CIGS Working Paper Series No. 25-006E

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February 4, 2025

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Monetary policy over the lifecycle[*][†]

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Abstract

We propose a quantitative model of monetary policy over the lifecycle with endogenous portfolio choices. Our model reproduces the average age profiles of asset portfolios, the empirical responses of aggregate variables, and the microeconomic responses of different age groups to a tightening in monetary policy. Households disagree about the benefits of a tighter monetary policy. Consumption and welfare of older age groups increase, but consumption and welfare of younger households fall.

Keywords: Monetary policy, Intergenerational redistribution, Lifecycle, Portfolio choice, Nominal government debt, Tobin effect.

Declaration of Interests: None.

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[*]These are our personal views and not those of the Bank of Japan.

[†]We thank Kosuke Aoki, Adrien Auclert, Marcus Hagedorn and Felix Wellschmied for their detailed comments. We also thank Fumio Hayashi, Jordi Galí, Carlos Garriga, Charles Horioka, Kosuke Aoki, Sagiri Kitao and seminar and conference participants at the Bank of Japan, FRB Kansas City, FRB Atlanta, Keio University, 2022 SED Meetings, the University of Virginia, the Swiss National Bank and the 23rd Annual Macro Conference. Braun is also a Senior Fellow at CIGS and part of his work on this project was done while employed by the Federal Reserve Bank of Atlanta. Braun also acknowledges support from JSPS KAKENHI Grant Number 24K04938.

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1 Introduction

This paper analyzes the impact of monetary policy on different age groups in a quantitative lifecycle framework with endogenous portfolio choices. Our results indicate that a tightening in monetary policy benefits older households but has negative effects on welfare of younger working-age households.

To understand why household exposures to monetary policy might differ by age of the household head, consider the situation of workers at two different stages of the lifecycle. On average, young working-age households have low earnings and low net worth and hold leveraged long positions in illiquid assets. They acquire loans from banks and use the proceeds to purchase tangible assets such as homes, cars, and televisions. Older working-age households, on the contrary, have high earnings, high net worth, and diversified portfolios of liquid and illiquid assets.

To illustrate why monetary policy has heterogeneous effects on different age groups, consider next how a tightening in monetary policy affects their situation. An increase in the policy interest rate increases nominal and real interest rates on liquid assets like deposits and government debt, increases borrowing expenses, and has a negative impact on the performance of stock and real estate markets. Subsequently, labor market conditions weaken and the growth rates of consumption, investment, and output decline. Empirical evidence for this narrative is provided by the large empirical literature on identified monetary policy shocks. Christiano et al. (1999) provides an overview of the structural vector autoregression (SVAR) literature, while Swanson (2024) summarizes the literature on high frequency identification (HFI) and we provide evidence in support of this narrative here.

According to this narrative, tighter monetary policy is bad news for younger households. Their borrowing expenses are higher, their debt burden has increased, the value of their tangible assets has fallen, and their labor earnings are depressed. Older working-age households and retirees, in contrast, are less impacted because they have more wealth, less debt, and less exposure to labor market risk. In fact, they may even benefit from a tightening in monetary policy if the share of liquid assets in their portfolio is sufficiently large.

To formalize and quantify these arguments, we start by providing empirical evidence that household portfolio exposures to monetary policy vary with age. Then we propose a model with endogenous portfolio choices that allows us to be more precise about the magnitude and persistence of intergenerational redistribution.[1] A surprise tightening in

[1] Our data is repeated cross-sections and data on holdings of tangible assets is only available at five year

monetary policy in our model is consistent with the above narrative. Consumption of a 31 year-old household in the impact year falls by 0.34 percent but consumption of a 71 year-old household increases by 0.09 percent. The 31 year-old experiences declines in labor income and capital losses on its leveraged holdings of illiquid assets, while, the 71 year-old benefits from a capital gain on its portfolio which includes large holdings of liquid assets.

In our model of endogenous portfolio choice, monetary policy has a persistent impact on interest rate spreads and changes household portfolio choice decisions. Household exposures to monetary policy differ by age and their investment strategies for reacting to a tighter monetary policy also depend on age. Older households' portfolio adjustments expand their investment opportunities, but investment opportunities of younger households contract and welfare increases for the former group but falls for the latter group.

According to our analysis, it is wise to ask "Whose consumption response?" when considering the effect of monetary policy on consumption. A tightening in monetary policy has heterogeneous impacts on household consumption. Aggregate consumption declines, but consumption of older age groups goes up instead, and it follows that aggregate consumption understates the large negative impact that the shock has on consumption of younger working-age households.

Monetary policy is special in this regard because a higher real interest rate on deposits and a smaller liquidity premium benefit savers but harm borrowers. Shocks to aggregate supply and demand also induce intergenerational redistribution but have qualitatively similar effects on the consumption of different age groups. For instance, consumption of all households increases in the short-run when technology improves in our model.

Our model also offers new insights into the transmission channels of monetary policy in the short-run and medium-term. Nominal price rigidities are particularly important for understanding the short-run effects of monetary policy on aggregate economic activity. Endogenous portfolios and asset substitution are the main transmission channels of monetary policy at medium-term frequencies.

Finally, our findings indicate that aggregate consumption is a poor indicator of the microeconomic impact of monetary policy shocks. Specifications of the model that have vastly different impacts on specific age groups produce essentially identical responses in aggregate consumption.

These results emerge in a general equilibrium lifecycle model that captures the main

frequencies. Thus, we use a model to analyze how shocks to monetary policy affect wealth accumulation of different age groups over their remaining life cycle.

features of household labor supply, saving, and portfolio choices over the lifecycle. The age profile of real wages is hump-shaped and individuals face idiosyncratic mortality risk that increases with age. Households make optimal portfolio choice decisions that reflect age-specific trade-offs between expected returns and their expected demand for liquidity.

Governments in industrialized countries provide public insurance to households for risks that are concentrated at the end of their lifecycle. We model a fiscal authority that taxes households on multiple margins and uses part of the proceeds to fund a pay-as-you-go public pension program.

Monetary policy affects the real economy in three ways. Nominal prices are costly to adjust, as in Rotemberg (1996), and monetary policy crowds out private capital formation via an asset substitution or Tobin effect channel. As in Hu et al. (2021), a surprise tightening of the nominal interest rate increases the real return on government debt and crowds out private capital formation even when prices are flexible. Finally, changes in the spread of interest rates on capital and government debt are persistent in our model even when government debt is a real asset because it is costly for households to adjust their holdings of illiquid assets (see also Luetticke, 2021).

Related literature One of the biggest reasons households save is to provision for retirement. Retirement is costly to reverse and mortality and medical expense risks are concentrated at the end of the lifecycle. The lifecycle framework is widely used in the household finance literature to analyze optimal saving and portfolio choice decisions (Gomes et al., 2021) and since the seminal work of Auerbach and Kotlikoff (1987), the general equilibrium lifecycle model has become the workhorse framework of macro public finance to analyze the role of public insurance for life expectancy and medical expense risks. Our lifecycle framework embeds the endogenous savings and endogenous portfolio decisions of household finance in a general equilibrium framework with public pensions and introduces nominal frictions and a monetary authority into the economy in a similar way to the Heterogeneous Agent New Keynesian (HANK) framework of Kaplan et al. (2018) by introducing five new parameters into a standard lifecycle model of the real side of the economy.[2] Two parameters govern the distinction between liquid and illiquid assets. The nominal side of the economy is determined by the adjustment costs on prices of intermediate goods producers

[2]It is an open question whether these distinct frameworks can be successfully linked because the HANK literature abstracts from the retirement savings decision and models uninsured earnings risk instead and it is unclear whether the empirical successes of these models obtain when the savings incentive is different.

(one parameter) and the central bank's nominal interest rate rule (two parameters). This parsimonious specification of liquidity, nominal price rigidities, and monetary policy allows the model to reproduce the empirical age profiles of net worth and liquid and illiquid asset holding, the empirical SVAR responses of aggregate variables to a tighter monetary policy, and microeconomic responses of cohort-specific consumption and disposable income to the same shock.

Our paper is related to the large theoretical literature on monetary policy in overlapping generation (OLG) models as in Wallace (1980). Recent papers, including Hu et al. (2021) and Sterk and Tenreyro (2018), have analyzed monetary policy in tractable OLG models and Angeletos et al. (2023) consider the interaction of public debt issue and monetary policy in a related finite horizon framework. Our OLG model reproduces a range of microeconomic and macroeconomic observations and fiscal and monetary policy jointly determine the price level. So it is a useful framework for analyzing the quantitative significance of the lifecycle for the transmission channels of monetary and fiscal policy.

Previous work has focused on subsets of four household risk exposures described by Auclert (2019). Kaplan et al. (2018) analyze substitution and household balance sheet exposures but abstract from unexpected inflation exposures. Hagedorn (2018) emphasizes that monetary and fiscal policy jointly determine the price level in incomplete market models with nominal government debt. Doepke and Schneider (2006) analyze unexpected inflation exposures by age of a household and Aoki et al. (2022) model how inflation influences household portfolio choices. Other research has analyzed how changes in real interest rate spreads influence household portfolio choices of liquid and illiquid assets. Berger et al. (2017) analyze how consumption decisions respond to changes in the real return on illiquid (housing) assets in a lifecycle framework. Garriga and Hedlund (2020) analyze the joint exposure of individual real asset cash flows and labor income cash flows in an infinite-horizon real economy with illiquid assets and leverage. Finally, Wong (2019) and Garriga et al. (2021) analyze nominal and real cash flow exposures to monetary policy. A merit of our framework is that all four exposures are active, and we analyze the individual and joint contributions of income, intertemporal substitution, inflation, and unhedged real interest rate risk to consumption at each point of the lifecycle.

Our paper is close in spirit to Glover et al. (2020) and Hur (2018), who find that the Great Recession produced persistent intergenerational redistribution. Both of these papers model endogenous household portfolio choice decisions for two assets, and households face aggregate risk. However, this additional detail in the risk environment creates trade-offs.

Hur (2018) models household asset allocation decisions to risky and safe assets in partial equilibrium and assumes that a model period is three years. Glover et al. (2020) assume that a model period is ten years, markets are complete, and the two assets are priced by the stochastic discount factor. They also hold the aggregate capital stock fixed and abstract from the nominal side of the economy. Our model period is one year, aggregate capital formation is endogenous, and we directly solve for the optimal portfolios of each household. Our model exhibits a rich array of interactions between real and nominal economic activity and fiscal and monetary policy that are absent in these two papers.

Our paper is also related to a recent paper by Bielecki et al. (2022) that analyzes monetary policy in a New Keynesian (NK) OLG framework with housing. Their model has more assets, but abstracts from the liquidity frictions that make portfolio choice endogenous in our model. In our model, households reduce their holdings of illiquid assets and make particularly large and heterogeneous adjustments to their holdings of government bonds when monetary policy is tightened. Households in Bielecki et al. (2022) participate in the government debt market, but cannot adjust their holdings of bonds when the central bank changes the interest rate on bonds. Not surprisingly, our model has different implications for how and why monetary policy affects the aggregate economy. These distinctions matter for the empirical performance of the model. Our model reproduces a broader range of empirical macro-responses including the relative magnitudes of the responses of consumption, output, investment. Our model is consistent with new empirical micro-evidence we provide on how consumption and disposable income of different age-groups respond to monetary policy.

Our model of household-level portfolio choice offers a novel resolution to puzzling properties of other HANK models. Countercyclical profits can produce counterfactual hours and output responses (Broer et al., 2020) and induce countercyclical responses in investment and stock prices (Kaplan et al., 2018) in response to monetary policy shocks. Common remedies include modeling habit persistence in consumption, adjustment costs on aggregate investment, nominal wage rigidities, and restricting how profits are distributed to shareholders. We abstract from consumption habits and aggregate adjustment costs on investment. Wages are also flexible, and we impose no constraints on the distribution of profits. Still, investment, hours, and stock prices all fall in our model when monetary policy is tightened.

The remainder of the paper is organized as follows. Section 2 motivates our analysis of monetary policy over the lifecycle and provides some intuition for the asset substitution channel of monetary policy. Section 3 describes the model and Section 4 explains how we parameterize the model. Section 5 reports the aggregate responses to a monetary policy shock in our model and in Japanese data. Section 6 documents the microeconomic properties of the model. We examine the differential impacts of monetary policy on consumption and welfare of different age groups and investigate the mechanisms through which monetary policy influences intergenerational consumption and wealth inequality. Section 7 links the microeconomic responses to the aggregate responses and documents the individual contributions of fiscal policy, nominal price frictions, and the asset substitution channel for our results. Section 8 conducts a robustness analysis and Section 9 contains our concluding remarks.

2 Motivation

2.1 Net worth, liquid and illiquid assets by age

The objective of this paper is to analyze how and why a household's exposure to monetary policy depends on its age. We motivate this analysis by showing that the size and composition of household net worth exhibits large variation over the lifecycle in Japanese data. Japan is interesting because the Japanese National Survey of Family Income and Expenditure (NSFIE), a nationally representative survey conducted every 5 years, provides detailed data on household holdings of tangible assets, including real estate and durable goods, as well as a range of financial asset categories by 10-year age group.

Table 1 reports our calculations of household net worth and holdings of liquid and illiquid assets by age group using data from the 2014 NSFIE. Age refers to the age of the household head and asset holdings are relative to peak pretax income over the lifecycle, which occurs in the group aged 50–59. Liquid assets are primarily deposits, and illiquid assets consist of less liquid real and financial assets like residential real estate and equity. Our strategy for classifying assets into these two categories follows Kaplan et al. (2018) with an important exception. We assign all household borrowing to liquid assets.[3] Thus, liquid asset holdings are net of all household borrowing, and illiquid asset holdings are gross. This way of organizing the data is consistent with the structure of our model.

[3] According to the April 2023 Bank of Japan Financial System report 64 percent of outstanding mortgages had floating interest rates in fiscal year 2015. By September 2022 the figure had risen to 75 percent. Prepayment penalties on Japanese residential mortgages are rare, but handling fees of between $50 and $100 are commonly charged for each prepayment. For these reasons, we treat mortgages as liquid liabilities. Appendix C provides more details on how we decompose total assets into liquid and illiquid categories.

Table 1: Household net worth, liquid and illiquid asset holdings by age in Japan

Age	Net worth	Liquid assets	Illiquid assets
Under 30	0.65	-0.08	0.73
30–39	1.60	-0.58	2.18
40–49	2.58	-0.31	2.90
50–59	4.52	0.76	3.76
60–69	6.29	1.70	4.60
70+	6.01	1.77	4.25

Note: Liquid assets are net of total borrowing and illiquid assets are gross. Net worth and assets are divided by peak income of the 50–59 year old age group. The main data source is the 2014 NSFIE. More details on the construction of the data can be found in Appendix C.

Table 1 has four main properties. First, net worth is hump-shaped. It increases steadily during working ages, reaching a zenith of more than six times peak income. Full public pensions become available at age 65 and the 60–69 age group has the highest net worth. Net worth then falls as households move into retirement and draw down their savings. Second, households aged 49 years or under have negative net holdings of liquid assets but positive holdings of illiquid assets. In other words, they are taking leveraged long positions in illiquid assets. Third, older households have positive holdings of liquid and illiquid assets. Fourth, net worth falls towards the end of life, but the decline is gradual. Net worth of Households in the 70+ age group is 6 times peak income.[4]

Households' other sources of income also vary by age. Real wages initially increase with age, peak around age 50, and then decline rapidly after age 55 (see Braun and Joines 2015).[5] After retirement public pensions are an important source of household income.

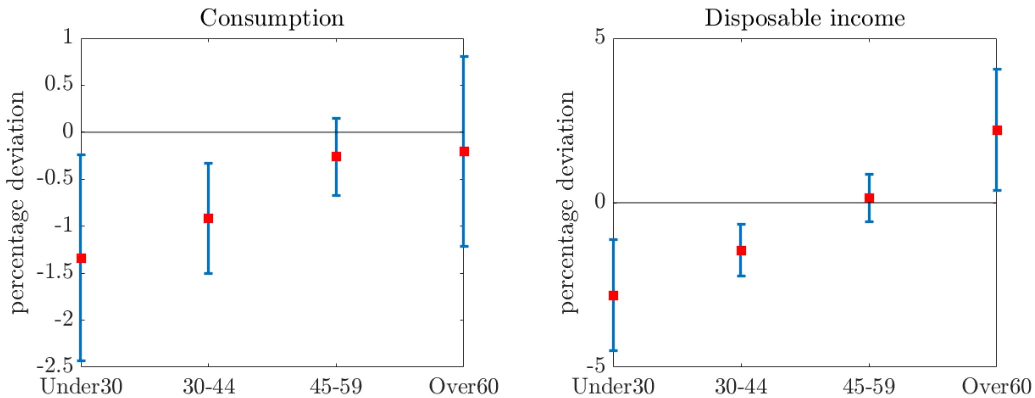
2.2 Household responses to monetary policy shocks by age

Monthly Japanese data on consumption and disposable income by age group are reported in the Family Income and Expenditure Survey (FIES), a separate nationally representative

[4]Our NSFIE data includes both couples and multi-generational households. Results reported in Murata (2019) suggest that net worth declines more rapidly with age in nuclear households consisting of an elderly couple.

[5]Many Japanese companies impose mandatory retirement at either 55 or 60 years of age on regular employees. Companies are required to offer a fixed-term contract after mandatory retirement until the employee qualifies for public pension benefits at the age of 65. However, salary and fringe benefits of re-employment contracts are often less attractive. A ruling by the Japanese supreme court in 2019 largely reaffirmed this practice (see Puckett 2019).

Figure 1: Responses by age group to a tightening in monetary policy



Note: Cumulative responses in the impact year; monthly Japanese data (FIES); the monetary policy shocks identified by Kubota and Shintani (2022); the sample period of 1992–2019. Vertical lines are 95 percent confidence intervals.

survey. The FIES has a smaller sample size and less comprehensive coverage of tangible assets compared to the NSFIE.[6] We use FIES data to analyze how the consumption and disposable income of different age groups respond to monetary policy shocks. Figure 1 reports the cumulative responses in the impact year of consumption and disposable income to a tightening of monetary policy.[7] The sample period is 1992–1999, consumption consists of nondurables plus services net of social expenditures, and the monetary policy shocks are taken from Kubota and Shintani (2022). They use a high-frequency identification (HFI) scheme similar to Gürkaynak et al. (2005) and estimate monetary policy shocks by measuring the response of high-frequency interest rate futures data to announced changes in monetary policy. More details on the construction of the data and empirical methodology can be found in Appendix F.

The results reported in the right panel of Figure 1 are consistent with our conjecture that household exposures to monetary policy vary by age and the estimated responses have relatively tight standard errors. Income falls sharply when monetary policy is tightened for households under age 30 and in the 30–44 age group. The magnitude of the decline in disposable income is smaller for older age groups and is about zero for households in the age 45–59 age group. Disposable income increases in the age 60+ group. This final result should be interpreted with some caution, because our FIES sample consists of working households, and many Japanese households in the 60+ age group are omitted from our sample because they are retired.

[6]The FIES is a nationally representative rotating panel that follows sampled households for 6 months. We use working households because FIES only reports monthly disposable income for working households.

[7]The responses of macroeconomic variables to this shock are reported in Section 5.

Our data do not allow us to directly measure how household hours and employment change, but labor risk is very different in Japan compared to the US. Large and small Japanese companies still offer implicit lifetime employment guarantees, and these practices are enshrined in Japanese labor laws which make it costly to fire regular employees. Braun et al. (2006) find that hours per worker are procyclical in Japan, but employment is countercyclical and the relative volatility of hours to output is much higher (0.54) than the relative volatility of employment to output (0.34). Nakamura et al. (2021) show that monetary policy shocks affect the unemployment rate of all ages by almost the same degree in Japan. Our interpretation is that Japanese households are less exposed to cyclical employment risk than US households. These empirical observations are reflected in our modeling choices. Our quantitative model allows for cyclical variation in hours per worker, but abstracts from unemployment risk.

Consumption responses by age group are reported in the left panel of Figure 1. The pattern of consumption responses is similar to that of disposable income, but the consumption responses are less precisely estimated. Consumption falls sharply in the youngest age group. This age group has the lowest net worth in Table 1 and limited access to credit.[8] Consumption of households in the 30–44 age group also exhibits large and statistically significant declines. Households in this age group have high levels of borrowing (Table 1). Finally, the consumption declines in the two older age groups are small and have very large standard errors. These age groups are better protected against shock because they have high net worth and a large share of liquid assets in their portfolios (Table 1) and the possibility of a positive consumption response cannot be excluded.

2.3 Asset-substitution channel of monetary policy

Another objective of this paper is to analyze the quantitative significance of the asset-substitution transmission channel of monetary policy. This transmission channel is either absent or weak in most other quantitative monetary models, so we illustrate how it works with a simple example. In a two-period flexible price OLG model with nominal government debt and capital accumulation along the lines of Hu et al. (2021) or Braun and Ikeda (2023), the equilibrium price level and capital stock are determined by the following two equations.

[8] In Japan, as in the US it takes time for a young household establish a credit record.

The first equation is the asset market clearing condition

$$\frac{d_t^n}{P_t} + k_{t+1} = w_t, \tag{1}$$

where d_t^n is the (exogenous) per capita stock of nominal government debt, P_t is the price level, k_{t+1} is per capita capital, $w_t = f(k_t) - k_t f'(k_t)$ is per capita income of the young, and $f(\cdot)$ is a standard production function with $f' > 0$ and $f'' < 0$. Per capita income in this model is also aggregate asset demand because households only consume in the second period of their life. The second equation is the Fisher equation

$$f'(k_{t+1}) = R_t \frac{P_t}{P_{t+1}}, \tag{2}$$

where R_t is the nominal interest rate on government debt set by the central bank, which we assume is also exogenous for the purposes of this discussion. Under the assumption that the stock of nominal government debt is fixed, Hu et al. (2021) provide a set of regularity conditions for existence and uniqueness of an equilibrium with the following properties. A temporary increase in R_t decreases k_{t+1}, which can be seen from equation (2) for given P_t/P_{t+1}. This in turn decreases P_t from equation (1) since w_t is predetermined and d_t^n is fixed. The decrease in P_t mitigates the increase in the real rate, $R_t P_t/P_{t+1}$, caused by the monetary tightening, but does not fully offset it. Thus, a higher nominal interest rate pushes the price level down and decreases private capital formation.[9] In the analysis that follows, we investigate the economic significance of the asset substitution channel in a quantitative lifecycle framework that features imperfect substitutability between government debt and illiquid assets and endogenous portfolio choices.

3 The Model

The model we propose strikes a compromise in terms of its microeconomic detail. On the one hand, it captures the main sources of income and savings over the lifecycle, and households solve dynamic portfolio allocation problems. On the other hand, it abstracts from idiosyncratic risk factors that produce cross-sectional heterogeneity within an age group such as uninsured earnings risk. Abstracting from uninsured earnings risk allows us

[9] Hagedorn (2021) refers to this model of price level determination as the Demand Theory of the Price Level (DTPL). Braun and Ikeda (2023) show that the DTPL and the Fiscal Theory of the Price Level have different implications for how the economy responds to a change in the population growth rate.

to easily discern how monetary policy affects households at different stages of the lifecycle. Furthermore, Japanese households' exposure to unemployment risk is low due to implicit lifetime employment guarantees and high firing costs (Section 2.2). Finally, there may not be all that much that a central bank can do to influence household exposures to countercyclical uninsured labor market risk. Previous work by Braun and Nakajima (2012) finds that modeling countercyclical uninsured earnings and asset income risk has a negligible effect on the properties of an optimal monetary policy.

The OLG economy considered here extends the OLG models of Braun et al. (2009) and Braun and Joines (2015) in two ways. First, households make endogenous portfolio choice decisions. They can save and/or borrow two assets that are imperfect substitutes because they offer different liquidity services. Illiquid assets such as homes and equity offer a higher steady state real return, but are costly to acquire and sell. Liquid assets offer a lower steady state return but are costless to adjust. Depending on where households are in their lifecycle, they choose to borrow liquid assets to purchase illiquid assets or hold positive amounts of both assets. Second, the model includes nominal government debt, nominal price rigidities, and a central bank that follows a nominal interest rate targeting rule.

3.1 Demographic structure

The economy has an OLG structure that evolves in discrete time with a period of one year. Let j denote the age of the individual as $j = 1, ..., J$. We start keeping track of individuals at age 21 and individuals survive until at most age 120. Thus, the model age of $j = 1$ corresponds to age 21, that of $J = 100$ corresponds to age 120 and $J = 100$ cohorts are active in a given year. We consider an economy with a stationary population distribution and no population growth. Let $N_{j,t}$ be the number of individuals of age j in period t. The population by age is constant, $N_{j,t} = N_j$, for all t, and the aggregate population is $N = \sum_{j=1}^{J} N_j$. The population of each age is defined recursively as $N_{j+1} = \psi_j N_j$, where ψ_j is the conditional probability that an individual of age j survives to the next year.[10]

[10]Note also that the unconditional probability of surviving from birth to age $j = 2, ..., J$ is given by $\Psi_j = \psi_{j-1}\Psi_{j-1}$ where $\Psi_1 = 1$ and that the population share of each cohort is given by $\mu_j = \frac{\Psi_j}{\sum_{j=1}^{J}\Psi_j}$.

3.2 Households

Individuals are organized into households. Each household consists of one individual (adult) and children. The number of children varies with the age of the adult and the age of the household is indexed by the age of the adult. Adults face mortality risk. We model state-dependent warm-glow bequests along the lines of Lockwood (2018). Let $z^i_{j,t} \in \{0, 1\}$ index the survival state for the i^{th} household where a value of zero denotes the death state. Death is the only source of idiosyncratic risk faced by households and there are only two types of households in any age cohort: surviving households ($z^i_{j,t} = 1$) and non-surviving households ($z^i_{j,t} = 0$). Households work, consume, and save. A household of age j in period t earns an after-tax wage rate of $(1 - \tau^w)w_t\epsilon_j$, where τ^w denotes a labor-income tax rate and ϵ_j is the efficiency of labor of an age-j household.[11] All cohorts face the same age-efficiency profile and the efficiency index ϵ_j is assumed to drop to zero for all $j \geq J_r$, where J_r is the mandatory retirement age.

Households provision for retirement by acquiring liquid and illiquid assets. They may save or borrow using either asset. The liquid asset earns the nominal interest rate R_{t-1} between period $t - 1$ to t and its after-tax real return is given by $\tilde{R}_{t-1}/\pi_t$, where $\tilde{R}_{t-1} \equiv 1 + (1 - \tau^a)(R_{t-1} - 1)$, $\pi_t = P_t/P_{t-1}$ and τ^a is the tax rate on asset income. The real return on the illiquid asset in period t is R^a_t and its after-tax return is $\tilde{R}^a_t \equiv 1 + (1 - \tau^a)(R^a_t - 1)$.

When we parameterize our model, we follow Glover et al. (2020) and Kaplan et al. (2018) and include tangible assets such as homes and durable goods and illiquid financial assets such as equities in our measure of illiquid assets. Following the example of this previous research, we also abstract from the service flow of utility services provided by tangible assets.[12] Thus, the benefit of holding illiquid assets is entirely pecuniary. Glover et al. (2020) model endogenous household portfolio decisions over the life cycle in a setting with shocks to aggregate technology and complete markets. Liquid and illiquid asset returns are priced off the stochastic discount factor under the assumption that technology takes on one of two states.

In our model liquid and illiquid assets are imperfect substitutes. In particular, households face costs of adjusting their holdings of illiquid assets. We assume that the adjustment

[11] Given that there is only one type of heterogeneity in a cohort, to conserve on notation we do not explicitly index the identity of each household of age j in period t in the ensuing discussion unless it is required to avoid confusion.

[12] Glover et al. (2020) and Kaplan et al. (2018) describe strategies for entertaining this possibility that leave their results intact.

cost function is quadratic and state dependent, but time and age invariant

$$\chi(a', a, z \in \{1, 0\}) = \frac{\gamma_a(z)}{2}(a' - a)^2, \tag{3}$$

where a is the level of assets that a household enters the period with and a' is the level of assets that the household leaves the period with. The state dependence arises because we assume that the costs of liquidating illiquid assets for a household that dies at the end of the current period are different from those of a household that survives until the next period. Thus, this cost function has two parameters. The first parameter $\gamma_a(1)$ governs the size of the adjustment cost for all surviving households and $\gamma_a(0)$ governs the size of adjustment costs for households who die in the current period.

We choose this specification of adjustment costs for the following reasons. First, a household's asset portfolio influences its overall exposure to monetary policy, and we want household portfolios to reflect their decisions. In our specification, their choices imply that liquid and illiquid assets have different endogenous steady state returns. In addition, the liquidity premium responds to changes in monetary policy as households strategically and dynamically adjust their holdings of the two assets.

Second, our Japanese data consist of average consumption and asset holdings for specific age groups, and the measured response of average consumption to a shock in monetary policy is smooth. Households in a given cohort in our model are also identical (apart from mortality risk) and with convex adjustment costs on illiquid assets, household consumption and asset allocation decisions respond smoothly to shocks to monetary policy.

Third, this assumption captures in a parsimonious way the frictions faced by Japanese households in acquiring and selling real estate and other illiquid assets.[13] The Japanese home ownership rate was about 61 percent in 2014. We allow the cost function to differ in the death year because the costs of liquidating real estate are high in Japan. As we explain in Appendix D.2 these costs are so significant that many inheritors have refused to claim their inheritance, and it is estimated that about 11 percent of the land mass in Japan has no (living) registered owner. The households in our model recognize this risk and adjust their portfolios accordingly as they age.

Fourth, positing convex adjustment costs facilitates the computation of household portfolios. Computing the steady state and dynamic optimal portfolios of liquid and illiquid

[13] Table 1 uses data from 2014 and the Japanese home ownership rate in that year was about 61 percent. The US home ownership rate in the same year was about 63 percent.

assets is burdensome due to the possibility of leverage. Imposing convexity avoids jumps and inaction regions in household policies that would make the computations even more delicate and ultimately less reliable.

Our assumption is different from some recent papers that model multiple assets in lifecycle frameworks. Glover et al. (2020) use age-dependent preference discount factors to pin down the steady state age profile of net worth and exogenous age-dependent parameters to pin down the steady state age profile of household holdings of risky assets because risky and safe assets are perfect substitutes in the steady state of their model. Bielecki et al. (2022) use age-dependent demand functions for housing to determine the steady state age profile of housing. They also assume that the age profiles of household (nominal) bond holdings are exogenous because bonds and other financial assets have the same real returns in the steady state of their model. As a result, households are not able to adjust their holdings of bonds when monetary policy increases the nominal interest rate. In our model, portfolio choices are endogenous and households adjust their holdings of liquid and illiquid assets when monetary policy changes. We will see in the results that follow that households choose to make large and heterogeneous adjustments to their holdings of liquid assets when monetary policy is changed if they are given the choice.

Given these definitions, the decisions of a surviving household of age-j in period t (i.e., a household with $z^i_{j,t} = 1$) are constrained by

$$\begin{aligned}(1+\tau^c)c_{j,t} &+ a_{j,t} + \chi(a_{j,t}, a_{j-1,t-1}, 1) + d_{j,t} \\ &\leq \tilde{R}^a_t a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t} d_{j-1,t-1} + (1-\tau^w) w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t(1), \end{aligned} \tag{4}$$

where $c_{j,t}$ is total household consumption for a household of age j in period t, τ^c is the consumption tax rate, $d_{j,t}$ denotes holdings of the liquid assets, expressed in terms of the final good, at the end of period t, $h_{j,t}$ denotes hours worked and $b_{j,t}$ denotes public pension (social security) benefits. A surviving household also receives a lump-sum transfer, $\xi_t(z)$, which consists of two components: a government transfer $\bar{\xi}_t$ that is received by all households and a bequest that is only received by the surviving households. Finally, $\chi(\cdot)$ is the cost of adjusting holdings of the illiquid asset.[14] Japanese households have to build a credit history before they can qualify for a loan to purchase an automobile and/or a home.

[14] We are omitting the dependence of individual choices on the survival event to save on notation. Formally, we have for $z^i_{j,t} \in \{0,1\}$: $c_{j,t}(z^i_{j,t})$, $b^q_{j,t}(z_{j,t})$, $a_{j,t}(z^i_{j,t})$, $d_{j,t}(z^i_{j,t})$, and $h_{j,t}(z^i_{j,t})$. In what follows, this dependence is only made explicit when necessary.

We recognize this constraint in the model by preventing households between the ages of 21 and 23 from borrowing.[15] Individuals aged 24 and older are free to borrow against their future earnings and they are also free to take leveraged long positions on illiquid assets, which, in equilibrium, have a higher return than liquid assets. Households of age $j \geq 24$ are constrained by the natural borrowing constraint. That is, households cannot borrow more today than they can repay if they experience the death event in the subsequent period.

A household that experiences the death event ($z^i_{j,t} = 0$) liquidates its holdings of illiquid assets and leaves its cash at hand as a bequest given by

$$b^q_{j,t} = \frac{1}{1+\tau^{b^q}} \Bigg(\tilde{R}^a_t a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t} d_{j-1,t-1} + (1-\tau^w) w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t(0) - \chi(0, a_{j-1,t-1}, 0) \Bigg). \tag{5}$$

where $b^q_{j,t}$ is the size of the bequest, τ^{b^q} is the tax rate in bequests and $\xi_t(0) = \bar{\xi}_t$ is a lump-sum transfer from the government. We assume that bequests are lump–sum transferred to all surviving households in the economy, so that the lump-sum transfer is given by

$$\xi_t(z) = \begin{cases} \bar{\xi}_t + B^q_t / (\sum_{j=1}^J \psi_j N_{j,t}), & \text{for } z = 1 \\ \bar{\xi}_t, & \text{for } z = 0 \end{cases} \tag{6}$$

where $B^q_t = \sum_{j=1}^J (1-\psi_j) b^q_{t,j} N_{j,t}$ is the aggregate bequests.

The period utility function of a household of age j in period t that survives to the next period is given by

$$u(c_{j,t}, b^q_{j,t}, h_{j,t}; \eta_j, z_{j,t} = 1) = \frac{\eta_j \left(c_{j,t}/\eta_j\right)^{1-\sigma}}{1-\sigma} - \frac{\upsilon}{1+1/\nu} h^{1+1/\nu}_{j,t}, \tag{7}$$

where $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution, $\nu > 0$ governs the Frisch elasticity of labor supply, $\upsilon > 0$ is a labor disutility parameter, and η_j is a family scale, which we assume is time-invariant. Thus, children influence the age profile of household consumption demand.

[15]This choice of ages allows the model to reproduce average borrowings of households in the 21-30 age group in our data.

The period utility function for a household that dies in the current period is:

$$u(c_{j,t}, b^q_{j,t}, h_{j,t}; \eta_j, z_{j,t} = 0) = \omega \frac{\eta_j \left(b^q_{j,t}/\eta_j\right)^{1-\sigma}}{1-\sigma} - \frac{\upsilon}{1+1/\nu} h_{j,t}^{1+1/\nu}. \tag{8}$$

and the intensity of the bequest motive is governed by ω. The size of the bequest consists of all wealth net of the costs of liquidating the household's holdings of illiquid assets. Our timing convention implies that households work prior to discovering that this is their death year.

We assume that working-age households belong to a labor union. The union respects their marginal utilities, and wages are flexible. We analyze the symmetric equilibrium. Thus, hours worked are identical for all workers in period t, $h_{j,t} = \bar{h}_t$ for all $j < J_r$ with $\bar{h}_t$ given by

$$(1-\tau^w)\bar{\epsilon}_t w_t = \upsilon \bar{\lambda}_t^{-1} \bar{h}_t^{\frac{1}{\nu}}, \tag{9}$$

where $\bar{\lambda}_t$ is the weighted average of the marginal utilities of working households and $\bar{\epsilon}$ is the weighted average of the efficiency of labor. More details on the labor supply decision and the derivation of equation (9) can be found in Appendix A.3. This specification implies that workers who experience a shock are unable to self-insure by adjusting their hours worked differently from the average worker. It is worth pointing out that earnings vary by age because the efficiency of a worker's labor depends on the worker's age.

Under these assumptions the household's optimal choices are given by the solution to

$$\begin{aligned} U_j(a_{j-1,t-1}, d_{j-1,t-1}, z) = \max_{\{c_{j,t}, a_{j,t}, d_{j,t}, b^q_{j,t}\}} \Big[& u_j\left(c_{j,t}, b^q_{j,t}, h_{j,t}; \eta_j, z\right) + \\ & \beta \left\{(1-\psi_j) U_{j+1}\left(a_{j,t}, d_{j,t}, 0\right) + \psi_j U_{j+1}\left(a_{j,t}, d_{j,t}, 1\right)\right\} \Big] \end{aligned} \tag{10}$$

subject to equations (4) and (5) for $z_{j,t} \in \{0, 1\}$ and for $j = 1, ..J-1$, and $z_{J,t} = 0$, where $\beta > 0$ is the preference discount factor and ψ_{j+1} is the conditional probability that a household of age $j+1$ survives to the next period.[16] Note that there are no restrictions on the sign or magnitude of asset holdings beyond the natural borrowing constraint.[17]

[16]There is a theoretical possibility that adjustment costs on illiquid assets could exceed the size of beginning of period illiquid assets. Our strategy for parameterizing the adjustment costs on illiquid assets rules out this possibility.

[17]In Section 4, we impose ad hoc borrowing constraints on the three youngest age groups to capture the fact that it takes time to establish a credit record. All other age groups are constrained by the natural borrowing constraint, as described here.

In equilibrium, the return on illiquid assets exceeds the return on liquid assets, and all private borrowing is in the form of liquid assets. Appendix A.1 reports the optimality conditions for the household problem. In addition, Appendix A.2 provides an analytical characterization of the liquidity premium and optimal portfolios for a simpler version of the household problem.

3.3 Production of goods and services

The production of goods and services is organized into four sectors. The intermediate goods sector uses labor and capital inputs to produce a continuum of intermediate goods. The final goods sector uses the output of the intermediate goods sector to produce a single final output good, the capital good sector converts the final good into capital, and the mutual fund sector invests in the market portfolio of capital in intermediate goods firms.

Final good sector. Firms in this sector are perfectly competitive and combine a continuum of intermediate goods, $\{Y_t(i)\}_{i\in(0,1)}$, to produce a homogeneous final good Y_t, using the production technology, $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\theta}} di\right]^{\theta}$ with $\theta > 1$. Let $P_t(i)$ denote the price of intermediate good i, and P_t denote the price of the final good. Final good firms are price takers in input markets and it follows that demand for intermediate good i is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{\theta-1}} Y_t. \tag{11}$$

The final good is either consumed by households or used as an input in the capital good sector.

Intermediate goods sector. Firms in this sector are monopolistically competitive and each firm produces a unique good indexed by $i \in (0,1)$. Intermediate goods firm i produces $Y_t(i)$ by combining capital $K_t(i)$ and effective labor $H_t(i)$ with a Cobb-Douglas production function

$$Y_t(i) = K_t(i)^{\alpha}(Z_t H_t(i))^{1-\alpha}, \quad 0 < \alpha < 1. \tag{12}$$

Intermediate goods firm i faces the demand curve (11), and sets its price $P_t(i)$ to maximize profits subject to a quadratic price adjustment cost function.

In a symmetric equilibrium, the optimality condition for intermediate goods producers can be expressed as

$$(\pi_t - 1)\pi_t = \frac{1}{\gamma}\frac{\theta}{\theta - 1}(mc_t - 1) + \Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(\pi_{t+1} - 1)\pi_{t+1}, \tag{13}$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate and mc_t is real marginal costs in period t. When deriving this expression, the steady state marginal cost distortion was eliminated by applying an output subsidy (see Appendix A.4 for details). Equation (13) is the New Keynesian Phillips curve. It relates the current inflation rate to the real marginal cost denoted by mc_t and the future inflation rate. Next, aggregate output is

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \tag{14}$$

where K_t denotes the aggregate capital and H_t denotes the aggregate effective labor. And aggregate profits, $\Omega_t \equiv \int_{i \in (0,1)} \Omega_t(i) di$, are

$$\Omega_t = \left[\theta - mc_t - \frac{\gamma}{2}(\pi_t - 1)^2\right] Y_t. \tag{15}$$

Capital good sector. Capital good firms are perfectly competitive and use a linearly homogeneous production technology to produce capital. The representative firm purchases $(1-\delta)K_t$ units of old (depreciated) capital from the mutual fund and I_t units of the final good from the final good firms, and uses the two inputs to produce K_{t+1} units of new capital that is sold back to the mutual fund. Thus, the conventional investment identity obtains

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{16}$$

Mutual fund sector. Our economy has two types of illiquid assets – capital and shares in intermediate goods firms – and there is no aggregate uncertainty in the model after time-zero. Thus, a no-arbitrage argument implies that the return on the two illiquid assets is the same in all periods except time-zero when their returns differ if an aggregate time-zero shock occurs. We allocate ownership and the potential time-zero capital gains and losses among households by assuming that households invest in a mutual fund produced by perfectly competitive financial service firms. Each firm holds the market portfolio of the two illiquid assets and pays households the market return on illiquid assets.

To derive the market return on illiquid assets, note that the return on capital in period

t is given by

$$R_t^k = r_t^k + 1 - \delta. \tag{17}$$

The one period return from investing one unit of the period $t-1$ final good in shares is

$$R_t^v = \frac{\Omega_t + V_t}{V_{t-1}}, \tag{18}$$

where V_t is the share price. We assume that the return on capital and equity is subject to a corporate tax as well as an asset-income tax paid by households. Liquid assets, in contrast, will consist primarily of government debt in equilibrium and are taxed once at the household level. To reduce the notational burden, we assume that corporate taxes are paid by the mutual fund. Let τ^k denote the corporate tax rate. Then, perfect competition leads to the no-arbitrage conditions

$$R_t^a - 1 = (1-\tau^k)(R_t^k - 1) = (1-\tau^k)(R_t^v - 1), \tag{19}$$

for all $t > 0$. From the second equality in equation (19) the share price is given by

$$V_t = \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{R_{t+j}^k} \right) \Omega_{t+i}. \tag{20}$$

Hence, the discount factor $\Lambda_{t,t+1}$ in equation (13) is given by $\Lambda_{t,t+1} = 1/R_{t+1}^k$.

Dynamic responses to shocks to monetary policy assume the economy is in a steady state in all periods prior to $t = 0$ and that an unexpected shock hits the economy in period $t = 0$. Equation (19) does not obtain in period $t = 0$ because the time-zero shock creates a wedge between the ex ante and ex post returns of each illiquid asset and thereby produces an unexpected capital gain or loss that generally differs across the two illiquid assets.

3.4 Government

The government consists of a central bank and a fiscal authority.

Central bank. The central bank sets the nominal interest rate R_t following a simple rule that depends on the current inflation rate and the past nominal interest rate

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\phi_\pi \log(\pi_t) + \epsilon_t, \tag{21}$$

where R is a constant and ϵ_t is a white noise monetary policy shock. The parameter ρ_r governs the inertia of the nominal interest rate, and the parameter $\phi_\pi > 1$ captures the central bank's stance on inflation. A high ϕ_π implies a strong anti-inflation stance and vice versa.

Fiscal authority. The fiscal authority raises revenue by taxing consumption, labor income, capital income, and mutual funds. Total tax revenue is

$$T_t = \sum_{j=1}^{J} \Big[\tau^c \psi_j c_{j,t} + (1-\psi_j)\tau^{bq} b^q_{j,t} + \tau^{ka}(R^k_t - 1)a_{j-1,t-1} + \tau^a \frac{(R_{t-1}-1)}{\pi_t} d_{j-1,t-1} + \tau^w w_t \epsilon_j \bar{h}_t\Big] N_{j,t}, \tag{22}$$

where $\tau^{ka} = \tau^a + \tau^k - \tau^a\tau^k$ is the total tax rate on illiquid assets.

Let D^n_t denote the face value of the nominal government debt issued in period t. Then aggregate government expenditures consist of government purchases G_t, nominal interest payments on its debt, net of new issuance, $(R_{t-1}D^n_{t-1} - D^n_t)/P_t$, subsidies to intermediate goods firms, $\tau^f Y_t = (\theta - 1)Y_t$, public pension benefits, $B_t \equiv \sum_{j=J_r}^{J} b_{j,t}N_{j,t}$, and government lump-sum transfers to households, $\bar{\Xi}_t \equiv \sum_{j=1}^{J} \bar{\xi}_t N_{j,t}$. It follows that the government flow budget constraint is given by

$$G_t + \frac{R_{t-1}D^n_{t-1} - D^n_t}{P_t} + \tau^f Y_t + B_t + \bar{\Xi}_t = T_t, \tag{23}$$

and the government bond market clearing condition is given by

$$\frac{D^n_t}{P_t} = D_t \equiv \sum_{j=1}^{J} \bar{d}_{j,t} N_{j,t}, \tag{24}$$

where $\bar{d}_{j,t} = \psi_j d_{j,t}(1) + (1-\psi_j)d_{j,t}(0)$ is the average government bond holdings of age-j households.[18]

To complete the description of the economy, we explain how fiscal imbalances induced by monetary policy shocks are resolved. The baseline specification fixes per capita nominal

[18]Because $d_{j,t}(0) = 0$, the aggregate bond can be arranged as

$$D_t \equiv \sum_{j=1}^{J} \left[\psi_j d_{j,t}(1) + (1-\psi_j)d_{j,t}(0)\right] N_{j,t} = \sum_{j=1}^{J} \psi_j d_{j,t}(1) N_{j,t} = \sum_{j=1}^{J} d_{j,t}(1) N_{j+1,t+1}.$$

government debt: $D_t^n/N = D_{-1}^n/N$, for $t \in \{0, 1, ...\}$. Consequently, changes in monetary policy affect the real values of government debt and tax revenues. We close the period-t government budget constraint (equation (23)) by adjusting the size of the lump-sum transfer $\bar{\xi}_t$. Changes in the timing of lump-sum transfers induce redistribution, and our decision to close the government budget constraint in this way is consistent with previous research by Sterk and Tenreyro (2018), Hagedorn et al. (2019) and Hu et al. (2021). In Section 7, we discuss how our results change if nominal government debt is adjusted instead.

The public pension benefit formula is the same as Braun et al. (2009). A household starts to receive a public pension benefit at the mandatory retirement age of J_r. The real size of the benefit during the household's retirement is constant at a level that is proportional to its average real wage income before retirement

$$b_{j,s+j-1} = \begin{cases} 0, & \text{for } j = 1, ..., J_r - 1 \\ \lambda \left(\frac{1}{J_r - 1} \sum_{j=1}^{J_r - 1} w_{s+j-1} \epsilon_j \bar{h}_{s+t-j} \right), & \text{for } j = J_r, ..., J, \end{cases} \tag{25}$$

where λ is the replacement ratio of the public pension benefit and s is the household's birth year. Thus, the public pension system implicitly assumes perfect inflation indexation of public pension benefits.

3.5 Competitive equilibrium

In the impulse response analysis that follows, we will assume that the shock arrives at the beginning of time zero and that households have perfect foresight about the subsequent evolution of prices and government policy.[19] Consequently, perfect foresight is assumed in the following definition of a competitive equilibrium.

Given prices, all firms maximize their profits, all households maximize their utility, and all markets clear. Appendix B provides specific details on the definition and algorithms used to compute a steady state and dynamic equilibria for this economy. Here we simply state the two market clearing conditions that have not yet been reported.

First, the aggregate household illiquid assets, denoted by $A_t \equiv \sum_{j=1}^{J} \bar{a}_{j,t} N_{j,t}$ with $\bar{a}_{j,t} = \psi_j a_{j,t}(1) + (1 - \psi_j) a_{j,t}(0)$, are equal to the sum of capital and the value of all ownership

[19] Boppart et al. (2017) provide a justification for using this approach in heterogeneous agent economies.

shares of intermediate goods firms

$$A_t = K_{t+1} + V_t. \tag{26}$$

Second, as shown in Appendix B.1, Walras' Law implies the market clearing condition for the final good

$$C_t + I_t + G_t = Y_t - \frac{\gamma}{2} (\pi_t - 1)^2 Y_t - X_t, \tag{27}$$

where $X_t = \sum_{j=1}^{J} \bar{\chi}_{j,t} N_{j,t}$ with $\bar{\chi}_{j,t} = \psi_j \chi_{j,t}(1) + (1 - \psi_j)\chi_{j,t}(0)$ is the aggregate cost of adjusting illiquid assets.

4 Model parameterization

4.1 Capital, saving and debt

Before calibrating the model, we first need to classify household assets and liabilities as liquid or illiquid. Our classification scheme is similar in spirit to that used by Kaplan et al. (2018). Table 2 provides an overview of the main components of the two categories. All variables are expressed as multiples of GDP.[20]

A comparison of the results in Table 2 with similar results for US data from the year 2004 reported in Kaplan et al. (2018) (see Table 2 of their paper) reveals some important distinctions between the US and Japan. The biggest difference between the two countries is that Japanese households hold a lot more liquid assets compared to Americans. The net stock of liquid assets in Japan is 1.73 but only 0.26 in the US. Japanese hold more deposits and currency than Americans. Deposits (plus currency) are 1.86 in Japan, but only 0.23 in the US. More than 90% of the Japanese government debt is held domestically. So, this difference between the two countries reflects the fact that Japanese households indirectly hold a large amount of government debt in the form of deposits.

The net stock of illiquid assets is about the same in Japan and the US. Illiquid assets are 3.0 in Japan and 2.9 in the US. However, Japanese households have smaller direct holdings of equity than Americans (0.49 in Japan versus 1.61 in the US.) and Japanese hold more tangible assets. Tangible assets are 1.53 in Japan versus 1.32 in the US. The remaining difference in the size of aggregate illiquid assets in the two countries relates to pension and life insurance assets which are included here but omitted in Kaplan et al. (2018). It

[20] See Appendix C for complete details.

Table 2: Aggregate stocks of liquid and illiquid assets relative to GDP in Japan

Liquid assets and liabilities	
Currency and domestic deposits	1.74
Bonds (total public and private)	0.052
Consumer credit	-0.069
Total net liquid assets as defined in Kaplan et al. (2018)	1.73
Total net liquid assets in our model	1.23
Illiquid assets and liabilities	
Household tangible assets	2.02
Equity and options	0.49
Insurance and private pensions	0.99
Mortgages and installment credit	-0.37
Other non-housing loans	-0.12
Total net illiquid assets as defined in Kaplan et al. (2018)	3.01
Total net illiquid assets in our model	3.50
Net worth	4.73

Note: The financial data are taken from the Flow of Funds Accounts (FFA) for the fiscal year 2014. The stock of household tangible assets is the 2014 end of calendar year value from the Japanese National Income and Product Accounts (NIPA). All variables are expressed as a multiple of GDP for the fiscal year 2014.

follows that Japanese households have higher aggregate net worth (relative to GDP) than Americans.[21] Aggregate household net worth in Japan is 4.73 times GDP and only 3.18 times GDP in the US.

An important distinction between our model and Kaplan et al. (2018) is why households save. Households save in the model of Kaplan et al. (2018) to self-insure against earnings risk. In our model, individuals save to smooth consumption over the lifecycle. Younger households borrow liquid assets to purchase illiquid assets because their mortality risk is low and leveraging up on illiquid assets increases the expected present value net worth. We consequently, assign all household borrowing to the liquid asset category.[22] This adjustment implies that aggregate liquid assets are 1.23 and that aggregate illiquid assets are 3.5.

[21] We define net worth to be the sum of illiquid and liquid assets and abstract from human wealth throughout the paper.

[22] See also footnote 3. The biggest component of illiquid liabilities using the Kaplan et al. (2018) scheme is mortgage debt.

4.2 Overview of the calibration

Our strategy for parameterizing most of the model's parameters follows Braun and Joines (2015) and is described in Appendix D. Here, we discuss the calibration of the parameters that govern endogenous household portfolio choices and nominal rigidities. Modeling these two mechanisms adds six new parameters. Three of the new parameters govern the nominal side of the economy. The degree of nominal price rigidities in the model is determined by γ, the cost of price adjustment for intermediate goods firms. We set $\gamma = 41.2$, which implies that intermediate goods prices adjust on average every 2 years.[23] We set the serial correlation parameter in the central bank's interest rate targeting rule, $\rho_r = 0.35 = (0.77)^4$ and the inflation elasticity is set to $\phi_\pi = 2$.

Modeling warm-glow preferences adds one new parameter, ω, that governs the weight of bequests in the utility function. This parameter helps the model reproduce the age profile of net worth for older households. Our specification of warm-glow preferences is based on Lockwood (2018) and we set this parameter to be consistent with his estimates.[24]

The final two parameters govern the costs that households face when adjusting their holdings of illiquid assets. These parameters govern the substitutability of liquid and illiquid assets and determine the (endogenous) steady state age profile of real asset holdings. They also induce endogenous time-varying liquidity premia that allow the model to account for the differential response of equity and bond prices to a tightening in monetary policy.

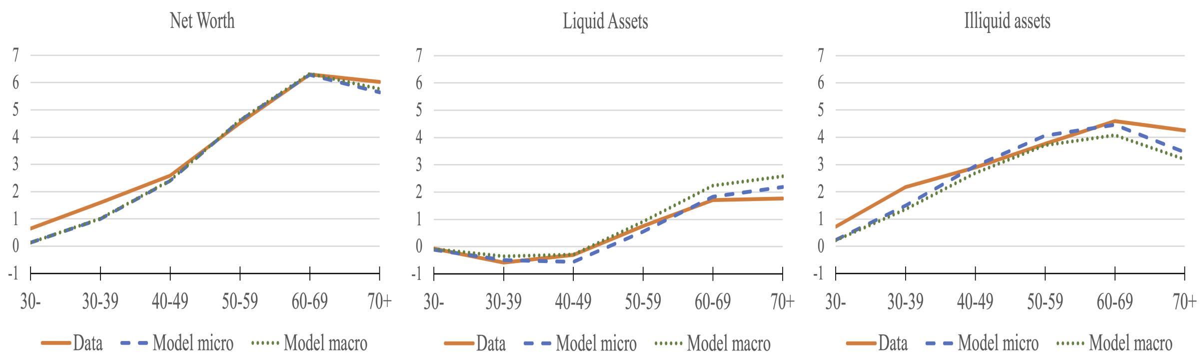
We parameterize the two adjustment cost parameters to fit the average age profiles of holdings of liquid and illiquid assets. The resulting estimates imply that adjustment costs are less than or equal to 2 percent of household illiquid asset holdings for surviving households and less than or equal to 15 percent of illiquid assets for non-surviving households. Appendix D contains more information about our parameterization of these two parameters.

Young Japanese households with short earnings and payment histories are not able to access credit to buy cars or homes unless they have a guarantor. We capture this in a simple way by imposing an ad hoc borrowing constraint on households between the age of 21 and 23 that prevents them from taking leveraged positions on illiquid assets. The specific choice of this age group allows the model to reproduce borrowing in our data for households between 21 and 29 years of age.

[23] Using a log-linearized version of the model, we can map back and forth between γ and the corresponding Calvo parameter and derive the average duration of price changes.

[24] Our setting of this parameter implies a weight on bequests of 0.9 using the notation in his paper.

Figure 2: Household net worth, liquid and illiquid asset holdings by age: model and data



Note: Liquid assets are net of total borrowing and illiquid assets are gross. Net worth and assets are relative to pretax income of the 50–59 year old age group. The "Model micro" results assume an aggregate debt–GDP ratio of 0.85. The "Model macro" results assume an aggregate debt–GDP ratio of 1.23. Details on the construction of the data can be found in Appendix C.

4.2.1 Net worth and asset portfolios by age group

Figure 2 compares the model's endogenous steady state age profiles of net worth, liquid assets, and illiquid assets with the NSFIE data we previously discussed in Table 1. The results are expressed as fractions of income of the 50–59 year old age group.[25] Our model is a general equilibrium model, and there is a gap between the implied aggregate debt–GDP ratio from our NSFIE data and the aggregate debt–GDP ratio from the Japanese Flow of Funds Accounts (see also our discussion in Appendix C). The former value is 0.85 and the latter value is 1.23. The model macro results assume a debt–GDP ratio of 1.23 and the model micro results assume it is 0.85 instead. Making this distinction has a negligible impact on the model's steady state age profiles of asset holdings and the dynamic results that follow.

As we discussed in Section 2, our data have four main properties. First, the age profile of net worth is hump-shaped. Second, households in the three youngest age groups hold leveraged long positions in illiquid assets. Third,older households have positive holdings of liquid and illiquid assets. Fourth, households gradually reduce their net worth beyond the age of 69 but their asset holdings remain high until late in life.

Our model reproduces all four properties of our data and does a good job of matching the magnitudes as well. The hump-shaped pattern of saving over the lifecycle produced by the model reflects primarily the hump-shaped age-earning profile and the fact that Japan's public pension insurance program provides incomplete coverage. The reason why younger households are taking leveraged long positions in illiquid assets in the model is because

[25]The lifetime peak in income occurs in the 50–59 year old age group in both the data and our model.

their mortality risk is relatively low, and thus investing in illiquid assets increases their expected present value net worth. In balance, the benefits of a higher expected return on illiquid assets exceed the cost of having to suddenly liquidate their holdings if they experience a death event. As households age they choose to pay off their debt and hold a diversified portfolio with positive positions of liquid and illiquid assets. Finally, net worth declines monotonically during retirement in the model beyond the age of 69. The pace of the decline is gradual in the model and net worth remains high until late in life. In our NSFIE sample, the average age of the 70+ age group is 76. Households of this age have net worth that is more than five times their peak earnings in the model.

The endogenous age profiles of liquid and illiquid asset holdings in our model and the data differ in two ways. First, the two oldest age groups of households in the model hold a higher share of their wealth in liquid assets compared to our data. We model a nuclear household. However, the NSFIE survey data consist of all households and in particular includes multigenerational households which are still relatively common in Japan, and have different age profiles of assets.[26] In 2015 just under 20 percent of households with a 65+ year old member had children and/or parents living with them. If we make some simple adjustments to the model's results by assuming an age difference of 28 years between adult children and their parents and then assigning 20 percent of younger households to the oldest age group, the model produces a portfolio share of illiquid assets for the oldest age group of 4.04 as compared to 4.24 in our NSFIE data. In what follows, we will abstract from cohabitation and report results for nuclear families since they are the largest group. Mitman et al. (2024) have recently proposed a theory of co-habitation of adults with their parents.

The second gap between our model and the data pertains to the two youngest age groups. Our model reproduces the amount of borrowing of these age groups in Japanese data. However, they have a bit more net worth and illiquid asset holdings in the data relative to the model, suggesting that they are able to leverage up more than the natural borrowing constraint permits. There is considerable stigma associated with default in Japan, but it is not unusual for lenders/landlords to require that a guarantor cosign a loan or lease. It is possible that this outside commitment is allowing younger households to leverage up more than the natural borrowing constraint permits.

[26]See Murata (2019). We thank Fumio Hayashi for making us aware of this measurement issue.

4.2.2 Marginal propensities to consume

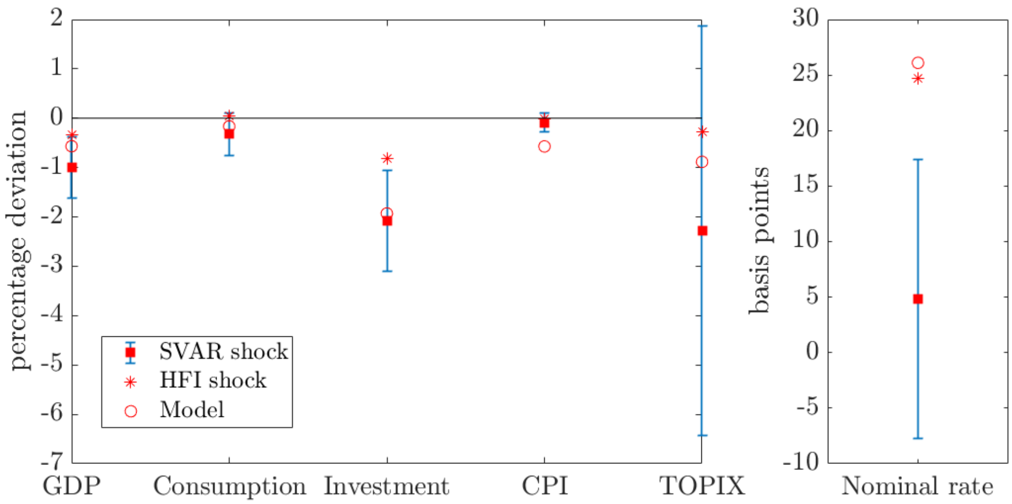
The average marginal propensity to consume (MPC) in our model is 0.11 for a one-year 50,000 yen increase in the lump-sum transfer and 0.16 if the transfer is increased by the same amount for two years. MPCs of this magnitude are consistent with recent estimates from Japanese data. Koga and Matsumura (2020) estimate that the average MPC out of transitory income shocks is 0.16 using data from the Japanese Household Panel Survey. Appendix E reports average MPCs by age for the one-year and two-year scenarios. The shape of the MPC age profile is similar in the two scenarios. For the one-year scenario the MPCs are largest (0.39) in the 21–30 year old age group because households between the ages of 21 and 23 are hand-to-mouth. MPCs decline in the second youngest age group to 0.05 and then gradually increase with age to a peak of about 0.24 for the oldest age group.

5 The aggregate effects of monetary policy

This section compares the response of aggregate variables in our model to a tighter monetary policy with Japanese data under two identification schemes. Our baseline specification assumes that the central bank follows an interest rate targeting rule and does not impose an effective lower bound (ELB) on the policy interest rate. Some justification for this assumption is in order because the empirical impulse responses are estimated on a sample period, when the Japanese policy interest rate was close to zero (between 1992–2019). We are implicitly assuming that the Bank of Japan was able to achieve its policy objectives because the unconventional monetary policy tools at its disposal allowed it to implement a negative (shadow) policy rate. This hypothesis is supported by research by Ikeda et al. (2024) who estimate the shadow policy rate at the ELB in Japan and find that unconventional monetary policy has had, if anything, stronger effects on inflation and output than conventional monetary policy at horizons of one year and beyond. We discuss the robustness of our conclusions to this maintained assumption in Section 8.

Figure 3 compares our model's responses to a 1 percent tightening in monetary policy in the impact year to the empirical responses of real GDP, consumption, investment, the CPI, and the nominal TOPIX stock price index in Japanese data. The empirical responses are cumulative responses of local projections estimated on quarterly data from 1992–2019. The SVAR results use monetary policy shocks estimated by Ikeda et al. (2024) who use an endogenous regime-switching VAR with sign restrictions. In contrast, the HFI results

Figure 3: Impulse responses by age to a tightening in monetary policy: data and model



The two data responses are cumulative effects over four quarters with the exception of TOPIX, which is the response of the stock price index in the impact quarter. The SVAR shocks are taken from Ikeda et al. (2024) and the HFI shocks are taken from Kubota and Shintani (2022). The sample period is 1992–2019 and the vertical lines are 95 percent confidence intervals for local projections with the SVAR shocks. "Model" refers to our baseline model.

use monetary policy shocks estimated by Kubota and Shintani (2022) who use the high frequency proxy variable identification scheme we discussed in Section 2 and Appendix F. The vertical lines in the figure are 95 percent confidence intervals for local projections with the SVAR shocks. We do not report confidence intervals for those with the HFI shocks because they are considerably larger and uninformative. The results labeled "Model" refer to our baseline model. The model and SVAR shock size are $\epsilon_t = 0.01$. However, the HFI scheme is conceptually different and the size of the HFI shock is adjusted (see Appendix F).

A comparison of the impact responses indicates that the model and data are in good accordance under either identification scheme. The signs and magnitudes of the responses of GDP, consumption, and investment in the model are all close to the empirical responses.[27] The model predicts a somewhat larger decline in the price level compared to the two empirical schemes, but the key take away is that the decline in the inflation rate is small in our model and in the data.

Observe next that the size of the 1-year nominal government bond yield (nominal inter-

[27]The measure of consumption in Figure 3 is based on the NIPA, while the measure of consumption by age group in Figure 1 is based on the FIES. This distinction is not important for our results. The empirical response of FIES-based average consumption to the SVAR shock is -0.60 and its response to the HFI shock is -0.34. Both estimates are within the confidence interval shown in Figure 3.

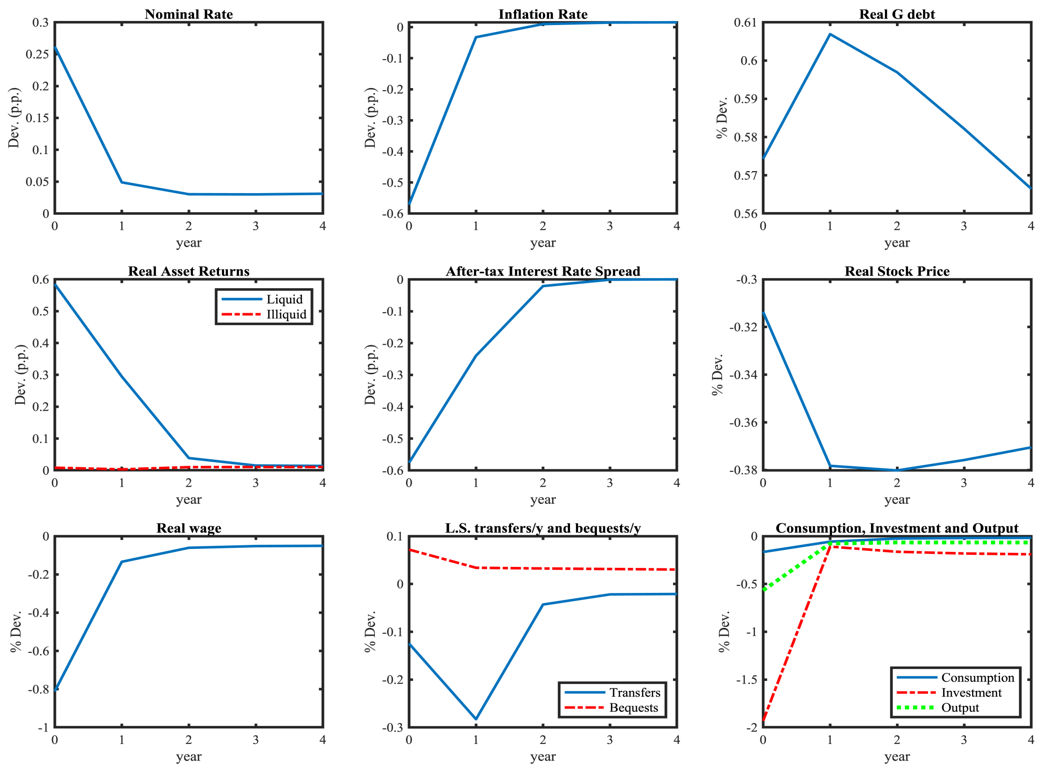
est rate) response is about the same for the HFI scheme (25 bp) and the model (28 bp), but is smaller using the SVAR identification scheme (5 bp). This difference between the two schemes reflects a conceptual difference between them. The SVAR shocks are estimated by imposing the ELB while our model and the HFI scheme do not impose this restriction. The HFI scheme identifies monetary policy shocks directly using interest rate futures in a narrow window around an announced change in monetary policy, but does not constrain the 1-year bond yield. Given that the 1-year yield was close to the ELB during most of our sample, its response to the HFI shock is best interpreted as the response of the shadow rate which is the relevant object to compare with our model.[28]

Perhaps the most interesting feature of our aggregate results is that the model reproduces the sign and magnitude of the empirical response of stock prices in Japanese data. The nominal stock price declines by 0.88 percent in the model, and reflects a 0.31 percent decline in the real stock price and a 0.57 percent decline in the inflation rate. For purposes of comparison, the (nominal) TOPIX declines by 0.30 percent using the HFI shock and by 2.30 percent using the SVAR shock.

The decline in the real stock price in our model reflects the combined effects of asset substitution analyzed in Tobin (1969) and Hu et al. (2021) and our model of portfolio choice. The asset substitution channel of monetary policy can be illustrated using equations (1) and (2) in the simple 2-period flexible price model. In the 2-period model, an increase in the nominal interest rate depresses the price level, which increases the real supply of government debt. Asset demand is fixed in the impact period, so the real return on government debt has to increase, and no arbitrage implies that the private capital stock has to fall to equate returns between bonds and capital. The same monetary policy transmission channel operates in our quantitative OLG model. The real supply of government debt has risen, and the real return on bonds has also increased because asset demand is downward sloping in our model. Illiquid assets are (imperfect) substitutes and their return must also increase as shown in Figure 4. Equity is an illiquid asset, and its price falls. In fact, as we show in Section 6.2 below, households of all ages respond to the monetary policy shock by reducing their holdings of illiquid assets, and this is why both stock prices and aggregate investment fall in our model. Figure 4 also shows that the decline in the interest

[28]The SVAR scheme, in contrast, imposes the ELB and measures the impact of unconventional monetary policy on economic activity indirectly by estimating the impact of shocks to the shadow policy rate. During most of our sample period, the 1-year yield is close to zero, so its response to shocks to the shadow policy rate is constrained by the ELB. This is why the average (local perturbation) response of the 1-year yield is smaller in the SVAR scheme than in the HFI scheme or our model.

Figure 4: Model impulse responses to a 1% tightening in monetary policy.



Note: Model impulse response functions in years 0–5, "real G debt" is real government debt and "L.S." is lump-sum.

rate spread is persistent and that real stock prices decline further in years one and two. This later result arises because it is costly for households to adjust their holdings of illiquid assets. If these costs were absent, returns on liquid and illiquid assets would be equalized from period one on and stock prices would rise towards zero from year one.

Producing plausible responses of aggregate investment and consumption to monetary policy shocks is a challenge for representative agent new Keynesian (RANK) models with capital formation. Christiano et al. (2005) and Smets and Wouters (2003) analyze monetary policy shocks in RANK models and appeal to aggregate adjustment costs on investment to reproduce the empirical magnitude and persistence of the decline in aggregate investment in US and European data, respectively. When adjustment costs on investment are omitted, standard RANK models overstate the response of investment and understate the response of consumption (Rupert and Šustek, 2019).

Generating empirically relevant aggregate investment responses is also a challenge in HANK models. Markups and profits increase in NK models when monetary policy is tightened, and the size and distribution of these profits matter when agents face uninsured earnings risk. Broer et al. (2020) produce an example (without capital formation) where the positive effect of higher profit income to households exactly offsets the decline in their labor income, leaving hours and output unchanged. They propose nominal wage rigidities as a solution because they reduce the size of the response of profits to the shock. The same issue arises in the HANK model of Kaplan et al. (2018). A tightening in monetary policy increases profits and aggregate investment, and stock prices increase when monetary policy is tightened if all profits are paid out to shareholders. They get investment to fall in their model by assuming that part of profits are reinvested in illiquid assets, but stock prices still increase when monetary policy is tightened. Alves et al. (2020) show that stock prices fall in a similar model when aggregate adjustment costs on investment are introduced. Our model has no adjustment costs on aggregate investment, no costs of adjusting nominal wages and all profits are paid out as dividends to shareholders, yet our model reproduces the sign and magnitude of the impact aggregate responses of output, investment, and prices. Moreover, as we show next, the declines in these variables are also persistent (Figure 4). Further discussion of the economic mechanisms responsible for our model's attractive empirical properties is provided in Section 7.

It will be helpful in our ensuing discussion of the microeconomic responses to monetary policy to know the complete range of aggregate disturbances experienced by households. Figure 4 provides this information. The figure has four notable properties. First, investment and real stock prices experience persistent declines, and the real returns on liquid and illiquid assets are persistently positive. Second, a tightening in monetary policy increases the real value of government debt because the stock of nominal debt is constant and the price level falls. Third, the real wage rate and lump-sum transfers exhibit persistent declines, and fourth, investment falls by more than output, and consumption falls by less than output. This final property of our model is consistent not only with our empirical results for Japan but also with the empirical findings in Christiano et al. (2005) for the US and Smets and Wouters (2003) for Europe.

To provide further evidence that the dynamic properties of our model are different from other RANK and HANK models, it is useful to compare our results with those of Bielecki et al. (2022). Their model has an OLG structure, but abstracts from the financial frictions that generate endogenous portfolio choices in our model. Instead, they rely on the conventional NK propagation mechanisms that we discussed above, including adjustment costs on investment, nominal wage rigidities, and consumption habits. One difference between our model and theirs is that a tightening in monetary policy in their model implies counterfactually that consumption declines more than output and output declines more than investment. A second difference is that the fit of their model deteriorates when we make their specification closer to ours by abstracting from adjustment costs on aggregate investment, consumption habits and nominal wage rigidities. Under this assumption, their model produces a counterfactual persistent increase in consumption when monetary policy is tightened.[29] In Section 6.2, we relate these differences in the aggregate dynamics of the two models to our specification of endogenous portfolio choice.

6 Monetary policy over the lifecycle

We now analyze how tighter monetary policy affects the situation of households of different ages. We begin by describing how households modify their consumption and portfolio choices in the impact year. Two novel properties of our model emerge. First, some age groups are negatively affected by the shock and reduce their consumption, while other age groups benefit and consume more (Section 6.1). Second, we show that the reason aggregate investment falls in our model is asset substitution. Tighter monetary policy narrows the excess return on illiquid assets, and all age groups choose to reduce their holdings of them (Section 6.2). Then in Section 6.3, we analyze the medium-term effects of tighter monetary policy on consumption, expected present value utility, and inequality.

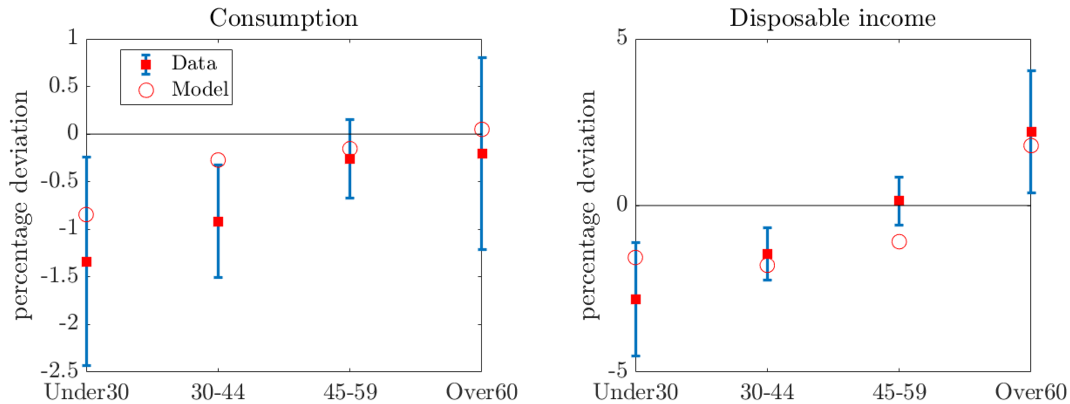
6.1 Impact responses of consumption by age

Model and data age profiles of consumption and disposable income responses to tighter monetary policy are reported in the left and right panels of Figure 5. The model reproduces the empirical responses of disposable income, which have relatively tight standard errors. Disposable income of the younger age groups declines in the data and model. Conversely, disposable income of the oldest age group increases, and our model successfully reproduces this observation.

The empirical responses for consumption have larger standard errors, but both the signs and the magnitudes of the model responses are also close to the data. Consumption of the youngest households experiences particularly large declines. The size of the consumption

[29] Code and plots of impulse responses for these scenarios of their model are included our replication package.

Figure 5: Responses to a tightening in monetary policy: model and data



Note: Cumulative responses in the impact year; monthly Japanese data (FIES) for working households; the monetary policy shocks identified by Kubota and Shintani (2022) ; the sample period of 1992–2019. Vertical lines are 95 percent confidence intervals."Over 60" for the "Model" refers to households between the ages of 60–74.

responses declines with age and the sign turns positive in the model as the household head approaches retirement.

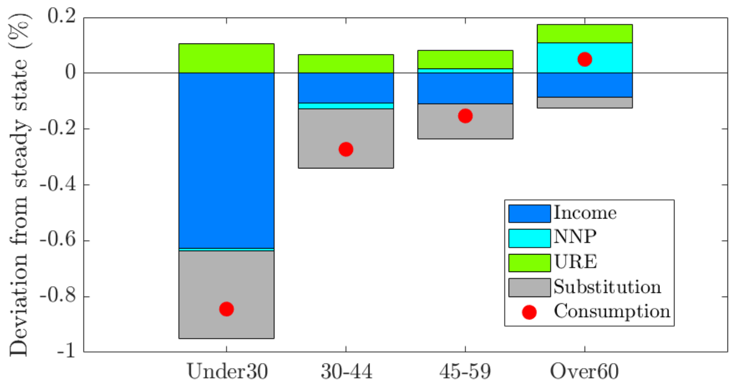
The biggest gap between the data and the model is that consumption of the 30–44 year old age group falls by more in the data. This age group has the highest ratio of borrowing relative to net worth in the model and the data. The model imposes the natural borrowing constraint and our steady state results suggest that they may be able to leverage up more than this constraint permits. If this is the case, then tighter monetary policy would produce an even larger loss in their portfolio.

Our finding that the consumption response of younger working-age households is particularly large is consistent with other results in the literature. Wong (2019) finds that consumption of younger working-age households is particularly sensitive to changes in monetary policy because interest rates have a large impact on their decision to purchase their first home and on their monthly expenses if they already own a home. Bachmann et al. (2021) find that consumption of younger working-age households is particularly sensitive to temporary changes in German value added taxes.

One difference between our results and other results in the literature is that the *sign* of the consumption response differs between age groups in our model and possibly also in Japanese data.[30] Other HANK models, in contrast, have the property that all households reduce their consumption on impact when monetary policy is tightened (e.g. Bielecki et al., 2022; Luetticke, 2021).

[30] The results using 5-year age bins suggest that the point estimate of the responses of consumption is positive for households in 50–54, 55–59, and 60–64 age groups, although the responses become noisier and there is possibility of zero responses. The results are available upon request.

Figure 6: Decomposition of the consumption responses in year 0 by age group



Note: "Consumption" denotes the consumption responses to a tightening in monetary policy in the model, shown in the left panel of Figure 5. "Over 60" corresponds to households with age 60–74.

Monetary policy risk exposures by age. Following Auclert (2019), we decompose the age profile of consumption responses in the impact period into four components that are shown in Figure 6.[31] The income component captures the impact of changes in labor and net government income while the net nominal position (NNP) component measures the effect of a surprise change in the inflation rate on the real value of nominal assets and liabilities, as emphasized in Doepke and Schneider (2006). The unhedged real interest rate exposure (URE) component captures how changes in the real returns on liquid and illiquid assets align with household portfolio plans. This channel plays a central role in the analyses of Guerrieri et al. (2020) and Garriga and Hedlund (2020).[32] Finally, the intertemporal substitution component captures the trade-off between consumption and saving arising from a change in the real interest rate. It is the main transmission mechanism in RANK models, but its role is smaller in HANK models as discussed by Kaplan et al. (2018).

Figure 6 shows that households in different age groups have fundamentally different exposures to monetary policy. The income and substitution effect components are negative for all age groups, but these two components are largest for households in the youngest age group. Households between the ages of 21 and 23 are borrowing-constrained, and this age group has the lowest net worth. It follows that their consumption plans are particularly sensitive to changes in labor income. Intertemporal substitution effects are also large for young households because they have the longest planning horizons. As households age,

[31] See Appendix G for the details of the decomposition.

[32] In our model, all financial and tangible assets are one-period assets. However, the URE component also includes future income in assets and future consumption plans in liabilities.

opportunities for intertemporal substitution decline and this component plays a smaller role in households' consumption decisions.

Younger households are reasonably well hedged against inflation risk. Their NNPs are negative but small in magnitude. Older households have large holdings of liquid assets, and tighter monetary policy benefits them. Their NNP components are positive, and NNP is the largest component in the 60+ age group.

Finally, observe that the URE component is positive and of a similar magnitude for all age groups. Households are actively adjusting the size and composition of the securities in their portfolios to take advantage of the movements in real interest rates. We now show that there are large differences in the way each age group makes these adjustments.

6.2 Impact responses of household net worth and optimal portfolios

A key distinction between our model and other NK models, that we discussed in Section 5, is the mechanism that determines the response of aggregate investment and stock prices to a tightening in monetary policy. In conventional NK models, including ours, a tightening in monetary policy increases the markup and profits, and this creates an incentive for households to invest more (see Broer et al., 2020). In our model, this incentive is complemented by heterogeneous portfolios and changes in the interest rate spread. Moreover, portfolio decisions are endogenous in our model and Table 3 shows that households are adjusting their portfolios in different ways to the shock.[33] Here, we summarize the main properties of Table 3. Although the size and nature of consumption exposures vary by age in our model, all households reduce their allocations to illiquid assets when monetary policy is tightened. Following the shock, the real supply of government debt is higher and demand for liquid and illiquid assets is downward sloping. Returns on both types of assets rise above their steady state levels in year one but the interest rate spread on illiquid assets narrows (Figure 4) and households of all ages choose to reduce their allocation to illiquid assets.

The sign of household adjustments to liquid assets differs by age and the size of these adjustments is larger than illiquid asset adjustments in most age groups. Households in the 26–30 age group are borrowers, and reduce both consumption and borrowing in response to the shock. Their net worth falls and they reduce their holdings of illiquid assets as well, but

[33] Recall that the steady state age-profile of asset holdings has two properties. First, surviving households of all ages hold positive amounts of illiquid assets. Second, younger working-age households have low net worth and borrow liquid assets to purchase illiquid assets.

Table 3: Portfolio adjustments by age to a tighter monetary policy

Age group	Liquid assets	Illiquid assets	Leverage	Net worth
21-25	5.64	-2.71	-3.19	-1.77
26-30	1.21	-1.76	0.55	-2.13
31-35	-0.46	-0.67	1.14	-1.16
36-40	-0.71	-0.40	1.11	-1.19
41-45	-1.01	-0.28	1.29	-0.50
46-50	-5.01	-0.22	4.62	-0.34
51-55	-0.32	-0.18	0.00	-0.18
56-60	0.29	-0.16	0.00	-0.03
61-65	0.53	-0.15	0.00	0.09
66-70	0.66	-0.15	0.00	0.14
71-75	0.68	-0.15	0.00	0.16
76-80	0.70	-0.17	0.00	0.19
81-85	0.69	-0.20	0.00	0.21
86-90	0.69	-0.26	0.00	0.25
91-95	0.73	-0.36	0.00	0.31
96+	0.76	-1.57	0.00	0.40

Note: These are asset holdings at the end of the impact year. Liquid, illiquid assets and net worth are expressed as percentage deviations from the steady state. The sign of the liquid asset response is adjusted so that a positive number implies that the household has increased holdings (or reduced borrowing for borrowers). The leverage ratio is the percentage change in the ratio of liquid to illiquid assets for households who are borrowers and is reported as a percentage change from steady state.

the leverage ratio on their portfolio increases. Households between the ages of 31–49 are also borrowers. They suffer from labor income declines and capital losses on their leveraged long positions on illiquid assets. Their consumption falls but they increase their borrowing and exit the period with a higher leverage ratio.

All households over the age of 50 have positive holdings of liquid and illiquid assets when the monetary policy shock arrives. However, the responses to the shock are heterogeneous in the 50+ age group. Households between 51–55 reduce their holdings of liquid and illiquid assets and their net worth falls. The remaining age groups respond to the shock by reallocating more of their wealth to liquid assets and saving more. From their perspective, liquid assets are now a more attractive investment. In addition, as we turn to discuss next, movements in interest rates are dovetailing with their preferred (age-dependent) asset allocation strategy and saving more expands their consumption opportunities in future periods. The oldest households have low net worth, but make large adjustments to their portfolio. Their mortality rate is high, they value bequests, and allocate more of their wealth to liquid assets to increase the size of their expected bequest.

We explained in Section 5 that our model has different aggregate dynamics compared to Bielecki et al. (2022). These differences in aggregate outcomes arise because households in their model make a single savings decision. Households face an exogenous time invariant age profile of bond holdings and consequently cannot adjust their bond holdings when the central bank changes the nominal interest rate on bonds.[34] The Taylor rule determines the nominal interest rate and nominal bonds are a more attractive investment than other financial assets when monetary policy is tightened, but the only way households can react is to reduce their holdings of financial assets and some households borrow financial assets to fund their exogenously given bond positions.[35]

6.3 Medium term effects of monetary policy on consumption, welfare and inequality

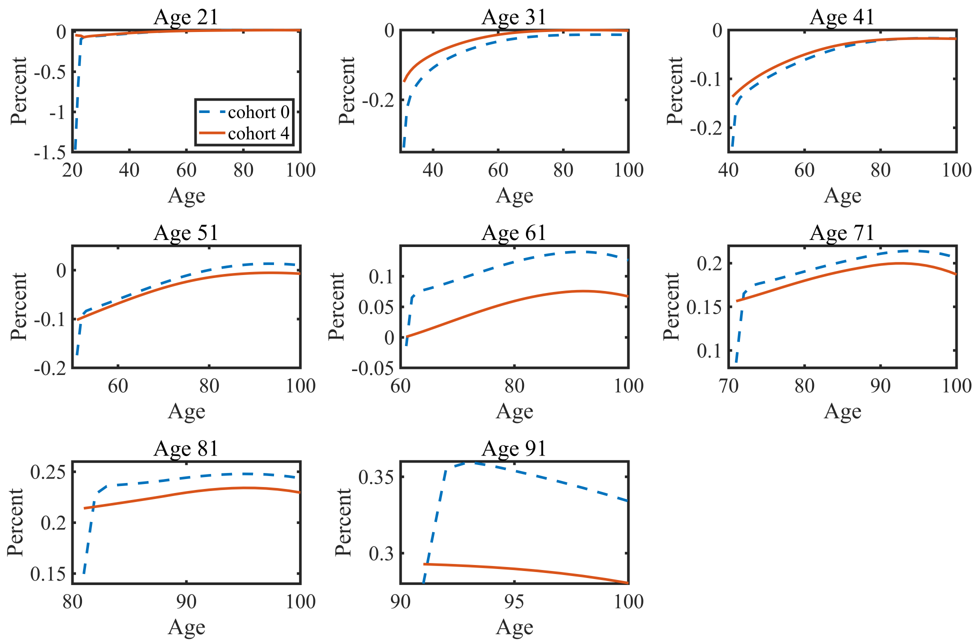
We now consider the medium-term effects of monetary policy on consumption, welfare, and inequality. Guerrieri et al. (2020) documents that changes in the price of illiquid assets such as housing have persistent impacts on consumption. Our model has a similar property; the sign and shape of a household's age profile of consumption responses depend on its age, and consumption of some age groups exhibits large persistent changes to monetary policy shocks.

Figure 7 displays age-consumption profiles of households in six different cohorts starting from the age specified in the title of the panel. The line labeled cohort 0 shows the age-consumption profile for the cohort with that age (e.g., 21 years in the upper left panel) in the year that the shock arrives. The consumption-age profiles are reported as the deviation of consumption at the age listed on the horizontal axis from its steady state level relative to steady state disposable income at the same age. Tighter monetary policy has a positive and very persistent impact on consumption of the age 71 cohort. Surprisingly, their peak consumption deviation occurs at age 94 or 23 years after the shock arrives. Appendix H decomposes the consumption responses of this cohort into three components: changes in cash flows from illiquid assets, changes in cash flows from liquid assets, and changes in net government cash flows. This decomposition indicates that liquid asset cash flows exhibit particularly large, positive, and persistent deviations from the steady state. As depicted

[34] The analogue of the second column of Table 3 in their model is a column of zeros.

[35] See our replication files.

Figure 7: Consumption-age profiles of selected cohorts.



Note: The figure reports consumption deviations from steady state as a percentage of steady state consumption for an household of the age listed on the horizontal axis. Cohort 0 has the age listed in the title of the panel when the shock arrives in time zero. Cohort 4 reaches the age specified in the title of the panel five years later.

in Figure 4, the real interest rate on liquid assets is persistently higher and the spread on illiquid assets is persistently lower. The age 71 cohort responds by increasing liquid asset holdings and decreasing illiquid asset holdings (Figure 11 in Appendix H). These portfolio adjustments allow them to enjoy higher consumption for many years.

Next, we describe how consumption-age profiles of other cohorts change. Consumption of the age 21 cohort experiences a large negative decline on impact, because it is borrowing constrained, and small persistent declines in subsequent years. Tighter monetary policy persistently depresses wages and the interest rate spread (Figure 4). Consequently, its preferred allocation strategy, which involves leveraging up on illiquid assets, now has a lower return, giving rise to the decrease in consumption.

Households in the age 31 and 41 cohorts experience the largest and most persistent consumption declines. They hold levered positions of illiquid assets when the shock hits (Figure 2) and experience capital losses. In subsequent years, they experience lower wages in the stage of their lifecycle when their labor efficiency units are highest and lower returns on their preferred investment strategy.

Members of the age 51 cohort also experience persistent declines in consumption. How-

Table 4: Expected present value utility of selected age groups in the year that monetary policy is tightened

Age at time of shock	21	31	41	51	61	71	81	91
Consumption Equivalent (%)	0.11	0.095	0.098	0.011	-0.48	-1.00	-1.54	-2.58

Note: Consumption equivalent variations of present value expected utility for different age groups relative to the steady state. A positive sign indicates that the cohort prefers the steady state allocation.

ever, this group is able to hedge the shock and eventually experiences consumption gains (conditional on surviving).

Households in the age 71, 81 and 91 cohorts all benefit from the shock. The size of the consumption response is larger in the older cohorts, and their consumption response peaks earlier. These cohorts benefit from capital gains in the impact period and higher returns on their preferred investment strategy, which involves gradually drawing down total asset holdings and allocating a larger share of their savings to liquid assets.

An alternative way to measure the size and persistence of cohort effects is to compare two cohorts who attain the same age in different years. The second line in each panel, labeled "cohort 4", shows the consumption-age profile of the cohort that reaches the age specified in the title of the panel five years later. A tightening in monetary policy has a similar (negative) impact on the consumption of the cohorts that reach the ages of 31, 41, and 51 five years later. For the other "cohort 4" age groups, the consumption-age profile deviations are smaller for most remaining periods of their life.

6.3.1 Welfare

Households value not only consumption, but also leisure and bequests, and it is conceivable that the consumption responses give a misleading picture about the welfare implications of monetary policy shocks. Table 4 reports compensating consumption variations in utility for selected cohorts. A positive value indicates that the cohort prefers their steady state allocations to the allocations in the shocked economy. Observe that the signs of the compensating variations are in good agreement with the signs and relative magnitudes of the consumption responses reported in Figure 7.

6.3.2 Intergenerational Inequality

The heterogeneous asset allocation and consumption responses that we have documented above imply that monetary policy influences intergenerational wealth and consumption

Table 5: Wealth, consumption and bequest intergenerational inequality

Year	0	1	2	3	4	5	6	7	8	9
Wealth Gini	0.13	0.12	0.11	0.096	0.085	0.075	0.065	0.056	0.047	0.039
Wealth Std. Dev.	0.94	0.85	0.75	0.65	0.56	0.47	0.39	0.32	0.25	0.19
Consumption Gini	-0.015	-0.041	-0.046	-0.047	-0.047	-0.047	-0.046	-0.045	0.043	-0.041
Consumption Std. Dev.	-0.005	-0.034	-0.039	-0.041	-0.041	-0.040	-0.039	-0.038	-0.037	-0.036
Bequest Gini	0.14	0.12	0.11	0.098	0.086	0.075	0.065	0.055	0.046	0.038
Bequest Std. Dev	0.96	0.86	0.76	0.66	0.58	0.50	0.43	0.36	0.30	0.24

Note: All numbers are percentage point deviations from the steady state value of the statistic listed in column one. For instance, a value of 0.13 implies an increase in the wealth Gini coefficient from 0.4090 to 0.4103.

inequality. Table 5 reports the direction, size, and persistence of inequality using Gini coefficients and cross-sectional standard deviations expressed as percentage point differences from steady state in each year. The wealth (net worth) Gini coefficient increases by 0.13 percentage points in the impact year and then gradually declines in subsequent years. Wealth inequality increases because older households have the highest wealth and experience positive portfolio returns, while younger households experience negative returns.

Consumption inequality, in contrast, declines by 0.015 percentage points on impact and exhibits small but persistent declines in subsequent years. The reason consumption and wealth inequality move in opposite directions is because households adjust expected bequests in different ways. Older households adjust their portfolios to increase their future wealth, and part of this benefit is reflected in higher bequests. Young households, on the contrary, are negatively impacted by the shock and smooth their consumption by reducing expected bequests. The magnitude of the increase in bequest inequality is large and about the same magnitude as the increase in wealth inequality. These results suggest that monetary policy has very persistent effects on inequality that can span generations.

Measuring inequality using the cross-sectional standard deviation metric yields qualitatively similar results, although some of the magnitudes change. The response of consumption inequality is similar under the two measures. However, wealth and bequest inequality exhibit larger increases using the standard deviation metric, increasing by 0.94 and 0.96 percentage points, respectively, in the impact year.

Table 6: Impact responses to tighter monetary policy in alternative scenarios.

Specification	Baseline	Higher govt debt	Fixed real govt debt	Lump-sum tax fixed	No illiquid assets	Flexible Price
Variable						
Output	-0.57	-0.60	-0.59	-0.30	-1.04	0.01
Investment	-1.93	-2.19	-2.06	-0.75	-3.98	0.08
Consumption	-0.17	-0.17	-0.17	-0.16	-0.16	-0.01
Real stock price	-0.31	-0.37	-0.16	-1.04	-0.63	-0.23
Real return on liquid assets	0.24	0.24	0.22	0.39	0.02	0.07
Interest rate spread	-0.24	-0.24	-0.21	-0.40	0.00	-0.07
Real govt debt	0.57	0.57	0.00	0.95	0.72	0.72
Illiquid asset demand	-0.24	-0.27	-0.19	-0.45	-0.49	-0.09
Higher net worth (ages)	59+	58+	None	63+	65–93	53+
Consumption Gini	-0.015	0.002	0.010	-0.041	0.032	-0.028

Note: Percentage deviations from steady state in the impact year except for the real return on liquid assets and the interest rate spread, which are percent changes from their steady state values in year one. The "Higher govt debt" scenario assumes a steady state debt–output ratio of 1.75, the "Fixed real govt debt" scenario assumes that the stock of real–per capita debt is fixed and that its real return in the impact period is predetermined. The "No illiquid assets" scenario sets the adjustment costs to illiquid assets to zero. The "Fixed lump-sum tax" scenario holds the size of the lump-sum transfer fixed for the first 100 years of the transition and the "Flexible price" specification assumes that it is costless for intermediate goods firms to adjust prices.

7 Inspecting the mechanisms

We have shown that our model reproduces the empirical age profiles of average net worth, liquid and illiquid asset holdings (Section 4.2.1), the microeconomic responses of consumption and disposable income to a monetary policy shock (Section 6.1), and the responses of a broad range of aggregate variables to a tightening in monetary policy (Section 5). It achieves these successes in new and distinct ways from the previous HANK literature. Households save in our model to provision for retirement in the face of uninsured mortality risk instead of earnings risk. We deliberately abstract from the mechanisms that have been used in previous NK models to produce plausible macroeconomic responses including aggregate adjustment costs on investment, habit persistence in consumption, and nominal wage rigidities. Households over the age of 23 are free to borrow up to the natural borrowing constraint, and all profits are paid out as dividends. Finally, liquid and illiquid assets are imperfect substitutes and households portfolio choices are endogenous. Given these large differences in our modeling strategy, it is useful to document the economic mechanisms underlying our results.

The starting point of our discussion of the transmission channels of monetary policy is to consider when money is neutral. Money is neutral in our model when government debt is real and in zero supply, liquid and illiquid assets are perfect substitutes, and prices are flexible. Consequently, we analyze the contribution of these mechanisms to our results by varying them one at a time and comparing the properties of the model to the baseline specification. Table 6 reports the impact responses of tighter monetary policy on output, consumption, investment, stock prices, the after-tax real return on liquid assets, the after-tax interest rate spread, the level of real government debt, the aggregate demand for illiquid assets, and two indicators of intergenerational redistribution. The first indicator lists the age groups that see their net worth increase, and the second indicator is the response of the consumption Gini.

Column 2 of Table 6 considers a scenario with a higher steady state government debt output ratio of 1.75 instead of 1.23.[36] The response of consumption and most other aggregate variables is essentially the same as the baseline. The main differences are that aggregate investment, stock prices, and illiquid asset demand decline more. The liquid asset is more abundant in this scenario, and household demand for illiquid assets is more elastic. Observe also that consumption inequality increases more in the impact period. Younger households experience larger capital losses and reduce their consumption by more. Older households, in contrast, see their net worth increase by more and increase their consumption by more, and aggregate consumption declines by 0.17 in both scenarios.

Government debt is a nominal asset in our model and shocks to monetary policy change the amount of real government debt in the economy and create capital gains or losses depending on the household's age. Column 3 of Table 6 controls for these effects by assuming that government debt is a real asset. Its per capita supply is held fixed and its real return is predetermined to prevent capital gains and/or losses on household liquid asset positions in the impact year. The responses of output, investment, and consumption are close to the baseline, but stock prices and illiquid asset demand fall by less, and the distributional consequences of the shock are very different. All age groups experience declines in net worth in the impact period, and all households reduce their expected bequests. However, consumption responses are heterogeneous. Households over the age of 75 increase their consumption, but those between 21 and 74 reduce their consumption. Older households in the latter group miss out on the capital gain on their liquid asset positions that occurs in the baseline, and their consumption response turns negative. Consumption of younger

[36] The model is recalibrated to reproduce the same steady state illiquid asset output ratio as the baseline. Thus, the return on capital is the same as the baseline. The steady state after-tax spread on illiquid assets is 0.39 percent as compared to 0.45 percent in the baseline. Per capita nominal debt is held fixed when monetary policy is shocked.

households declines in both scenarios, but the decline is smaller when government debt is real because they do not experience a loss on their debt position in the impact period. Asset substitution effects are operative here even though government debt is real because liquid and illiquid assets are imperfect substitutes for households. The real return on liquid assets still increases in period 1, and this puts upward pressure on the return on illiquid assets, and investment and stock prices continue to fall.

The reaction of government borrowing to tighter monetary policy also matters in our model because households have finite lives and different planning horizons. In the baseline scenario, lump-sum taxes adjust each period to clear the government budget constraint. Instead, column 4 of Table 6 fixes lump-sum taxes for the first 100 years of the transition and allows (nominal) government to increase until year 100 at which point lump-sum taxes adjust to clear the government budget constraint. Since the youngest cohort survives for at most 100 years, all cohorts alive at the time of the shock avoid the reduction in transfers that occurs in the baseline specification.

Observe that real government debt increases by 0.95 percent, which is larger by two-thirds than its baseline response. Household demand for the liquid asset is interest rate elastic, and the return on liquid assets increases more than in the baseline. Higher government debt also crowds out demand for illiquid assets and the spread falls more than the baseline. Demand for illiquid assets now declines by 0.45 percent and stock prices fall by -1.04 percent on impact. Investment also declines, but the decline is smaller than the baseline. Investment and stock prices are forward-looking and have the same one-period holding returns, but the stream of future payoffs is different. The output decline is also smaller in this scenario than in the baseline, but aggregate consumption declines by about the same amount in the two scenarios. Finally, transfers and thus redistribution are very different. Lump-sum transfers fall in the baseline but are fixed in this scenario. Consequently, there is more redistribution to low net worth households here and consumption inequality declines by considerably more.

Kekre and Lenel (2022) analyze how shocks to monetary policy influence the risk environment and the market price of risk in a HANK model with heterogeneous preferences for risk, endogenous portfolios, and leverage. They find that modeling exposures to risk enhances the transmission of monetary policy to the real sector through its effect on equilibrium risk premia. Our model also has endogenous portfolios and leverage, and monetary policy also influences the equilibrium liquidity premium. In addition, our model reproduces the empirical responses of stock prices and the short-term rate to a tightening in monetary policy in Japanese data (Figure 3). Consequently, it is useful to analyze how the transmission of monetary policy changes when the distinction between liquid and illiquid assets is removed. This is accomplished by setting the costs of adjusting illiquid assets to zero.[37]

Removing household portfolio choices strengthens the transmission of monetary policy in our model. Investment and output decline by twice as much even though the year one real return on liquid assets increases by less: 0.02 in this scenario versus 0.24 in the baseline. Now, liquid and illiquid assets are perfect substitutes after the impact period, and the returns on capital and shares in intermediate goods producers have to fully match the higher real return on liquid assets. It follows that aggregate demand for illiquid assets falls more than in the baseline. Consumption inequality increases in this scenario because low net worth households are more exposed to the shock. Young households do not enjoy the benefits of taking leveraged long positions in illiquid assets and they reduce their consumption by more than in the baseline. Consumption of households older than age 86 increases in the baseline, but declines in this scenario. They particularly benefit from lending liquid assets to young households in the baseline, and this opportunity is absent here.

Finally, we consider a scenario with flexible prices. The transmission of monetary policy is much weaker when prices are flexible. Consumption and aggregate demand for illiquid assets exhibit small declines, while output and investment increase slightly. The one variable that responds more strongly is the real stock of government debt, which exhibits a large increase. Since nominal per capita debt is fixed, the increase is entirely due to a large decline in the price level. Even though most aggregate responses are small, the distributional consequences are very different. Gini consumption falls more and net worth increases for all households older than age 52.

Based on this analysis, we conclude that nominal price rigidities on intermediate goods producers and our model of household portfolio choice are the two most important factors contributing to our empirical results. Both mechanisms influence the size of the responses of output and investment and also how the shock affects net worth and consumption of different age groups. How the government closes its budget constraint also matters for investment, output, and stock prices. But the differences with the baseline are smaller,

[37] In this parameterization of the model the age distribution of portfolio shares of liquid and illiquid assets is indeterminate. We assume that households save in a single mutual fund that holds the market portfolio of government debt, capital, and equity. The preference discount factor is adjusted so that aggregate steady state holdings of capital, equity, and government debt relative to output are the same as in the baseline economy.

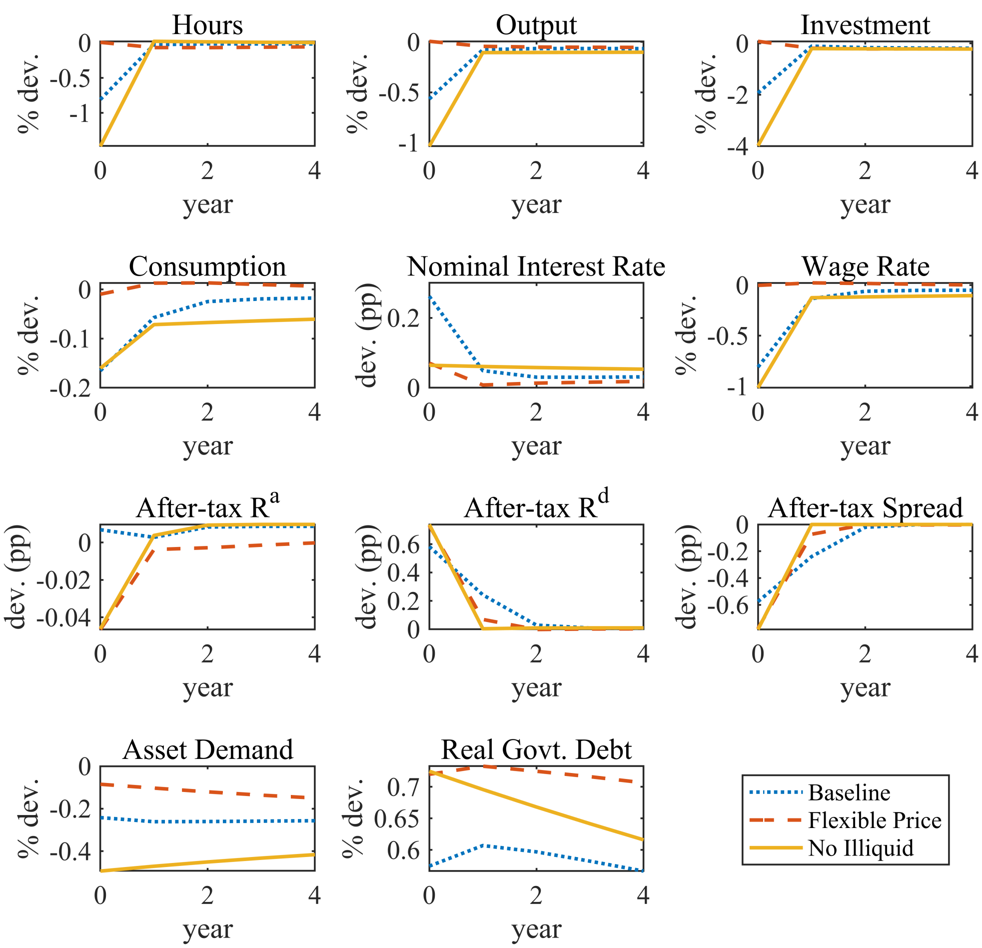
even though the scenario we consider is rather extreme. All of the fiscal imbalances created by the shock are accommodated by higher government borrowing and the resulting debt–output ratio increases from 1.23 to a peak of 1.32 before it is stabilized after 100 years. Less fiscal accommodation would produce impulse responses that are closer to the baseline scenario.

Figure 8 reports the full impulse response functions for the baseline, the no illiquid asset, and the flexible price scenarios. A comparison of the baseline with the flexible price specification reinforces our conclusion that nominal rigidities play a central role in the model's empirical performance. The decline in consumption is more persistent when they are present, and hours and wages respond to the shock. A comparison of the baseline with the no illiquid asset scenario indicates that the micro financial frictions are particularly important for investment, hours, and output, which all decline by less. There is also less substitution between private assets and government debt when financial frictions are present. Aggregate demand for private assets (capital and shares in intermediate goods firms) falls by less, and demand for government debt increases by less in the baseline. Finally, the size of the wealth effect is smaller, as evidenced by the faster recovery of consumption in the baseline specification. In the no illiquid asset specification, private assets experience a large capital loss when the shock arrives with their return falling by 0.046 percentage points. Modeling endogenous portfolio decisions also has different implications for the impact of monetary policy on consumption inequality at medium term horizons. The consumption Gini declines by 0.047 percentage points in year 5 in the baseline (see Table 5). However, when we abstract from portfolio choice decisions, the consumption Gini increases by 0.020 percentage points instead.

8 Robustness

We have found that nominal rigidities are particularly important to understand the short-run response of aggregate variables to a tightening in monetary policy. However, the asset substitution channel is stronger for other impulses. Braun and Ikeda (2023) show that the asset substitution transmission channel of monetary policy is more powerful than the NK transmission channel when the shock is a persistent change in the age distribution. In Appendix I.1 we show that the asset substitution channel is also a powerful transmission channel of monetary policy when the economy experiences a persistent improvement in technology.

Figure 8: Responses of aggregate variables to a tightening in monetary policy: baseline, flexible price, and no illiquid asset specifications



Note: Responses to an innovation of size 0.01 in the interest rate targeting rule.

In our baseline analysis, the government budget constraint is closed by adjusting the lump-sum transfer in each period. This government transfer provides a consumption floor that is particularly valuable to households that survive to very old ages. These households also have relatively high MPCs so it is possible that how we treat the oldest households matters for the results. We investigate this possibility by considering a scenario where the lump-sum transfer is held fixed at its steady state value for 76+ year old households (see Table 11 in Appendix I.2). Changing the government financing scheme in this manner does not alter our findings.

When comparing impulse response functions in our model with Japanese data, we have maintained the assumption that unconventional policies are equally effective in stabilizing the economy when the nominal interest rate is at its effective lower bound. We investigate the robustness of our conclusions to this assumption by considering a monetary policy shock under the assumption that monetary policy is exogenous ($\phi_\pi = 0$). Under this assumption, the monetary policy authority exogenously increases the nominal interest rate but does not react to the decline in the inflation rate. A comparison of the results in Table 10 in Appendix I.2 with Figure 5 indicates that the fit of the model improves in several dimensions. Aggregate consumption now declines by nearly twice as much on impact, the real price of equity declines by more, and the inflation rate falls by less than the baseline.

We have seen that a household's exposure to a tightening in monetary policy depends on its age, and the liquidity premium is endogenous in our model. Given the large differences in their wealth, asset holdings, and exposure to labor market risk, it is possible that an easing in monetary policy has a larger impact on economic activity compared to a tightening. A negative shock to the policy rate of the same magnitude produces small but meaningful differences in the absolute magnitudes of the aggregate responses (Table 10 in Appendix I.2). Hours per worker and wages respond by more, but consumption, investment, and output exhibit smaller responses to the shock. The response of the consumption Gini is also larger rising by 0.020 percentage points as compared to a decline of 0.015 percentage points in the baseline.

9 Conclusion

We have proposed a parsimonious model of liquidity, endogenous household portfolio decisions, and nominal frictions into a computable OLG model. Our framework produces empirically relevant macro and micro responses to monetary policy shocks and has rich implications for how households of different ages cope with and adjust their portfolios to shocks to monetary policy. In our current research, we analyze how central bank and fiscal authority feedback rules influence aggregate economic activity during a demographic transition to an older age distribution. Demographic change is a gradual but persistent impulse, and our preliminary results suggest that the reaction of government policy has large and persistent consequences for macroeconomic activity during Japan's demographic transition.

10 Declaration of Generative AI and AI-assisted technologies in the writing process

Statement: During the preparation of this work the author(s) used Writefull in Overleaf to check grammar and to propose alternative formulations of several awkward sentencess. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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Appendixes (For online publication)

A Model

A.1 Household problem

The derivations here are based on the household problem (10). Note, however, that the warm-glow utility function used in the following is more general than the one used in the main text. The derivations here assume that warm-glow preferences are given by

$$u(c_{j,t}, b^q_{j,t}, \bar{h}_{j,t}; \eta_j, z_{j,t} = 0) = \left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\right)^{\sigma} \eta_j \frac{\left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\underline{c} + b^q_{j,t}/\eta_j\right)^{1-\sigma}}{1-\sigma} \tag{A.1}$$

and allow for the possibility that bequests are a superior good. The specification in the main text and in the calibrated version of the model assumes that $\underline{c} = 0$. In this situation it is more parsimonious to represent the preference weight by a single parameter.[38] Let $\lambda_{j,t}(z)$ denote a Lagrange multiplier on the budget constraint (4) for $z = 1$ or (5) for $z = 0$. With no ad-hoc borrowing constraint, the first-order conditions with respect to $c_{j,t}$, $b^q_{j,t}$, $a_{j,t}$ and $d_{j,t}$ are given respectively by

$$\lambda_{j,t}(1) = \frac{1}{(1+\tau^c_t)} \left(\frac{c_{j,t}}{\eta_j}\right)^{-\sigma}, \tag{A.2}$$

$$\lambda_{j,t}(0) = \frac{1}{(1+\tau^{bq}_t)} \left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\right)^{\sigma} \left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\underline{c} + b^q_{j,t}/\eta_j\right)^{-\sigma}, \tag{A.3}$$

$$\begin{aligned} &\lambda_{j,t}(1)\left(1+\gamma_a(1)\Delta a_{j,t}\right) \\ &\quad = \beta\left[(1-\psi_{j+1,t+1})\frac{\partial U_{j+1}(a_{j,t}, d_{j,t}, 0)}{\partial a_{j,t}} + \psi_{j+1,t+1}\frac{\partial U_{j+1}(a_{j,t}, d_{j,t}, 1)}{\partial a_{j,t}}\right], \end{aligned} \tag{A.4}$$

$$\lambda_{j,t}(1) = \beta\left[(1-\psi_{j+1,t+1})\frac{\partial U_{j+1}(a_{j,t}, d_{j,t}, 0)}{\partial d_{j,t}} + \psi_{j+1,t+1}\frac{\partial U_{j+1}(a_{j,t}, d_{j,t}, 1)}{\partial d_{j,t}}\right], \tag{A.5}$$

where $\Delta a_{j,t} = a_{j,t}(1) - a_{j-1,t-1}$. For $z = 0$, conditions (A.4) and (A.5) are replaced by $a_{j,t} = 0$ and $d_{j,t} = 0$. The envelope conditions imply

$$\frac{\partial U_{j+1}(a_{j,t}, d_{j,t}, 1)}{\partial a_{j,t}} = \lambda_{j+1,t+1}(1)\left(\tilde{R}^a_{t+1} + \gamma_a(1)\Delta a_{j+1,t+1}\right), \tag{A.6}$$

$$\frac{\partial U_{j+1}(a_{j,t}, d_{j,t}, 0)}{\partial a_{j,t}} = \lambda_{j+1,t+1}(0)\left(\tilde{R}^a_{t+1} - \gamma_a(0)a_{j,t}\right), \tag{A.7}$$

$$\frac{\partial U_{j+1}(a_{j,t}, d_{j,t}, 1)}{\partial d_{j,t}} = \lambda_{j+1,t+1}(1)\frac{\tilde{R}_t}{\pi_{t+1}}, \tag{A.8}$$

[38] If we define $\omega = (\tilde{\omega}/(1-\tilde{\omega}))^{\sigma}$ in the preferences (A.1) with $\underline{c} = 0$, the preferences are reduced to those in the main text (8).

$$\frac{\partial U_{j+1}(a_{j,t}, d_{j,t}, 0)}{\partial d_{j,t}} = \lambda_{j+1,t+1}(0)\frac{\tilde{R}_t}{\pi_{t+1}}, \tag{A.9}$$

Rearranging conditions (A.2)-(A.9) yields

$$\begin{aligned}
&\frac{(1+\gamma_a(1)\Delta a_{j,t})}{(1+\tau_t^c)}\left(\frac{c_{j,t}}{\eta_{j,t}}\right)^{-\sigma} \\
=&\beta\Bigg[\frac{(1-\psi_{j+1,t+1})}{(1+\tau_{t+1}^{bq})}\left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\right)^{\sigma}\left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\underline{c}+b^q_{j+1,t+1}/\eta_{j+1}\right)^{-\sigma}\left(\tilde{R}^a_{t+1}-\gamma_a(0)a_{j,t}\right) \\
+&\frac{\psi_{j+1,t+1}}{(1+\tau_{t+1}^c)}\left(\frac{c_{j+1,t+1}}{\eta_{j+1}}\right)^{-\sigma}\left(\tilde{R}^a_{t+1}+\gamma_a(1)\Delta a_{j+1,t+1}\right)\Bigg],
\end{aligned} \tag{A.10}$$

$$\begin{aligned}
\frac{1}{(1+\tau_t^c)}\left(\frac{c_{j,t}}{\eta_{j,t}}\right)^{-\sigma} =&\beta\Bigg[\frac{1-\psi_{j+1,t+1}}{(1+\tau_{t+1}^{bq})}\left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\right)^{\sigma}\left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\underline{c}+b^q_{j+1,t+1}/\eta_{J+1}\right)^{-\sigma} \\
&+\frac{\psi_{j+1,t+1}}{(1+\tau_{t+1}^c)}\left(\frac{c_{j+1,t+1}}{\eta_{j+1}}\right)^{-\sigma}\Bigg]\frac{\tilde{R}_t}{\pi_{t+1}}.
\end{aligned} \tag{A.11}$$

Since the ad-hoc borrowing constraint that is imposed for $j = 1, 2, 3$ is binding under the parameter values of the model, equation (A.11) is replaced with $d_{j,t} = 0$ for $j = 1, 2, 3$. In the state $z = 0$, the household is in the final period of life and bequeaths all of its wealth

$$\begin{aligned}
b^q_{j,t}(0) = \frac{1}{(1+\tau_t^{bq})}\Bigg(&\tilde{R}^a_t a_{j-1,t-1} - \chi(0, a_{j-1,t-1}, 0) + \frac{\tilde{R}_{t-1}}{\pi_t}d_{j-1,t-1} \\
&+(1-\tau^w)w_t\epsilon_j h_{j,t} + b_{j,t} + \xi_t(0)\Bigg).
\end{aligned} \tag{A.12}$$

A.2 Liquidity premium and asset portfolios

Suppose that adjustment costs on illiquid assets are the same for surviving and non-surviving households: $\gamma_a(1) = \gamma_a(0) \equiv \gamma_a$ and that the utility function for bequests is given by (8). In addition, consider a stationary equilibrium so that time subscripts are dropped. Then, combining equations (A.10) and (A.11) yields

$$\Delta a_{j+1} = \frac{\tilde{R}}{\pi\kappa_{j+1}}\Delta a_j - \frac{1}{\gamma_a\kappa_{j+1}}\left(\tilde{R}^a - \frac{\tilde{R}}{\pi}\right) + \frac{1-\kappa_{j+1}}{\kappa_{j+1}}a_j, \tag{A.13}$$

where $\Delta a_j \equiv a_j(1) - a_{j-1}$ and $0 < \kappa_{j+1} < 1$, given by

$$\kappa_{j+1} = \frac{\psi_{j+1}\left(c_{j+1}(1)/\eta_{j+1}\right)^{-\sigma}}{(1-\psi_{j+1})\omega\left(b^q_{j+1}/\eta_{j+1}\right)^{-\sigma} + \psi_{j+1}\left(c_{j+1}/\eta_{j+1}\right)^{-\sigma}}.$$

From equation (A.13) we can establish two results. In doing so, we guess and verify that the initial real asset holding is positive $a_1 > 0$. Since $a_0 = 0$, an increase in the real asset in the initial age is also positive: $\Delta a_1 > 0$. The first result is that the interest rate spread between illiquid assets and liquid assets has to be positive

$$\tilde{R}^a - \frac{\tilde{R}}{\pi} > 0.$$

To show this, suppose contrarily that the interest rate spread is non-positive: $\tilde{R}^a - \tilde{R}/\pi \leq 0$. Then, because $\Delta a_1 > 0$, equation (A.13) implies $\Delta a_j > 0$ for all $j = 2, ..., J$, so that $a_J > 0$, violating the terminal condition. Hence, the interest rate spread has to be positive in equilibrium. Second, because of the positive interest rate spread, individuals will prefer to borrow at the same interest rate as the liquid assets. The second result implies $a_j \geq 0$ for all $j = 1, ..., J$. Hence, the household may leverage illiquid assets by borrowing liquid assets: $a_j > 0$ and $d_j < 0$. Since the problem of the illiquid asset choice is smooth in the initial age, illiquid asset holdings are positive in the initial age $a_1 > 0$, verifying the guess assumed in this discussion.

A.3 Labor supply decision

Working households belong to a labor union and work for an identical amount of hours $h_{j,t} = \bar{h}_t$ for all $j = 1, ..., J_r - 1$. The labor union consists of a continuum of union groups $l \in (0, 1)$ and distributes total hours worked $\sum_{j=1}^{J_r-1} \bar{h}_t N_{j,t}$ among union groups. Each union group l has a one-to-one linear technology that transforms hours per worker into specific labor supply per worker $\tilde{h}_t(l)$. An employment agency combines a continuum of specific labor and produces homogeneous labor $\tilde{H}_t = \tilde{h}_t \sum_{j=1}^{J_r-1} N_{j,t}$ following the technology: $\tilde{h}_t = \left[\int_0^1 \tilde{h}_t(l)^{\frac{1}{\theta_w}} dl\right]^{\theta_w}$, where $\theta_w > 1$ is a wage markup. The employment agency is competitive and it follows that demand for specific labor l is given by

$$\tilde{h}_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\frac{\theta_w}{\theta_w - 1}} \tilde{h}_t, \tag{A.14}$$

where $W_t(l)$ is nominal wage for specific labor supply $\tilde{h}_t(l)$.

Each union group chooses $W_t(l)$ to maximize the benefits of the members of the labor union, i.e. working households. Then, the problem of the union group is given by

$$\max_{\{W_t(l)\}} \quad (1 - \tau^w) \sum_{j=1}^{J_r-1} \frac{W_t(l)}{P_t} \epsilon_j \tilde{h}_t(l) \mu_{j,t}^w - \sum_{j=1}^{J_r-1} \left(\frac{\psi_{j,t}}{\lambda_{j,t}(1)} + \frac{1 - \psi_{j,t}}{\lambda_{j,t}(0)}\right) \frac{\upsilon}{1 + \frac{1}{\nu}} \tilde{h}_t(l)^{1+\frac{1}{\nu}} \mu_{j,t}^w,$$

subject to the labor demand curve (A.14), where $\lambda_{j,t}(z)$ with $z \in \{0, 1\}$ is a Lagrange multiplier, given by equation (A.2), and $\mu^w_{j,t} = N_{j,t}/\sum_{j=1}^{J_r-1} N_{j,t}$ is the ratio of population with age j to the working population. The second term in the problem is the weighted average of disutility of supplying labor, which is transformed into the units of the final good, over working households. This statement of the problem weights the disutilities of both surviving households and also households who experience a death shock. Our baseline specification assumes that only surviving households receive weight and $\psi_{j,t}$ is set to 1 for all j in the problem.[39] Then the first-order condition of this problem is equation (9) with $\bar{\lambda}_t$ and $\bar{\epsilon}_t$ are given, respectively, by

$$\bar{\lambda}_t = \left[\sum_{j=1}^{J_r-1}\left(\frac{\psi_{j,t}}{\lambda_{j,t}(1)} + \frac{1-\psi_{j,t}}{\lambda_{j,t}(0)}\right)\mu^w_{j,t}\right]^{-1}$$

$$\bar{\epsilon}_t = \sum_{j=1}^{J_r-1}\epsilon_j\mu^w_{j,t}.$$

A.4 Intermediate goods firms

The i-th intermediate goods firm's problem can be solved in two steps. First, it minimizes the real costs of production, $w_t\bar{H}_t(i) + r^k_t K_t(i)$, subject to equation (12), where w_t is the real wage on labor input, and r^k_t is the rental rate of capital. Let $K_t \equiv \int_{i\in(0,1)} K_t(i)di$ and $H_t = \int_{i\in(0,1)} H_t(i)di$ denote aggregate capital and aggregate raw hours worked, respectively. Then the cost minimizing input demands can be expressed as

$$w_t = mc_t(1-\alpha)\left(\frac{K_t}{Z_tH_t}\right)^{\alpha} Z_t, \tag{A.15}$$

$$r^k_t = = mc_t\alpha\left(\frac{K_t}{Z_tH_t}\right)^{\alpha-1}, \tag{A.16}$$

where mc_t is real marginal cost.

Second, the firm chooses $P_t(i)$ to maximize the present value of profits, $\Omega_t(i)+\Lambda_{t,t+1}\Omega_{t+1}(i)+ ...$, subject to equation (11), where the discount factor, $\Lambda_{t,t+1}$, is derived from preferences in the next subsection, and where the period-t profits, $\Omega_t(i)$, are given by

$$\Omega_t(i) = (1+\tau^f)\frac{P_t(i)}{P_t}Y_t(i) - mc_tY_t(i) - \frac{\gamma}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 Y_t, \quad \gamma > 0. \tag{A.17}$$

In equation (A.17) τ^f is a subsidy and the quadratic term is the price adjustment cost. We

[39] The small fraction of workers who experience the death event prefer to supply much less or even zero labor because they optimally choose to consume all of their wealth. If we weight these workers too, a suitable adjustment to η produces identical results.

assume that the subsidy is set at $\tau^f = \theta - 1$, so that marginal cost is one in the steady state. The optimality condition for the firm's price setting problem is:

$$\gamma\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)\frac{Y_t}{P_{t-1}(i)} = \frac{\theta}{\theta-1}\frac{mc_t Y_t}{P_t} - \frac{1+\tau^f}{\theta-1}\frac{Y_t}{P_t} + \Lambda_{t,t+1}\gamma\left(\frac{P_{t+1}(i)}{P_t(i)} - 1\right)\frac{P_{t+1}(i)}{P_t(i)^2}Y_{t+1}.$$

In a symmetric equilibrium the previous equation simplifies to equation (13), which is a nonlinear version of the NK Phillips curve.

B Competitive equilibrium

B.1 Derivation of the final goods market equilibrium condition

To derive the final goods market clearing condition, observe that the household budget constraints given by equations (4)–(5) hold with equality in equilibrium and can be summed over j, to obtain

$$\begin{aligned}
&\sum_{j=1}^{J} N_{j,t}\left[\psi_j(1+\tau_t^c)c_{j,t} + (1+\tau^{b^q})(1-\psi_j)b_{j,t}^q + \psi_j a_{j,t}(1) + \psi_j d_{j,t}(1)\right] + X_t \\
=&\tilde{R}_t^a \sum_{j=1}^{J} a_{j-1,t-1}(1)N_{j,t} + \frac{\tilde{R}_{t-1}}{\pi_t}\sum_{j=1}^{J} d_{j-1,t-1}(1)N_{j,t} + (1-\tau^w)w_t H_t + B_t + \Xi_t, \\
=&\tilde{R}_t^a A_{t-1} + \frac{\tilde{R}_{t-1}}{\pi_t}D_{t-1} + (1-\tau^w)w_t H_t + B_t + \Xi_t,
\end{aligned}$$

where $H_t = \sum_{j=1}^{J}\epsilon_j \bar{h}_t N_{j,t}$, $\Xi_t = \bar{\Xi}_t + B_t^q$ and

$$X_t = \sum_{j=1}^{J}\left[\psi_{j,t}\chi\left(a_{j,t}(1), a_{j-1,t-1}(1), 1\right) + (1-\psi_{j,t})\chi\left(0, a_{j-1,t-1}(1), 0\right)\right]N_{j,t}$$

with $a_{0,t-1} = 0$. Substituting the government budget constraint, equation (23), into the previous equation and using the formulas for the after-tax interest rates we obtain

$$\begin{aligned}
&(1+\tau_t^c)C_t + \tau_t^{b^q}B_t^q + A_t + X_t \\
=&[1 + (1-\tau^{ka})(R_t^k - 1)]A_{t-1} - \tau^a\frac{R_t}{\pi_t}D_{t-1} + (1-\tau^w)w_t H_t + T_t - G_t - \tau^f Y_t.
\end{aligned}$$

Further substituting the tax equation (22) for T_t into this condition yields

$$C_t + A_t + X_t = R_t^k A_{t-1} + w_t H_t - G_t - \tau^f Y_t.$$

Recall that equation (19) implies $R_t^k = (\Omega_t + V_t)/V_{t-1}$. Thus, income from illiquid assets can be expressed as

$$\begin{aligned} R_t^k A_{t-1} =& R_t^k (K_t + V_{t-1}), \\ =& [r_t^k + 1 - \delta] K_t + \Omega_t + V_t \\ =& r_t^k K_t + K_{t+1} - I_t + \Omega_t + V_t. \end{aligned}$$

Substituting (A.15), (A.16), (15), and (26) for w_t, r_t^k, Ω_t, and A_t, respectively, into the budget constraint yields[40]

$$C_t + I_t + G_t = Y_t - \frac{\gamma}{2} (\pi_t - 1)^2 Y_t - X_t, \tag{B.1}$$

B.2 Stationary equilibrium

In the economy with non-zero population growth, the aggregate variables such as Y_t grow at the rate of the population growth of $n_t = N_t/N_{t-1}$ in a stationary equilibrium. We scale aggregate variables in capital letters by using population and denote corresponding per-capita variables as their small letters, e.g., $y_t = Y_t/N_t$.

In a stationary equilibrium the inflation rate is at its target value of $\pi = 1$. This implies that the marginal cost is unity, $mc = 1$, from equation (13), and the price adjustment cost is zero. Instead of pinning down per-capita hours worked $h = H/N$, we normalize it to be unity in a stationary equilibrium and set the coefficient of disutility of labor, υ, to satisfy $h = 1$. Note that hours in efficiency units per total population and hours per working population are related as follows: $h = \sum_{j=1}^{J_r-1} \epsilon_j \bar{h} \mu_j$. Under the normalization of $h = 1$, hours per working population is given by

$$\bar{h} = \frac{1}{\sum_{j=1}^{J_r-1} \epsilon_j \mu_j}.$$

Only when $\sum_{j=1}^{J_r-1} \epsilon_j \mu_j = 1$ e.g. for normalization, the two variables coincide: $h = \bar{h} = 1$.

[40] Substituting equation (26) into the aggregate household budget constraint yields

$$C_t + I_t + G_t = r_t^k K_t + w_t H_t + \Omega_t - \tau^f Y_t - X_t.$$

Substituting (15) into this equation

$$C_t + I_t + G_t = r_t^k K_t + w_t H_t + \left[\theta - mc_t - \frac{\gamma}{2} (\pi_t - 1)^2\right] Y_t - \tau^f Y_t - X_t.$$

Note that equations (A.16) and (A.15) imply $r_t^k K_t + w_t H_t = mc_t Y_t$. And note that $\tau^f = \theta - 1$. Substituting these into the above equation yields equation (B.1).

Fix r^k, $\tilde{R}$, $\bar{\xi}(0)$, and $\xi(1)$. The return on capital is $R^k = r^k + 1 - \delta$. After the deduction of the capital income tax, the return on illiquid assets is given by $R^a = 1 + (1-\tau^k)(R^k - 1)$. The interest rate income tax τ^a is also imposed. So, the after-tax return is $\tilde{R}^a = 1 + (1-\tau^a)(R^a - 1)$. The capital-labor ratio is given by $K/H = (r^k/\alpha)^{-1/(1-\alpha)}$ from equation (A.16). The output is given by $y = (K/H)^\alpha h$ with $h = 1$ by normalization. The real wage is given by $w = (1-\alpha)(K/H)^\alpha$ from equation (A.15). Then the pension benefit b is

$$b = \lambda \frac{1}{j_r - 1} \sum_{j=1}^{j_r - 1} w\epsilon_j h_j,$$

where $h_j = \bar{h}$. Holdings of liquid assets are $d = (D/Y)y$, where D/Y is a targeted net government debt output ratio. The government spending is $g = (G/Y)y$, where G/Y is a targeted government expenditure output ratio.

Consider a backward shooting algorithm for b_J^q and $a_{J-1}(1)$. Fix $b_J^q(0) > 0$ and $a_{J-1}(1) \geq 0$. Consumption $c_{J-1}(1)$ is given by equation (A.11) with $\psi_J = 0$ as

$$c_{J-1}(1) = \left(\frac{(1+\tau_t^c)\beta\tilde{R}}{\pi(1+\tau^{b^q})} \right)^{-\frac{1}{\sigma}} \eta_{J-1} \left(\frac{\tilde{\omega}}{1-\tilde{\omega}} \right)^{-1} \left(\frac{\tilde{\omega}}{1-\tilde{\omega}} \underline{c} + b_J^q/\eta_J \right) \tag{B.2}$$

From equation (A.10) with $\psi_J = 0$, we obtain $a_{J-2}(1)$ as

$$a_{J-2} = a_{J-1} - \frac{1}{\gamma_a(1)} \left[\frac{\beta(1+\tau^c)}{(1+\tau^{b^q})} \left(\frac{\tilde{\omega}}{1-\tilde{\omega}} \right)^{\sigma} \left(\frac{\frac{\tilde{\omega}}{1-\tilde{\omega}}\underline{c} + b_J^q/\eta_J}{c_{J-1}(1)} \eta_{J-1} \right)^{-\sigma} \left(\tilde{R}^a - \gamma_a(0) a_{J-1} \right) - 1 \right].$$

Equation (A.12) is used to find liquid asset holding $d_{J-1}(1)$

$$d_{J-1} = \frac{1}{\tilde{R}} \left\{ (1+\tau^{b^q}) b_J^q + \chi(0, a_{J-1}(1), 0) - \tilde{R}^a a_{J-1}(1) - \left[(1-\tau^w) w\epsilon_J \bar{h} + b + \xi(0) \right] \right\}.$$

From the budget constraint (4), the liquid asset holding $d_{J-2}(1)$ is given as

$$\begin{aligned} d_{J-2} = \frac{1}{\tilde{R}} \Big\{ & c_{J-1}(1) + a_{J-1}(1) + \chi(a_{J-1}(1), a_{J-2}(1), 1) + d_{J-1}(1) \\ & - \tilde{R}^a a_{J-2}(1) - \left[(1-\tau^w) w\epsilon_{J-1} \bar{h} + b + \xi(1) \right] \Big\}. \end{aligned} \tag{B.3}$$

Then, b_{J-1}^q is given by equation (A.12). With b_{J-1}^q and c_{J-1} on hand, c_{J-2} is given by equation (A.11) as

$$c_j = \left(\frac{1}{(1+\tau^c)\beta\tilde{R}} \right)^{\frac{1}{\sigma}} \eta_j \left[\frac{(1-\psi_{j+1})}{(1+\tau_{t+1}^{b^q})} \left(\frac{\tilde{\omega}}{1-\tilde{\omega}} \right)^{\sigma} \left(\frac{\tilde{\omega}}{1-\tilde{\omega}} \underline{c} + b_{j+1,t+1}^q/\eta_{j+1} \right)^{-\sigma} \right.$$

$$+\frac{\psi_{j+1,t+1}}{(1+\tau^{c}_{t+1})}\left(\frac{c_{j+1,t+1}}{\eta_{j+1}}\right)^{-\sigma}\Bigg]^{-\frac{1}{\sigma}}, \tag{B.4}$$

for $j = J-2$. Then, Δa_j is given by equation (A.10) and the real asset holding in the previous period is given by $a_{j-1} = a_j - \Delta a_j$ for $j = J-2$. Then, the liquid asset holding in the previous period is given by (B.3),

$$\begin{aligned}a_{j-1} =& a_j - \frac{1}{\gamma_a(1)}\Bigg[\beta(1+\tau^c)\frac{(1-\psi_{j+1})}{(1+\tau^{bq})}\left(\frac{\tilde{\omega}}{1-\tilde{\omega}}\right)^{\sigma}\left(\frac{\frac{\tilde{\omega}}{1-\tilde{\omega}}\underline{c}+b^q_{j+1}/\eta_{j+1}}{c_j(1)}\eta_j\right)^{-\sigma}\left(\tilde{R}^a-\gamma_a(0)a_j\right)\\ &+\beta\psi_{j+1}\left(\frac{c_{j+1}}{c_j}\frac{\eta_j}{\eta_{j+1}}\right)^{-\sigma}\left(\tilde{R}^a+\gamma_a(1)\Delta a_{j+1}\right)-1\Bigg].\end{aligned}$$

Bequests b^q_{j-1} are given by equation (A.12), and the consumption $c_{j-1}(1)$ of surviving households is given by equation (B.4). Continuing this process for $j = J-3,...,1$ yields the initial asset holdings of $a_{-1}(1)$ and $d_3(1)$. Since the ad-hoc borrowing constraint $d_j \geq 0$ is imposed for $j = 1, 2, 3$, its initial condition is $d_3(1)$. Bequests at age J, b^q_J, and the real asset holdings in period $J-1$, $a_{J-1}(1)$, are adjusted so as to satisfy $d_3(1) = 0$ and $a_{-1}(1) = 0$. Having found individual bequests, per-capita bequests are then given by:

$$b^q = \sum_{j=1}^{J}(1-\psi_j)b^q_j\mu_j.$$

The loop for r^k, $\tilde{R}$, $\xi(0)$ and $\xi(1)$ is closed as follows. Let τ_t denote per capita tax revenue: $\tau_t = T_t/N_t$. From equation (23), the updated value of the lump-sum transfers $\xi'(0)$ and $\xi'(1)$ are given by

$$\begin{aligned}\xi'(0) =& \tau - g - \left(\frac{R}{\pi n}-1\right)d - \tau^f y - b\sum_{j=j_r}^{J}\mu_j,\\ \xi'(1) =& \xi'(0) + b^q/\sum_{j=1}^{J}\psi_j\mu_j.\end{aligned}$$

where $R = 1 + (\tilde{R}-1)/(1-\tau^a)$, n is the gross growth rate of population, and τ is given by equation (22) as

$$\tau = \sum_{j=1}^{J}(\tau^c\psi_j c_j + \tau^{b^q}(1-\psi_t)b^q_j)\mu_j + \tau^{ka}(r^k-\delta)\sum_{j=1}^{J}a_{j-1}(1)\mu_j$$

$$+\tau^a \frac{R-1}{\pi} \sum_{j=1}^{J} d_{j-1}(1)\mu_j + \tau^w w \sum_{j=1}^{J} \epsilon_j \bar{h} \mu_j. \tag{B.5}$$

From equation (24),

$$d_t = \sum_{j=1}^{J} d_{j,t}(1)\mu_{j+1,t+1} n_{t+1},$$

with $d_{J,t}(1) = 0$, so that in evaluating τ, $\sum_{j=1}^{J} d_{j-1}(1)\mu_j$ in equation (B.5) should be set at

$$\sum_{j=1}^{J} d_{j-1}(1)\mu_j = d/n.$$

The capital stock is given by $k = (a - v)/n$ from equation (26), where the illiquid asset holding is given by $a \equiv \sum_{j=1}^{J} a_{j-1}(1)\mu_j/n$ and the value of firms v is given by combining equation (20) and (15) as

$$v = \frac{(\theta - 1)y}{R^k/n - 1}.$$

The updated value of r^k is given by

$$r^{k\prime} = \alpha k^{\alpha-1} h^{1-\alpha},$$

with $h = 1$. The value of the liquid asset holding, implied from the household optimization problem is $d' = \sum_{j=1}^{J} d_{j-1}(1)\mu_j/n$. The corresponding value that is consistent with the target value of debt output ratio is d. Since households are willing to hold liquid assets more as the interest rate increases, adjust the updated value of $\tilde{R}'$ as follows: increase (decrease) $\tilde{R}'$ if $d' < d$ ($d' > d$). In doing so, make sure that there is a positive spread between $\tilde{R}^a$ and $\tilde{R}/\pi$: $\tilde{R}^a - \tilde{R}/\pi > 0$.

Guessed values for r^k, $\tilde{R}$, and $\bar{\xi}$ are adjusted until $|r^k - r^{k\prime}| + |d - d'| + |\xi(0) - \xi'(0)| + |\xi(1) - \xi'(1)|$ becomes close enough to zero. This completes the description of the computation of the stationary equilibrium.

B.3 Dynamic equilibrium

Consider the computation of a transition path from period t_s to period t_e. Without loss of generality we assume that the economy reaches a stationary equilibrium in period t_e. In what follows we will assume that the initial condition is also a steady state, that is, that the economy is in steady state in period $t_s - 1$. In order to induce a transition we assume that in period t_s an MIT shock arrives. The MIT shock could consist of a perturbation to the central bank's nominal interest rate targeting rule, and/or some element of fiscal policy.

In the main text we have assumed that the population distribution is stationary with zero population growth in the dynamic equilibrium. However, here we relax that assumption and allow for the possibility of time-varying population during the transition. In particular, the size of the population in period t follows $N_t = n_t N_{t-1}$ where n_t is the population growth rate and survival probabilities ψ_t can depend on t during the transition. In what follows our use of the term steady state when referring to the initial or terminal condition consists of a steady state population distribution with a constant population as well as the steady state price system and allocations that we defined in the previous section.

The aggregate state variables in period $t = t_s$ consist of the per-capita capital stock k_t, per-capita value of equity in intermediate goods firms v_{t-1}, per capita government debt d_{t-1}, and the nominal interest rate R_{t-1}. The initial conditions for age-specific variables are liquid asset holdings $\{d_{j,t-1}\}_{j=1}^J$ and illiquid asset holdings $\{a_{j,t-1}\}_{j=1}^J$. Recall that illiquid assets consist of the capital stock and equity in intermediate goods firms and that each age-group holds the market portfolio of the two underlying assets. The market portfolio shares of capital stock and equity are k_{t_s}/a_{t_s-1} and v_{t_s-1}/a_{t_s-1}, respectively, where $a_{t_s-1} = k_{t_s} + v_{t_s-1}$.

We solve this two point boundary problem by guessing and verifying a path for $\{r_t^k, \pi_t, \xi_t(z), \gamma_{y,t+1}, \gamma_{k,t+1}\}_{t=t_s}^{t_e}$ and $R_{t_s}^a$, where $\gamma_{y,t+1} = y_{t+1}/y_t$, $\gamma_{k,t+1} = k_{t+1}/k_t$, and $R_{t_s}^a$ is the ex post return on the market portfolio of illiquid assets in the initial period.

Since the initial condition is a steady state, a sequence of capital is given by $k_{t+1} = \gamma_{k,t+1} k_t$ with $k_{t_s} = k$. From the law of motion for capital (16), the newly produced investment good in per capita is given by

$$i_t = n_{t+1} k_{t+1} - (1-\delta) k_t.$$

where $i_t \equiv I_t/N_t$. The real return on illiquid assets is given by the arbitrage condition (19) as $R_{t+1}^a = 1 + (1-\tau^k)(R_{t+1}^k - 1)$, where $R_{t+1}^k = r_{t+1}^k + 1 - \delta$. Its after-tax return is given as $\tilde{R}_s^a = 1 + (1-\tau^a)(R_s^a - 1)$ for $s = t_s+1, t_s+2, \ldots$. For $s = t_s$, the guessed value of $R_{t_s}^a$ is used to compute its after-tax return. Compute the nominal interest rate R_t using the monetary policy rule (21) for $t = t_s, ..., t_e$. Compute the marginal cost mc_t using the Phillips curve (13) as

$$mc_t = 1 + \frac{\gamma(\theta-1)}{\theta}\left[(\pi_t - 1)\pi_t - \frac{\gamma_{Y,t+1} n_{t+1}}{R_{t+1}^k}(\pi_{t+1} - 1)\pi_{t+1}\right].$$

for $t = t_s, ..., t_e$. Compute k_t/h_t from equation (A.16) and compute w_t from equation (A.15). Since k_t is known, hours per capita in efficiency units h_t can be computed from the ratio k_t/h_t. Hours per working population is given by $\bar{h}_t = h_t / \sum_{j=1}^{J_r-1} \epsilon_j \mu_{j,t}$.

Given these prices and aggregate variables, solve the household problem for those with age $j = 2, ..., J$ in period $t = t_s$ and those who are born in period $t = t_s, ..., t_e$. The solution yields $\{a_{j,t}(z), d_{j,t}(z), c_{j,t}, b^q_{j,t}\}_{j=1}^J$ for $t = t_s, ..., t_e$ and for $z \in \{0, 1\}$. From these individual decisions, the illiquid asset holding, the liquid asset holding, and consumption in per capita terms are given, respectively, by $a_t = \sum_{j=1}^J [\psi_{j,t} a_{j,t}(1) + (1 - \psi_{j,t}) a_{j,t}(0)] \mu_{j,t}$, $d_t = \sum_{j=1}^J [\psi_{j,t} d_{j,t}(1) + (1 - \psi_{j,t}) d_{j,t}(0)] \mu_{j,t}$, and $c_t = \sum_{j=1}^J c_{j,t} \mu_{j,t}$, where $\mu_{j,t} = N_{j,t}/N_t$ is the share of population with age j.

Now we are in a position to aggregate the economy and derive conditions to confirm whether the initially guessed values for $R^a_{t_s}$ and $\{r^k_t, \pi_t, \xi_t(z), \gamma_{y,t+1}, \gamma_{k,t+1}\}_{t=t_s}^{t_e}$ are in an equilibrium. By using the endogenously computed d_t and the exogenously given $d^n_t = D^n_t / N_t$, the price level P_t can be computed from the liquid asset market clearing condition (24) for all t. This yields the updated sequence of the inflation rate, $\pi'_t = P_t/P_{t-1}$ for all t, where P_{t_s-1} is in the initial steady state. From equation (24), using $d_{0,t-1}(1) = d_{J,t-1}(1) = 0$, the liquid asset market clearing condition is written as

$$\frac{d^n_{t-1}}{P_{t-1}} = \sum_{j=1}^J d_{j-1,t-1}(1) \mu_{j,t} n_t.$$

Then, the liquid asset term in the tax equation (22) can be written as

$$\begin{aligned}
\tau^a \frac{R_{t-1} - 1}{\pi_t} \sum_{j=1}^J d_{j-1,t-1}(1) N_{j,t} =& \tau^a \frac{R_{t-1} - 1}{P_t/P_{t-1}} \sum_{j=1}^J d_{j-1,t-1}(1) \mu_{j,t} N_t \\
=& \tau^a \frac{R_{t-1} - 1}{P_t} \frac{d^n_{t-1}}{n_t} N_t.
\end{aligned}$$

From equations (22) and (23), the updated value of the lump-sum transfers $\xi'_t(0)$ and $\xi'_t(1)$ are given by: for $t = t_s + 1, ..., t_e$

$$\begin{aligned}
\xi'_t(0) =& \sum_{j=1}^J [\psi_{j,t} \tau^c c_{j,t} + (1 - \psi_{j,t}) \tau^{b^q} b^q_{j,t}] \mu_{j,t} + \tau^{ka} (R^k_t - 1) \sum_{j=1}^J a_{j-1,t-1}(1) \mu_{j,t} + \tau^a \frac{(R_{t-1} - 1) \frac{d^n_{t-1}}{n_t}}{P_t} \\
&+ \tau^w w_t \sum_{j=1}^J \epsilon_j \bar{h}_t \mu_{j,t} - g_t - \frac{R_{t-1} d^n_{t-1}/n_t - d^n_t}{P_t} - \tau^f y_t - \sum_{j=j_r}^J b_{j,t} \mu_{j,t}, \qquad \text{(B.6)} \\
\xi'_t(1) =& \xi'_t(0) + \frac{\sum_{j=1}^J (1 - \psi_{j,t}) \tau^{b^q} b^q_{j,t} \mu_{j,t}}{\sum_{j=1}^J \psi_{j,t} \mu_{j,t}}. \qquad \text{(B.7)}
\end{aligned}$$

where $g_t = G_t/N_t$ and $\tau^{ka} = \tau^k + \tau^a - \tau^k\tau^a$. In period $t = t_s$ when the monetary policy shock hits, the returns earned by investing in capital and ownership shares can be different so that the lump-sum transfer is given by:

$$\xi'_{t_s}(0) = \sum_{j=1}^{J}\left[\psi_{j,t_s}\tau^c c_{j,t_s} + (1-\psi_{j,t_s})\tau^{b^q} b^q_{j,t_s} + \frac{\tau^{ka}}{1-\tau^k}(R^a_{t_s}-1)a_{j-1,t_s-1} + \tau^w w_{t_s}\epsilon_j \bar{h}_{t_s}\right]\mu_{j,t_s}$$
$$+\tau^a\frac{(R_{t_s-1}-1)d^n_{t_s-1}/n_{t_s}}{P_{t_s}} - g_{t_s} - \frac{R_{t_s-1}d^n_{t_s-1}/n_{t_s} - d^n_{t_s}}{P_{t_s}} - \tau^f y_{t_s} - \sum_{j=j_r}^{J} b_{j,t_s}\mu_{j,t_s},$$

and $\xi'_{t_s}(1)$ can be derived as in equation (B.7).

From equation (26), the updated value of the capital stock is given by

$$k'_{t+1} = (a_t - v_t)/n_{t+1},$$

where $v_t \equiv V_t/N_t$ is the per-capita value of the sum of the intermediate goods firms and investment good firms. Its aggregate value V_t is given by (20) with Ω_t given by (15). Thus, v_t can be computed forward using

$$v_t = \frac{\left[\theta - mc_{t+1} - \frac{\gamma}{2}(\pi_{t+1}-1)^2\right]y_{t+1}}{R^k_{t+1}/n_{t+1}} + \frac{v_{t+1}}{R^k_{t+1}/n_{t+1}},$$

With a sequence of k'_{t+1} in hand, the updated gross growth rate of capital is simply computed as $\gamma'_{k,t+1} = k'_{t+1}/k'_t$. From the wage Phillips curve (9), the updated value of the hours worked $\bar{h}'_t$ can be computed. The updated value for the net return on capital $r^{k\prime}_t$ is given by equation (A.16). The updated output is given by

$$y'_t = (k'_t)^\alpha\left(\sum_{j=1}^{J}\epsilon_j\bar{h}'_t\mu_{j,t}\right)^{1-\alpha},$$

for all $t = t_s, ..., t_e$. Then, the updated value of $\gamma_{y,t+1}$ is given by $\gamma'_{y,t+1} = y'_{t+1}/y'_t$. The updated value of $R^a_{t_s}$ is the weighted average of the returns on capital and equity

$$R^{a\prime}_{t_s} = 1 + (1-\tau^k)\left\{\frac{k_{t_s}n_{t_s}}{k_{t_s}n_{t_s}+v_{t_s-1}}(r^k_{t_s}+1-\delta)\right.$$
$$\left.+\frac{v_{t_s-1}}{k_{t_s}n_{t_s}+v_{t_s-1}}\frac{\left(v_{t_s} + \left[\theta - mc_{t_s} - \frac{\gamma}{2}(\pi_{t_s}-1)^2\right]y_{t_s}\right)n_{t_s}}{v_{t_s-1}} - 1\right\},$$

where $k_{t_s} = k$ and $v_{t_s-1} = v$. Finally, the updated value of pension benefits is computed as:

$$b_{j,t} = \lambda \left(\frac{1}{j_r - 1} \sum_{i=1}^{j_r - 1} w_{t-j+i} \epsilon_i \bar{h}_{t-j+i} \right).$$

for $j = j_r, ..., J$ and $t = t_s, t_s + 1, ...$

The values of $\{r_t^k, \pi_t, \xi_t(z), \gamma_{Y,t+1}\}_{t=t_s}^{t_e}$ and $R_{t_s}^a$ are adjusted until $\max_{t \in \{t_s,...,t_e\}}(|r_t^k - r_t^{k\prime}| + |\pi_t - \pi_t'| + |\xi_t(0) - \xi_t(0)'| + |\xi_t(1) - \xi_t(1)'| + |\gamma_{Y,t+1} - \gamma_{Y,t+1}'| + |\gamma_{k,t+1} - \gamma_{k,t}'|) + |R_{t_s}^a - R_{t_s}^{a\prime}|$ becomes close enough to zero. This completes the computation of the transition.

C Data measures of liquid and illiquid assets

Here we provide more detail on how we construct the aggregate stocks of liquid and illiquid assets. We have two alternative data sources. First, we can construct them using aggregate data from the Japanese Flow of Funds Accounts (FFA) for financial variables and data from the Japanese National Income and Product Accounts (NIPA) accounts for aggregate holdings of physical assets. Secondly, we derive aggregate stocks from the NSFIE, which is nationally representative and conducted once every five years. We describe each of these strategies in turn. Table 7 reports assets and liabilities as a fraction of GDP. The GDP shares in the column with the heading FFA/NIPA are constructed using 2014 fiscal year FFA data for the household sector from the Bank of Japan and using 2014 NIPA accounts for calendar year GDP and holdings stocks of physical assets held by households. We discuss construction of the data in this column first. In Japanese FFA data the household sector consists of households and private unincorporated enterprises. Aggregate liquid assets consist of household holdings of cash, domestic deposits, and public and private debt securities. This amounts to 1.79 times GDP. Liquid liabilities, which consist of consumer credit, are 0.069 of GDP. Net liquid assets relative to GDP are then 1.73 using the classification scheme of Kaplan et al. (2018). As explained in the main text it is more suitable in our model to treat all household borrowing as liquid borrowing. Under this assumption net liquid assets are 1.23.

Illiquid assets have two components: physical and financial assets. Physical assets are the end-of-calendar-year stock of household (and unincorporated private business) non-financial assets taken from the 2014 NIPA and amount to 2.02 times GDP. Financial assets include household holdings of: non-life insurance reserves, life insurance reserves, annuity entitlements, private pensions (defined benefit and defined contribution), and equity and options from the FFA. The resulting magnitude of illiquid financial assets is 3.50 times

Table 7: Liquid and illiquid assets relative to GDP in FFA/NIPA and NSFIE

	FFA/NIPA	NSFIE
A. Liquid assets and liabilities		
Liquid assets		
Currency and domestic deposits	1.74	0.99
Debt securities (total public and private)	0.052	0.047
Total liquid assets	1.79	1.04
Liquid liabilities		
Consumer credit	-0.069	-0.025
Total liquid liabilities	-0.069	-0.025
Net liquid assets: Kaplan et al. (2018)	1.73	1.01
Net liquid assets: Our model	1.23	0.85
B. Illiquid assets and liabilities		
Illiquid assets		
Physical assets	2.02	2.17
Equity and options	0.49	0.19
Insurance	0.99	0.30
Non-life insurance reserves	0.09	NA
Life insurance	0.88	0.30
Life insurance reserves	0.39	NA
Annuity entitlements	0.18	NA
Private pensions	0.31	NA
Total illiquid assets	3.50	2.66
Illiquid liabilities		
Installment credit and net non-financial sector loans	-0.00	-0.01
Mortgages	-0.37	-0.34
Other loans fin. inst. and non-home loans public inst.	-0.11	NA
Total illiquid liabilities	-0.49	-0.35
Net illiquid assets: Kaplan et al. (2018)	3.01	2.31
Net illiquid assets: Our model	3.50	2.66
C. Net worth	4.73	3.32

Note: Data are expressed as a multiple of GDP. Results under the heading FFA/NIPA are based on Flow of Funds and NIPA aggregate data. Data under the heading NSFIE are constructed from the National Family Income and Expenditure Survey and the Family Income and Expenditure Survey.

GDP. Illiquid liabilities consist of net non-financial sector loans, installment credit and non-financial sector loans, mortgages, and other loans by financial institutions plus non-housing loans by public financial institutions. Total illiquid liabilities constructed in this way amount to 0.49 of GDP. Net illiquid assets are then 3.01 using the classification scheme of Kaplan et al. (2018) and 3.50 using our classification scheme. Summing together net liquid and illiquid assets implies that total household net-worth is 4.73 times GDP.

An alternative way to construct these aggregates is to aggregate up micro survey data. Our primary source for the results reported in the final column of Table 7 is the 2014 NSFIE. An attractive feature of this survey is that it reports family income, financial and physical assets, and liabilities by 10 year age group of the household head. This survey is large and nationally representative but is only conducted at 5 year intervals. The most recent survey for which data is publicly available as of 2020 is from the year 2014. This is why we have chosen to use the year 2014 as our reference point.

A much smaller household survey called the Family Income and Expenditure Survey (FIES) is conducted on a quarterly basis. This survey provides more detail on the composition of financial assets and liabilities than the NSFIE but doesn't report results for holdings of physical assets. We use data from the FIES 2019 end of year survey to impute shares of financial liquid assets and liabilities by 10-year age group when these categories are not available in the NSFIE. Namely, we impute liquid and illiquid shares of financial securities and equity using their shares in the 2019 FIES for each 10 year age group. The 2014 NSFIE reports mortgages but does not provide detail on other liabilities. We thus impute non-mortgage liabilities by 10-year age group using their shares in the 2019 FIES. A final issue is that holdings of cash are not reported in the NSFIE. We impute cash holdings assuming that they are 9% of deposits, which is the ratio of cash to deposits in the aggregate FFA data.

With household level data on liquid and illiquid assets in hand, we then construct aggregate NSFIE assets by multiplying average per-household levels of each variable times the total number of households in Japan in 2014 and then dividing by calendar year nominal GDP for the year 2014. A comparison of the two columns of data in Table 7 reveals that the size of net liquid and illiquid assets is smaller in the NSFIE than in the FFA/NIPA. For instance, liquid assets are 1.79 times GDP in the FFA/NIPA and only 1.04 times GDP in the NFSIE. Deposits in the FFA, in particular, are much higher than in the NSFIE.

Illiquid assets are also smaller in the NSFIE as compared to the FFA. Physical assets are about the same in the two datasets, but, holdings of life insurance and equity are much smaller in the NSFIE. A final and smaller difference is that illiquid liabilities are also smaller in the NSFIE. This second difference appears to be primarily due to non–mortgage lending by financial institutions and non-housing loans by public financial institutions, categories that are not broken out in the NSFIE. These differences translate into smaller average net worth and a larger share of illiquid assets in total (net) assets. Net worth is 3.32 in the NSFIE while it is 4.73 in the FFA data. Using definitions suitable for our model, the share of illiquid assets in total assets is 0.76 for the NSFIE and 0.74 for the FFA.

Takayama and Kitamura (1994) report that the size of average household financial assets is substantially smaller in the NSFIE survey than the FFA data using the NSFIE in earlier years. They suggest three reasons for the gap. First, a disproportionately large number of high wealth households may be refusing to participate in the NSFIE. Second, some selfemployed respondents may be confused about what to report and are not reporting assets and liabilities of their businesses. Third, there may be measurement error because the household sector is treated as a residual in the SNA/FFA commodity flow method.

The results in Table 7 are consistent with the claim that the NSFIE may not be adequately capturing assets and *liabilities* of proprietorships. Not only are deposits small in the NSFIE, but the NSFIE doesn't appear to be capturing other loans of financial institutions and non-home loans of public institutions. We conjecture that most of these loans are to proprietorships. Finally, our results also suggest that respondents in the NSFIE are also under-stating the value of their private pensions and annuities.

D Calibration of the model

Table 8 reports the entire parameterization of the model. Our calibration strategy follows Braun and Joines (2015). In particular, we adjust Japanese NIPA data to recognize some differences between our model and the data. For instance, our model has no external sector and no government investment. The specific adjustments follow the strategy of Hayashi and Prescott (2002).

Starting with demographics, we assume that new households are formed at age 21 and the size of the household is parameterized in the same way as Braun et al. (2009). One important difference is that we assume ($J_r = 70$) that is the labor efficiencies are positive until the age of 90. Our age profiles of labor efficiency and net worth are constructed from the NSFIE and that data shows that labor income declines beyond 65 but is positive until relatively late in the lifecycle. We assume that households receive their public pensions from the age 68. This is two years older than the age where one can qualify for full public pension benefits in Japan and is chosen to be consistent with the effective labor-market exit age in 2014 for Japan estimated by the OECD.[41] Finally, the maximum lifespan is set to 120 years ($J = 100$).

We use the same depreciation rate as Braun and Joines (2015). But we use a smaller value of the capital share parameter $\alpha = 0.3$. This value in conjunction with the rest of our parameterization results in an after-tax return on illiquid assets of 1.93% per annum. The parameter θ governs the elasticity of substitution of intermediate goods. We set this parameter to produce a gross markup of 1.05 in a steady state if there were no government subsidies. We adjust the preference discount factor to produce and aggregate stock of

[41] See Pensions at a Glance OECD, 2015.

Table 8: Parameterization of the model

Parameter	Description	Value
Demographics		
J_r	Retirement age	70 (Age 90)
J	Maximum lifespan	100 (Age 120)
$\{\psi_j\}_{j=1}^{J}$	Survival probabilities	Braun and Joines (2015)
Technology		
θ	Gross markup	1.05
γ	Price adjustment cost	41.2
α	Capital share parameter	0.30
δ	Depreciation rate	0.102
$\gamma_a(0)$	Cost of adjusting illiquid assets in death year	0.058
$\gamma_a(1)$	Cost of adjusting illiquid assets in non-death year	0.322
Preferences		
σ	Inverse elasticity of intertemporal substitution	3
ω	Preference weight on bequests	729
ν	Frisch labor supply elasticity	2
υ	Preference weight on leisure	6.9
β	Preference discount factor	0.985
Monetary Policy		
ρ_r	Interest rule persistence	0.35
ϕ_π	Interest rule inflation elasticity	2
Fiscal Policy		
τ^c	Consumption tax rate	0.05
τ^k	Corporate tax rate	0.35
τ^a	Tax rate on asset income	0.2
τ^w	Tax rate on labor income	0.232
τ^{bq}	Tax rate on bequests	0.15
τ^f	Subsidy to intermediate goods firms	$\theta - 1$
G/Y	Government share of output	0.16
λ	Public pension replacement ratio	0.084
D/Y	Net government debt output ratio	1.23

illiquid assets of 3.5 and the resulting value of β is 0.985.

Many real business cycle and NK models assume that preferences are additively separable in consumption and leisure and posit log-preferences over consumption. Lifecycle analyses though often set the relative-risk aversion coefficient on consumption at a higher level of about 3 (see Brown and Finkelstein, 2008). We set σ to this value. Hours worked in our model are determined in a way that is close to the representative agent framework and there is no distinction between extensive and intensive labor supply decisions. It follows that the Frisch labor supply elasticity in our model, given by ν, reflects the combined effects of adjustments along both margins. We set this parameter to 2.[42]

Next consider the fiscal policy variables. We report steady state age profiles of asset holdings for two settings of the the net Japanese government debt ratio. The "Model micro" results assume that it is 0.85 which is the value that emerges when we aggregate our micro data. The "Model macro" results assume it is 1.23, which is the size of net liquid assets held by households in the 2014 Japanese FFA data as we discussed above in Appendix C. Given the concerns we raised there about the measurement of liquid assets in the NSFIE, the steady state results reported below in Table 9 and the impulse responses use the "Model macro" value of 1.23.

Intermediate goods firms receive a subsidy that is chosen so that the steady state markup is zero. Capital income is taxed twice in Japan. The overall tax rate on capital income faced by households in the model is 48%, which is a combination of a corporate profits tax rate (τ^k) of 35% and a 20% personal tax rate on asset income (τ^a). The consumption tax rate is set to 5% and the labor income tax rate is set to 23.2%. The personal tax rate on labor income, government purchases relative to output and the replacement rate of the public pension are calibrated using the same targets as Braun and Joines (2015).

Bequest taxes are progressive ranging from 10 percent to 55 percent of assets in excess of the exemption.[43] Holdings of real estate and equity are registered and face the full burden of this tax. Based on these considerations we set the bequest tax rate to 15 percent.

The specific values of the two parameters that govern the age profile of adjustment costs on illiquid assets were selected after experimenting with a range of values for each parameter. We varied $\gamma_a(1)$ so that the initial cost of acquiring illiquid assets for 21 year old households ranged from 2% to 5% of their end of period holdings of illiquid assets. The parameter $\gamma_a(0)$ was chosen by varying the maximum costs for a household in their death year of liquidating all illiquid assets from 10% to 25% of illiquid assets. The baseline targets of 2 percent of illiquid assets for survivors and 15 percent of assets for non-survivors help the model to account for the long number of years that working-age households borrow and create a market price of risk on illiquid assets is an attractive proposition for older households. In fact, if we remove the warm-glow bequests, households who outlive the average life expectancy choose to leverage up on illiquid assets.

[42]We assign zero weight to individuals in their death year when computing $\bar{\lambda}$ in equation (9). If we include them and set $\nu = 1$, the model results are essentially indistinguishable from what is reported here.

[43]The formula for exemptions provides a base exemption of \$30,000 plus \$6,000 per legal heir.

D.1 Aggregate moments

Table 9 reports the main aggregate steady state moments of the calibrated model. The steady state of our model reproduces the main features of the Japanese economy including the composition of aggregate output, the size of aggregate net worth and the main revenue sources and expenditures of the public sector. Lump-sum transfers are used to close the government budget constraint and amount to 1.7 percent of output. The pre-tax premium

Table 9: Model aggregate steady state moments

Variable	description	steady state value
$R^k - 1$	return on capital	3.72%
$R/\pi - 1$	return on government debt	1.86%
$\tilde{R}^a - 1$	After–tax return on illiquid assets	1.93%
$\tilde{R}/\pi - 1$	After–tax return on liquid assets	1.48%
D^G/Y	Gross liquid assets relative to output	1.39
$(D^G - D)/Y$	Private borrowing relative to output	0.16
$(A + D - D^G)/Y$	Net stock of illiquid assets relative to output	3.34
$(A + D)/Y$	Net worth	4.73
K/Y	Capital-output ratio	2.16
V/Y	Value of shares relative to output	1.34
$\Xi(1)/Y$	Lump–sum transfers relative to output, survivors	0.108
$\Xi(1)/Y$	Lump-sum transfers relative to output, non-survivors	0.017
B/Y	Social Security outlays relative to output	0.084
C/Y	Consumption share of output	0.56
I/Y	Investment share of output	0.20
$\bar{\gamma}_a/y$	Financial services share of output	0.009

on illiquid assets is 1.87 percent in the model. It consists of a liquidity premium of 0.45 percent and a tax treatment premium of 1.41 that arises because illiquid assets are subject to both the corporate profits tax and the household tax on asset income. It is worth emphasizing that the steady state after tax returns on liquid and illiquid assets are not equalized and thus face a nontrivial asset allocation decision among the two asset classes. Given that there is no aggregate uncertainty in the model and no individual-specific earnings risk, or limited participation effects, it is to be expected that the excess return on illiquid assets in the model is smaller than the excess return on equity in Japanese data which Damodaran (2020) estimates to be 5.4 percent.

The model does a good job of reproducing the age profiles of household borrowing in the NSFIE. However, it understates the aggregate amount of household borrowing in

Japanese FFA data. In the model, household borrowing is 16 percent of GDP whereas in our FFA data it is 56 percent. This result is interesting because the model imposes a loose (natural) borrowing constraint on individual borrowing. Personal defaults are rare in Japan but do occur and our model does not allow for this possibility. The model also abstracts from individual earnings and medical expense risk and introducing other risk factors could produce higher aggregate borrowing in the model.

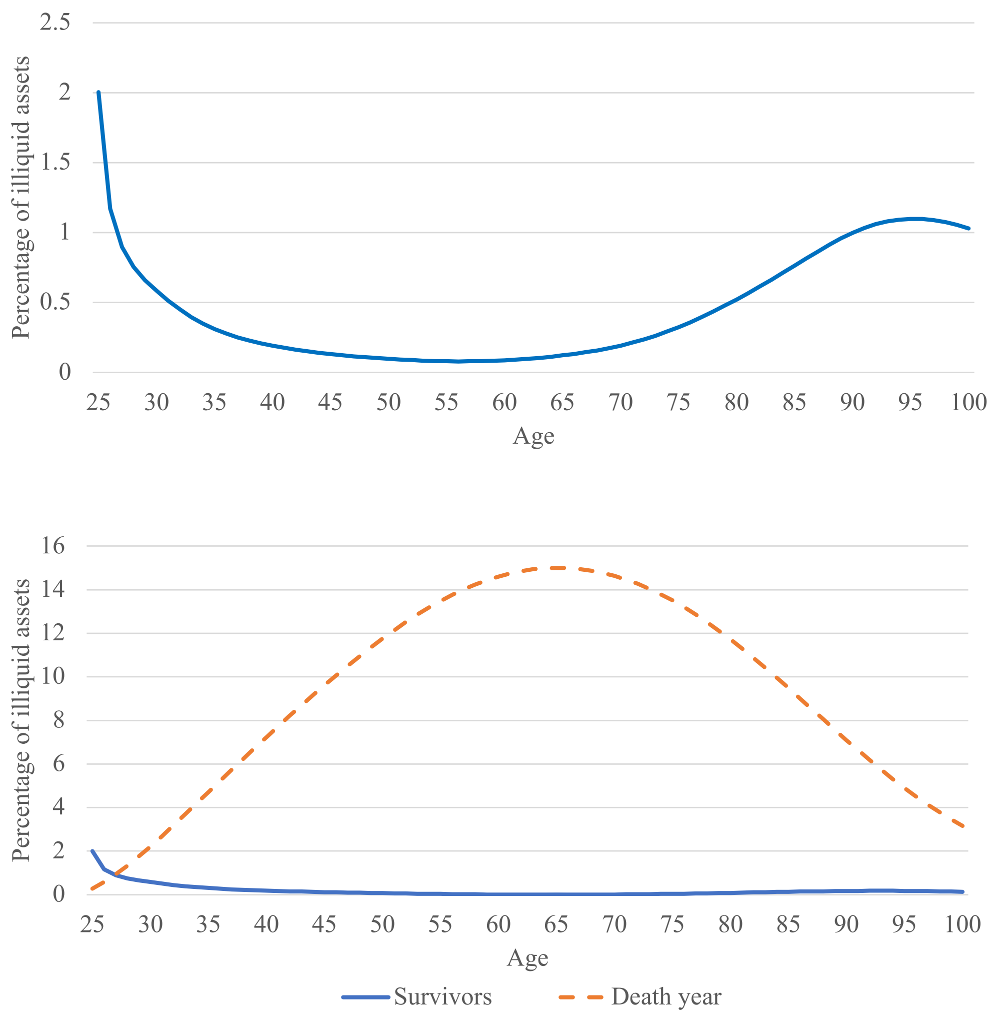
Finally, the model steady state implies a pretax real return on government debt of 1.86 percent. This figure is close to current yield on a 20-year Japanese government bond which was 1.69 percent on August 21 2024 but, higher than the 20-year bond yield was in 2015. We are not too concerned about this gap between the model and Japanese data because this interest rate is also the interest rate on private loans in the model.

D.2 Costs of adjusting illiquid assets

Here we report the size of the baseline steady state age profile of adjustment costs and provide some intuition for how they work. Figure 9 reports the age profile of adjustment costs on illiquid assets relative to total assets using the baseline parameterization of the model which assumes that the maximum cost at any age is two percent of assets conditional on surviving and 20 percent of assets conditional on it being the death year. The right panel of the figure reports the adjustment costs by survival state. It has two main properties. Newly formed households of age 21 have the highest adjustment costs conditional on survival. This is because they enter the economy with no assets. Newly retired households (age 68) have the highest adjustment costs if they experience a death event shock at this age because they have the largest holdings of illiquid assets. Put differently, a 68 year old household who discovers that this is the last year of its life and quickly liquidates its large stock of illiquid assets pays a fee of 20 percent of total assets.

The upper panel of Figure 9, which shows the age profile of the average costs of adjusting illiquid assets, has two modes. The first mode of two percent of illiquid assets occurs at age 21 and is a cost of acquiring illiquid assets. The adjustment costs then fall sharply and are less than 0.5 percent of total assets until age 76. The second mode in average adjustment costs occurs at age 95 and is 1.1 percent of illiquid assets. Households older than age 76 face an interesting trade off. On the one hand, they are attracted to the higher return offered by illiquid assets. On the other hand, mortality risk is increasing and it is costly for them to have to rapidly liquidate their entire holdings of illiquid assets if they discover that this is their death year.

Figure 9: Adjustment costs on illiquid assets relative to illiquid asset holdings by age



Having described the size of the adjustment costs by age, we are now in a position to discuss what they represent and how we parameterized them. The adjustment cost for survivors captures in a simple way that young households may not be very sophisticated purchasers of a home or car and allocate more resources to acquiring them. The peak cost occurs at age 21 and is two percent of total assets. For purposes of comparison taxes, broker commissions and other fees of purchasing a home in Japan are about 6-7 percent of the purchase price. The second and larger cost is the cost of liquidating illiquid assets in the death year. In the model households of age 65 pay the peak cost of 15 percent of illiquid assets. We believe that this is a simple way to capture the following considerations. Average commissions for mediating residential real estate sales in Japan are 4.1 percent (see Shirakawa and Okoshi (2017)). Construction costs in Japan are commensurate with costs in other advanced economies, but land prices are relatively high. Moreover, residential structure lifespans are as short as 15 years and it is common to demolish the existing structure at the time of sale (see Koo and Sasaki (2008)). These factors are likely to be particularly important for a 65 year old household that passes away. On average the age of the home is young, but the resale value of the structure is low or even negative if it has to be demolished. The model assumes that cost of liquidating illiquid assets is born by the owner of the asset at the time of death. In practice, the costs are born by heirs who choose to register as an inheritor. We abstract from this decision and assume that bequests net of liquidation costs are lump–sum transferred to surviving households. So this distortion is reflected in the size of the lump–sum transfer. It is clear though that the costs of liquidating real estate associated with an estate are significant in Japan. Unclaimed land is estimated to constitute 11 percent of Japan's total land mass and is larger than the prefecture of Shikoku. Unclaimed land is an obstacle to development and is a major public policy issue in Japan (See The Tokyo Foundation for Economic Research, 2018, for more details).

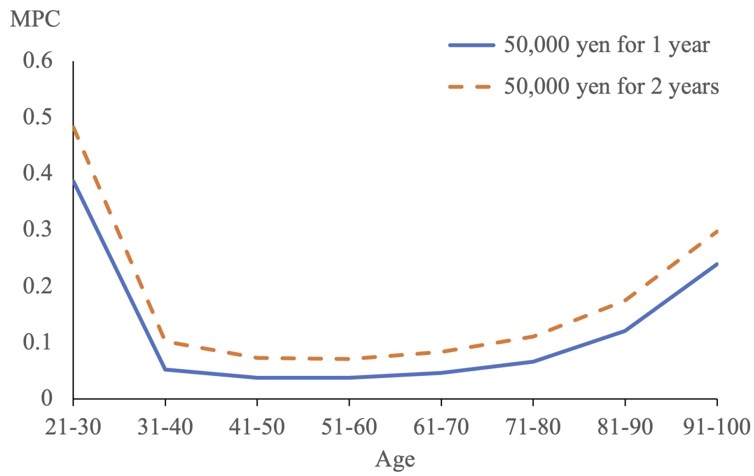
E Marginal propensities to consume

Figure 10 reports the marginal propensity to consume for households with ages between 21–100 for two scenarios. The first scenario increases the lump-sum transfer by 50,000 yen for one year and the second scenario increases it by the same amount for two years. These are partial equilibrium marginal propensities to consume that hold prices and government policy variables fixed at their steady state values. Households optimally choose how to divide up the bonus to their income among consumption in all future periods of their life. The marginal propensities to consume are calculated as expected values over the two survival states.

Figure 10 shows that the pattern of the marginal propensity to consume is u-shaped. It is high for the youngest age group because individuals aged 21-23 are borrowing constrained. It is low for households between the ages of 31 and 70 but then increases beyond the age of 70. The main reason household MPCs increase beyond the age of 70 is because their planning horizon is becoming shorter.[44] They face relatively high mortality risk and if this is their death year, they consume all of their wealth. Average life expectancy in the model is 83 years. So the fraction of older households with high MPCs is small. Finally,

[44] We don't report MPCs for households over age 100 in Figure 10 because their share of the population is very small.

Figure 10: Partial equilibrium marginal propensities to consume by age



the MPCs reported here treat bequests as part of consumption. If we compute MPCs for consumption of survivors instead, the MPCs are essentially unchanged for all but the two oldest age groups. Under this assumption the MPC associated with a two-year lump-sum transfer falls from 0.17 to 0.12 for the 81-90 year old age group from from 0.30 to 0.15 for the 91-100 year old age group.

F Empirical impulse responses

Local projections. Let Y_t denote a variable of our interest in period t. For example, it is the real consumption for a specific age group or the GDP. Since we are interested in the cumulative effects of a monetary policy shock over a year while the data we use are monthly or quarterly, we consider local projections with the following specification:

$$\frac{\sum_{h=0}^{H-1}\log(Y_{t+h})}{H} = \alpha + \beta\epsilon_t + \sum_{k=1}^{K}\gamma_k X_{t-k} + u_t \tag{F.1}$$

where H is the horizon over which the effect of the monetary policy shock ϵ_t is estimated, including its impact effect in period t, and X_{t-k} is a column vector that consists of lagged variables.[45] Coefficient β captures the effect of the monetary policy shock of $\epsilon_t = 1$ as

[45] Even if an independent variable in (F.1) is given as $\log(\sum_{h=0}^{H-1} Y_{t+h})$, it would not change our result significantly since it is equivalent to using the left-hand-side of (F.1) up to the first-order approximation and under the assumption of $1/H = Y_{t+j}/\sum_{t=0}^{H-1} Y_{t+h}$ for all $j = 0, .., H-1$.

shown by

$$\beta = \frac{1}{H}\sum_{h=0}^{H-1}\left\{E_t\left[\log(Y_{t+h})|\epsilon_t = 1\right] - E_t\left[\log(Y_{t+h})|\epsilon_t = 0\right]\right\}$$

For monthly data we set $H = 12$ and for quarterly data we set $H = 4$. The lagged variables include $\log(Y_{t-k})$. Following the VAR analyses conducted by Miyao (2002) and Kubota and Shintani (2024) for Japan, we include a stock price index in X_{t-k} for the impulse responses of the macro variables shown in Figure 3. We set the number of lags as $K = 12$ for monthly data and $K = 4$ for quarterly data. Since the lagged variables are included in our specification of local projections, the standard error of the estimate of β can be computed by using the usual heteroscedasticity-robust Eicker-Huber-White standard error, as shown by Montiel Olea and Plagborg-Møller (2021).

Monetary policy shocks. We use two measures of monetary policy shocks. The first measure is a surprise to interest rate futures before and after a monetary policy announcement, estimated by Kubota and Shintani (2022) using high-frequency data for Japan. Specifically, we use the target factor proposed by Gürkaynak et al. (2005), which reflects the expectations up to one year ahead, aggregated over monthly frequency and used by Kubota and Shintani (2024). For estimation with quarterly data, we simply sum up the monthly monetary policy shocks over the corresponding quarter.

Our second measure of monetary policy shocks is that estimated by Ikeda et al. (2024). They use a non-linear SVAR and identify monetary policy shocks for Japan by combining an identification method that takes advantage of the presence of the effective lower bound, newly proposed by Mavroeidis (2021), with sign restrictions. Since the monetary policy shocks are not point-identified but identified as a set, we use the middle value of the set as our monetary policy measure.

The model and SVAR shock size is set at $\epsilon_t = 0.01$ in Figures 3 and 5. Since the unit of the HFI shock differs from that of the model shock and the SVAR shock, the size of the HFI shock is adjusted. Specifically, in Figure 3, to make it comparable to the SVAR shock, ϵ_t is set in a way that equates the shock size relative to the shock volatility between the two shocks: $0.01/\text{SD(SVAR shock)} = \epsilon_t/\text{SD(HFI shock)}$, where $\text{SD}(\cdot)$ denotes a standard deviation. In Figure 5, to make it comparable to the model, the size of the HFI shock is set so that the 1-year response of the nominal interest rate coincides with that of the model.

Since the first measure of monetary policy shocks is available from 1992 to 2019, the sample period of our analyses in empirical responses is set to 1992–2019. This sample stating year happens to coincide with the period of the burst of the so-called asset price "bubble" in Japan and the starting period of the so-called "lost decade."

Data. For the data by age group, used in Figure 1, the data source is the Family Income and Expenditure Survey (FIES). We use the disposable income reported in the FIES, which is available only for the category of working households. The consumption used in Figure 1 is non-durable consumption for working households. Specifically, our measure of non-durable consumption is defined as total consumption net of durable consumption and unknown consumption and gifts or transfers in the category of other consumption. Our measure of durable consumption consists of repairs and maintenance (house, garden and other facilities and their repair), household durable goods, interior furnishings and decorations, bedding, domestic utensils, private transportation (purchases of automobiles and motorcycles, and their maintenance fees and services), text books, and entertainment durables. The coverage of the FIES changed slightly in 2000: before 2000 the FIES excludes fishery and farmer households, but after 2000 it includes those households. We use year-on-year growth rates to connect the series before 2000 with those after 2000. Both consumption and disposable income are deflated by using the CPI index.

For the aggregate data used in Figure 3, GDP is the real GDP, consumption is the real private consumption, investment is the sum of the real private non-residential investment and the real private residential investment, CPI is the CPI index excluding the effect of changes in consumption tax rates, TOPIX is a stock price index that measures the overall trend in the stock market in Japan, covering an extensive proportion of the Japanese stock market, and the nominal rate is one-year government bond yield.

G The decomposition of consumption responses

In applying Auclert (2019)'s decomposition in the case of a persistent monetary policy shock, let us first define:

$$\tilde{a}_{j,t+1} \equiv \tilde{R}^a_{t+1} a_{j,t}, \quad \tilde{D}_{j,t+1} \equiv \tilde{R}_t D_{j,t}, \quad y_{j,t}(z) \equiv b_{j,t} + \xi_t(z) - \chi_{j,t}(z), \quad q_{t,t+1} \equiv \frac{1}{\tilde{R}^a_{t+1}},$$

$$Q_{t,t+1} \equiv \frac{1}{\tilde{R}_t}, \quad \tilde{c}_{j,t} \equiv (1+\tau^c) c_{j,t}, \quad \tilde{b}^q_{j,t} = (1+\tau^{b^q}) b^q_{j,t}, \quad \tilde{w}_{j,t} \equiv (1-\tau^w) w_t \epsilon_j,$$

where $z \in \{0, 1\}$ is a state of survival ($z = 1$) or death ($z = 0$). Then, the budget constraint (4) for a surviving household can be written as

$$P_t\tilde{c}_{j,t} = P_t y_{j,t}(1) + P_t\tilde{w}_{j,t}h_{j,t} + \tilde{D}_{j-1,t} - Q_{t,t+1}\tilde{D}_{j,t+1} + P_t\tilde{a}_{j-1,t} - P_t q_{t,t+1}\tilde{a}_{j,t+1}. \tag{G.1}$$

In the Auclert decomposition, the effect of wage changes on consumption through changes in labor supply is classified as a part of substitution effects. In our decomposition, this effect is classified into the wealth effect as an increase in wage income. In addition, since we focus on the consumption responses of survived households and they face mortality risk from the next period, our decomposition takes into account mortality risk in discounting future. Then, applying the Auclert decomposition yields the response of consumption by an age-j household at time 0, given by

$$d\tilde{c}_{j,0} = \underbrace{MPC_{j,0}d\Omega_{j,0}}_{\text{Wealth effect}} + \underbrace{\tilde{c}_{j,0}\sum_{s\geq 0}\epsilon_{j,q_s}\frac{dq_s}{q_s}}_{\text{Substitution effect}}, \tag{G.2}$$

where MPC_j is the partial equilibrium marginal propensity to consume for age-j household, $d\Omega_{j,0}$ is a change in wealth, $\epsilon_{j,q_s} \equiv \frac{\partial\tilde{c}_{j,0}}{\partial q_s}\frac{q_s}{\tilde{c}_{j,0}}$ is the Hicksian elasticity of age-j household consumption in period 0 with respect to the discount factor q_s, and $q_s \equiv q_{-1,s}$. The change in wealth consist of four components: the unearned income (UI), the earned income (EI), the unhedged interest rate exposure (URE), and the net nominal position (NNP).

The effect of the UI is given by

$$d(UI_{j,0}) = dy_{j,0}(1) + \sum_{s\geq 1}\psi_{j,j+s-1}q_s\left[\psi_{j+s-1,j+s}dy_{j+s,s}(1) + (1-\psi_{j+s-1,j+s})dy_{j+s,s}(0)\right],$$

where $\psi_{j,j+s-1}$ denotes the surviving probability from age j to age $j+s-1$. The effect of earned income change is given by

$$d(EI_{j,0}) = \tilde{w}_{j,0}h_{j,0}\left(\frac{d\tilde{w}_{j,0}}{\tilde{w}_{j,0}} + \frac{dh_{j,0}}{h_{j,0}}\right) + \sum_{j\geq 1}\psi_{j,j+s-1}q_s\tilde{w}_{j+s,s}h_{j+s,s}\left(\frac{d\tilde{w}_{j+s,s}}{\tilde{w}_{j+s,s}} + \frac{dh_{j+s,s}}{h_{j+s,s}}\right).$$

Next, the effect of the URE is given by

$$\begin{aligned}
d(URE_{j,0}) = &\left(\frac{\tilde{D}_{j-1,0}}{P_0} + \tilde{a}_{j-1,0}\right)\frac{dq_0}{q_0} + \left(y_{j,0}(1) + \tilde{w}_{j,0}h_{j,0} - \tilde{c}_{j,0}\right)dq_0 \\
&+ \sum_{s\geq 1}\psi_{j,j+s-1}\Big\{\psi_{j+s-1,j+s}\left[y_{j+s,s}(1) - \tilde{c}_{j+s,s}\right] \\
&+ (1-\psi_{j+s-1,j+s})\left[y_{j+s,s}(0) - \tilde{b}^q_{j+s,s}\right] + \tilde{w}_{j+s,s}h_{j+s,s}\Big\}dq_s.
\end{aligned}$$

Finally, the effect of the NNP is given by

$$d(NNP) = -\frac{\tilde{D}_{j-1,0}}{P_0}\frac{dP_0}{P_0}.$$

To summarize, the change in the wealth is captured by the three components:

$$d\Omega_{j,0} = \underbrace{d(UI_{j,0}) + d(EI_{j,0})}_{\text{Income}} + \underbrace{d(URE_{j,0}}_{\text{URE}} + \underbrace{d(NNP_{J,0})}_{\text{NNP}}.$$

In calculating the decomposition, the MPCs for an increase in the lump-sum transfer by 50,000 yen, studied in Appendix E, are used. The substitution effect can be calculated as $d\tilde{c}_{j,0} - MPC_{j,0}d\Omega_{j,0}$ according to equation (G.2).

H The effects of a tighter monetary policy on age-consumption profile of a 71 year old household.

This appendix provides more details about why a tightening in monetary policy produces the persistent and hump-shaped responses of consumption for households who are close to retirement age, as shown in Figure 7, by analyzing the response of the cohort that is aged 71 when monetary policy is tightened.

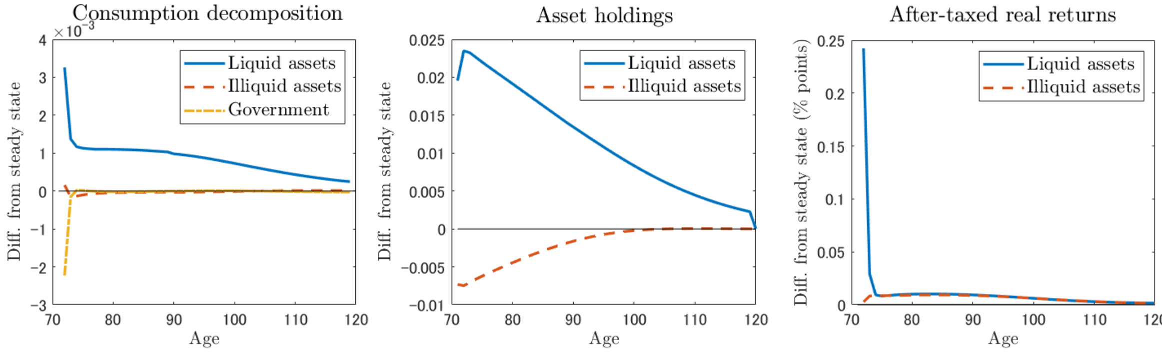
Consumption of a household with age $j \geq 71$ in period t is given by

$$(1+\tau_c)c_{j,t} = \underbrace{(\tilde{R}^a_t - 1)a_{j-1,t-1} - \triangle a_{j,t} - \chi_{j,t}}_{\text{Illiquid assets}} + \underbrace{(\tilde{R}_t - 1)d_{j-1,t-1} - \triangle d_{j,t}}_{\text{Liquid assets}} + \underbrace{b_{j,t} + \xi_t}_{\text{Government}}, \quad \text{(H.1)}$$

where $\triangle a_{j,t} = a_{j,t} - a_{j-1,t-1}$, $\triangle d_{j,t} = d_{j,t} - d_{j-1,t-1}$, and $\chi_{j,t}$ denotes the cost of adjusting illiquid assets. The left panel of Figure 11 plots a decomposition of the consumption response into an illiquid asset cash flow component, $(\tilde{R}^a_t - 1)a_{j-1,t-1} - \triangle a_{j,t} - \chi_{j,t}$, a liquid asset cash flow component, $(\tilde{R}_t - 1)d_{j-1,t-1} - \triangle d_{j,t}$, and the government transfer component, $b_{j,t} + \xi_t$, in terms of a differences from steady state, from $t = 1$ (age 72) onward, where the impact period of $t = 0$ (age 71) is omitted because the responses in period $t = 0$ are much greater than those in the remaining periods. The left panel indicates that the liquid asset cash flow component is driving the persistent and hump-shaped response of consumption deviations from steady state.

In response to a tightening in monetary policy, the household reduces its holdings of illiquid assets and increases its holdings of liquid assets (the middle panel of Figure 11) in a persistent fashion. The household tilts its portfolio to liquid assets because liquid assets

Figure 11: The background of the consumption response of households aged 71



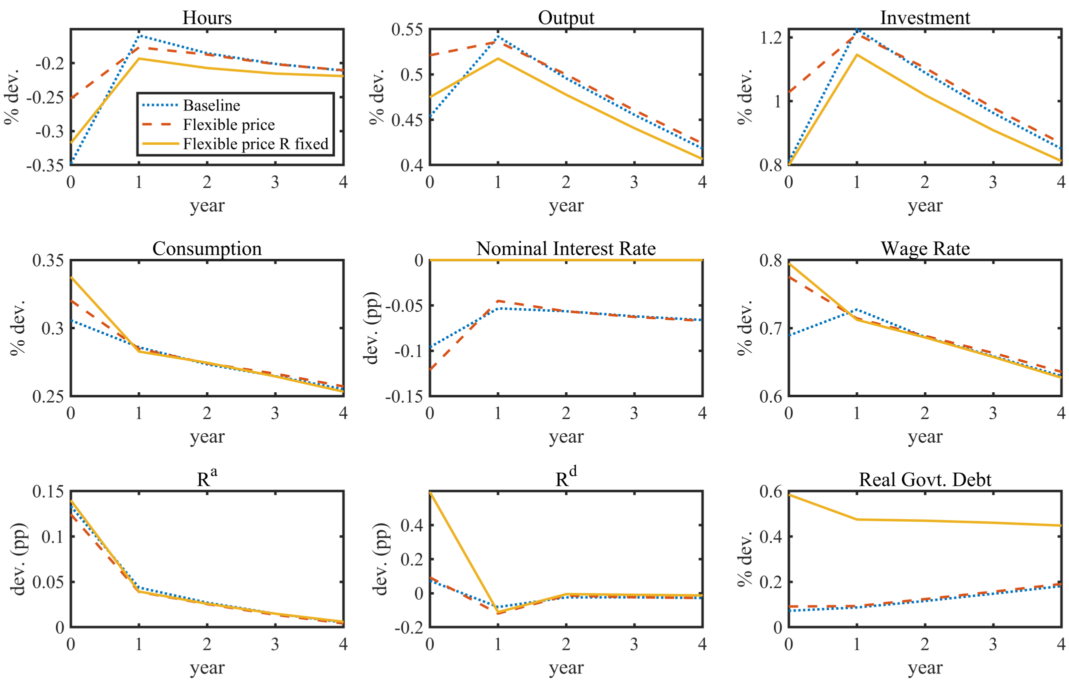
offer a higher after-taxed real return relative to steady state (the right panel of Figure 11). The real return on liquid assets increases significantly in the initial few periods because the nominal interest rate set by the central bank is persistent to some degree and thus the nominal rate is kept higher than the steady state level in response to a tightening in monetary policy.

I Robustness Results

I.1 Technology shocks

Figure 12 reports aggregate responses to an improvement in technology for three scenarios. The "Flexible price" scenario allows monetary policy to respond to the technology shock, and the "Flexible price R fixed" scenario holds the policy rate fixed. Output, investment, and wages increase more in the baseline compared to the flexible price scenario, indicating that nominal price rigidities are important in the short-run for this shock as well. However, NK price rigidities are much less important after the impact period because aggregate outcomes in the baseline and flexible price specifications are nearly identical. Next, compare the "Flexible price" specification with the "Flexible price R fixed" scenario and observe that the responses of hours, output, investment, consumption, the real return on liquid assets, and government debt are very different in the two scenarios. The lack of response of monetary policy produces a stronger decline in the liquidity premium on impact and also alters the investment opportunities in subsequent periods. These comparisons indicate that the response of monetary policy to the technology shock has important non-neutralities and that the source of the non-neutralities is asset substitution.

Figure 12: Responses of aggregate variables to an improvement in technology: *Baseline*, *Flexible price* and *Flexible price, R fixed* specifications.



Note: Responses to an innovation of size 0.01 in the state of technology. Persistence parameter is 0.9.

I.2 Monetary policy shocks

Table 10: Impact responses of aggregate variables to monetary policy innovation under alternative scenarios.

A. Response of goods and labor market variables

Scenario	Y	C	I	H	w	r^k	π
Baseline	-0.566	-0.165	-1.93	-0.808	-0.811	-0.225	-0.571
Lump-sum	-0.567	-0.165	-1.93	-0.808	-0.807	-0.224	-0.569
$\phi_\pi = 0$	-0.785	-0.306	-2.36	-1.12	-1.32	-0.338	-0.598
Negative	0.655	0.149	1.67	0.937	0.836	0.248	0.594

B. Response of financial and fiscal variables

Scenario	R	r^d	r^a	Spread	V	Ω	ξ	d
Baseline	0.262	0.585	0.009	-0.577	-0.314	19.03	-0.991	0.574
Lump-sum	0.264	0.583	0.005	-0.578	-0.326	18.95	-0.991	0.573
$\phi_\pi = 0$	1.02	0.612	-0.026	-0.638	-0.699	30.58	-0.992	0.601
Negative	-0.235	-0.602	-0.041	0.560	0.304	-23.32	-0.990	-0.591

Note: "Baseline" is the baseline scenario. "Lump-sum" assumes that lump-sum taxes are fixed at their steady state level for households aged 76+ and $\phi_\pi = 0$ assumes that the coefficient on the inflation rate in the interest rate targeting rule is 0. "Negative" assumes that the monetary policy innovation is -0.01. The real asset returns and spread are pre-tax, ex-post impact period responses.

Table 11: Wealth and consumption inequality under alternative scenarios

Wealth Inequality					
Year	0	1	2	3	4
Baseline	0.13	0.12	0.11	0.096	0.085
Lump-sum	0.13	0.12	0.11	0.097	0.086
$\phi_\pi = 0$	0.22	0.20	0.17	0.15	0.13
Negative	-0.13	-0.12	-0.11	-0.095	-0.084
Consumption Inequality					
Year	0	1	2	3	4
Baseline	-0.015	-0.041	-0.046	-0.047	-0.048
Lump-sum	-0.017	-0.047	-0.052	-0.054	-0.054
$\phi_\pi = 0$	0.036	-0.032	-0.052	-0.057	-0.058
Negative	0.020	0.043	0.047	0.049	0.049

Note: The results are percentage point changes in the Gini coefficients from their steady state values, associated with a shock of size 0.01 to monetary policy.