Foreign Reserve Accumulation, Foreign Direct Investment, and Economic Growth

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February 18, 2019

Abstract

This paper develops a quantitative small-open-economy model to assess the optimal pace of foreign reserve accumulation by emerging and developing countries. In the model, reserve accumulation depreciates the real exchange rate and attracts foreign direct investment (FDI) inflows, which promotes productivity growth through endogenous firm dynamics. The economy is also subject to sudden stops in the form of an occasionally binding constraint on foreign borrowing, and accumulated reserves are used to prevent severe economic downturns. The model shows that two factors are the key determinants of the optimal pace of reserve accumulation: the elasticity of the foreign borrowing spread with respect to foreign debt, and the entry cost for FDI entry. The model suggests that these two factors can explain a substantial amount of the cross-country variation in the observed pace of reserve accumulation.

Keywords: Foreign reserve accumulation, foreign direct investment, sudden stop, endogenous growth, real exchange rate, gross capital flows
JEL code: F23, F31, F32, F41, F43

*I am grateful to Felipe Saffie for his advice and guidance on this project. I would like to thank Kosuke Aoki, Boragan Aruoba, Rasmus Fatum, Tomohiro Hirano, Sebnem Kalemli-Ozcan, Ryo Kato, Nuno Limao, John Shea, Ina Simonovska, Luminita Stevens, Nao Sudo, and seminar participants at the Economic Graduate Students Conference at Washington University in St. Louis, 2017 Fall Midwest Macroeconomics Meetings, 2018 SED Annual Meetings, Bank of Japan, Bank of Korea, and the University of Maryland for useful comments and discussions. All errors are my own. The views expressed in this paper are my own and do not necessarily reflect the official views of the Bank of Japan. Email: hidehiko.matsumoto@boj.or.jp.

1 Introduction

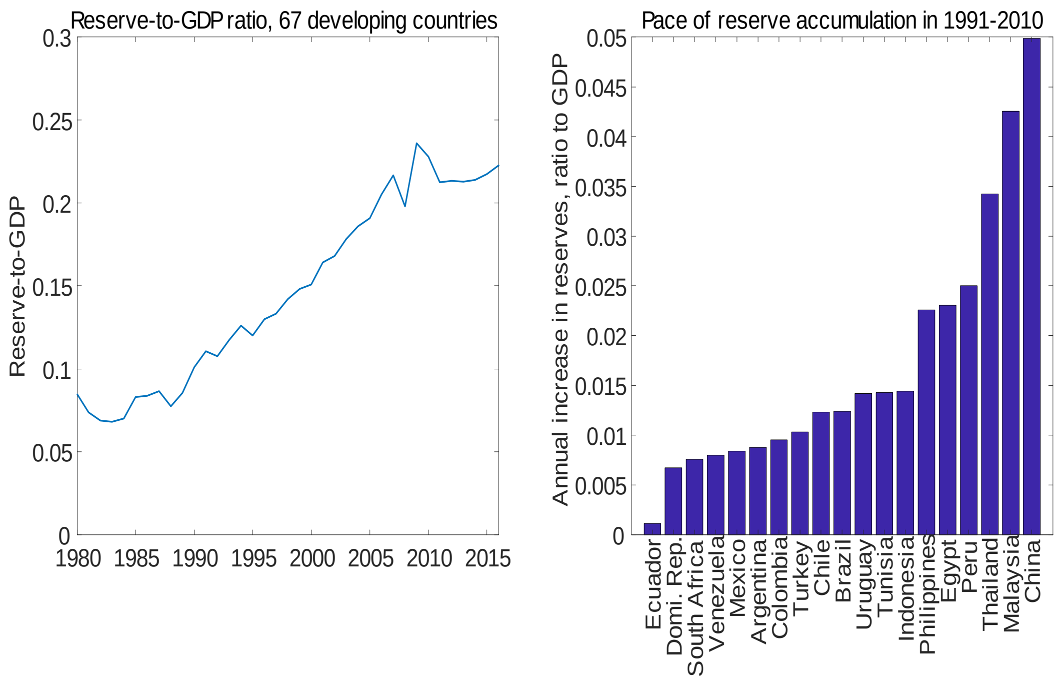
The active accumulation of foreign reserves by emerging economies, especially those in East and Southeast Asia, is one of the most prominent developments in the international financial system over the past 25 years. The left panel in Figure 1 shows that the average reserve-to-GDP ratio across 67 emerging and developing countries has increased from lower than 10% before 1990 to almost 25% by 2010. A large volume of literature investigates the motives for this active reserve accumulation, and identifies two main motives: a precautionary motive to prevent severe economic downturns caused by sudden stops in capital inflows, and a growth strategy through exchange rate depreciation and export promotion.[1]

Although these benefits of reserve accumulation are well understood, there is a wide cross-country variation in how actively each country pursues these benefits. The right panel in Figure 1 shows the average annual change in reserve holdings in terms of the ratio to GDP for 19 selected emerging and developing countries in 1991-2010. It illustrates that Asian countries such as China, Malaysia, and Thailand have been accumulating reserves equivalent to $3.5-5\%$ of GDP on average, while many Latin American countries are accumulating reserves less than 1% of GDP. In spite of active research on the optimal reserve policy throughout the past decade, the literature still tells little about what the optimal pace of reserve accumulation for each country is, and why different countries accumulate reserves at different paces.

To answer the above questions, this paper develops a quantitative small-open-economy model of reserve accumulation. The main novelty of this paper is twofold. First, the model incorporates the key benefits and costs of reserve accumulation into a unified quantitative framework. On the benefit side, reserve accumulation promotes productivity growth in the tradable sector through real exchange rate depreciation. When a sudden stop in capital inflows occurs, accumulated reserves are used to mitigate the negative effects on the economy. Most existing theoretical papers studying optimal reserve policy incorporate only one of these two effects. On the cost side, reserve accumulation takes away resources that could otherwise

[1] Ghosh, Ostry, and Tsangarides (2017) provide empirical evidence to show that the motives of reserve accumulation have been shifting over time. They show that the precautionary motive for current account shocks was important in the 1980s, but since the 1990s, the precautionary motive for capital account shocks and the currency-depreciation motive have become more important.

Figure 1: Reserve accumulation by emerging and developing countries



have been used by domestic agents, and reduces consumption in the short run and crowds out investment.[2] The optimal pace of reserve accumulation in the model is determined by the trade-off between these benefits and costs.

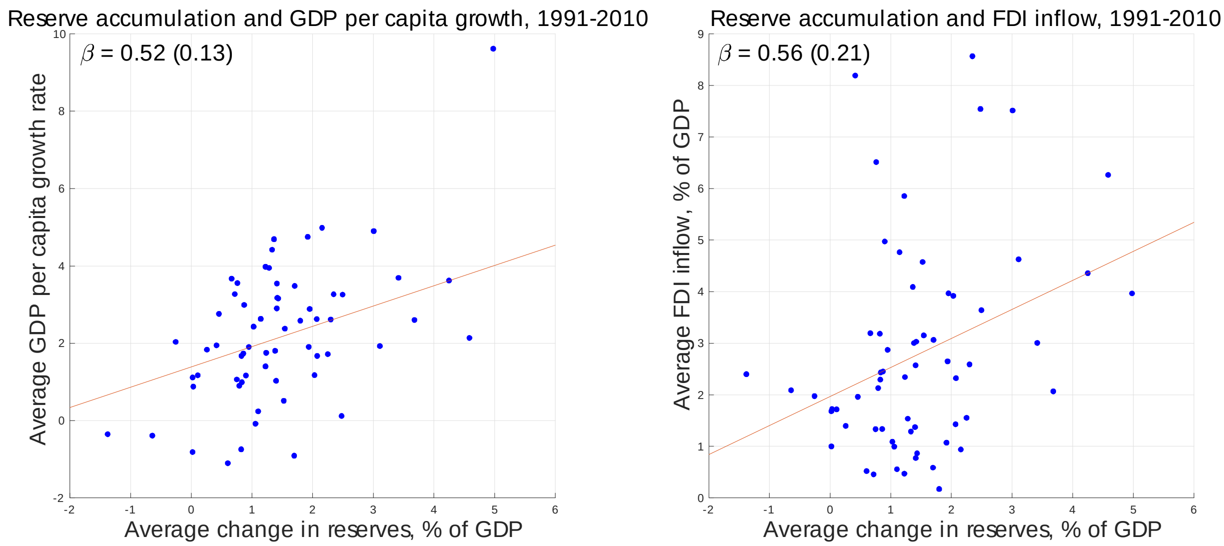
The second novel contribution is that instead of trying to identify the unique optimal pace of reserve accumulation for a representative emerging economy, this paper asks why different countries accumulate reserves at different paces.[3] As shown in the main analysis, it is crucially important to take into account country-specific characteristics in considering the optimal reserve policy. One of the key messages of this paper is that it is not optimal for Latin American countries to accumulate reserves at a pace similar to Asian countries.

The model describes a small open economy with tradable and non-tradable sectors. Corresponding to the two benefits of reserve accumulation mentioned above, two features are introduced into the model: endogenous growth with foreign direct investment (FDI) entry,

[2] Crowding-out of investment resulting from reserve accumulation is documented by Reinhart, Reinhart, and Tashiro (2016) at the macro level, and shown empirically by Cook and Yetman (2012) at the micro level.

[3] Benigno and Fornaro (2012) and Korinek and Servén (2016) are two examples of the papers that study the optimal pace of reserve accumulation for a representative emerging country.

Figure 2: Reserve accumulation, GDP per capita growth, FDI inflow



and sudden stops in capital inflows. Endogenous growth is modeled as a version of the Schumpeterian growth model in which intermediate goods-producing firms in the tradable sector invest in innovation and increase productivity. FDI is introduced in order to capture the idea that reserve accumulation promotes growth in part by attracting FDI from developed countries as well.[4] Sudden stops are modeled as an occasionally binding borrowing constraint on private foreign debt and working capital financing.

The government reserve policy consists of two interventions. First, in normal times when the borrowing constraint is not binding, the government collects taxes on tradable goods to accumulate reserves. Since tradable goods become relatively scarce compared with non-tradable goods, the relative price of non-tradable goods falls, which is real exchange rate depreciation. Real depreciation increases the relative profitability of the tradable sector, and induces a labor shift to the tradable sector. This labor shift brings higher profits for intermediate firms, which induces more investment and attracts FDI, and promotes productivity growth. Figure 2 presents some empirical evidence that is in line with this mechanism. The left panel shows that countries with a faster pace of reserve accumulation

[4] Dooley, Folkerts-Landau, and Garber (2007) and Dooley, Folkerts-Landau, and Garber (2014) argue that Asian countries' growth strategy is to repress real wages by foreign exchange rate intervention and to attract FDI.

are likely to have faster growth in GDP per capita. The right panel indicates a significant positive correlation between the pace of reserve accumulation and the FDI inflow-to-GDP ratio.[5]

Second, when the borrowing constraint becomes binding, the government provides accumulated reserves to bail out private agents. Government bailouts relax the binding borrowing constraint and prevent severe economic downturns. In addition to the contemporaneous effects, bailouts induce firms' investment and attract FDI ex ante in normal times, because investment and FDI entry are forward-looking decisions, and anticipation of future bailouts increases the expected future profits for firms.

Fast and stable growth by reserve accumulation comes at a cost. As the government collects tax revenue to accumulate reserves, private agents borrow more from abroad to compensate for the loss of resources. In the model, the interest rate on foreign borrowing is debt-elastic, and the higher debt-to-GDP ratio leads to a higher interest rate spread. This higher interest rate spread causes two costs of reserve accumulation. First, it prevents private agents from fully offsetting reserve accumulation by increasing foreign borrowing, thus lowering consumption in the short run. The optimal reserve policy therefore balances the costs of short-run austerity with the benefits of higher long-run consumption. Second, the higher spread discourages investment in capital and innovation, a form of crowding-out, which reduces the growth-promoting effect of reserve accumulation.

In the quantitative analysis, the model is calibrated to the average of the 19 emerging and developing countries in the right panel of Figure 1.[6] Since the benefit of receiving FDI is the crucial part of the quantitative analysis, parameters related to FDI are set to match several targets taken from empirical papers on FDI.

The first important result using this model is that the optimal pace of reserve accumulation and its welfare impact depend crucially on two characteristics of each country: the debt-elasticity of the foreign borrowing spread, and the FDI entry cost. In countries where

[5] There are also several empirical papers that document the correlations between reserve accumulation and real depreciation, and real depreciation and GDP per capita growth. See the literature review for details.

[6] These countries are chosen for two reasons. First, the development levels of these countries are similar in the sense that most of them were lower middle-income countries in 1991 and became upper middle-income countries by 2010. The second reason is the data availability for the foreign borrowing spread and FDI inflow by sectors.

the spread has a higher debt-elasticity, even small increase in private debt increases the spread substantially, which increases the costs of short-run lower consumption and crowding-out of investment. In countries with a higher FDI entry cost, reserve accumulation is not as effective in attracting FDI and the growth-promoting effect is therefore limited. In this case, the optimal pace of reserve accumulation is slower, and the welfare gain is limited.

Given these results, this paper takes these two factors into account when evaluating the pace of reserve accumulation for each of the 19 countries. The debt-elasticity of the foreign borrowing spread is estimated using panel regression for the sample countries. Because the debt-elasticity of the spread is significantly associated with the default history of each country, the sample 19 countries are divided into 5 groups according to the number of past defaults, and the elasticity is estimated for each group. The FDI entry cost parameter is adjusted to match the FDI inflow-to-GDP ratio for each country in the data.[7] Then the optimal pace of reserve accumulation is derived for each country and compared with the actual pace of reserve accumulation in the data. The second important result is that many of the sample countries are roughly in line with the optimal pace in the model, suggesting that these two factors can explain a substantial amount of the observed cross-country variation in the pace of reserve accumulation. The average optimal pace in the model across the 19 countries is 1.43% of GDP while it is 1.71% in the data, and the correlation coefficient between the optimal and actual pace across the countries is 0.73. A few countries including China, however, have been accumulating reserves much faster than the optimal pace suggested by the model.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model. Section 4 discusses the mechanisms of how the reserve policy works in the model. Section 5 presents the calibration of the model and the quantitative analysis. Section 6 studies the key determinants of the optimal pace of reserve accumulation. Section 7 evaluates the actual pace of reserve accumulation by developing countries. Section 8 concludes.

[7] As an alternative approach, the appendix of this paper explicitly estimates the FDI entry cost using the Starting a Business Index from the World Bank's Doing Business Surveys.

2 Related Literature

Foreign reserve accumulation by developing countries has been an active research area in the last decade. One strand of literature focuses on the growth-promoting effect. As an empirical motivation, Aguiar and Amador (2011) find that there is a positive correlation between government net foreign asset growth and GDP growth across developing countries, which is in stark contrast to the predictions of neo-classical growth models. Gourinchas and Jeanne (2013) and Alfaro, Kalemli-Ozcan, and Volosovych (2014) show that this correlation is driven mainly by reserve accumulation by the public sector. Regarding the mechanism through which reserve accumulation promotes growth, Levy-Yeyati, Sturzenegger, and Gluzmann (2013) shows that reserve accumulation causes real exchange rate depreciation and that real depreciation in turn promotes economic growth. Rodrik (2008) also shows that real depreciation promotes economic growth and further presents suggestive evidence that a labor shift to the industry sector is the channel through which real depreciation promotes growth. In line with these empirical studies, Aizenman and Lee (2010) and Korinek and Servén (2016) develop models with a learning-by-doing externality in the tradable sector and study the optimal reserve policy to promote growth. The present model shares with these papers the premise that reserve accumulation causes real depreciation and a labor shift to the tradable sector, but differs in that productivity increases through firms' endogenous investment and FDI entry. Attracting FDI is another proposed channel through which reserve accumulation promotes productivity growth. Dooley, Folkerts-Landau, and Garber (2007) and Dooley, Folkerts-Landau, and Garber (2014) argue that Asian countries' growth strategy has been to repress real wages through foreign exchange rate intervention in order to attract FDI from developed countries.

Another strand of literature studies the precautionary benefits of reserve accumulation. Jeanne and Rancière (2011) model reserve accumulation as an insurance contract that pays off in a sudden stop and quantify the optimal amount of reserve holdings. Bianchi, Hatchondo, and Martinez (2016) and Hernández (2017) build a sovereign default model in which the government holds reserve assets to insure against future defaults and loss of access to international financial markets. Hur and Kondo (2016) develop a model of bank-run style sudden stops and show that a small increase in rollover risk can explain the outburst of sudden stop episodes in the late 1990s and the following active reserve accumulation by developing countries in the 2000s.

All of these theoretical papers focus on either the growth-promoting effect or the precautionary effect of reserve accumulation, but not both. The model in this paper incorporates both effects into a unified framework and studies the interaction between the growth-promoting effect and the precautionary effect. In this sense, the present model is similar to the model in Benigno and Fornaro (2012). Benigno and Fornaro (2012) develop a stylized model in which reserve accumulation in normal times and bailouts during crisis induce private agents to use more imported inputs, which improves productivity through a learning-by-doing type assumption. In the present model, in contrast, productivity increases through full-fledged endogenous firm dynamics. This framework captures two important effects of reserve policy that are absent in Benigno and Fornaro (2012). First, anticipation of future bailouts during crises induces firms to invest more in normal times ex ante, because bailouts stabilize the economy, and firms' investment is forward-looking. Second, reserve accumulation takes away some domestic resources from private agents, causing crowding-out of domestic investment. The analyses in this paper show that these two effects constitute an important part of the growth-promoting effect and the welfare impact of reserve policy.

Studies focused on the cross-country differences in reserve accumulation are scarce. Obstfeld, Shambaugh, and Taylor (2010) consider the risk of simultaneous domestic capital flight and sudden stops of foreign capital inflows, and show that the size of the banking sector has a strong explanatory power for cross-country differences in the size of reserve holdings. Aguiar and Amador (2011) develop a neo-classical growth model with political frictions and show that the degree of political friction matters for the speed of net public debt reduction by the government. The present paper focuses on two other factors that can explain cross-country variation in the pace of reserve accumulation, the debt-elasticity of the spread on foreign borrowing and the FDI entry cost.

The structure of the model rests on two strands of literature. First, the endogenous growth framework is based on a version of the Schumpeterian growth model developed by Ates and Saffie (2016). They incorporate the growth model developed by Klette and Kortum

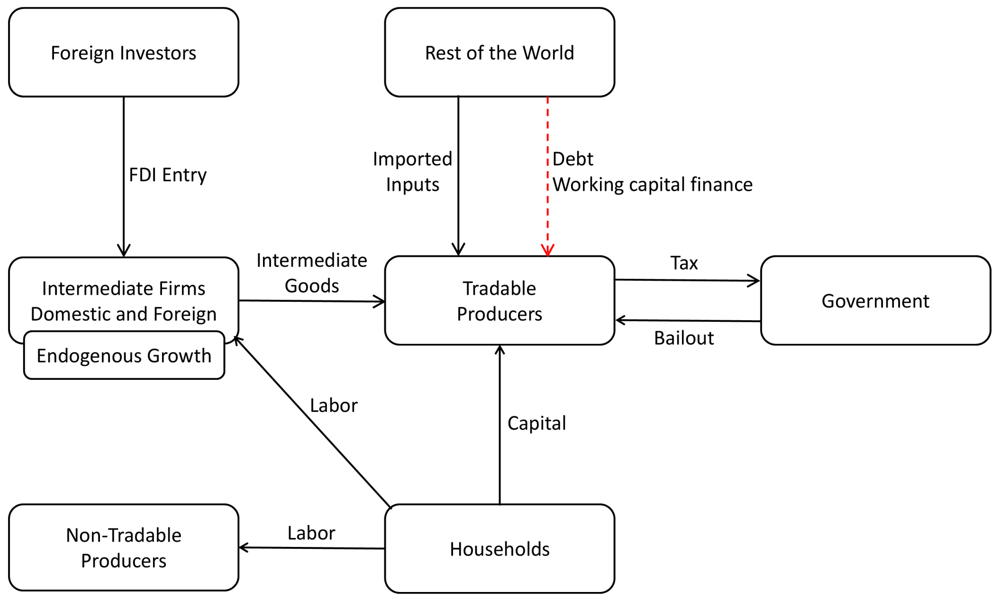
(2004) into a DSGE framework and introduce heterogeneous innovations. The present model extends the framework in Ates and Saffie (2016) by introducing FDI and innovation by foreign firms. Second, sudden stops are modeled as an occasionally binding constraint on foreign borrowing, following Mendoza (2010). The difference from Mendoza (2010) is that the fraction of capital used as collateral is stochastic and the collateral value of capital is set at book value rather than market value, similarly to Bianchi (2016). The present model differs from this literature in combining endogenous growth and non-linear crisis dynamics caused by an occasionally binding constraint.

Lastly, this paper shares with several recent papers the feature that crises have a permanent negative impact on productivity. Queralto (2015) builds a model based on the Comin and Gertler (2006) version of the product-variety expansion model and shows that the model can explain the persistent negative effect of the 1998 sudden stop on productivity in Korea. Gornemann (2015) also adopts the product-variety expansion model and develops a model that captures a very persistent negative effect of sovereign default on productivity. Ates and Saffie (2016) introduce heterogeneous firms and financial selection into the Schumpeterian growth model and show that firm heterogeneity has a persistent effect on productivity growth after sudden stops.

3 Model

The model framework is based on an infinite-horizon small open economy with tradable and non-tradable sectors. Figure 3 presents a diagram for the model economy. There are six agents in the model: tradable goods producers, non-tradable goods producers, domestic and foreign intermediate firms, foreign investors, households, and the government. There are two driving forces of economic growth: capital accumulation by households, and endogenous productivity growth in the intermediate sector. Unlike standard business cycle models, the economy starts with scarce capital, accumulates capital and grows fast toward the balanced growth path. The focus of the model is how the reserve policy should be conducted during the transition period.

Figure 3: Model overview



3.1 Tradable Goods Producers

Tradable goods are the numeraire of this model economy, and their price is normalized at one. The representative tradable goods producer uses capital K_t^D, unit-mass variety of intermediate goods $\{y_t(i)\}_{i=0}^1$, and imported inputs M_t to produce output Y_t^T following the Cobb-Douglas production function:

$$Y_t^T = (K_t^D)^\alpha (I_t^M)^\theta (M_t)^{1-\alpha-\theta}, \tag{1}$$

where I_t^M is the composite of intermediate goods:

$$I_t^M = \exp\left[\int_0^1 \ln y_t(i) di\right]. \tag{2}$$

Before production materializes, a fixed fraction ϕ of the cost of intermediate goods and imported inputs needs to be paid. This payment is financed by within-period borrowing from abroad with no interest cost. In addition, the tradable goods producer borrows from abroad using one-period non-contingent debt B_t on behalf of households.[8] Foreign borrowing

[8] This assumption is just to simplify the algebra. It would be an equivalent model even if households borrow directly from abroad.

is subject to an occasionally binding borrowing constraint. Specifically, the borrowing limit is given by $\kappa_t K_{t-1}$, where κ_t is a collateral shock and takes either of two values, κ_H or κ_L, following a two-state Markov process. K_{t-1} is the capital stock of this country at the beginning of period t. κ_H is the value in normal times, and it is large enough that the borrowing constraint never binds. With exogenous probability, κ_t switches from κ_H to κ_L, which is small enough that the borrowing constraint may bind depending on the state of the economy. In particular, in the transition when capital is scarce, the borrowing limit is low but there is a large incentive to borrow from abroad to accumulate capital. In this case, a collateral shock κ_L is likely to cause a binding borrowing constraint. As shown below, the binding borrowing constraint endogenously generates drops in output, consumption, investment in capital and innovation, FDI inflows, and intermediate firms' profits. In the long run when capital has reached a steady state condition, the borrowing limit is large enough that the borrowing constraint never binds even with κ_L.

Given these settings, the maximization problem by the representative tradable goods producer is as follows:

$$\max_{\{K_t^D, \{y_t(i)\}_{i=0}^1, M_t, B_t\}_{t=0}^{\infty}} E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \lambda_t \Pi_t^T,$$

subject to the production function (1) and

$$\Pi_t^T = Y_t^T - r_t K_t^D - \int_0^1 p_t(i) y_t(i) di - P^M M_t - B_t + R_{t-1} B_{t-1} - T_t + V_t, \tag{3}$$

$$-B_t + \phi \left[\int_0^1 p_t(i) y_t(i) di + P^M M_t \right] - V_t \leq \kappa_t K_{t-1}, \tag{4}$$

where λ_t is the marginal utility of tradable goods consumption by households, $p_t(i)$ is the price of intermediate goods i, P^M is the price of imported inputs, and R_{t-1} is the gross interest rate on foreign debt repaid at period t. T_t is a lump-sum tax that the government collects to accumulate reserves, and V_t is a bailout by the government, which are explained in detail in a later section. For each period, the tradable goods producer chooses capital demand K_t^D, intermediate goods demand $\{y_t(i)\}_{i=0}^1$, imported inputs M_t, and foreign debt B_t to maximize the expected profit discounted by household's discount rate adjusted by the marginal utility λ_t. Let μ_t denote the Lagrange multiplier on the borrowing constraint (4).

The first-order conditions with respect to the choice variables are as follows:

$$K_t^D : r_t = \alpha \frac{Y_t^T}{K_t^D}, \tag{5}$$

$$y_t(i) : p_t(i) \left(1 + \phi \frac{\mu_t}{\lambda_t}\right) = \theta \frac{Y_t^T}{y_t(i)}, \tag{6}$$

$$M_t : P^M \left(1 + \phi \frac{\mu_t}{\lambda_t}\right) = (1 - \alpha - \theta) \frac{Y_t^T}{M_t}, \tag{7}$$

$$B_t : \lambda_t - \mu_t = \beta R_t E_t(\lambda_{t+1}), \tag{8}$$

$$\mu_t : \mu_t \left(B_t + \phi \left[\int_0^1 p_t(i) y_t(i) di + P^M M_t\right] - V_t - \kappa_t K_{t-1}\right) = 0,\ \mu_t \geq 0. \tag{9}$$

The first three equations are the demand functions for capital, intermediate goods, and imported inputs. When the borrowing constraint is slack, $\mu_t = 0$ and the demand functions for intermediate goods (6) and imported inputs (7) are the standard ones equating the price and the marginal products. When the borrowing constraint binds, strictly positive μ_t appears as the external financing premium on working capital payments, which increases the effective cost of inputs. Equation (8) is the Euler equation with respect to foreign debt. Given that λ_t is the marginal utility of tradable goods consumption by households, it is the standard Euler equation except when the Lagrange multiplier on the borrowing constraint μ_t appears. This term captures the external financing premium on foreign debt when the borrowing constraint binds, which increases the effective real interest rate on foreign debt, as explained in Mendoza (2010). The last equation (9) is the complementary slackness condition for the borrowing constraint.

The gross interest rate on foreign borrowing R_t is endogenously determined. Following Schmitt-Grohé and Uribe (2003), R_t is a function of the aggregate debt-to-GDP ratio:

$$R_t = \overline{R} + \psi_b \left(\exp\left(\frac{B_t}{GDP_t} - \overline{b}\right) - 1\right), \tag{10}$$

where GDP is given by $Y_t^T - P^M M_t + P_t^N Y_t^N M$, with $P_t^N Y_t^N$ being the non-tradable goods price times output.[9] As shown in Schmitt-Grohé and Uribe (2003), this formulation guarantees that the debt-to-GDP ratio will converge to the given value $\bar{b}$, and R_t will be $\overline{R}$ in the long run, so that the balanced growth path is uniquely pinned down. In this model, however, the debt-elastic spread plays an important role in determining the cost of reserve accumulation, namely, lower consumption in the short run and crowding-out of investment. Section 4.2 discusses these key mechanisms and their implications for the optimal policy.

3.2 Intermediate Goods Producing Firms

There is a unit-mass variety of differentiated intermediate goods in the tradable sector, indexed by i. Following the version of the Schumpeterian growth model developed by Klette and Kortum (2004), a firm is defined as a collection of one or more product line(s) among these differentiated goods. Each firm produces the product line(s) using labor, and innovates over other product lines. The production technology is given by:

$$y_t(i) = a_t(i)\ell_t(i), \tag{11}$$

where $a_t(i)$ is the labor productivity and $\ell_t(i)$ is labor input. Labor productivity $a_t(i)$ is heterogeneous across i, and improves over time by entry and innovations by domestic and foreign firms. The dynamics of the intermediate sector such as entry and innovation are laid out in the next subsection. This subsection focuses on the static part of the sector and shows that only the productivity leader produces goods for each product line, and that profit is determined by the size of the productivity lead by the productivity leader over rival firms.

As shown in equation (6), demand for each product line from the tradable goods producer is a unit-elastic demand function. This implies that the solution to the profit-maximization problem for a monopolist is to set the price infinitely high. In the Schumpeterian growth model, however, there are rival firms that can produce the same type of good with lower productivity, who could steal the market by setting a price slightly below the monopoly price. Therefore, through Bertrand competition, the profit-maximizing behavior by the

[9] Following Schmitt-Grohé and Uribe (2003), it is assumed that tradable goods producers do not internalize the effect of debt on the interest rate.

productivity leader is to set the price equal to the marginal cost of the closest rival firm and monopolize the demand.

Let $1+\sigma_t(i)$ denote the productivity lead by the leader for product line i over the closest rival. The productivity level of the closest rival is then given by $a_t(i)/\left(1+\sigma_t(i)\right)$, and the optimal price for the leader $p_t(i)$ is:

$$p_t(i) = \frac{W_t}{a_t(i)/\left(1+\sigma_t(i)\right)}, \tag{12}$$

where W_t is a real wage. The corresponding profit for the leader $\pi_t(i)$ can be written as follows:

$$\pi_t(i) = \frac{\theta Y_t^T}{1+\phi\mu_t/\lambda_t} - W_t \ell_t(i) = \frac{\sigma_t(i)}{1+\sigma_t(i)} \frac{\theta Y_t^T}{1+\phi\mu_t/\lambda_t}. \tag{13}$$

The first expression implies that larger demand from the tradable goods producer and a cheaper real wage bring higher profits. The second expression indicates that profit for each product line depends on the productivity lead by the leader over the closest rival $\sigma_t(i)$. Note also that the borrowing constraint on the tradable goods producer affects profit through the Lagrange multiplier μ_t. When the borrowing constraint binds, the tradable goods producer faces a higher effective cost of buying intermediate goods, demand for them thereby being reduced. Equation (13) shows that the smaller demand directly translates into lower profit for the intermediate goods producing firms.

The next subsection on firm dynamics shows that there are only two cases for the productivity lead by the leader, depending on whether the leader is a domestic firm or a foreign firm. The productivity lead by a domestic leader is denoted by σ_t^D, and it is σ_t^F for a foreign leader. Therefore, there are only two types of profits:

$$\pi_t(i) = \begin{cases} \frac{\sigma_t^D}{1+\sigma_t^D} \frac{\theta Y_t^T}{1+\phi\mu_t/\lambda_t} & \text{if leader is a domestic firm} \\ \frac{\sigma_t^F}{1+\sigma_t^F} \frac{\theta Y_t^T}{1+\phi\mu_t/\lambda_t} & \text{if leader is a foreign firm} \end{cases}.$$

It follows from the first expression in (13) that labor hired by each product line $\ell_t(i)$ also depends only on the leader type. More explicitly, labor hired by each product line is given as follows:

$$\ell_t(i) = \frac{\pi_t(i)}{\sigma_t(i) W_t}. \tag{14}$$

Hereafter let π_t^D and π_t^F denote a profit for each domestic and foreign product line, and ℓ_t^D and ℓ_t^F denote labor hired by each domestic and foreign product line respectively.

3.3 Innovation and Firm Dynamics

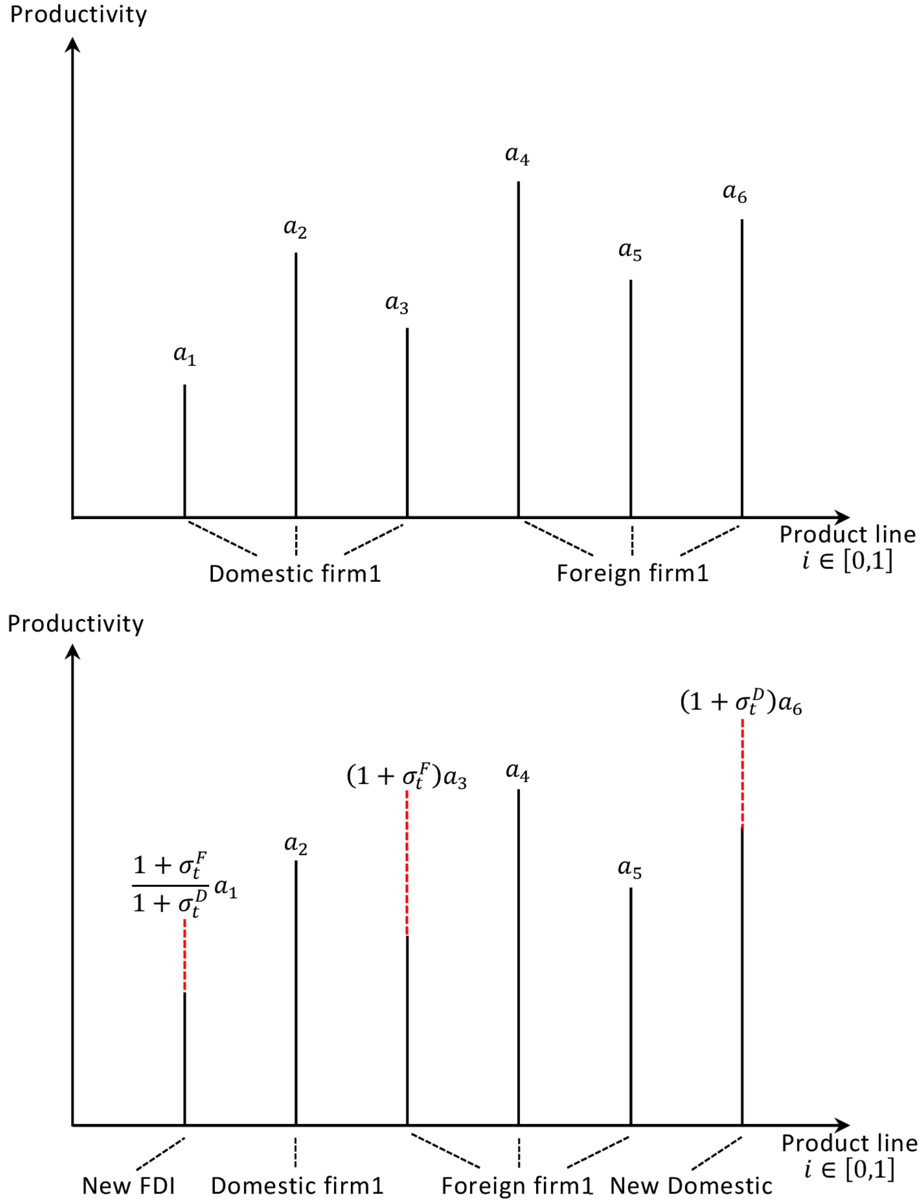
The firm dynamics in this economy are characterized by four different types of innovations: domestic entry, FDI entry, domestic incumbent innovation, and foreign incumbent innovation. Figure 4 illustrates an example of the evolution of firms from one period to the next. The upper panel shows six product lines with productivity a_1 to a_6. The first three lines are owned and produced by domestic firm 1, and the latter three lines by foreign firm 1. In the next period, depicted in the lower panel, three things are happening: First, for product line 1, foreign investors acquire the line from domestic firm 1 via FDI entry. It is assumed that FDI entry takes place only on domestic product lines. FDI entry improves productivity of the recipient product line by the factor of $(1+\sigma_t^F)/(1+\sigma_t^D)$. Second, for product line 3, foreign firm 1 succeeds in innovation and replaces domestic firm 1 as the productivity leader. Incumbent firms always innovate over product lines owned by other firms because innovating on its own product line would give a lower benefit with the same cost. Domestic incumbent innovation improves the productivity by a factor of $(1+\sigma_t^D)$, and by $(1+\sigma_t^F)$ for foreign incumbent innovation. Third, for product line 6, domestic entry occurs and a new domestic firm replaces foreign firm 1 as the productivity leader. Domestic entry may occur for any product line. Through such entry and innovation, firms compete with each other and endogenously expand, shrink, enter and exit, and increase the overall productivity of the intermediate sector.

This framework captures the several features of FDI entry and foreign firms documented by empirical studies: (1) Most FDI entry is through the acquisition of domestic firms rather than greenfield investment.[10] (2) Some domestic firms are forced to exit through competition with foreign firms.[11] Later sections will also show: (3) Foreign firms innovate more often

[10] Barba-Navaretti and Venables (2004) show that 90% of FDI is in the form of acquisitions.

[11] See Aitken and Harrison (1999).

Figure 4: Firm dynamics



than domestic firms.[12] (4) In a crisis, foreign firms invest more in innovation than domestic firms and survive better.[13]

The productivity improvement process explained above can be summarized as follows:

$$a_t(i) = \begin{cases} (1+\sigma_t^D)a_{t-1}(i) & \text{if domestic entry or innovation} \\ (1+\sigma_t^F)/(1+\sigma_t^D)a_{t-1}(i) & \text{if FDI entry} \\ (1+\sigma_t^F)a_{t-1}(i) & \text{if foreign innovation} \\ a_{t-1}(i) & \text{if nothing happens} \end{cases}.$$

Under this process, the productivity lead over the closest rival is determined by whether the productivity leader is a domestic firm or a foreign firm. In the former case the productivity lead is by a factor of $(1+\sigma_t^D)$, and in the latter case it is by a factor of $(1+\sigma_t^F)$. Combined with the discussion in the previous subsection, this implies that there are only two types of profit for each product line, π_t^D or π_t^F, depending on whether the productivity leader is a domestic firm or a foreign firm.

It is assumed that the productivity step size σ_t^D and σ_t^F decline as capital accumulates. Specifically, the following functional forms are assumed:

$$\sigma_t^D = \sigma^D \left(\frac{k_{ss}}{k_{t-1}}\right)^{\iota}, \tag{15}$$

$$\sigma_t^F = \sigma^F \left(\frac{k_{ss}}{k_{t-1}}\right)^{\iota}, \tag{16}$$

where $k_{ss} = K_{t-1}/A_t$ is the capital stock normalized by the aggregate productivity along the balanced growth path, which is constant, and $k_{t-1} = K_{t-1}/A_t$ is the same variable in the transition, which trends upward over time. $\iota > 0$ and $\sigma^F > \sigma^D$ are parameters. In the transition in which capital is scarce, $k_{ss}/k_{t-1} > 1$ and the step sizes are large. As capital accumulates and k_{t-1} gets close to k_{ss}, k_{ss}/k_{t-1} declines toward 1 and the step sizes converge to σ^D and σ^F along the balanced growth path.[14] The idea behind this assumption is that

[12] See Arnold and Javorcik (2009) and Guadalupe, Kuzmina, and Thomas (2012).

[13] See Alfaro and Chen (2012).

[14] This assumption implies that the growth-promoting effect of reserve accumulation declines over time. Without this assumption, it would be optimal for the government to keep accumulating reserves permanently to promote productivity growth, which is unrealistic.

capital scarcity indicates the technological distance from the world frontier, and when there is a large distance, the economy can grow faster through the catching-up effect. The following subsections explain each type of entry and innovation in turn.

3.3.1 Innovation by Foreign Incumbent Firms

Consider a foreign firm that owns n product lines. As seen before, operating profit depends only on the owner type and is independent of the individual firms's productivity. Therefore the total operating profit of this firm is $n\pi_t^F$. It is assumed that a firm with n product lines has n opportunities for innovation. The idea behind this assumption is that an incumbent firm's innovation is based on and spins off from the existing technology in practice. For each innovation opportunity, firms invest final tradable goods to create the innovation. The success probability of innovation i_t^F for each innovation opportunity is a concave function of tradable goods Z_t^F that a firm invests:

$$i_t^F = \eta^F \left(\frac{Z_t^F}{A_t} \right)^{1-\rho}, \tag{17}$$

where $\eta^F > 0$ is the productivity coefficient and $0 < \rho < 1$ is the parameter that governs the concavity. As is common in Schumpeterian growth models, innovation is undirected in the sense that innovation is equally likely to apply to any product line. This feature is preserved in this model by the structure that the operating profit is independent of productivity. Undirected innovations by many firms imply that each product line faces the same replacement probability. Let d_t denote this probability, and define as $P(i, n, p)$ the probability of having i successes in n trials for a binomial process with success probability p. Namely,

$$P(i, n, p) = \binom{n}{i} p^i (1-p)^{1-i}.$$

The value of a foreign firm with n product lines can be written in a recursive form as follows:

$$V_t^F(n) = \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} \left[\sum_{i=0}^{n} P(i, n, i_t^F) \left(\sum_{j=0}^{n} P(j, n, d_t) E_t \left(V_{t+1}^F(n+i-j) \right) \right) \right] \right\}.$$

The first two terms are the operating profit minus the innovation investment cost. R^F is the world interest rate, and foreign investors who own foreign firms discount future profits by this rate. The bracketed term is the expected value of this firm in the next period. The first summation adds up the expected value over the $n+1$ cases for the number of successful innovations, from 0 to n. The second summation adds up over the $n+1$ cases for the number of replacements, again from 0 to n. Thus for a foreign firm with n product lines, there are $(n+1)^2$ different possible combinations of the number of successful innovations and replacements. Note, however, that the expected value of the firm in each case, $E_t\left(V_{t+1}^F(n+i-j)\right)$, depends only on the number of the product lines, $n+i-j$, and not on the specific combination of the number of innovations and replacements. For example, 3 innovations and 2 replacements will give the same expected value as 4 innovations and 3 replacements, namely $E_t(V_{t+1}^F(n+1))$.

Following Ates and Saffie (2016), it can be shown by using a guess-and-verify method that the value of a foreign firm with n product lines is equal to n times the value of a foreign firm with a single product line:

$$V_t^F(n) = nV_t^F(1).$$

The formal proof is left to the Appendix. This linear relation enables aggregation of the firm dynamics in a tractable way without keeping track of the firm size distribution. The value of a foreign firm with a single product line is given by:

$$V_t^F(1) = \max_{Z_t^F}\left\{\pi_t^F - Z_t^F + \frac{1}{R^F}(1+i_t^F-d_t)E_t(V_{t+1}^F(1))\right\}. \tag{18}$$

Taking into account equation (17), the first-order condition with respect to Z_t^F gives the optimal investment condition:

$$\eta^F(1-\rho)\left(\frac{Z_t^F}{A_t}\right)^{-\rho}\frac{1}{A_t}\frac{1}{R^F}E_t(V_{t+1}^F(1)) = 1. \tag{19}$$

Since $0<\rho<1$, investment Z_t^F and the probability of successful innovation i_t^F are increasing in the expected value of a product line next period.

3.3.2 FDI Entry

FDI entry takes the form of the acquisition of a domestic-owned product line by foreign investors. There are infinitely many foreign investors who can consider acquiring a product line and entering this country via FDI. There are three types of cost for FDI entry. First, foreign investors need to pay a fixed fraction $0 < \lambda < 1$ of the discounted expected value of a product line $(1/R^F)E_t(V^F_{t+1}(1))$ to the domestic firm that owns the product line. λ can be interpreted as the negotiation power of the domestic owner firm against foreign investors.[15] Second, there is a fixed entry cost $A_t C^F$. This is in line with Helpman, Melitz, and Yeaple (2004) in that FDI entry is characterized by a large fixed entry cost. Third, there is a congestion cost of entry, which is linearly increasing in the aggregate number of product lines acquired by FDI in each period. Since there is an infinite number of potential foreign investors, FDI entry continues until the congestion cost pushes down the net expected profit of entry to zero. Therefore FDI entry in each period, denoted by e^F_t, is determined by the following zero-profit condition:

$$A_t \chi^F \left(\frac{e^F_t}{1 - \theta_{t-1}} \right) = (1 - \lambda) \frac{1}{R^F} E_t(V^F_{t+1}(1)) - A_t C^F, \tag{20}$$

where χ^F is a coefficient on the congestion cost. The denominator $1 - \theta_{t-1}$ is the fraction of domestic-owned product lines at the beginning of period t. This is introduced to capture the idea that it is more costly to find a good product line to acquire as the number of domestic-owned product lines falls. Both the congestion cost and the fixed entry cost increase over time along with the aggregate productivity of the economy A_t, so that FDI entry e^F_t will be constant in the long run. Similarly to innovation by foreign incumbent firms, FDI entry e^F_t is an increasing function of the expected value of a product line.

[15] For the domestic owner firm to be willing to sell a product line to foreign investors, the incentive compatibility condition must be satisfied. This condition is given by $\lambda(1/R^F)E_t(V^F_{t+1}(1)) \geq E_t(\Lambda_{t,t+1} V^D_{t+1}(1))$ where the right-hand side is the expected value of a domestic-owned product line discounted by the households' stochastic discount factor. Because this condition is always satisfied under the calibrated parameter values, it is not considered explicitly here.

3.3.3 Innovation by Domestic Incumbent Firms

Characterization of domestic incumbent firms is similar to foreign incumbent firms, but different in one key aspect: there is the possibility that product lines are acquired by foreign investors via FDI. Consider a domestic firm with n product lines. This firm has n innovation opportunities, as assumed in the case of foreign firms. For each opportunity, the firm invests Z_t^D units of tradable goods, and the probability of successful innovation i_t^D is given by the following equation:

$$i_t^D = \eta^D \left(\frac{Z_t^D}{A_t}\right)^{1-\rho}, \tag{21}$$

with $0 < \rho < 1$. Let Q_t denote the price that foreign investors pay to the domestic owner firm to acquire a product line via FDI. From the last subsection this is given by:

$$Q_t = \lambda \frac{1}{R^F} E_t(V_{t+1}^F(1)). \tag{22}$$

Using Q_t, the replacement probability d_t, and the probability for a binomial process $P(i,n,p)$, the value of a domestic firm with n product lines can be recursively written as follows:

$$\begin{aligned} V_t^D(n) \;=\; & \max_{Z_t^D} \Bigg\{ n\pi_t^D - nZ_t^D \\ & + \left[\sum_{i=0}^{n} P(i,n,i_t^D) \left\{ \sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left[\Lambda_{t,t+1} V_{t+1}^D(n+i-j-k)\right] \right) \right\} \right] \\ & + \left[\sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \Bigg\}. \end{aligned}$$

Compared with the value of a foreign firm, the additional terms are the third summation in the second line and the third line. The third summation in the second line adds up the expected value over the $n-j+1$ cases for the number of product lines acquired via FDI by foreign investors, from 0 to $n-j$, given that j product lines are replaced. $e_t^F/(1-\theta_{t-1})$ is the probability that each product line is acquired via FDI by foreign investors. Note that the expected value of the firm in the next period is discounted by the households' stochastic discount factor $\Lambda_{t,t+1}$. Note also that the number of product lines the firm owns in the next period is given by $n+i-j-k$. The third line adds up the acquisition price of FDI entry over the same $n-j+1$ cases given j replacements. Using the same guess-and-verify method, it can be shown that a linear relation holds for the value of a domestic firm:

$$V_t^D(n) = nV_t^D(1),$$

and the value of a domestic firm with a single product line is given by:

$$V_t^D(1) = \max_{Z_t^D} \left\{ \pi_t^D - Z_t^D + \left[i_t^D + (1-d_t)\left(1 - \frac{e_t^F}{1-\theta_{t-1}}\right) \right] E_t(\Lambda_{t,t+1} V_{t+1}^D(1)) + (1-d_t)\frac{e_t^F}{1-\theta_{t-1}} Q_t \right\}. \tag{23}$$

The first-order condition with respect to Z_t^D gives the optimal condition for domestic innovation investment:

$$\eta^D (1-\rho) \left(\frac{Z_t^D}{A_t} \right)^{-\rho} \frac{1}{A_t} E_t(\Lambda_{t,t+1} V_{t+1}^D(1)) = 1. \tag{24}$$

3.3.4 Domestic Entry

Finally, entry of new domestic firms comes from innovation by households and poaches a product line from incumbent firms. Households invest Z_t^E units of tradable goods to create new firms. The number of firms created from Z_t^E units of investment is given by:

$$e_t^D = \eta^E \left(\frac{Z_t^E}{A_t} \right)^{1-\rho}. \tag{25}$$

The optimal investment Z_t^E satisfies that the marginal benefit of investment is equal to the marginal cost, therefore:

$$\eta^E (1-\rho) \left(\frac{Z_t^E}{A_t} \right)^{-\rho} \frac{1}{A_t} E_t(\Lambda_{t,t+1} V_{t+1}^D(1)) = 1. \tag{26}$$

3.4 Aggregation and Productivity Growth

This subsection characterizes how firm dynamics translate into macroeconomic dynamics, specifically the transition of the share of product lines owned by foreign firms, and productivity growth.

First, replacement of a product line happens for three different reasons: domestic incumbent innovations, foreign incumbent innovations, and domestic entry. Thus the replacement rate d_t is the sum of these three probabilities:

$$d_t = (1 - \theta_{t-1}) i_t^D + \theta_{t-1} i_t^F + e_t^D \tag{27}$$

Note that the successful innovation probabilities by incumbents, i_t^D and i_t^F, are multiplied by the share of domestic-owned and foreign-owned product lines respectively. Next, the share of product lines owned by foreign firms, θ_t, increases for two reasons: foreign incumbent innovation over domestic-owned product lines, and FDI entry. θ_t decreases for two reasons: domestic incumbent innovation and domestic entry over foreign-owned product lines. The transition of θ_t is thus given by the following law of motion:

$$\begin{aligned} \theta_t &= \theta_{t-1} + \theta_{t-1}(1 - \theta_{t-1}) i_t^F + e_t^F - \theta_{t-1}(1 - \theta_{t-1}) i_t^D - \theta_{t-1} e_t^D \\ &= \theta_{t-1} + e_t^F - \theta_{t-1} e_t^D + (i_t^F - i_t^D)\theta_{t-1}(1 - \theta_{t-1}). \end{aligned} \tag{28}$$

Next, the expressions for aggregate productivity and its growth can be obtained in the following way: First, combine (13) and (14) to obtain the ratio between labor input by domestic-owned product lines ℓ_t^D and foreign-owned product lines ℓ_t^F:

$$\frac{\ell_t^D}{\ell_t^F} = \frac{1 + \sigma_t^F}{1 + \sigma_t^D}.$$

Combining this with total labor hired by the intermediate firms in the tradable sector $L_t^T = (1 - \theta_{t-1})\ell_t^D + \theta_{t-1}\ell_t^F$, the following expressions for labor input by domestic-owned and foreign-owned product lines are obtained:

$$\ell_t^D = \frac{1 + \sigma_t^F}{(1 - \theta_{t-1})\sigma_t^F + \theta_{t-1}\sigma_t^D} L_t^T,$$

$$\ell_t^F = \frac{1 + \sigma_t^D}{(1 - \theta_{t-1})\sigma_t^F + \theta_{t-1}\sigma_t^D} L_t^T.$$

Plugging these equations and the production function for intermediate goods (11) into (2), the composite of intermediate goods I_t^M can be written as follows:

$$I_t^M = A_t L_t^T \frac{(1+\sigma_t^D)^{\theta_{t-1}}(1+\sigma_t^F)^{1-\theta_{t-1}}}{\theta_{t-1}(1+\sigma_t^D) + (1-\theta_{t-1})(1+\sigma_t^F)}, \tag{29}$$

where aggregate productivity A_t is given by:

$$A_t = \exp\left[\int_0^1 \ln a_t(i) di\right].$$

As is clear from this expression, aggregate productivity A_t grows as productivity of each product line $a_t(i)$ improves. Using the four different innovation rates and the innovation step sizes, the growth rate of A_t is characterized as follows:

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \left(\frac{1+\sigma_t^F}{1+\sigma_t^D}\right)^{e_t^F} (1+\sigma_t^D)^{e_t^D} (1+\sigma_t^D)^{(1-\theta_{t-1})i_t^D} (1+\sigma_t^F)^{\theta_{t-1} i_t^F}. \tag{30}$$

The four terms in the right-hand side correspond respectively to FDI entry, domestic entry, domestic incumbent innovation, and foreign incumbent innovation. This completes the characterization of firm dynamics and its effect on aggregate productivity growth.

3.5 Non-Tradable Goods Producers

The non-tradable goods producer hires labor from households and produces non-tradable goods. The production function is given as follows:

$$Y_t^N = A_t (L_t^N)^{1-\alpha^N}, \tag{31}$$

where $0 < 1 - \alpha^N < 1$ is the labor share in non-tradable goods production. It is assumed that total factor productivity in non-tradable goods production increases at the same rate as aggregate productivity in the tradable sector. This assumption comes from the empirical fact that productivity spillovers from multinational firms to domestic firms happens through worker mobility.[16] Since this spillover to non-tradable goods production is not internalized by innovation investment decisions, it works as an externality that may cause too little

[16] Dasgupta (2012) reviews the relevant empirical literature.

innovation. This spillover guarantees that production of tradable goods and non-tradable goods will grow at the same rate in the long run. Let P_t^N denote the non-tradable goods price. Since the law of one price holds for tradable goods between this country and the rest of the world, the non-tradable goods price P_t^N determines the real exchange rate of this country. Thus P_t^N can be considered the real exchange rate of this economy, and an increase in P_t^N is real appreciation. The first-order condition of the non-tradable goods producer is given by:

$$W_t = P_t^N A_t (1-\alpha^N)(L_t^N)^{-\alpha^N}. \tag{32}$$

Labor is assumed to be mobile between the tradable and non-tradable sectors. Therefore, the real wage W_t is common in both sectors. Using P_t^N and W_t the profit for non-tradable goods producer is given by:

$$\Pi_t^N = P_t^N Y_t^N - W_t L_t^N, \tag{33}$$

which is paid to households.

3.6 Households

The representative household consumes tradable goods C_t^T and non-tradable goods C_t^N, supplies labor L_t elastically, accumulates and rents capital K_t to the tradable goods producer, and invests Z_t^E units of tradable goods in domestic entry. They receive the wage income $W_t L_t$, capital income $r_t K_{t-1}$, and profits from tradable goods producers Π_t^T, non-tradable goods producers Π_t^N, and domestic intermediate goods producing firms $(1-\theta_{t-1})(\pi_t^D - Z_t^D)$. They also receive FDI inflow $e_t^F Q_t^F$, which is revenue from the sales of domestic-owned product lines to foreign investors. The representative household's optimization problem is then given as follows:

$$\max_{\{C_t^T, C_t^N, L_t, K_t\}_{t=0}^{\infty}} E_0 \sum\nolimits_{t=0}^{\infty} \left[\ln C_t - \psi (L_t)^{\omega}\right],$$

$$C_t = \left[(\gamma)^{1/\varepsilon}(C_t^T)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{1/\varepsilon}(C_t^N)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{34}$$

subject to

$$C_t^T + P_t^N C_t^N + I_t + Z_t^E = W_t L_t + r_t K_{t-1} + \Pi_t^T + \Pi_t^N + (1-\theta_{t-1})(\pi_t^D - Z_t^D) + e_t^F Q_t^F, \tag{35}$$

$$I_t = K_t - (1-\delta)K_{t-1} + \frac{\psi_k}{2} K_{t-1} \left(\frac{K_t}{K_{t-1}} - (1+\overline{g}) \right)^2, \tag{36}$$

where γ is a parameter to determine the weight of tradable goods in composite consumption C_t, ε is the constant elasticity of substitution between tradable and non-tradable consumption, and δ is the depreciation rate of capital. Capital accumulation is subject to a capital adjustment cost that slows down the transition process, and ψ_k is the parameter that governs the size of this cost. The functional form of the capital adjustment cost is taken from Neumeyer and Perri (2005) to be consistent with long-run growth of the economy.

Optimal investment in domestic entry Z_t^E is determined by equation (25). The first-order conditions for the rest of the choice variables can be summarized as follows:

$$\frac{C_t^T}{C_t^N} = \frac{\gamma}{1-\gamma} (P_t^N)^\varepsilon, \tag{37}$$

$$\psi \omega (L_t)^{\omega - 1} = \frac{W_t}{C_t} \left(\gamma \frac{C_t}{C_t^T} \right)^{1/\varepsilon}, \tag{38}$$

$$\lambda_t \left[1 + \psi_k \left(\frac{K_t}{K_{t-1}} - (1+\overline{g}) \right) \right] = \beta E_t \left[\lambda_{t+1} \left\{ r_{t+1} + 1 - \delta - \frac{\psi_k}{2} \left((1+\overline{g})^2 - \left(\frac{K_{t+1}}{K_t} \right)^2 \right) \right\} \right], \tag{39}$$

where λ_t is the marginal utility of tradable consumption given by:

$$\lambda_t = \frac{1}{C_t} \left(\gamma \frac{C_t}{C_t^T} \right)^{1/\varepsilon}. \tag{40}$$

The stochastic discount factor $\Lambda_{t,t+1}$ is then given by $\Lambda_{t,t+1} = \beta \lambda_{t+1} / \lambda_t$. Equation (37) relates the optimal ratio of tradable and non-tradable goods consumption to the real exchange rate. Equation (38) gives the optimal labor supply L_t, and equation (39) is the Euler equation with respect to capital.[17]

[17] It is assumed that households do not internalize the fact that capital accumulation will relax the borrowing limit on foreign debt and working capital financing.

3.7 Government

The government in this model engages in a reserve policy to improve household's welfare. The reserve policy consists of two types of interventions: reserve accumulation in normal times and bailouts during crisis.

When $\kappa_t = \kappa_H$ and the borrowing constraint is loose, the government collects T_t units of tradable goods through a lump-sum tax and accumulates reserves. In general T_t can be any function of the state of the economy, but in this paper only a simple tax rule is considered in which the government collects a fixed fraction τ of tradable goods output Y_t^T each period. The government keeps accumulating τY_t^T units of reserves each period until it becomes suboptimal to do so. There are two reasons why accumulating reserves becomes suboptimal at some point in the transition. First, the benefit from attracting FDI becomes smaller as capital accumulates and the step size $\sigma_t^F(k_{t-1})$ becomes smaller. Second, as capital accumulates, the collateral value becomes large enough at some point that the borrowing constraint never binds and there is no need for bailouts. For these reasons it is optimal for the government to stop accumulating reserves once capital is sufficiently accumulated.[18]

Bailouts in sudden stops are modeled as follows. The borrowing constraint being binding implies that the tradable goods producer cannot borrow as much as they would if the borrowing constraint was loose. Let $Y_t^{T,\text{loose}}$ denote tradable goods output when the constraint is loose. The shortage of foreign borrowing, denoted by S_t, can be written as follows:

$$S_t = \max\left\{-B_t + \phi(1-\alpha)\, Y_t^{T,\text{loose}} - \kappa_L K_{t-1}, 0\right\}.$$

The first two terms are the borrowing amount when the constraint is loose, and $\kappa_L K_{t-1}$ is the borrowing limit, so that the gap is the shortage of foreign borrowing. A negative gap implies that the borrowing limit $\kappa_L K_{t-1}$ is large enough to cover the necessary amount of borrowing, and in this case a max operator sets $S_t = 0$. When S_t is positive, i.e. the borrowing constraint binds without a bailout, the government gives reserves to cover the

[18] It is also optimal for the government not to rebate reserves after reserve accumulation stops, because rebating reserves would reduce the growth rate. The redundant reserves are lost from the economy, but the welfare loss is limited because the value of goods received after 30 periods or further in the future is heavily discounted.

shortage up to the amount of reserves at hand. The size of a bailout, denoted by V_t, is given as:

$$V_t = \min\left\{S_t, R^F F_{t-1}\right\},$$

where $R^F F_{t-1}$ is the amount of reserves at the beginning of period t.

Given the tax rule in normal times and bailouts in sudden stops, the amount of reserves F_t follows the transition equation given as follows:

$$F_t = \begin{cases} R^F F_{t-1} + T_t & \text{when } \kappa_t = \kappa_H \\ R^F F_{t-1} - V_t & \text{when } \kappa_t = \kappa_L \end{cases}. \tag{41}$$

3.8 Market Clearing Conditions

To close the model, this subsection lists the market clearing conditions. The capital market, labor market, and non-tradable goods market clearing conditions are given as follows:

$$K_{t-1} = K_t^D, \tag{42}$$

$$L_t = L_t^T + L_t^N, \tag{43}$$

$$Y_t^N = C_t^N, \tag{44}$$

and labor in the tradable sector satisfies:

$$L_t^T = (1-\theta_{t-1})\ell_t^D + \theta_{t-1}\ell_t^F. \tag{45}$$

This completes the exposition of the model economy. The appendix formally defines the equilibrium of the model economy and the stationarized equilibrium conditions that are used to solve the model numerically.

4 Discussion on Reserve Policy

This section elaborates on the key mechanism of the model, namely how the reserve policy attracts FDI, promotes growth, and improves welfare. The section starts by describing the main benefit of the reserve policy and explains how the reserve policy may improve welfare. The section then discusses the cost of the reserve policy, and explains the key trade-off that the reserve policy faces, i.e. lower consumption in the short run and higher consumption in the long run.

4.1 Benefits of Reserve Policy

The benefits of a reserve policy are that it promotes productivity growth by inducing more domestic entry and incumbent firms' investment, and it also attracts FDI. Both reserve accumulation in normal times and bailouts during crises induce these firm dynamics.

In normal times when the borrowing constraint is loose, the government collects taxes from private agents to accumulate reserves. As some resources are taken away by the government, households reduce tradable goods consumption.[19] This reduction in tradable goods consumption leads to a fall in non-tradable goods price through equation (37), which is real exchange rate depreciation. Real depreciation in turn affects the relative profitability between the tradable and non-tradable sectors, and causes a labor shift across sectors. To see this, combine equations (1), (7), (13), (29) and (32) to obtain the wage equality condition across sectors:

$$W_t = A_t F(K_{t-1}, \theta_{t-1}) \left(L_t^T\right)^{-\frac{\alpha}{\alpha+\theta}} = P_t^N A_t (1-\alpha^N)(L_t - L_t^T)^{-\alpha^N}, \tag{46}$$

where $F(K_{t-1}, \theta_{t-1})$ is a function of the state variables. Note that production is concave in labor in both sectors, and thus the marginal product of labor is decreasing in labor in both sectors. This implies that a reduction in P_t^N causes a labor shift from the non-tradable sector to the tradable sector through (46). As more labor becomes available in the tradable sector, profit for each firm in the intermediate sector increases, which induces more entry and investment by firms, and attracts FDI entry. This mechanism is in line with the empirical

[19] Private agents have an incentive to borrow more from abroad to compensate for the loss of resources. As shown in Jeanne (2012), if reserve accumulation is completely offset by private foreign borrowing, there would be no effect on the consumption path, and thus reserve accumulation would not cause real depreciation. In this model, the debt-elastic foreign spread prevents full offset. As private agents borrow more from abroad, the interest spread rises through equation (10) and makes foreign borrowing more costly. Therefore the offset is only partial.

findings by Rodrik (2008) that real depreciation promotes productivity growth by shifting more production resources to the tradable sector.

When the borrowing constraint binds, bailouts by the government help the tradable goods producer finance working capital payments, and prevent sharp drops in profits for intermediate firms, as shown in (13). As explained in the previous sections, firms' investment and FDI entry decisions are forward-looking, and anticipation of future bailouts increases the expected value of firms ex ante and induces more investment and FDI entry.

The reason why these government interventions may improve households' welfare is because private agents do not internalize the fact that their actions affect FDI entry decisions by foreign investors, thus there is an externality in the model. The reserve policy corrects this externality through the two channels explained above, namely reserve accumulation in normal times and bailouts during crisis. However, whether the reserve policy actually improves welfare depends on its cost. The next subsection discusses this point.

4.2 Costs of Reserve Policy

There are two types of costs associated with the reserve policy in the model. First, as explained above, reserve accumulation takes away some tradable goods from private agents, so that consumption of tradable goods becomes lower in the short run. Consumption of non-tradable goods also becomes lower because reserve accumulation shifts labor to the tradable sector. As is clear from the discussion in the previous subsection, lower consumption and a labor shift to the tradable sector are the essential parts of the mechanism of how reserve policy works. In this sense these are the unavoidable costs of the reserve policy. At the cost of short-run lower consumption, the reserve policy promotes productivity growth and households enjoy higher consumption in the long run. Therefore, for the reserve policy to improve welfare, the long-run gain of higher consumption must exceed the short-run loss of lower consumption. This is the key trade-off that the reserve policy faces.

The second cost is a crowding-out effect of reserve accumulation. Reinhart, Reinhart, and Tashiro (2016) show that there is a strong negative correlation between the reserve-to-GDP ratio and the investment-to-GDP ratio in Asian countries after 2000, suggesting that active reserve accumulation crowds out investment. Cook and Yetman (2012) use micro-level data to show empirically that reserve accumulation reduces bank lending in emerging Asian countries. In the current model, the crowding-out effect results from the debt-elastic spread on foreign borrowing. As the government accumulates reserves by collecting taxes, private agents borrow more from abroad to compensate for the loss of resources, at least partially. Higher borrowing then increases the interest rate on foreign debt through equation (10).

A higher interest rate causes crowding-out of investment in both capital and innovation in the intermediate sector. On the one hand, a higher interest rate requires a higher capital return through the arbitrage condition, thereby discouraging capital investment. On the other hand, a higher interest rate reduces the stochastic discount factor by households, which in turn reduces the expected value of product lines in the intermediate sector. To see this point, arrange the Euler equation with respect to foreign debt (8) to obtain the expression for the stochastic discount factor as follows:

$$E_t(\Lambda_{t,t+1}) = \frac{\beta E_t(\lambda_{t+1})}{\lambda_t} = \left(1 - \frac{\mu_t}{\lambda_t}\right)\frac{1}{R_t}. \tag{47}$$

This equation indicates that a higher interest rate on foreign debt R_t reduces the stochastic discount factor, implying that households evaluate future tradable consumption less highly compared with tradable consumption today. Since investment in domestic firm entry and innovation are forward-looking, the lower stochastic discount factor leads to lower investment.

In summary, the reserve policy brings higher consumption in the long run at the cost of lower consumption in the short run. The optimal reserve policy is one that achieves a balance between the marginal gain and the marginal loss. The policy analysis section shows that the debt-elasticity of the spread ψ_b and the FDI entry cost affect the optimal reserve policy and its welfare gain.

5 Quantitative Analysis

This section calibrates the model parameters, demonstrates the baseline simulation, and discusses the model features. The model is solved numerically by two steps: First, the equilibrium conditions are divided by productivity level A_t to stationarize the equations.

In the second step, the stationarized model is solved globally using a version of the policy function iteration algorithm to deal with the occasionally binding borrowing constraint. The stationarized equilibrium conditions and the details of the solution procedure are left to the appendix.

5.1 Calibration

One period in the model is meant to be annual. There are 30 parameters to be determined in the model, except the debt-elasticity of spread ψ_b which is estimated from the data below. Conventional parameters are set to the conventional values in the literature, and the other parameters are calibrated to target the data for a sample of 19 developing countries from 1991-2010.[20] Table 1 presents 17 externally-determined parameter values. Parameters regarding preferences are set to the conventional values in the literature. The discount factor $\beta = 0.96$ and gross return on the safe asset $R^F = 1.02$ are standard values for annual models. The weight on tradable goods in consumption $\gamma = 0.34$ is set following Mendoza (2005) and Durdu, Mendoza, and Terrones (2009). The elasticity of substitution between tradable and non-tradable goods in consumption, $\varepsilon = 0.6$, is in the middle of the range discussed in Mendoza (2005). The coefficient on labor disutility $\psi = 0.525$ is set so that labor supply in the long run is equal to 1. The parameter for the labor supply elasticity $\omega = 1.455$ is set following Mendoza (1991).

Regarding the production parameters, capital's share in tradable production $\alpha = 0.3$ and the capital depreciation rate $\delta = 0.1$ are set to the conventional values. The imported input price P^M is set to be 1, and labor's share in non-tradable production $1 - \alpha^N = 0.75$ is taken from Schmitt-Grohé and Uribe (2016). The share of intermediate goods $\theta = 0.54$ is set so that the imported inputs-to-GDP ratio matches the data at 10%. The fraction of the input cost subject to the working capital requirement ϕ is determined by the method adopted in Mendoza (2010), in which the ratio of domestic credit to private firms to GDP is used as a proxy for working capital. This ratio is 47% on average for the sample countries, and this

[20] The countries are the 19 countries in the right panel of Figure 1. These counties are chosen based on the data availability and due to similar development levels. See footnote 6.

Table 1: Externally-determined parameters

Variable		Value	Source and Target
β	Discount factor	0.96	Standard
R^F	Return on reserve asset	1.02	Standard
γ	Tradable share in consumption	0.34	Mendoza (2005)
ε	CES between T and NT	0.6	Middle value in literature
ψ	Labor disutility	0.525	Unit labor supply
ω	Frisch elasticity $1/(\omega-1)$	1.455	Mendoza (1991)
α	Capital share	0.3	Standard
θ	Intermediate input share	0.54	Imported input/GDP 10%
P^M	Imported input price	1	Normalized value
$1-\alpha^N$	Labor share in non-tradable	0.75	Schmitt-Grohe Uribe (2016)
δ	Capital depreciation rate	0.1	Standard
ϕ	Share of input subject to WK	1.05	Private credit/GDP 47%
$\bar{b}$	Long-run debt/GDP	-0.36	Data
$\overline{R}$	Long-run interest rate	1.0635	Consistent with BGP growth
P_{HL}, P_{LH}	Transition matrix of κ_t	0.080, 0.851	Frequency and duration of SS
ρ	Concavity of innovation investment	0.5	Akcigit and Kerr (2015)

results in $\phi = 1.05$.[21] The long-run debt-to-GDP ratio $\bar{b}$ is set to the average of the sample countries in recent years at 36%. The long-run interest rate on foreign borrowing $\overline{R}$ is set to be consistent with the long-run growth rate satisfying $\beta\overline{R} = 1 + \bar{g}$, where the long-run growth rate $\bar{g}$ is determined below.

The only uncertainty in the model is a stochastic borrowing limit coefficient κ_t. κ_t follows a two-state Markov process with a 2×2 transition matrix. Following Jeanne and Rancière (2011), the average frequency and duration of sudden stop episodes are derived in the following way: a given country in a given year is in a sudden stop if the capital inflow-to-GDP ratio drops more than 5% from the previous year. Using the same sample of 33 countries as in Jeanne and Rancière (2011) over the period of 1980-2009, the unconditional probability of sudden stops is 8.6%, and each sudden stop episode continues for two years with probability 14.9%. Accordingly, the transition matrix is set so that $P_{HL} = 0.080$ and $P_{LH} = 0.851$.

For the parameters related to innovation and FDI entry, the concavity parameter governing investment, ρ, is set to 0.5 following Akcigit and Kerr (2015) and their literature review.

[21] This value is higher than 0.26 in Mendoza (2010), but close to 1 in Neumeyer and Perri (2005) and 1.25 in Uribe and Yue (2006).

Table 2: Jointly-determined parameters

	Variable	Value	Target	Model
η^D	Domestic innovation coeff.	0.20	Manu. R&D/GDP 2.4%	2.4%
η^E	Domestic entry coeff.	0.72	Domestic entry rate 8.11%	8.11%
η^F	Foreign innovation coeff.	0.17	Relative innovation rate 1.387	1.387
χ^F	Coeff. of FDI congestion cost	0.60	FDI value-added in manu. 32.25%	32.25%
λ	Share of FDI firm value paid	0.71	Manu. FDI inflow/GDP 1.57%	1.57%
C^F	Fixed entry cost	0.16	Value of a line/fixed cost 1.1	1.1
σ^D	Domestic innovation size	0.24	Long-run growth rate 2.1%	2.1%
σ^F	Foreign innovation size	0.36	11% productivity gain upon FDI entry	

The eight parameters regarding growth, $\eta^D, \eta^E, \eta^F, \chi^F, \lambda, C^F, \sigma^D, \sigma^F$, are jointly determined to match eight moments in the data to those in the model in the long run, as summarized in Table 2. Each of the following moments is tightly related to the above eight parameters in the same order. (1) The ratio of R&D expenditure in the manufacturing sector to GDP is closely related to η^D, which governs the scale of domestic innovation. This ratio is in general small in developing countries and high in developed countries. The long-run ratio in the model is set to match the average of developed countries in the recent data, which is 2.4%. (2) The domestic entry rate is closely related to η^E. The data is taken from The World Development Indicators. The average of the sample countries is 8.11%. (3) The innovation rate of foreign firms relative to domestic firms identifies η^F, which governs foreign incumbents' innovation. Guadalupe, Kuzmina, and Thomas (2012) document that foreign firms in Spain conduct product innovations 1.387 times more often than domestic firms, which is set as the calibration target. (4) The value added share of foreign firms in the manufacturing sector identifies χ^F, the congestion cost of FDI entry. This target is meant to pin down the economic presence of foreign firms in the tradable sector in the model. Ramstetter (2009) reports this value for Malaysia, Thailand, and Vietnam, and Ramondo (2009) reports this for Chile. The average of these four countries is 32.25%, which is set as the target. (5) The ratio of FDI inflows to the manufacturing sector to GDP helps to pin down the cost of acquisition for FDI. Because the data for FDI inflows by sector are limited, the sample is extended to twelve countries including six out-of-sample developing countries.[22] The average of the ratio

[22] The in-sample countries are Colombia, Ecuador, Malaysia, Mexico, Turkey, and Uruguay, and the out-of-sample countries are Costa Rica, Hungary, Pakistan, Poland, Romania, and Vietnam. The data can be found at the International Trade Centre's website.

Table 3: Parameters related to transitional dynamics

Variable		Value	Target
κ_L	Borrowing limit coefficient	0.85	SS dynamics
ψ_k	Capital adjustment cost	15	Drop in investment in SS
ι	Exponent on catch-up term	0.25	Avg. growth 3% in transition

of FDI inflows to the manufacturing sector to GDP for these countries is 1.57%, which is set as the target. (6) There is no data or reliable estimation for the fixed cost of FDI entry C^F. The ratio of the expected value of a foreign product line to the fixed entry cost determines the elasticity of FDI inflows to the expected value of a line, as suggested in equation (20). This ratio is set to target 1.1, so that the dynamics of FDI inflows during sudden stops match the data as shown in the next subsection. (7) The long-run growth rate helps to pin down σ^D, the productivity gain of domestic innovation. The long-run growth rate is set to 2.1%, which is the average growth rate of developed countries in recent years. (8) The productivity gain from FDI entry identifies σ^F. Arnold and Javorcik (2009) and Guadalupe, Kuzmina, and Thomas (2012) estimate the productivity gain from FDI entry in Indonesia and Spain using the propensity score matching method to mitigate the cherry-picking effect of FDI entry choice. These papers show that in the year of entry, firm productivity increases by 11%. Hence σ^F is set so that $(1+\sigma^F)/(1+\sigma^D) = 1.11$. When each country's reserve policy is evaluated below, the FDI congestion cost χ^F and the fixed entry cost C^F are adjusted across countries to match variation in the relevant moments.

Finally, the borrowing limit coefficient κ_L, the capital adjustment cost parameter ψ_k, and the exponent on the catch-up term ι are determined to target the model behavior in the transition, because these parameters are irrelevant along the balanced growth path. κ_L and ψ_k are set to match the sudden stop dynamics of the model with the data shown in the next subsection. ι governs the growth rate of the economy in the transition, and thus is set to match the average growth rate of the model economy in the first 30 periods with the average growth rate of the sample countries from 1980 to 2010. The average GDP growth rate in the data is about 3%. With $\iota = 0.25$, along with the initial capital holdings $k_{-1} = 0.45k_{ss}$, the average growth rate of the first 30 periods is 3%. In determining these parameter values, the pace of capital accumulation is not targeted. According to the Penn World Table, the capital holdings of the sample countries in 1980 is 28% of that in 2010. In the model, the initial capital K_{-1} is about 28% of the capital holdings at period 30.

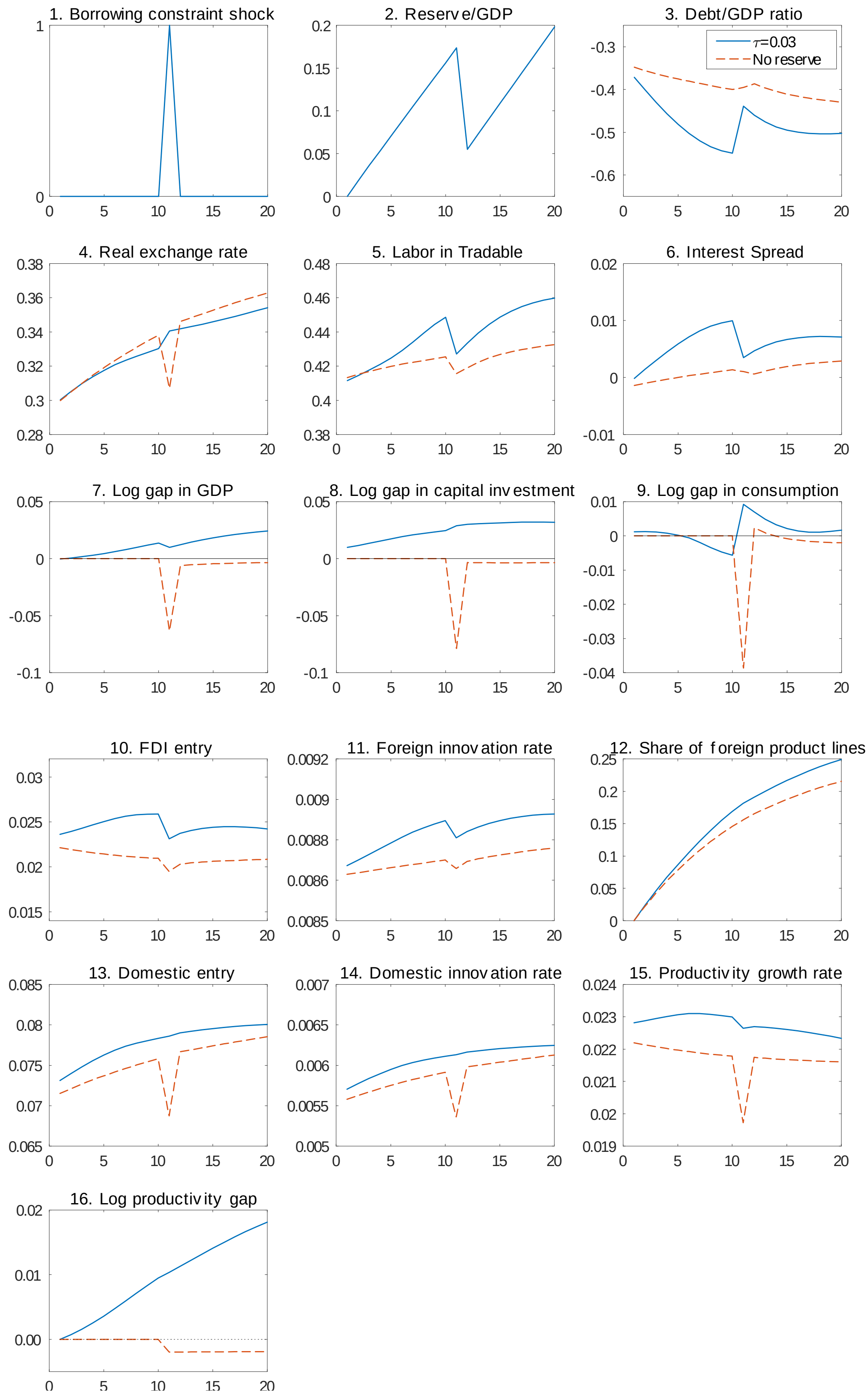
5.2 Quantitative Performance of the Model

This subsection documents the quantitative performance of the model and demonstrates the role of the reserve policy. It starts by showing a sample dynamic simulation path to give an idea of what the transition dynamics look like. Figure 5 presents a sample dynamic path of the model. The debt-elasticity of spread is set at $\psi_b = 0.0561$, which is a middle value of the 19 sample countries estimated below. Initial capital is 45% of its long-run level as measured by the productivity-adjusted value, initial debt is 33% of GDP to match with the data in 1980, and the initial share of foreign-owned product lines is 0. The solid lines are the paths with reserve policy, and the dashed lines are the paths without reserve policy. The tax rate τ, which is the pace of reserve accumulation, is set to 3%.

As the first panel shows, there is a borrowing constraint shock at period 11. The second panel shows that reserve is accumulated in normal times, and used for a bailout when the borrowing constraint shock hits the economy. Responding to reserve accumulation by the government, private agents borrow more from abroad to compensate for the loss of resources, as displayed in panel 3. As discussed in the previous section, reserve accumulation causes real depreciation and a labor shift to the tradable sector in panel 4 and 5. Panel 4 also shows that a bailout by the government prevents a sharp drop in real exchange rate in a crisis. Panel 6 shows that as private agents borrow more from abroad, the interest spread on foreign borrowing rises.

Panels 7, 8, and 9 show the log gaps in GDP, capital investment, and consumption compared with the path without shocks or reserve policy. When the borrowing constraint shock hits the economy, all three variables drop sharply without policy intervention. But as shown in panels 7 and 8, reserve policy induces faster growth of GDP and capital investment, and a bailout in crisis cancels the negative effect of the shock on these variables. Panel 9 shows that consumption becomes lower in the short run but higher in the long run with reserve policy, which is the key trade-off that reserve policy faces, as discussed in the previous section.

Figure 5: Model simulation (ψ_b=0.0561)



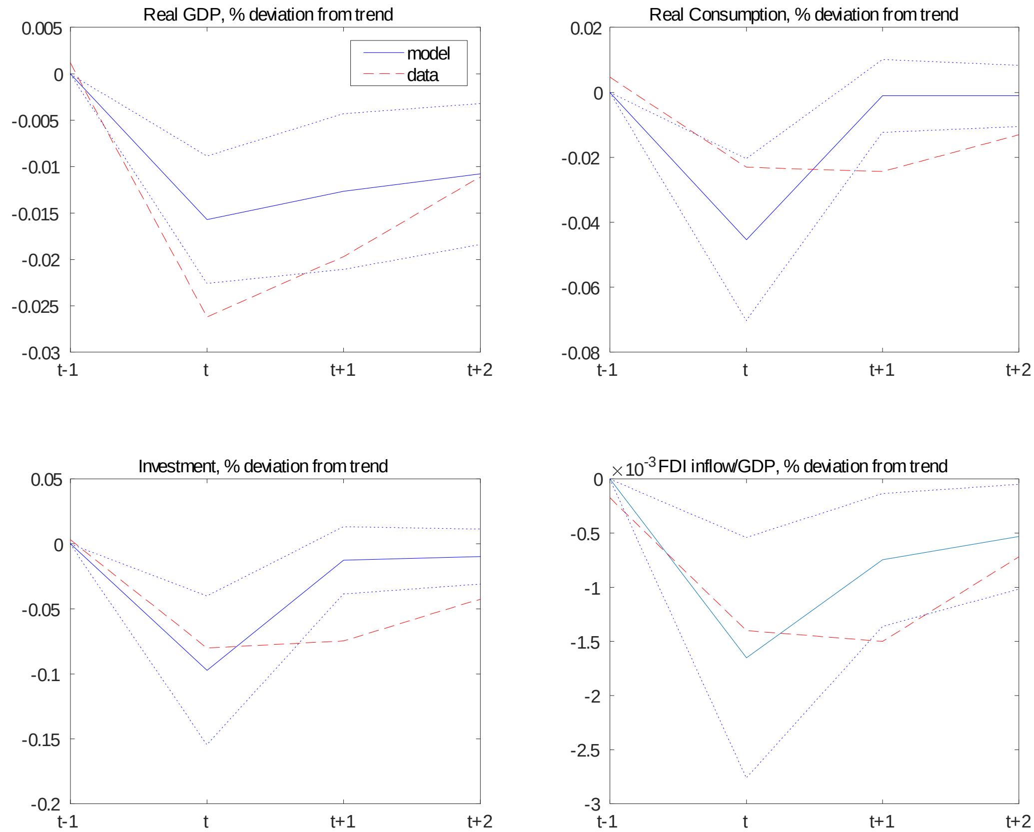
Turning to firm dynamics and innovation, panels 10, 11, 13, and 14 show that FDI entry, foreign innovations, domestic entry, and domestic innovations all drop in a crisis without policy intervention. These panels also indicate that the size of the drop is much larger for domestic innovations compared with foreign innovations. This difference comes from the fact that the stochastic discount factor by domestic households drops due to the binding borrowing constraint as shown in equation (47), and thus domestic firms reduce their investment substantially, while foreign firms discount future profits by the fixed rate $1/R^F$. These different responses by domestic and foreign firms are consistent with the empirical facts documented by Alfaro and Chen (2012). Reserve policy induces more entry and innovations in normal times and mitigates the shocks from crisis by a bailout. As a result, productivity grows faster and more stably with reserve policy as shown in panel 15. The last panel shows that the reserve policy in this simulation increases the productivity level by 1.8% in 20 periods. It also shows that a sudden stop has a permanent negative effect on productivity without policy intervention. This is in line with empirical evidence shown in Cerra and Saxena (2008) that crises in general have a very persistent negative effect on economic growth.

To evaluate the quantitative performance of the model in capturing sudden stop dynamics, Figure 6 compares the dynamics of the key variables in the data and the model. The data dynamics show the average percentage deviations of the variables from the HP-filtered smooth trends. The sudden stop episodes are taken from the same samples that are used to determine the Markov transition matrix in calibration.[23] The model dynamics are created as follows: the model is simulated 1,000 times with stochastic shocks, starting from the same initial state as the above sample simulation, and the first sudden stop episode for each simulation is picked up. For each sudden stop episode, the percentage deviation of the variables from the smooth path without shocks is computed, and the average of all the sudden stop episodes across 1,000 simulations is taken.

Figure 6 shows that there is a small underprediction of a drop in real GDP and a small overprediction of a drop in real consumption, but drops in capital investment and FDI

[23]The data for GDP, consumption, and investment are taken from the World Development Indicators. The FDI inflow-to-GDP ratio is computed using the data from Broner, Didier, Erce, and Schmukler (2013).

Figure 6: Sudden stop dynamics; model and data



inflow-to-GDP ratio are well captured in the model. Overall the model replicates well the quantitative dynamics of the average sudden stop episodes in the data.

6 Determinants of Optimal Accumulation Pace

This section studies how the optimal reserve policy is determined, and shows the first main result of the paper: the debt-elasticity of the interest rate spread and the FDI entry cost are key determinants of the optimal pace of reserve accumulation. As discussed in the model section, the analysis considers a simple policy rule that the government collects a fixed fraction τ of tradable output every period to accumulate reserves. The government optimally stops accumulating reserves when capital holdings adjusted by the productivity level reach 90% of the balanced growth level.[24] This level is reached at around period 35 of simulation, and the economy will follow a smooth balanced growth path in the following periods.

6.1 Role of Debt-Elasticity of Spread

This subsection shows how the debt-elasticity of spread ψ_b affects the optimal reserve policy. This elasticity is empirically estimated for the sample 19 developing countries in the next section. The result indicates that the elasticity varies from 0.0223 to 0.0899 across countries, which implies that if the debt-to-GDP ratio increases by 10%, the spread increases by 22.3 to 89.9 basis points. The analysis below uses these estimated elasticities.

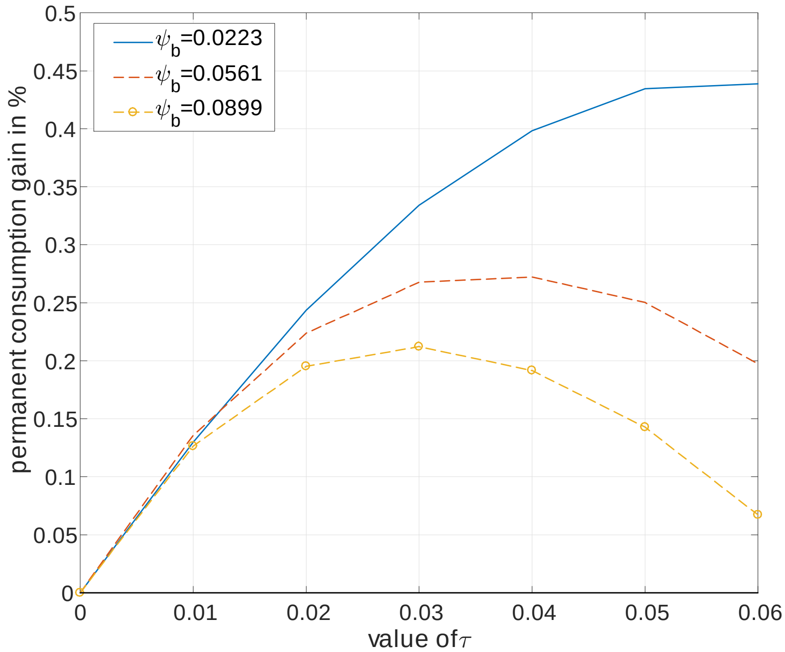
The optimal pace of reserve accumulation is derived as follows: Given each value of ψ_b, the model is solved and simulated with reserve policies $\tau = 0.01$ to 0.06 for 300 periods 100,000 times, and the expected utility over these simulations is computed. The optimal pace of reserve accumulation is the value of τ that gives the highest expected utility.

The result is presented in Figure 7. The welfare gain by each reserve policy on the vertical axis is evaluated in terms of the permanent consumption gain in percentage compared with the economy without reserve policy, as is common in the literature. The figure clearly indicates that the optimal pace of reserve accumulation τ is faster with lower debt-elasticity of spread ψ_b, and the associated welfare gain is larger.

To understand the role of ψ_b, Figure 8 plots the dynamics of key variables in simulations with $\psi_b = 0.0223$ and $\psi_b = 0.0899$. This figure is created by simulating the model with tax rate $\tau = 0.04$ for both economies 100,000 times with stochastic shocks and taking the average path of each variable. The first panel shows that the debt-to-GDP ratio becomes much larger with low elasticity of the spread ψ_b. This difference comes from the difference in foreign borrowing spread plotted in the second panel. Under high ψ_b, the interest spread goes up quickly, which prevents private agents from increasing foreign borrowing. As a result, consumption becomes lower in the short run under high ψ_b, while it barely declines under

[24] To be precise, it is assumed that the government reduces the tax rate linearly from τ to 0 as capital accumulates from 85% to 90% of the productivity-adjusted long-run level. This assumption avoids a clear kink in the decision rule and makes numerical solution easier and more accurate.

Figure 7: Welfare impact with different ψ_b

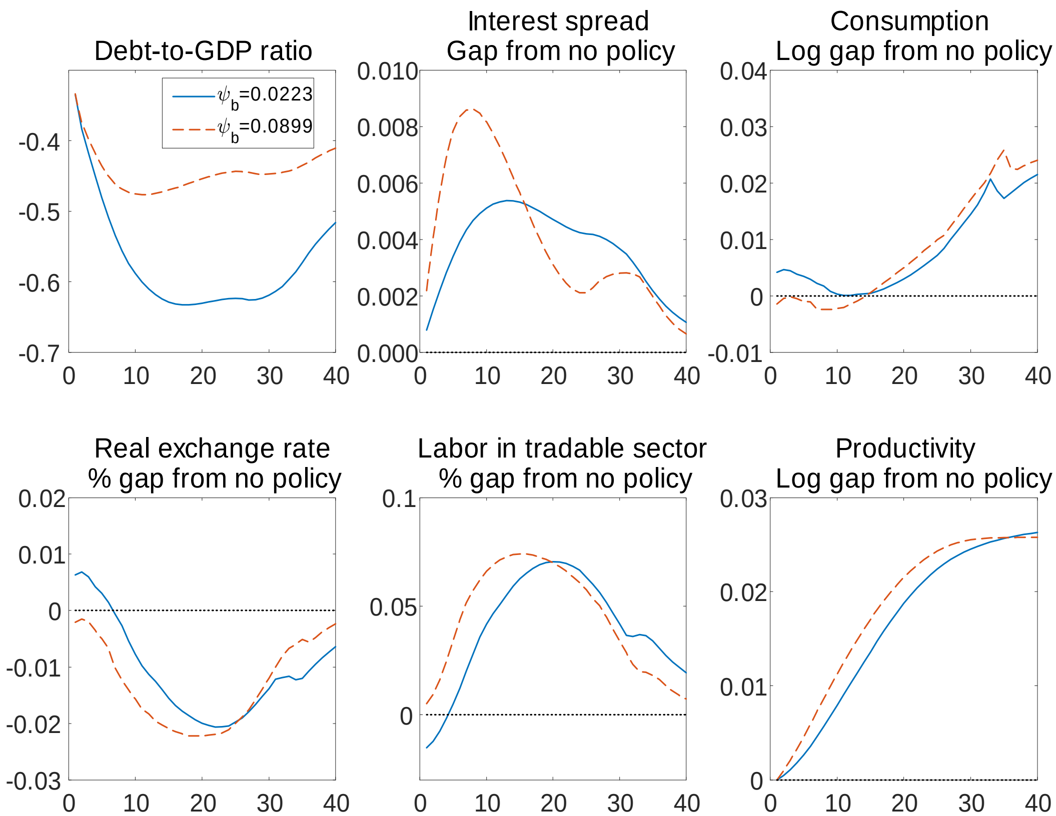


low ψ_b. This implies that the short-run cost of reserve accumulation becomes higher as the debt-elasticity of the spread ψ_b becomes higher.

Another implication of different ψ_b is its effect on productivity growth. As consumption becomes lower under high ψ_b, the real exchange rate depreciates more and a larger amount of labor shifts to the tradable sector, as the first and second panels in the second row show. This brings higher profits for intermediate firms and induces more entry and innovation, which leads to faster productivity growth as shown in the last panel. However, there is a counteracting effect on productivity growth, which is crowding-out by the high interest spread. A high interest spread implies high costs for domestic investment, thus it discourages domestic investment in both capital and innovation.

To quantify the crowding-out effect of a high interest spread on productivity growth, consider the following three economies: the first one is without reserve policy, the second one is with reserve policy, and the last one is with reserve policy and innovation rates manipulated as follows: when the borrowing constraint is not binding, the expected value of a domestic product line is taken from the economy with reserve policy, but it is multiplied by

Figure 8: Effects of ψ_b on transitional dynamics



the interest rate with policy and then divided by the interest rate without policy. Namely,

$$E_t(\Lambda_{t,t+1}V_{t+1}^D(1))\big|_{\text{manipulated}} = E_t(\Lambda_{t,t+1}V_{t+1}^D(1))\big|_{\text{policy}} \frac{R_t^{\text{policy}}}{R_t^{\text{nopolicy}}}$$

Since the stochastic discount factor $E_t(\Lambda_{t,t+1})$ is the inverse of the interest rate when the borrowing constraint is not binding, this manipulation gives approximately the expected value of a domestic product line with policy discounted by the low interest rate without policy. Domestic investment in entry and innovation are then computed by the first order conditions (24) and (26) using this manipulated value of a product line, and consumption is adjusted according to changes in investment. This manipulation is intended to remove the crowding-out effect of a high interest rate on domestic investment on entry and innovation, preserving the effect of a labor shift to the tradable sector by reserve accumulation.

Figure 9: Productivity loss by crowding-out

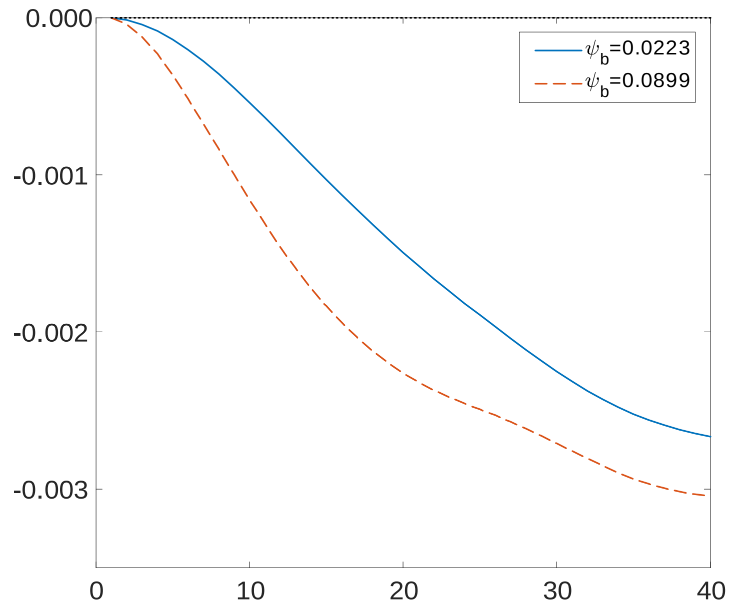


Table 4: Welfare loss by crowding-out

$\psi_b \setminus \tau$	0.01	0.02	0.03	0.04	0.05	0.06
0.0223	-0.040	-0.070	-0.097	-0.123	-0.140	-0.155
0.0561	-0.053	-0.092	-0.121	-0.145	-0.164	-0.181
0.0899	-0.060	-0.102	-0.131	-0.152	-0.177	-0.193

Figure 9 plots the log gaps in productivity in the economy with normal reserve policy compared with the economy with a manipulated firm value (no crowding-out), computed from the average of 100,000 stochastic simulations. It suggests that the productivity loss by crowding-out is 0.26% under low ψ_b and 0.30% under high ψ_b at period 40. These numbers may look small, but the welfare impact is not. Table 4 presents the welfare losses caused by these productivity losses, which are the gaps in the expected utility in the economy with normal reserve policy compared with the economy without crowding-out. It shows that the welfare loss by crowding-out in terms of permanent consumption is not small, and can be larger than 0.1% as τ becomes higher than 0.03. It also indicates that the welfare loss becomes larger as the debt-elasticity of the spread becomes higher.

In short, high debt-elasticity of the spread ψ_b prevents private agents from offsetting reserve accumulation by borrowing from abroad, increasing the short-run cost of lower consumption. This short-run lower consumption causes larger real depreciation and labor shift to the tradable sector, but the positive effect of this labor shift on productivity growth is counteracted by a higher interest spread, reducing the long-run benefit of higher consumption. This implies that the high debt-elasticity of the spread worsens the trade-off that reserve policy faces. In this case, the optimal pace of reserve accumulation becomes slower to reduce the short-run cost and achieve a balance between the short-run cost and the long-run benefit.

6.2 Role of the FDI Entry Cost

Another key determinant of the optimal reserve policy in the model is the FDI entry cost. There is a vast literature on the determinants of FDI inflows, and many factors have been identified as significant determinants, such as the host country's institutions, relative labor endowments, and so on.[25] FDI entry costs in the model can be interpreted as these implicit factors that affect and govern the size of FDI inflows to the country.[26]

Comparison of two countries with different FDI entry cost χ^F helps to show how the FDI entry cost affects the optimal pace of reserve accumulation. One country's FDI entry cost is set to target the FDI inflow-to-GDP ratio of 1.09%, which is the average of the 19 sample countries.[27] The other country's FDI entry cost is lower, and set to target 1.5 times the FDI inflow-to-GDP ratio, at 1.63%. The fixed entry cost C^F is also adjusted to keep the ratio of the expected value of a product line to the fixed entry cost at 1.1, and the innovation step sizes σ^D and σ^F are adjusted to have the same long-run growth rate at 2.1%, keeping the relative size $(1+\sigma^F)/(1+\sigma^D) = 1.11$ unchanged. The other parameters are left unchanged.

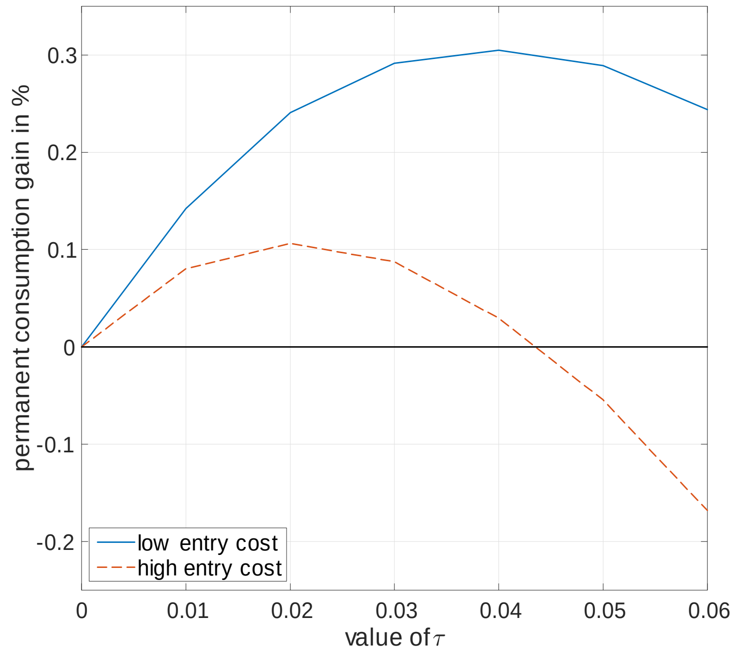
Figure 10 presents the welfare impact of reserve policy in these two countries. It is clear that the optimal pace of reserve accumulation is faster for a country with low FDI entry cost, and the welfare gain is substantially larger. This result suggests that attracting FDI is

[25] Blonigen (2005) and Blonigen and Piger (2011) review the literature on the determinants of FDI.

[26] As an alternative approach, the appendix explicitly estimates the FDI entry cost using the Starting a Business Index from the World Bank's Doing Business Surveys.

[27] Since the data for the manufacturing sector's share of FDI inflow is not available for every country in the sample, it is assumed that each country has the same share 40.7%, which is the average of the twelve countries used in calibration.

Figure 10: Welfare impact with different FDI entry costs



an important channel through which the reserve policy improves welfare. If some factors of the country impede FDI inflow and the reserve policy is not effective in attracting FDI, the optimal pace of reserve accumulation is slower, and the welfare impact is likely to be limited.

The appendix of this paper conducts a decomposition analysis of the effect of reserve policy on growth and welfare. To summarize: (1) about 68% of the productivity gain by reserve policy comes from reserve accumulation in normal times, and 32% from bailouts in crisis; (2) about 60% of the productivity gain by reserve policy comes from higher domestic entry and innovation, and 40% comes from higher FDI entry and foreign innovation; (3) 64% of welfare gain by bailouts comes from financing working capital payment, and 36% from rebating accumulated reserves to private agents.

7 Evaluation of Reserve Policy

This section conducts the second main analysis of the paper: evaluation of the actual reserve policies of developing countries. The first subsection estimates the debt-elasticity of the spread for developing countries from the data. Then the second subsection derives the optimal pace of reserve accumulation for each country and compares it with the actual pace observed in the data.

7.1 Estimation of Debt-Elasticity of Spread

There is a large amount of literature on the determinants of the foreign borrowing spread in developing countries, but there are not many studies that estimate the debt-elasticity of the spread for each country.[28] Reinhart and Rogoff (2009) argue that the history of default affects how foreign investors see each developing country when they consider lending, and a history of serial default implies lower thresholds for safe lending. According to the data in the Chartbook for their book, Reinhart (2010), the number of defaults for the 19 sample countries before 1994 varies from 0 to 9. Accordingly, the 19 sample countries are divided into five groups depending the number of past defaults, 0 or 1, 2 or 3, 4 or 5, 6 or 7, and 8 or 9, and each group is assigned a number from 0 to 4. Then consider the following regression:

$$S_{i,t} = \beta_0 + \beta_1 \text{debtGDP}_{i,t} + \beta_2 (\text{debtGDP}_{i,t} \times \text{Default}_i) + \alpha_i + \tau_t + \varepsilon_{i,t},$$

where $S_{i,t}$ is the interest rate spread on external borrowing in percent, Default_i is the indicator for the default history from 0 to 4, α_i is a country-specific fixed effect, τ_t is a time-specific fixed effect, and $\varepsilon_{i,t}$ is an error term. The data for the spread is taken from JP Morgan's EMBI Global, as is common in the literature.[29] Since the time period of the available data is different across countries, this is an unbalanced panel regression with 19 countries with maximum time period 1994-2015. The debt-to-GDP ratio is computed using the data from Lane and Milesi-Ferretti (2017). The data is annual and the total number of observations is 323.

The result is presented in Table 5. This result implies that the debt-elasticity of the spread for countries with 0 or 1 default is 0.0223, and the elasticity increases by 0.0169 as

[28] One of the main questions in the literature is whether a developing country's spread is determined by global factors or the country's fundamentals. See for example Kennedy and Palerm (2014) and their literature review.

[29] It would be better to use the data for private foreign debt spreads, but this is not available for many developing countries. As many papers show, private and public spreads have very high correlations.

Table 5: Estimation of debt-elasticity of spread

Explanatory Variables	Coefficient (S.E.)	t-value
β_1: Debt-GDP ratio	0.0223 (0.0161)	1.39
β_2: Debt-GDP ratio $\times$ Default	0.0169*** (0.0049)	3.45

the number of defaults increases: 0.0392 for 2 or 3 defaults, 0.0561 for 4 or 5 defaults, 0.0730 for 6 or 7 defaults, and 0.0899 for 8 or 9 defaults.[30] This result is similar to the results of other papers that include more controls, such as 0.0447 in Dell'Erba, Hausmann, and Panizza (2013) and 0.0567 in Kennedy and Palerm (2014).

7.2 Evaluation of Each Country's Reserve Policy

Given the estimated debt-elasticity of the spread, this subsection evaluates each country's reserve policy. The analysis proceeds as follows: (1) For each country, the FDI entry cost parameters and innovation step sizes are adjusted to match the FDI inflow–to-GDP ratio in the model to the data.[31] (2) Given the new parameters and the estimated ψ_b, the model is numerically solved and simulated, and the optimal reserve accumulation pace τ that maximizes household's expected utility is derived. (3) The optimal pace in the model is then compared with the reserve accumulation pace in the data.[32] The reserve accumulation pace in the data is obtained by computing reserve increase-to-GDP ratio every year from 1991 to 2010 and taking the average across the years. The data is from the World Development Indicators. The results are presented in Figure 11 and Table 6.

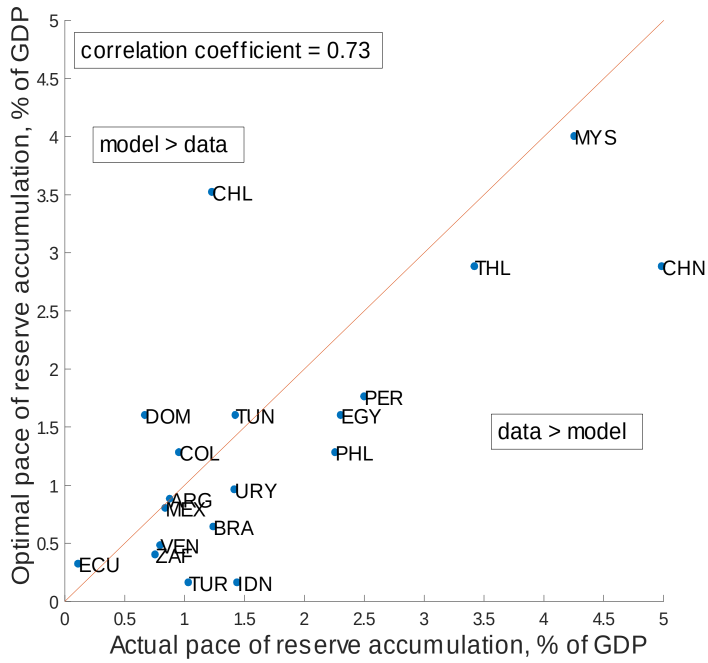
Figure 11 plots each country in the 45-degree line diagram, in which the observed pace of reserve accumulation is on the horizontal axis and the optimal pace in the model is on the vertical axis. Most countries are located close to the 45 degree line, implying that the observed pace is close to the optimal pace. The average optimal pace across the 19 countries is 1.43% of GDP while it is 1.71% in the data, and the correlation coefficient between the optimal and actual paces across the countries is 0.73. On the other hand, it is also observed

[30] Although the constant term is not significant, it would become at least 10% significant with a larger number of countries.

[31] As in the previous section, it is assumed that 40.7% of FDI inflow goes to the manufacturing sector for each country. Accordingly, the target is 40.7% of the FDI–to-GDP ratio for each country.

[32] In the model, a 1% tax on tradable output corresponds to 0.63% of GDP on average in the first 30 periods.

Figure 11: Pace of reserve accumulation, model and data



that two countries have large deviations. Chile seems to be accumulating reserves much less actively compared with the optimal pace, while China seems to be accumulating reserves too actively.

Table 6 presents more detailed results including the welfare gain/loss by the actual and optimal pace of reserve accumulation. It can be observed that the welfare gain from the actual policy is close to the optimal level for most countries. Looking at each country in detail, Chile and China have much room for welfare improvement. Indonesia and Turkey may be losing welfare by accumulating reserve too actively compared with the optimal pace. Regarding the debt-elasticity of the spread, most Latin American countries have high elasticity because of their default history. This reduces the optimal pace of reserve accumulation, but most Latin American countries are actually in line with the optimal pace.

This overall result suggests that the debt-elasticity of the foreign borrowing spread and the FDI entry cost can explain a substantial amount of the cross-country variation in the pace of reserve accumulation. The appendix shows that if the debt-elasticity of the spread is fixed at 0.0561 for every country and only the FDI entry cost is adjusted across countries, the

Table 6: Comparison of Reserve Accumulation Pace and Welfare

Country	Accum. Pace (% of GDP)		Welfare (%)		Elasticity of Spread	FDI Inflow / GDP (%)
	Actual	Optimal	Actual	Optimal		
Argentina	0.88	0.88	0.06	0.06	0.0730	2.44
Brazil	1.24	0.64	0.02	0.04	0.0899	2.34
Chile	1.23	3.52	0.34	0.55	0.0899	5.85
China	4.99	2.88	0.22	0.36	0.0392	3.96
Colombia	0.96	1.28	0.10	0.10	0.0730	2.86
Dominican Rep.	0.67	1.60	0.10	0.14	0.0730	3.19
Ecuador	0.11	0.32	0.01	0.01	0.0899	1.71
Egypt	2.31	1.60	0.11	0.13	0.0392	2.58
Indonesia	1.44	0.16	-0.11	0.00	0.0392	0.86
Malaysia	4.26	4.00	0.54	0.54	0.0223	4.35
Mexico	0.84	0.80	0.05	0.05	0.0899	2.42
Peru	2.50	1.76	0.15	0.18	0.0899	3.63
Philippines	2.26	1.28	0.00	0.05	0.0223	1.55
South Africa	0.76	0.40	0.01	0.02	0.0392	1.33
Thailand	3.42	2.88	0.25	0.26	0.0223	3.00
Tunisia	1.43	1.60	0.15	0.15	0.0561	3.02
Turkey	1.03	0.16	-0.06	0.00	0.0561	1.08
Uruguay	1.42	0.96	0.04	0.06	0.0899	2.56
Venezuela	0.80	0.48	0.02	0.03	0.0899	2.12

correlation coefficient between the optimal and actual paces reduces from 0.73 to 0.43. This implies that FDI entry cost may be slightly more important in explaining the cross-country variation, but the debt-elasticity of the spread also makes a substantial contribution.

8 Conclusion

Active accumulation of foreign reserves by developing countries has been both an active research area and a central area of policy debate in the past decade. However, the literature still tells little about the optimal reserve policy for individual countries and the reason for the wide variation in the amount and pace of reserve accumulation across countries. This paper contributes to the literature by developing a quantitative model of reserve accumulation and studying the determinants of the optimal pace of reserve accumulation. The model incorporates the key benefits and costs of reserve accumulation. On the benefit side, endogenous growth and sudden stops are introduced to incorporate the growth-promoting effect and the precautionary effect of reserve accumulation. FDI inflow is also introduced into the endogenous growth framework, which constitutes an important channel through which reserve accumulation promotes growth and welfare. On the cost side, reserve accumulation in the model causes crowding-out of investment.

Using the model, this paper identified two factors that are important determinants of the optimal pace of reserve accumulation: the debt-elasticity of the foreign borrowing interest rate spread, and the FDI entry cost. In countries where debt-elasticity of the spread is high, active reserve accumulation causes a large drop in consumption in the short run, and also severely crowds out investment, which reduces the growth-promoting effect. In countries with high FDI entry costs, reserve policies are not effective in attracting FDI, and the growth-promoting effect is limited. In these cases, the optimal pace of reserve accumulation is slower, and the welfare gain is limited.

Taking into account differences in the debt-elasticity of the spread and the FDI entry cost across countries, most developing countries are roughly in line with the optimal pace of reserve accumulation suggested by the model. This result implies that these two factors can explain a substantial amount of the cross-country variation in the observed pace of reserve accumulation.

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Appendix

A Equilibrium and Stationarized Equilibrium

This section defines the equilibrium of the model economy and the stationarized equilibrium.

A.1 Equilibrium

Definition: *The equilibrium of the model economy is defined by the initial states A_0, $R_{-1}B_{-1}$, K_{-1}, θ_{-1}, F_{-1}, κ_{-1}, the stochastic process $\{\kappa_t\}_{t=0}^{\infty}$, the government policy rules $\{T_t, V_t\}_{t=0}^{\infty}$ and the following:*

1. *Tradable goods producers: Given prices $\{r_t, W_t, R_t\}_{t=0}^{\infty}$ and the government policy rules $\{T_t, V_t\}_{t=0}^{\infty}$, $\{K_t^D, M_t, B_t, I_t^M, Y_t^T, \Pi_t^T, \mu_t\}_{t=0}^{\infty}$ satisfy (1), (3), (5), (7), (8). (9), (29).*

2. *Foreign intermediate goods producing firms: Given prices $\{W_t\}_{t=0}^{\infty}$ and tradable goods output $\{Y_t^T\}_{t=0}^{\infty}$, $\{\ell_t^F, Z_t^F, \pi_t^F, i_t^F, V_t^F, \sigma_t^F\}_{t=0}^{\infty}$ satisfy (13), (14), (16), (17), (18), (19).*

3. *Domestic intermediate goods producing firms: Given prices $\{W_t\}_{t=0}^{\infty}$ and tradable goods output $\{Y_t^T\}_{t=0}^{\infty}$, $\{\ell_t^D, Z_t^D, \pi_t^D, i_t^D, V_t^D, \sigma_t^D\}_{t=0}^{\infty}$ satisfy (13), (14), (15), (21), (23), (24).*

4. *Foreign investors: $\{e_t^F, Q_t^F\}_{t=0}^{\infty}$ satisfy (20) (22).*

5. *Non-tradable goods producers: Given prices $\{W_t, P_t^N\}_{t=0}^{\infty}$, $\{Y_t^N, L_t^N, \Pi_t^N\}_{t=0}^{\infty}$ satisfy (31), (32), (33).*

6. *Households: Given prices $\{r_t, W_t, P_t^N\}_{t=0}^{\infty}$, $\{C_t, C_t^T, C_t^N, L_t, K_t, Z_t^E, I_t, e_t^E, \lambda_t\}_{t=0}^{\infty}$ satisfy (25), (26), (34), (35), (36), (37), (38), (39), (40).*

7. *Foreign reserves: $\{F_t\}_{t=0}^{\infty}$ follows the transition equation given by (41).*

8. *Aggregate variables $\{A_t, \theta_t, d_t\}_{t=0}^{\infty}$ satisfy (27), (28), (30).*

9. *Prices $\{r_t, W_t, P_t^N, R_t\}_{t=0}^{\infty}$ and labor in tradable sector $\{L_t^T\}_{t=0}^{\infty}$ satisfy (10), (42), (43), (44), (45).*

A.2 Stationarized Equilibrium

To stationarize the model, the equilibrium conditions are divided by aggregate productivity A_t. Let the lower-case letters denote stationarized variables, and use g_t to denote the productivity growth rate A_{t+1}/A_t. After some arrangements to reduce the number of equations, the following is the complete list of equations to characterize the stationarized equilibrium of the model:

Tradable goods producers

$$y_t^T = \left(\frac{k_{t-1}}{1+g_{t-1}}\right)^{\alpha} (i_t^M)^{\theta} (m_t)^{1-\alpha-\theta}$$

$$i_t^M = L_t^T \frac{(1+\sigma_D)^{\theta_{t-1}}(1+\sigma_F)^{1-\theta_{t-1}}}{\theta_{t-1}(1+\sigma_D)+(1-\theta_{t-1})(1+\sigma_F)}$$

$$w_t = \frac{\theta_{t-1}(1+\sigma_D)+(1-\theta_{t-1})(1+\sigma_F)}{(1+\sigma_D)(1+\sigma_F)} \frac{\theta y_t^T}{L_t^T} \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$r_t^k = \alpha \frac{y_t^T}{k_{t-1}/(1+g_{t-1})}$$

$$(1-\alpha-\theta)\frac{y_t^T}{m_t} = P^M \left(1+\phi\frac{\mu_t}{\lambda_t}\right)$$

$$1-\frac{\mu_t}{\lambda_t} = \beta R_t E_t \left(\frac{\lambda_{t+1}}{\lambda_t(1+g_t)}\right)$$

$$R_t = \overline{R} + \psi_b \left(\exp\left(\frac{b_t}{gdp_t} - \overline{b}\right) - 1\right)$$

$$\mu_t \left[-b_t + \frac{\phi(1-\alpha)y_t^T}{1+\phi\mu_t/\lambda_t} - \kappa_t\right] = 0$$

Foreign intermediate goods producing firms

$$\sigma_t^F = \sigma^F \left(\frac{k_{ss}}{k_{t-1}}\right)^{\iota}$$

$$\pi_t^F = \frac{\sigma_t^F}{1+\sigma_t^F}\theta y_t^T \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$v_t^F = \pi_t^F - z_t^F + \left[i_t^F + (1-d_t)\right](1+g_t)\frac{1}{R^F}E_t(v_{t+1}^F)$$

$$i_t^F = \eta^F (z_t^F)^{1-\rho}$$

$$\eta^F (1-\rho)(z_t^F)^{-\rho}(1+g_t)\frac{1}{R^F}E_t(v_{t+1}^D) = 1$$

Domestic intermediate goods producing firms

$$\sigma_t^D = \sigma^D \left(\frac{k_{ss}}{k_{t-1}}\right)^{\iota}$$

$$\pi_t^D = \frac{\sigma_t^D}{1+\sigma_t^D}\theta y_t^T \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$v_t^D = \pi_t^D - z_t^D + \left[i_t^D + (1-d_t)\left(1-\frac{e_t^F}{1-\theta_{t-1}}\right)\right](1+g_t)E_t(\Lambda_{t,t+1}v_{t+1}^D) + (1-d_t)\frac{e_t^F}{1-\theta_{t-1}}q_t^F$$

$$i_t^D = \eta^D (z_t^D)^{1-\rho}$$

$$\eta^D (1-\rho)(z_t^D)^{-\rho}(1+g_t)E_t(\Lambda_{t,t+1}v_{t+1}^D) = 1$$

FDI entry

$$\frac{e_t^F}{1-\theta_{t-1}} = \chi^F\left[(1-\lambda)(1+g_t)\frac{1}{R^F}E_t(v_{t+1}^F) - c^F\right]$$

$$q_t^F = \lambda(1+g_t)\frac{1}{R^F}E_t(v_{t+1}^F)$$

Aggregate variables

$$d_t = e_t^D + (1-\theta_{t-1})i_t^D + \theta_{t-1}i_t^F$$

$$\theta_t = \theta_{t-1} + e_t^F - \theta_{t-1}e_t^D + (i_t^F - i_t^D)\theta_{t-1}(1-\theta_{t-1})$$

$$1+g_t = \left(\frac{1+\sigma_t^F}{1+\sigma_t^D}\right)^{e_t^F}(1+\sigma_t^D)^{e_t^D}(1+\sigma_t^F)^{(1-\theta_{t-1})i_t^D}(1+\sigma_t^F)^{\theta_{t-1}i_t^F}$$

Non-tradable goods producers

$$y_t^N = (L_t - L_t^T)^{1-\alpha^N}$$

$$w_t = P_t^N(1-\alpha^N)(L_t - L_t^T)^{-\alpha^N}$$

Households

$$c_t^T + b_t + k_t + z_t^E = y_t^T - P^M m_t - \theta_{t-1}\pi_t^F - (1-\theta_{t-1})z_t^D + (1-\delta)\frac{k_{t-1}}{1+g_{t-1}} + \frac{R_{t-1}b_{t-1}}{1+g_{t-1}} + e_t^F q_t^F - \tau_t - \psi_t^K$$

$$\frac{c_t^T}{y_t^N} = \frac{\gamma}{1-\gamma}(P_t^N)^\varepsilon$$

$$\psi\omega(L_t)^{\omega-1} = \frac{w_t}{c_t}\left(\gamma\frac{c_t}{c_t^T}\right)^{\frac{1}{\varepsilon}}$$

$$1+\psi_k\left(\frac{k_t(1+g_{t-1})}{k_{t-1}} - (1+\overline{g})\right) = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t(1+g_t)}\left\{r_{t+1} + 1 - \delta - \frac{\psi_k}{2}\left((1+\overline{g})^2 - \left(\frac{k_{t+1}(1+g_t)}{k_t}\right)^2\right)\right\}\right]$$

$$\lambda_t = \frac{1}{c_t}\left(\gamma\frac{c_t}{c_t^T}\right)^{1/\varepsilon}$$

Domestic firm entry

$$e_t^D = \eta^E(z_t^E)^{1-\rho}$$

$$\eta^E(1-\rho)(z_t^E)^{-\rho}(1+g_t)E_t(\Lambda_{t,t+1}v_{t+1}^D) = 1$$

Foreign reserves transition

$$f_t = R^F\frac{f_{t-1}}{1+g_{t-1}} + \tau_t - v_t$$

The stationarized equilibrium is characterized by 33 variables $\{y_t^T$, k_t, g_t, i_t^M, m_t, L_t^T, θ_t, w_t, μ_t, λ_t, r_t^k, σ_t^F, π_t^F, v_t^F, z_t^F, i_t^F, σ_t^D, π_t^D, v_t^D, z_t^D, i_t^D, e_t^F, q_t^F, d_t, R_t, y_t^N, L_t, P_t^N, c_t^T, b_t, z_t^E, e_t^D, $f_t\}_{t=0}^\infty$ and the above 33 equations, given the initial state $R_{-1}b_{-1}/(1+g_{-1})$, $k_{-1}/(1+g_{-1})$, θ_{-1}, $f_{-1}/(1+g_{-1})$, κ_{-1}, the government policy $\{\tau_t, v_t\}_{t=0}^\infty$, and the stochastic process $\{\kappa_t\}_{t=0}^\infty$.

B Numerical Solution

This section sketches the numerical solution method and presents the accuracy of the solution.

B.1 Solution Method

The solution method is a version of the policy function iteration, modified to deal with the occasionally binding constraint. Below is the procedure to obtain the numerical solution.

1. Set the equally-spaced grid points for the endogenous state variables, foreign debt $R_{t-1}b_{t-1}/(1+g_{t-1})$, capital $k_{t-1}/(1+g_{t-1})$, share of product lines owned by foreign firms θ_{t-1}, and foreign reserve holdings $f_{t-1}/(1+g_{t-1})$. The number of grid points is set to 31 for debt, capital, and reserves, and 5 for the share of foreign product lines. There are also 2 states for the borrowing limit κ_t.

2. For each grid point, set the initial guess for 5 variables: b_t, z_t^D, z_t^F, L_t^T, and the right-hand side of the Euler equation with respect to capital (RHSEE).

3. For each grid point, do the following:

   (a) Leave the 5 variables for which the guess was made as unknown variables, and express all the other endogenous variables in terms of the state variables and 5 unknowns. In this process, first assume that the borrowing constraint is not binding and proceed. Later check if the constraint is satisfied. If it is not satisfied, recalculate all the variables using the binding borrowing constraint. The other endogenous variables, which include next-period state variables, are now functions of the 5 variables.

   (b) Using multi-dimensional linear interpolation over the next-period state variables and the guess for the 5 variables (b_t, z_t^D, z_t^F, L_t^T, RHSEE), compute all the endogenous variables next period. Then calculate all the forward-looking expectation terms, such as the right-hand side of the Euler equations and the value functions.

   (c) All the equilibrium conditions are now the functions of the initial 5 unknowns. There are 4 equations that are not used in step (a), and the explicit expression for RHSEE, thus 5 equations in total. Solve for the 5 unknowns using non-linear solver.

4. Check the gap between the guess and the newly-obtained values for the 5 variables. If they are close enough, stop. If not, update the guess by the newly-obtained values, and go back to step 3. Repeat this process until the gap becomes sufficiently small.

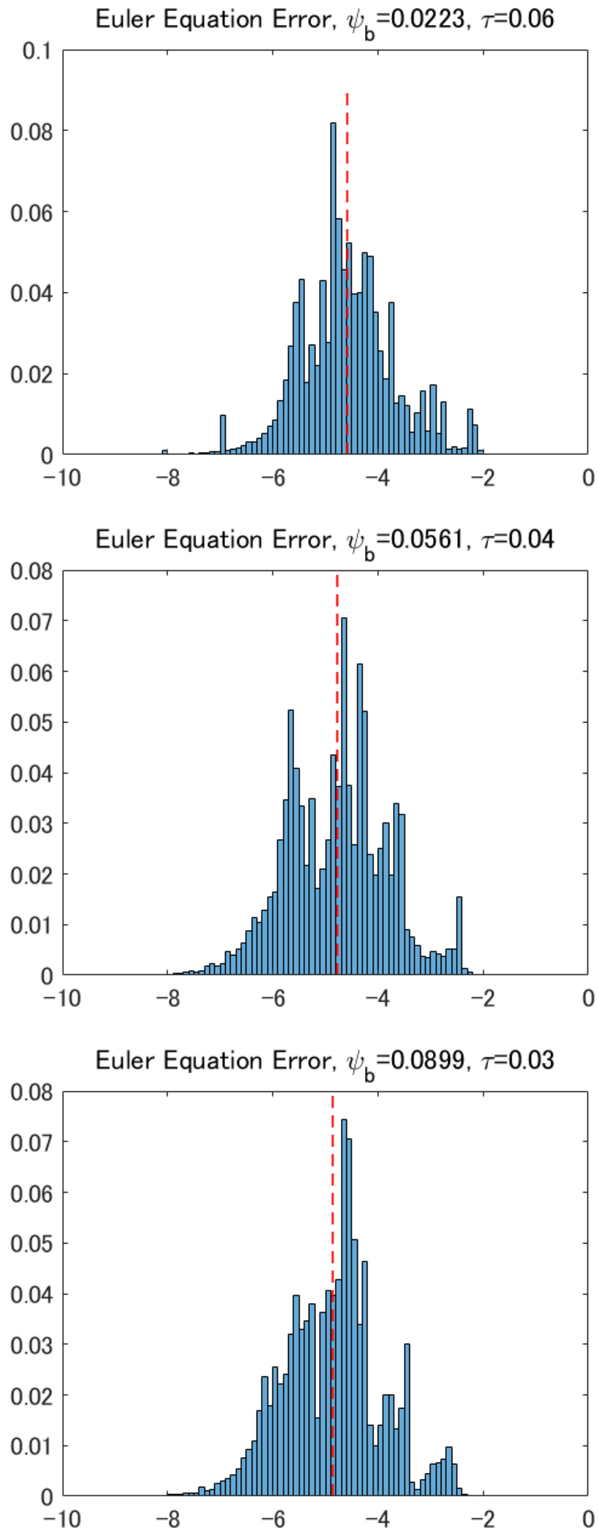
B.2 Accuracy of the Solution

This subsection presents the accuracy of the numerical solution obtained by the above method. Following Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006), the Euler equation error of the solution is computed. Here the Euler equation with respect to foreign borrowing is used, because it is subject to the occasionally binding borrowing constraint, and thus likely to cause a larger error. For each value of ψ_b and τ, the model is simulated for 50 periods with the initial states used in the main analysis and stochastic shocks to the borrowing constraint. The reason for stopping simulation at period 50 is because the economy after period 50 follows a smooth path with no borrowing constraint binding, and thus errors are very small. This simulation is repeated 10,000 times. For each period t in each simulation i, the Euler equation error defined as follows is computed:

$$error_{t,i} = \log_{10}\left[1 - \frac{c_{t,i}^{T,EE}}{c_{t,i}^{T}}\right],$$

where $c_{t,i}^{T}$ is tradable consumption computed directly from the decision rules, and $c_{t,i}^{T,EE}$ is computed by using the Euler equation with respect to foreign borrowing. Figure 12 plots the distribution of the Euler equation errors obtained by this method. As a reference, the distributions of the Euler equation errors for the models with three different ψ_b with the corresponding optimal τ are plotted. For each case, the average error is smaller than -4 and the maximum error is smaller than -2, which is reasonably small when compared with the literature. With different values of ψ_b and τ, the distributions of errors are similar.

Figure 12: Euler Equation Error



C Proof of Linearity in Value Functions

This section shows the detailed procedure of the guess-and-verify method to prove the linear relation in value functions for intermediate producing firms.

C.1 Foreign Firms

The value of a foreign firm with a single product line is given as follows:

$$V_t^F(1) = \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[\sum_{i=0}^{1} P(i,1,i_t^F) \left(\sum_{j=0}^{1} P(j,1,d_t) E_t \left(V_{t+1}^F(1+i-j) \right) \right) \right] \right\}.$$

There are four cases next period, depending on whether innovation is successful or not, and replacement happens or not. Writing out all four cases and noting $V_{t+1}^F(0) = 0$,

$$\begin{aligned} V_t^F(1) &= \max_{Z_t^F} \Big\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[P(0,1,i_t^F) P(0,1,d_t) E_t \left(V_{t+1}^F(1) \right) \right] \\ &\qquad + P(1,1,i_t^F) P(1,1,d_t) E_t \left(V_{t+1}^F(1) \right) \\ &\qquad + P(1,1,i_t^F) P(0,1,d_t) E_t \left(V_{t+1}^F(2) \right) \Big\} \\ &= \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[(1-i_t^F)(1-d_t) E_t \left(V_{t+1}^F(1) \right) \right] + i_t^F d_t E_t \left(V_{t+1}^F(1) \right) + i_t^F (1-d_t) E_t \left(V_{t+1}^F(2) \right) \right\}. \end{aligned}$$

Guess the linear relation $V_t^F(n) = nV_t^F(1)$. Using the linear relation $V_{t+1}^F(2) = 2V_{t+1}^F(1)$,

$$\begin{aligned} V_t^F(1) &= \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[(1-i_t^F)(1-d_t) E_t \left(V_{t+1}^F(1) \right) \right] + i_t^F d_t E_t \left(V_{t+1}^F(1) \right) + 2 i_t^F (1-d_t) E_t \left(V_{t+1}^F(1) \right) \right\} \\ &\quad \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[(1+i_t^F - d_t) E_t \left(V_{t+1}^F(1) \right) \right] \right\}, \end{aligned}$$

which is equation (18) in the main text. Next, the value of a foreign firm with n product lines is given as:

$$V_t^F(n) = \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} \left[\sum_{i=0}^{n} P(i,n,i_t^F) \left(\sum_{j=0}^{n} P(j,n,d_t) E_t \left(V_{t+1}^F(n+i-j) \right) \right) \right] \right\}.$$

Using the linear relation $V_{t+1}^F(n+i-j) = (n+i-j)V_{t+1}^F(1)$,

$$\begin{aligned}
V_t^F(n) &= \max_{Z_t^F}\left\{n\pi_t^F - nZ_t^F + \frac{1}{R^F}\left[\sum_{i=0}^{n} P(i,n,i_t^F)\left(\sum_{j=0}^{n} P(j,n,d_t)(n+i-j)E_t\left(V_{t+1}^F(1)\right)\right)\right]\right\} \\
&= \max_{Z_t^F}\left\{n\pi_t^F - nZ_t^F + \frac{1}{R^F}E_t\left(V_{t+1}^F(1)\right)\left[\sum_{i=0}^{n} P(i,n,i_t^F)\sum_{j=0}^{n} P(j,n,d_t)(n+i-j)\right]\right\}.
\end{aligned}$$

Inside the brackets can be written as follows:

$$\sum_{i=0}^{n} P(i,n,i_t^F)\sum_{j=0}^{n} P(j,n,d_t)(n+i-j) = n+\sum_{i=0}^{n} P(i,n,i_t^F)i-\sum_{j=0}^{n} P(j,n,d_t)j = n+ni_t^F-nd_t.$$

Note that the last two terms are just the expected number of successes for each binomial process. Thus $V_t^F(n)$ can be written as follows:

$$\begin{aligned}
V_t^F(n) &= \max_{Z_t^F}\left\{n\pi_t^F - nZ_t^F + \frac{1}{R^F}n(1+i_t^F-d_t)E_t\left(V_{t+1}^F(1)\right)\right\} \\
&= n\max_{Z_t^F}\left\{\pi_t^F - Z_t^F + \frac{1}{R^F}(1+i_t^F-d_t)E_t\left(V_{t+1}^F(1)\right)\right\} \\
&= nV_t^F(1).
\end{aligned}$$

This verifies that the initial guess $V_t^F(n) = nV_t^F(1)$ is correct.

C.2 Domestic Firms

Similarly to foreign firms, the proof starts with the value of a domestic firm with a single product line, this time taking into account acquisition by foreign investors:

$$\begin{aligned}
V_t^D(1) &= \max_{Z_t^D}\{\pi_t^D - Z_t^D \\
&\quad + \left[\sum_{i=0}^{1} P(i,1,i_t^D)\left\{\sum_{j=0}^{1} P(j,1,d_t)\left(\sum_{k=0}^{1-j} P\left(k,1-j,\frac{e_t^F}{1-\theta_{t-1}}\right)E_t\left[\Lambda_{t,t+1}V_{t+1}^D(1+i-j-k)\right]\right)\right\}\right] \\
&\quad + \left[\sum_{j=0}^{1} P(j,1,d_t)\left(\sum_{k=0}^{1-j} P\left(k,1-j,\frac{e_t^F}{1-\theta_{t-1}}\right)kQ_t\right)\right]\Bigg\}.
\end{aligned}$$

There are now six cases next period: Whether innovation is successful or not, and whether the product line is replaced, acquired, or survives. Writing out the second line,

$$
\begin{aligned}
& \sum_{i=0}^{1} P(i,1,i_t^D)\left\{\sum_{j=0}^{1} P(j,1,d_t)\left(\sum_{k=0}^{1-j} P\left(k,1-j,\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left[\Lambda_{t,t+1}V_{t+1}^D(1+i-j-k)\right]\right)\right\} \\
= \; & P(0,1,i_t^D)P(0,1,d_t)P\left(0,1,\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right) \\
& + P(1,1,i_t^D)P(0,1,d_t)P\left(0,1,\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1}V_{t+1}^D(2)\right) \\
& + P(1,1,i_t^D)P(0,1,d_t)P\left(1,1,\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right) \\
& + P(1,1,i_t^D)P(1,1,d_t)E_t\left(\Lambda_{t,t+1}V_{t+1}^D(2)\right).
\end{aligned}
$$

Guess the linear relation $V_t^D(n) = nV_t^D(1)$. Using the linear relation $V_{t+1}^D(2) = 2V_{t+1}^D(1)$,

$$
\begin{aligned}
= \; & \left[P(0,1,i_t^D)P(0,1,d_t)P\left(0,1,\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right)\right] \\
& + \left[P(1,1,i_t^D)P(0,1,d_t)P\left(0,1,\frac{e_t^F}{1-\theta_{t-1}}\right) 2E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right)\right] \\
& + \left[P(1,1,i_t^D)P(0,1,d_t)P\left(1,1,\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right)\right] \\
& + \left[P(1,1,i_t^D)P(1,1,d_t)E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right)\right] \\
= \; & (1-i_t^D)(1-d_t)\left(1-\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right) \\
& + 2i_t^D(1-d_t)\left(1-\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right) \\
& + i_t^D(1-d_t)\frac{e_t^F}{1-\theta_{t-1}} E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right) \\
& + i_t^D d_t E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right) \\
= \; & i_t^D + (1-d_t)\left(1-\frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1}V_{t+1}^D(1)\right).
\end{aligned}
$$

Next, writing out the third line,

$$
\sum_{j=0}^{1} P(j,1,d_t)\left(\sum_{k=0}^{1-j} P\left(k,1-j,\frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t\right) = (1-d_t)\frac{e_t^F}{1-\theta_{t-1}} Q_t.
$$

Therefore $V_t^D(1)$ can be written as follows:

$$
V_t^D(1) = \max_{Z_t^D}\left\{\pi_t^D - Z_t^D + \left[i_t^D + (1-d_t)\left(1-\frac{e_t^F}{1-\theta_{t-1}}\right)\right] E_t(\Lambda_{t,t+1}V_{t+1}^D(1)) + (1-d_t)\frac{e_t^F}{1-\theta_{t-1}} Q_t\right\},
$$

which is equation (23) in the main text. Next, the value of a domestic firm with n product lines is given as:

$$
\begin{aligned}
V_t^D(n) &= \max_{Z_t^D} \Big\{ n\pi_t^D - nZ_t^D \\
&+ \left[\sum_{i=0}^{n} P(i,n,i_t^D) \left\{ \sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left[\Lambda_{t,t+1} V_{t+1}^D(n+i-j-k)\right] \right) \right\} \right] \\
&+ \left[\sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \Big\}.
\end{aligned}
$$

Using the linear relation $V_{t+1}^D(n+i-j-k) = (n+i-j-k)V_{t+1}^D(1)$,

$$
\begin{aligned}
V_t^D(n) &= \max_{Z_t^D} \Big\{ n\pi_t^D - nZ_t^D \\
&+ \left[\sum_{i=0}^{n} P(i,n,i_t^D) \left\{ \sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) E_t\left[\Lambda_{t,t+1} V_{t+1}^D(1)\right] \right) \right\} \right] \\
&+ \left[\sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \Big\} \\
&= \max_{Z_t^D} \Big\{ n\pi_t^D - nZ_t^D \\
&+ E_t\left[\Lambda_{t,t+1} V_{t+1}^D(1)\right] \left[\sum_{i=0}^{n} P(i,n,i_t^D) \sum_{j=0}^{n} P(j,n,d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) \right] \\
&+ Q_t \left[\sum_{j=0}^{n} P(j,n,d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k \right] \Big\}.
\end{aligned}
$$

The bracketed term in the second line is:

$$
\begin{aligned}
& \sum_{i=0}^{n} P(i,n,i_t^D) \sum_{j=0}^{n} P(j,n,d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) \\
&= n + ni_t^D - nd_t - \sum_{j=0}^{n} P(j,n,d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k \\
&= n + ni_t^D - nd_t - \sum_{j=0}^{n} P(j,n,d_t) \left[(n-j) \frac{e_t^F}{1-\theta_{t-1}} \right] \\
&= n + ni_t^D - nd_t - n\frac{e_t^F}{1-\theta_{t-1}} + nd_t \frac{e_t^F}{1-\theta_{t-1}} \\
&= n \left[i_t^D + (1-d_t) \left(1 - \frac{e_t^F}{1-\theta_{t-1}} \right) \right].
\end{aligned}
$$

The bracketed term in the last line is:

$$\sum_{j=0}^{n} P(j, n, d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k = n\left[(1-d_t)\frac{e_t^F}{1-\theta_{t-1}}\right].$$

Therefore $V_t^D(n)$ can be written as follows:

$$\begin{aligned} V_t^D(n) &= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\ &\quad + n\left[i_t^D + (1-d_t)\left(1 - \frac{e_t^F}{1-\theta_{t-1}}\right)\right] E_t\left[\Lambda_{t,t+1} V_{t+1}^D(1)\right] \\ &\quad \left. + n\left[(1-d_t)\frac{e_t^F}{1-\theta_{t-1}}\right] Q_t \right\} \\ &= nV_t^D(1). \end{aligned}$$

This verifies that the initial guess $V_t^D(n) = nV_t^D(1)$ is correct.

D Alternative Policy and Decomposition

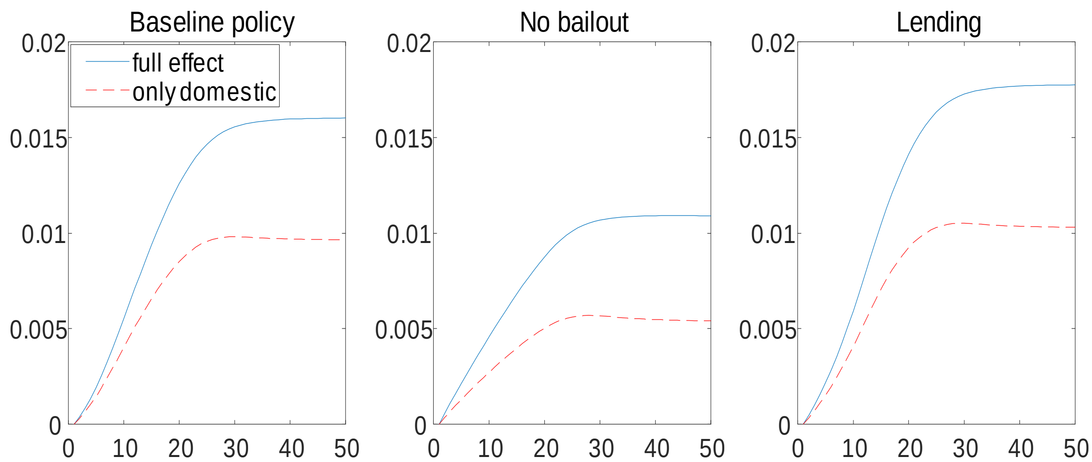
The analysis in the main text considers a reserve policy which consists of reserve accumulation in normal times and bailouts during crisis. This section studies two other different types of reserve policy schemes and decomposes the policy effect on productivity and welfare into different channels.

The first policy scheme considered in this section is a "no-bailout" scheme, in which the government accumulates reserves but never uses them for bailouts. The second one is a "lending" scheme, in which the government provides accumulated reserves to private agents to help finance working capital payments, but private agents need to repay these reserves to the government after production. In the analysis below, the FDI entry cost parameters are set to target 40.7% of the average FDI-to-GDP ratio for the 19 sample countries, which is 1.09%, and the debt-elasticity of the spread is set at the middle value $\psi_b = 0.0561$.

D.1 Policy Effect on Productivity

Figure 13 shows the log gaps in productivity with each policy scheme compared with the case without reserve policy, computed by taking the average of 100,000 stochastic simula-

Figure 13: Effect of three policy schemes on productivity

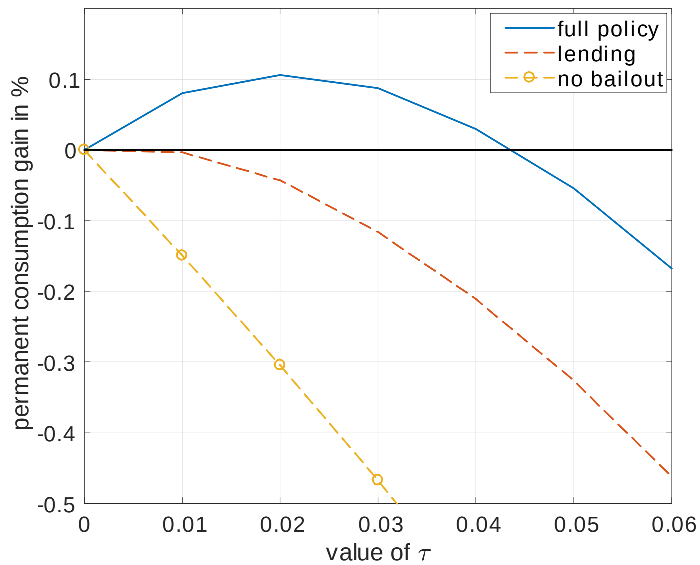


tions. The reserve accumulation pace τ is set at the average of the optimal accumulation paces across the 19 countries, which is $\tau = 0.022$. There are three observations. First, the no-bailout scheme promotes productivity growth only through the channel of reserve accumulation causing real depreciation. Therefore, the gap between the impact of the baseline policy and the no-bailout scheme is the effect of policy on growth through bailouts, including anticipation of bailouts. This implies that 68% of the growth-promoting effect of reserve policy comes from the real depreciation channel, and 32% comes from the bailout channel.

Second, productivity gain by the lending scheme is slightly higher than the baseline policy. This implies two things. First, note that the lending scheme helps finance working capital payment, but does not rebate accumulated reserves. This means that the growth-promoting effect through bailouts mainly comes from helping working capital payment and not from rebate of reserves. Second, slightly higher productivity gain by the lending scheme suggests the existence of moral-hazard borrowing by private agents. In the baseline policy, private agents anticipate that they will receive a bailout from the government when the borrowing constraint binds. This induces private agents to borrow more in normal times, which mitigates the effect of reserve accumulation on real exchange rate and a labor shift, reducing the effect on productivity growth slightly compared with the lending scheme.

Third, the red dotted lines in Figure 13 are created by fixing FDI entry and foreign innovation rates at the rates without policy. Thus, these lines show the productivity gain

Figure 14: Effect of three policy schemes on welfare



by each policy scheme only through higher domestic entry and innovation rates. The ratio of productivity gain by only domestic factors to the total gain is 60.2%, 49.5%, and 58.1% respectively from the left in Figure 13.

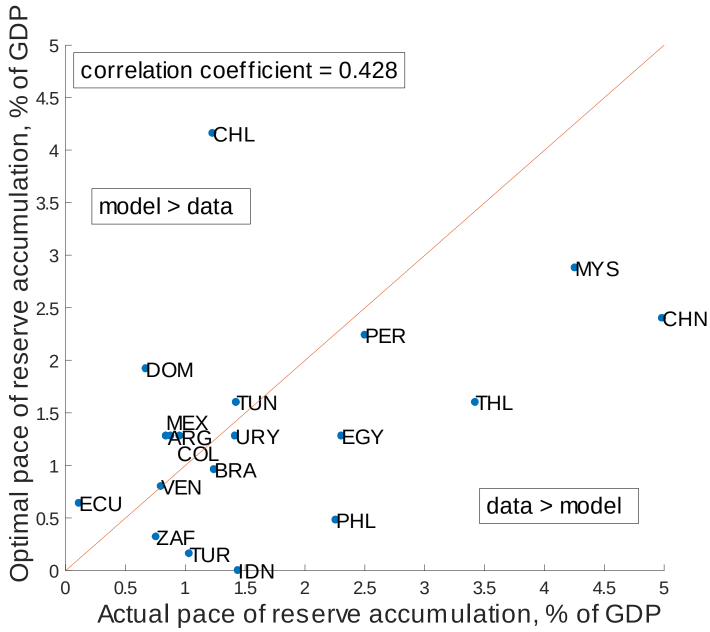
D.2 Policy Effect on Welfare

Figure 14 plots the welfare gain/loss by the above three policy schemes with accumulation paces τ from 0.01 to 0.06. The welfare gain/loss is again measured in terms of the permanent consumption gain/loss in percentage. The first observation is that the no-bailout scheme never improves welfare, and the gap in welfare between the baseline policy and the no-bailout scheme is substantial. This gap in welfare comes from whether the government uses accumulated reserves for bailout or not.

The second observation is that the welfare impact by the lending scheme is substantially better than the no-bailout scheme, although it is still negative. The difference between the lending scheme and the no-bailout scheme is whether the government helps finance working capital payment using reserves or not.

These observations imply that the welfare impact of bailouts by the government can be divided into two channels: one works through rebating reserves, which is the gap between the

Figure 15: Pace of reserve accumulation, $\psi_b = 0.0561$



baseline policy and the lending scheme, and the other works through helping working capital payment, which is the gap between the lending scheme and the no-bailout scheme. Figure 14 shows that for any τ, about 64% of the welfare impact of bailouts comes from helping working capital finance (the gap between lending and no-bailout), and the remaining 36% comes from rebating (the gap between baseline and lending.)

E Relative Importance of Two Determinants

This section studies the relative importance of the two determinants of the optimal pace of reserve accumulation by conducting the following analysis: For each country, the FDI entry cost is adjusted in the same way as the main analysis, but the debt-elasticity of the spread is constant at $\psi_b = 0.0561$ across countries. Then the optimal pace of reserve accumulation is derived for each country and compared with the observed pace.

Figure 15 plots the result. The correlation coefficient between the optimal and actual paces is 0.43 compared with 0.73 when both are adjusted. This result may suggest that the variation in the FDI entry cost is relatively more important, but the variation in the elasticity of the spread also makes a substantial contribution. Another observation is that actively reserve-accumulating countries such as Malaysia and Thailand have a much lower optimal pace compared with the main analysis and also the data. This implies that Asian countries are benefitting substantially from the low elasticity of the spread.

F Policy Evaluation with Estimated FDI Entry Cost

This section presents an alternative analysis of reserve policy evaluation. In the main text, the FDI entry cost parameters are adjusted to target the FDI inflow-to-GDP ratio for each country, and the reserve policy is evaluated. This section estimates the FDI entry cost for each country using the Starting a Business Index from the World Bank's Doing Business Surveys, and evaluates each country's pace of reserve accumulation.

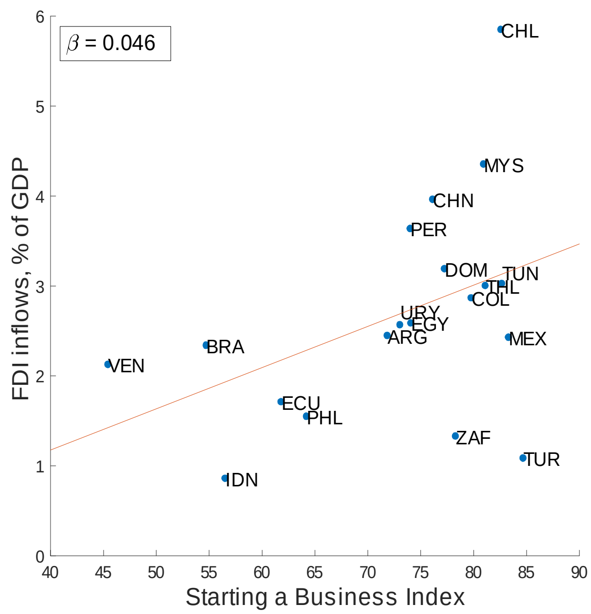
F.1 Estimation of FDI Entry Cost

The Starting a Business Index measures the effective cost of starting a new business in each country by taking into account the minimum capital requirement, number of procedures, and time and cost to start up a new business. The Index takes a value between 0 and 100, with a higher value implying lower cost to start a new business. The Index is created with the focus on domestically-owned firms, but the analysis below uses this Index as a proxy for the cost for foreign investors to start a new business.

To confirm that this Index can be used as a proxy for FDI entry cost, Figure 16 plots each country's Index on the horizontal axis and the FDI inflow-to-GDP ratio on the vertical axis. The FDI inflow-to-GDP ratio is the average for 1991-2010, and the Starting a Business Index is the average over the maximum available period for each country, which is 2004-2017 for most countries. The red line is the simple regression line whose slope is 0.046 with almost 7% significant. This slope would become more significantly positive as more developing countries are included.

Given this result, the FDI entry cost parameters are estimated using the Starting a Business Index in the following way. First, assume that the congestion cost coefficient for

Figure 16: Starting a Business Index and FDI-to-GDP ratio



FDI entry is a function of the Index:

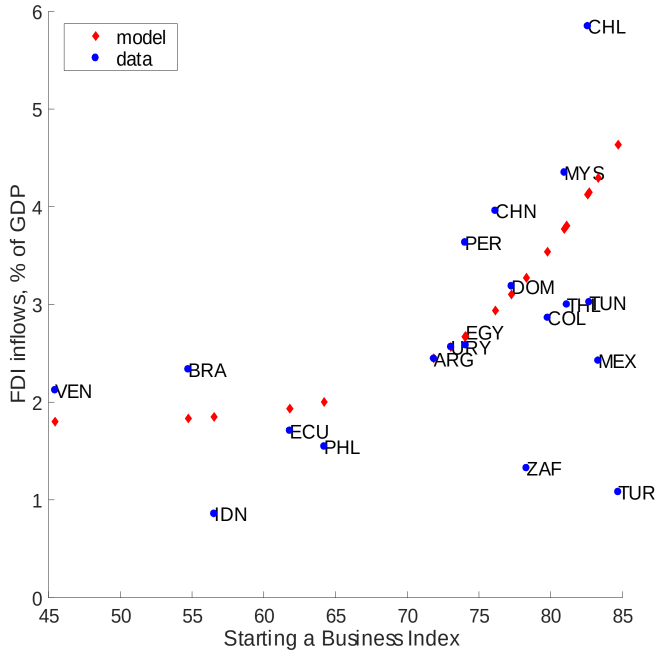
$$1/\chi_i^F = \beta_0 + \beta_1(\text{Index}_i)^{\beta_2},$$

where χ_i^F is the congestion cost coefficient for country i, and Index_i is the Starting a Business Index for the same country. The reason for taking the inverse of χ_i^F is that a higher Index implies lower cost. Then three parameters β_0, β_1, β_2 are chosen to minimize the sum of squared gaps in the FDI inflow-to-GDP ratios between the model and the data across countries. Namely,

$$\min_{\beta_0,\beta_1,\beta_2} \sum_{i=1}^{19} \left[\left(\frac{\text{FDI}}{\text{GDP}}\right)_i^{\text{data}} - \left(\frac{\text{FDI}}{\text{GDP}}\right)_i^{\text{model}} \right]^2 .$$

Following the analysis in the main text, the fixed entry cost C^F is adjusted to keep the firm value-to-fixed entry cost ratio at 1.1, and the step sizes σ^D and σ^F are adjusted to have the same long-run growth rate as in the baseline model. The result is $\beta_0 = 0.61$, $\beta_1 = 9.38$, and $\beta_2 = 10.61$. Figure 17 plots the FDI inflow-to-GDP ratios using the estimated FDI entry cost χ_i^F, along with the ratios in the data. Each country has two points, a blue circle that

Figure 17: FDI-to-GDP ratios based on estimated entry cost



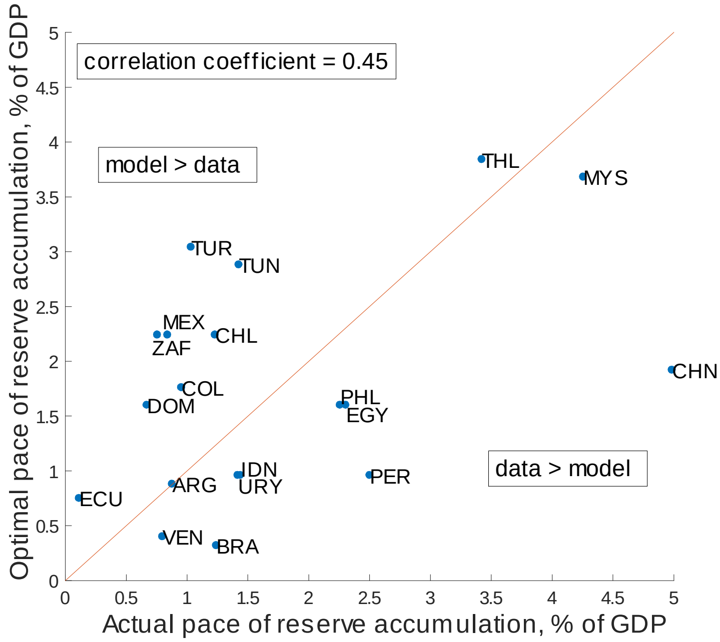
indicates the data FDI-to-GDP ratio, and a red diamond that is the FDI-to-GDP ratio based on the estimated FDI entry cost, both of which have the same Index value on the horizontal axis. Large gaps between the data points and the model points in the vertical distance imply that those countries have a high FDI-to-GDP ratio and a low Index, or the opposite.

F.2 Evaluation of Each Country's Reserve Policy

This subsection evaluates each country's reserve policy using the estimated FDI entry cost. The optimal pace of reserve accumulation and the welfare gain/loss for each country are computed in the same way as in the main text. The results are presented in Figure 18 and Table 7. Figure 18 shows that there are more gaps between the actual pace and the optimal pace compared with the analysis in the main text. But the correlation between them is still 0.45. The average of the optimal pace is 1.78% of GDP compared with 1.71% in the data.

Table 7 shows that welfare gains for most countries are close to the optimal level, suggesting that most countries are still not very far from the optimal pace. But China has an even larger gap between the optimal and actual pace, compared with the main analysis. Due

Figure 18: Pace of reserve accumulation based on estimated FDI entry cost



to this large gap, China has a large welfare loss by the actual pace of reserve accumulation. This is because the Starting a Business Index is relatively low in China, and the estimated FDI entry cost is relatively high. Another clear difference from the main analysis is Turkey, which has now a large welfare gain and the optimal pace is much faster than that in the main analysis. Figure 17 shows that Turkey has a very high Starting a Business Index, which implies a low FDI entry cost.

Table 7: Comparison of Reserve Accumulation Pace and Welfare

Country	Accum. Pace (%)		Welfare (%)		Elasticity of	Estimated
	Actual	Optimal	Actual	Optimal	Spread	FDI/GDP
Argentina	0.88	0.88	0.06	0.06	0.0730	2.45
Brazil	1.24	0.32	-0.04	0.02	0.0899	1.83
Chile	1.23	2.24	0.22	0.25	0.0899	4.12
China	4.99	1.92	-0.16	0.18	0.0392	2.94
Colombia	0.96	1.76	0.15	0.19	0.0730	3.53
Dominican Rep.	0.67	1.60	0.10	0.14	0.0730	3.10
Ecuador	0.11	0.75	0.01	0.02	0.0899	1.93
Egypt	2.31	1.60	0.12	0.14	0.0392	2.67
Indonesia	1.44	0.96	0.03	0.05	0.0392	1.85
Malaysia	4.26	3.68	0.41	0.42	0.0223	3.77
Mexico	0.84	2.24	0.18	0.28	0.0899	4.29
Peru	2.50	0.96	-0.06	0.07	0.0899	2.67
Philippines	2.26	1.60	0.09	0.10	0.0223	2.00
South Africa	0.76	2.24	0.13	0.23	0.0392	3.27
Thailand	3.42	3.84	0.43	0.43	0.0223	3.80
Tunisia	1.43	2.88	0.27	0.33	0.0561	4.15
Turkey	1.03	3.04	0.24	0.42	0.0561	4.63
Uruguay	1.42	0.96	0.04	0.06	0.0899	2.56
Venezuela	0.80	0.40	-0.01	0.01	0.0899	1.80