we can use micro-estimates of occupational employment and wages, which have been studied extensively in labor economics, to study how occupational choices aggregate up to macro-level sectoral shifts. The main finding in this regard is that micro-level productivities and elasticities in the labor market are important for understanding sectoral shifts at the aggregate level.

This is of particular empirical relevance for the U.S. The 1980s marks a starting point of rising labor market inequality, much of which can be attributed to polarization. It was also the starting point of a clear rise in manufacturing productivity (Herrendorf et al., 2014) and the rise of low-skill service jobs (Autor and Dorn, 2013). As of yet, polarization and structural change have been treated separately, and vertical polarization has not been considered. We provide a unified explanation with occupation-level productivity enhancements among middle-skill jobs as the single driving force.

When we quantify the model also allowing for exogenous sector-specific TFP growth, we find that occupation-specific TFP growth alone can still account for more than half of structural change. Conversely, we find that exogenous sector-specific TFP growth is mostly important for accounting for vertical polarization, but not horizontal polarization. However, sector-specific TFP growth cannot generate within-sector polarization, horizontal or vertical. Lastly, we document that while the routinization index commonly used in the polarization literature is positively correlated with occupation-specific TFP growth, the correlation is about 0.5, indicating that other forces may be at work.

Related Literature Task-based models are found in Acemoglu and Autor (2011) and Goos et al. (2014) to explain polarization in employment shares, but neither addresses wage polarization. None of them relate polarization to structural change across macroeconomic sectors, nor treat managers as an occupation that is qualitatively different from workers.

The manager-level technology in our model is an extension of the span-of-control model of Lucas (1978), in which managers hire workers to produce output. However, unlike all existing variants of the span-of-control model, in our model managers organize tasks instead of workers. That is, instead of deciding how many workers to hire, they decide on the quantities of each task to use in production, and for each task, how much skill to hire (rather than how many homogeneous workers). Moreover, rather than assuming a Cobb-Douglas technology between managerial talent and workers, we assume a CES technology between managerial talent and tasks.[1]

Our model is closely related to the rapidly growing literature in international trade that use assignment models to explain inequality between occupations and/or industries (Burstein et al., 2015; Lee, 2015). The majority of such models follow in the tradition of Roy: all workers have as many types of skills as there are available industry/occupation combinations, and select themselves into the job they in which they have a comparative advantage. To make the model tractable, they typically employ a Fréchet distribution which collapses the model into an empirically testable set of equations for each industry and/or occupation pair. While the manager-worker division in our model is also due to Roy-selection, but the horizontal sorting of workers into tasks is qualitatively different. In addition, we assume only 2-skill types, which is arguably more suitable for studying the endogenous formation of skills.

Since Ngai and Pissarides (2007), most production-driven models of structural change rely on exogenously evolving sectoral productivities. Closer to our model is Acemoglu and Guerrieri (2008), in which the capital-intensive sector (in the sense of

[1]Starting with the standard span-of-control model, we incorporate (i) non-unitary elasticity between managers and workers, (ii) heterogeneity in worker productivity as well as in managerial productivity, (iii) multiple worker tasks or occupations, and (iv) multiple sectors.

having a larger capital share in a Cobb-Douglas technology) vanishes in the limiting balanced growth path. While sectors in their model differ in how intensively they use capital and labor, in our model they differ in how intensively they use different tasks. By contrasting different types of labor, rather than capital and labor, we can connect structural change—which happens across sectors—to labor market inequality across occupations. Moreover, as mentioned above, unlike any other existing explanation of structural change, our model implies that it is certain occupations, not broadly-defined sectors, that may vanish in the limit.

While we are the first to build a model in which individuals with different skills sort themselves into different occupations, which in turn are used as production inputs in multiple sectors,[2] there have been recent attempts such as Buera and Kaboski (2012) and Buera et al. (2015), in which multiple sectors use different combinations of heterogeneous skills as production inputs. One important distinction is that we separate worker human capital (continuous distribution) and skill levels of a task or occupation (discrete). More important, our driving force (i.e., routinization) is specific to a task or occupation, not to workers' human capital levels. This way, not only can we address broader dimensions of wage inequality and use micro-labor estimates to discipline our model,[3] but also represent sectoral TFPs endogenously by aggregating over equilibrium occupational choices, rather than relying on exogenously evolving worker-skill specific productivities.

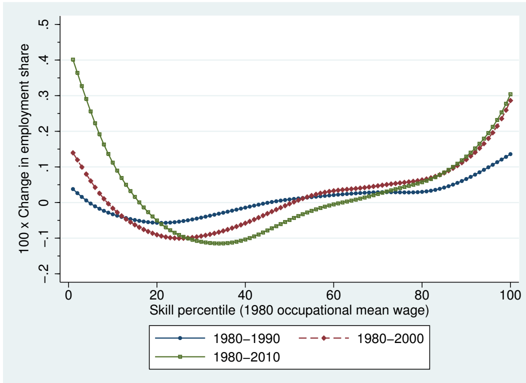
The rest of the paper is organized as follows. In section 2, we summarize three groups of facts: horizontal polarization, structural change, and the rise of managers. In section 3, we present the model and solve for its equilibrium allocation. Section 4 performs comparative statics on the equilibrium, which demonstrate that routinization leads to horizontal and vertical polarization, and ultimately structural change. Section 5 characterizes the BGP of the dynamic economy and studies local stability. Section 6 calibrates the model to data from 1980-2010, and also quantifies how important routinization may have been in explaining recent trends in structural change. Section 7 concludes.

2 Facts

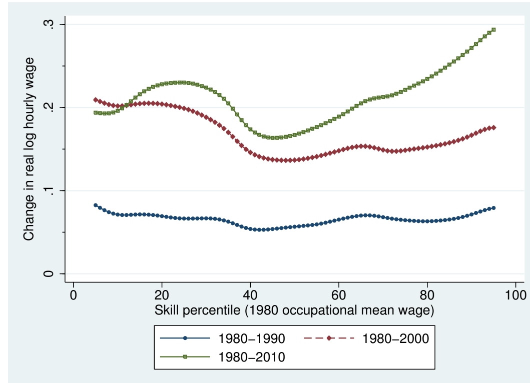
1. Jobs and wages have polarized, figure 1 Source: U.S. Census and ACS 2010, replicated and extended from Autor and Dorn (2013) and extended to 2010. Occupations are ranked by their 1980 mean wage.

[2] Bárány and Siegel (2015) build a model in which occupations are tied to sectors.

[3] Autor et al. (2006); Acemoglu and Autor (2011) show that residual wage inequality controlling for education groups is much larger than between-group inequality.

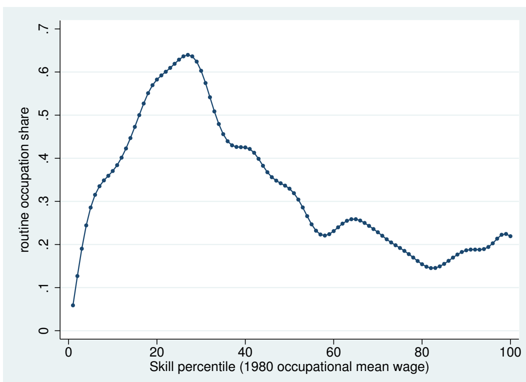


(a) Employment Polarization

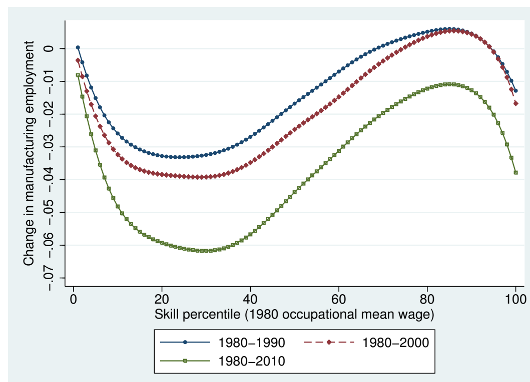


(b) Wage Polarization

Fig. 1: Job and Wage Polarization, 30 years.



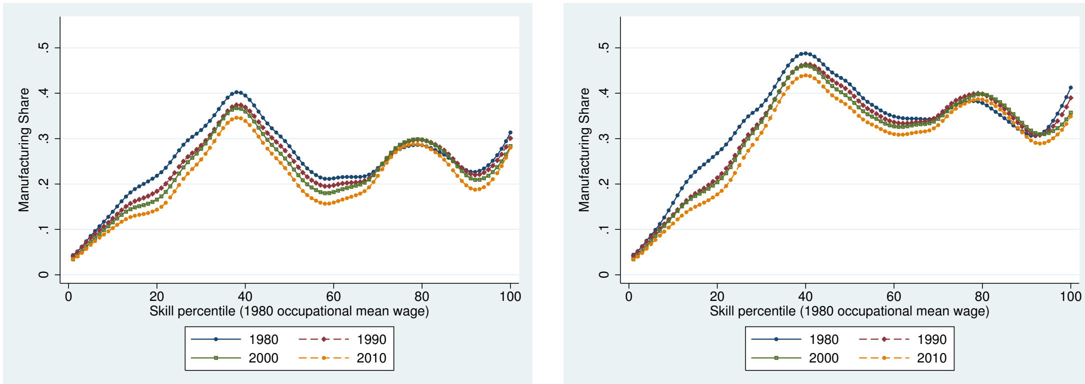
(a) Routinization



(b) Manufacturing Employment

Fig. 2: Routinization and Structural Change, 30 years.

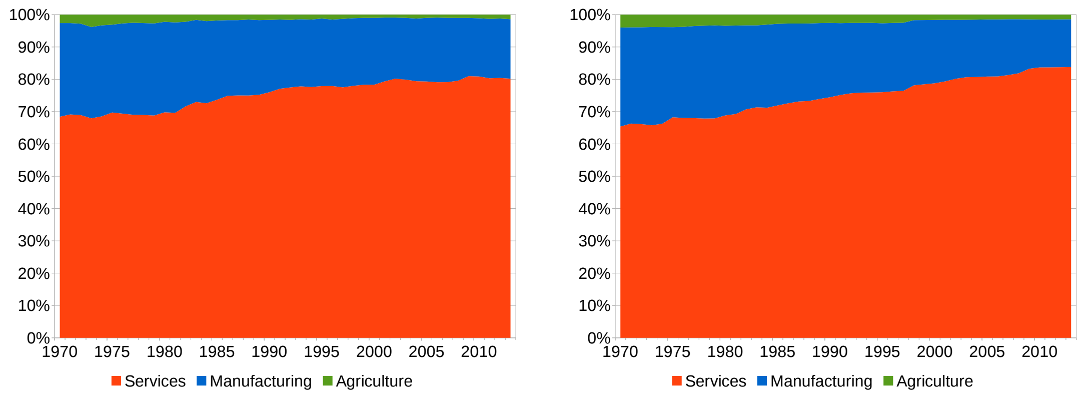
2. Routinizable jobs correlate with structural change, figure 2. Source: U.S. Census and ACS 2010. Left panel replicated from Autor and Dorn (2013), right panel shows change in manufacturing employment by occupation cell.
3. Manufacturing has relatively higher share of intermediate occupations, of which low-to-middle skill jobs have been relatively shrinking, figure 3. The right panel includes mining and construction into manufacturing. Source: U.S. Census and ACS 2010.
4. Structural change continues in the U.S., figure 4. Source: BEA (value-added), NIPA Table 6 (persons involved in production). Manufacturing includes mining and construction.
5. Both employment share and relative wages for managers are increasing: figure 5. See appendix for definition of managers in the census.



(a) Manufacturing

(b) Manufacturing+Mining+Construction

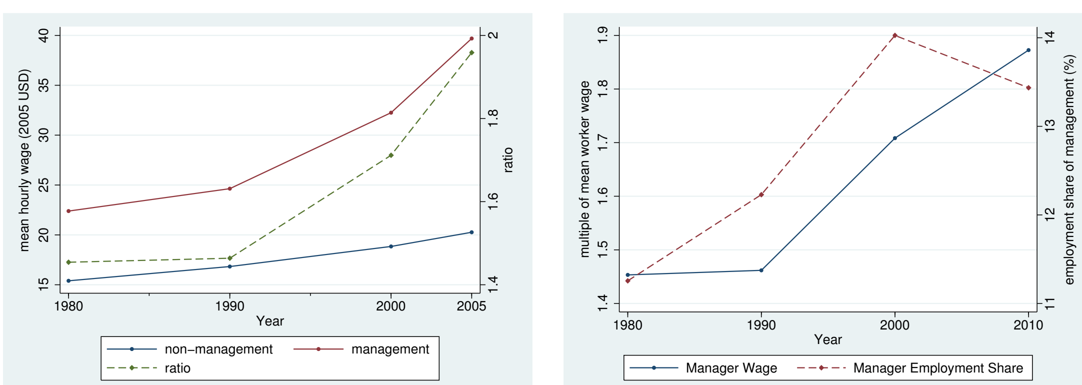
Fig. 3: Manufacturing employment shares across skill percentiles.



(a) GDP, Value-Added

(b) Aggregate Employment Shares

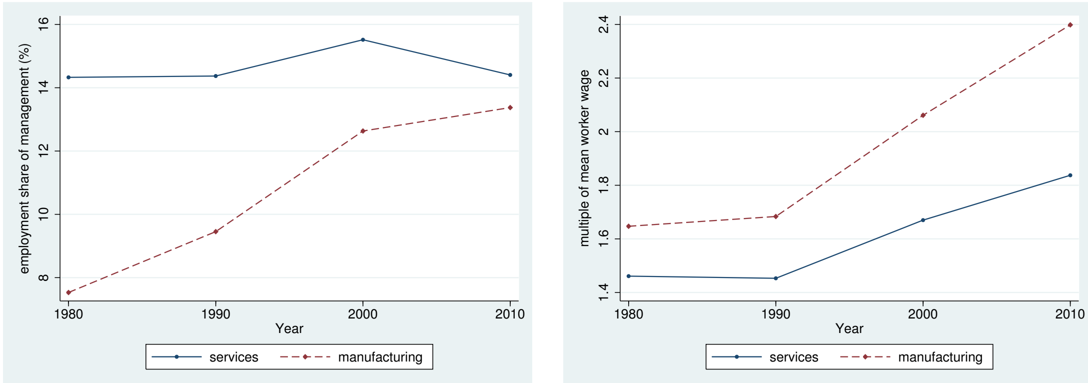
Fig. 4: Structural Change, 1970-2013.



(a) Wage Levels

(b) Relative Employment and Wages

Fig. 5: Managers vs Workers



(a) Manager Employment Share

(b) Relative Manager Wage

Fig. 6: Managers by Sector

6. Moreover, this occurs faster in the manufacturing sector: figure 6.

3 Model

There are a continuum of individuals each endowed with two types of skill, (h, z). Human capital, h, is used to produced tasks. Management, z, is a special skill for organizing tasks. WLOG we assume that the mass of individuals is 1, with associated distribution function μ.

There are 2 sectors $i \in \{m, s\}$.[4] In each sector, goods are produced in teams. A single manager uses her own skill and physical capital to organize three types of tasks $j \in \{0, 1, 2\}$ (e.g., low-, medium-, high-skill occupations; or manual, routine, abstract tasks). Each task requires both physical and human capital, and how much of each is allocated to each task is decided by the manager. Aggregating over the goods produced by all managers within a sector yields total sectoral output.

Within a sector, a better manager can produce more goods with the same amount of tasks, but task intensities may differ across sectors. Specifically, we assume that

$$y_i(z) = \left[\eta_i^{\frac{1}{\omega}} x_z(z)^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} x_h(z)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{1a}$$

$$x_z(z) = M_z k^{\alpha} z^{1-\alpha}, \qquad x_h(z) = \left[\sum_{j=0}^{2} \nu_{ij}^{\frac{1}{\sigma}} \tau_{ij}(z)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{1b}$$

$$\tau_{ij}(z) = M_j \int_{\mathrm{h_{ij}(z)}} t_j(k, h) d\mu, \tag{1c}$$

[4] In our application, the two sectors stand in for "manufacturing" and "services," respectively. However, I analytical model can be extended to incorporate any countably finite number N of sectors; we use the subscripts m and s to avoid confusing them with the subscripts for tasks.

$$t_0(k,h) = k^\alpha \bar{h}^{1-\alpha}, \quad t_1(k,h) = k^\alpha h^{1-\alpha} \quad t_2(k,h) = k^\alpha (h-\chi)^{1-\alpha}, \tag{1d}$$

with $\sum_j \nu_{ij} = 1$. The $t_j(\cdot)$'s are the amounts of task output produced by an individual with human capital h and physical capital k, the latter of which is allocated by the manager. Integrating over individual task outputs over the set of workers hired by a manager of skill z for task j in sector i, $\mathrm{h_{ij}(z)}$, yields a task aggregate $\tau_{ij}(z)$. The substitutability between tasks is captured by the elasticity parameter σ, and ω captures the elasticity between all workers and managers.

For task (or occupation) 0, a worker's own human capital is irrelevant for production: all workers' effective skill input becomes $\bar{h}$. This is to capture manual jobs that do not depend on skills. For task 2, some of your skills become useless and effective skill input becomes $h - \chi$. This is to capture analytic jobs, for which lower levels of skill are redundant. We will refer to the managerial task as "task z," which is vertically differentiated from tasks $j \in \{0,1,2\}$, which are horizontally differentiated. The M_j's, $j \in \{0,1,2,z\}$, capture task-specific TFP's, which are sector-neutral.

Several points are in order. As is the case with most models of sorting workers into tasks, the worker side of our model can be viewed as a special case of Costinot and Vogel (2010). However, we model managers and have more than one sector. In contrast to Acemoglu and Autor (2011), we have a continuum of skills rather than tasks, and a discrete number of tasks rather than skills. While the implications are comparable, our formulation is more suitable for exploring employment shares across tasks (which are discrete in the data). The model is also comparable to Goos et al. (2014), who show (empirically) that relative price changes in task-specific capital, representing routinization, can drive employment polarization. However, they do not model skill and hence cannot explain wage polarization.

Now let $\mathcal{H}_{ij}$ denote the set of individuals working in sector $i \in \{m,s\}$ on task $j \in \{0,1,2\}$. We can define

- $\mathcal{H}_i = \cup_{j\in\{0,1,2\}} \mathcal{H}_{ij}$: set of individuals who work as workers in sector $i \in \{m,s\}$,
- $\mathcal{H}_j = \cup_{i\in\{m,s\}} \mathcal{H}_{ij}$: set of individuals who work as workers on task $j \in \{0,1,2\}$,
- $\mathcal{H} = \cup_{i\in\{m,s\}} \mathcal{H}_i = \cup_{j\in\{0,1,2\}} \mathcal{H}_j$: set of all individuals who work as workers,
- $\mathcal{Z} = \mathcal{Z}_m \cup \mathcal{Z}_s$: set of individuals who work as managers in sector $i \in \{m,s\}$.

Output in each sector is then

$$Y_i = \int_{\mathcal{Z}_i} y_i(z) d\mu, \tag{2}$$

and a final good is produced by combining output from both sectors according to a CES aggregator:

$$Y = G(Y_m, Y_s) = \left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{3}$$

where $\gamma_M + \gamma_S = 1$ and we will assume $\epsilon < 1$.[5]

3.1 Planner's Problem

We assume complete markets for solve a static planner's problem. A planner allocates aggregate capital K and all individuals into sectors $i \in \{m, s\}$ and tasks $j \in \{0, 1, 2, z\}$. The objective is to maximize current output (3) subject to (1))-(2) and

$$K = K_m + K_s \equiv \left\{ \int_{\mathcal{Z}_m} \left[\sum_{j \in \{0,1,2,z\}} k_{mj}(z) \right] + \int_{\mathcal{Z}_s} \left[\sum_{j \in \{0,1,2,z\}} k_{sj}(z) \right] \right\} d\mu$$

$$H_{ij} \equiv \int_{\mathcal{H}_{ij}} h d\mu = \int_{\mathcal{Z}_i} h_{ij}(z) d\mu, \quad j \in \{0, 1, 2\},$$

where K_i is the amount of capital allocated to sector i, H_{ij} the total amount of human capital allocated to task j in sector i, and $(k_{ij}(z), h_{ij}(z))$ is the amounts of physical and human capital allocated to task j in sector i under a manager with skill z, where $j \in \{0, 1, 2, z\}$ for k and $j \in \{0, 1, 2\}$ for h.

For existence of a solution, we assume that

Assumption 1 *The population means of both skills are finite, that is,*

$$\int z d\mu < \infty, \quad \int h d\mu < \infty.$$

and

Assumption 2 *There exists a strictly positive mass of individuals who do not lose all of their h-skill by working in task 2, i.e. $\mu(h > \chi) > 0$.*

The following assumption is needed for uniqueness:

Assumption 3 *The measure $\mu(z, h)$ is continuous and has a connected support on $(h, z) \in [0, h_u) \times [0, z_u)$, where $x_u \leq \infty$ is the upperbound of skill $x \in \{h, z\}$; i.e. $\mu(h, z) > 0$ on $(0, 0) \leq (h, z) < (h_u, z_u) \leq \infty$.*

Along with assumption 2, this implies $\chi < h_u$. Before showing existence and uniqueness of the solution, we first characterize the solution in the following order:

1. Characterize optimal physical capital allocations across tasks within a sector.
2. Characterize optimal human capital (h) allocations across tasks within a sector.
3. Characterize optimal labor (manager-worker) allocations within a sector.
4. Solve for optimal capital and labor allocations across sectors.

[5] The estimated ϵ between the manufacturing and service sector (broadly defined) is close to zero, as we show in section 6.1.

Capital allocation within sectors Thanks to the HD1 assumptions, we can write sectoral technologies as

$$Y_i = \left[\eta_i^{\frac{1}{\omega}} X_{iz}^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} X_{ih}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{5a}$$

$$X_{iz} = M_z K_{iz}^{\alpha} Z_i^{1-\alpha}, \qquad X_{ih} = \left(\sum_j \nu_{ij}^{\frac{1}{\sigma}} T_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{5b}$$

where X_{ih} is a sectoral task aggregate and

$$T_{i0} = M_0 K_{i0}^{\alpha}\left[\bar{h}\mu(\mathcal{H}_{i0})\right]^{1-\alpha}, \quad T_{i1} = M_1 K_{i1}^{\alpha} H_{i1}^{1-\alpha},$$
$$T_{i2} = M_2 K_{i2}^{\alpha}\left[H_{i2} - \chi\mu(\mathcal{H}_{i2})\right]^{1-\alpha}.$$

Given sectoral capital K_i, the planner equalizes marginal product across tasks:

$$MPK_{i0} = MPK_{i1} = MPK_{i2}$$
$$\Rightarrow \quad \frac{MPT_{i0}\cdot\alpha T_{i0}}{K_{i0}} = \frac{MPT_{i1}\cdot\alpha T_{i1}}{K_{i1}} = \frac{MPT_{i2}\cdot\alpha T_{i2}}{K_{i2}}$$
$$\Rightarrow \quad \frac{MPT_{i1}\cdot T_{i1}}{MPT_{i0}\cdot T_{i0}} = \frac{K_{i1}}{K_{i0}} \quad \equiv \quad \pi_{i1} = \left(\frac{\nu_{i1}}{\nu_{i0}}\right)^{\frac{1}{\sigma}}\cdot\left(\frac{T_{i1}}{T_{i0}}\right)^{\frac{\sigma-1}{\sigma}} \tag{6a}$$
$$\frac{MPT_{i2}\cdot T_{i2}}{MPT_{i1}\cdot T_{i1}} = \frac{K_{i2}}{K_{i1}} \quad \equiv \quad \pi_{i2} = \left(\frac{\nu_{i2}}{\nu_{i1}}\right)^{\frac{1}{\sigma}}\cdot\left(\frac{T_{i2}}{T_{i1}}\right)^{\frac{\sigma-1}{\sigma}}, \tag{6b}$$

where MPT_{ij} is the marginal product of T_{ij} w.r.t. X_i, and π_{ij} is the capital input ratio in tasks $j \in \{1,2\}$ and $j-1$. Due to the Cobb-Douglas assumption, π_{ij} divided by task output ratios is the marginal rate of technological substitution ($MRTS$) between tasks j and $j-1$; furthermore, π_{ij} divided by either factor input ratios in tasks j and $j-1$ measures the $MRTS$ of that factor between tasks j and $j-1$. (For capital, this is equal to 1.) Given (6) we can write

$$X_{ih} = \nu_{i0}^{\frac{1}{\sigma-1}} \underbrace{(1+\pi_{i1}+\pi_{i1}\pi_{i2})}_{\equiv\Pi_{ih}}{}^{\frac{\sigma}{\sigma-1}} T_{i0}. \tag{7}$$

Of course, MPK must also be equalized across the managerial task and the rest:

$$MPK_{iz} = MPK_{i0}$$
$$\Rightarrow \quad \frac{MPX_{iz}\cdot\alpha X_{iz}}{K_{iz}} = \frac{MPX_{ih}\cdot MPT_{i0}\cdot\alpha T_{i0}}{K_{i0}}$$
$$\Rightarrow \quad \frac{MPX_{iz}\cdot X_{iz}}{MPX_{ih}\cdot MPT_{i0}\cdot T_{i0}} = \frac{K_{iz}}{K_{i0}} \quad \equiv \quad \pi_{iz} = \left(\frac{\eta_i}{1-\eta_i}\right)^{\frac{1}{\omega}}\cdot\left(\frac{X_{iz}}{X_{ih}}\right)^{\frac{\omega-1}{\omega}}\cdot\Pi_{ih}, \tag{8}$$

which then allows us to write, using (7),

$$Y_i = (1-\eta_i)^{\frac{1}{\omega-1}}\left[1+\pi_{iz}/\Pi_{ih}\right]^{\frac{\omega}{\omega-1}} X_{ih}$$
$$= (1-\eta_i)^{\frac{1}{\omega-1}}\left[1+\pi_{iz}/\Pi_{ih}\right]^{\frac{\omega}{\omega-1}} \nu_{i0}^{\frac{1}{\sigma-1}} \Pi_{ih}^{\frac{\sigma}{\sigma-1}} T_{i0} \tag{9}$$

Sorting skills across worker tasks within sectors Given the sectoral production function (5), we can now decide how to allocate worker skills h to worker tasks $j \in \{0, 1, 2\}$. Since skill doesn't matter in task 0 and some becomes irrelevant in task 2, there is positive sorting of workers into tasks; i.e. there will be thresholds $(\hat{h}_1, \hat{h}_2)$ s.t. all workers with $h \le \hat{h}_1$ work in task 0 and those with $h > \hat{h}_2$ work in task 2. Note that these thresholds must be equal across sectors, hence are not subscripted by i.

For each threshold, it must be that the marginal product of the threshold worker is equalized in either task:

$$MPT_{i0} \cdot \frac{(1-\alpha)T_{i0}}{\bar{h}\mu(\mathcal{H}_{i0})} \cdot \bar{h} = MPT_{i1} \cdot \frac{(1-\alpha)T_{i1}}{H_{i1}} \cdot \hat{h}_1,$$
$$MPT_{i1} \cdot \frac{(1-\alpha)T_{i1}}{H_{i1}} \cdot \hat{h}_2 = MPT_{i2} \cdot \frac{(1-\alpha)T_{i2}}{H_{i2} - \chi\mu(\mathcal{H}_{i2})} \cdot (\hat{h}_2 - \chi)$$

using assumption 3, so

$$\hat{h}_1 = \frac{\bar{h}_1 L_{i1}}{\pi_{i1} L_{i0}}, \qquad 1 - \frac{\chi}{\hat{h}_2} = \frac{(\bar{h}_2 - \chi) L_{i2}}{\pi_{i2} \bar{h}_1 L_{i1}}, \tag{10}$$

where $L_{ij} \equiv \mu(\mathcal{H}_{ij})$ and $\bar{h}_j \equiv H_{ij}/L_{ij}$; that is, we are assuming

Assumption 4 *The means of skills in tasks $j \in \{0, 1, 2, z\}$, that is, $(\bar{h}_j, \bar{z})$, are equal across sectors $i \in \{m, s\}$.*

This is an assumption is needed because we assume discrete tasks; it can be thought of as the limit of vanishing supermodularity within segments of a continuum of tasks. Assumption 1 also guarantees that all objects are finite and well-defined. Using (10) we can reformulate (6) as

$$\pi_{i1} = \frac{\nu_{i1}}{\nu_{i0}} \cdot \left[\frac{M_1}{M_0} \left(\frac{\hat{h}_1}{\bar{h}} \right)^{1-\alpha} \right]^{\sigma-1}, \quad \pi_{i2} = \frac{\nu_{i2}}{\nu_{i1}} \cdot \left[\frac{M_2}{M_1} \left(1 - \frac{\chi}{\hat{h}_2} \right)^{1-\alpha} \right]^{\sigma-1}. \tag{11}$$

Sorting managers and workers within a sector Now we know how to allocate $K_i, \mathcal{H}_i$ within a sector, but we still need to know how to divide individuals into managers and workers; that is, determine $\mathcal{Z}_i \cup \mathcal{H}_i$ given a mass of individuals within a sector.

Since individuals are heterogeneous in 2 dimensions, the key is to get a cutoff rule $\tilde{z}_j(h)$ s.t. for every h, individuals with z above $\tilde{z}_j(h)$ become managers and below become workers. Since the h-skill is used differently across tasks, we need to get 3 such rules for each sector; however the rule must be identical across sectors.

For $h \le \hat{h}_1$, this rule is simple. For these workers, h does not matter, so $\tilde{z}_0(h) = \hat{z}$, i.e., is constant. The constant is chosen so that the marginal product of the threshold manager is equalized in either task:

$$MPX_{iz} \cdot \frac{(1-\alpha)X_{iz}}{Z_i} \cdot \tilde{z}_0(h) = MPX_{ih} \cdot MPT_{i0} \cdot \frac{(1-\alpha)T_{i0}}{\bar{h}\mu(\mathcal{H}_{i0})} \cdot \bar{h}$$

$$\Rightarrow \quad \tilde{z}_0(h) = \hat{z} = \frac{Z_i}{\pi_{iz} L_{i0}} = \frac{\bar{z} L_{iz}}{\pi_{iz} L_{i0}}, \tag{12}$$

where $L_{iz} = \mu(\mathcal{Z}_i)$ and $\bar{z} = Z_i / L_{iz}$ (which is equal across sectors by assumption 4). Then from (8) we can write

$$\pi_{iz} = \frac{\eta_i \nu_{i0}^{\frac{1-\omega}{\sigma-1}}}{1-\eta_i} \cdot \left[\frac{M_z}{M_0} \left(\frac{\hat{z}}{\bar{h}} \right)^{1-\alpha} \right]^{\omega-1} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{\sigma-1}}. \tag{13}$$

For $h \in (\hat{h}_1, \hat{h}_2]$, the rule is linear:

$$MPX_{iz} \cdot \frac{(1-\alpha) X_{iz}}{Z_i} \cdot \tilde{z}_1(h) = MPX_{ih} \cdot MPT_{i1} \cdot \frac{(1-\alpha) T_{i1}}{H_{i1}} \cdot h$$
$$\Rightarrow \quad \frac{\tilde{z}_1(h)}{h} = \phi_1 = \frac{\pi_{i1} Z_i}{\pi_{iz} H_{i1}} = \frac{\pi_{i1} \bar{z} L_{iz}}{\pi_{iz} \bar{h}_1 L_{i1}}.$$

and finally for $h > \hat{h}_2$, the rule is affine:

$$MPX_{iz} \cdot \frac{(1-\alpha) X_{iz}}{Z_i} \cdot \tilde{z}_2(h) = MPX_{ih} \cdot MPT_{i2} \cdot \frac{(1-\alpha) T_{i2}}{H_{i2} - \chi \mu(\mathcal{H}_{i2})} \cdot (h - \chi)$$
$$\Rightarrow \quad \frac{\tilde{z}_2(h)}{h-\chi} = \phi_2 = \frac{\pi_{i1} \pi_{i2} Z_i}{\pi_{iz} \left(H_{i2} - \chi L_{i2} \right)} = \frac{\pi_{i1} \pi_{i2} \bar{z} L_{iz}}{\pi_{iz} (\bar{h}_2 - \chi) L_{i2}}.$$

Observe that

$$\hat{z} = \phi_1 \hat{h}_1, \qquad 1 - \chi / \hat{h}_2 = \phi_1 / \phi_2, \tag{14}$$

so $(\hat{h}_1, \hat{h}_2, \hat{z})$ completely determine the ϕ_j's, and all objects are well defined given assumption 2, since all tasks are essential.

Sectoral production function and allocation across sectors Equations (10)-(13) completely describe the task thresholds. What is important here is that all these thresholds are determined independently of the amount of physical capital. To see this more clearly, rewrite (9) to obtain

$$Y_i = \psi_i \cdot [1 + \pi_{iz} / \Pi_{ih}]^{\frac{\omega}{\omega-1}} \, \Pi_{ih}^{\frac{\sigma}{\sigma-1}} M_0 K_{i0}^{\alpha} L_{i0}^{1-\alpha}$$
$$\psi_i \equiv (1-\eta_i)^{\frac{1}{\omega-1}} \nu_{i0}^{\frac{1}{\sigma-1}} \bar{h}^{1-\alpha}$$

and furthermore since

$$K_i = K_{i0} \underbrace{(\Pi_{ih} + \pi_{iz})}_{\Pi_{K_i}} \tag{15}$$

$$L_i = L_{i0} \underbrace{\left[1 + (\hat{h}_1 / \bar{h}_1) \pi_{i1} + \frac{1 - \chi / \hat{h}_2}{(\bar{h}_2 - \chi) / \hat{h}_1} \cdot \pi_{i1} \pi_{i2} + (\hat{z} / \bar{z}) \pi_{iz} \right]}_{\Pi_{L_i}}, \tag{16}$$

we obtain

$$Y_i = \underbrace{M_0 \psi_i \cdot \Pi_{ih}^{\frac{\omega-\sigma}{(\omega-1)(\sigma-1)}} \Pi_{K_i}^{\frac{\omega}{\omega-1}-\alpha} \Pi_{L_i}^{\alpha-1}}_{\Phi_i:\ \text{TFP}} K_i^{\alpha} L_i^{1-\alpha}. \tag{17}$$

Note that sectoral TFP, Φ_i, can be decomposed into 3 parts: M_0, that is common across both sectors, ψ_i, which is sector-specific but exogenous, and the parts determine by $(\Pi_{ih}, \Pi_{K_i}, \Pi_{L_i})$, which is sector-specific and endogenously determined by $(\hat{h}_1, \hat{h}_2, \hat{z})$. Furthermore, since the thresholds depend only on the relative masses of individuals across tasks *within* a sector, they do not depend on the employment size of the sector (nor capital). Hence even as K_i or L_i changes, these thresholds do not as long as the distribution of skills remains constant.

Sectors only differ in how intensely they use each task, i.e., the mass of individuals allocated to each task. As usual, these masses are determined so that the MPK and MPL are equalized across sectors:

$$\kappa \equiv \frac{K_s}{K_m} = \left(\frac{\gamma_s}{\gamma_m}\right)^{\frac{1}{\epsilon}} \left(\frac{Y_s}{Y_m}\right)^{\frac{\epsilon-1}{\epsilon}} = \frac{L_s}{L_m} \tag{18}$$

where κ is capital input ratios between sectors m and s.

3.2 Existence and Uniqueness

Having characterized the optimal allocation (which is identical to the equilibrium allocation), we can now establish existence and uniqueness:

Theorem 1 *Under assumptions 1-4, the solution to the planner's problem exists and is unique.*

Proof: First define a renormalization of Π_{L_i}:

$$\tilde{\Pi}_{L_i} \equiv (1-\eta_i)\nu_{i0} M_0^{\sigma-1} \bar{h}^{(\sigma-1)(1-\alpha)} \Pi_{L_i} = \sum_{j=0,1,2,z} V_{ij},$$

where the weights V_{ij} are

$$V_{i0} \equiv (1-\eta_i)\nu_{i0} M_0^{\sigma-1} \cdot \frac{\bar{h}^{\alpha+\sigma(1-\alpha)}}{\bar{h}}, \tag{19a}$$

$$V_{i1} \equiv (1-\eta_i)\nu_{i1} M_1^{\sigma-1} \cdot \frac{\hat{h}_1^{\alpha+\sigma(1-\alpha)}}{\bar{h}_1}, \tag{19b}$$

$$V_{i2} \equiv (1-\eta_i)\nu_{i2} M_2^{\sigma-1} \cdot \frac{\left[\hat{h}_1(1-\chi/\hat{h}_2)\right]^{\alpha+\sigma(1-\alpha)}}{\bar{h}_2 - \chi}, \tag{19c}$$

$$V_{iz} \equiv \eta_i \tilde{\Pi}_{ih}^{\frac{\sigma-\omega}{\sigma-1}} M_z^{\omega-1} \cdot \frac{\hat{z}^{\alpha+\omega(1-\alpha)}}{\bar{z}} \tag{19d}$$

and $\tilde{\Pi}_{ih}$ is a renormalization of Π_{ih}:

$$\tilde{\Pi}_{ih} \equiv \nu_{i0} M_0^{\sigma-1} \bar{h}^{(\sigma-1)(1-\alpha)} \cdot \Pi_{ih}.$$

Note that the only differences in the V_{ij}'s across sectors i comes from the task intensity parameters ν_{ij}, η_i (since Π_{ih} is also a function only of the ν_{ij}'s in equilibrium). The total amount of labor in each task j can be expressed as

$$L_j = \sum_{i \in \{m,s\}} \frac{V_{ij}}{\tilde{\Pi}_{L_i}} \cdot L_i, \quad \text{where} \quad L_m = 1/(1+\kappa),\ L_s = \kappa/(1+\kappa) \tag{20}$$

for $j \in \{0, 1, 2, z\}$. This system of equations that solves the planner's problem are also the equilibrium market clearing conditions; the LHS is the labor supply and RHS demand for each task j. Since $\bar{h}_j L_j = H_j$, $\bar{z} L_z = Z$, $\sum_j L_j = 1$ and $\kappa = \kappa(\hat{h}_1, \hat{h}_2, \hat{z})$ is a function of $(\hat{h}_1, \hat{h}_2, \hat{z})$ from (18), the solution to $(\hat{h}_1, \hat{h}_2, \hat{z})$ is found from the system of three equations

$$\log \hat{z} = \frac{(1-\omega)\log M_z + \log Z - \log\left[\sum_i \frac{\eta_i \tilde{\Pi}_{ih}^{\frac{\sigma-\omega}{\sigma-1}}}{\tilde{\Pi}_{L_i}} \cdot L_i\right]}{\alpha + \omega(1-\alpha)} \tag{21a}$$

$$\log \hat{h}_1 = \frac{(1-\sigma)\log M_1 + \log H_1 - \log\left[\sum_i \frac{(1-\eta_i)\nu_{i1}}{\tilde{\Pi}_{L_i}} \cdot L_i\right]}{\alpha + \sigma(1-\alpha)} \tag{21b}$$

$$\begin{aligned} \log \hat{h}_2 =& \log(\hat{h}_2 - \chi) \\ &- \frac{(1-\sigma)\log\left(\frac{M_2}{M_1}\right) + \log\left(\frac{H_2 - \chi L_2}{H_1}\right) - \log\left[\sum_i \frac{(1-\eta_i)\nu_{i1}}{\tilde{\Pi}_{L_i}} \cdot L_i \Big/ \sum_i \frac{(1-\eta_i)\nu_{i1}}{\tilde{\Pi}_{L_i}} \cdot L_i\right]}{\alpha + \sigma(1-\alpha)} \end{aligned} \tag{21c}$$

where

$$\begin{aligned} Z(\hat{h}_1, \hat{h}_2, \hat{z}) &= \left[\int^{\hat{h}_1} \int_{\hat{z}} + \int_{\hat{h}_1}^{\hat{h}_2} \int_{\phi_1(\hat{z}, \hat{h}_1)\cdot h} + \int_{\hat{h}_2} \int_{\phi_2(\hat{h}_1, \hat{h}_2, \hat{z})\cdot(h-\chi)}\right] z dF(z|h) dG(h) \\ H_1(\hat{h}_1, \hat{h}_2, \hat{z}) &= \int_{\hat{h}_1}^{\hat{h}_2} hF\left(\phi_1(\hat{h}_1, \hat{z}) \cdot h | h\right) dG(h) \\ H_2(\hat{h}_1, \hat{h}_2, \hat{z}) &= \int_{\hat{h}_2} hF\left(\phi_2(\hat{h}_1, \hat{h}_2, \hat{z}) \cdot (h-\chi) | h\right) dG(h) \end{aligned}$$

and $G(h)$ is the marginal distribution of h, and $F(z|h)$ the distribution of z conditional on h; that is

$$\mu(\tilde{h}, \tilde{z}) = \int^{\tilde{h}} \int^{\tilde{z}} dF(z|h) dG(h).$$

Existence is straightforward. Note that the domain of $\hat{z} \in [0, z_u)$, $\hat{h}_1 \in [0, \hat{h}_2)$, and $\hat{h}_2 \in [\hat{h}_1, h_u)$. Hence holding other variables fixed,

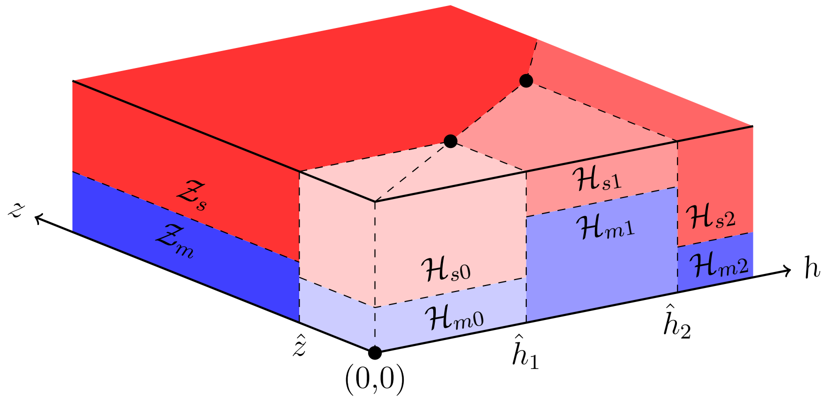


Fig. 7: Equilibrium

1. as $\hat{z} \to 0$, LHS of (21a) approaches $-\infty$, while RHS approaches ∞. Conversely, as the LHS approaches $\log z_u$, RHS approaches $-\infty$.
2. as $\hat{h}_1 \to 0$, LHS of (21b) approaches $-\infty$, while RHS remains finite. Conversely, as LHS approaches $\log \hat{h}_2$, RHS approaches $-\infty$.
3. as $\hat{h}_2 \to \max\{\hat{h}_1, \chi\}$, LHS of (21c) remains finite, while RHS approaches $-\infty$. Conversely, as LHS approaches $\log h_u$, RHS becomes larger than LHS.

Assumption 3 ensures that all RHS's are continuous; hence a solution exists. The assumption also ensures that the mapping in the RHS is monotone; so for any guess of the two other thresholds, each threshold is found uniquely as a function of the other two. That is, the RHS of system (21) with respect to $(\log \hat{z}, \log \hat{h}_1, \log \hat{h}_2)$ is a contraction; so the fixed point to the system is unique. □

The equilibrium skill allocation is depicted in figure 8. The thresholds determine the tasks, and employment is split across sectors while preserving the means for each task. The different masses of sectoral employment across tasks are due to the task intensity parameters ν_{ij}, η_i.

3.3 Equilibrium wages and prices

Since there are no frictions, the planner's allocation coincides with a competitive equilibrium. Hence, the solution $(\hat{h}_1, \hat{h}_2, \hat{z})$ gives all the information needed to derive equilibrium prices. The price of the final good can be normalized to 1:

$$P = 1 = \left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \qquad p_i = \left[Y_i/\gamma_i Y\right]^{-\frac{1}{\epsilon}}. \tag{22}$$

Sectoral output prices can be obtained by applying the sectoral output in (15)-(17) in (22). The interest rate R is given either by the dynamic law of motion for aggregate capital, or fixed in a small open economy. So w_0 is

$$w_0 = \frac{1-\alpha}{\alpha} \cdot \frac{K_{i0}}{L_{i0}} \cdot \frac{R}{\bar{h}}$$

where the capital-labor ratio can be found from (15)-(16).

Given w_0, indifference across tasks for threshold workers imply

$$w_0 \bar{h} = w_1 \hat{h}_1, \quad w_1 \hat{h}_2 = w_2(\hat{h}_2 - \chi) \tag{23a}$$

$$\Rightarrow \quad w_0/w_1 = \hat{h}_1/\bar{h}, \quad w_1/w_2 = 1 - \chi/\hat{h}_2. \tag{23b}$$

and likewise, the threshold manager implies a "managerial efficiency wage"

$$w_z \hat{z} = w_0 \bar{h} \quad \Rightarrow \quad w_0/w_z = \hat{z}/\bar{h}. \tag{23c}$$

Hence, relative wages for task j are simply the inverse of the thresholds.

4 Comparative Statics

The sectoral technology representation (17) implies that this model has similar implications as Ngai and Pissarides (2007): the sector with the larger TFP shrinks. The major difference is that these TFP's are endogenous.

What is more interesting is the implications of growth in task-specific TFP's—this is equivalent to the price of task-specific capital falling in Goos et al. (2014)—or changes in the distribution for skill. In particular, we are interested in the effect of routinization, which we model as an increase in the task 1's TFP, M_1. This is illustrated in a series of comparative statics, which is possible since the equilibrium is unique and skill distribution continuous (under assumption 3). To simplify notation, define the elasticities of the thresholds w.r.t. M_1:

$$\Delta_{h_1} \equiv \frac{d \log \hat{h}_1}{d \log M_1}, \quad \Delta_{h_2} \equiv \tilde{\chi} \cdot \frac{d \log \hat{h}_2}{d \log M_1}, \quad \Delta_z \equiv \frac{d \log \hat{z}}{d \log M_1}, \quad \text{where } \tilde{\chi} \equiv \frac{\chi}{\hat{h}_2 - \chi} > 0.$$

Similarly define Δ_x as the elasticity of any variable x with respect to M_1. Given $(\Delta_{h_1}, \Delta_{h_2}, \Delta_z)$ we know what happens to all the other variables of interest since

$$\Delta_{\phi_1} = \Delta_z - \Delta_{h_1}, \quad \Delta_{\phi_2} = \Delta_{\phi_1} - \Delta_{h_2},$$
$$\Delta_{W_1} = -\Delta_{h_1}, \qquad \Delta_{W_2} = -\Delta_{h_2}, \qquad \Delta_{W_z} = -\Delta_z.$$

where W_j's are the wage ratios

$$W_1 = w_1/w_0, \quad W_2 = w_2/w_1, \quad W_z = w_z/w_0.$$

We proceed as follows:

1. approximate the change in thresholds $(\hat{h}_1, \hat{h}_2, \hat{z})$ within a sector, taking the distribution of skill in sector i, μ_i, as given;
2. given the comparative statics in the thresholds, approximate the change in employment shares across tasks within a sector, taking μ_i as given;
3. approximate the differences in polarization across sectors holding L_i constant;
4. approximate the change in employment shares across sectors.

4.1 Wage and Job Polarization

To approximate the change in thresholds, we will first focus on the within sector allocation of skill implied by (10) and (12):

$$\hat{h}_1 \cdot \pi_{i1}(\hat{h}_1) = H_{i1}(\hat{h}_1, \hat{h}_2, \hat{z}) \Big/ L_{i0}(\hat{z}, \hat{h}_1) \tag{24a}$$

$$\left(1 - \chi_h/\hat{h}_2\right) \cdot \pi_{i2}(\hat{h}_2) = \left[H_{i2}(\hat{h}_1, \hat{h}_2, \hat{z}) - \chi L_{i2}(\hat{h}_1, \hat{h}_2, \hat{z})\right] \Big/ H_{i1}(\hat{h}_1, \hat{h}_2, \hat{z}) \tag{24b}$$

$$\hat{z} \cdot \pi_{iz}(\hat{h}_1, \hat{h}_2, \hat{z}) = Z_i(\hat{h}_1, \hat{h}_2, \hat{z}) \Big/ L_{i0}(\hat{z}, \hat{h}_1) \tag{24c}$$

where (π_{ij}, π_{iz}) are defined in (11) and (13), and (ϕ_1, ϕ_2) are defined in (14). The masses and skill aggregates are defined over a sector-specific distribution μ_i, which is taken as given.

For the approximation, we will assume that $\Delta_{L_{ij}} \to 0$ for $j \in \{0, 1, 2, z\}$. This implies that the density function is sufficiently small everywhere, which we assume to ignore the indirect effects of M_1 on (L_{ij}, H_{ij}, Z_i) that arise from changes in the thresholds. This can be thought of a limiting case of either when skills are discrete (Goos et al., 2014),[6] or when there are both a continuum of tasks and skills which are matched assortatively (Costinot and Vogel, 2010). Within the context of our model, it can be understood as approximating the equilibrium response using only the response of labor demand (the LHS's), while keeping labor supply (the RHS's) fixed. We can then show that

Proposition 1 (Routinization and Polarization) *Suppose there is an increase in M_1, and that $\Delta_{L_{ij}} \to 0$ for $j \in \{0, 1, 2, z\}$. Then*

1. $\Delta_{h_1} \approx -\Delta_{h_2} > 0$ *iff* $\sigma < 1$, *and*
2. $\Delta_{\phi_1} < \{\Delta_{h_1}, \Delta_z \approx \Delta_{\phi_2}\} < 0$ *if* $\omega < \sigma < 1$.

This implies that capital and labor flow out of task 1 (job polarization), relative wages decline in task 1 (wage polarization), and both the employment share and wages of

[6] In fact, they assume that wages are fixed and labor is inelastically supplied.

managers increase (vertical polarization).

Proof: Under the assumption (or, holding labor supply fixed), the comparative static is identical across sectors. System (24) becomes

$$\Delta_{h_1} + \Delta_{\pi_{i1}} \approx 0, \quad \Delta_{h_2} + \Delta_{\pi_{i2}} \approx 0, \quad \Delta_z + \Delta_{\pi_{iz}} \approx 0,$$

where

$$\begin{aligned}\Delta_{\pi_{i1}} &= (\sigma - 1)\left[(1-\alpha)\Delta_{h_1} + 1\right], \quad \Delta_{\pi_{i2}} = (\sigma - 1)\left[(1-\alpha)\Delta_{h_2} - 1\right], \\ \Delta_{\pi_{iz}} &= (\omega - 1)(1-\alpha)\Delta_z + \frac{\sigma - \omega}{\sigma - 1} \cdot \frac{\pi_{i1}(1+\pi_{i2})\Delta_{\pi_{i1}} + \pi_{i1}\pi_{i2}\Delta_{\pi_{i2}}}{\Pi_{ih}}\end{aligned} \tag{25}$$

Hence we obtain that

$$\Delta_{h_1} \approx -\Delta_{h_2} \approx \frac{1-\sigma}{\alpha + \sigma(1-\alpha)} > 0, \quad \Delta_{W_1} < 0, \quad \Delta_{W_2} > 0 \quad \Leftrightarrow \quad \sigma < 1.$$

Furthermore if $\omega < \sigma < 1$,

$$\begin{aligned}\Delta_z &\approx \frac{\sigma - \omega}{(\sigma - 1)\left[\alpha + \omega(1-\alpha)\right]} \cdot \frac{\pi_{i1}}{\Pi_{ih}} \cdot \Delta_{h_1} < 0, \\ \Delta_{\phi_1} &< 0, \quad \Delta_{\phi_2} \approx \Delta_z < 0, \quad \text{and} \quad \Delta_{W_z} > 0.\end{aligned} \tag{26}$$

□

The change in thresholds makes it easier to analyze what happens to employment shares by task. If $\sigma < 1$, and holding management employment shares constant, employment and payroll in task 1 shrinks while they increase in tasks 0 and 2. Hence, similarly as in Goos et al. (2014), we get employment polarization only when tasks are complementary, i.e. $\sigma < 1$; we also get wage polarization even with endogenous choice of tasks. At the same time, capital flows out to the other tasks as well.

Furthermore if $\omega < \sigma$, we also find that the mass and wage of managers increase relative to all workers. But while it is clear that the thresholds move in a direction that continues to shrink L_{i1}, it is unclear what happens to L_{i0} and L_{i2}, since both tasks 0 and 2 gain employment from task 1 but lose employment to managers.

So let us think about the (supply side) changes in L_{ij}, $j \in \{0, 1, 2, z\}$, arising from the change in thresholds within a sector i, still taking the sectoral distribution μ_i as given. To sign the $\Delta_{L_{i0}}, \Delta_{L_{i2}}$, we need additional parametric restrictions for sufficiency:

Lemma 1 *Suppose the skill distribution in sector i is uniform and that $\omega < \sigma < 1$. A sufficient condition for employment in tasks 0 and 2 to rise is*

$$\sigma - \omega < (1 - \sigma)\left[\alpha + \omega(1-\alpha)\right].$$

So if

$$\frac{\sigma-(1-\sigma)\alpha}{1+(1-\sigma)(1-\alpha)}<\omega<\sigma,$$

all employment shares except task 1's increase. This also implies that the average skill of task 1 workers rises.

Proof: Using the approximations from Proposition 1 and (24), we can approximate

$$\Delta_{L_{i0}}-\Delta_{L_{i1}}\approx\Delta_{\bar{h}_1},\quad \Delta_{L_{i0}}-\Delta_{L_{i2}}\approx\Delta_{\bar{h}_2-\chi},\quad \Delta_{L_{i0}}-\Delta_{L_{i1}}\approx\Delta_{\bar{z}}.$$

Since $\{\Delta_z,\Delta_{h_2}\}$ <0, we know that $\{\Delta_{\bar{z}},\Delta_{\bar{h}_2-\chi}\}$ <0, that is, the average skill of managers, and workers in task 2, become diluted. We cannot sign $\Delta_{\bar{h}_1}$; however, under the uniform distribution assumption

$$\Delta_{L_{i0}}=\Delta_z+\Delta_{h_1}\approx\left[1-\frac{\sigma-\omega}{(1-\sigma)\left[\alpha+\omega(1-\alpha)\right]}\cdot\frac{\pi_{i1}}{\Pi_{ih}}\right]\Delta_{h_1}$$

using (26). Since π_{i1}/Π_{ih} is a fraction bounded above by 1, the condition in the lemma guarantees that $\Delta_{L_{i0}}>0$, so

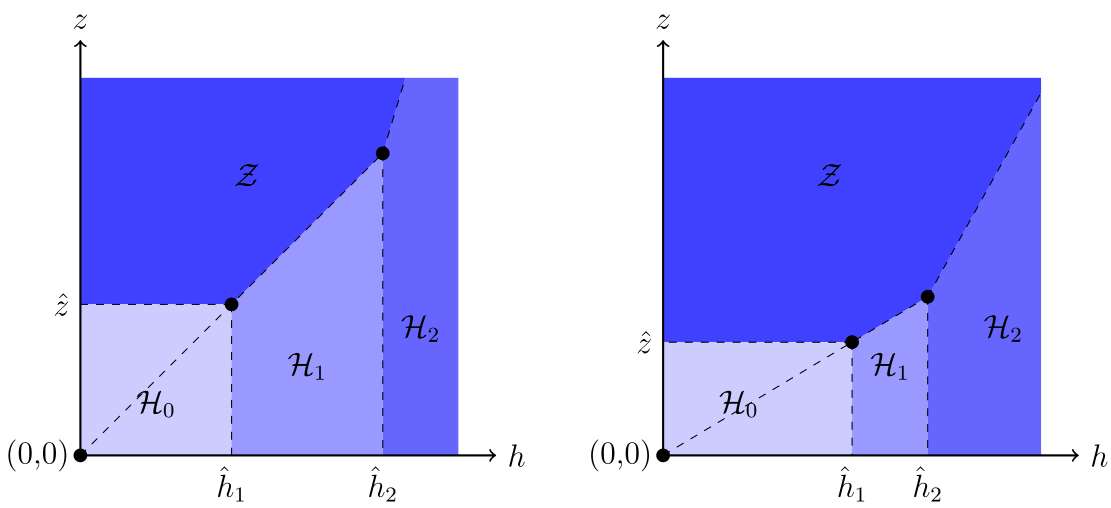
$$\Delta_{L_{i1}}<0<\Delta_{L_{i0}}<\{\Delta_{L_{i2}},\Delta_{L_{iz}}\},$$

although we can still not order the last two. □

This is intuitive. If tasks were substitutes, task 1 would crowd out all other tasks, including managers. As task 1 becomes the dominant occupation, wages also increase. However, when tasks are complements, workers need to flow to the other tasks, and for this to happen relative wages must decline in task 1. Moreover, if management is more complementary with tasks than tasks are among themselves, more individuals must become managers—and in equilibrium, manager wages must increase. The within-sector comparative static is depicted in figure 8.

4.2 Structural change

Previous models of structural change either rely on a special non-homogeneous form of demand (rise in income shifting demand for service products) or relative technology differences across sectors (rise in manufacturing productivity relative to services, combined with complementarity between the two types of goods, shifting production to services). Our model is also technology driven, but transformation arises from a skill neutral increase in task productivities, or routinization. Most importantly, in contrast to recent papers arguing that sectoral productivity differences can explain the skill premia or polarization (Bárány and Siegel, 2015; Buera et al., 2015), we argue



(a) Equilibrium wt low M_1

(b) Equilibrium wt high M_1

Fig. 8: Comparative Static, Within-Sector

exactly the opposite—that routinization can explain sectoral productivity differences and structural change.

To begin this analysis, note that from (25),

$$\Delta_{\Pi_{ih}} \equiv \frac{d\log \Pi_{ih}}{d\log M_1} = \frac{d\log \tilde{\Pi}_{ih}}{d\log M_1} \approx \frac{(1-\sigma)\left[\alpha + \omega(1-\alpha)\right]}{\sigma - \omega} \cdot \Delta_z, \tag{27}$$

and using Proposition 1, the $\Delta_{V_{ij}}$'s can be approximated from (19) as

$$\Delta_{V_{i0}} = 0, \qquad \Delta_{V_{i1}} \approx -\Delta_{\bar{h}_1} < 0 \quad \text{by Lemma 1}, \tag{28a}$$

$$\Delta_{V_{i2}} \approx -\Delta_{\bar{h}_2 - \chi} > 0, \qquad \Delta_{V_{iz}} \approx \Delta_{\bar{z}} > 0. \tag{28b}$$

So the $\Delta_{V_{ij}}$'s are sector-neutral and can be ordered as

$$\Delta_{V_1} < \{0 = \Delta_{V_0}\} < \{\Delta_{V_2}, \Delta_{V_z}\}.$$

Also note that

$$\Delta_{\Pi_{L_i}} \equiv \frac{d\log \Pi_{L_i}}{d\log M_1} = \frac{d\log \tilde{\Pi}_{L_i}}{d\log M_1} = \frac{\sum_{j=0,1,2,z} V_{ij}\Delta_{V_j}}{\tilde{\Pi}_{L_i}} = \sum_{j=0,1,2,z} \frac{L_{ij}}{L_i} \cdot \Delta_{V_j}. \tag{29}$$

Decomposing Polarization The change in the total amount of labor in each task, expressed in (20), can be decomposed similarly as in Goos et al. (2014):[7]

$$\Delta_{L_j} = \sum_{i\in\{m,s\}} \frac{L_{ij}}{L_j} \cdot \left[\Delta_{V_j} - \Delta_{\Pi_{L_i}} + \Delta_{L_i}\right]$$

[7] However, our decomposition differs from theirs. Their thought experiment is to separate the effects from keeping industry output fixed and when it is allowed to vary. Ours is to separate the effect from keeping sectoral employment fixed and when allowing it to vary.

$$= \sum_{i\in\{m,s\}} \frac{L_{ij}}{L_j} \cdot \Big[\underbrace{\Delta_{V_j} - \sum_{j'=0,1,2,z} \frac{L_{ij'}}{L_i} \cdot \Delta_{V_{j'}}}_{B_{ij}} + \Delta_{L_i} \Big] \quad \text{by (29)}, \quad j \in \{0,1,2,z\}. \tag{30}$$

A change in the V_{ij}'s occurs even holding L_i's constant, shifting the term B_{ij}. This leads to "within-sector polarization," as we saw in the previous subsection. In particular, from (28), the Δ_{V_j}'s are sector-neutral and common across sectors. So any difference in how the share of task j employment evolves across sectors depends on the weighted average of the Δ_{V_j}'s by the employment shares of all tasks within a sector, L_{ij}/L_i.

Holding L_i's constant, we know from Lemma 1 that task 1 is shrinking and other tasks growing within-sectors. Now we can compare the $\Delta_{\Pi_{L_i}}$'s across sectors, which is the weighted average of within-sector employment shifts as seen in (29). Thus

Lemma 2 *The weighted average of within-sector employment share changes, $\Delta_{\Pi_{L_i}}$, is smaller in the sector with a larger within-sector employment share in task 1, and larger in the sector with larger shares in all other tasks. That is,*

$$L_{s1}/L_s < L_{m1}/L_m \quad \Rightarrow \quad \Delta_{\Pi_{L_s}} > \Delta_{\Pi_{L_m}}. \tag{31}$$

This implies that, holding sectoral employment shares constant, manufacturing polarizes more compared to services.[8]

The term Δ_{L_i} in (30) captures structural change. To compute Δ_{L_i}, rewrite capital input (or employment) ratios in (18) using (17):

$$\kappa = \left(\frac{\gamma_s}{\gamma_m}\right) \left[\left(\frac{1-\eta_s}{1-\eta_m}\right)^{\frac{1}{\omega-1}} \left(\frac{\nu_{s0}}{\nu_{m0}}\right)^{\frac{1}{\sigma-1}} \cdot \left(\frac{\Pi_{sh}}{\Pi_{mh}}\right)^{\frac{\omega-\sigma}{(\omega-1)(\sigma-1)}} \times \left(\frac{\Pi_{K_s}}{\Pi_{K_m}}\right)^{\frac{\omega}{\omega-1}-\alpha} \left(\frac{\Pi_{L_s}}{\Pi_{L_m}}\right)^{\alpha-1} \right]^{\epsilon-1}. \tag{32}$$

So relative employment is completely determined by the relative endogenous TFP ratio between the two sectors. Since the elasticities of Π_{ih} are sector-neutral (both change at the negative rate of $\hat{z}$), we obtain

$$\Delta_\kappa \approx (1-\epsilon) \left[\left(\alpha - \frac{\omega}{\omega-1}\right) \left(\Delta_{\Pi_{K_s}} - \Delta_{\Pi_{K_m}}\right) + (1-\alpha) \left(\Delta_{\Pi_{L_s}} - \Delta_{\Pi_{L_m}}\right) \right]. \tag{33}$$

[8] Of course, the assumption in the lemma is a condition on employment shares, which are endogenous. However, the condition holds throughout our observation period in the data, so our analysis is valid. Alternatively, we could assume $\nu_{m1} >> \nu_{s1}$ and $\eta_m << \eta_s$. The astute reader would have already noticed that what the task-specific TFP's effectively do is shift the relative employment shares over time as if the parameters ν_{ij}, η_i were changing.

So if $(\Delta_{\Pi_{K_i}}, \Delta_{\Pi_{L_i}})$ are larger in services, employment shifts to services; that is, routinization (a rise in M_1) leads to structural change. We have already seen that $\Delta_{\Pi_{L_i}}$ is smaller in the manufacturing when (31) holds. Now from (27),

$$\Delta_{\Pi_{K_i}} \approx \frac{\Delta_z}{\Pi_{K_i}} \cdot \left[\pi_{iz} + \Pi_{ih} \cdot \frac{(1-\sigma)[\alpha + \omega(1-\alpha)]}{\sigma - \omega}\right] < 0,$$

so under the assumption in Lemma 1, $\Delta_{\Pi_{K_2}} > \Delta_{\Pi_{K_1}}$ if

$$\frac{\pi_{sz}}{\Pi_{sh}} > \frac{\pi_{mz}}{\Pi_{mh}} \quad \Leftrightarrow \quad \frac{\eta_s}{1-\eta_s} \cdot (\nu_{s0}\Pi_{sh})^{\frac{1-\omega}{\sigma-1}} > \frac{\eta_m}{1-\eta_m} \cdot (\nu_{m0}\Pi_{mh})^{\frac{1-\omega}{\sigma-1}},$$

which holds when the manager share of capital is larger in services, or $\eta_1 << \eta_2$. Hence, both because of shifts in labor *and* capital, structural change occurs toward services.

To understand why capital reallocation matters for structural change, note that we can write change in sectoral employment shares as

$$\Delta_{L_m} = -L_s \cdot \Delta_\kappa < 0, \qquad \Delta_{L_s} = L_m \cdot \Delta_\kappa > 0, \tag{34}$$

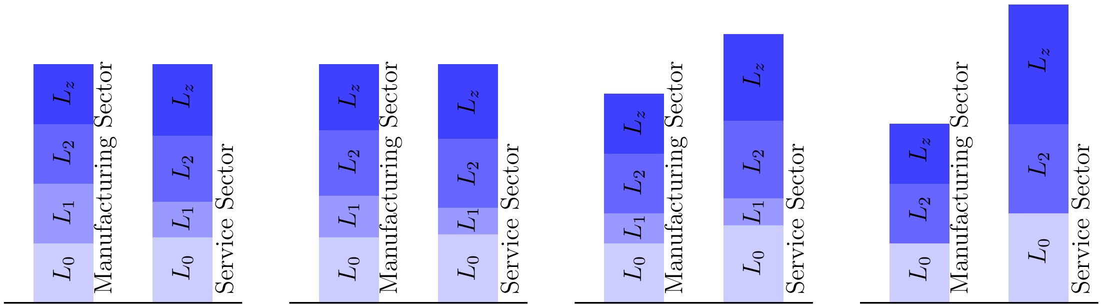
or plugging in Δ_κ from (33),

$$\begin{aligned}\Delta_{L_i} \approx (1-\epsilon)\Bigg[(1-\alpha)\Big(\underbrace{\Delta_{\Pi_{L_i}} - \sum_{i'} L_{i'}\Delta_{\Pi_{L_{i'}}}}_{C_{L_i}}\Big) \\ + \left(\alpha + \frac{\omega}{1-\omega}\right)\Big(\underbrace{\Delta_{\Pi_{K_i}} - \sum_{i'} K_{i'}\Delta_{\Pi_{K_{i'}}}}_{C_{K_i}}\Big)\Bigg].\end{aligned}$$

This makes clear that structural change in our model is due to a reallocation of both labor *and* capital, in contrast to Goos et al. (2014). The reason that capital matters in our model is because labor in our model is in skill units, which is different from employment shares. However, given sectoral capital K_i within a sector, physical capital does not affect employment shares as it is simply allocated to equalize its MRTS with the MRTS of skills across tasks; only when we let factors move across sectors does its effect appear in the model. Also note that the term C_{L_i} can be written as

$$C_{L_i} = \sum_j \frac{L_{ij}}{L_i} \cdot \Delta_{V_j} - \sum_{i'} L_i \cdot \left(\sum_{j'} \frac{L_{i'j'}}{L_{i'}} \Delta_{V_{j'}}\right),$$

which is the "between-sector" counterpart to the within-sector component B_{ij}: that is, C_{L_i} captures the average change in employment in sector i compared to the weighted average across sectors. The contribution from capital, C_{K_i}, is additional.



(a) Low M_1 (b) High M_1, κ fixed (c) High M_1 (d) BGP m_1

Fig. 9: Comparative Statics, Across-Sectors

Of course, from (30), structural change also contributes to polarization. To see this, rewrite (30) using (34) as

$$\Delta_{L_j} = \Delta_{V_j} - \sum_{i \in \{m,s\}} \frac{L_{ij}}{L_j} \Delta_{\Pi_{L_i}} + \left[\frac{L_{sj}}{L_j} L_m - \frac{L_{mj}}{L_j} L_s \right] \Delta_\kappa \tag{35}$$

$$\Rightarrow \quad L_j \left(\Delta_{L_j} - \Delta_{V_j} \right) = - \sum_{i \in \{m,s\}} L_{ij} \Delta_{\Pi_{L_i}} + \left[\frac{L_{sj}}{L_s} - \frac{L_{mj}}{L_m} \right] L_m L_s \Delta_\kappa .$$

Thus,

Lemma 3 *Suppose lemma 4.2 holds. Then structural change also contributes to polarization.*

Proof: The term in the square brackets in (35) is negative for $j = 1$, and positive for all other tasks, under lemma 4.2. □

This is intuitive. Manufacturing has a larger within-sector employment share in task 1 (that is, if it is more routine-intense), employing more for that task. So in addition to task 1 shrinking in both sectors, if sector 1 also shrinks (structural change), there is even more polarization.

Lemmas 4.2 and 3 are depicted in the first 3 subplots in figure 9. In figure (a), manufacturing is depicted as having a higher share in task 1, and services in task z. As we move from (a) to (b), sectoral employment shares are held fixed, and task 1 shrinks in both sectors. The change in employment shares is larger in manufacturing due to lemma . This leads to structural change in (c), according to lemma 3. Because manufacturing has a higher share in task 1, shrinking its size contributes to polarization.

Polarization or Structural Change? One may argue that it is not task productivities that lead to structural change, but advances in sector-specific productivities that lead to polarization. While it is most likely in reality that both forces are in play, in the context of our model, as long as technologies are either task- or sector-specific (that is, there are no task- *and* sector-specific technologies), sector-specific productivity shifts does not lead to polarization within sectors.

Suppose that in addition to the M_j's, there were A_i sectoral TFP's, so that $Y_i = A_i \Phi_i K_i^{\alpha} L_i^{1-\alpha}$. As in Ngai and Pissarides (2007), a rise in A_m changes κ at a rate of $1-\epsilon$, that is, manufacturing shrinks. But it is easily seen that none of the thresholds change, and hence neither do the Φ_i's (the endogenous sectoral TFP's). So polarization can only arise by the reallocation of labor across sectors that use different mixes of tasks. To be precise, from (30),

$$\frac{d\log L_j}{d\log A_m} = (1-\epsilon)\cdot\frac{d\log L_j}{d\log\kappa} = (1-\epsilon)\left[\frac{L_{sj}}{L_j}L_m - \frac{L_{mj}}{L_j}L_s\right] < 0. \tag{36}$$

Note that $d\log L_j/d\log\kappa$ is equal to the term in square brackets in (35), and negative under assumption (31). Hence, polarization only occurs because manufacturing shrinks. The reason is that In our micro-founded model, tasks are aggregated up into sectoral output, not the other way around.

Equation (36) also puts a bound on how much sectoral shifts alone can account for job polarization. For example, in the data, manufacturing employment fell from approximately 33% to 19% from 1980 to 2010. If this were solely due to a change in A_m, this means that (denoting empirical values with hats):

$$\frac{d\log\hat{\kappa}}{d\log A_m} \approx 14/67 + 14/33 \approx 0.63$$

which means that

$$\frac{d\hat{L}_j}{d\log A_m} \approx 0.63\left[\hat{L}_{sj}\hat{L}_m - \hat{L}_{mj}\hat{L}_s\right] = 0.63\left[\frac{\hat{L}_{sj}}{\hat{L}_j}0.33 - \frac{\hat{L}_{mj}}{\hat{L}_j}0.67\right].$$

In Section 6, we measure the employment share of routine, manufacturing jobs and routine, service jobs (as a share of total employment; that is, L_{m1} and L_{s1}) in 1980 were 26% and 33%, respectively (refer to Table 2). So

$$\frac{d\hat{L}_j}{d\log A_m} = 0.63\left[0.33\cdot 0.33 - 0.26\cdot 0.67\right] = -0.04,$$

that is, a change in A_m alone would imply an approximately 4 percentage point drop in routine jobs from 1980 to 2010. As shown in Table 2, the actual drop was 13 percentage points.

5 Dynamics

The above result implies that on a dynamic path in which M_1 grows at a constant rate, polarization happens faster than structural change. This implies that in the limit, task

1 vanishes, structural change ceases, but both sectors still employ non-trivial amounts of labor, unlike previous models of structural change.

Such a dynamic version of the model is a straightforward extension of the neoclassical growth model. Assume that aggregate labor L grows at rate n, and a representative household with CRRA preferences

$$\int_0^\infty \exp(-\rho t) \cdot \frac{c(t)^{1-\theta} - 1}{1-\theta} dt$$

where $c_t = C_t/L_t$, and a law of motion for aggregate capital

$$\dot{K}_t = Y_t - \delta K_t - C_t,$$

and for simplicity let us assume that $\dot{M}_1/M_1 = m_1$ and $M_0 = M_2 = M_z$, $\dot{M}_0/M_0 = m$. Then from (18), we can also write the aggregate production function as

$$Y_t = Y_{st} \cdot \left[\gamma_m^{\frac{1}{\epsilon}} \left(\frac{Y_{mt}}{Y_{st}} \right)^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} = \Phi_{st} \cdot L_{st} K_t^\alpha L_t^{1-\alpha} \cdot \left(\gamma_s^{\frac{1}{\epsilon}} L_{st}^{-1} \right)^{\frac{\epsilon}{\epsilon-1}}$$
$$= \Phi_{st} \cdot (L_{st}/\gamma_s)^{\frac{1}{1-\epsilon}} \cdot K_t^\alpha L_t^{1-\alpha}$$

where Φ_{st}, L_{st}, the endogenous sectoral TFP and employment share of services at time t, are functions of $(\hat{z}_t, \hat{h}_{1t}, \hat{h}_{2t})$.

Now define

$$\Phi_t^{1-\alpha} \equiv \Phi_{st} \cdot (L_{st}/\gamma_s)^{\frac{1}{1-\epsilon}}$$

the endogenous aggregate (Harrod-neutral) TFP and its growth rate $g_t \equiv \dot{\Phi}_t/\Phi_t$. As in the RCK model, define the normalized consumption and capital per efficiency unit of labor

$$\hat{c}_t \equiv C_t/\Phi_t L_t \qquad \hat{k}_t \equiv K_t/\Phi_t L_t,$$

so output per efficiency unit of labor is

$$\hat{y}_t \equiv Y_t/\Phi_t L_t = f(\hat{k}_t) \equiv \hat{k}_t^\alpha.$$

The dynamic equilibrium is characterized by

$$\dot{\hat{c}}_t = \frac{1}{\theta} \cdot \left[f'(\hat{k}_t) - (n + \delta + \rho + g_t\theta) \right] \cdot \hat{c}_t$$
$$\dot{\hat{k}}_t = f(\hat{k}_t) - (n + \delta + g_t)\hat{k}_t - \hat{c}_t$$
$$g_t \equiv \frac{\dot{\Phi}_t}{\Phi_t} = g\left(\hat{h}_{1t}, \hat{h}_{2t}, \hat{z}_t \right).$$

So instead of having sectoral shares as in Acemoglu and Guerrieri (2008), we have endogenously evolving TFP which pins down the sectoral shares at every instant. Using (17) and (33), the endogenous growth rate g_t becomes

$$(1-\alpha)g_t = m + \sum_i L_{it} \cdot \left[\frac{\omega - \sigma}{(\omega-1)(\sigma-1)} \cdot \frac{\dot{\Pi}_{ih}}{\Pi_{ih}} + \left(\frac{\omega}{\omega-1} - \alpha \right) \cdot \frac{\dot{\Pi}_{K_i}}{\Pi_{K_i}} + (\alpha - 1) \cdot \frac{\dot{\Pi}_{L_i}}{\Pi_{L_i}} \right].$$

On a BGP, g_t must be constant. Hence it must be that $(\hat{h}_1, \hat{h}_2, \hat{z})$ no longer evolve: Clearly this happens when $\hat{h}_1 = \hat{h}_2$, or from (10)-(11),

$$\hat{h}_2 - \chi = \frac{(\bar{h}_2 - \chi)L_{i2}}{L_{i0}} \cdot \left[\frac{\nu_{i0}}{\nu_{i2}} \cdot \left(\frac{\hat{h}_2 - \chi}{\bar{h}} \right)^{(1-\alpha)(1-\sigma)} \right]$$

$$\Rightarrow \qquad \frac{\nu_{i0} L_{i2}}{\nu_{i2} L_{i0}} = \frac{(\hat{h}_2 - \chi)^{\sigma + \alpha(1-\sigma)}}{\bar{h}_2 - \chi},$$

assuming $\bar{h} = 1$. Then $\hat{z}$ is determined by (12), sectoral-task employment masses are determined by (Π_{K_i}, Π_{L_i}) according to (32), and on a BGP

$$g^* = \frac{m}{1-\alpha}.$$

The long-run dynamics is depicted in figure 9(d), where both polarization and structural change continue until task 1 vanishes.

6 Quantitative Analysis

In the quantitative analysis, we assume that there are 10 rather than 3 (horizontally differentiated) worker tasks. All worker tasks $j \geq 2$ are characterized by a skill lost parameter χ_j. (The characterization of the equilibrium is exactly the same as before.) Each task broadly corresponds to a 1-digit occupation category in the census, as shown in Table 2. The 10 occupation groups can broadly be further be broadly grouped into low-, medium- and high skill tasks, or manual, routine, and abstract jobs, according to the mean wages of each occupation group and routinization indices.

The objective is to quantify how much of the observed changes in employment and wage shares from 1980 to 2010 can be explained by task-level productivity growth. To do so, we first calibrate the skill distribution over (h, z) to 1980 data. Then we choose all other model parameters, except for the elasticities of substitutions between tasks, (σ, ω), to fit the 1980 data exactly. Lastly, the elasticities are calibrated jointly with the growth rates of M_j's, $j \in \{0, 1, \ldots, 9, z\}$, to the time trends in employment and wage shares from 1980 to 2010. When doing so we also include a manufacturing-specific, exogenous sectoral TFP A_m, and calibrate its growth rate from 1980 to 2010. This allows us to quantify how the growth in the M_j's contributes to explaining the data,

	(1)	(2)	(3)
γ	0.371 **	0.346 **	0.258 **
	(0.003)	(0.005)	(0.004)
ϵ	0.003 **	0.002 **	0.003
	(0.000)	(0.000)	(0.004)
AIC	-550.175	-551.264	-550.866
RMSE_1	0.106	0.106	0.106
RMSE_2	0.039	0.039	0.039

Standard errors in parentheses
† $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table 1: Aggregate Production Function

that is, the role of task-level productivity growth in explaining polarization, structural change, and sectoral and total output growth.

When obtaining our empirical moments, we subsume mining and construction into manufacturing, and government into services, and the agricultural sector is dropped. That is, aggregate output and capital correspond to the moments computed without agriculture, as is the case for employment shares.[9]

6.1 Aggregate Production Function

The aggregate production function is estimated outside of the model. For the estimation, we only look at the manufacturing and service sectors, where manufacturing includes mining and construction, and government is included in services. We estimate the parameters (γ, ϵ) from the system of equations

$$\log\left(\frac{p_m Y_m}{PY}\right) = \log\gamma + (1-\epsilon)\log p_m - \log\left[\gamma p_m^{1-\epsilon} + (1-\gamma)p_s^{1-\epsilon}\right] + u_1$$
$$\log(Y) = c + \frac{\epsilon}{\epsilon-1}\log\left[\gamma^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}}\right] + u_2$$

where $\gamma \equiv \gamma_m$, using non-linear SUR (seemingly unrelated regression), on all years of real and nominal sectoral output observed in the BEA/NIPA.

Real production by sector is computed by a cyclical expansion procedure as in (Herrendorf et al., 2014) using production value-added to merge lower level industries (as opposed to consumption value-added, as in their analysis).[10] The price indices are implied from nominal versus real sectoral quantities. We try different base years as

[9]Agriculture shares are only about 2% of both total output and employment, and has been more or less flat since the 90s.
[10]The constant c is included since it is not levels, but relative changes that identifies ϵ.

Ranked by mean wage (except management)	SOC Code	Employment Shares 1980	2010	Manufacturing		Total Wage Shares 1980	2010	Manufacturing	
Low Skill Services	400	**10.44**	**13.92**	**0.59**	**0.23**	**6.75**	**7.60**	**0.52**	**0.16**
Middle Skill		**59.09**	**46.48**	**25.86**	**12.93**	**53.43**	**35.90**	**24.76**	**10.02**
Administrative Support	300	16.57	14.13	3.47	1.53	12.90	9.60	2.90	1.15
Machine Operators	700	9.81	3.75	8.79	3.02	8.21	2.39	7.37	1.91
Transportation	800	8.73	6.64	3.80	2.28	7.73	4.15	3.37	1.46
Sales	240	7.87	9.37	0.79	0.62	7.40	8.45	1.06	0.85
Technicians	200	3.23	3.86	1.00	0.57	3.35	4.33	1.13	0.66
Mechanics & Construction	500	7.91	6.02	4.44	3.19	8.40	4.88	4.91	2.61
Miners & Precision Workers	600	4.97	2.71	3.58	1.73	5.43	2.10	4.03	1.38
High Skill		**19.22**	**26.16**	**3.87**	**3.64**	**24.20**	**33.98**	**6.07**	**5.51**
Professionals	40	11.02	16.51	1.73	1.45	13.36	20.78	2.59	2.12
Management Support	20	8.20	9.65	2.14	2.20	10.84	13.20	3.48	3.39
Management	1	**11.26**	**13.44**	**2.47**	**2.59**	**15.62**	**22.52**	**4.22**	**5.81**

Table 2: Occupation Groups used for Calibration

well: 1947, 1980, and 2005, corresponding to columns (1)-(3) in Table 1. As shown there, the values are in a similar range as in Herrendorf et al. (2014), although ϵ is not significant with 2005 as a base year. For the calibration, we will use take the values of (γ, ϵ) in column (1) values as a benchmark.

The capital income share α is computed as 1-(labor income/total income), and fixed at 0.360. Since we do not model investment, for the calibration we also need the level of total capital stock (for manufacturing and services) for each decade, which we take from the Fixed Asset Table and is directly plugged into the model.

6.2 Occupations

For the calibration, we broadly categorize all occupations in the census into 11 broad categories, as summarized in Table 2. The left panel describes their SOC code with a short job description, and the middle and right panels their employment and total wage shares in 1980 and 2010, respectively. For the employment and wage share panels, the first two columns shows the size of each occupation in 1980 and 2010 as a fraction of total employment/wages. The next two columns show the size of each occupation within manufacturing as a fraction of total employment/wages.

The RTI indices of each task is tabulated in Table 4, along with model parameters calibrated from the data.

6.3 Calibration

Given the aggregate production function, we find the rest of the parameters by fitting an initial equilibrium to 1980 data, while finding (σ, ω), the growth rate of task-TFP growth, m_j, and the growth rate of exogenous manufacturing-specific sectoral TFP growth, a_m, to fit time trends in aggregate employment and wage shares by task and sector. All parameters have been specified except for the skill distribution, which we

	Parameter	Value	Target
From data	k_t (2)		BEA/NIPA
	α	0.361	
	γ	0.348	Estimated in section 6.1
	ϵ	0.004	
Fit to 1980	$M_j \equiv M$	1.054	Output per worker, normalization
	A_m	1.015	Manufacturing employment share
	ν_{ij} (20)	Table 4	Employment shares by task/sector
	$\eta_i(2)$		Manager share by sector
	χ_j (8)	Table 4	Wage shares by task, 1980
	a	8.171	
	γ_h	0.289	
	γ_z	1.000	Normalizations;
	$\bar{h}$	1.000	Not separately identified from M_j
Fit to 2010	σ	0.261	Output per worker growth,
	ω	0.150	income/employment
	m_j	Table 4	shares by task/sector
	a_m	0.044	

Table 3: Parameters

All parameters valued 1 are normalized.

assume to be a type IV bivariate Pareto (Arnold, 2014), with the c.d.f. given by:

$$\mu(h,z) = 1 - \left[1 + h^{1/\gamma_h} + z^{1/\gamma_z}\right]^{-a}.$$

We normalize $\gamma_z = 1$, since we cannot separately identify both skills from the skill-specific TFP's. For the same reason, we normalize $\bar{h}$ and $M_j \equiv M_0$ for all $j \in \{0, 1, \ldots, z\}$. All model parameters are summarized in table 3.

The calibrated skill loss parameters χ_j and employment weights (η_i, ν_{ij}) for each occupation are tabulated in Table 4, along with the task-level productivity growth rates (m_j) implied by the model. The RTI indices and productivity growth rates are also visualized in Figure 14.

Calibrating the distribution The model is fitted exactly to 1980 moments. For any guess of (γ_h, a, χ), we can find $(\hat{h}_{jt}, \hat{z}_t)$, $j \in \{0, \ldots 9\}$ for $t = 1980$ that exactly match observed employment shares, by integrating over the guessed skill distribution. Given the thresholds, we then compute the model-implied income shares using (23),

$$\frac{w_1 H_1}{w_0 H_0} = \frac{H_1}{\hat{h}_1 L_0}, \quad \frac{w_2(H_2 - \chi_2 L_2)}{w_1 H_1} = \frac{H_2 - \chi_2 L_2}{H_1(1 - \chi_2/\hat{h}_2)}, \quad \frac{w_z Z}{w_0 H_0} = \frac{Z}{\hat{z} L_0},$$

Ranked by mean wage (except management)	χ_j	Emp Wgts (ν_{ij}, η_i) Manu.	Serv.	m_j	RTI
Low Skill Services	**-**	**0.021**	**0.177**	**0.000**	**-0.426**
Middle Skill		**0.817**	**0.508**		
Administrative Support	-	0.090	0.171	0.015	2.428
Machine Operators	0.005	0.259	0.015	0.043	0.609
Transportation	0.012	0.119	0.078	0.021	-0.586
Sales	0.018	0.026	0.119	0.002	0.494
Technicians	0.024	0.034	0.038	-0.001	-0.074
Mechanics & Construction	0.031	0.157	0.061	0.014	-0.637
Miners & Precision Workers	0.037	0.131	0.026	0.031	0.642
High Skill		**0.162**	**0.317**		
Professionals	0.044	0.068	0.182	-0.007	-0.671
Management Support	0.050	0.095	0.135	0.001	-0.657
Management	**-**	**0.058**	**0.098**	**0.000**	**-1.122**

Table 4: Calibrated Employment Weights and Growth Rates

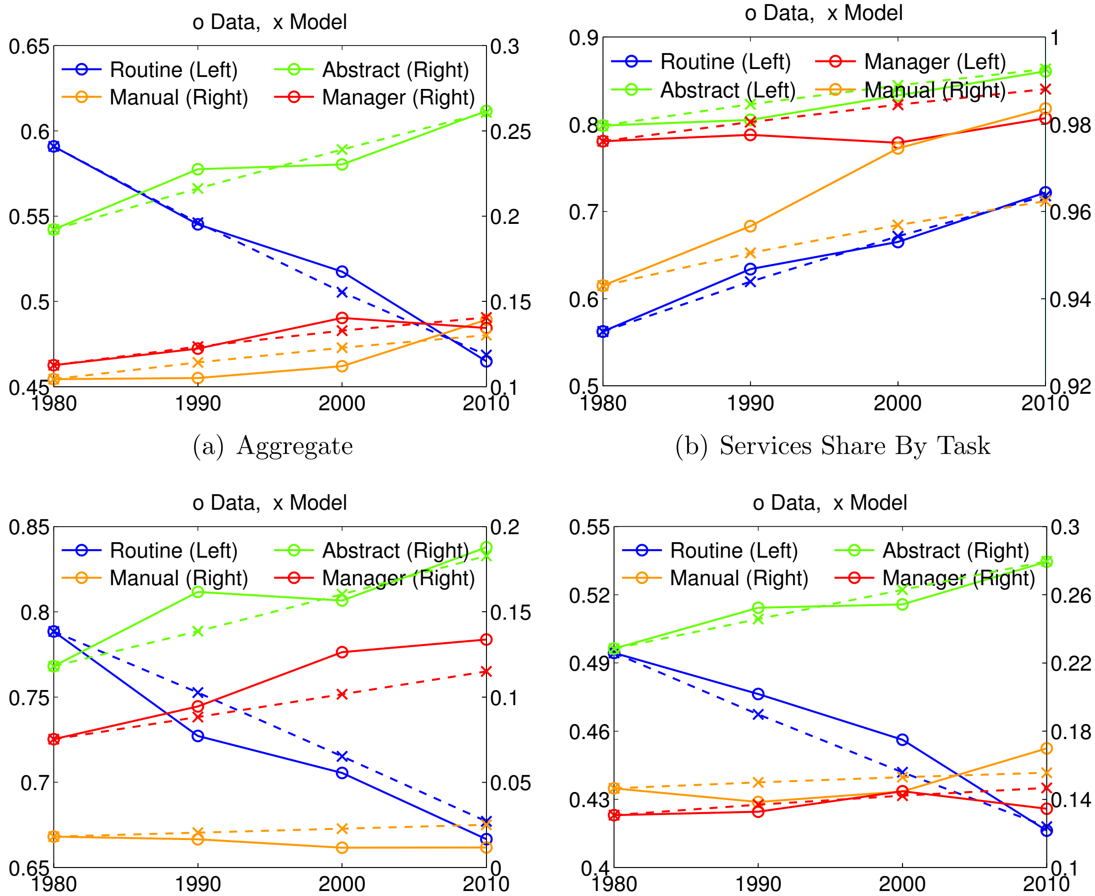
and similarly for $j \in \{3, \dots 9\}$. The LHS is the ratio of total wagebills by task, which we observe from the data, while the RHS is a function only of the thresholds, which themselves are functions of (γ_h, a, χ_j). Hence, we iterate over (γ_h, a, χ_j) so that the model-implied ratios match observed average wage ratios exactly. Then for the rest of the calibration, we can fix these three parameters and ensure that the other parameters generate an equilibrium that yields the implied thresholds.

Calibrated within the Model The rest of the model must be calibrated jointly, since (σ, ω) are only identified over time trends (as should be apparent from our comparative statics). Hence, they are calibrated together with the growth rates as follows:

1. Guess $(\sigma, \omega, m_j, a_m)$. For each year 1980, 1990, 2000, and 2010, take k_t, capital per worker, directly from the data.
2. Given the guess, calibrate the parameters in the second panel of table 3 to exactly fit 1980 moments.
3. Given the 1980 equilibrium, calibrate the parameters in the third panel of table 3 to fit time trends.

When we calibrate the 1980 equilibrium, we take the distributional parameters, in addition to the implied thresholds, as given. Then given a guess for M_0, we can plug in the observed employment shares from 1980, for each task by sector using (10)-(13) and (18). From this, we can obtain all the η_i's and $\nu'_{ij}s$, in addition to A_m. We then iterate over M_0 so that output per worker in 1980 is exactly equal to 1.

Given values for $(\sigma, \omega, m_j, a_m)$, all the other parameters are now fixed to 1980.



(a) Aggregate

(b) Services Share By Task

(c) Manufacturing

(d) Services

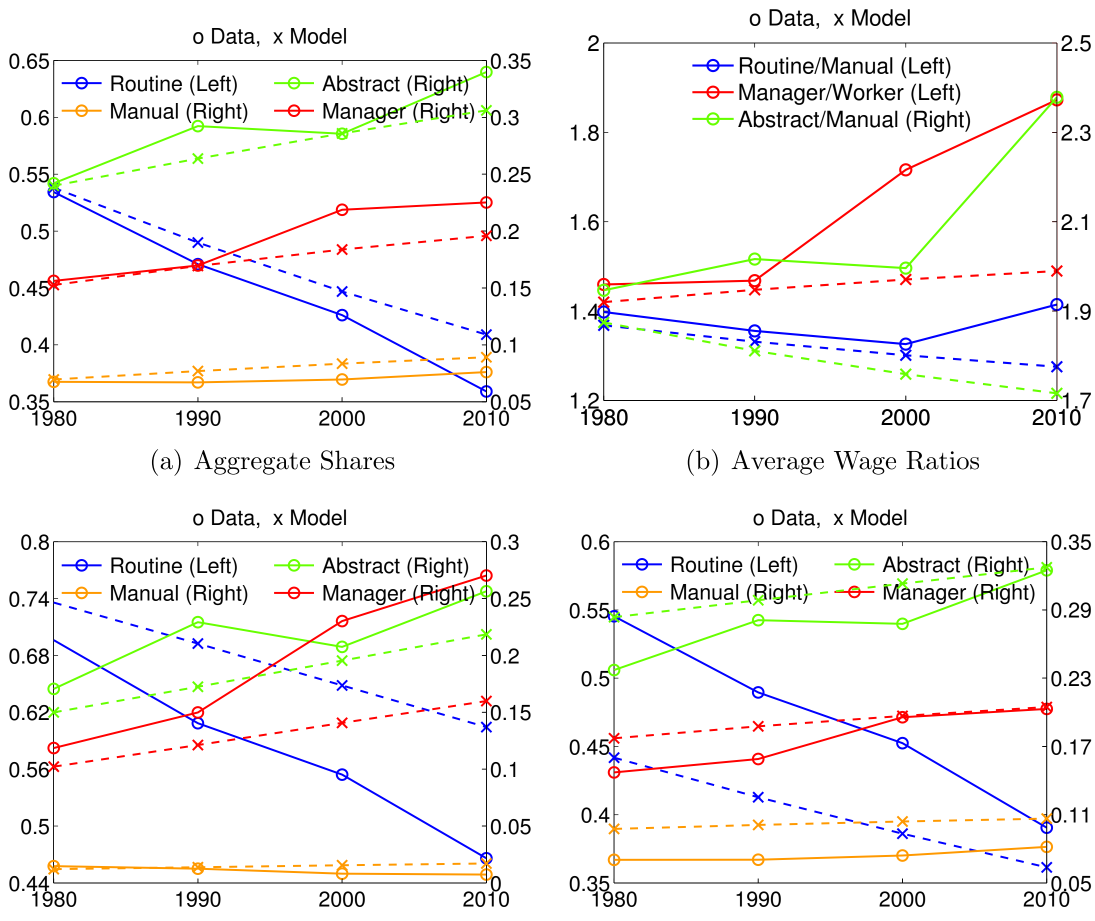
Fig. 10: Data vs. Model, Employment Shares by Task

We then fit these 14 parameters to fit the trends in output per worker growth and employment/wage shares by task and by sector, taking the value of capital per worker for each year, k_t, $t = 1990, 2000, 2010$, as given and fixed from the data.

6.4 Model Fit

Figure 10 plots the model implied trends in employment shares across tasks, in aggregate and by sector. Aggregate task shares (in the bottom) are targeted, so it is not surprising that we almost exactly fit the trend. Figures 10(a)-(b) were not targeted, and the model still performs well, although we underestimate the rise and fall of managerial and manual tasks in manufacturing, respectively. In services, the model overstates the increase in managers.

Figure 11 plots the same graphs as in Figure 10 for income shares. Again, aggregate shares are targeted, so it is not so surprising that Figure 11(a) fits well—although the



(a) Aggregate Shares (b) Average Wage Ratios (c) Manufacturing Shares (d) Services Shares

Fig. 11: Data vs. Model, Incomes by Task

fit is not as good as employment shares. Surprisingly, the bottom two panels which show income shares *within* manufacturing and services, also fit well despite they were not targeted. On the other hand, the model predicts that the average wage of manual workers rise relative to abstract workers, contrary to the data.

Overall, the model targeted only to aggregate moments delivers a good fit by task even within sectors. Next, we turn to how much of each of these trends can explain other outcomes.

6.5 Counterfactuals

We do two counterfactuals. First, we set $a_m = 0$, which gives the model's predictions in the absence of any sector-specific TFP growth. Then we set $m_j = m$ and recalibrate m to match output per worker growth, which gives the model's predictions in the absence of any task-specific TFP growth. Then we again set $a_m = 0$, but recalibrate all the m_j's to match the change in employment shares; this gives the model the best change to explain the data in the absence of any sector-specific TFP growth. The results are plotted against the benchmark in figure 12. In the top panels, note that structural change alone can account for about 15–20 percent of horizontal and vertical polarization. However, remember that with a_m, there are no changes in employment shares *within* sectors. On the other hand, 12(c) and (e) shows that polarization alone accounts for about 70 percent of structural change in aggregate, and also among routine jobs. In management, polarization accounts for about 15% of structural change. In fact, with only sector-specific TFP growth, we overshoot structural change in management, because the shift in management is much less than in routine jobs in the data.

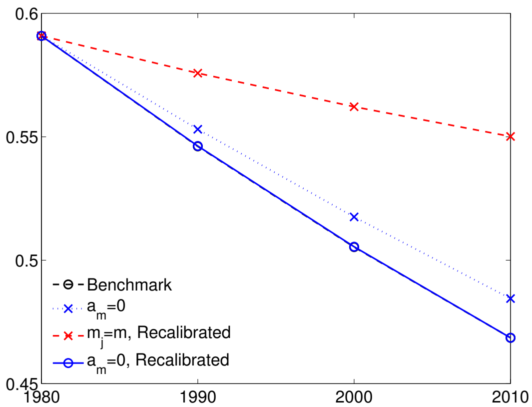
Lastly in Figure 13, we show the benchmark and counterfactual (log) GDP per worker, for aggregate and by sector. For both the aggregate and manufacturing, the benchmark model lines up almost perfectly with the data; it is slightly less than the data for services; this is also the case with only a sector-specific TFP growth. In contrast, if there were only task-specific TFP growth, the model lines up better with services but underestimates manufacturing.

In sum, task-specific TFP growth can account for 60-70 percent of the observed structural change between 1980 and 2010. Due to the vertical and horizontal polarization induced by routinization, employment shifts to the sector that uses the routine task less and management more intensively. Conversely, while sector-specific productivities are still required to fully account for structural change, we have shown both analytically and quantitatively that contrary to the data, it does not cause any polarization within sectors. However, it can also account for 15-30 percent of polarization observed in the aggregate data.

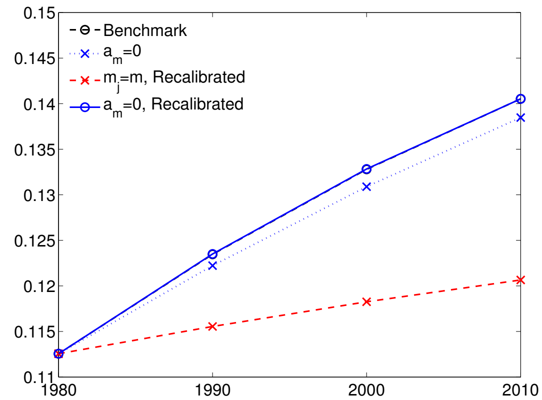
How much is actually routinization? The correlation between the task-TFP growth rates and RTI measure used in Autor and Dorn (2013) is above 0.4, and while not perfectly in line, it is visually clear that there is a strong relationship between the two in Figure 14. The rest we suspect comes from off-shoring and endogenously changes in the distribution of skill, from which we have abstracted from.

6.6 Long-Run Growth Path

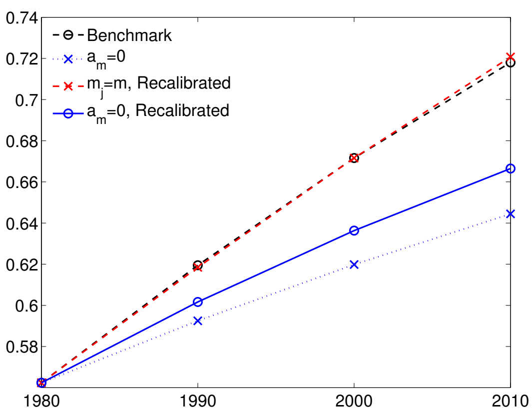
Lastly, we show the long-run dynamics of the model, assuming that the economy starts in 1980. We assume that the CRRA coefficient $\theta = 2$, as is standard in the data, and target an asymptotic interest rate of 2%, implying an approximately equal discount rate ρ. The depreciation rate is set to $\delta = 0.065$, as computed from the NIPA accounts. As can be seen, both routine and manufacturing continue to decline, with the former at a faster rate. Likewise, managerial employment continues to rise, dominating the



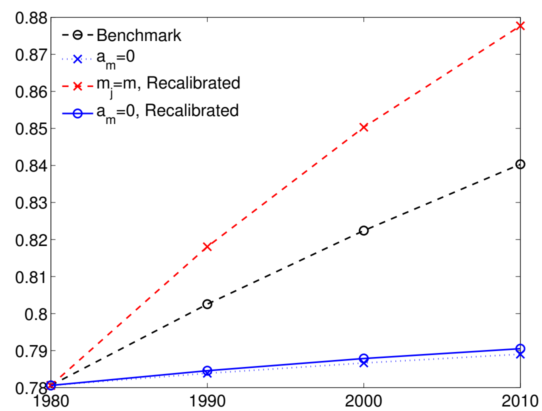
(a) Routine Employment Share



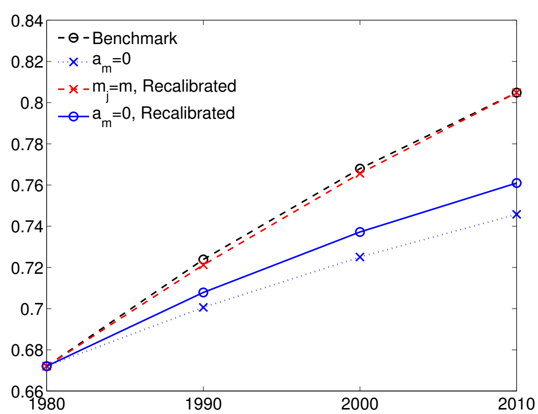
(b) Manager Employment Share



(c) Routine Services Share

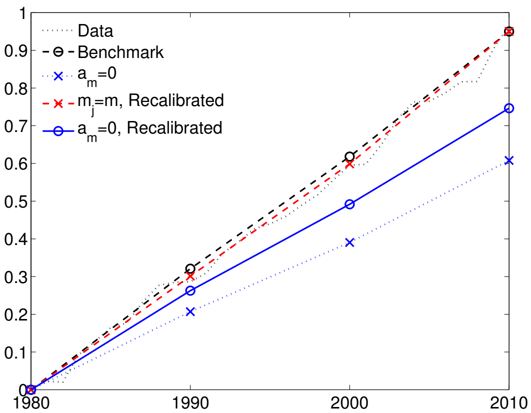


(d) Manager Services Share

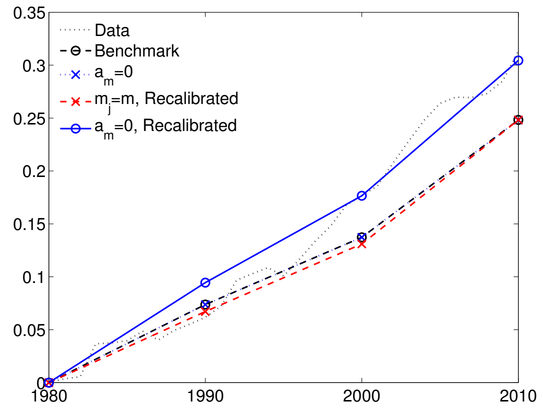


(e) Service Employment Share

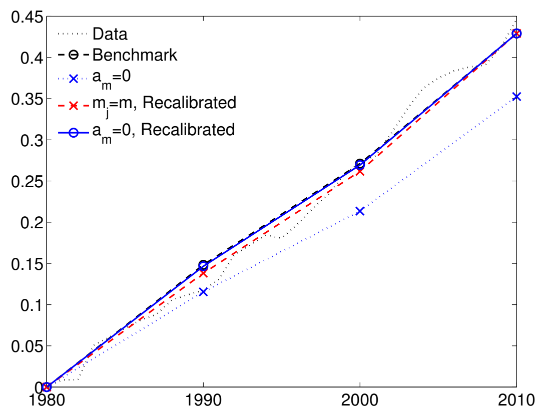
Fig. 12: Benchmark vs. Counterfactuals, Polarization and Structural Change



(a) Log GDP per Worker, Manufacturing



(b) Log GDP per Worker, Services



(c) Log GDP/Worker, Aggregate

Fig. 13: Benchmark vs. Counterfactuals, Output

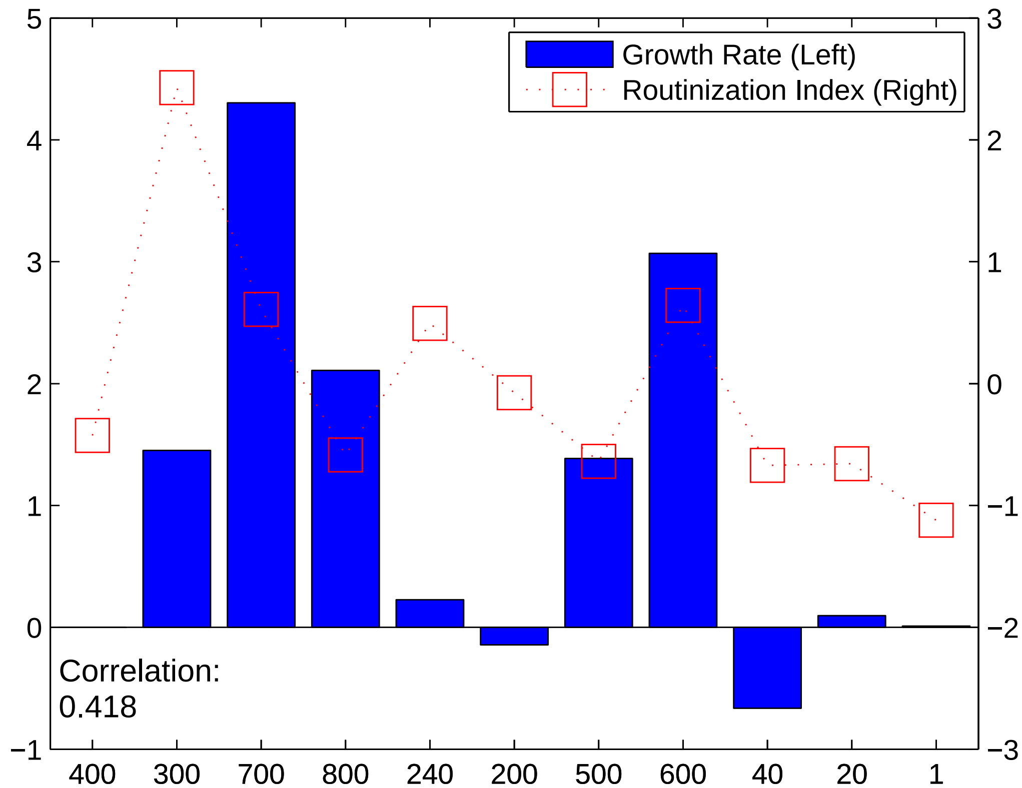


Fig. 14: Task TFP growth and Routinization

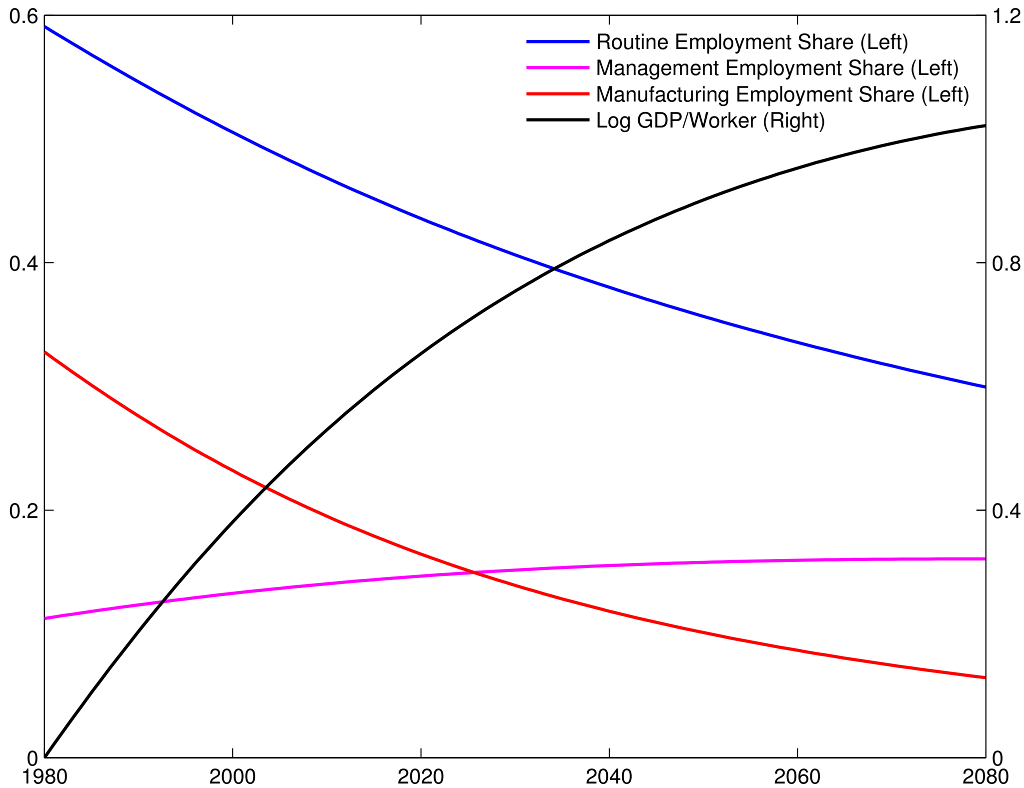


Fig. 15: Long-run Dynamics

economy. Finally, note that the first 150 years displays structural change and near balanced growth, consistently with the Kuznets and Kaldor facts.

7 Conclusion

We presented a new model which encompasses job polarization, structural change, and a modified span of control technology. We showed analytically and the quantitatively that the model can be a useful tool for analyzing macroeconomic dynamics.

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