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Tax Ironing and Its Implications for US Business Cycles*

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Abstract

This paper investigates the macroeconomic effects of tax ironing, the misperception of the average tax rate as the marginal tax rate. Existing micro evidence indicates that about 43 percent of US taxpayers iron. Using US tax law and household microdata, I measure the tax progressivity perceived by correct perceivers and ironers and incorporate these measures into a medium-scale DSGE model with tax ironing. Based on the estimated model, I find that ironing raises steady-state economic activity by about one percent and modestly improves steady-state welfare. In contrast, its effects on business-cycle moments are tiny. Allowing for tax ironing barely improves the model's marginal likelihood.

Keywords: Tax ironing; tax progressivity; tax misperception; business cycles; Bayesian estimation

JEL Classifications: E32, E62, H24, D91

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1 Introduction

The purpose of this paper is to investigate the macroeconomic effects of tax ironing. Tax ironing is the misperception of the average tax rate as the marginal tax rate. Rees-Jones and Taubinsky (2020) estimate that about 43 percent of US taxpayers iron. Under progressive taxation, ironers perceive a lower marginal tax rate than they actually face and may consequently supply more labor, potentially affecting the aggregate economy.

Using US tax law and household microdata, I measure the tax progressivity perceived by correct perceivers and ironers. The analysis uses 14 tax-unit categories based on filing status, number of children, and, for joint filers, number of earners. I find that perceived progressivity differs between correct perceivers and ironers. This result is inconsistent with the constant residual progression (CRP) tax function commonly used in the literature.

I then develop a medium-scale DSGE model with tax ironing and the measured progressivity. The model features a representative household consisting of correct perceivers and ironers. I set the share of ironers at 43 percent, based on existing micro evidence. As in standard medium-scale DSGE models, the model includes habit persistence, investment adjustment costs, variable capital utilization, nominal price and wage stickiness with partial inflation indexation, and a range of structural shocks.

I estimate the model using mixed-frequency Bayesian methods. The data consist of US quarterly macroeconomic series from 1984Q1 to 2015Q4 and the CBO's annual series of the average marginal tax rate on labor income. During the zero lower bound period, the shadow rate replaces the federal funds rate. I find that the posterior distributions of the parameters are very similar with and without tax ironing, and that allowing for tax ironing barely improves the marginal likelihood.

Through counterfactual simulations, I find that tax ironing raises steady-state hours, output, consumption, investment, and capital by about 1.05 percent and improves steady-state welfare by 0.23 percent in consumption-equivalent terms. The welfare gain arises because ironing partly offsets the inefficiently low labor supply caused by labor income taxation and the wage markup. In contrast, its effects on business-cycle moments are tiny. The standard deviations of the observed variables change by at most 0.11 percent, while their persistence and comovement remain nearly unchanged.

The different effects on steady-state levels and business cycles arise from two properties of the perceived net-of-tax rate: its level and its income elasticity. Tax ironing raises the perceived net-of-tax rate and thereby increases steady-state labor supply. Business-cycle dynamics, in contrast, depend on perceived tax progressivity through wage setting. Although ironing changes perceived progressivity under the US tax system, its effect on aggregate wage setting is small. To first order, ironing raises the progressivity entering the aggregate wage Phillips curve by only about 0.015. These results are robust to a wider income range, an ironing share of 62 percent (the upper bound reported by Rees-Jones and Taubinsky

(2020)), and separate households for ironers and correct perceivers.

Related Literature This paper builds on micro evidence that taxpayers misperceive nonlinear tax schedules. de Bartolome (1995) provides early experimental evidence that many people treat the average tax rate as if it were the marginal rate, and Liebman and Zeckhauser (2004) define ironing as responding to the average rather than the marginal price of a nonlinear schedule. Rees-Jones and Taubinsky (2020) measure the prevalence of ironing in the US federal income tax and examine its implications for optimal tax progressivity, and Farhi and Gabaix (2020) study how behavioral misperceptions, including ironing, affect optimal taxation. These studies ask how individuals perceive taxes and how taxes should be designed. I take their perception rule and the estimated share of ironers as given and ask how ironing affects aggregate activity and business cycles under the existing schedule.

The paper is also related to studies of how taxes affect aggregate fluctuations. Braun (1994) and McGrattan (1994) study distortionary taxation and tax rate fluctuations in quantitative business cycle models. Mattesini and Rossi (2012) examine how progressive labor income taxation affects wage setting and the transmission of shocks in a New Keynesian model; in this paper, too, progressivity matters through wage setting. McKay and Reis (2016) and Jang et al. (2023) study tax and transfer systems in models with heterogeneous households, and Mertens and Montiel Olea (2018) distinguish changes in marginal and average tax rates in US time series. These studies examine the effects of taxes and transfers, whereas this paper focuses on the effects of tax-rate misperception.

The power tax schedule of Bénabou (2002) and Heathcote et al. (2017) has constant residual progression and summarizes progressivity with one parameter, which makes it convenient in macroeconomics. Rather than imposing this form, I measure perceived progressivity on actual US tax law.

Organization of the Rest of the Paper Section 2 defines and measures perceived progressivity, and Section 3 presents the model and the neutrality result under constant residual progression. Section 4 reports the estimates, the model comparison and the counterfactual, and Section 5 concludes.

2 Tax Schedules and Tax Ironing

This section defines the tax objects that enter the macroeconomic model, characterizes the schedules under which correct perceivers and ironers perceive the same progressivity, describes the measurement on the US tax schedule and explains where the measured values enter the model.

2.1 Tax Rates and Perceived After-Tax Pay

Let x be a taxpayer's labor income and $\mathcal{T}(x)$ the tax attributable to it: the taxpayer's tax liability minus the liability it would owe with no labor income, so that $\mathcal{T}(0) = 0$. The marginal and average tax rates are

$$M(x) = \mathcal{T}'(x), \qquad A(x) = \frac{\mathcal{T}(x)}{x}. \tag{1}$$

A taxpayer who perceives the schedule correctly treats $M(x)$ as its marginal tax rate and takes $1 - M(x)$ as the share of an additional dollar that it keeps. A full ironer treats the average rate $A(x)$ as its marginal rate and takes $1 - A(x)$. Following Rees-Jones and Taubinsky (2020) and Farhi and Gabaix (2020), the perceived marginal rate of a taxpayer with ironing weight $\omega \in \{0, 1\}$ is

$$M^P(x) = (1 - \omega)M(x) + \omega A(x). \tag{2}$$

Perception affects only decisions. Correct perceivers and ironers alike pay the tax $\mathcal{T}(x)$ that the actual schedule prescribes.

Two features of perceived after-tax pay must be kept apart. The first is its level at a given income: under a progressive schedule $M(x) > A(x)$, so an ironer perceives a higher retention rate than a correct perceiver at the same income. The second is its income elasticity, the percent change in perceived retention when the taxpayer's own income rises by one percent. The progressivity measures of this paper are not tax rates. They are the negatives of the income elasticities of the two net-of-tax rates, $1 - M$ and $1 - A$:

$$\tau_M(x) \equiv -\frac{d\log\big(1 - M(x)\big)}{d\log x} = \frac{x\mathcal{T}''(x)}{1 - M(x)}, \qquad \tau_A(x) \equiv -\frac{d\log\big(1 - A(x)\big)}{d\log x} = \frac{M(x) - A(x)}{1 - A(x)}. \tag{3}$$

The subscript names the tax rate whose net-of-tax rate is measured. τ_M, the progressivity of the net-of-marginal tax rate, is the percent decline of the correct perceiver's retention rate $1 - M$ when income rises by one percent. τ_A, the progressivity of the net-of-average tax rate, is the same decline for the ironer's retention rate $1 - A$, and it equals the gap between the marginal and the average rate, scaled by the net-of-average rate. Because both rates belong to the same schedule, the second equality in (3) holds at every income, and

$$\frac{1 - M(x)}{1 - A(x)} = 1 - \tau_A(x). \tag{4}$$

τ_A therefore governs both the level difference between the two perceived retention rates at a given income and the income elasticity of the ironer's rate, while τ_M governs the income elasticity of the correct perceiver's rate. In the model, the level of perceived retention determines the steady state, and its income elasticity enters the wage dynamics.

By (3), the ratio of the two perceived retention rates, $(1 - M)/(1 - A)$, changes with log income at the rate $\tau_A - \tau_M$. When $\tau_A > \tau_M$, the perceived retention rate of an ironer has a more negative income elasticity than that of a correct perceiver at the same income.

2.2 Constant Residual Progression

The ratio $(1-M)/(1-A)$ is residual income progression, the elasticity of after-tax income with respect to pre-tax income, which goes back to Musgrave and Thin (1948), so τ_A is one minus residual income progression. A schedule has constant residual progression (CRP) if after-tax income is a power function of pre-tax income, $x-\mathcal{T}(x)=\eta x^{1-\tau}$, as in Bénabou (2002) and Heathcote et al. (2017); its progressivity parameter τ equals one minus residual income progression, and the same schedule is used, for example, by Bayer et al. (2024). Under CRP, $1-M(x)=\eta(1-\tau)x^{-\tau}$ and $1-A(x)=\eta x^{-\tau}$. The two retention rates differ by the constant factor $1-\tau$, and both have the income elasticity $-\tau$, so $\tau_M=\tau_A=\tau$ at every income. Ironing then raises the level of perceived retention by the factor $1/(1-\tau)$ at every income and leaves its elasticity unchanged. The next proposition shows that only this family has the property.

Proposition 1. *Let $\mathcal{T}$ be twice differentiable on an interval of incomes on which $1-M(x)>0$ and $1-A(x)>0$. Then $\tau_M(x)=\tau_A(x)$ at every income in the interval if and only if there are $\eta>0$ and $\tau<1$ such that $x-\mathcal{T}(x)=\eta x^{1-\tau}$ on the interval. In that case $\tau_M=\tau_A=\tau$.*

Proof. By (3), $d\log\big[(1-M)/(1-A)\big]/d\log x=\tau_A-\tau_M$. Hence $\tau_M=\tau_A$ on the interval if and only if $(1-M)/(1-A)$ is constant there, which by (4) can be written as $1-\tau$. Let $R(x)=x-\mathcal{T}(x)$ be after-tax income. Then $R'=1-M$ and $R/x=1-A$, so the condition is $R'(x)=(1-\tau)R(x)/x$, whose positive solutions are $R(x)=\eta x^{1-\tau}$. Conversely, this R gives $\tau_M=\tau_A=\tau$. □

Proposition 1 concerns the schedule and a taxpayer's perception of it and does not depend on the macroeconomic model. What equal measures imply for the macroeconomy depends on how perceived retention enters the model. In the model of Section 3, the level of perceived retention determines the steady state and its income elasticity enters the wage Phillips curve, and Proposition 2 there shows that under CRP ironing changes steady state levels but not the first-order dynamics. Actual schedules are not of this form, so I measure both on the actual schedule.

2.3 Measuring Progressivity for the Macroeconomic Model

This subsection measures, for groups of tax units, the two progressivity measures that the model uses for correct perceivers and for ironers. The construction evaluates the statutory schedule on the incomes of US tax units.

The measurement uses the 2015 federal tax code and the 2015 distribution of labor income. The choice of 2015 aligns the cross-sectional measurement with the last observation of the average marginal tax rate on labor income that Congressional Budget Office (2019) computes from tax return data; in that report the rates for later years are projections. The year 2015 is therefore the common endpoint of the measured tax schedule, the income distribution and the macroeconomic sample.

The income data are the 2016 Annual Social and Economic Supplement of the Current Population Survey (CPS ASEC). As documented by U.S. Census Bureau (2016), its income questions refer to the previous calendar year, here 2015. I form tax units with the procedure of the taxdata project described by Frailey (2021), which groups the persons of each household into tax units by assigning spouses and dependents to the household reference person and to the other filers of the household. I add one rule: when a child could be claimed by more than one filer in the household, the child's parent claims the child. I keep units with wage and salary income of at least $1,000. Taxes are the federal individual income tax after credits, including the refundable earned income tax credit and child tax credit, plus employee and employer payroll taxes for Social Security and Medicare; state taxes are not included. The evidence of Rees-Jones and Taubinsky (2020) concerns the federal income tax; I apply the same perception rule to this combined schedule. I compute tax liabilities under 2015 law with version 4.3.4 of Tax-Calculator, the microsimulation model of the Policy Simulation Library (Policy Simulation Library, 2024).

The schedule a tax unit faces depends on its filing status, its number of children and, for married couples, the number of earners. I divide tax units into cells by filing status, number of children, from zero to three or more, and, for married couples filing jointly, one or two earners. The 14 cells with at least half a percent of total wage and salary income contain 99.99 percent of it, and the four excluded cells are single filers and married couples filing separately with children; this coverage of the cells differs from the coverage of the income grid within each cell, discussed below. For each cell I evaluate the statutory schedule of a representative unit, with the filing status and the median number of children of the cell, on a fine grid of wages; for two-earner couples, wages are split between the spouses in their average proportion. The income x on the horizontal axis of the schedule is labor compensation: wages and salaries plus the employer share of payroll taxes, which is half of the combined employee and employer Social Security and Medicare tax on wages. The tax attributable to labor income is the tax at a given wage minus the tax of the same unit with zero wages, and the average tax rate is this tax divided by x. The marginal tax rate is the change in tax divided by the change in x when wages rise by 0.1 percent. Self-employment income is not part of x, and other income, such as interest, dividends and capital gains, does not enter the cell schedules: they are those of units with wage income only, and a tax unit enters the income distribution below through its wage compensation alone. The end points of the grid are set from unweighted percentiles of compensation in the cell: the lower end is the larger of $2,000 and the 0.5th percentile, the upper end is the 99.5th percentile, and each is widened by ten percent. The grid itself is a log grid of wages, so the evaluated range of x lies somewhat above these amounts. Within each cell, about 2 to 8 percent of wage and salary income belongs to units with compensation above the evaluated range.

Along the smooth parts of the schedule, the measures (3) are numerical derivatives of $-\log(1-M)$

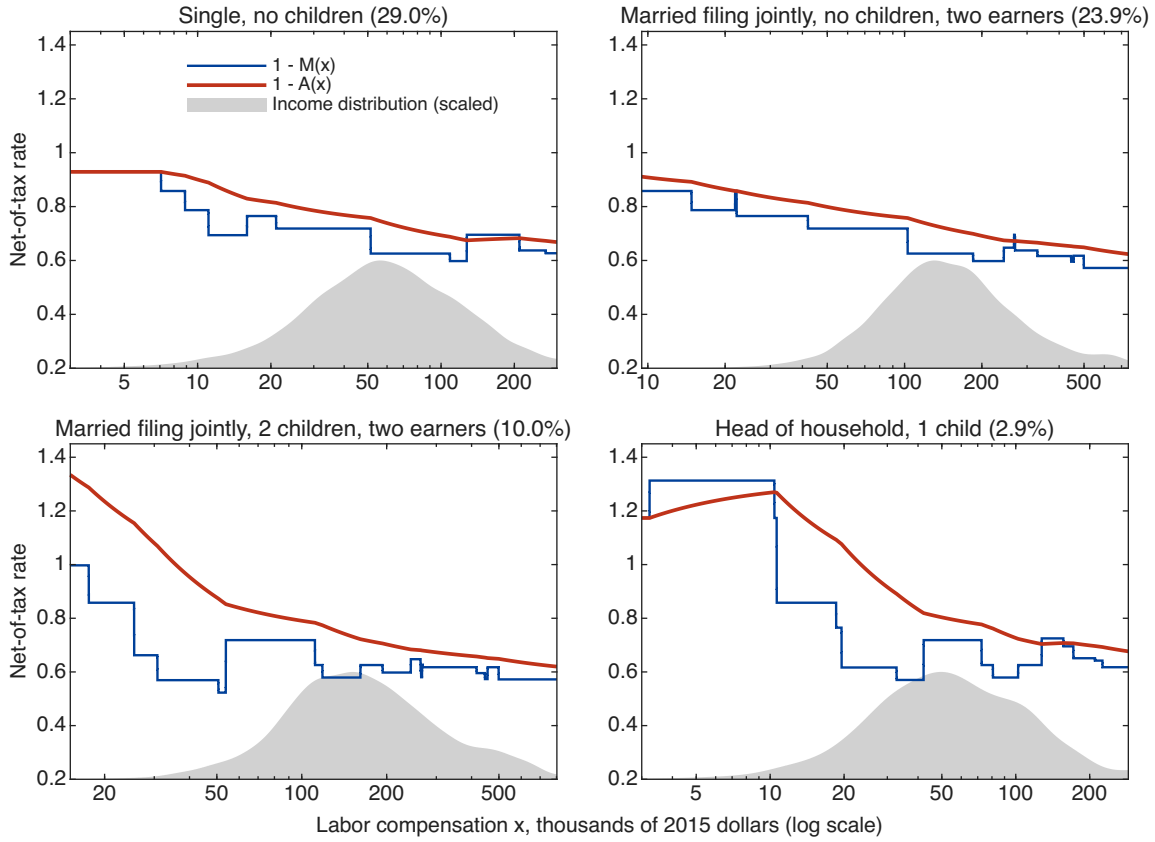


Figure 1: Evaluated 2015 Schedules for Four Cells

Notes: Representative tax unit of each cell under 2015 law. Net-of-tax rates above one reflect refundable credits. Income distribution: kernel density of $\log x$ in the cell, rescaled within each panel and not on the scale of the tax rates. In parentheses: share of the cell in the wage and salary income of the 14 cells.

and $-\log(1-A)$ with respect to $\log x$ on the grid. The statutory schedule also has breakpoints at which the marginal rate jumps, such as bracket thresholds, the phase-in and phase-out ranges of credits and the Social Security earnings cap; at a breakpoint $-\log(1-M)$ jumps, while $-\log(1-A)$ is continuous and changes slope. Almost all of τ_M comes from these jumps, and τ_A comes from the slopes. To average within each cell, I estimate the distribution of $\log x$ with a Gaussian kernel density in which each tax unit is weighted by its CPS sample weight times its wage and salary income, with the bandwidth from the rule of thumb of Silverman (1986). The slopes are integrated against this density, and each jump is weighted by the density at its breakpoint. The density is normalized over the evaluated range of x, so the measures are averages over that range rather than over the whole income distribution. Weighting by wage income makes the measures describe the progressivity faced by a dollar of labor income, as in the model, where the wages of the two perception types aggregate with their shares of labor income. The result is a pair $(\tau_{M,h}, \tau_{A,h})$ for each cell h.

Table 1 reports these cell averages. Figure 1 shows what the averages do not: the shape of the schedule

of a representative unit and where the income distribution of the cell lies on it. It plots the evaluated schedules for the three cells with the largest shares of labor income and for the head-of-household cell with the largest share. The curves are the rates of the representative unit of each cell computed with Tax-Calculator, not the rates of individual CPS units; the CPS provides the income distribution. The net-of-marginal rate $1 - M$ is a step function, constant between breakpoints and jumping at them. The net-of-average rate $1 - A$ is continuous and changes slope at the breakpoints. A fall in $1 - M$ is a rise in the marginal rate. The marginal rate need not rise with income: it falls, for example, where the phase-out range of a credit ends or above the Social Security earnings cap, so $1 - M$ can also jump up. A value of $1 - A$ above one means that the tax attributable to labor income is negative, whereas $1 - M$ above one means a negative marginal rate; for married couples with two children and two earners, it is mainly $1 - A$ that exceeds one. Childless single filers can also receive the earned income tax credit, but it is much smaller than for units with children, and in the evaluated range of that cell both rates stay below one. Because x includes the employer share of payroll taxes, the breakpoints lie at values of x somewhat above the statutory thresholds in wages. The vertical distance between the two curves is the difference in the level of perceived retention at a given income, not the difference between τ_M and τ_A.

The result of the measurement is, for each of the 14 cells, the pair $(\tau_{M,h}, \tau_{A,h})$ and the share s_h of the cell in the wage and salary income of the 14 cells, normalized to sum to one. These values are fixed inputs of the model. Section 4.1 combines them into the two measures of the model's representative schedule.

Statutory marginal rates are constant within most income intervals, so τ_M is not the elasticity that one taxpayer at a particular income faces inside such an interval. Within each cell, τ_M combines the local slopes and the jumps at breakpoints with the income-weighted density. The model combines the cell values by (35) and uses the result as the local elasticity of its representative schedule.

Heterogeneity in income and family structure enters the two values through the cell schedules and the income distributions within the cells. The macroeconomic model does not track either. The heterogeneity it keeps is the perception of the schedule, with two types, correct perceivers and ironers.

The perception rule (2), the survey data, the construction of tax units and the calculation of tax liabilities follow existing work and tools. The contribution of the measurement is to evaluate the statutory 2015 schedule over the observed income distribution separately for correct perceivers and for ironers, and to map the two measures into the wage setting of the model through (35) in Section 4.1.

2.4 Measured Values and the Model's Tax Inputs

Table 1 reports the cell values. $\tau_{A,h}$ exceeds $\tau_{M,h}$ in 13 of the 14 cells. The two measures are nearly identical for married couples filing jointly without children, regardless of whether they have one or two

Table 1: Progressivity by Cell of Tax Units, 2015

Filing status	Children	Earners	Share (%)	τ_M	τ_A
Single	0		29.0	0.0713	0.0904
Married filing jointly	0	1	9.3	0.0729	0.0747
	0	2	23.9	0.1034	0.1028
	1	1	3.1	0.0440	0.1291
	1	2	9.5	0.0840	0.1267
	2	1	3.4	0.0471	0.1602
	2	2	10.0	0.0837	0.1435
	3+	1	2.1	0.0799	0.1824
	3+	2	3.4	0.0832	0.1741
Married filing separately	0		1.0	0.0950	0.0968
Head of household	0		0.6	0.0817	0.0864
	1		2.9	0.0464	0.1893
	2		1.4	0.0851	0.2419
	3+		0.6	0.2111	0.2639
All 14 cells, income-weighted mean			100.0	0.0811	0.1150

Notes: Share: share of the cell in the wage and salary income of the 14 cells.

earners. These two cells account for 33.1 percent of the wage and salary income of the 14 cells, while the differences are generally larger for tax units with children. Weighted by the cells' shares of labor income, the means are 0.0811 for τ_M and 0.1150 for τ_A. The cell values are averages over the income distribution within each cell; they are not the elasticities of one statutory schedule at a particular income.

The model has one representative schedule. Its two progressivity measures combine the cell values with the weights that wage setting implies (Section 4.1) and are the local properties of this schedule. They are applied at a base income inside the model, the steady state labor income of correct perceivers relative to trend; the base is not matched to a dollar income in the CPS. At the base, the identity (4) gives a perceived retention rate of an ironer about 1.13 times that of a correct perceiver. This ratio describes the representative schedule of the model, not individual taxpayers or an average over the microdata. The level of the marginal rate comes from a different source. The 1984–2015 mean of the CBO series, M^* percent, is a fixed input that translates movements in the observed rate into movements of net-of-tax rates; it is not a microdata average, and it is not estimated. Section 3 makes these connections precise.

In the model the tax schedule matters through wage setting. The members of the model's representative family set their wages on the basis of their perceived marginal retention rate. The level of perceived retention determines how much they work in the steady state, and its income elasticity, $-\tau_M$ for correct perceivers and $-\tau_A$ for ironers, enters the wage Phillips curve of each type.

The estimation holds the 2015 cell values fixed over the sample. Movements in the CBO marginal tax rate series shift the level of the schedule, through the observation equation and the tax shock of the model, and leave its curvature unchanged. The two measures are built from the same CPS sample, the same cells and the same income-weighted distributions. I do not estimate separate cells or income distributions for

correct perceivers and for ironers, or how ironing is related to them. These common weights belong to the measurement. In the model, the labor incomes of the two types and the ironers' share of labor income are determined in equilibrium (Section 3).

The measured values differ, with $\tau_{A,h} > \tau_{M,h}$ in all but one cell: the retention rate that an ironer perceives falls faster with income than the one that a correct perceiver perceives. Under CRP the two would fall at the same rate. The combined measures of the model's representative schedule differ as well: at the estimated parameters, τ_A exceeds τ_M by about 0.034. By Proposition 1, this schedule does not have constant residual progression, and Proposition 2 does not apply to it. Ironing can then affect the business cycle in the model. The difference between the two measures does not by itself imply that these effects are large. Their size depends on how the measures enter wage setting and on the estimated parameters, and Section 4 evaluates it quantitatively.

3 The Macroeconomic Model

The model is a medium-scale DSGE model in the tradition of Christiano et al. (2005) and Smets and Wouters (2007), with habit formation, investment adjustment costs, variable capital utilization, staggered price and wage setting with partial indexation, and a monetary policy rule. Its distinctive features are a progressive labor income tax and the perception of it. The economy consists of a representative family, whose members supply differentiated labor services and set their own wages; a competitive labor packer; a competitive final-good firm and a continuum of monopolistically competitive intermediate-good firms; a government; and a central bank. Labor-augmenting technology Z_t grows stochastically, $Z_t/Z_{t-1} = z^* e^{z_t^z}$, where $z^* = 1 + \bar{z}/100$ is the steady state gross growth factor, $\bar{z}$ is trend growth in percent per quarter and z_t^z is a technology shock. Real variables that grow are divided by Z_t in the analysis.

Throughout, θ denotes the share of ironers among the members of the family, s_I the ironers' share of labor income, which is determined in equilibrium, and b habit persistence. The symbol τ with a subscript M, A, C, I or j denotes progressivity as defined in Section 2.1; for the two perception types, $\tau_C = \tau_M$ for correct perceivers and $\tau_I = \tau_A$ for ironers. $M(x)$ is the marginal rate of the schedule at income x and M_t the aggregate marginal tax rate. Inflation is written Π_t in the nonlinear conditions, with gross steady state value Π^*, $\hat{\pi}_t$ in the linearized system and $\bar{\pi}$ for its steady state in percent per quarter. The shocks z_t^x enter the nonlinear conditions as log deviations; in the linearized system of Appendix A they are expressed in percent, like the hatted variables.

At the beginning of each period all shocks are realized and observed. Members that may reset their wages and firms that may reset their prices do so; capital services are rented, labor is hired and production takes place; the central bank sets the nominal interest rate; and the family receives the income of its members, pays taxes and chooses consumption, investment and bond holdings. Expectations are rational

and are formed with the information of the current period. In the tax-ironing model, ironing members misperceive their marginal tax rate but not prices, wages, interest rates or the shock processes. The RE model and the tax-ironing model have the same equations except for the perception of the tax schedule. Expectations are rational in both, and each model is linearized around its own steady state. Members pool their income within the family, so the two perception types differ in wage setting and labor supply but not in consumption; the model does not include the asset and income risk heterogeneity of heterogeneous-agent New Keynesian models.

The family and its members: The household sector is one representative family with a continuum of members $h \in [0,1]$. A share θ of the members are full ironers and the rest perceive the schedule correctly; perception is a permanent trait of a member. The family has preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{z_t^b} \left[\frac{\left(C_t - b\, C_{t-1}\right)^{1-\sigma}}{1-\sigma} - Z_t^{1-\sigma} \int_0^1 \frac{\ell_t(h)^{1+\chi}}{1+\chi}\, dh \right], \tag{5}$$

where β is the discount factor, b is habit persistence, σ is the inverse intertemporal elasticity of substitution, χ is the inverse Frisch elasticity of labor supply and z_t^b is a preference shock. Scaling the disutility of work by $Z_t^{1-\sigma}$ allows a balanced growth path with stationary hours. The family pools the actual after-tax labor income of its members, and its budget constraint is

$$C_t + I_t + \frac{B_t}{P_t} = \int_0^1 \Big[W_t(h)\ell_t(h) - Z_t\, \mathcal{T}_t\big(x_t(h)\big) \Big] dh + R_t^k u_t K_{t-1} + \frac{R_{t-1}^n B_{t-1}}{P_t} + D_t + T_t^r, \tag{6}$$

with $x_t(h) \equiv W_t(h)\ell_t(h)/Z_t$, where $W_t(h)$ is the real wage of member h's labor type, $\ell_t(h)$ its hours, I_t investment, B_t nominal bonds, R_t^n the gross nominal interest rate, P_t the price level, R_t^k the real rental rate of capital services, u_t utilization, K_{t-1} the capital stock installed at the end of $t-1$, D_t dividends and T_t^r a lump-sum transfer. Each member's labor income relative to trend, $x_t(h)$, is taxed by the schedule $\mathcal{T}_t$, which keeps tax rates stationary along the balanced growth path; the family's total income is not taxed as one income.

The head of the family chooses consumption, bonds, investment, capital and utilization. Each member sets the nominal wage of its own labor type, as described below, and acts for the family: it values its perceived after-tax labor income at the family's marginal utility of wealth, not by its own consumption. Income pooling within the family and this division of decisions are assumptions of the model. They make consumption common to all members, as insurance across households does in Erceg et al. (2000). The family is an aggregate construct; it does not correspond to a tax unit of Section 2.3.

The tax schedule and its perception: The schedule actually paid is

$$x - \mathcal{T}_t(x) = e^{\nu_t}\big[x - \mathcal{T}(x)\big], \qquad 1 - M_t = e^{\nu_t}(1 - \bar{M}), \tag{7}$$

where x is a member's labor income relative to trend, $\mathcal{T}(x)$ is the representative schedule of the model and ν_t shifts after-tax income in proportion at every income. The representative schedule is not the statutory schedule. It is described by its local properties at a base income, the steady state income x_C of correct perceivers in the tax-ironing model. At x_C the net-of-marginal and the net-of-average rate have the elasticities $-\tau_M$ and $-\tau_A$, which combine the 2015 cell values of Section 2.4 by (35), and by (4) their ratio is $\big(1-M(x_C)\big)/\big(1-A(x_C)\big) = 1-\tau_A$. In the RE model the elasticity $-\tau_M$ applies at the steady state income of its members. The relative steady state level effects reported in this paper and the first-order dynamics do not require the levels $M(x_C)$ and $A(x_C)$; they use only this ratio and the two elasticities. The absolute steady state levels of hours and output do depend on the level of perceived retention, but they are not used: hours enter the estimation demeaned, and the level effects are ratios between the two models under the same schedule. The welfare comparison of Section 4.3 is the exception. It requires the absolute level of hours and therefore an additional assumption on the marginal rate at x_C.

M_t is the CBO average marginal tax rate, a fraction, with sample mean $\bar{M} = M^*/100$, where M^* is the mean in percent. Because ν_t scales every net-of-marginal rate by the same factor, it scales any average of these rates with fixed weights by that factor, which gives the second equation in (7). The level $\bar{M}$ is used only to translate a percentage point movement in M_t into the common percent change of net-of-tax rates; the model does not set the marginal rate of the representative schedule at x_C equal to $\bar{M}$. A tax shock scales the net-of-marginal and the net-of-average rate by the same factor and leaves the measures (3) unchanged. The cell values behind the two measures are held at their 2015 values over the sample, and the tax rate moves only the level of the schedule.

Ironing members have $\omega = 1$ in (2) and correct perceivers $\omega = 0$. In the RE model $\theta = 0$; in the tax-ironing model $\theta = 0.43$, a value taken from micro evidence (Section 4.1). A member sets its wage using the perceived liability $\mathcal{T}_t^P$, whose derivative is the perceived marginal rate $M_t^P(x)$. The perceived marginal retention rate of a member is $q_t^P(x) = 1 - M_t^P(x) = e^{\nu_t} q^P(x)$, with $q^P(x) = 1 - M(x)$ for a correct perceiver and $q^P(x) = 1 - A(x)$ for an ironer. The family's other decisions do not involve the tax rate: once wages are set, labor income enters the conditions for consumption, saving, investment and utilization as a lump sum. Perception therefore affects only the choice of the wage. The tax actually paid, the budget constraint (6), the government budget and the resource constraint follow the actual schedule.

Let Λ_t be the marginal utility of wealth. The conditions for consumption and bonds are

$$\Lambda_t = e^{z_t^b}\big(C_t - bC_{t-1}\big)^{-\sigma} - \beta b\,\mathbb{E}_t\Big[e^{z_{t+1}^b}\big(C_{t+1} - bC_t\big)^{-\sigma}\Big], \tag{8}$$

$$\Lambda_t = \beta\,\mathbb{E}_t\Big[\Lambda_{t+1}\frac{R_t^n}{\Pi_{t+1}}\Big], \tag{9}$$

where $\Pi_t = P_t/P_{t-1}$ is gross inflation.

Labor aggregation and wage setting: A competitive packer combines the labor services of all types, $\ell_t = \left[\int_0^1 \ell_t(h)^{1/(1+\lambda_w)} dh\right]^{1+\lambda_w}$, where λ_w is the steady state net wage markup. The demand for type h and the wage index are

$$\ell_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\varepsilon_w} \ell_t, \qquad W_t = \left[\int_0^1 W_t(h)^{1-\varepsilon_w} dh\right]^{\frac{1}{1-\varepsilon_w}}, \qquad \varepsilon_w = \frac{1+\lambda_w}{\lambda_w}. \tag{10}$$

For ironers ($j = I$, measure θ) and correct perceivers ($j = C$, measure $1-\theta$), define type wage indexes $W_{j,t}$ and type labor composites $\ell_{j,t} = (W_{j,t}/W_t)^{-\varepsilon_w}\ell_t$. Then

$$W_t^{1-\varepsilon_w} = \theta\, W_{I,t}^{1-\varepsilon_w} + (1-\theta)\, W_{C,t}^{1-\varepsilon_w}, \qquad W_t\ell_t = \theta\, W_{I,t}\ell_{I,t} + (1-\theta)\, W_{C,t}\ell_{C,t}, \tag{11}$$

and the ironers' share of labor income in the steady state is $s_I \equiv \theta W_I \ell_I/(W\ell)$. The share s_I is an equilibrium outcome and differs from the population share θ.

Member h supplies the labor service of type h and sets its nominal wage, as in Erceg et al. (2000). Let $j \in \{I, C\}$ be its perception type. Each period it can reset its wage with probability $1-\xi_w$. A wage that is not reset is indexed to trend growth and to a geometric average of past and steady state inflation, $\mathcal{X}_{t,k} = \prod_{i=1}^{k} z^* \Pi_{t+i-1}^{\gamma_w}(\Pi^*)^{1-\gamma_w}$, where $\Pi^* = 1 + \bar{\pi}/100$. The member takes the family's marginal utility of wealth Λ_t as given. When it can reset, the member chooses the wage $W_t^{o,n}(h)$ to maximize its perceived after-tax labor income, valued at the family's marginal utility of wealth, net of its disutility of work, over the periods in which the wage remains in force:

$$\max_{W_t^{o,n}(h)} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\xi_w)^k \left[\Lambda_{t+k} Z_{t+k}\left(x_{t+k}(h) - \mathcal{T}_{t+k}^P\left(x_{t+k}(h)\right)\right) - \frac{1+\lambda_{j,t+k}^w}{1+\lambda_w}\, e^{z_{t+k}^b} Z_{t+k}^{1-\sigma} \frac{\ell_{t+k}(h)^{1+\chi}}{1+\chi}\right], \tag{12}$$

where $\ell_{t+k}(h)$ follows (10). Without the factor $(1+\lambda_{j,t}^w)/(1+\lambda_w)$, (12) collects the terms of (5), subject to (6) and (10), that depend on the reset wage, with labor income evaluated after the perceived liability $\mathcal{T}^P$ rather than the actual one. The factor is a wage markup wedge. It equals one in the steady state and affects only wage setting: it does not enter the budget constraint, the resource constraint or the flexible-wage economy. With the constant demand elasticity ε_w, it makes $1+\lambda_{j,t}^w$ the desired gross markup of the perceived after-tax real wage over the marginal rate of substitution between leisure and consumption. The first-order condition is

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\xi_w)^k\, \ell_{t+k}(h)\Lambda_{t+k} \left[q_{t+k}^P\left(x_{t+k}(h)\right) \frac{W_t^{o,n}(h)\mathcal{X}_{t,k}}{P_{t+k}} - (1+\lambda_{j,t+k}^w) \frac{e^{z_{t+k}^b} Z_{t+k}^{1-\sigma} \ell_{t+k}(h)^{\chi}}{\Lambda_{t+k}}\right] = 0. \tag{13}$$

The perceived retention rate multiplies the reset wage and is evaluated at the member's own labor income at every future date. Through this term the level of perceived retention moves the wage the member sets, and its income elasticity determines how the reset wage responds to the member's own hours.

The desired markup is driven by the wage markup shock z_t^w, which is common to all members:

$$\log \frac{1+\lambda_{j,t}^w}{1+\lambda_w} = \frac{z_t^w}{\kappa_w(\tau_j)}, \qquad \kappa_w(\tau) = \frac{K_w}{\Gamma_w + \tau}, \qquad K_w = \frac{(1-\xi_w)(1-\xi_w z^*/r^*)\lambda_w}{\xi_w}, \tag{14}$$

where $\Gamma_w = \Gamma_w(\chi) = \lambda_w + \chi(1+\lambda_w)$, $r^* = (z^*)^\sigma/\beta$ is the steady state gross real interest rate, and τ_j is the progressivity perceived by type j, $\tau_C = \tau_M$ and $\tau_I = \tau_A$. The coefficient $\kappa_w(\tau_j)$ is the slope of the wage Phillips curve of type j that follows from (13). A higher τ flattens the curve, because a member who raises its wage lowers its labor income and thereby raises its perceived retention. How much it flattens the curve depends on χ through $\Gamma_w(\chi)$, and Section 4.1 uses this relation to combine the measured cell values. The loading $1/\kappa_w(\tau_j)$ measures the shock in units of its effect on that curve: z_t^w shifts the wage Phillips curve of each type by the same amount, and the desired markup moves more for the type whose curve is flatter. A common movement in the desired markup of both types would instead shift the two curves in proportion to their slopes. In the RE model all members have $\tau_j = \tau_M$.

In the steady state, with $w_j = W_j/Z$ and $\lambda = \Lambda Z^\sigma$, condition (13) reduces to

$$q_j\, w_j\, \lambda = (1+\lambda_w)\, \ell_j^{\chi}, \qquad q_I = 1 - A(x_I), \qquad q_C = 1 - M(x_C). \tag{15}$$

Labor demand gives $w_j = w(\ell_j/\ell)^{-1/\varepsilon_w}$, so labor incomes satisfy $x_I/x_C = (q_I/q_C)^{k_I}$ with $k_I = (\varepsilon_w - 1)/(\varepsilon_w\chi + 1)$. To express q_I/q_C, I use the local representation of the schedule at the base income x_C, where $q_C/\big(1-A(x_C)\big) = 1-\tau_A$. For the ironers' income x_I the model uses the first-order approximation $\log\big(1-A(x_I)\big) \approx \log\big(1-A(x_C)\big) - \tau_A \log(x_I/x_C)$, so that

$$\log\frac{q_I}{q_C} \approx -\log(1-\tau_A) - \tau_A \log\frac{x_I}{x_C}. \tag{16}$$

With $x_I/x_C = (q_I/q_C)^{k_I}$, the log relative labor income $u_I = \log(x_I/x_C)$ and the ironers' share of labor income are

$$u_I = \frac{-k_I \log(1-\tau_A)}{1 + k_I\tau_A}, \qquad s_I = \frac{\theta\, e^{u_I}}{\theta\, e^{u_I} + 1 - \theta}. \tag{17}$$

These closed forms define the steady state of the model, and Dynare evaluates them for every set of parameter values. They are exact if the net-of-average rate has the constant elasticity $-\tau_A$ over the interval between x_C and x_I. For one smooth schedule, a constant elasticity of $1 - A$ on an interval implies $\tau_M = \tau_A$ there by (4), which is the case of constant residual progression discussed in Section 2.2. With the measured values, which differ, (16) and (17) are first-order approximations in the income difference between the two types. This difference is small: at the posterior mean of the tax-ironing model, ironers earn 2.46 percent more than correct perceivers. Because $\tau_A > 0$, ironers perceive a higher return to work, supply more hours at a slightly lower wage and earn a higher labor income, so $s_I > \theta$.

Firms: A competitive firm produces final goods from intermediate goods,

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\lambda_t^p}}\, df\right]^{1+\lambda_t^p}, \tag{18}$$

where λ_t^p is the net price markup with steady state value λ_p; its movements are driven by the price markup shock defined in (22) below. Profit maximization of the final-good firm gives the demand for

good f, $Y_t(f) = \left[P_t(f)/P_t\right]^{-(1+\lambda_t^p)/\lambda_t^p} Y_t$, where $P_t(f)$ is the price of good f. Intermediate-good firm f produces with

$$Y_t(f) = K_t^S(f)^\alpha \left(Z_t \ell_t(f)\right)^{1-\alpha} - \Phi_t, \qquad \Phi_t = \phi\, \bar{y}\, Z_t, \tag{19}$$

where $K_t^S(f)$ are capital services and α is the elasticity of gross production $F_t(f) = K_t^S(f)^\alpha \left(Z_t \ell_t(f)\right)^{1-\alpha}$ with respect to capital services. Output $Y_t(f)$ is gross production net of the fixed cost Φ_t, which is a share ϕ of the economy's own steady state detrended output $\bar{y}$. Each firm rents capital services and hires the labor composite at the common rental rate and wage, so all firms choose the same ratio of capital services to labor. Cost minimization then gives, in terms of aggregates,

$$\frac{W_t\, \ell_t}{R_t^k\, u_t K_{t-1}} = \frac{1-\alpha}{\alpha}, \qquad mc_t = \left(\frac{W_t}{(1-\alpha) Z_t}\right)^{1-\alpha} \left(\frac{R_t^k}{\alpha}\right)^\alpha. \tag{20}$$

The labor tax is levied on the members' labor income, so the firm's labor cost is the pre-tax wage. As in Calvo (1983), a firm resets its price with probability $1 - \xi_p$, and a price that is not reset is indexed to $\Pi_{t-1}^{\gamma_p} (\Pi^*)^{1-\gamma_p}$. The family owns the firms, so a firm values future real profits with the family's stochastic discount factor $\beta^k \Lambda_{t+k}/\Lambda_t$. A firm that resets its price in period t chooses P_t^o to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \xi_p)^k \frac{\Lambda_{t+k}}{\Lambda_t} \left[\frac{P_t^o}{P_{t+k}} \prod_{i=1}^{k} \Pi_{t+i-1}^{\gamma_p} (\Pi^*)^{1-\gamma_p} - mc_{t+k} \right] Y_{t+k}(f),$$

where $Y_{t+k}(f)$ is the demand for its good at the indexed price. Real profits also subtract $mc_{t+k} \Phi_{t+k}$, which does not depend on the price and is omitted. The first-order condition is

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \xi_p)^k \frac{\Lambda_{t+k} Y_{t+k}(f)}{\lambda_{t+k}^p} \left[\frac{P_t^o}{P_{t+k}} \prod_{i=1}^{k} \Pi_{t+i-1}^{\gamma_p} (\Pi^*)^{1-\gamma_p} - (1 + \lambda_{t+k}^p)\, mc_{t+k} \right] = 0. \tag{21}$$

In the steady state real marginal cost is $mc = 1/(1+\lambda_p)$. The price markup shock z_t^p moves the markup according to

$$\log \frac{1+\lambda_t^p}{1+\lambda_p} = \frac{z_t^p}{\kappa_p}, \qquad \kappa_p = \frac{(1-\xi_p)(1-\xi_p z^*/r^*)}{\xi_p}, \tag{22}$$

where κ_p is the slope of the price Phillips curve that follows from (21), so z_t^p is measured in units of its effect on that curve.

In the steady state aggregate gross production is $F = (1+\phi)Y$ and factor payments are $mc\, F$, so profits net of the fixed cost, which the family receives as dividends D, are, as a share of output,

$$\frac{D}{Y} = 1 - \frac{1+\phi}{1+\lambda_p}. \tag{23}$$

I impose zero profits in the steady state, which requires

$$\phi = \lambda_p. \tag{24}$$

The fixed cost then absorbs the revenue from the steady state markup. The markup itself is not zero: λ_p is the net price markup, it is estimated, and price exceeds marginal cost. Profits are zero only in the

steady state; along the business cycle, marginal cost, the markup and output move, and profits are not zero period by period. With (24), the payments to labor and capital are the shares $1-\alpha$ and α of output. The fixed-cost ratio ϕ is not a separate parameter. It equals λ_p for any parameter values and has no prior or estimate of its own.

Capital, investment and utilization: Capital accumulates according to

$$K_t = \big[1-\delta(u_t)\big]K_{t-1} + \left[1 - S\left(\frac{e^{z_t^i} I_t}{z^* I_{t-1}}\right)\right] I_t, \qquad S(x) = \frac{\zeta}{2}(x-1)^2, \tag{25}$$

where z_t^i is an investment efficiency shock and ζ is the curvature of the adjustment cost. Higher utilization raises depreciation: $\delta(u)$ is increasing and convex with $\delta(1) = \delta$, and $\mu \equiv \delta'(1)/\delta''(1)$. Utilization is paid for through faster depreciation of the installed stock rather than through a cost in goods. With Tobin's q denoted Q_t and $X_t = e^{z_t^i} I_t/(z^* I_{t-1})$, the conditions for utilization, investment and capital are

$$R_t^k = Q_t\, \delta'(u_t), \tag{26}$$

$$1 = Q_t\big[1 - S(X_t) - S'(X_t)X_t\big] + \beta\, \mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} S'(X_{t+1}) X_{t+1} \frac{I_{t+1}}{I_t}\right], \tag{27}$$

$$Q_t = \beta\, \mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\Big(R_{t+1}^k u_{t+1} + Q_{t+1}\big[1-\delta(u_{t+1})\big]\Big)\right]. \tag{28}$$

Monetary and fiscal policy: The central bank sets the gross nominal interest rate according to

$$\frac{R_t^n}{\bar{R}^n} = \left(\frac{R_{t-1}^n}{\bar{R}^n}\right)^{\phi_r} \left[\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\phi_y}\right]^{1-\phi_r} \left(\frac{Y_t/Y_t^f}{Y_{t-1}/Y_{t-1}^f}\right)^{\phi_{\Delta y}} e^{z_t^r}, \tag{29}$$

where $\bar{R}^n = r^*\,\Pi^*$ is the steady state nominal rate, r^* is the steady state gross real interest rate, Y_t^f is output in the flexible-price, flexible-wage economy defined below, and z_t^r is a monetary policy shock. The rule follows Smets and Wouters (2007). The parameter ϕ_r governs interest rate smoothing. The central bank responds to current inflation with ϕ_π and to the output gap Y_t/Y_t^f with ϕ_y, and it also responds to the change in the output gap with $\phi_{\Delta y}$; this last response is not scaled by $1-\phi_r$.

Exogenous spending, which corresponds to government purchases plus net exports in the data, is $G_t = g_y\, \bar{y}\, Z_t\, e^{z_t^g}$, where z_t^g is an exogenous spending shock. Exogenous spending and the fixed cost are proportional to the economy's own steady state output. Labor tax revenue is the liability actually paid, and a lump-sum transfer balances the budget each period, $T_t^r = \int_0^1 Z_t \mathcal{T}_t(x_t(h))\, dh - G_t$. There is no government debt; bonds are private claims in zero net supply.

Shocks: The model has seven structural shocks, common to all members. The technology shock z_t^z moves the growth rate of labor-augmenting technology. The preference shock z_t^b scales period utility in (5). The exogenous spending shock z_t^g moves G_t. The investment efficiency shock z_t^i shifts the adjustment

cost in (25). The price markup shock z_t^p moves the desired price markup through (22). The wage markup shock z_t^w moves the desired wage markup of each perception type through (14); it is the only shock with a type-specific loading, $1/\kappa_w(\tau_j)$. The monetary policy shock z_t^r enters the rule (29). Each follows a first-order autoregression,

$$z_t^x = \rho_x \, z_{t-1}^x + \sigma_x \, \varepsilon_t^x, \qquad x \in \{z, b, g, i, p, w, r\}. \tag{30}$$

The aggregate marginal tax rate M_t is exogenous and moves the schedule through ν_t in (7). Its deviation from the sample mean in percentage points, $\hat{M}_t = 100(M_t - \bar{M})$, follows $\hat{M}_t = \rho_M \hat{M}_{t-1} + \sigma_M \varepsilon_t^M$. The eight innovations are independent standard normal.

Equilibrium and the flexible economy: An equilibrium consists of allocations and prices that satisfy the family's conditions (8)–(9) and (26)–(28), wage setting (13) with the markup (14) for each perception type together with (10)–(11), the firms' conditions (20)–(22), the policy rules and shock processes, and market clearing,

$$C_t + I_t + G_t = Y_t, \qquad Y_t \, \Delta_t^p = \left(u_t K_{t-1}\right)^{\alpha} \left(Z_t \ell_t\right)^{1-\alpha} - \Phi_t, \qquad B_t = 0, \tag{31}$$

where $\Delta_t^p = \int_0^1 \left[P_t(f)/P_t\right]^{-(1+\lambda_t^p)/\lambda_t^p} df$ is price dispersion, equal to one in the steady state and to first order. The second condition aggregates production over the intermediate-good firms with the clearing conditions for capital services and labor, $u_t K_{t-1} = \int_0^1 K_t^S(f)\, df$ and $\ell_t = \int_0^1 \ell_t(f)\, df$. The output gap in (29) is measured relative to a flexible-price, flexible-wage economy with the same parameters, the same tax schedule and the same perception regime, but without nominal rigidities and without markup shocks.

Steady state and linearization: In the steady state the Euler equation gives the gross real interest rate $r^* = (z^*)^{\sigma}/\beta$, so the real interest rate $\bar{r} = 100(r^* - 1)$, in percent per quarter, is determined by β, σ and trend growth. The capital conditions give the rental rate $\bar{r}^k = r^* - 1 + \delta$. Price setting gives $mc = 1/(1 + \lambda_p)$, and cost minimization then fixes the real wage and the capital-labor ratio. With zero profits (24), capital income is the share α of output, so the capital-output ratio is $k/y = \alpha z^*/\bar{r}^k$, and $i/y = \left(1 - (1-\delta)/z^*\right)k/y$ and $c/y = 1 - i/y - g_y$; none of these ratios depends on λ_p. Because exogenous spending and the fixed cost are proportional to output, the great ratios c/y, i/y and k/y do not depend on labor supply, and output, consumption, investment and capital are proportional to hours. In this model the real wage and the great ratios are therefore the same in the RE and the tax-ironing model, and all quantities move in proportion to hours.

At the same parameter values, ironing raises hours. With the schedule held fixed in levels, the log ratio of hours in the tax-ironing model to hours in the RE model is

$$\log \frac{\ell^{\mathrm{TI}}}{\ell^{\mathrm{RE}}} = \Upsilon \, \frac{\chi + 1/\varepsilon_w + \tau_M(1 - 1/\varepsilon_w)}{\sigma + \chi + \tau_M}, \qquad \Upsilon = \frac{\varepsilon_w}{\varepsilon_w - 1} \log\left(\theta e^{u_I} + 1 - \theta\right), \tag{32}$$

which is derived in the same local representation as (17), with perceived retention log-linear in income; for the measured schedule it is also a first-order approximation. Ironers work more, so output and consumption rise and the common marginal utility of wealth falls; correct perceivers respond to this income effect by working slightly less, and aggregate hours rise.

I log-linearize each model around its own steady state. Appendix A presents the complete linearized system. A hat denotes 100 times the log deviation of a variable from its steady state, except for the tax rate: $\hat{M}_t$ is the deviation of the marginal tax rate from its mean in percentage points, and $\hat{M}_t/(1-\bar{M}) = -100\,\nu_t$, in percent, is to first order the corresponding percent decline of every net-of-tax rate and of after-tax income at every income: a rise in the marginal tax rate by one percentage point lowers them by $1/(1-\bar{M})$ percent.

Progressivity enters the linearized wage equations through the perceived retention rate. To first order around the steady state income x_j of type j,

$$\log q_t^P(x) \approx \log q_j + \nu_t - \tau_j \log \frac{x}{x_j}, \tag{33}$$

because τ_j is the negative of the income elasticity of the retention rate that type j perceives. Labor income relative to trend is the real wage relative to trend times hours, so in percent deviations $\hat{x} = \hat{w} + \hat{\ell}$, and the perceived after-tax return to work, $\hat{q}^P + \hat{w}$, equals $(1-\tau_j)\hat{w} - \tau_j\hat{\ell} - \frac{\hat{M}_t}{1-\bar{M}}$. Setting it against the marginal rate of substitution, $\chi\hat{\ell} - \hat{\lambda} + z_t^b$, gives the bracket of the wage Phillips curve of each type in Appendix A, with the coefficient $1-\tau_j$ on the wage and $\chi+\tau_j$ on hours; with flexible wages the bracket is zero. What is held fixed is the coefficient τ_j of this first-order approximation, not the marginal tax rate, which varies with income along the schedule. Under a proportional tax $\mathcal{T}(x) = tx$, $M(x) = A(x) = t$ and the retention rate $1-t$ does not vary with income, so $\tau_M = \tau_A = 0$ for any $t > 0$ and the coefficients are 1 and χ; the rate t affects only the level of retention. Under CRP, $\tau_M = \tau_A = \tau > 0$, so equal measures do not imply a proportional tax. The difference between τ_M and τ_A is the difference between the income elasticities of retention perceived by the two types.

The shocks z_t^x enter the linearized system in percent. Because of the loadings in (22) and (14), the price markup shock enters the price Phillips curve with a unit coefficient, and the wage markup shock enters the wage Phillips curve of each type with a unit coefficient.

Constant residual progression in the model: Perception enters the dynamics through wage setting. In the linearized wage Phillips curve of each type, (A.16), the perceived progressivity of the type, τ_A for ironers and τ_M for correct perceivers, enters twice. It raises the target wage, because a higher income lowers perceived retention, and it lowers the slope of the curve, because a member that raises its wage lowers its own labor income and raises its perceived retention. Type wages aggregate with the labor income share s_I. When the two types perceive the same progressivity, which by Proposition 1 is the case

under CRP, the following holds.

Proposition 2. *Suppose that $\tau_M = \tau_A$ and that, at the same parameter values and with the same shock processes, including the tax shock in (7) that moves after-tax income in proportion at every income, the linearized equilibrium of each model has a unique stable solution. Then the first-order dynamics of the tax-ironing model around its own steady state coincide with those of the RE model around its own steady state, and every moment of the observables, measured around each model's own steady state, is the same in the two models. The steady state levels of hours, output, consumption, investment and capital are higher in the tax-ironing model whenever $\theta > 0$ and $\tau_A > 0$.*

Proof. With $\tau_I = \tau_C$, the wage Phillips curves (A.16) of the two types and their flexible-wage conditions (A.29) have the same coefficients. Then $\hat{w}_{I,t} = \hat{w}_{C,t} = \hat{w}_t$ and $\hat{\ell}_{I,t} = \hat{\ell}_{C,t} = \hat{\ell}_t$ satisfy the wage block of the tax-ironing model, which coincides with the wage block of the RE model. All other equations and their coefficients are the same in the two models, because the steady state ratios of consumption, investment and capital to output, c/y, i/y and k/y, and the steady state real wage do not depend on perception. Since the stable solution is unique, it is this one. The level result follows from (17) and (32) with $u_I > 0$. □

Proposition 2 concerns the first-order approximation, the shocks of the model and a tax shock that moves after-tax income in proportion at every income; it does not apply to reforms that change the progressivity of the schedule. Within this model, under CRP, ironing affects levels and not relative fluctuations. The proposition uses the definitions of Section 2.1 and the model, not the data or the estimates. The measured values of Section 2.4 differ, so it does not apply to the estimated model, and Section 4 evaluates the effects of ironing quantitatively.

4 Empirical Results

4.1 Data and Estimation Strategy

I estimate the model on eight quarterly US series for 1984Q1–2015Q4, 128 quarters. Output, consumption and investment are nominal GDP, personal consumption expenditures and gross private domestic investment from the national accounts of the Bureau of Economic Analysis, divided by the GDP deflator and the civilian labor force. The real wage is nonfarm business hourly compensation divided by the GDP deflator. Hours are employment times average weekly hours in the nonfarm business sector divided by the labor force, in logs and demeaned over the sample. Inflation is the quarterly log change of the GDP deflator. The nominal interest rate is the effective federal funds rate, a quarterly average divided by four; from 2008Q4 the series is the shadow rate of Wu and Xia (2016). The labor market series and the in-

terest rate are from FRED. The tax rate is the economywide average marginal tax rate on labor income of Congressional Budget Office (2019). It combines the federal individual income tax and payroll taxes and weights the marginal rates of taxpayers by their labor income, which makes it a suitable observable for the aggregate labor supply and wage setting of the model. It is one value per calendar year, placed in the fourth quarter; the other three quarters are treated as missing, and the Kalman filter skips them. The estimation sample ends in 2015Q4 because 2015 is the last year for which Congressional Budget Office (2019) computes the rate from tax return data; the values that the report gives for later years are projections. The same year is the base year of the tax law and of the CPS income distribution with which Section 2.3 measures progressivity, so the measurement and the macroeconomic sample end in the same year.

The observation equations are

$$\begin{bmatrix} 100\Delta \log Y_t \\ 100\Delta \log C_t \\ 100\Delta \log I_t \\ 100\Delta \log W_t \\ 100 \log \ell_t \\ 100\Delta \log P_t \\ 100 \log R_t^n \\ M_t^{\text{obs}} \end{bmatrix} = \begin{bmatrix} \bar{z} \\ \bar{z} \\ \bar{z} \\ \bar{z} \\ \bar{\ell} \\ \bar{\pi} \\ \bar{r} + \bar{\pi} \\ M^* \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + z_t^z \\ \hat{c}_t - \hat{c}_{t-1} + z_t^z \\ \hat{\imath}_t - \hat{\imath}_{t-1} + z_t^z \\ \hat{w}_t - \hat{w}_{t-1} + z_t^z \\ \hat{\ell}_t \\ \hat{\pi}_t \\ \hat{r}_t^n \\ \frac{1}{4}\left(\hat{M}_t + \hat{M}_{t-1} + \hat{M}_{t-2} + \hat{M}_{t-3}\right) \end{bmatrix}, \tag{34}$$

where the left side is in percent. The constants $\bar{z}$, $\bar{\pi}$ and $\bar{\ell}$ are estimated: $\bar{z}$ is trend growth and $\bar{\pi}$ steady state inflation, both in percent per quarter, and $\bar{\ell}$ is a free constant for the demeaned hours series, not the steady state hours of the model. The real interest rate $\bar{r}$, in percent per quarter, is not estimated separately; it follows from $\bar{z}$, β and σ through the gross steady state values used in the model, $z^* = 1 + \bar{z}/100$, $r^* = (z^*)^\sigma/\beta$ and $\bar{r} = 100(r^* - 1)$, and $\Pi^* = 1 + \bar{\pi}/100$. The tax observation is the annual rate, the average of the four quarterly marginal rates of the calendar year, and is used only in fourth quarters; it enters in percentage points with no perception factor. As described in Section 2.4, the cell values behind τ_M and τ_A are held at their 2015 values over the sample, and the CBO rate moves only the level of the schedule. The comparison of the two models is conditional on this specification.

Table 2 lists the parameters that are fixed in both models. The depreciation rate and the wage markup take conventional values. The capital share is estimated, and the fixed-cost ratio is not in the table because zero profits (24) set $\phi = \lambda_p$. The ratio of government purchases plus net exports to GDP, which is the steady state share g_y of exogenous spending in the model, and the mean marginal tax rate are sample means. The progressivity and the income shares of the 14 cells are the 2015 values of Table 1, and the population share of full ironers is the estimate of Rees-Jones and Taubinsky (2020) from micro data on

Table 2: Fixed Parameters

Symbol	Description	Value	Source or basis
Panel A. Technology, preferences and fiscal policy			
δ	Depreciation rate	0.025	Conventional value
λ_w	Net wage markup	0.20	Conventional value
g_y	Exogenous spending share of GDP	0.1666	Sample mean, 1984Q1–2015Q4
Panel B. Tax schedule			
$\tau_{j,h}, s_h$	Cell progressivity and income share	Table 1	2015 law, 2016 CPS ASEC
M^*	Mean marginal tax rate (%)	28.38	Sample mean, Congressional Budget Office (2019)
Panel C. Perception			
θ	Population share of ironers	0.43	Rees-Jones and Taubinsky (2020)

tax perception; I do not estimate it from the macroeconomic series. The ironers' share of labor income s_I is not a parameter; it follows from (17) given the other parameters, and it is 43.60 percent at the posterior mean of the parameters of the tax-ironing model.

The model uses one value of each progressivity measure, so the cell values are combined. The cells are a device of the measurement: the model does not contain 14 groups of households or 14 wage Phillips curves, only the two perception types, and the combination yields the two measures of its representative schedule. The combination follows the role of progressivity in wage setting. By (14), the wage Phillips curve of workers who perceive progressivity τ has a slope proportional to $1/(\Gamma_w(\chi) + \tau)$. I combine the cells so that the slope at the combined value equals the average of the cell slopes weighted by the cells' shares of labor income:

$$\Gamma_w(\chi) = \lambda_w + \chi(1 + \lambda_w), \qquad \tau_j(\chi) = \Big[\sum_{h=1}^{14} \frac{s_h}{\Gamma_w(\chi) + \tau_{j,h}}\Big]^{-1} - \Gamma_w(\chi), \qquad j \in \{M, A\}. \tag{35}$$

The cell values $\tau_{M,h}$ and $\tau_{A,h}$ and the shares s_h are fixed inputs, while τ_M and τ_A are derived from the estimated inverse Frisch elasticity χ. In the estimation they are recomputed from (35) at every candidate value of χ, in the search for the posterior mode and in every draw of the sampler, and the counterfactuals evaluate them at the parameter values used. Both models use the same combination; the RE model uses only τ_M. At the posterior mean of the tax-ironing model, $\tau_M = 0.0810$ and $\tau_A = 0.1147$, and over the 90 percent posterior interval of χ either changes by less than 0.0002. They agree with the income-weighted means of the cell values to three decimal places. With χ inside this interval, halving or doubling the bandwidth of the density estimate changes τ_M by at most 0.004 and τ_A by at most 0.001.

The remaining 35 parameters are estimated, with the same priors in both models. Table 3 lists the priors by group: preferences and technology, nominal rigidities, the monetary policy rule, steady state values, and the persistence and innovation standard deviations of the shocks. The priors for the parameters of preferences, technology, nominal rigidities and the policy rule follow common choices in the literature

Table 3: Prior Distributions

Parameter		Distribution	Mean	SD
Preferences and technology				
$100(\beta^{-1}-1)$	Discount rate	Gamma	0.25	0.10
σ	Utility curvature	Gamma	1.00	0.375
b	Habit	Beta	0.70	0.10
χ	Inverse Frisch elasticity	Gamma	2.00	0.75
ζ	Adjustment cost	Gamma	4.00	1.50
μ	Utilization	Gamma	1.00	0.20
α	Capital share	Normal†	0.30	0.05
Nominal rigidities				
γ_w	Wage indexation	Beta	0.50	0.15
ξ_w	Calvo, wages	Beta	0.50	0.10
γ_p	Price indexation	Beta	0.50	0.15
ξ_p	Calvo, prices	Beta	0.75	0.10
λ_p	Price markup‡	Gamma	0.15	0.05
Monetary policy rule				
ϕ_r	Interest rate smoothing	Beta	0.75	0.10
ϕ_π	Inflation response	Gamma	1.50	0.25
ϕ_y	Output gap response	Gamma	0.12	0.05
$\phi_{\Delta y}$	Gap change response	Normal†	0.125	0.05
Steady state				
$\bar{z}$	Trend growth	Gamma	0.398	0.05
$\bar{\pi}$	Inflation	Gamma	0.554	0.05
$\bar{\ell}$	Hours constant	Normal	0.00	0.05
Persistence				
ρ_z	Technology	Beta	0.50	0.20
ρ_b	Preference	Beta	0.50	0.20
ρ_g	Exogenous spending	Beta	0.50	0.20
ρ_i	Investment efficiency	Beta	0.50	0.20
ρ_p	Price markup	Beta	0.50	0.20
ρ_w	Wage markup	Beta	0.50	0.20
ρ_r	Monetary policy	Beta	0.50	0.20
ρ_M	Tax rate	Beta	0.50	0.20
Standard deviation of innovations				
σ_z	Technology	Inv. gamma	0.50	∞
σ_b	Preference	Inv. gamma	0.50	∞
σ_g	Exogenous spending	Inv. gamma	0.50	∞
σ_i	Investment efficiency	Inv. gamma	0.50	∞
σ_p	Price markup	Inv. gamma	0.50	∞
σ_w	Wage markup	Inv. gamma	0.50	∞
σ_r	Monetary policy	Inv. gamma	0.50	∞
σ_M	Tax rate	Inv. gamma	0.50	∞

Notes: Same priors in both models, before the restriction to the admissible region (Section 4.1). σ_x is in percent and σ_M in percentage points. †The bounds $[0.05, 0.50]$ for α and $[0.001, 0.5]$ for $\phi_{\Delta y}$ are used only in the search for the posterior mode. ‡Net price markup, equal to the fixed-cost ratio ϕ.

on estimated DSGE models, such as Smets and Wouters (2007); the prior for the price markup follows Justiniano et al. (2010). The capital share has a normal prior with mean 0.30 and standard deviation 0.05, as in Smets and Wouters (2007). Because $\phi = \lambda_p$, the fixed-cost ratio has no prior of its own. The discount factor is estimated through the discount rate $100(\beta^{-1} - 1)$, in percent per quarter, which has a gamma prior with mean 0.25 and standard deviation 0.10, as in Smets and Wouters (2007). The priors for trend growth and inflation are centered on their sample means, and the prior for the hours constant on zero. The persistence of each shock has a beta prior with mean 0.5 and standard deviation 0.2, and each shock scale has an inverse gamma prior with mean 0.5 and two degrees of freedom. The tax rate process has the same priors as the other shocks.

The joint prior is restricted to the region $\beta(z^*)^{1-\sigma} < 1$, equivalently $r^* > z^*$, in which discounted utility along the balanced growth path is finite. Outside this region the prior density is zero. Within it the prior density is divided by the prior probability of the region, 0.94, so that it integrates to one. The prior means and standard deviations in Table 3 are those before this restriction. The normalizing constant is the same in both models.

The likelihood of the linearized model is evaluated with the Kalman filter. I compute the posterior mode of each model by numerical optimization and draw from its posterior with the random-walk Metropolis–Hastings algorithm, using 2 chains of 1,000,000 draws for each model and discarding the first half of each chain.

4.2 Posterior Estimates and Model Comparison

Table 4 reports, for each model, the posterior mean and the 90 percent highest posterior density interval. Tables 3 and 4 use short names. σ is the inverse of the intertemporal elasticity of substitution, $\zeta = S''(1)$ is the curvature of the investment adjustment cost, $\mu = \delta'(1)/\delta''(1)$ governs the cost of utilization, and ϕ_r, ϕ_π, ϕ_y and $\phi_{\Delta y}$ are the coefficients of the policy rule (29). The persistence ρ_x and the innovation standard deviation σ_x of each shock are those of (30). The posterior distributions of the two models are very close, as expected, because the models differ only in the perceived progressivity of the ironers. Wages and prices are sticky, and the posterior mean of the inverse Frisch elasticity, 3.92, means that labor supply responds little to the wage on the intensive margin. The posterior mean of the net price markup λ_p, which by (24) is also the fixed-cost ratio ϕ, is 0.23 in both models. The posterior mean of the capital share is 0.14, so with zero profits the labor share of output is 0.86.

Table 5 compares the two models by their marginal likelihoods. The main estimate is the modified harmonic mean of Geweke (1999), computed from the retained posterior draws. Two adjustments account for the restriction of the prior. Both log marginal likelihoods include the normalization of the prior over the admissible region. The weighting function of the modified harmonic mean, a truncated normal

Table 4: Posterior Estimates

Parameter		RE model Mean	RE model 90% HPD	Tax-ironing model Mean	Tax-ironing model 90% HPD
Preferences and technology					
$100(\beta^{-1}-1)$	Discount rate	0.22	[0.10, 0.34]	0.22	[0.09, 0.34]
σ	Utility curvature	1.01	[0.68, 1.33]	1.01	[0.69, 1.34]
b	Habit	0.63	[0.56, 0.71]	0.64	[0.56, 0.71]
χ	Inverse Frisch elasticity	3.92	[2.51, 5.26]	3.92	[2.48, 5.28]
ζ	Adjustment cost	7.63	[4.68, 10.64]	7.62	[4.59, 10.51]
μ	Utilization	0.93	[0.65, 1.22]	0.93	[0.64, 1.21]
α	Capital share	0.14	[0.13, 0.15]	0.14	[0.12, 0.15]
Nominal rigidities					
γ_w	Wage indexation	0.44	[0.20, 0.66]	0.44	[0.20, 0.66]
ξ_w	Calvo, wages	0.75	[0.69, 0.82]	0.75	[0.69, 0.82]
γ_p	Price indexation	0.45	[0.28, 0.62]	0.46	[0.29, 0.63]
ξ_p	Calvo, prices	0.97	[0.96, 0.98]	0.97	[0.96, 0.98]
λ_p	Price markup‡	0.23	[0.14, 0.33]	0.23	[0.13, 0.33]
Monetary policy rule					
ϕ_r	Interest rate smoothing	0.85	[0.80, 0.89]	0.85	[0.80, 0.89]
ϕ_π	Inflation response	1.38	[1.03, 1.71]	1.37	[1.03, 1.70]
ϕ_y	Output gap response	0.16	[0.09, 0.22]	0.16	[0.09, 0.22]
$\phi_{\Delta y}$	Gap change response	0.23	[0.18, 0.29]	0.23	[0.17, 0.29]
Steady state					
$\bar{z}$	Trend growth	0.39	[0.33, 0.45]	0.39	[0.33, 0.45]
$\bar{\pi}$	Inflation	0.58	[0.51, 0.64]	0.58	[0.52, 0.64]
$\bar{\ell}$	Hours constant	0.001	[−0.082, 0.083]	0.002	[−0.079, 0.084]
Persistence					
ρ_z	Technology	0.17	[0.07, 0.27]	0.17	[0.07, 0.27]
ρ_b	Preference	0.93	[0.90, 0.97]	0.93	[0.90, 0.96]
ρ_g	Exogenous spending	0.99	[0.97, 1.00]	0.99	[0.97, 1.00]
ρ_i	Investment efficiency	0.63	[0.53, 0.73]	0.63	[0.53, 0.74]
ρ_p	Price markup	0.18	[0.04, 0.32]	0.18	[0.04, 0.33]
ρ_w	Wage markup	0.09	[0.01, 0.16]	0.09	[0.01, 0.16]
ρ_r	Monetary policy	0.51	[0.41, 0.62]	0.51	[0.41, 0.62]
ρ_M	Tax rate	0.92	[0.87, 0.97]	0.92	[0.87, 0.97]
Standard deviation of innovations					
σ_z	Technology	0.68	[0.60, 0.76]	0.68	[0.60, 0.76]
σ_b	Preference	3.04	[2.04, 4.03]	3.02	[2.04, 3.98]
σ_g	Exogenous spending	1.54	[1.38, 1.69]	1.53	[1.37, 1.69]
σ_i	Investment efficiency	4.62	[3.75, 5.45]	4.63	[3.74, 5.47]
σ_p	Price markup	0.15	[0.12, 0.18]	0.15	[0.12, 0.18]
σ_w	Wage markup	0.84	[0.74, 0.95]	0.84	[0.74, 0.94]
σ_r	Monetary policy	0.13	[0.11, 0.15]	0.13	[0.11, 0.15]
σ_M	Tax rate	0.60	[0.46, 0.74]	0.60	[0.45, 0.74]

Notes: Posterior mean and 90 percent highest posterior density interval. σ_x is in percent and σ_M in percentage points. ‡Net price markup, equal to the fixed-cost ratio ϕ.

Table 5: Marginal Likelihoods and Bayes Factors

Method	Log marginal likelihood		Difference	Bayes factor
	RE model	Tax-ironing model	(tax-ironing − RE)	(tax-ironing/RE)
Modified harmonic mean	−660.83	−660.75	0.08	1.09
Laplace (reference)	−661.00	−660.95	0.05	1.05

density, must integrate to one over the region in which the posterior density is positive, where the parameters lie in the prior support and the admissible region and the model has a unique stable solution. I rescale it by its probability in that region. The Laplace approximation at the posterior mode is shown for reference; apart from the normalization of the prior, it does not account for the restriction of the parameter space. By both methods the tax-ironing model has a slightly higher marginal likelihood, and the Bayes factors are close to one, so the macroeconomic series do not distinguish the two models. I assign no prior probabilities to the models, so the Bayes factors are not posterior odds. The marginal likelihoods come from the estimation of each model and do not involve the common parameter values of Section 4.3.

This result is consistent with the level effect reported in Section 4.3. Output, consumption, investment and the real wage are observed in growth rates, in which steady state levels cancel, and the level of hours is absorbed by the constant $\bar{\ell}$. The likelihood therefore carries little information about steady state levels and compares mainly the dynamics of the two models, although the labor income share s_I and the perceived progressivity of each type enter the coefficients of the dynamic equations. The comparison is between two specifications, $\theta = 0$ and $\theta = 0.43$. It is not an estimate of θ, and similar marginal likelihoods at these two values do not show that the likelihood is flat in θ.

4.3 Macroeconomic Effects of Tax Ironing

The two models are estimated separately, so their posterior estimates differ slightly. A comparison of the moments of the two estimated models would mix the effect of ironing with these differences. To isolate the effect of ironing, I solve both models at the same parameter values, the posterior mean parameters of the tax-ironing model,[1] with the same fixed inputs, tax schedule, shock processes and shock covariance, and change only the perception of the schedule. Each model is solved around its own steady state. Table 6 reports the results.

Panel A shows the level effect. Tax ironing raises each of steady state hours, output, consumption, investment and capital by the same 1.05 percent, and the real wage is unchanged. The five quantities rise in the same proportion because, at the same parameter values, the real wage and the steady state ratios c/y, i/y and k/y are the same in the two models, so each quantity is proportional to hours. Behind the

[1] The results are robust to evaluating both models at the posterior mode rather than the posterior mean of the tax-ironing model. The steady state level effect is 1.10 percent, compared with 1.05 percent at the posterior mean, while the effects on business cycle moments remain small.

Table 6: Macroeconomic Effects of Tax Ironing

Panel A. Steady state

	RE	Tax-ironing	Effect (%)
Hours, output, consumption, investment and capital	100.00	101.05	1.05
Real wage	100.00	100.00	0.00
Steady state welfare (CEV)	–	–	0.23

Panel B. Business cycle moments of the quarterly observables

	SD			AR(1)			Corr. with output		
	RE	TI	Change (%)	RE	TI	Diff. ($\times 10^{-3}$)	RE	TI	Diff. ($\times 10^{-3}$)
$100\Delta \log Y_t$	0.73	0.73	−0.01	0.48	0.48	−0.02	1.00	1.00	–
$100\Delta \log C_t$	0.79	0.79	0.02	0.58	0.58	−0.01	0.57	0.57	0.03
$100\Delta \log I_t$	4.24	4.24	−0.01	0.58	0.58	−0.04	0.54	0.54	−0.25
$100\Delta \log W_t$	0.92	0.92	0.04	0.11	0.11	0.25	0.15	0.15	−0.14
$100 \log \ell_t$	2.00	2.00	−0.05	0.96	0.96	−0.02	0.07	0.07	0.06
$100\Delta \log P_t$	0.24	0.24	0.11	0.65	0.65	0.81	−0.11	−0.11	−0.16
$100 \log R_t^n$	0.46	0.46	0.11	0.97	0.97	0.04	−0.12	−0.12	−0.07

Notes: Welfare: consumption equivalent, assuming a marginal tax rate of 28.38 percent at the base income (Appendix B). Panel B: theoretical moments of the observables of (34) with all eight shocks, in percent per quarter (hours in percent). SD: standard deviation; AR(1): first-order autocorrelation; Corr. with output: correlation with $100\Delta \log Y_t$. TI: tax-ironing model. Change (%): percent change of the SD from RE to TI. Diff.: TI minus RE, in units of 10^{-3}.

aggregate, ironers work more than members in the RE model, while correct perceivers work slightly less: higher consumption lowers the marginal utility of wealth, and this income effect outweighs the higher demand for their labor. The ironers' share of labor income, 43.60 percent, exceeds their population share of 43 percent. The level effect is not a treatment effect estimated directly from macroeconomic series. It is a counterfactual computed from the measured tax schedule, the measured share of ironers and the estimated model.

The last row of Panel A compares welfare. Higher activity does not by itself imply higher welfare, because members also work more. I evaluate the two steady states with the preferences (5), in which the actual rather than the perceived tax rates matter, and measure the difference by the consumption equivalent: the percent change in the whole consumption path of the RE steady state that makes the family as well off as in the tax-ironing steady state. Unlike the level effect, this comparison depends on the absolute level of hours and therefore on the level of the marginal tax rate at the base income, which the rest of the paper does not use. With the CBO mean marginal rate of 28.38 percent, an assumption rather than an estimate, welfare rises by 0.23 percent of consumption. Consumption rises by 1.05 percent, and the additional disutility of work offsets about three quarters of this gain. The gain arises because the labor income tax and the wage markup that members charge when they set their wages hold hours in the RE steady state below the level at which the consumption value of an additional hour equals its disutility. Nominal wage rigidity plays no role in this comparison, because wages are indexed to trend growth and steady state inflation. Ironing lowers the marginal tax rate that 43 percent of members perceive and offsets

part of the wedge. The welfare gain is therefore a result for this model, in which existing distortions hold hours too low. In a labor market with a different structure or different distortions, for example one in which firms rather than workers set wages, the welfare effect of ironing could differ in size and even in sign. This is the interaction between a misperception and an existing tax distortion that Farhi and Gabaix (2020) study in the design of optimal taxes, and Rees-Jones and Taubinsky (2020) quantify how ironing strengthens the case for progressive taxation in a calibrated model of the US tax system. Here the schedule is held fixed, and the comparison only evaluates the steady state welfare effect of ironing in the estimated macroeconomic model. The gain is positive when the marginal rate at the base income exceeds about 8.6 percent, and it remains between 0.23 and 0.46 percent when ironers and correct perceivers form separate households or when the steady state level of exogenous spending, rather than its share of output, is held fixed. Appendix B gives the details.

Panel B reports theoretical moments with all eight shocks and shows that relative fluctuations change little. The standard deviations of the seven observables change by at most 0.11 percent, and their first-order autocorrelations and correlations with output growth change by less than 10^{-3}. Ironing does affect the dynamics, because the perceived retention rates of the two types have different income elasticities, but the effects on the volatility, persistence and comovement of the observed series are small. Section 4.4 explains why.

These moments describe fluctuations relative to each model's own steady state. Measured in common units the picture differs. The standard deviation of detrended output relative to its own steady state is slightly lower in the tax-ironing model, but measured in units of the steady state output of the RE model it is about one percent higher, because the steady state level of output is about one percent higher. The statements about fluctuations in this paper concern relative variability, persistence and comovement, not the absolute size of fluctuations. The tax rate follows the same exogenous process in both models, so its own standard deviation and autocorrelation are unchanged, and only its correlation with output growth changes slightly.

4.4 Interpretation and Robustness

The counterfactual shows two effects with different sources. The level effect comes from the level of perceived retention. At the base income the retention rate that an ironer perceives is about 1.13 times that of a correct perceiver. Ironers therefore value an additional hour of work more, and by (15) they supply more hours at a slightly lower wage; at the posterior mean they earn 2.46 percent more than correct perceivers. Higher output and consumption lower the family's marginal utility of wealth, and correct perceivers respond to this income effect by working slightly less. In the aggregate, hours rise by 1.05 percent, and output, consumption, investment and capital rise in proportion. This effect requires only

that the average rate lie below the marginal rate. It is present under any progressive schedule, including CRP, as Proposition 2 states.

The effects on business cycle moments come from the income elasticity of perceived retention. The measured schedule does not have constant residual progression, so Proposition 2 does not apply, and Table 6 shows that these effects are not zero. Why they are small can be seen from the wage block, the only part of the linearized system in which the two models differ. Weight the wage Phillips curves (A.16) of the two types by s_I and $1 - s_I$ and substitute the type hours from (A.17). The relative wage of each type, $\hat{w}_{j,t} - \hat{w}_t$, enters its curve with the coefficient $\kappa_w(\tau_j)\left[(\chi + \tau_j)\varepsilon_w + 1 - \tau_j\right] = K_w/\lambda_w$, which is the same for both types, so these terms cancel in the weighted sum. The result is the wage Phillips curve (A.15) of the RE model with τ_M replaced by

$$\bar{\tau} = \left[\frac{s_I}{\Gamma_w + \tau_A} + \frac{1 - s_I}{\Gamma_w + \tau_M}\right]^{-1} - \Gamma_w, \tag{36}$$

both in the slope and in the bracket. The flexible wage conditions (A.29), which determine the output gap in the policy rule, aggregate in the same way, and all other linearized equations are the same in the two models. Appendix A gives the steps. The aggregate variables of the tax-ironing model therefore satisfy the linearized RE model with perceived progressivity $\bar{\tau}$ instead of τ_M. When both have unique stable solutions, the observables of the tax-ironing model have the same first-order dynamics and moments around its steady state as this RE system. The equivalence concerns fluctuations, not steady state levels. The wages of the two types differ, but their difference does not feed back into the aggregates. Under CRP, $\bar{\tau} = \tau_M$, as in Proposition 2. This aggregation differs from the combination of the cells in (35): (35) builds the model's τ_M and τ_A from the measurement, while (36) aggregates the wage setting of the two perception types within the model.

The size of $\bar{\tau} - \tau_M$ and of its effect on the coefficients follows from the existing estimates. At the posterior mean of the tax-ironing model, $\Gamma_w = \lambda_w + \chi(1+\lambda_w)$ is about 4.91, and s_I is 43.60 percent. Then $\bar{\tau}$ is about 0.096, compared with $\tau_M = 0.0810$ and $\tau_A = 0.1147$. For first-order aggregate fluctuations, ironing is therefore equivalent to raising the perceived progressivity of the RE model by about 0.015, which is close to s_I times the difference between τ_A and τ_M. This increase changes the coefficients of the wage Phillips curve little. Because $\kappa_w(\tau)$ is proportional to $1/(\Gamma_w + \tau)$, the wage Phillips curve of ironers is only about 0.67 percent flatter than that of correct perceivers, and the aggregate slope $\kappa_w(\bar{\tau})$ is about 0.3 percent lower than $\kappa_w(\tau_M)$. In the bracket, the coefficient on hours, $\chi + \tau$, rises by about 0.4 percent, because τ is small relative to χ, whose posterior mean is 3.92. The coefficient on the real wage, $1 - \tau$, falls by about 1.6 percent.

These coefficient comparisons explain why the effects in Table 6 can be small: ironing changes the coefficients of the aggregate wage block by less than two percent. They do not determine each moment. How the changes in the slope and in the bracket move the volatility, persistence and comovement of each

observable depends on the rest of the model and on the shock processes, and Table 6 reports the result of solving the model. I do not decompose the changes in the moments into the contributions of these coefficients. The two results should be kept apart. Under CRP the effects on relative fluctuations would be zero by Proposition 2. Under the measured schedule they are not zero, and that they are small is a quantitative result of the measured progressivity, the share of ironers and the estimated model.

Three robustness checks hold the estimated parameters fixed and solve both models again; none is a re-estimation. The first extends the income range of the measurement. Extending the income grid of each cell to the observed range of compensation lowers both τ_M and τ_A. With $\theta = 0.43$ and the other parameters fixed, the level effect becomes about 1.03 percent instead of 1.05 percent, the welfare gain about 0.22 instead of 0.23 percent, and the effects on business cycle moments remain small. Details are reported in Appendix C.

The second raises the share of ironers from 43 to 62 percent, the upper end of the 95 percent confidence interval of the estimate of Rees-Jones and Taubinsky (2020), holding the other parameters fixed. A larger share of ironers is expected to increase the departure from the RE model. The level effect rises from 1.05 to 1.52 percent, so the level effect of about one percent is the result for $\theta = 0.43$, and the welfare gain rises from 0.23 to 0.33 percent. The effects on business cycle moments remain small: the largest change in a standard deviation is 0.16 percent, and the largest changes in an autocorrelation and in a correlation with output growth are 1.17×10^{-3} and 0.36×10^{-3}. Details are reported in Appendix D.

These results describe tax ironing in a model in which the members of a representative family pool their income, so perception acts through wage setting and labor supply. Household heterogeneity matters for the business cycle through other channels. McKay and Reis (2016) find that tax and transfer programs affect aggregate volatility mainly through their effects on inequality and social insurance, and Jang et al. (2023) show that the progressivity of taxes and transfers at the bottom of the income distribution shapes the cyclical volatility of hours through employment choices. These channels work through the tax and transfer system itself, whereas tax ironing works through how households perceive the schedule. Introducing heterogeneity does not by itself imply that the cyclical effects of ironing are larger. Their aggregate size depends on which households iron and on how differences in perception, the schedules they face and their responses line up. The cell schedules in Table 1 and Figure 1 differ, but this paper does not estimate how ironing is related to them.

The third check relaxes the pooling of income. In the baseline, ironers and correct perceivers belong to one representative family that pools their income. In the extension they form two separate households that do not pool income and trade a one-period nominal bond. The structural parameters and shock processes are those of the baseline, at the posterior mean of the tax-ironing model, and the model is not re-estimated. The steady state level effect and the effects on business cycle moments hardly change.

The level effect is 1.0518 percent instead of 1.0515 percent, the welfare gain is 0.23 percent as in the baseline, and the changes in the standard deviations of the observables differ from the baseline only in the fourth decimal place. The steady state welfare gain thus remains positive in all three checks. Details are reported in Appendix E.

5 Conclusion

This paper examined the macroeconomic implications of tax ironing documented in microeconomic studies. By combining measures of perceived tax progressivity derived from US tax law and household microdata with a medium-scale DSGE model, I evaluated the effects of tax ironing on steady-state economic activity, welfare, and business cycles.

The results show that tax ironing raises steady-state economic activity by about one percent and modestly improves steady-state welfare. This welfare gain depends on the labor supply distortions present in the model. In contrast, the effects on business-cycle moments are tiny. The posterior distributions of the parameters are very similar with and without tax ironing, and allowing for ironing barely improves the marginal likelihood. The counterfactual results are robust to alternative income ranges, a higher share of ironers, and separate households for ironers and correct perceivers.

An important implication is that widespread tax ironing need not have large business-cycle effects. While its impact on steady-state labor supply comes from the level of the perceived net-of-tax rate, its impact on fluctuations depends on perceived progressivity. The measured difference between correct perceivers and ironers is inconsistent with the CRP tax function, but allowing for this difference produces only small changes in business-cycle dynamics. Beyond the present model, higher labor supply induced by ironing could affect long-run growth through R&D, while ironing of progressive capital income taxes could influence investment and capital accumulation. Thus, explicitly modeling tax ironing may not be essential for business-cycle analysis in a standard medium-scale DSGE framework, but its effects on steady-state levels should not be overlooked.

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Appendix A The Linearized Model

This appendix lists the complete linearized system that is estimated. A hat denotes 100 times the log deviation of a variable from the steady state of its own model, except for the tax rate: $\hat{M}_t$ is the deviation of the marginal tax rate from its mean in percentage points, and $\hat{M}_t/(1-\bar{M})$ is, to first order, the percent decline of the net-of-tax rates. The coefficients are steady state objects: z^*, r^*, $\bar{r}^k = r^* - 1 + \delta$, $c_y = c/y$, $i_y = i/y$, s_I, the fixed-cost ratio $\phi = \lambda_p$ from (24), and

$$\kappa_p = \frac{(1-\xi_p)(1-\xi_p z^*/r^*)}{\xi_p}, \qquad \kappa_w(\tau) = \frac{K_w}{\Gamma_w + \tau},$$
$$K_w = \frac{(1-\xi_w)(1-\xi_w z^*/r^*)\lambda_w}{\xi_w}, \qquad \Gamma_w = \lambda_w + \chi(1+\lambda_w). \tag{A.1}$$

These are the slopes of (22) and (14). The two models share every equation except the wage block. The RE model has one wage-setting type with perceived progressivity τ_M. The tax-ironing model has full ironers, with perceived progressivity $\tau_I = \tau_A$, and correct perceivers, with $\tau_C = \tau_M$; s_I is given by (17). The variable $\hat{\lambda}_t$ is the marginal utility of wealth, $\hat{Q}_t$ Tobin's q, $\hat{u}_t$ utilization, $\hat{r}^k_t$ the rental rate, $\widehat{mc}_t$ real marginal cost and $\hat{r}^n_t$ the nominal interest rate.

A.1 Sticky-price, sticky-wage economy

Marginal utility of wealth, with internal habit:

$$\Big(1-\frac{b}{z^*}\Big)\Big(1-\frac{b}{r^*}\Big)\hat{\lambda}_t = -\sigma\Big(\hat{c}_t - \frac{b}{z^*}(\hat{c}_{t-1} - z^z_t)\Big) + \Big(1-\frac{b}{z^*}\Big)z^b_t + \frac{b}{r^*}\mathbb{E}_t\Big[\sigma\Big(\hat{c}_{t+1} + z^z_{t+1} - \frac{b}{z^*}\hat{c}_t\Big) - \Big(1-\frac{b}{z^*}\Big)z^b_{t+1}\Big]. \tag{A.2}$$

Consumption Euler equation, utilization, investment and capital accumulation:

$$\hat{\lambda}_t = \mathbb{E}_t\hat{\lambda}_{t+1} - \sigma\mathbb{E}_t z^z_{t+1} + \hat{r}^n_t - \mathbb{E}_t\hat{\pi}_{t+1}, \tag{A.3}$$

$$\hat{u}_t = \mu\big(\hat{r}^k_t - \hat{Q}_t\big), \tag{A.4}$$

$$\zeta\big(\hat{\imath}_t - \hat{\imath}_{t-1} + z^z_t + z^i_t\big) = \hat{Q}_t + \zeta\,\frac{z^*}{r^*}\,\mathbb{E}_t\big(\hat{\imath}_{t+1} - \hat{\imath}_t + z^z_{t+1} + z^i_{t+1}\big), \tag{A.5}$$

$$\hat{k}_t = \frac{1-\delta}{z^*}\big(\hat{k}_{t-1} - z^z_t\big) - \frac{\bar{r}^k}{z^*}\hat{u}_t + \Big(1 - \frac{1-\delta}{z^*}\Big)\hat{\imath}_t. \tag{A.6}$$

Tobin's q, marginal cost, cost minimization and production:

$$\hat{Q}_t = \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \sigma\mathbb{E}_t z^z_{t+1} + \frac{1}{r^*}\Big(\bar{r}^k\,\mathbb{E}_t\hat{r}^k_{t+1} + (1-\delta)\mathbb{E}_t\hat{Q}_{t+1}\Big), \tag{A.7}$$

$$\widehat{mc}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}^k_t, \tag{A.8}$$

$$\hat{w}_t - \hat{r}^k_t = \hat{u}_t + \hat{k}_{t-1} - \hat{\ell}_t - z^z_t, \tag{A.9}$$

$$\hat{y}_t = (1+\phi)\Big[(1-\alpha)\hat{\ell}_t + \alpha\big(\hat{u}_t + \hat{k}_{t-1} - z^z_t\big)\Big]. \tag{A.10}$$

Price Phillips curve, resource constraint, monetary policy rule and the tax rate:

$$\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} = \frac{z^*}{r^*}\mathbb{E}_t\big(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\big) + \kappa_p\widehat{mc}_t + z^p_t, \tag{A.11}$$

$$\hat{y}_t = c_y\hat{c}_t + i_y\hat{\imath}_t + g_y z^g_t, \tag{A.12}$$

$$\hat{r}^n_t = \phi_r\hat{r}^n_{t-1} + (1-\phi_r)\big[\phi_\pi\hat{\pi}_t + \phi_y(\hat{y}_t - \hat{y}^f_t)\big] + \phi_{\Delta y}\big[(\hat{y}_t - \hat{y}^f_t) - (\hat{y}_{t-1} - \hat{y}^f_{t-1})\big] + z^r_t, \tag{A.13}$$

$$\hat{M}_t = \rho_M\hat{M}_{t-1} + \sigma_M\varepsilon^M_t. \tag{A.14}$$

Wage block of the RE model:

$$\begin{aligned}\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t - \gamma_w\hat{\pi}_{t-1} + z^z_t = {}& \frac{z^*}{r^*}\mathbb{E}_t\big(\hat{w}_{t+1} - \hat{w}_t + \hat{\pi}_{t+1} - \gamma_w\hat{\pi}_t + z^z_{t+1}\big)\\ &+ \kappa_w(\tau_M)\Big[(\chi+\tau_M)\hat{\ell}_t - \hat{\lambda}_t - (1-\tau_M)\hat{w}_t + z^b_t + \frac{\hat{M}_t}{1-\bar{M}}\Big] + z^w_t.\end{aligned} \tag{A.15}$$

Wage block of the tax-ironing model, for $j \in \{I, C\}$ with $\tau_I = \tau_A$ and $\tau_C = \tau_M$:

$$\begin{aligned}\hat{w}_{j,t} - \hat{w}_{j,t-1} + \hat{\pi}_t - \gamma_w\hat{\pi}_{t-1} + z^z_t = {}& \frac{z^*}{r^*}\mathbb{E}_t\big(\hat{w}_{j,t+1} - \hat{w}_{j,t} + \hat{\pi}_{t+1} - \gamma_w\hat{\pi}_t + z^z_{t+1}\big)\\ &+ \kappa_w(\tau_j)\Big[(\chi+\tau_j)\hat{\ell}_{j,t} - \hat{\lambda}_t - (1-\tau_j)\hat{w}_{j,t} + z^b_t + \frac{\hat{M}_t}{1-\bar{M}}\Big] + z^w_t,\end{aligned} \tag{A.16}$$

$$\hat{\ell}_{j,t} = \hat{\ell}_t - \varepsilon_w\big(\hat{w}_{j,t} - \hat{w}_t\big), \qquad \hat{w}_t = s_I\,\hat{w}_{I,t} + (1-s_I)\,\hat{w}_{C,t}. \tag{A.17}$$

In the brackets of (A.15) and (A.16), τ enters through the local expansion (33) of perceived retention; under a proportional tax $\tau = 0$ and the coefficients $1-\tau$ and $\chi+\tau$ reduce to 1 and χ. The wage markup shock z^w_t enters the wage Phillips curve of each type with a unit coefficient because of the loading $1/\kappa_w(\tau_j)$ in (14); the price markup shock z^p_t enters the price Phillips curve with a unit coefficient because of (22).

A.2 Flexible-price, flexible-wage economy

The flexible economy has no nominal rigidities and no markup shocks. Its variables carry the superscript f, and $\hat{r}^{r,f}_t$ is its real interest rate.

$$\begin{aligned}\Big(1-\frac{b}{z^*}\Big)\Big(1-\frac{b}{r^*}\Big)\hat{\lambda}^f_t = {}& -\sigma\Big(\hat{c}^f_t - \frac{b}{z^*}\big(\hat{c}^f_{t-1} - z^z_t\big)\Big) + \Big(1-\frac{b}{z^*}\Big)z^b_t\\ &+ \frac{b}{r^*}\,\mathbb{E}_t\Big[\sigma\Big(\hat{c}^f_{t+1} + z^z_{t+1} - \frac{b}{z^*}\hat{c}^f_t\Big) - \Big(1-\frac{b}{z^*}\Big)z^b_{t+1}\Big],\end{aligned} \tag{A.18}$$

$$\hat{\lambda}_t^f = \mathbb{E}_t \hat{\lambda}_{t+1}^f - \sigma \mathbb{E}_t z_{t+1}^z + \hat{r}_t^{r,f}, \tag{A.19}$$

$$\hat{u}_t^f = \mu\big(\hat{r}_t^{k,f} - \hat{Q}_t^f\big), \tag{A.20}$$

$$\zeta\big(\hat{\imath}_t^f - \hat{\imath}_{t-1}^f + z_t^z + z_t^i\big) = \hat{Q}_t^f + \zeta \frac{z^*}{r^*} \mathbb{E}_t\big(\hat{\imath}_{t+1}^f - \hat{\imath}_t^f + z_{t+1}^z + z_{t+1}^i\big), \tag{A.21}$$

$$\hat{k}_t^f = \frac{1-\delta}{z^*}\big(\hat{k}_{t-1}^f - z_t^z\big) - \frac{\bar{r}^k}{z^*}\hat{u}_t^f + \Big(1 - \frac{1-\delta}{z^*}\Big)\hat{\imath}_t^f, \tag{A.22}$$

$$\hat{Q}_t^f = \mathbb{E}_t \hat{\lambda}_{t+1}^f - \hat{\lambda}_t^f - \sigma \mathbb{E}_t z_{t+1}^z + \frac{1}{r^*}\Big(\bar{r}^k\, \mathbb{E}_t \hat{r}_{t+1}^{k,f} + (1-\delta)\mathbb{E}_t \hat{Q}_{t+1}^f\Big), \tag{A.23}$$

$$0 = (1-\alpha)\hat{w}_t^f + \alpha \hat{r}_t^{k,f}, \tag{A.24}$$

$$\hat{w}_t^f - \hat{r}_t^{k,f} = \hat{u}_t^f + \hat{k}_{t-1}^f - \hat{\ell}_t^f - z_t^z, \tag{A.25}$$

$$\hat{y}_t^f = (1+\phi)\Big[(1-\alpha)\hat{\ell}_t^f + \alpha\big(\hat{u}_t^f + \hat{k}_{t-1}^f - z_t^z\big)\Big], \tag{A.26}$$

$$\hat{y}_t^f = c_y \hat{c}_t^f + i_y \hat{\imath}_t^f + g_y z_t^g. \tag{A.27}$$

Flexible wage of the RE model:

$$(1-\tau_M)\hat{w}_t^f = (\chi + \tau_M)\hat{\ell}_t^f - \hat{\lambda}_t^f + z_t^b + \frac{\hat{M}_t}{1-\bar{M}}. \tag{A.28}$$

Flexible wages of the tax-ironing model, for $j \in \{I, C\}$:

$$(1-\tau_j)\hat{w}_{j,t}^f = (\chi + \tau_j)\hat{\ell}_{j,t}^f - \hat{\lambda}_t^f + z_t^b + \frac{\hat{M}_t}{1-\bar{M}}, \tag{A.29}$$

$$\hat{\ell}_{j,t}^f = \hat{\ell}_t^f - \varepsilon_w\big(\hat{w}_{j,t}^f - \hat{w}_t^f\big), \qquad \hat{w}_t^f = s_I\, \hat{w}_{I,t}^f + (1-s_I)\, \hat{w}_{C,t}^f. \tag{A.30}$$

A.3 Shock processes and observation equations

$$z_t^x = \rho_x z_{t-1}^x + \sigma_x \varepsilon_t^x, \qquad x \in \{z, b, g, i, p, w, r\}. \tag{A.31}$$

The tax rate follows the process given in Appendix A.1, and the eight innovations are independent standard normal. The observation equations are (34); the tax observation is used in fourth quarters only.

A.4 Aggregation of the wage block of the tax-ironing model

This section derives the aggregation used in Section 4.4. Substituting $\hat{\ell}_{j,t} = \hat{\ell}_t - \varepsilon_w(\hat{w}_{j,t} - \hat{w}_t)$ from (A.17) into the bracket of (A.16) gives

$$\kappa_w(\tau_j)\Big[(\chi+\tau_j)\hat{\ell}_t - \hat{\lambda}_t - (1-\tau_j)\hat{w}_t + z_t^b + \frac{\hat{M}_t}{1-\bar{M}}\Big] - \kappa_w(\tau_j)\big[(\chi+\tau_j)\varepsilon_w + 1 - \tau_j\big]\big(\hat{w}_{j,t} - \hat{w}_t\big).$$

Since $\varepsilon_w = (1+\lambda_w)/\lambda_w$, $(\chi+\tau_j)\varepsilon_w + 1 - \tau_j = (\Gamma_w + \tau_j)/\lambda_w$, so the coefficient on the relative wage is K_w/λ_w for both types. By (A.17), $s_I(\hat{w}_{I,t} - \hat{w}_t) + (1-s_I)(\hat{w}_{C,t} - \hat{w}_t) = 0$. Weight (A.16) by the constants s_I for $j = I$ and $1 - s_I$ for $j = C$ and add. The left side and the expectation term are linear

in $\hat{w}_{j,t}$ with the same coefficients for both types, and z_t^w enters both curves with a unit coefficient, so by (A.17) these terms become the same terms in $\hat{w}_t$. The relative wage terms cancel, and the result is

$$\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t - \gamma_w \hat{\pi}_{t-1} + z_t^z = \frac{z^*}{r^*}\mathbb{E}_t\left(\hat{w}_{t+1} - \hat{w}_t + \hat{\pi}_{t+1} - \gamma_w \hat{\pi}_t + z_{t+1}^z\right) + \bar{\kappa}\left[(\chi + \bar{\tau})\hat{\ell}_t - \hat{\lambda}_t - (1-\bar{\tau})\hat{w}_t + z_t^b + \frac{\hat{M}_t}{1-\bar{M}}\right] + z_t^w,$$

with $\bar{\kappa} = s_I\kappa_w(\tau_A) + (1-s_I)\kappa_w(\tau_M)$ and $\bar{\tau} = \left[s_I\kappa_w(\tau_A)\tau_A + (1-s_I)\kappa_w(\tau_M)\tau_M\right]/\bar{\kappa}$. Because $\kappa_w(\tau)(\Gamma_w + \tau) = K_w$ for every τ, $\bar{\kappa}(\Gamma_w + \bar{\tau}) = K_w$, so $\bar{\kappa} = \kappa_w(\bar{\tau})$ and $\bar{\tau}$ is given by (36). The aggregate curve is (A.15) with τ_M replaced by $\bar{\tau}$.

The same substitution in (A.29) gives

$$\frac{\Gamma_w + \tau_j}{\lambda_w}\left(\hat{w}_{j,t}^f - \hat{w}_t^f\right) = (\chi + \tau_j)\hat{\ell}_t^f - \hat{\lambda}_t^f - (1-\tau_j)\hat{w}_t^f + z_t^b + \frac{\hat{M}_t}{1-\bar{M}}.$$

Since $\lambda_w\kappa_w(\tau_j)/K_w = \lambda_w/(\Gamma_w + \tau_j)$, multiplying this condition by $\lambda_w s_I\kappa_w(\tau_A)/K_w$ for $j = I$ and by $\lambda_w(1-s_I)\kappa_w(\tau_M)/K_w$ for $j = C$ and adding gives

$$s_I\left(\hat{w}_{I,t}^f - \hat{w}_t^f\right) + (1-s_I)\left(\hat{w}_{C,t}^f - \hat{w}_t^f\right) = \frac{\lambda_w\bar{\kappa}}{K_w}\left[(\chi + \bar{\tau})\hat{\ell}_t^f - \hat{\lambda}_t^f - (1-\bar{\tau})\hat{w}_t^f + z_t^b + \frac{\hat{M}_t}{1-\bar{M}}\right].$$

The left side is zero by the flexible wage index in the block of (A.29). Dividing by $\lambda_w\bar{\kappa}/K_w > 0$ gives

$$(1-\bar{\tau})\hat{w}_t^f = (\chi + \bar{\tau})\hat{\ell}_t^f - \hat{\lambda}_t^f + z_t^b + \frac{\hat{M}_t}{1-\bar{M}},$$

which is the flexible wage of the RE model with τ_M replaced by $\bar{\tau}$.

The two models share all other linearized equations, with the same coefficients: the conditions of the family and the firms, the resource constraints, the monetary policy rule, in which the output gap is output relative to the flexible economy, the shock processes and the observation equations. Hence the aggregate variables of the tax-ironing model satisfy the linearized RE model with $\bar{\tau}$ in place of τ_M in both wage blocks, at the same parameter values and shock processes, and the type wages do not enter this system. When both systems have unique stable solutions, the observables of the tax-ironing model, measured around its own steady state, have the first-order dynamics and moments of this RE system. The result concerns first-order fluctuations, not steady state levels, which the level of perceived retention determines through (32). With $\tau_A = \tau_M$, $\bar{\tau} = \tau_M$. The aggregation equivalence is also verified numerically at the posterior mean: the tax-ironing model and the corresponding RE model with progressivity $\bar{\tau}$ produce identical first-order aggregate dynamics up to numerical precision.

Appendix B Steady State Welfare Analysis

This appendix describes the welfare row of Table 6. Both models are evaluated in their steady states at the posterior mean parameters of the tax-ironing model, the parameter values of Table 6. Along the balanced growth path $C_t - bC_{t-1} = Z_t\, c\, (1 - b/z^*)$, where $c = C_t/Z_t$, and both terms of (5) grow with $Z_t^{1-\sigma}$. Lifetime utility is therefore $Z_0^{1-\sigma} u/\big(1 - \beta(z^*)^{1-\sigma}\big)$, with

$$u = \frac{\big[(1 - b/z^*)\, c\big]^{1-\sigma}}{1-\sigma} - \theta\, \frac{\ell_I^{1+\chi}}{1+\chi} - (1-\theta)\, \frac{\ell_C^{1+\chi}}{1+\chi},$$

where ℓ_I and ℓ_C are the hours of ironers and correct perceivers; in the RE model all members work the same hours. The factor in front of u is the same in both models. Perception does not enter u. It affects welfare only through the hours that members choose when they set wages, and the tax revenue is returned to the family. The consumption equivalent g is the proportional change in the whole consumption path of the RE steady state, $C_t \to (1+g)C_t$ for all t, that makes the family as well off as in the tax-ironing steady state. Because the change applies to C_t and C_{t-1} alike, it scales the habit-adjusted consumption term by $(1+g)^{1-\sigma}$, and g solves $(1+g)^{1-\sigma} u_c^{\mathrm{RE}} - v^{\mathrm{RE}} = u^{\mathrm{TI}}$, where u_c and v are the consumption and labor terms of u.

The level effects of Table 6 do not depend on the level of the marginal tax rate at the base income, but welfare does. The level of hours in the RE steady state follows from the wage condition (15), $q\, w\, \lambda = (1+\lambda_w)\ell^{\chi}$ with $q = 1 - M$, and the disutility of work depends on that level. I set the marginal rate at the base income to the CBO mean marginal rate, $M(x_C) = 0.2838$. This value is an additional assumption of the welfare calculation. It is not an estimate of the marginal rate of the representative schedule, and Table B.1 shows how the results change with it.

At this baseline, consumption and composite hours are 1.05 percent higher in the tax-ironing steady state than in the RE steady state. Ironers work 2.73 percent more and correct perceivers 0.21 percent less than RE members. The consumption equivalent is 0.23 percent: the higher consumption alone is worth 1.05 percent of consumption, and the higher disutility of work costs 0.82 percent.

The sign of the gain follows from the wedge between the private return to work and its value to the family in the RE steady state. Across steady states, raising all hours by one percent, with capital adjusting in proportion, raises output and consumption by one percent. The ratio of the marginal disutility of these hours to the marginal utility of the additional consumption is

$$\Omega = \frac{q}{1+\lambda_w} \cdot \frac{1-\alpha}{c_y} \cdot \frac{1 - b/r^*}{1 - b/z^*}.$$

The factors q and $1+\lambda_w$ reflect the labor income tax and the wage markup. With zero profits (24), the wage equals the labor share $1-\alpha$ of output per hour; the fixed cost is proportional to the economy's own steady state output, so the price markup does not enter Ω. The share c_y of output is consumed, and the last factor,

close to one, reflects habit formation and discounting. Nominal rigidities do not enter, because wages and prices are indexed to steady state inflation. At the baseline, $\Omega = 0.716 \times 0.833 \times 1.222 \times 1.004 = 0.73$. Hours are too low in the RE steady state, and the additional hours of ironers raise welfare. To first order, the gain is $1 - \Omega$ times the 1.05 percent increase in hours, or 0.28 percent. The curvature of utility in consumption and hours and the unequal hours of the two types reduce it to 0.23 percent.

Table B.1 reports the consumption equivalent for four levels of $M(x_C)$. It also reports two changes of the model: the two-household model of Appendix E, and steady state exogenous spending fixed at its level in the RE steady state instead of proportional to the economy's own steady state output. With spending proportional to output, the gain is positive when $M(x_C)$ exceeds 8.6 percent in the representative family and 7.8 percent in the two-household model. With spending fixed, all additional output that is not invested is consumed, c_y in Ω is replaced by $1 - i_y$, and the gain is about 0.2 percentage points larger and positive at every level in the table. In the two-household model at the baseline level, ironers gain 0.55 percent with spending proportional to output and 0.78 percent with spending fixed. Correct perceivers lose 0.01 percent in the first case, because the equal per capita tax that finances the additional spending absorbs their higher labor income, and gain 0.21 percent in the second. Which group gains thus depends on the fiscal rule and the distribution of income; the aggregate result does not.

The comparison is between steady states. It does not include the transition between them, in which capital is accumulated or run down, and it treats exogenous spending as not valued by the family.

Table B.1: Steady State Welfare Gains from Tax Ironing

	Representative family		Two households	
Marginal tax rate at the base income, $M(x_C)$ (%)	G/Y fixed	G fixed	G/Y fixed	G fixed
0	−0.10	0.14	−0.09	0.15
10	0.02	0.25	0.02	0.26
28.38 (baseline)	0.23	0.45	0.23	0.46
45	0.42	0.63	0.42	0.64
$M(x_C)$ above which the gain is positive (%)	8.55	–	7.84	–

Notes: Consumption equivalent in percent. Two households: social welfare is the population-weighted sum of the utilities of the two households. –: positive at every level from 0 to 60 percent.

Appendix C Measurement Check: The Income Range of the Grid

The measurement of Section 2.3 evaluates the schedule of each cell on a log grid of wages whose end points are set from unweighted percentiles of compensation x, and it normalizes the income-weighted density over the evaluated range of x. Because x exceeds wages by the employer share of payroll taxes, the evaluated range lies somewhat above the amounts used to set the end points. Within each cell, about 2 to 8 percent of wage and salary income belongs to units above the evaluated range and less than 0.1 percent to units below it. This is distinct from the coverage of the 14 cells, which contain 99.99 percent of wage and salary income.

Tables C.1 and C.2 compare the results when only the income range of the grid changes. The cells, the 2015 law, the representative tax units, the income and tax definitions, the weights, the bandwidths, the grid step, the detection of breakpoints and the combination (35) are held fixed. G0 is the current range. G2 extends the grid of each cell to the observed range of compensation in the cell; part of the estimated kernel density still lies outside it. G3 extends G2 by four bandwidths at each end and contains almost all of the estimated density; outside the observed range it extrapolates the density and adds no observations. Most of the change comes from the normalization of the density: the wider ranges add high incomes, where the slopes and jumps of the schedule are smaller than on average, and both measures fall.

For each range I solve the RE model and the tax-ironing model again at the posterior mean parameters of the tax-ironing model, the parameter values of Table 6, and change only the cell values, which (35) combines at the posterior mean of χ. The ironers' labor income share s_I, the steady states and the decision rules of both models are recomputed; the other parameters, the shock processes and the observation equations are unchanged. Panel A, with the current range, reproduces Table 6. The comparison is a sensitivity of the measurement at fixed parameters, not a result of re-estimation, and the estimation and the results of Section 4 use the cell values of the current range.

The level effect falls from about 1.05 to 1.03 percent when the range is extended. The effects on business cycle moments change little: in every panel of Table C.2 the largest change in a standard deviation is about 0.11 percent, in price inflation, and the largest changes in the autocorrelations and in the correlations with output growth are below 10^{-3}. Widening the income range keeps a level effect of about one percent and small effects on business cycle moments.

Table C.1: Steady State Effects of Tax Ironing by Evaluation Range

	RE	Tax-ironing	Effect (%)
Panel A. G0: current range, $\tau_M = 0.0810$, $\tau_A = 0.1147$			
Hours, output, consumption, investment and capital	100.00	101.05	1.05
Real wage	100.00	100.00	0.00
Steady state welfare (CEV)	–	–	0.23
Panel B. G2: observed range of compensation, $\tau_M = 0.0784$, $\tau_A = 0.1126$			
Hours, output, consumption, investment and capital	100.00	101.03	1.03
Real wage	100.00	100.00	0.00
Steady state welfare (CEV)	–	–	0.22
Panel C. G3: G2 plus four bandwidths at each end, $\tau_M = 0.0780$, $\tau_A = 0.1122$			
Hours, output, consumption, investment and capital	100.00	101.03	1.03
Real wage	100.00	100.00	0.00
Steady state welfare (CEV)	–	–	0.22

Notes: Welfare: consumption equivalent as in Table 6, with $M(x_C) = 0.2838$ at the base income of each range.

Table C.2: Effects of Tax Ironing on Business Cycle Moments by Evaluation Range

	SD			AR(1)			Corr. with output		
	RE	TI	Change (%)	RE	TI	Diff. ($\times 10^{-3}$)	RE	TI	Diff. ($\times 10^{-3}$)
Panel A. G0: current range									
$100\Delta \log Y_t$	0.73	0.73	−0.01	0.48	0.48	−0.02	1.00	1.00	–
$100\Delta \log C_t$	0.79	0.79	0.02	0.58	0.58	−0.01	0.57	0.57	0.03
$100\Delta \log I_t$	4.24	4.24	−0.01	0.58	0.58	−0.04	0.54	0.54	−0.25
$100\Delta \log W_t$	0.92	0.92	0.04	0.11	0.11	0.25	0.15	0.15	−0.14
$100 \log \ell_t$	2.00	2.00	−0.05	0.96	0.96	−0.02	0.07	0.07	0.06
$100\Delta \log P_t$	0.24	0.24	0.11	0.65	0.65	0.81	−0.11	−0.11	−0.16
$100 \log R_t^n$	0.46	0.46	0.11	0.97	0.97	0.04	−0.12	−0.12	−0.07
Panel B. G2: observed range of compensation									
$100\Delta \log Y_t$	0.73	0.73	−0.01	0.48	0.48	−0.02	1.00	1.00	–
$100\Delta \log C_t$	0.79	0.79	0.02	0.58	0.58	−0.01	0.57	0.57	0.03
$100\Delta \log I_t$	4.24	4.24	−0.01	0.58	0.58	−0.04	0.54	0.54	−0.26
$100\Delta \log W_t$	0.92	0.92	0.04	0.11	0.11	0.25	0.15	0.15	−0.14
$100 \log \ell_t$	2.00	2.00	−0.05	0.96	0.96	−0.02	0.07	0.07	0.06
$100\Delta \log P_t$	0.24	0.24	0.11	0.65	0.65	0.82	−0.11	−0.11	−0.16
$100 \log R_t^n$	0.46	0.46	0.11	0.97	0.97	0.04	−0.12	−0.12	−0.07
Panel C. G3: G2 plus four bandwidths at each end									
$100\Delta \log Y_t$	0.73	0.73	−0.01	0.48	0.48	−0.02	1.00	1.00	–
$100\Delta \log C_t$	0.79	0.79	0.02	0.58	0.58	−0.01	0.57	0.57	0.03
$100\Delta \log I_t$	4.24	4.24	−0.01	0.58	0.58	−0.04	0.54	0.54	−0.25
$100\Delta \log W_t$	0.92	0.92	0.04	0.11	0.11	0.25	0.15	0.15	−0.14
$100 \log \ell_t$	2.00	2.00	−0.05	0.96	0.96	−0.02	0.07	0.07	0.06
$100\Delta \log P_t$	0.24	0.24	0.11	0.65	0.65	0.82	−0.11	−0.11	−0.16
$100 \log R_t^n$	0.46	0.46	0.11	0.97	0.97	0.04	−0.12	−0.12	−0.07

Notes: Columns and units as in Panel B of Table 6.

Appendix D Sensitivity to the Share of Ironers

The baseline sets the population share of full ironers to $\theta = 0.43$, the estimate of Rees-Jones and Taubinsky (2020). A larger share of ironers is expected to increase the departure from the RE model. Tables D.1 and D.2 therefore repeat the comparison of Table 6 at $\theta = 0.62$, the upper end of the 95 percent confidence interval of this estimate, which Rees-Jones and Taubinsky also use in their own revenue and welfare calculations. Both models are solved at the posterior mean parameters of the tax-ironing model, with τ_M and τ_A at the posterior mean of χ, and only θ changes. For $\theta = 0.62$, the ironers' labor income share s_I, the steady state and the decision rules of the tax-ironing model are recomputed. The RE model has no ironers and is the same in both comparisons. Panel A, with $\theta = 0.43$, reproduces Table 6. The larger share is an alternative value of a fixed input taken from the micro evidence; the comparison is not a re-estimation of the model, and it does not give a confidence interval for the effects in this model.

Ironers earn 2.46 percent more than correct perceivers in both cases, and s_I rises from 43.60 to 62.57 percent. The level effect rises from 1.05 to 1.52 percent. With $\theta = 0.62$, the largest change in a standard deviation, in price inflation, is 0.16 percent instead of 0.11 percent. The largest change in an autocorrelation, also in price inflation, is 1.17×10^{-3} instead of 0.81×10^{-3} and exceeds 10^{-3}. The largest change in a correlation with output growth, in investment growth, is 0.36×10^{-3} instead of 0.25×10^{-3}. These changes remain small relative to the moments themselves.

Table D.1: Steady State Effects of Tax Ironing by Share of Ironers

	RE	Tax-ironing	Effect (%)
Panel A. $\theta = 0.43$*, baseline*			
Hours, output, consumption, investment and capital	100.00	101.05	1.05
Real wage	100.00	100.00	0.00
Steady state welfare (CEV)	–	–	0.23
Panel B. $\theta = 0.62$*, upper end of the 95 percent confidence interval*			
Hours, output, consumption, investment and capital	100.00	101.52	1.52
Real wage	100.00	100.00	0.00
Steady state welfare (CEV)	–	–	0.33

Notes: Welfare: consumption equivalent as in Table 6, with $M(x_C) = 0.2838$ at the base income of each case.

Table D.2: Effects of Tax Ironing on Business Cycle Moments by Share of Ironers

	SD			AR(1)			Corr. with output		
	RE	TI	Change (%)	RE	TI	Diff. ($\times 10^{-3}$)	RE	TI	Diff. ($\times 10^{-3}$)
Panel A. $\theta = 0.43$									
$100\Delta \log Y_t$	0.73	0.73	−0.01	0.48	0.48	−0.02	1.00	1.00	–
$100\Delta \log C_t$	0.79	0.79	0.02	0.58	0.58	−0.01	0.57	0.57	0.03
$100\Delta \log I_t$	4.24	4.24	−0.01	0.58	0.58	−0.04	0.54	0.54	−0.25
$100\Delta \log W_t$	0.92	0.92	0.04	0.11	0.11	0.25	0.15	0.15	−0.14
$100 \log \ell_t$	2.00	2.00	−0.05	0.96	0.96	−0.02	0.07	0.07	0.06
$100\Delta \log P_t$	0.24	0.24	0.11	0.65	0.65	0.81	−0.11	−0.11	−0.16
$100 \log R_t^n$	0.46	0.46	0.11	0.97	0.97	0.04	−0.12	−0.12	−0.07
Panel B. $\theta = 0.62$									
$100\Delta \log Y_t$	0.73	0.73	−0.02	0.48	0.48	−0.03	1.00	1.00	–
$100\Delta \log C_t$	0.79	0.79	0.02	0.58	0.58	−0.01	0.57	0.57	0.04
$100\Delta \log I_t$	4.24	4.24	−0.02	0.58	0.58	−0.05	0.54	0.54	−0.36
$100\Delta \log W_t$	0.92	0.92	0.06	0.11	0.11	0.36	0.15	0.15	−0.20
$100 \log \ell_t$	2.00	2.00	−0.07	0.96	0.96	−0.02	0.07	0.07	0.09
$100\Delta \log P_t$	0.24	0.24	0.16	0.65	0.65	1.17	−0.11	−0.11	−0.23
$100 \log R_t^n$	0.46	0.46	0.15	0.97	0.97	0.06	−0.12	−0.12	−0.10

Notes: Columns and units as in Panel B of Table 6.

Appendix E The Two-Household Extension

In the baseline, ironers and correct perceivers are members of one representative family that pools their labor income. This appendix treats them as two separate households, with population shares of 43 and 57 percent. Each household makes its own consumption and saving decisions, with habit in its own consumption, and its members set wages on the basis of the household's own marginal utility of wealth. Misperception still affects only wage setting.

Separating the households requires three assumptions that the baseline does not need. First, the households trade a one-period nominal bond in zero net supply. Steady state bond holdings are zero, and a small quadratic holding cost keeps the bond position stationary; the results are insensitive to its size. Second, a common fund owns capital and firms, and both households hold equal per capita shares in it. The fund makes its investment decisions with the population-weighted marginal utility of wealth of the two households. Third, labor income tax revenue is rebated lump sum to the household that pays it, and government purchases are financed by equal per capita lump-sum taxes. The structural parameters and shock processes are those of the baseline, at the posterior mean of the tax-ironing model, and the model is not re-estimated.

Tables E.1 and E.2 repeat the comparison of Table 6 for the baseline and for the two-household model. Without income pooling, ironers consume 2.4 percent more than correct perceivers in the steady state.

The resulting income effect lowers the hours of ironers relative to the baseline and raises those of correct perceivers, and the two changes nearly offset in the aggregate. The level effect is 1.0518 percent instead of 1.0515 percent. The effects on business cycle moments change only in the fourth decimal place of the changes in standard deviations, and the differences in autocorrelations and in correlations with output growth change by less than 10^{-5}. Because the two households have different marginal utilities of wealth, the aggregation of Section 4.4 holds only approximately in this model, but the departure is negligible.

The aggregate results hardly change, but the two types are affected differently. Relative to the RE model, ironers work 2.40 percent more and consume 2.43 percent more, while the hours and consumption of correct perceivers rise by only 0.03 and 0.02 percent (Table E.1). Higher consumption is not the same as higher welfare, because ironers also work more. At the baseline level of $M(x_C)$, with spending proportional to output, ironers gain 0.55 percent of consumption and correct perceivers lose 0.01 percent (Appendix B).

Table E.1: Steady State Effects of Tax Ironing by Household Structure

	RE	Tax-ironing	Effect (%)
Panel A. Baseline: one family, pooled labor income			
Hours, output, consumption, investment and capital	100.00	101.05	1.0515
Real wage	100.00	100.00	0.0000
Steady state welfare (CEV)	–	–	0.2270
Ironers' share of labor income (%)	0.00	43.60	–
Hours per ironer	100.00	102.73	2.7339
Hours per correct perceiver	100.00	99.79	−0.2145
Consumption per ironer	100.00	101.05	1.0515
Consumption per correct perceiver	100.00	101.05	1.0515
Panel B. Two households, no income pooling			
Hours, output, consumption, investment and capital	100.00	101.05	1.0518
Real wage	100.00	100.00	0.0000
Steady state welfare (CEV)	–	–	0.2318
Ironers' share of labor income (%)	0.00	43.48	–
Hours per ironer	100.00	102.40	2.4023
Hours per correct perceiver	100.00	100.03	0.0349
Consumption per ironer	100.00	102.43	2.4261
Consumption per correct perceiver	100.00	100.02	0.0150

Notes: Without ironing the two households are identical, so the RE columns are the same in both panels. Per ironer and per correct perceiver: members of the two groups, which perceive correctly in the RE model. Welfare in Panel B: population-weighted sum of the utilities of the two households.

Table E.2: Effects of Tax Ironing on Business Cycle Moments by Household Structure

	SD			AR(1)			Corr. with output		
	RE	TI	Change (%)	RE	TI	Diff. ($\times 10^{-3}$)	RE	TI	Diff. ($\times 10^{-3}$)
Panel A. Baseline: one family, pooled labor income									
$100\Delta \log Y_t$	0.73	0.73	−0.0137	0.48	0.48	−0.022	1.00	1.00	–
$100\Delta \log C_t$	0.79	0.79	0.0157	0.58	0.58	−0.006	0.57	0.57	0.028
$100\Delta \log I_t$	4.24	4.24	−0.0126	0.58	0.58	−0.038	0.54	0.54	−0.251
$100\Delta \log W_t$	0.92	0.92	0.0386	0.11	0.11	0.251	0.15	0.15	−0.140
$100 \log \ell_t$	2.00	2.00	−0.0480	0.96	0.96	−0.015	0.07	0.07	0.064
$100\Delta \log P_t$	0.24	0.24	0.1129	0.65	0.65	0.810	−0.11	−0.11	−0.159
$100 \log R_t^n$	0.46	0.46	0.1075	0.97	0.97	0.044	−0.12	−0.12	−0.071
Panel B. Two households, no income pooling									
$100\Delta \log Y_t$	0.73	0.73	−0.0137	0.48	0.48	−0.023	1.00	1.00	–
$100\Delta \log C_t$	0.79	0.79	0.0153	0.58	0.58	−0.006	0.57	0.57	0.029
$100\Delta \log I_t$	4.24	4.24	−0.0127	0.58	0.58	−0.038	0.54	0.54	−0.249
$100\Delta \log W_t$	0.92	0.92	0.0385	0.11	0.11	0.250	0.15	0.15	−0.139
$100 \log \ell_t$	2.00	2.00	−0.0477	0.96	0.96	−0.015	0.07	0.07	0.064
$100\Delta \log P_t$	0.24	0.24	0.1127	0.65	0.65	0.808	−0.11	−0.11	−0.159
$100 \log R_t^n$	0.46	0.46	0.1072	0.97	0.97	0.044	−0.12	−0.12	−0.072

Notes: Columns and units as in Panel B of Table 6. The RE columns are the same in both panels.