Fragility in modeling consumption tax revenue

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Abstract

This study shows that the shape of the tax revenue curve for consumption tax and its boundedness are sensitive to (i) the functional form of utility and (ii) the use of tax revenue in a neoclassical general equilibrium model. The tax revenue curve for consumption tax cannot be hump-shaped if the utility function is King–Plosser–Rebelo utility with constant labor supply elasticity. Conversely, the curve can be hump-shaped if the utility function is additively separable in consumption and labor supply or if the utility function is Greenwood–Hercowitz–Huffman, both of which are popular in the literature. The use of tax revenue also has significant effects on the tax revenue curve for consumption tax. If the tax revenue is mainly used as a lump-sum transfer to households, Then the tax revenue is likely to be unbounded, whereas it is likely to be bounded and the tax revenue curve is likely to be hump-shaped if the tax revenue is mainly used as government consumption.

Keywords: tax revenue; consumption tax

JEL classification: E62; H20; H30

1 Introduction

The main objective of this study is to show that there is fragility in modeling consumption tax revenue. In other words, the shape of the tax revenue curve for consumption tax (whether it is hump-shaped or not) and its boundedness (whether tax revenue is bounded or unbounded) are very sensitive to (i) the specification of the functional form of the utility, and (ii) the assumption of the use of tax revenue in standard macroeconomic general equilibrium models.

Consumption tax has been receiving attention as both a policy and academic issue. For example, in Japan, the increasing consumption tax rate is an important policy topic. In many European countries, the standard value-added tax (VAT) rate is greater than or equal to 20%, and consumption tax revenue is an important source of government revenue. In academics, macroeconomists often employ models with consumption tax to analyze fiscal problems, like fiscal sustainability and fiscal reform. In the applied works using models, the theoretical property of the tax revenue curve for consumption tax (shape of tax revenue curve and its boundedness) is an important determinant of the results.

This study shows, on the one hand, that the tax revenue curve for consumption tax cannot be hump-shaped if the utility function is the traditional King–Plosser–Rebelo (KPR) type with constant labor supply elasticity. On the other hand, the tax revenue curve for consumption tax can be hump-shaped if the utility function is additively separable in consumption and labor supply or if the utility function is a Greenwood–Hercowitz–Huffman (GHH) type, both of which are popular in macroeconomic literature. The use of tax revenue also has significant effects on the tax revenue curve for consumption tax. If tax revenue is mainly used as a lump-sum transfer to households, then it is likely to be unbounded, whereas it is likely to be bounded and the tax revenue curve is likely to be hump-shaped if the tax revenue is mainly used as government consumption.

The key parameters for the hump-shaped tax revenue curve and the boundedness

of tax revenue are (i) labor supply elasticity and (ii) relative risk aversion (RRA) as a curvature of the utility. For the hump-shaped tax revenue curve for consumption tax, the labor supply elasticity should be greater than one in the case of GHH utility. In addition, the RRA should be less than one in the case of additively separable utility. An increase in the consumption tax rate decreases the relative price of leisure with respect to consumption goods. Then, both labor supply and consumption decrease through the substitution effect. The RRA and the labor supply elasticity are generally related to the elasticity of consumption and labor supply with respect to the consumption tax rate. In general equilibrium, consumption and labor supply mutually influence each other through the resource constraint and the production function. Then, the elasticity of equilibrium consumption depends on the RRA and the labor supply elasticity. In this study, the elasticity of equilibrium consumption with respect to the consumption tax rate is derived under the general functional form of utility.

It is not uncommon for the macroeconomic literature with representative agent models to employ high labor supply elasticity. Then, a hump-shaped tax revenue curve for consumption tax is likely to arise if the utility function is GHH. On the other hand, it is not standard in the literature to set a small RRA. Then, even if utility is additively separable, the tax revenue curve for consumption tax is not hump-shaped under standard parameter values.

Our basic results are obtained as the property of the consumption tax revenue curve for consumption tax under a simple static general equilibrium model, where tax revenue is used as a lump-sum transfer to households or it is used as government consumption. However, the results can be easily extended to (i) the property of the total tax revenue curve for consumption tax, which includes both labor income and capital income tax revenue, (ii) the property of the tax revenue curve under alternative uses of tax revenue, and (iii) the property of the tax revenue curve for consumption tax under a dynamic model à la Trabandt and Uhlig (2011).

Note that the models in this study are plain vanilla frictionless neoclassical ones. Some frictions, like home production and tax evasion, would be sources of hump-shaped tax revenue curve for consumption tax. However, this study focuses on economies without such frictions, and investigates the shape of the tax revenue curve for consumption tax in a standard neoclassical model. According to our results, the tax revenue curve for consumption tax is hump-shaped only if the RRA is greater than one. If the tax revenue is used as a lump-sum transfer to households, the inverse of labor supply elasticity also should be greater than one for the hump-shaped tax revenue curve for consumption tax. These parameter values are not standard in the literature, although some works do support them. Moreover, our results imply that some additional frictions, such as home production and tax evasion, should be included in the models to generate the hump-shaped tax revenue curve for consumption tax, as long as standard utility function parameter values are employed.[1]

Related Literature: The tax revenue curve itself is investigated by many researchers as the Laffer curve. The followings studies investigate the tax revenue curve theoretically. Ireland (1994) focuses on the tax revenue curve for capital income tax using an AK model. Schmitt-Grohè and Uribe (1997) investigate the tax revenue curve for labor income tax in a neoclassical growth model. Recent studies by Trabandt and Uhlig (2011, 2013) compute the tax revenue curves for consumption, labor, and capital taxes for the US and EU using a neoclassical growth model. Nutahara (2015) applies the model of Trabandt and Uhlig (2011) to the Japanese economy, and derives the tax revenue curves for consumption, labor, and capital income taxes. Fève, Matheron, and Sahuc (2013) analyze the tax revenue curves for consumption, labor, and capital taxes in an incomplete-market economy. Holter, Krueger, and Stepanchuk (2014) investigate the effect of household heterogeneity and a progressive tax scheme on the peak tax rate

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[1]For example, Baydur and Yilmaz (2017) employ home production to generate a hump-shaped tax revenue curve for VAT.

of the tax revenue curve for income tax using an overlapping-generations model. Badel and Huggett (2017) compute the peak tax rate of the Laffer curve for labor income tax by the sufficient statistic approach.

This study is closely related to Trabandt and Uhlig (2011, 2013) and Nutahara (2015), who estimate the tax revenue curve for consumption tax using neoclassical growth models. Those studies employ KRP utility and use the numerical method to find that the tax revenue curve for consumption tax is monotonically increasing. Kobayashi (2014) investigates whether consumption tax revenue is bounded using a neoclassical growth model with log utility function. He finds that although the fixed supply of production factor affects the boundedness of consumption tax revenue, the tax revenue curve for consumption tax continues to be monotonically increasing in his model. In the incomplete market model with log utility function of Fève, Matheron, and Sahuc (2018), the tax revenue curve for consumption tax is not hump-shaped. To the best of our knowledge, one of the novel contributions of the present study is the finding that the tax revenue curve for consumption tax can be hump-shaped if the utility is additively separable or GHH.[2]

This study is also related to the literature on the fiscal limit. The fiscal limit is the maximum government debt–GDP ratio that can be sustained without appreciable risk of default or higher inflation. While the fiscal limit naturally is related to the maximum size of government tax revenue, most existing research on the fiscal limit excludes the role of consumption tax. For example, Bi (2012) and Leeper (2013) consider models with labor income tax. Leeper and Walker (2011) consider a model in which the lump-sum tax responds to the government debt-to-GDP ratio. However, to calculate the fiscal limit under realistic situations, it is important to consider consumption tax revenue.

Our finding has implications for the literature of fiscal reform, because consumption

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[2]Hiraga and Nutahara (2018) investigate the theoretical cause of the difference in the shapes of the Laffer curve for consumption and labor income taxes.

tax is receiving a lot of attention as a useful tool to finance government expenditure such as in Braun and Joines (2015), Hansen and Imrohoroglu (2016), and Kitao (2018). This literature highlights consumption tax because the welfare loss from consumption tax is less than that from other distortionary taxes, and because the tax revenue curves for other taxes, like labor income tax, are hump-shaped and the tax revenue bounded. According to our finding, consumption tax might not be useful if the tax revenue curve for consumption tax is hump-shaped.

The remainder of the paper is organized as follows. Section 2 introduces the simple static model and shows the main results. Some extensions are also shown in this section. Section 3 extends the result of Section 2 to a dynamic setting à la Trabandt and Uhlig (2011). Section 4 discusses the results. Section 5 concludes.

2   Simple static economy

This section, in which a simple static economy is assumed, characterizes the tax revenue curve for consumption tax.

2.1   Model

Representative households supply labor n to firms and earn wage rate w. Households also receive government transfers s. Let τ^c denote consumption tax. The budget constraint of households is

$$(1 + \tau^c)\, c \leq wn + s, \tag{1}$$

where c denotes consumption.

The firms are perfectly competitive. Their production function is

$$y = n, \tag{2}$$

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where y denotes output.

The government budget constraint is

$$s + g = T, \tag{3}$$

where g is government consumption, and total tax revenue T is defined by

$$T = \tau^c c. \tag{4}$$

Since there is no investment, the resource constraint of this closed economy is

$$y = c + g. \tag{5}$$

Three popular utility functions are considered. All have constant labor supply elasticity, and the first two have constant RRA. The first utility is

$$U^{KPR} = \frac{1}{1 - \eta} \Big\{ c^{1-\eta} \upsilon(n) - 1 \Big\},$$

where

$$\upsilon(n) \equiv \left[1 - \kappa(1 - \eta) n^{1+\lambda} \right]^{\eta},$$

which is the traditional KPR utility function with constant labor supply elasticity. The specification of $\upsilon(n)$ is for constant labor supply elasticity. The parameters η and $1/\lambda$ are the RRA and the labor supply elasticity,[3] respectively. This utility function is employed by Basu and Kimball (2005), Shimer (2009), Trabandt and Uhlig (2011), Kim and Katayama (2013), and Katayama and Kim (2018). The second utility is additively separable such that

$$U^{AS} = \frac{c^{1-\eta} - 1}{1 - \eta} - \kappa n^{1+\lambda},$$

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[3]In this study, $1/\lambda$ is called "labor supply elasticity," although it is often interpreted as "Frisch elasticity" in the literature. A discussion on this topic appears in Section 4.

where η and $1/\lambda$ are still the RRA and the labor supply elasticity, respectively. This type of utility function is also very popular in the literature, for example, Gali (2008). If $\eta = 1$, U^{KPR} and U^{AS} are identical. Otherwise, these two specifications are different. The last utility is

$$U^{GHH} = \frac{1}{1 - \bar{\eta}} \left\{ \left(c - \kappa n^{1+\lambda} \right)^{1-\bar{\eta}} - 1 \right\},$$

which is GHH utility. GHH utility implies that there are no income effects on labor supply, and it provides tractability. This type of utility is employed by Jaimovich and Rebelo (2009), Gertler, Kiyotaki, and Queralto (2012), and Korinek and Simsek (2016). The parameter $1/\lambda$ is still the labor supply elasticity, but the parameter $\bar{\eta}$ is no longer the RRA. The RRA is given by

$$-\frac{cU^{GHH}_{cc}}{U_c} = \bar{\eta} \times \frac{c}{c - \kappa n^{1+\lambda}}.$$

The following two fiscal policy schemes (use of tax revenue) are considered.

Definition 1. *Scheme (1): The total tax revenue is used as a lump-sum transfer to households.*

$$s = T, \qquad g = 0$$

Definition 2. *Scheme (2): The total tax revenue is used as government consumption.*

$$g = T, \qquad s = 0$$

2.2 Scheme (1): Tax revenue is used as a lump-sum transfer

The key element in this scheme is the elasticity of aggregate consumption to the consumption tax rate. If it is greater than one, an increase in the consumption tax rate increases consumption tax revenue, and vice versa. In this model, consumption equals labor supply by the resource constraint and production function.

KPR utility: In the case of the KPR utility function, the optimization condition for the consumption–labor choice is

$$\eta(1+\lambda)\left(\frac{\kappa c n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right) = \frac{1}{1+\tau^c}w. \tag{6}$$

Solving this condition yields

$$c = n = \left[\tau^c \eta\kappa(1+\lambda) + \kappa(\eta\lambda+1)\right]^{-1/(1+\lambda)}, \tag{7}$$

and the elasticity of consumption to the consumption tax rate is

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{\tau^c \eta\kappa}{\tau^c \eta\kappa(1+\lambda) + \kappa(\eta\lambda+1)}. \tag{8}$$

It is easily shown that $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is increasing in τ^c, $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = 0$ if $\tau^c = 0$, and $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ converges to $\frac{1}{1+\lambda} < 1$ as τ^c approaches infinity. Then, the KPR utility function cannot generate a hump-shaped consumption tax revenue curve for consumption tax, as in Proposition 1.

Proposition 1. *Suppose that the utility function is KPR: U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (1) is monotonically increasing. Consumption tax revenue is unbounded except for $\lambda = 0$.*

Proof. See Appendix A. □

The unboundedness of the tax revenue could be perceived as strange. This result comes from the fiscal policy scheme. Under Scheme (1), all tax revenue is transferred back to the household as lump-sum transfer s. This lump-sum transfer enables households to pay an infinite amount of consumption tax in the model.

Additive separable utility: In the case of the additively separable utility function, the optimization condition for the consumption–labor choice is

$$\kappa(1+\lambda)c^{\eta}n^{\lambda} = \frac{1}{1+\tau^c}w. \tag{9}$$

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Solving this condition yields

$$c = n = [\kappa(1 + \lambda)(1 + \tau^c)]^{-1/(\eta+\lambda)}, \tag{10}$$

and the elasticity of aggregate consumption to the consumption tax rate is

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \cdot \frac{1}{\eta + \lambda}. \tag{11}$$

It is shown that $\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is increasing in τ^c, $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = 0$ if $\tau^c = 0$, and $\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ converges to $\frac{1}{\eta+\lambda}$ as τ^c approaches infinity. Therefore, the Laffer curve for consumption tax can be hump-shaped if $\frac{1}{\eta+\lambda}$ is greater than one.

The following is a formal statement of a necessary and sufficient condition for a hump-shaped consumption tax revenue curve for consumption tax.

Proposition 2. *Suppose that the utility function is additively separable: U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (1) is hump-shaped if and only if $\eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. Consumption tax revenue is bounded if and only if $\eta + \lambda \leq 1$. Otherwise, it is unbounded.*

Proof. See Appendix B. □

The condition $\eta + \lambda < 1$ for the hump-shaped consumption tax revenue curve can be understood by the optimization condition for the consumption–labor choice, Equation (9). The consumption tax revenue curve can be hump-shaped if an increase in the consumption tax rate reduces the labor supply by a sufficient amount. The key parameter is the inverse of λ, that is, the labor supply elasticity to the effective after-tax wage rate $w/(1 + \tau^c)$, which is also interpreted as the relative price of leisure with respect to consumption. Then, a low value of λ implies a highly distorted increase in the consumption tax rate. In general equilibrium, consumption c is closely related to labor supply n through the resource constraint and the production function. In the current setting,

$c = n$. Then, the RRA η works as the inverse of the aggregate labor supply elasticity. As a result, the inverse of $\eta + \lambda$ is the elasticity of the aggregate labor supply in general equilibrium, as in (10). Then, the inverse of $\eta + \lambda$ is the maximum of the elasticity of consumption, since $c = n$.

GHH utility: In the case of the GHH utility function, the optimization condition for the consumption–labor choice is

$$\kappa(1 + \lambda)n^{\lambda} = \frac{1}{1 + \tau^c}w. \tag{12}$$

Solving this condition yields

$$c = n = [\kappa(1 + \lambda)(1 + \tau^c)]^{-1/\lambda}, \tag{13}$$

and the elasticity of aggregate consumption to the consumption tax rate is

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \cdot \frac{1}{\lambda}. \tag{14}$$

It is easily shown that $\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is increasing in τ^c, $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = 0$ if $\tau^c = 0$, and $\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ converges to $\frac{1}{\lambda}$ as τ^c approaches infinity. Therefore, the Laffer curve for consumption tax can be hump-shaped if $\frac{1}{\lambda}$ is greater than one.

The following is a formal statement of a necessary and sufficient condition for a hump-shaped consumption tax revenue curve for consumption tax.

Proposition 3. *Suppose that the utility function is GHH: U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (1) is hump-shaped if and only if $\lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\lambda}{1-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\lambda \leq 1$. Otherwise, it is unbounded.*

Proof. See Appendix C. □

In the case of GHH utility, the consumption tax revenue curve is hump-shaped if and only if $\lambda < 1$. This is the case when the labor supply elasticity is greater than one. Therefore, GHH utility is most likely to generate a hump-shaped tax revenue curve for consumption tax in the abovementioned three specifications of utility.

Perspectives from general functional form of utility: Propositions 1, 2, and 3 are derived under the three specifications of utility. Here, the consumption tax revenue curve under the general form of utility is investigated under Scheme (1).

Suppose the utility function $U(c, n)$ with standard assumptions $U_c > 0$, $U_{cc} < 0$, $U_n < 0$, and $U_{nn} \leq 0$. The consumption–labor choice condition is

$$-\frac{U_n}{U_c} = \frac{1}{1 + \tau^c} w. \tag{15}$$

As in Appendix D, the elasticity of consumption with respect to the consumption tax rate is given by

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1}. \tag{16}$$

This form tell us that the elasticity of consumption consists of four parts. The first is the RRA $-cU_{cc}/U_c$. The second is the analogue of labor supply nU_{nn}/U_n. The last two parts are about the cross-derivatives of utility with respect to consumption and labor supply: cU_{cn}/U_n and $-nU_{cn}/U_c$. Note that there is no uncertainty in this economy and the attitude to risk itself is not important. The RRA η is interpreted as an index of the curvature of the utility function.

In the case of KPR utility U^{KPR},

$$-\frac{cU_{cc}}{U_c} = \eta, \qquad \frac{nU_{nn}}{U_n} = \lambda + \frac{(1 - \eta)^2 \kappa(1 + \lambda)n}{1 - \kappa(1 - \eta)n^{1+\lambda}},$$

$$\frac{cU_{cn}}{U_n} = 1 - \eta, \qquad -\frac{nU_{cn}}{U_c} = \frac{\eta\kappa(1 - \eta)(1 + \lambda)n^{1+\lambda}}{1 - \kappa(1 - \eta)n^{1+\lambda}}.$$

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By equation (7), it is shown that $n \to 0$ as $\tau^c \to \infty$. Then, the elasticity of consumption goes to $1/(1 + \lambda)$ as $\tau^c \to \infty$. In the case of additively separable utility U^{AS},

$$-\frac{cU_{cc}}{U_c} = \eta, \qquad \frac{nU_{nn}}{U_n} = \lambda, \quad \text{and} \quad \frac{cU_{cn}}{U_n} = 0, \quad -\frac{nU_{cn}}{U_c} = 0,$$

and then,

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \times \frac{1}{\eta + \lambda}.$$

In the case of GHH utility U^{GHH},

$$-\frac{cU_{cc}}{U_c} = \bar{\eta} \frac{c}{c - \kappa n^{1+\lambda}}, \qquad \frac{nU_{nn}}{U_n} = \lambda + \kappa \eta \frac{n^{1+\lambda}}{c - \kappa n^{1+\lambda}},$$

$$\frac{cU_{cn}}{U_n} = -\bar{\eta} \frac{c}{c - \kappa n^{1+\lambda}}, \qquad -\frac{nU_{cn}}{U_c} = -\kappa \bar{\eta} \frac{n^{1+\lambda}}{c - \kappa n^{1+\lambda}},$$

and then,

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \times \frac{1}{\lambda}.$$

Note that the RRA is canceled out by the term cU_{cn}/U_n in the case of KPR and GHH utility, while it affects the elasticity of consumption generally.

2.3 Scheme (2): Tax revenue is used as government consumption

The resource constraint and the production function imply $n = c + g = (1 + \tau^c)c$, since government consumption g equals the total tax revenue $T = \tau^c c$ in the case of Scheme (2).

KPR utility: In the case of KPR utility, the closed form of consumption is

$$c = \{[\eta(1 + \lambda) + (1 - \eta)]\kappa\}^{-\frac{1}{1+\lambda}} (1 + \tau^c)^{-1}, \tag{17}$$

and the elasticity of consumption is

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c}. \tag{18}$$

Then, the analogue of Proposition 1 is the following.

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Proposition 4. *Suppose that the utility function is KPR: U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (2) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. See Appendix E. □

In the case of KPR utility, the consumption tax revenue curve is still monotonically increasing under any parameter values. However, consumption tax revenue is bounded under Scheme (2), whereas it is unbounded under Scheme (1). Therefore, the use of tax revenue has a significant effect on the boundedness of tax revenue.

The difference in Propositions 1 and 4 comes from the negative income effect under Scheme (2). Under Scheme (1), the tax payments of households are transferred back as a lump-sum transfer, and household income is not hurt. On the contrary, tax payments are used as government consumption under Scheme (2), and household income decreases, placing downward pressure on consumption as normal goods.

Additively separable utility: In this case, if the utility function is additively separable, consumption is given by

$$c = [\kappa(1 + \lambda)]^{-\frac{1}{\eta+\lambda}} \cdot [1 + \tau^c]^{-\frac{1+\lambda}{\eta+\lambda}}, \tag{19}$$

and the elasticity of consumption is

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \times \frac{1 + \lambda}{\eta + \lambda}.$$

Then, the analogue of Proposition 2 is the following.

Proposition 5. *Suppose that the utility function is additively separable: U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (2) is hump-shaped if and only if $\eta < 1$, and revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. Consumption tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. See Appendix F. □

According to Proposition 5, the RRA should be less than one for the hump-shaped consumption tax revenue curve for consumption tax, whereas the sum of the RRA and labor supply elasticity should be less than one under Scheme (1). Then, a hump-shaped consumption tax revenue curve is more likely to arise under Scheme (2). This is because the tax payment is used as government consumption under Scheme (2), placing a downward pressure on consumption, as in the case of KPR utility.

Even under Scheme (2), the tax revenue is unbounded if $\eta > 1$. Under Scheme (1), paying an infinite amount of consumption tax is possible because the tax revenue is transferred back to the household. Under Scheme (2), the labor supply is given by

$$n = (1 + \tau^c)c = [\kappa(1 + \lambda)]^{-\frac{1}{\eta+\lambda}} \cdot [1 + \tau^c]^{\frac{\eta-1}{\eta+\lambda}} . \tag{20}$$

If $\eta > 1$, the labor supply is increasing in τ^c, and it diverges to infinity as $\tau^c \to \infty$, whereas consumption converges to zero. This result implies that the labor income of household wn is also unbounded, and an infinite amount of tax payment is possible. This unboundedness of labor supply may be unrealistic. However, it could occur in theory if utility is the popular additively separable kind and if the RRA is not an unusual value.

GHH utility: If the utility function is additively separable, then consumption is given by

$$c = [\kappa(1 + \lambda)]^{-\frac{1}{\lambda}} \cdot [1 + \tau^c]^{-\frac{1+\lambda}{\lambda}} , \tag{21}$$

and the elasticity of consumption is

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \times \frac{1 + \lambda}{\lambda}.$$

The elasticity of consumption is greater than one if the consumption tax rate is sufficiently high. Then, the tax revenue curve for consumption tax is hump-shaped under

any parameter values in the case of GHH utility. The analogue of Proposition 3 is the following.

Proposition 6. *Suppose that the utility function is GHH: U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (2) is hump-shaped, and the revenue is maximized at $\tau^c = \lambda$. The consumption tax revenue is bounded.*

Proof. See Appendix G. □

Under Scheme (2), the tax revenue curve for consumption tax is hump-shaped under any parameter values, whereas the labor supply elasticity should be greater than one to generate a hump-shaped tax revenue curve under Scheme (1). This result also comes from downward pressure on consumption by the negative income effects under Scheme (2).

Perspectives from general functional form of utility: As in the previous subsection, the consumption tax revenue curve under the general form of utility is investigated under Scheme (2). Appendix D derives the elasticity of consumption with respect to the consumption tax rate as

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1+\tau^c} \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1} \times \left[1 + \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c} \right]. \quad (22)$$

As in the case in which the tax revenue is used as a lump-sum transfer, the elasticity of consumption consists of three parts: the RRA $-cU_{cc}/U_c$, its analogue of labor supply nU_{nn}/U_n, and the cross-derivative of utility with respect to consumption and labor supply cU_{cn}/U_n and $-nU_{cn}/U_c$, respectively.

The results under Schemes (1) and (2) are summarized in Table 1. The functional form of utility is important for the shape of the tax revenue curve for consumption tax. The tax revenue curve can be hump-shaped if utility if additively separable or GHH, whereas it cannot be hump-shaped if utility is KPR. The use of tax revenue is particularly

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important for the boundedness of tax revenue, especially in the case of KPR utility. In the cases of additively separable and GHH utility, a hump-shaped consumption tax revenue curve is likely to arise if the tax revenue is used as government consumption.

[Insert Table 1]

2.4   Total tax revenue curve with labor income tax

The baseline result considers the consumption tax revenue curve. By introducing labor income tax τ^n, the tax revenue curve refers to total tax revenue, including labor income tax revenue.

In this case, the budget constraint of a household becomes

$$(1 + \tau^c)c \leq (1 - \tau^n)wn + s, \tag{23}$$

and the total tax revenue is

$$T = \tau^c c + \tau^n wn. \tag{24}$$

The total tax revenue curve for consumption tax is the relationship between the consumption tax rate, τ^c, and the total tax revenue, T. In this situation, Scheme (1) is naturally redefined so that the total tax revenue is used as a lump-sum transfer. Scheme (2) is also redefined so that the total tax revenue is used as government consumption.

The properties of the total tax revenue curve are similar to those of the consumption tax revenue curve. The details are described in Section A.2 of the appendix of Hiraga and Nutahara (2019).

KPR utility: In the case of KPR utility, it is shown that a hump-shaped total tax revenue curve cannot be generated under either redefined Scheme (1) or (2). The total tax revenue is unbounded except for $\lambda = 0$ under the redefined Scheme (1), whereas it is bounded under the redefined Scheme (2).

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Additively separable utility:　In the case of additively separable utility U^{AS}, the condition $\eta + \lambda < 1$ is necessary but not sufficient for the hump-shaped total tax revenue curve under the redefined Scheme (1). The total tax revenue curve for consumption tax is monotonically decreasing if the labor income tax rate is sufficiently high. If $\eta + \lambda < 1$, the total tax revenue curve is hump-shaped, but the peak tax rate becomes negative. Then, in the area of $\tau^c \geq 0$, the total tax revenue curve is monotonically decreasing. The total tax revenue is bounded if and only if $\eta + \lambda \leq 1$. Under the redefined Scheme (2), the condition $\eta < 1$ is a necessary condition for the hump-shaped total tax revenue curve. The total tax revenue is bounded if and only if $\eta \leq 1$.

GHH utility:　In the case of GHH utility U^{GHH}, the condition $\lambda < 1$ is necessary but not sufficient for the hump-shaped total tax revenue curve under the redefined Scheme (1). If the labor income tax rate is sufficiently high, the total tax revenue curve is monotonically decreasing, as in the case of additively separable utility. The total tax revenue is bounded if and only if $\lambda \leq 1$. Under the redefined Scheme (2), the total tax revenue curve for consumption tax is hump-shaped, and the total tax revenue is bounded.

2.5　Alternative fiscal policy schemes

Schemes (1) and (2) are two extreme cases in which all tax revenue is used as a lump-sum transfer or as government consumption. In Section A.3 of the appendix of Hiraga and Nutahara (2019), some modified versions are considered, where tax revenue is used for both a lump-sum transfer and government consumption.

Constant g/y scheme and constant s/y scheme:　A modification of Scheme (1) is the case in which the ratio of government consumption to output, g/y, is constant and the rest of tax revenue is used as a lump-sum transfer. Under this scheme, the change in tax revenue is mainly adjusted by the lump-sum transfer. The analogue of Scheme (2) is the

case in which the ratio of the lump-sum transfer to output, s/y, is constant and the rest of tax revenue is used as government consumption. Under this scheme, the change in tax revenue is mainly adjusted by government consumption. It is shown that Propositions 1–6 still hold under these two modified schemes. The consumption tax revenue curve for consumption tax cannot be hump-shaped if the utility is KPR, whereas it can be hump-shaped if the utility is additively separable or GHH. Then, even if tax revenue is used for both lump-sum transfer and government consumption, our results are robust.

Constant g scheme and constant s scheme: The following alternative fiscal policy schemes can be considered. As a modified Scheme (1), the level of government spending g is constant and the rest of tax revenue is used as a lump-sum transfer. As a modified Scheme (2), the level of lump-sum transfer s is constant and the rest of tax revenue is used as government consumption. These two schemes are similar to those employed by Trabandt and Uhlig (2011). Under these two schemes, the consumption tax revenue curve still cannot be hump-shaped in the case of KPR utility, but the consumption tax revenue is bounded. A necessary and sufficient condition for a hump-shaped consumption tax revenue curve is $\eta < 1$ under both modified Schemes (1) and (2) if the utility is additively separable. The consumption tax revenue is bounded if and only if $\eta \leq 1$. If the utility is GHH, the consumption tax revenue curve for consumption tax is hump-shaped under both Schemes (1) and (2), and the revenue is bounded.

The main changes of the results occur under the modified Scheme (1). The changes come from the assumption of constant g. An increase in the consumption tax rate decreases labor supply and output. Then, the ratio of government consumption to output increases, which implies that the ratio of the lump-sum transfer to output decreases. This can be interpreted as a negative income effect, and places downward pressure on consumption. Then, a hump-shaped consumption tax revenue curve is likely to arise in the cases of additively separable and GHH utility, and tax revenue is bounded in the case of

KPR utility.

3 Extension to the dynamic economy à la Trabandt and Uhlig (2011)

In this section, the result of Section 2 is extended to a neoclassical growth model à la Trabandt and Uhlig (2011).

3.1 Model

The representative households hold capital stock k_{t-1} and debt b_{t-1} as assets at the beginning of the period. They supply labor n_t and capital stock k_{t-1} to firms, and earn wage rate w_t, rental rate of capital d_t, and interest on debt at the rate of R_t^b. They also receive government transfers s_t and transfers from abroad m_t. The latter can be interpreted as net imports, as discussed by Trabandt and Uhlig (2011). Let τ_t^c, τ_t^n, and τ_t^k denote the consumption tax, labor tax, and capital tax rates, respectively. The budget constraint of households is

$$(1 + \tau_t^c)c_t + x_t + b_t \leq (1 - \tau_t^n)w_t n_t + (1 - \tau_t^k)(d_t - \delta)k_{t-1} + \delta k_{t-1} + R_t^b b_t + s_t + m_t, \quad (25)$$

where c_t denotes consumption, δ the depreciation rate of capital, and x_t investment. The capital stock evolves according to the following equation.

$$k_t = (1 - \delta)k_{t-1} + x_t. \quad (26)$$

The firms are perfectly competitive. Their production function is

$$y_t = \xi^t k_{t-1}^\theta n_t^{1-\theta}, \quad (27)$$

where ξ denotes the technology growth rate, and θ the capital share in production. The profit-maximization problem implies

$$w_t = (1 - \theta)\frac{y_t}{n_t} \qquad \text{and} \tag{28}$$

$$d_t = \theta\frac{y_t}{k_{t-1}}. \tag{29}$$

The government budget constraint is

$$g_t + s_t + R_t^b b_{t-1} \leq b_t + T_t, \tag{30}$$

where g_t denotes government consumption, and total tax revenue T_t is defined by

$$T_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k (d_t - \delta)k_{t-1}. \tag{31}$$

The resource constraint of this economy is

$$y_t = c_t + x_t + g_t - m_t. \tag{32}$$

The KPR utility function for this dynamic economy is

$$\mathbb{U}^{KPR} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\eta} \left\{ c_t^{1-\eta} \left[1 - \kappa(1-\eta)n_t^{1+\lambda} \right]^{\eta} - 1 \right\} + v(g_t) \right],$$

where $v(\cdot)$ is an increasing function.[4] The additively separable utility function is

$$\mathbb{U}^{AS} = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\eta} - 1}{1-\eta} - \kappa\psi^{t(1-\eta)}n_t^{1+\lambda} + v(g_t) \right].$$

The preference over labor supply shifts with the level of technology, $\psi^{t(1-\eta)} \equiv \xi^{t(1-\eta)/(1-\theta)}$, to guarantee the existence of a balanced growth path, as employed by Erceg, Guerrieri, and Gust (2006). The GHH utility function is

$$\mathbb{U}^{GHH} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\bar{\eta}} \left\{ \left(c_t - \kappa\psi^t n_t^{1+\lambda} \right)^{1-\bar{\eta}} - 1 \right\} + v(g_t) \right].$$

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[4]Following Trabandt and Uhlig (2011), it is assumed that g yields utility. However, this assumption does not matter for the main results of this study.

The preference shift parameter for balanced growth path is $\psi^t \equiv \xi^{t/(1-\theta)}$ in this case.

Following Trabandt and Uhlig (2011), the total tax revenue curve for consumption tax is given by the relationship between tax revenue and the tax rate on the balanced growth path. With regard to the use of tax revenue, the following two schemes are considered.

Definition 3. *Scheme (3): The ratio of government bonds to output, b/y, and the ratio of government consumption to output, g/y, are constant. The remainder of the changes in tax revenue is adjusted by lump-sum transfer to households.*

$$s = T - (R^b - 1)b - g, \qquad g/y = \phi_{gy}, \qquad b/y = \phi_{by}$$

Definition 4. *Scheme (4): The ratio of government bonds to output, b/y, and the ratio of lump-sum transfer to output, s/y, are constant. The remainder of the changes in tax revenue is adjusted by government consumption.*

$$g = T - (R^b - 1)b - s, \qquad s/y = \phi_{sy}, \qquad b/y = \phi_{by}$$

These two schemes are natural extensions of Schemes (1) and (2) in the static model, but they differ from the assumptions of Trabandt and Uhlig (2011). Trabandt and Uhlig (2011) employ the assumption that $g_t = \psi^t \bar{g}$ and $b_t = \psi^t \bar{b}$ instead of Scheme (3). The constant steady-state ratio of government consumption to GDP in Scheme (3) is interpreted as government control g_t/y_t as in Hayashi and Prescott (2002). These assumptions of constant steady-state ratios are used to prove the propositions in this section. Under Scheme (3), an increase in the consumption tax rate decreases both output and government consumption. This decrease in government consumption implies a positive wealth effect, and consumption increases. Therefore, the Laffer curve for consumption tax is less likely to be hump-shaped than under the assumptions employed by Trabandt and Uhlig (2011).

In addition, the following assumption is imposed for technical reasons.

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Assumption 1. *The ratio of net imports to GDP is constant: $m/y = \phi_{my}$.*

The results on the consumption tax revenue curve are the same as in the static model. If the utility is KPR, the consumption tax revenue curve is never hump-shaped. If the utility is additively separable or GHH, the consumption tax revenue curve can be hump-shaped. In the case of the additively separable utility, a necessary and sufficient condition for a hump-shaped consumption tax revenue curve is $\eta + \lambda < 1$ under Scheme (3), and $\eta < 1$ under Scheme(4) In the case of GHH utility, a necessary and sufficient condition for a hump-shaped consumption tax revenue curve is $\eta + \lambda < 1$ under Scheme (3). Under Scheme (4), the consumption tax revenue curve is hump-shaped if the utility is GHH. The formal propositions on the consumption tax revenue curve are described in Section B.3 of the separate appendix of Hiraga and Nutahara (2019).

The rest of this section describes the results on the total tax revenue curve for consumption tax.

3.2 Scheme (3): Changes in total tax revenue are adjusted by lump-sum transfer

Propositions 7, 8, and 9 refer to the total tax revenue curve under Scheme (3).

Proposition 7. *Suppose that the utility function is KPR: $\mathbb{U}^{KPR}$. The total tax revenue curve for consumption tax under Scheme (3) is monotonically increasing if and only if*

$$\tau^n(1 - \theta) + \tau^k(d - \delta)\left(\frac{k}{y}\right) \leq \frac{1 - \eta}{\eta}(1 - \theta)(1 - \tau^n) + (1 + \lambda)\left(\frac{c}{y}\right),$$

where

$$d = \frac{1}{1 - \tau^k}\left[\frac{\psi^\eta}{\beta} - 1 + \delta\right], \quad \frac{k}{y} = \frac{\theta}{d}, \quad and \quad \frac{c}{y} = 1 - [\psi - (1 - \delta)]\frac{\theta}{d} - \phi_g + \phi_m.$$

Otherwise, the total tax revenue curve for consumption tax is U-shaped.

The total tax revenue is unbounded except for $\lambda = 0$.

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Proof. See Appendix I. □

Proposition 8. *Suppose that the utility function is additively separable:* $\mathbb{U}^{AS}$. *The total tax revenue curve for consumption tax under Scheme (3) is hump-shaped if and only if*

$$\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] < \eta + \lambda < 1,$$

where

$$d = \frac{1}{1-\tau^k}\left[\frac{\psi^\eta}{\beta} - 1 + \delta\right], \quad \frac{k}{y} = \frac{\theta}{d}, \quad \text{and} \quad \frac{c}{y} = 1 - [\psi - (1-\delta)]\frac{\theta}{d} - \phi_g + \phi_m.$$

and the revenue is maximized at $\tau^c = \frac{1}{1-\eta-\lambda}\left\{(\eta+\lambda) - \left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right]\right\}$. *Otherwise, the total tax revenue curve for consumption tax is*

- *U-shaped if* $\eta + \lambda > 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] > \eta + \lambda$.

- *monotonically increasing if* $\eta + \lambda > 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] \leq \eta + \lambda$.

- *monotonically increasing if* $\eta + \lambda = 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] < \eta + \lambda$.

- *flat if* $\eta + \lambda = 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] = \eta + \lambda$.

- *monotonically decreasing if* $\eta + \lambda = 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] > \eta + \lambda$.

- *monotonically decreasing if* $\eta + \lambda < 1$ *and* $\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] \geq \eta + \lambda$.

The total tax revenue is bounded if and only if $\eta + \lambda \leq 1$. *Otherwise, it is unbounded.*

Proof. See Appendix J. □

Proposition 9. *Suppose that the utility function is GHH:* $\mathbb{U}^{GHH}$. *The total tax revenue curve for consumption tax under Scheme (3) is hump-shaped if and only if*

$$\left(\frac{y}{c}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{k}{y}\right)\right] < \lambda < 1,$$

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where

$$d = \frac{1}{1 - \tau^k} \left[\frac{\psi^\eta}{\beta} - 1 + \delta \right], \quad \frac{k}{y} = \frac{\theta}{d}, \quad and \quad \frac{c}{y} = 1 - [\psi - (1 - \delta)] \frac{\theta}{d} - \phi_g + \phi_m.$$

and the revenue is maximized at $\tau^c = \frac{1}{1-\lambda} \left\{ \lambda - \left(\frac{y}{c} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{k}{y} \right) \right] \right\}.$
Otherwise, the total tax revenue curve for consumption tax is

- *U-shaped if* $\lambda > 1$ *and* $\left(\frac{y}{c} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{k}{y} \right) \right] > \lambda.$

- *monotonically increasing if* $\lambda > 1$ *and* $\left(\frac{y}{c} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{k}{y} \right) \right] \leq \lambda.$

- *monotonically increasing if* $\lambda = 1$ *and* $\left(\frac{y}{c} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{k}{y} \right) \right] < \lambda.$

- *flat if* $\lambda = 1$ *and* $\left(\frac{y}{c} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{k}{y} \right) \right] = \lambda.$

- *monotonically decreasing if* $\lambda = 1$ *and* $\left(\frac{y}{c} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{k}{y} \right) \right] > \lambda.$

- *monotonically decreasing if* $\lambda < 1$ *and* $\left(\frac{y}{c} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{k}{y} \right) \right] \geq \lambda.$

The total tax revenue is bounded if and only if $\lambda \leq 1$*. Otherwise, it is unbounded.*

Proof. See Appendix K. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

Propositions 7, 8, and 9 imply that the total tax revenue curve can be hump-shaped if the utility function is additively separable or GHH, whereas it cannot be if the utility function is KPR. As in the static economy, $\eta + \lambda < 1$ and $\lambda < 1$ are the keys for the hump-shaped Laffer curve in the case of additively separable and GHH utility, respectively. The boundedness of tax revenue is the same as in the static economy: it is bounded if and only if $\eta + \lambda \leq 1$ and $\lambda \leq 1$ in the case of the additively separable and GHH utility, respectively, and it is unbounded except for $\lambda = 0$ in the case of KPR utility.

Propositions 7, 8, and 9 imply that the total tax revenue curve can take various shapes, for example, U-shaped. Under this situation, the total tax revenue is decreasing if the consumption tax rate is low, and increasing if the consumption tax rate is

sufficiently high. A U-shaped total tax revenue curve for consumption tax is generated if the consumption tax revenue curve is monotonically increasing and the labor income and capital income tax rates are sufficiently high. Decreases in these tax revenues associated with an increase in the consumption tax rate dominate the increase in consumption tax revenue if the consumption tax rate is low. In addition, a monotonically decreasing tax revenue curve is possible in the case of additively separable and GHH utility. This occurs in the case in which the consumption tax revenue curve is hump-shaped and the labor income and capital income tax rates are sufficiently high.

3.3   Scheme (4): The changes in total tax revenue is adjusted by government consumption

Propositions 10, 11, and 12 are the analogues of Propositions 7, 8, and 9 under Scheme (4), respectively.

Proposition 10. *Suppose that the utility function is KPR: $\mathbb{U}^{KPR}$. The total tax revenue curve for consumption tax under Scheme (4) is monotonically increasing. The total tax revenue is bounded.*

Proof. See Appendix L. □

Proposition 11. *Suppose that the utility function is additively separable: $\mathbb{U}^{AS}$. The total tax revenue curve for consumption tax under Scheme (4) is hump-shaped if and only if*

$$\frac{-\lambda\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}{\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}} < \eta < 1,$$

where

$$d = \frac{1}{1-\tau^k}\left[\frac{\psi^\eta}{\beta} - 1 + \delta\right], \quad \frac{k}{y} = \frac{\theta}{d}, \quad and \quad (1+\tau^c)\frac{c}{y} = 1 - [\psi - (1-\delta)]\frac{\theta}{d} + \phi_m,$$

and the revenue is maximized at $\tau^c = \frac{\frac{\eta+\lambda}{1-\eta}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}{\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}$.

Otherwise, the total tax revenue curve for consumption tax is

- *monotonically increasing if $\eta \geq 1$.*

- *monotonically decreasing if $\eta < 1$ and $\eta \leq \frac{-\lambda\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)+\tau^n(1-\theta)+\tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}{\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)+\tau^n(1-\theta)+\tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}.$*

The total tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.

Proof. See Appendix M. □

Proposition 12. *Suppose that the utility function is GHH: $\mathbb{U}^{GHH}$. The total tax revenue curve for consumption tax under Scheme (4) is hump-shaped if and only if*

$$\lambda > \frac{\tau^n(1-\theta)+\tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}{(1+\tau^c)\frac{\tilde{c}}{\tilde{y}}},$$

where

$$d = \frac{1}{1-\tau^k}\left[\frac{\psi^\eta}{\beta}-1+\delta\right], \quad \frac{k}{y}=\frac{\theta}{d}, \quad and \quad (1+\tau^c)\frac{c}{y}=1-[\psi-(1-\delta)]\frac{\theta}{d}+\phi_m,$$

and the revenue is maximized at $\tau^c = \frac{\lambda(1+\tau^c)\frac{\tilde{c}}{\tilde{y}}-\tau^n(1-\theta)-\tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}{(1+\tau^c)\frac{\tilde{c}}{\tilde{y}}+\tau^n(1-\theta)+\tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}.$

Otherwise, the total tax revenue curve for consumption tax is monotonically decreasing. The total tax revenue is bounded.

Proof. See Appendix N. □

As in the static economy, the condition $\eta < 1$ is necessary to generate the hump-shaped Laffer curve for consumption tax in the case of additively separable utility, and the total tax revenue is bounded if and only if $\eta \leq 1$. The Laffer curve for consumption tax cannot be hump-shaped in the case of KPR utility. However, the total tax revenue is bounded under Scheme (4).

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4 Discussion

4.1 Likelihood of a hump-shaped tax revenue curve for consumption tax

According to the propositions in Sections 2 and 3, to generate a hump-shaped tax revenue curve for consumption tax, $\eta + \lambda < 1$ is necessary in the case of additively separable utility, and $\lambda < 1$ is necessary in the case of GHH utility. For these conditions, η and λ should be less than one. The likelihood of this condition is discussed in this subsection.

It is not standard to set $\eta < 1$ in the literature. The log utility is the case of $\eta = 1$. In many business cycle models, η is set to be $\eta \geq 1$. Chetty (2006) may support the condition $\eta < 1$. He estimates the RRA using 33 existing estimates of wage and income elasticities. The average of his estimated RRAs is 0.71, and they range from 0.15 to 1.78 in the additive utility case. However, it should be noticed that Chetty's result depends on a small estimate of labor supply elasticity, which contradicts the condition $\eta + \lambda < 1$. It is possible to interpret η as the inverse of the intertemporal elasticity of substitution (IES), because the RRA is the same as the inverse of the IES under our specification of the additively separable utility function. In this case, the empirical results of Mulligan (2002), Vissing-Jorgensen and Attanasio (2003), Sakuragawa and Hosono (2010), and Gruber (2013) support an IES value greater than one (or $\eta < 1$). However, according to the meta analysis of Havranek, Horvath, Irsova, and Rusnak (2015), the mean estimate of the IES reported in existing empirical studies is 0.5.

On the one hand, it is not uncommon for the macroeconomic literature with representative agent models to set $\lambda < 1$. For example, Prescott (1986) sets $\lambda = 1/2$. On the other hand, empirical evidence from micro data implies that Frisch elasticity is very small, and does not seem to support $\lambda < 1$. However, the parameter λ should not be restricted by evidence on the Frisch elasticity, as claimed by Christiano, Trabandt, and Walentin (2010). As in the seminal works of Hansen (1985) and Rogerson (1988), even

if the individual elasticity of labor supply is zero, the aggregate labor supply can be sensitive to changes in the real wage rate. Recently, Keane and Rogerson (2011, 2012) claim that small micro and large macro elasticities of labor supply are consistent if human capital accumulation and the intensive and extensive margins are controlled. For example, Imai and Keane (2004) estimate the labor supply elasticity (inverse of λ) as 3.8 using a model with human capital accumulation. Christiano, Trabandt, and Walentin (2010) estimate this parameter for the US economy by using Bayesian impulse response matching, and find that λ is around 0.1.

These empirical results suggest that $\lambda < 1$ is possible. Then, if the utility is GHH, the tax revenue curve for consumption tax can be hump-shaped. The results also suggest that $\eta + \lambda < 1$ is not unrealistic, following the standard parameter values. Then, if additively separable utility is employed, some additional features, like home production or market failures, are necessary to generate a hump-shaped tax revenue curve for consumption tax.

4.2 Numerical results of the total tax revenue curve for consumption tax

Sections 2 and 3 characterize the shape of the tax revenue curve for consumption tax and show that the tax revenue curve can be hump-shaped in the cases of additively separable and GHH utility. This subsection presents numerical results. As the baseline, the total tax revenue curve in the dynamic model under Scheme (3) is considered.

The parameter values except for the utility function are the same as those employed by Trabandt and Uhlig (2011) for the US economy. The capital share in the production function θ is 0.35. The depreciation rate of capital δ is 0.083. The steady-state ratio of debt to output b/y is 0.63. The steady-state ratio of government expenditure to output g/y is 0.08. The steady-state ratio of transfer from abroad to output m/y is 0.04. The balanced growth parameter ψ is 1.02. The steady-state real interest rate R is 1.04. The

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steady-state labor supply h is 0.2. The steady-state capital income tax rate is 0.36, labor income tax is 0.28, and consumption tax rate is 0.05. The utility function parameters are set such that $\eta = 2$ and $\lambda = 1/2$. Under these parameter values, the tax revenue curve for consumption tax can be hump-shaped if the utility is GHH, but it cannot be if the utility is additively separable.

Figure 1 shows a numerical example of the total tax revenue curves for consumption tax in the dynamic model. The procedure to calculate the tax revenue curves is described in Appendix O. The horizontal axes show the consumption tax rate. The vertical axes show normalized tax revenue. The tax revenue is normalized such that total tax revenue in the case of the baseline tax rates is 100. The circles denote the peak tax rates that maximize the total tax revenue. The vertical dotted lines show the baseline consumption tax rate of 5%. The total tax revenue curve for consumption tax is hump-shaped in the case of the GHH utility function, and is monotonically increasing for KPR and additively separable utility. The peak tax rate that maximizes the total tax revenue of the additively separable utility is 22.4%, whereas the consumption tax revenue is maximized at 100%, that is, $\lambda/(1 - \lambda)$. It is surprising that the maximized total tax revenue is 102.8 in the case of GHH utility, as this result implies that there is little room to increase the total tax revenue even if the consumption tax peaks.

[Insert Figure 1]

Figure 2 shows numerical examples of the consumption tax revenue curves for consumption tax in the dynamic model. Each line shows the case of the inverse of the labor supply elasticity $\lambda = 1/2$, 1, and 2. The RRA η is set to be 2. The consumption tax revenue curve for consumption tax is hump-shaped only in the case of the GHH utility function with $\lambda = 1/2$, and the others are monotonically increasing. In the case of GHH utility with $\lambda = 1/2$, the peak tax rate that maximizes consumption tax revenue is maximized at 100%, that is, $\lambda/(1 - \lambda)$. The maximum tax revenue is about 400 even in the case of GHH utility with $\lambda = 1/2$, whereas the total tax revenue is only 102.8 in Figure

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1. This difference comes from the decreases in the labor income and capital income taxes in the case of total tax revenue.

[Insert Figure 2]

5   Concluding remarks

This study has shown that the shape of the tax revenue curve for consumption tax and the boundedness of tax revenue are sensitive to the choice of utility function and the assumption of the use of tax revenue. The tax revenue curve for consumption tax cannot be hump-shaped if the utility function is of the KPR type with constant labor supply elasticity, whereas the curve can be humps-haped if the utility function is additively separable in consumption and labor supply, or GHH. The key parameters for the hump-shaped tax revenue curve are the labor supply elasticity and the RRA. The use of tax revenue also has significant effects on the tax revenue curve for consumption tax. If the tax revenue is mainly used as a lump-sum transfer to households, the tax revenue is likely to be unbounded, whereas the revenue is likely to be bounded and the tax revenue curve is likely to be hump-shaped if the tax revenue is mainly used as government consumption.

Of course, there are some limitations and room to extend our models in order to capture the actual economy. For simplicity of analysis, the economies considered in this study are quite simple neoclassical frictionless representative-agent ones. The heterogeneity of households is important to focus on the redistribution effect of consumption tax. In addition, household production and tax evasion behavior are important Factors in estimating realistic tax revenue curves. The current study focused only on the balanced growth path of the tax revenue curve, whereas The transition dynamics of tax revenue are also important for future analysis.

The main contribution of this study is to show that a model with consumption tax may be sensitive to the choice of the functional form of utility and the assumption of the

use of tax revenue. Therefore, the results of the study indicate that robustness checks are essential in applications using models with consumption tax.

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Appendix

A   Proof of Proposition 1

Proof. It is obvious that

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c \eta \kappa}{\tau^c \eta \kappa (1 + \lambda) + \kappa(\eta\lambda + 1)} \leq 1.$$

It is also obvious that the tax revenue is unbounded if $\lambda > 0$ by the elasticity of consumption. If $\lambda = 0$, by (7), the following is obtained

$$\lim_{\tau^c \to \infty} \tau^c c = \frac{1}{\eta \kappa},$$

$\square$

B   Proof of Proposition 2

Proof. Note that

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 = \frac{1}{\eta + \lambda} \cdot \frac{1}{1 + \tau^c} \left[(1 - \eta - \lambda)\tau^c - (\eta + \lambda) \right].$$

Suppose that $\eta + \lambda = 1$. In this case, $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 < 0$ and the consumption tax revenue is monotonically increasing.

Suppose $\eta + \lambda \neq 1$. In this case,

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 = \left(\frac{1 - \eta - \lambda}{\eta + \lambda} \right) \left(\frac{1}{1 + \tau^c} \right) \left(\tau^c - \frac{\eta + \lambda}{1 - \eta - \lambda} \right).$$

If $\eta + \lambda > 1$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \leq 1$.

If $\eta + \lambda < 1$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \leq 1$ for $\tau^c \leq (\eta + \lambda)/(1 - \eta - \lambda)$, and $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| > 1$ for $\tau^c > (\eta + \lambda)/(1 - \eta - \lambda)$.

The elasticity of consumption implies that the consumption tax revenue is bounded if

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$\eta + \lambda < 1$ and it is unbounded if $\eta + \lambda > 1$. In the case of $\eta + \lambda = 1$, by (10), the following is obtained

$$\lim_{\tau^c \to \infty} \tau^c c = \frac{1}{\kappa(1 + \lambda)}.$$

□

C   Proof of Proposition 3

Proof. Note that

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 = \frac{1}{\lambda} \cdot \frac{1}{1 + \tau^c} \left[(1 - \lambda)\tau^c - \lambda \right].$$

Suppose that $\lambda = 1$. In this case, $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 < 0$ and the consumption tax revenue is monotonically increasing.

Suppose $\lambda \neq 1$. In this case,

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 = \left(\frac{1 - \lambda}{\lambda} \right) \left(\frac{1}{1 + \tau^c} \right) \left(\tau^c - \frac{\lambda}{1 - \lambda} \right).$$

If $\lambda > 1$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \leq 1$.

If $\lambda < 1$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \leq 1$ for $\tau^c \leq (\lambda)/(1 - \lambda)$, and $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| > 1$ for $\tau^c > (\lambda)/(1 - \lambda)$.

The elasticity of consumption implies that the consumption tax revenue is bounded if $\lambda < 1$ and it is unbounded if $\lambda > 1$. In the case of $\lambda = 1$, by (13), the following is obtained

$$\lim_{\tau^c \to \infty} \tau^c c = \frac{1}{\kappa(1 + \lambda)}.$$

□

D Elasticity of consumption with respect to consumption tax rate under general form of utility

Suppose the general utility function $U(c, n)$. Then, the consumption–labor choice condition is

$$-\frac{U_n}{U_c} = \frac{1}{1 + \tau^c} w.$$

In equilibrium, $w = 1$, and then,

$$-U_n = (1 + \tau^c)^{-1} U_c.$$

Taking the total derivative yields

$$-U_{cn}\frac{dc}{d\tau^c} - U_{nn}\frac{dn}{d\tau^c} = -(1 + \tau^c)^{-2} U_c + (1 + \tau^c)^{-1}(U_{cc}\frac{dc}{d\tau^c} + U_{cn}\frac{dn}{d\tau^c})$$

$$\iff -cU_{cn}\frac{dc/c}{d\tau^c/\tau^c} - nU_{nn}\frac{dn/n}{d\tau^c/\tau^c} = -\frac{\tau^c}{(1 + \tau^c)^2} U_c + \frac{1}{1 + \tau^c}\left(cU_{cc}\frac{dc/c}{d\tau^c/\tau^c} + nU_{cn}\frac{dn/n}{d\tau^c/\tau^c}\right).$$

By using $-U_n = (1 + \tau^c)^{-1} U_c$,

$$-cU_{cn}\frac{dc/c}{d\tau^c/\tau^c} - nU_{nn}\frac{dn/n}{d\tau^c/\tau^c} = \frac{\tau^c}{1 + \tau^c} U_n + \frac{1}{1 + \tau^c}\left(cU_{cc}\frac{dc/c}{d\tau^c/\tau^c} + nU_{cn}\frac{dn/n}{d\tau^c/\tau^c}\right)$$

$$\iff -\frac{cU_{cn}}{U_n}\frac{dc/c}{d\tau^c/\tau^c} - \frac{nU_{nn}}{U_n}\frac{dn/n}{d\tau^c/\tau^c} = \frac{\tau^c}{1 + \tau^c} + \frac{1}{1 + \tau^c}\left(\frac{cU_{cc}}{U_n}\frac{dc/c}{d\tau^c/\tau^c} + \frac{nU_{cn}}{U_n}\frac{dn/n}{d\tau^c/\tau^c}\right)$$

$$\iff -\frac{cU_{cn}}{U_n}\frac{dc/c}{d\tau^c/\tau^c} - \frac{nU_{nn}}{U_n}\frac{dn/n}{d\tau^c/\tau^c} = \frac{\tau^c}{1 + \tau^c} - \frac{cU_{cc}}{U_c}\frac{dc/c}{d\tau^c/\tau^c} - \frac{nU_{cn}}{U_c}\frac{dn/n}{d\tau^c/\tau^c}$$

$$\iff \left[-\frac{cU_{cc}}{U_c} + \frac{cU_{cn}}{U_n}\right]\frac{dc/c}{d\tau^c/\tau^c} + \left[\frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}\right]\frac{dn/n}{d\tau^c/\tau^c} = -\frac{\tau^c}{1 + \tau^c}.$$

(i) In the case in which the tax revenue is used as a lump-sum transfer, $c = n$, and then $\frac{dc/c}{d\tau^c/\tau^c} = \frac{dn/n}{d\tau^c/\tau^c}$. It follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = -\frac{\tau^c}{1 + \tau^c} \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}.$$

(ii) If the case in which the tax revenue is used as government spending $(1+\tau^c)c = n$,

$$\frac{dn}{d\tau^c} = \frac{d(1+\tau^c)c}{d\tau^c}$$
$$= (1+\tau^c)\frac{dc}{d\tau^c} + c,$$

and then,

$$n\frac{dn/n}{d\tau^c/\tau^c} = (1+\tau^c)c\frac{dc/c}{d\tau^c/\tau^c} + \tau^c c,$$
$$\Longleftrightarrow \quad \frac{dn/n}{d\tau^c/\tau^c} = (1+\tau^c)\left(\frac{c}{n}\right)\frac{dc/c}{d\tau^c/\tau^c} + \tau^c\left(\frac{c}{n}\right),$$
$$\Longleftrightarrow \quad \frac{dn/n}{d\tau^c/\tau^c} = \frac{dc/c}{d\tau^c/\tau^c} + \frac{\tau^c}{1+\tau^c}.$$

It follows that

$$\left[-\frac{cU_{cc}}{U_c} + \frac{cU_{cn}}{U_n}\right]\frac{dc/c}{d\tau^c/\tau^c} + \left[\frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}\right]\frac{dn/n}{d\tau^c/\tau^c} = -\frac{\tau^c}{1+\tau^c}$$
$$\Longleftrightarrow \quad \left[-\frac{cU_{cc}}{U_c} + \frac{cU_{cn}}{U_n}\right]\frac{dc/c}{d\tau^c/\tau^c} + \left[\frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}\right]\left[\frac{dc/c}{d\tau^c/\tau^c} + \frac{\tau^c}{1+\tau^c}\right] = -\frac{\tau^c}{1+\tau^c}$$
$$\Longleftrightarrow \quad \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c} + \frac{cU_{cn}}{U_n}\right]\frac{dc/c}{d\tau^c/\tau^c} = -\frac{\tau^c}{1+\tau^c}\left[1 + \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}\right].$$

Finally, the following is obtained

$$\frac{dc/c}{d\tau^c/\tau^c} = -\frac{\tau^c}{1+\tau^c}\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1} \times \left[1 + \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}\right].$$

E   Proof of Proposition 4

Proof. It is obvious that

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{\tau^c}{1+\tau^c} \leq 1$$

Since

$$c = \{[\eta(1+\lambda) + (1-\eta)]\kappa\}^{-\frac{1}{1+\lambda}}(1+\tau^c)^{-1},$$

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the limit of consumption tax revenue is given as

$$\lim_{\tau^c \to \infty} \tau^c c = \{[\eta(1+\lambda) + (1-\eta)]\kappa\}^{-\frac{1}{1+\lambda}}.$$

□

F Proof of Proposition 5

Proof. Note that

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\eta+\lambda} \cdot \frac{1}{1+\tau^c} [(1-\eta)\tau^c - (\eta+\lambda)].$$

Suppose that $\eta = 1$. In this case, $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 < 0$ and consumption tax revenue is monotonically increasing.

Suppose $\eta \neq 1$. In this case,

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \left(\frac{1-\eta}{\eta+\lambda}\right)\left(\frac{1}{1+\tau^c}\right)\left(\tau^c - \frac{\eta+\lambda}{1-\eta}\right).$$

If $\eta > 1$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \leq 1$.

If $\eta < 1$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq (\eta+\lambda)/(1-\eta)$, and $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > (\eta+\lambda)/(1-\eta)$.

The elasticity of consumption implies that consumption tax revenue is bounded if $\eta + \lambda < 1$ and it is unbounded if $\eta + \lambda > 1$. In the case of $\eta + \lambda = 1$, by (19), the following is obtained

$$\lim_{\tau^c \to \infty} \tau^c c = \frac{1}{\kappa(1+\lambda)}.$$

□

G Proof of Proposition 6

Proof. Note that

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\lambda} \cdot \frac{1}{1+\tau^c} [\tau^c - \lambda].$$

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Therefore, $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq \lambda$, and $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > \lambda$.

The elasticity of consumption implies that the consumption tax revenue is bounded. $\square$

H Equilibrium system of the dynamic model

The equilibrium system of the dynamic model is

$$(1 + \tau_t^c)\lambda_t = U_c(c_t, n_t),$$

$$\lambda_t(1 - \tau_t^n)w_t = -U_n(c_t, n_t),$$

$$\lambda_t = \beta E_t \left\{\lambda_{t+1}\left[(1 - \delta) + (1 - \tau_{t+1}^k)(d_{t+1} - \delta) + \delta\right]\right\},$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1}R_{t+1}^b\right],$$

$$k_t = (1 - \delta)k_{t-1} + x_t,$$

$$y_t = \xi^t \left[k_{t-1}\right]^\theta n_t^{1-\theta},$$

$$w_t = (1 - \theta)\frac{y_t}{n_t},$$

$$d_t = \theta\frac{y_t}{k_{t-1}},$$

$$y_t = c_t + x_t + g_t - m_t,$$

$$T_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k(d_t - \delta)k_{t-1},$$

where if the utility function is KPR $\mathbb{U}^{KPR}$, the marginal utility is defined by

$$U_c(c_t, n_t) \equiv (c_t)^{-\eta} \left[1 - \kappa(1 - \eta)n_t^{1+\lambda}\right]^\eta,$$

$$U_n(c_t, n_t) \equiv -\eta(1 + \lambda)\left\{(c_t)^{1-\eta}\left[1 - \kappa(1 - \eta)n_t^{1+\lambda}\right]^{\eta-1}\kappa n_t^\lambda\right\}$$

if the utility function is the additively separable $\mathbb{U}^{AS}$, the marginal utility is defined by

$$U_c(c_t, n_t) \equiv (c_t)^{-\eta},$$

$$U_n(c_t, n_t) \equiv -\kappa\psi^{t(1-\gamma)}(1 + \lambda)n_t^\lambda$$

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and if the utility function is GHH $\mathbb{U}^{GHH}$, the marginal utility is defined by

$$U_c(c_t, n_t) \equiv (c_t - \kappa \psi^t n_t^{1+\lambda})^{-\bar{\eta}},$$

$$U_n(c_t, n_t) \equiv -\kappa \psi^t (1 + \lambda) n_t^{\lambda} (c_t - \psi^t \kappa n_t^{1+\lambda})^{-\bar{\eta}}$$

Then, the equilibrium system is detrended by $\psi = \xi^{1/(1-\theta)}$, and $a_t/\psi^t \equiv \tilde{a}_t$ (except for $\tilde{k}_{t-1} \equiv k_{t-1}/\psi^t$ and λ). The detrended equilibrium system is

$$(1 + \tau_t^c)\tilde{\lambda}_t = U_c(\tilde{c}_t, n_t),$$

$$\tilde{\lambda}_t(1 - \tau_t^n)\tilde{w}_t = -U_n(\tilde{c}_t, n_t),$$

$$\tilde{\lambda}_t = \beta \psi^{-\eta} E_t \left\{ \tilde{\lambda}_{t+1} \left[(1 - \delta) + (1 - \tau_{t+1}^k)(d_{t+1} - \delta) + \delta \right] \right\},$$

$$\tilde{\lambda}_t = \beta \psi^{-\eta} E_t \left[\tilde{\lambda}_{t+1} R_{t+1}^b \right],$$

$$\psi \tilde{k}_t = (1 - \delta)\tilde{k}_{t-1} + \tilde{x}_t,$$

$$\tilde{y}_t = \left[\tilde{k}_{t-1} \right]^{\theta} n_t^{1-\theta},$$

$$\tilde{w}_t = (1 - \theta)\frac{\tilde{y}_t}{n_t},$$

$$d_t = \theta \frac{\tilde{y}_t}{\tilde{k}_{t-1}}.$$

$$\tilde{y}_t = \tilde{c}_t + \tilde{x}_t + \tilde{g}_t - \tilde{m}_t,$$

$$\tilde{T}_t = \tau_t^c \tilde{c}_t + \tau_t^n \tilde{w}_t n_t + \tau_t^k (d_t - \delta)\tilde{k}_{t-1}.$$

On the balanced growth path, the system becomes

$$(1 + \tau^c)\tilde{\lambda} = U_c(\tilde{c}, n),$$

$$\tilde{\lambda}(1 - \tau^n)\tilde{w} = -U_n(\tilde{c}, n),$$

$$1 = \beta\psi^{-\eta}\left[(1 - \delta) + (1 - \tau^k)(d - \delta) + \delta\right],$$

$$1 = \beta\psi^{-\eta}R^b,$$

$$\psi\tilde{k} = (1 - \delta)\tilde{k} + \tilde{x},$$

$$\tilde{y} = \left[\tilde{k}\right]^\theta n^{1-\theta},$$

$$\tilde{w} = (1 - \theta)\frac{\tilde{y}}{n},$$

$$d = \theta\frac{\tilde{y}}{\tilde{k}}.$$

$$\tilde{y} = \tilde{c} + \tilde{x} + \tilde{g} - \tilde{m}_t,$$

$$\tilde{T} = \tau_t^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d - \delta)\tilde{k}.$$

Scheme (3): Changes in tax revenue are adjusted by lump-sum transfer: Under Scheme (3), $\tilde{g}/\tilde{y} = \phi_g$ and $\tilde{m}/\tilde{y} = \phi_m$ are constant. Then, the balanced growth path values are obtained by

$$R^b = \frac{\psi^\eta}{\beta},$$

$$d = \frac{1}{1 - \tau^k}\left[R^b - 1 + \delta\right],$$

$$\frac{\tilde{k}}{\tilde{y}} = \frac{\theta}{d},$$

$$\frac{\tilde{x}}{\tilde{y}} = [\psi - (1 - \delta)]\frac{\tilde{k}}{\tilde{y}},$$

$$\frac{\tilde{c}}{\tilde{y}} = 1 - \frac{\tilde{x}}{\tilde{y}} - \frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{m}}{\tilde{y}},$$

$$\frac{n}{\tilde{y}} = \left[\frac{\tilde{y}}{\tilde{k}}\right]^{\theta/(1-\theta)},$$

$$\tilde{w} = (1 - \theta)\frac{\tilde{y}}{\tilde{n}},$$

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From this system, the following lemma and corollary are obtained from the balanced growth path equilibrium system.

Lemma 1. *On the balanced growth path, the dividend (d), capital–output ratio ($k/y = \tilde{k}/\tilde{y}$), investment–output ratio ($x/y = \tilde{x}/\tilde{y}$), consumption–output ratio ($c/y = \tilde{c}/\tilde{y}$), and labor–output ratio ($n/\tilde{y}$) are independent of the consumption tax rate (τ^c).*

Remark 1. *The elasticity of consumption with respect to the consumption tax rate equals that of output:*

$$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| = \left| \frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c} \right|.$$

Scheme (4): Changes in tax revenue are adjusted by government consumption:
By the budget constraint, the following is obtained

$$\tilde{T} = \tau^c \tilde{c} + \tau^n \tilde{w} n + \tau^k (d - \delta) \tilde{k}.$$

Dividing this equation by $\tilde{y}$ yields

$$\frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{s}}{\tilde{y}} + (R^b - 1)\frac{\tilde{b}}{\tilde{y}} = \tau^c \frac{\tilde{c}}{\tilde{y}} + \tau^n \tilde{w} \frac{\tilde{n}}{\tilde{y}} + \tau^k (d - \delta)\frac{\tilde{k}}{\tilde{y}}$$

Since $n/\tilde{y}$ and $\tilde{k}/\tilde{y}$ are independent of τ^c,

$$\frac{\tilde{g}}{\tilde{y}} = \tau^c \frac{\tilde{c}}{\tilde{y}} + constant.$$

The resource constraint can be rewritten as

$$c + i + g - m = y$$

$$\iff \quad \frac{\tilde{c}}{\tilde{y}} + \frac{\tilde{i}}{\tilde{y}} + \tau^c \frac{\tilde{c}}{\tilde{y}} + const - \frac{\tilde{m}}{\tilde{y}} = 1.$$

Since $\tilde{i}/\tilde{y}$ and $\tilde{m}/\tilde{y}$ is independent of τ^c,

$$(1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} = constant.$$

Therefore, the following lemma holds.

Lemma 2. $(1 + \tau^c)\tilde{c}/\tilde{y}$ *is independent of* τ^c

By Lemma 2, the following is obtained.

Remark 2. *The elasticity of consumption with respect to the consumption tax rate equals that of* $y/(1 + \tau^c)$*:*

$$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| = \left| \frac{d(1 + \tau^c)^{-1}\tilde{y}/(1 + \tau^c)^{-1}\tilde{y}}{d\tau^c/\tau^c} \right|$$

I   Proof of Proposition 7

Proof. The total tax revenue is

$$\tilde{T} = \tau^c \tilde{c} + \tau^n \tilde{w}n + \tau^k(d - \delta)\tilde{k}$$

$$= \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}} \right) + \tau^n(1 - \theta) + \tau^k(d - \delta)\left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \tilde{y}.$$

By Remark 1 of Appendix H, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left(\frac{\tilde{c}}{\tilde{y}} \right)\tilde{y} + \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}} \right) + \tau^n(1 - \theta) + \tau^k(d - \delta)\left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \frac{d\tilde{y}}{d\tau^c}.$$

The optimization condition for the consumption–labor choice,

$$\eta(1 + \lambda)\left\{ \frac{\kappa \tilde{c}n^\lambda}{1 - \kappa(1 - \eta)n^{1+\lambda}} \right\} = \frac{1 - \tau^n}{1 + \tau^c}(1 - \theta)\frac{\tilde{y}}{n},$$

yields

$$\eta(1 + \lambda)\left\{ \frac{\kappa \left(\frac{\tilde{c}}{\tilde{y}} \right)\left(\frac{n}{\tilde{y}} \right)^{1+\lambda} \tilde{y}^{1+\lambda}}{1 - \kappa(1 - \eta)\left(\frac{n}{\tilde{y}} \right)^{1+\lambda} \tilde{y}^{1+\lambda}} \right\} = \frac{1 - \tau^n}{1 + \tau^c}(1 - \theta),$$

and

$$\tilde{y} = \left(\frac{\tilde{y}}{n} \right)(\kappa)^{-1/(1+\lambda)}\left[(1 - \eta) + \frac{1}{1 - \theta}\left(\frac{\tilde{c}}{\tilde{y}} \right)\eta(1 + \lambda)\frac{1 + \tau^c}{1 - \tau^n} \right]^{-1/(1+\lambda)}.$$

Taking the first-order derivative of τ^c, the following is obtained

$$\frac{d\tilde{y}}{d\tau^c} = -\frac{1}{1-\theta}\left(\frac{\tilde{c}}{n}\right)\left(\frac{1}{1-\tau^n}\right)\eta\kappa^{-1/(1+\lambda)}\left[(1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\left(\frac{1+\tau^c}{1-\tau^n}\right)\right]^{-1/(1+\lambda)-1}.$$

Then,

$$\frac{d\tilde{T}}{d\tau^c} = \left(\frac{\tilde{c}}{n}\right)\kappa^{-1/(1+\lambda)}\left[(1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)(1+\tau^c)\right]^{-1/(1+\lambda)-1}\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\left(\frac{1}{1-\tau^n}\right)\eta\times$$
$$\left\{\tau^c\lambda - \left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) - \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n) - (1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)\right]\right\}.$$

If $\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) < \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n) + (1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)$, then $\frac{d\tilde{T}}{d\tau^c} > 0$.

If $\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) = \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n) + (1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)$, then $\frac{d\tilde{T}}{d\tau^c} \geq 0$.

If $\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) > \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n) + (1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)$,

then

$\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c < \frac{1}{\lambda}\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) - \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n) - (1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)\right]$,

$\frac{d\tilde{T}}{d\tau^c} = 0$ for $\tau^c = \frac{1}{\lambda}\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) - \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n) - (1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)\right]$, and

$\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c > \frac{1}{\lambda}\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right) - \frac{1-\eta}{\eta}(1-\theta)(1-\tau^n) - (1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)\right]$.

For the boundedness of the total tax revenue, see the proof on consumption tax revenue in Section B.3.1 of the appendix of Hiraga and Nutahara (2019).

□

J Proof of Proposition 8

Proof. The total tax revenue is

$$\tilde{T} = \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d-\delta)\tilde{k}$$
$$= \left[\tau^c\left(\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\tilde{y}.$$

By Remark 1 of Appendix H, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left(\frac{\tilde{c}}{\tilde{y}}\right)\tilde{y} + \left[\tau^c\left(\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\frac{d\tilde{y}}{d\tau^c}.$$

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By the optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)\tilde{c}^\eta n^\lambda = \frac{1-\tau^n}{1+\tau^c}\tilde{w},$$

it follows that

$$\kappa(1+\lambda)\left(\frac{\tilde{c}}{\tilde{y}}\right)^\eta\left(\frac{n}{\tilde{y}}\right)^\lambda \tilde{y}^{\eta+\lambda} = \frac{1-\tau^n}{1+\tau^c}(1-\theta)\left(\frac{\tilde{y}}{n}\right),$$

and

$$\tilde{y} = (1+\tau^c)^{-1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.$$

Taking the first-order derivatives of τ^c yields

$$\frac{d\tilde{y}}{d\tau^c} = -\frac{1}{\eta+\lambda}(1+\tau^c)^{-1/(\eta+\lambda)-1}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.$$

Then,

$$\frac{d\tilde{T}}{d\tau^c} = (1+\tau^c)^{-1/(\eta+\lambda)-1}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times$$

$$\left(\frac{\tilde{c}}{\tilde{y}}\right)\left\{1+\tau^c\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right) - \frac{1}{\eta+\lambda}\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}.$$

Suppose that $\eta+\lambda = 1$. Then,

$$\frac{d\tilde{T}}{d\tau^c} = (1+\tau^c)^{-1/(\eta+\lambda)-1}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times$$

$$\left(\frac{\tilde{c}}{\tilde{y}}\right)\frac{1}{\eta+\lambda}\left\{\eta+\lambda - \left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}.$$

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] < \eta+\lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$.
If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] = \eta+\lambda$, then $\frac{d\tilde{T}}{d\tau^c} \geq 0$.
If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] > \eta+\lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$.

Suppose that $\eta + \lambda \neq 1$. It follows that

$$\frac{d\tilde{T}}{d\tau^c} = (1 + \tau^c)^{-1/(\eta+\lambda)-1}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}\left(\frac{\tilde{c}}{\tilde{y}}\right)\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right)\times$$
$$\left[\tau^c - \frac{1}{\eta+\lambda-1}\left\{\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] - (\eta+\lambda)\right\}\right].$$

Suppose that $\eta + \lambda > 1$.

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] < \eta + \lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] = \eta + \lambda$, then $\frac{d\tilde{T}}{d\tau^c} = 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] > \eta + \lambda$,

then

$\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c < \frac{1}{\eta+\lambda-1}\left\{\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] - (\eta+\lambda)\right\}$,

$\frac{d\tilde{T}}{d\tau^c} = 0$ for $\tau^c = \frac{1}{\eta+\lambda-1}\left\{\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] - (\eta+\lambda)\right\}$, and

$\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c > \frac{1}{\eta+\lambda-1}\left\{\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] - (\eta+\lambda)\right\}$.

Suppose that $\eta + \lambda < 1$.

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] > \eta + \lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] = \eta + \lambda$, then $\frac{d\tilde{T}}{d\tau^c} = 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right] < \eta + \lambda$,

then

$\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c < \frac{1}{1-\eta-\lambda}\left\{(\eta+\lambda) - \left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}$,

$\frac{d\tilde{T}}{d\tau^c} = 0$ for $\tau^c = \frac{1}{1-\eta-\lambda}\left\{(\eta+\lambda) - \left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}$, and

$\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c > \frac{1}{1-\eta-\lambda}\left\{(\eta+\lambda) - \left(\frac{\tilde{y}}{\tilde{c}}\right)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\left(\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}$.

For the boundedness of the total tax revenue, see the proof on consumption tax revenue in Section B.3.1 in the appendix of Hiraga and Nutahara (2019). □

K   Proof of Proposition 9

Proof. The total tax revenue is

$$\tilde{T} = \tau^c \tilde{c} + \tau^n \tilde{w} n + \tau^k (d - \delta) \tilde{k}$$

$$= \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}} \right) + \tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \tilde{y}.$$

By Remark 1 of Appendix H, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left(\frac{\tilde{c}}{\tilde{y}} \right) \tilde{y} + \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}} \right) + \tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \frac{d\tilde{y}}{d\tau^c}.$$

By the optimization condition for the consumption–labor choice,

$$\kappa (1 + \lambda) n^\lambda = \frac{1 - \tau^n}{1 + \tau^c} \tilde{w},$$

it follows that

$$\kappa (1 + \lambda) \left(\frac{n}{\tilde{y}} \right)^\lambda \tilde{y}^\lambda = \frac{1 - \tau^n}{1 + \tau^c} (1 - \theta) \left(\frac{\tilde{y}}{n} \right),$$

and

$$\tilde{y} = (1 + \tau^c)^{-1/\lambda} \left[\frac{1 - \theta}{\kappa (1 + \lambda)} (1 - \tau^n) \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/\lambda}.$$

Taking the first-order derivatives of τ^c yields

$$\frac{d\tilde{y}}{d\tau^c} = -\frac{1}{\lambda} (1 + \tau^c)^{-1/\lambda - 1} \left[\frac{1 - \theta}{\kappa (1 + \lambda)} (1 - \tau^n) \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/\lambda}.$$

Then,

$$\frac{d\tilde{T}}{d\tau^c} = (1 + \tau^c)^{-1/\lambda - 1} \left[\frac{1 - \theta}{\kappa (1 + \lambda)} (1 - \tau^n) \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/\lambda} \times$$

$$\left(\frac{\tilde{c}}{\tilde{y}} \right) \left\{ 1 + \tau^c \left(\frac{\lambda - 1}{\lambda} \right) - \frac{1}{\lambda} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \right\}.$$

Suppose that $\lambda = 1$. Then,

$$\frac{d\tilde{T}}{d\tau^c} = (1 + \tau^c)^{-1/\lambda - 1} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/\lambda} \times$$

$$\left(\frac{\tilde{c}}{\tilde{y}} \right) \left\{ 1 - \frac{1}{\lambda} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \right\}.$$

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] < \lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] = \lambda$, then $\frac{d\tilde{T}}{d\tau^c} \geq 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] > \lambda$, then $\frac{d\tilde{T}}{d\tau^c} < 0$.

Suppose that $\lambda \neq 1$. It follows that

$$\frac{d\tilde{T}}{d\tau^c} = (1 + \tau^c)^{-1/\lambda - 1} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/\lambda} \times$$

$$\left(\frac{\tilde{c}}{\tilde{y}} \right) \left(\frac{\lambda - 1}{\lambda} \right) \left\{ \tau^c - \frac{1}{\lambda - 1} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] + \frac{\lambda}{\lambda - 1} \right\}.$$

Suppose that $\lambda > 1$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] < \lambda$, then $\frac{dT}{d\tau^c} > 0$ for $\tau^c \geq 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] > \lambda$,

then

$\frac{dT}{d\tau^c} < 0$ for $\tau^c < \frac{1}{\lambda - 1} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - \frac{\lambda}{\lambda - 1}$,

$\frac{dT}{d\tau^c} = 0$ for $\tau^c = \frac{1}{\lambda - 1} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - \frac{\lambda}{\lambda - 1}$, and

$\frac{dT}{d\tau^c} > 0$ for $\tau^c > \frac{1}{\lambda - 1} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - \frac{\lambda}{\lambda - 1}$.

Suppose that $\lambda < 1$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] > \lambda$, then $\frac{dT}{d\tau^c} < 0$ for $\tau^c \geq 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] < \lambda$,

then

$\frac{dT}{d\tau^c} > 0$ for $\tau^c < \frac{1}{\lambda - 1} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - \frac{\lambda}{\lambda - 1}$,

$\frac{dT}{d\tau^c} = 0$ for $\tau^c = \frac{1}{\lambda - 1} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - \frac{\lambda}{\lambda - 1}$, and

$\frac{dT}{d\tau^c} < 0$ for $\tau^c > \frac{1}{\lambda - 1} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - \frac{\lambda}{\lambda - 1}$.

It is obvious that the total tax revenue is bounded.

$\square$

L   Proof of Proposition 10

Proof. The total tax revenue is

$$\tilde{T} = \tau^c \tilde{c} + \tau^n \tilde{w} n + \tau^k (d - \delta) \tilde{k}$$

$$= \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}} \right) + \tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \tilde{y}.$$

By the consumption–labor choice condition, $\tilde{y}$ is obtained

$$\eta (1 + \lambda) \left\{ \frac{\kappa \tilde{c} n^\lambda}{1 - \kappa (1 - \eta) n^{1+\lambda}} \right\} = \frac{1 - \tau^n}{1 + \tau^c} (1 - \theta) \frac{\tilde{y}}{h}$$

$$\iff \quad \eta (1 + \lambda) \left\{ \frac{\kappa \left(\frac{\tilde{c}}{\tilde{y}} \right) \left(\frac{n}{\tilde{y}} \right)^\lambda}{\tilde{y}^{-1-\lambda} - \kappa (1 - \eta) \left(\frac{n}{\tilde{y}} \right)^{1+\lambda}} \right\} = \frac{1 - \tau^n}{1 + \tau^c} (1 - \theta) \frac{\tilde{y}}{n}$$

$$\iff \quad \tilde{y} = \left(\frac{\tilde{y}}{n} \right) (\kappa)^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left((1 + \tau^c) \frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \frac{1}{1 - \tau^n} \right]^{-1/(1+\lambda)}.$$

Then, $\tilde{y}$ is independent of τ^c. Therefore, the shape of the total tax revenue curve is the same as that of the consumption tax revenue curve.

Now, the following is obtained

$$(1 + \tau^c)^{-1} \tilde{y} = (1 + \tau^c)^{-1} \left(\frac{\tilde{y}}{n} \right) (\kappa)^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left((1 + \tau^c) \frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \frac{1 + \tau^c}{1 - \tau^n} \right]^{-1/(1+\lambda)},$$

and then,

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d(1 + \tau^c)^{-1} \tilde{y}/(1 + \tau^c)^{-1} \tilde{y}}{d\tau^c/\tau^c}$$

$$= -\frac{\tau^c}{1 + \tau^c}.$$

$\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is increasing in τ^c. If $\tau^c = 0$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = 0$. If $\tau^c \to \infty$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \to$ 1. Therefore, the consumption tax revenue curve and the total tax revenue curve are monotonically increasing.

The boundedness of tax revenue is proved as follows.

Letting $\phi = (1 + \tau^c)\tilde{c}/\tilde{y}$, consumption tax revenue is given by

$$\tau^c c = \tau^c \phi \tilde{y}(1 + \tau^c)^{-1}$$

$$= \phi \frac{\tau^c}{1 + \tau^c}\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1 - \eta) + \frac{1}{1 - \theta}\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\eta(1 + \lambda)\frac{1 + \tau^c}{1 - \tau^n}\right]^{-1/(1+\lambda)}$$

This converges to

$$\phi\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1 - \eta) + \frac{1}{1 - \theta}\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\eta(1 + \lambda)\frac{1 + \tau^c}{1 - \tau^n}\right]^{-1/(1+\lambda)}$$

as $\tau^c \to \infty$.

$\square$

M   Proof of Proposition 11

Proof. By the consumption–labor choice condition,

$$\kappa(1 + \lambda)\tilde{c}^\eta n^\lambda = \frac{1 - \tau^n}{1 + \tau^c}\tilde{w}$$

$$\Longleftrightarrow \quad \kappa(1 + \lambda)\tilde{y}^{\eta+\lambda}\left(\frac{\tilde{c}}{\tilde{y}}\right)^\eta\left(\frac{n}{\tilde{y}}\right)^\lambda = \frac{1 - \tau^n}{1 + \tau^c}(1 - \theta)\frac{\tilde{y}}{n}$$

$$\Longleftrightarrow \quad \tilde{y} = (1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)}\left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.$$

The total tax revenue is given as

$$\tilde{T} = \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d - \delta)\tilde{k}$$

$$= \left[\tau^c\frac{\tilde{c}}{\tilde{y}} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]\tilde{y}$$

$$= \left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{\tau^c}{1 + \tau^c} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]\tilde{y}$$

$$= \left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{\tau^c}{1 + \tau^c} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]$$

$$\times (1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)}\left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.$$

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Using the fact that $\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)$ is independent of τ^c, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{1}{(1 + \tau^c)^2}(1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)}$$

$$\times \left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}$$

$$-\frac{1-\eta}{\eta+\lambda}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{\tau^c}{1+\tau^c} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]$$

$$\times (1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)-1}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}$$

$$= (1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)-2}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times$$

$$\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) - \frac{1-\eta}{\eta+\lambda}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{\tau^c}{1+\tau^c} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right](1 + \tau^c)\right]$$

$$= (1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)-2}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times$$

$$\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) - \frac{1-\eta}{\eta+\lambda}\left\{\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\tau^c + (1 + \tau^c)\left[\tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]\right\}\right]$$

$$= (1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)-2}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times$$

$$\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \frac{\eta-1}{\eta+\lambda}\left\{\tau^c\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right] + \tau^n\theta + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right\}\right].$$

Suppose that $\eta = 1$, then $\frac{d\tilde{T}}{d\tau^c} > 0$.

Suppose $\eta \neq 1$. In this case, the following is obtained

$$\frac{d\tilde{T}}{d\tau^c} = (1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)-2}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times$$

$$\left[\frac{\eta-1}{\eta+\lambda}\right]\left[\frac{\eta+\lambda}{\eta-1}\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^c\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right] + \tau^n\theta + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]$$

$$= (1 + \tau^c)^{-(1-\eta)/(\eta+\lambda)-2}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)} \times$$

$$\left[\frac{\eta-1}{\eta+\lambda}\right]\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right][\tau^c - \Omega],$$

where

$$\Omega = \frac{\frac{\eta+\lambda}{1-\eta}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) - \tau^n(1-\theta) - \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}{\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}.$$

Suppose that $\eta > 1$. This implies that $\Omega < 0$, and then, $\frac{d\tilde{T}}{d\tau^c} > 0$.

Suppose that $\eta < 1$.

If $\Omega > 0$, then

$\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c < \Omega$,

$\frac{d\tilde{T}}{d\tau^c} = 0$ for $\tau^c = \Omega$, and

$\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c > \Omega$.

If $\Omega \le 0$, then $\frac{d\tilde{T}}{d\tau^c} \le 0$.

Finally, $\Omega > 0$ if and only if

$$\frac{\eta+\lambda}{1-\eta}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) > \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}$$

$$\Longleftrightarrow \quad (\eta+\lambda)\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) > (1-\eta)\left[\tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}\right]$$

$$\Longleftrightarrow \quad \eta\left[\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}\right] > -\lambda\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}$$

$$\Longleftrightarrow \quad \eta > \frac{-\lambda\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}{\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n(1-\theta) + \tau^k(d-\delta)\frac{\tilde{k}}{\tilde{y}}}.$$

For the boundedness of tax revenue, see Section B.3.2 of the appendix of Hiraga and Nutahara (2019). □

N   Proof of Proposition 12

Proof. By the consumption–labor choice condition,

$$\kappa(1 + \lambda)n^\lambda = \frac{1 - \tau^n}{1 + \tau^c}\tilde{w}$$

$$\Longleftrightarrow \quad \kappa(1 + \lambda)\tilde{y}^\lambda \left(\frac{n}{\tilde{y}}\right)^\lambda = \frac{1 - \tau^n}{1 + \tau^c}(1 - \theta)\frac{\tilde{y}}{n}$$

$$\Longleftrightarrow \quad \tilde{y} = (1 + \tau^c)^{-1/\lambda}\left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}.$$

The total tax revenue is given by

$$\tilde{T} = \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d - \delta)\tilde{k}$$

$$= \left[\tau^c\frac{\tilde{c}}{\tilde{y}} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]\tilde{y}$$

$$= \left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{\tau^c}{1 + \tau^c} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]\tilde{y}$$

$$= \left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{\tau^c}{1 + \tau^c} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right] \times (1 + \tau^c)^{-1/\lambda}\left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}.$$

Using the fact that $\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)$ is independent of τ^c, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{1}{(1 + \tau^c)^2}(1 + \tau^c)^{-1/\lambda}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}$$

$$- \frac{1}{\lambda}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{\tau^c}{1 + \tau^c} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right](1 + \tau^c)^{-1/\lambda-1}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}$$

$$= (1 + \tau^c)^{-1/\lambda-2}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda} \times$$

$$\left\{\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) - \frac{1}{\lambda}(1 + \tau^c)\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\frac{\tau^c}{1 + \tau^c} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right]\right\}$$

$$= (1 + \tau^c)^{-1/\lambda-2}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda} \times$$

$$\left\{\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) - \frac{1}{\lambda}\left[\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\tau^c + (1 + \tau^c)\left(\tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right)\right]\right\}$$

$$= (1 + \tau^c)^{-1/\lambda-2}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}\left(\frac{1}{\lambda}\right) \times$$

$$\left\{\lambda\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right) - \left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\tau^c - (1 + \tau^c)\left(\tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right)\right\}$$

$$= (1 + \tau^c)^{-1/\lambda-2}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}\left(\frac{1}{\lambda}\right) \times$$

$$\left\{\lambda(1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} - \tau^n(1 - \theta) - \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}} - \left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right)\tau^c\right\}$$

$$= -(1 + \tau^c)^{-1/\lambda-2}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}\left(\frac{1}{\lambda}\right) \times$$

$$\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}\right)(\tau^c - \Sigma),$$

where

$$\Sigma = \frac{\lambda(1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} - \tau^n(1 - \theta) - \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}}{(1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} + \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}}.$$

If $\Sigma \leq 0$, then $\frac{dT}{d\tau^c} \leq 0$.

If $\Sigma > 0$, then

$\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c < \Sigma$,

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$\frac{d\tilde{T}}{d\tau^c} = 0$ for $\tau^c = \Sigma$, and

$\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c > \Sigma$.

Finally, $\Sigma > 0$ if and only if

$$\lambda(1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} > \tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}$$

$$\iff \quad \lambda > \frac{\tau^n(1 - \theta) + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}}{(1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}}.$$

It is obvious that the total tax revenue is bounded. □

O   Procedure for numerical calculations

Given the steady-state labor supply $n = 0.2$, the parameter of disutility of labor, κ, is calibrated as follows. First, the steady-state values are calculated by

$$d = \frac{1}{1 - \tau^k}\left[\frac{\psi^\eta}{\beta} - 1\right] + \delta,$$

$$\frac{\tilde{k}}{\tilde{y}} = \frac{\theta}{d},$$

$$\frac{\tilde{x}}{\tilde{y}} = [\psi - (1 - \delta)]\frac{\tilde{k}}{\tilde{y}},$$

$$\frac{\tilde{c}}{\tilde{y}} = 1 - \frac{\tilde{x}}{\tilde{y}} - \frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{m}}{\tilde{y}},$$

$$\frac{n}{\tilde{y}} = \left[\frac{\tilde{k}}{\tilde{y}}\right]^{-\theta/(1-\theta)},$$

$$\tilde{y} = n \times \left(\frac{n}{\tilde{y}}\right)^{-1}.$$

If the utility is KPR $\mathbb{U}^{KPR}$, κ is given by

$$\kappa = \tilde{y}^{-(1+\lambda)}\left(\frac{\tilde{y}}{n}\right)^{1+\lambda}\left[(1 - \eta) + \frac{1}{1 - \theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1 + \lambda)\frac{1 + \tau^c}{1 - \tau^n}\right]^{-1}.$$

If the utility is additively separable $\mathbb{U}^{AS}$, κ is given by

$$\kappa = \frac{1 - \theta}{\tilde{y}^{\eta+\lambda}(1 + \lambda)}\frac{1 - \tau^n}{1 + \tau^c}\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}.$$

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If the utility is GHH $\mathbb{U}^{GHH}$, κ is given by

$$\kappa = \frac{1 - \theta}{\tilde{y}^{\lambda}(1 + \lambda)} \frac{1 - \tau^n}{1 + \tau^c} \left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}.$$

Given the value of κ, if the utility is KPR $\mathbb{U}^{KPR}$, the output is given by

$$\tilde{y} = \left(\frac{\tilde{y}}{n}\right) \kappa^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}}\right) \eta(1 + \lambda) \frac{1 + \tau^c}{1 - \tau^n}\right]^{-1/(1+\lambda)}.$$

If the utility is additively separable $\mathbb{U}^{AS}$, the output is given by

$$\tilde{y} = (1 + \tau^c)^{-1/(\eta+\lambda)} \left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n) \left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta} \left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.$$

If the utility is GHH $\mathbb{U}^{GHH}$, the output is given by

$$\tilde{y} = (1 + \tau^c)^{-1/\lambda} \left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n) \left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}.$$

The associated capital stock and consumption are

$$\tilde{k} = \frac{\tilde{k}}{\tilde{y}} \times \tilde{y}, \quad \text{and} \quad \tilde{c} = \frac{\tilde{c}}{\tilde{y}} \times \tilde{y},$$

respectively. Finally, the total tax revenue is given by

$$T = \tau^c \tilde{c} + \tau^n \tilde{w} n + \tau^k (d - \delta) \tilde{k}$$

$$= \tau^c \tilde{c} + \tau^n (1 - \theta) \tilde{y} + \tau^k (d - \delta) \tilde{k}.$$

Table 1: Main results on tax revenue curve for consumption tax in the static model

|		KPR	Additively separable	GHH
(1)	Hump-shaped revenue curve?	NO	YES iff $\eta + \lambda < 1$	YES iff $\lambda < 1$
	Bounded revenue?	NO	YES iff $\eta + \lambda \leq 1$	YES iff $\lambda \leq 1$
(2)	Hump-shaped revenue curve?	NO	YES iff $\eta < 1$	YES
	Bounded revenue?	YES	YES iff $\eta \leq 1$	YES

Note: (1) The case in which the tax revenue is used as lump-sum transfer; (2) the case in which tax revenue is used as government spending. The parameter η is the RRA, and the parameter λ is the inverse of the labor supply elasticity.

Figure 1: Numerical example of the total tax revenue curve for consumption tax: $\eta = 2$ and $\lambda = 1/2$
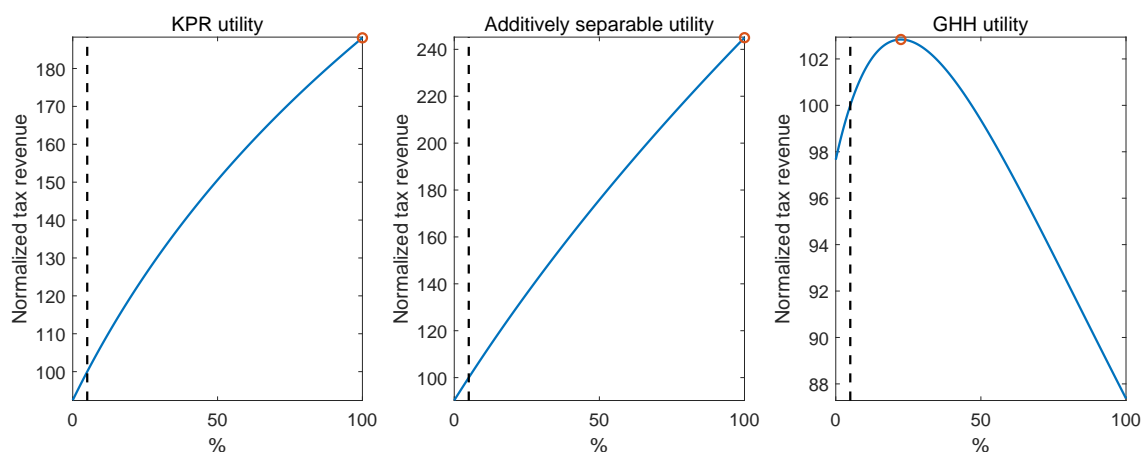
Note: The horizontal axes show consumption tax rates. The vertical axes show total tax revenue, which is normalized to 100 at the baseline tax rates ($\tau^c = 5\%$).

Figure 2: Numerical examples of the consumption tax revenue curve for consumption tax
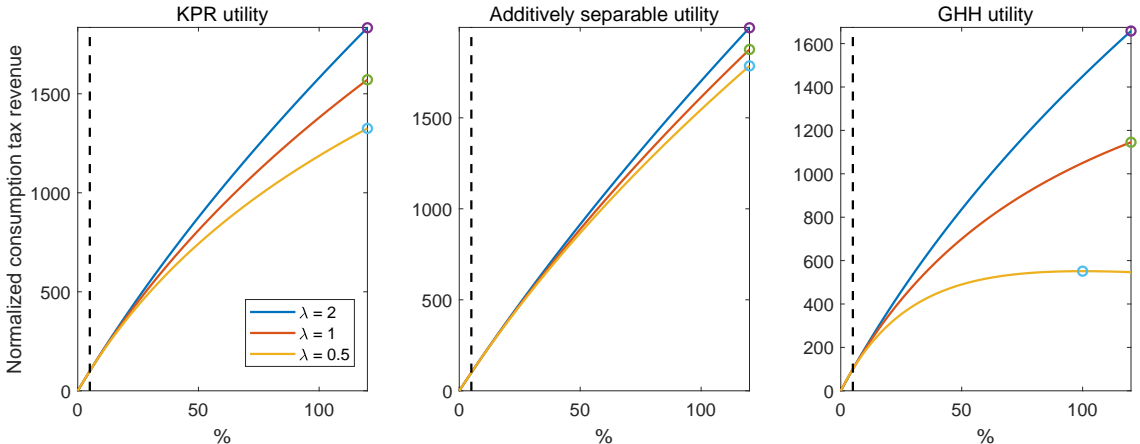
Note: The horizontal axes show consumption tax rates. The vertical axes show consumption tax revenue, which is normalized to 100 at the baseline tax rates ($\tau^c = 5\%$).

Appendices: Fragility in modeling consumption tax revenue

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February 24, 2019

Abstract

This appendix contains supplemental results on "Fragility in modeling consumption tax revenue" by Kazuki Hiraga and Kengo Nutahara.

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Contents

A   Static economy

A.1   Static model with consumption and labor income taxes

Representative households supply labor n to firms and earn wage rate w. They also receive government transfers s. Let τ^c and τ^n denote consumption and labor income taxes, respectively. The budget constraint of households is

$$(1 + \tau^c)\, c \leq (1 - \tau^n)wn + s,$$

where c denotes consumption.

The firms are perfectly competitive. Their production function is

$$y = n,$$

where y denotes output.

The government budget constraint is

$$s + g = T,$$

where g is government consumption. Total tax revenue T is defined as

$$T = \tau^c c + \tau^n wn.$$

Since there is no investment, the resource constraint of this closed economy is

$$y = c + g.$$

Three types of utility functions are considered.

$$U^{KPR} = \frac{1}{1-\eta}\left\{ c^{1-\eta}\left[1 - \kappa(1-\eta)n^{1+\lambda}\right]^{\eta} - 1 \right\}.$$

$$U^{AS} = \frac{c^{1-\eta} - 1}{1 - \eta} - \kappa n^{1+\lambda}.$$

$$U^{GG} = \frac{1}{1-\eta}\left\{ \left(c - \kappa n^{1+\lambda}\right)^{1-\eta} - 1 \right\}.$$

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A.2 Total tax revenue curve for consumption tax

In the main text, the consumption tax revenue curve is investigated. Here, we investigate the total tax revenue curve, which also includes labor income tax revenue.

In the case of the total tax revenue curve, fiscal policy schemes are the following.

Definition A. 1. *Scheme (1′): Tax revenue is used as a lump-sum transfer to households.*

$$s = \tau^c c + \tau^n wn, \qquad g = 0$$

Definition A. 2. *Scheme (2′): Tax revenue is used as government consumption.*

$$g = \tau^c c + \tau^n wn, \qquad s = 0$$

A.2.1 Scheme (1′): Tax revenue is used as a lump-sum transfer

Propositions A.1, A.2, and A.3 are the analogues of Propositions 1, 2, and 3 in the main text.

Proposition A. 1. *Suppose that the utility function is KPR; U^{KPR}. The total tax revenue curve for consumption tax under Scheme (1′) is monotonically increasing. The total tax revenue curve is unbounded except for $\lambda = 0$.*

Proof. By the optimization condition for the consumption–labor choice,

$$\eta(1 + \lambda)\left(\frac{\kappa c n^\lambda}{1 - \kappa(1 - \eta)n^{1+\lambda}}\right) = \frac{1 - \tau^n}{1 + \tau^c}w,$$

it follows that

$$c = (1 - \tau^n)^{1/(1+\lambda)}\left[\tau^c \eta\kappa(1 + \lambda) + \kappa(\eta\lambda + 1) - \tau^n\kappa(1 - \eta)\right]^{-1/(1+\lambda)}.$$

The total tax revenue is

$$T = \tau^c c + \tau^n wn$$

$$= (\tau^c + \tau^n)(1 - \tau^n)^{1/(1+\lambda)}\left[\tau^c \eta\kappa(1 + \lambda) + \kappa(\eta\lambda + 1) - \tau^n\kappa(1 - \eta)\right]^{-1/(1+\lambda)}.$$

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Then,

$$
\frac{dT}{d\tau^c} = (1 - \tau^n)^{1/(1+\lambda)} \left[\tau^c \eta \kappa (1 + \lambda) + \kappa(\eta\lambda + 1) - \tau^n \kappa(1 - \eta) \right]^{-1/(1+\lambda)-1}
$$

$$
\times \left[\tau^c \eta \kappa \lambda + \kappa(\eta\lambda + 1 - \tau^n) \right] > 0.
$$

Since the consumption tax revenue is bounded if and only if $\lambda = 0$, as in Proposition 1 in the main text, the total tax revenue is also bounded if and only if $\lambda = 0$. □

Proposition A. 2. *Suppose that the utility function is additively separable, U^{AS}. The total tax revenue curve for consumption tax under Scheme (1′) is hump shaped if and only if $\tau^n < \eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta + \lambda - \tau^n}{1 - \eta - \lambda}$. If $\eta + \lambda \leq \tau^n < 1$, the total tax revenue curve for consumption tax is monotonically decreasing. Otherwise, the total tax revenue curve for consumption tax is monotonically increasing. The total tax revenue is bounded if and only if $\eta + \lambda \leq 1$. Otherwise, it is unbounded.*

Proof. The optimization condition for the consumption–labor choice,

$$
\kappa(1 + \lambda)c^\eta n^\lambda = \frac{1 - \tau^n}{1 + \tau^c} w,
$$

indicates that

$$
c = \left[\frac{\kappa(1 + \lambda)}{1 - \tau^n}(1 + \tau^c) \right]^{-\frac{1}{\eta+\lambda}}.
$$

The total tax revenue is

$$
T = \tau^c c + \tau^n wn
$$

$$
= (\tau^c + \tau^n) \left[\frac{\kappa}{1 - \tau^n}(1 + \tau^c) \right]^{-\frac{1}{\eta+\lambda}},
$$

and thus,

$$
\frac{dT}{d\tau^c} = \left[\frac{\kappa(1 + \lambda)}{1 - \tau^n}(1 + \tau^c) \right]^{-\frac{1}{\eta+\lambda}-1} \left(\frac{\kappa(1 + \lambda)}{1 - \tau^n} \right) \left[\tau^c \left(\frac{\eta + \lambda - 1}{\eta + \lambda} \right) + \frac{\eta + \lambda - \tau^n}{\eta + \lambda} \right].
$$

Suppose that $\eta + \lambda = 1$; then, $\frac{dT}{d\tau^c} > 0$.

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Suppose that $\eta + \lambda \neq 1$; then,

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}-1}\left(\frac{\kappa(1+\lambda)}{1-\tau^n}\right)\left(\frac{\eta+\lambda-1}{\eta+\lambda}\right)\left[\tau^c - \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}\right].$$

Suppose that $\eta + \lambda > 1$; then, $\frac{dT}{d\tau^c} > 0$.

Suppose that $\eta + \lambda < 1$.

$\frac{dT}{d\tau^c} > 0$ for $\tau^c < \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$,

$\frac{dT}{d\tau^c} = 0$ for $\tau^c = \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$, and

$\frac{dT}{d\tau^c} < 0$ for $\tau^c > \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$.

Since the consumption tax revenue is bounded if and only if $\eta + \lambda \leq 1$, as in Proposition 2, the total tax revenue is also bounded if and only if $\eta + \lambda \leq 1$. $\qquad\square$

Proposition A. 3. *Suppose that the utility function is additively separable, U^{GHH}. The total tax revenue curve for consumption tax under Scheme (1′) is hump shaped if and only if $\tau^n < \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\lambda-\tau^n}{1-\lambda}$. If $\lambda \leq \tau^n < 1$, the total tax revenue curve for consumption tax is monotonically decreasing. Otherwise, the total tax revenue curve for consumption tax is monotonically increasing. The total tax revenue is bounded if and only if $\lambda \leq 1$. Otherwise, it is unbounded.*

Proof. The optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)n^\lambda = \frac{1-\tau^n}{1+\tau^c}w,$$

indicates that

$$c = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\lambda}}.$$

The total tax revenue is

$$T = \tau^c c + \tau^n wn$$

$$= (\tau^c + \tau^n)\left[\frac{\kappa}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\lambda}},$$

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and thus,

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\lambda}-1}\left(\frac{\kappa(1+\lambda)}{1-\tau^n}\right)\left[\tau^c\left(\frac{\lambda-1}{\lambda}\right)+\frac{\lambda-\tau^n}{\lambda}\right].$$

Suppose that $\lambda = 1$; then, $\frac{dT}{d\tau^c} > 0$.

Suppose that $\lambda \neq 1$; then,

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\lambda}-1}\left(\frac{\kappa(1+\lambda)}{1-\tau^n}\right)\left(\frac{\lambda-1}{\lambda}\right)\left[\tau^c - \frac{\lambda-\tau^n}{1-\lambda}\right].$$

Suppose that $\lambda > 1$; then, $\frac{dT}{d\tau^c} > 0$.

Suppose that $\lambda < 1$.

$\frac{dT}{d\tau^c} > 0$ for $\tau^c < \frac{\lambda-\tau^n}{1-\lambda}$,

$\frac{dT}{d\tau^c} = 0$ for $\tau^c = \frac{\lambda-\tau^n}{1-\lambda}$, and

$\frac{dT}{d\tau^c} < 0$ for $\tau^c > \frac{\lambda-\tau^n}{1-\lambda}$.

Since the consumption tax revenue is bounded if and only if $\lambda \leq 1$, as in Proposition 3 of the main text, the total tax revenue is also bounded if and only if $\lambda \leq 1$. $\qquad\square$

As for the consumption tax revenue curve, the total tax revenue curve for consumption tax is monotonically increasing. in the case of the KPR utility function. In the case of the additively separable utility function U^{AS}, the condition $\eta + \lambda < 1$ is necessary for a hump-shaped total tax revenue curve for consumption tax. Note that the total tax revenue curve might be monotonically decreasing if the labor income tax rate is sufficiently high ($\eta + \lambda \leq \tau^n$). This is interpreted as the case in which there is a negative peak consumption tax rate that maximizes the total tax revenue ($\tau^c = \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$) of the hump-shaped total tax revenue curve. In the case of GHH utility, the condition $\lambda < 1$ is still necessary for a hump-shaped tax revenue curve. The total tax revenue curve also might be monotonically decreasing if the labor income tax rate is sufficiently high ($\lambda \leq \tau^n$), since the peak tax rate is negative.

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A.2.2 Scheme (2′): Tax revenue is used as government consumption

Under Scheme (2′), since $g = \tau^c c + \tau^n wn$ and $n = c + g$, the following is obtained:

$$n = \frac{1 + \tau^c}{1 - \tau^n} c.$$

Propositions A.4, A.5, and A.6 are analogues of Propositions 4, 5, and 6 in the main text.

Proposition A. 4. *Suppose that the utility function is KPR, U^{KPR}. The total tax revenue curve for consumption tax under Scheme (2′) is monotonically increasing. The total tax revenue is bounded.*

Proof. By the optimization condition for the consumption–labor choice, the following is obtained:

$$c = \{[\eta(1 + \lambda) + (1 - \eta)]\kappa\}^{-\frac{1}{1+\lambda}} (1 - \tau^n)(1 + \tau^c)^{-1}.$$

Then,

$$n = \frac{1 + \tau^c}{1 - \tau^n} c$$
$$= \{[\eta(1 + \lambda) + (1 - \eta)]\kappa\}^{-\frac{1}{1+\lambda}}$$

This implies that labor supply is independent from both τ^c and τ^n. The total tax revenue is

$$T = \tau^c c + \tau^n wn$$
$$= \tau^c c + \tau^n \{[\eta(1 + \lambda) + (1 - \eta)]\kappa\}^{-\frac{1}{1+\lambda}}$$

Therefore, the shape of the total tax revenue curve is the same as that of the consumption tax revenue curve.

Since the consumption tax revenue is bounded if and only if $\lambda = 0$, as in Proposition 4 in the main text, the total tax revenue is also bounded if and only if $\lambda = 0$. □

Proposition A. 5. *Suppose that the utility function is additively separable, U^{AS}. The total tax revenue curve for consumption tax under Scheme (2′) is hump shaped if and only if*

$$(1 + \lambda)\tau^n - \lambda < \eta < 1,$$

and the revenue is maximized at $\tau^c = \frac{(1+\lambda)\tau^n - (\eta+\lambda)}{\eta - 1}$.
Otherwise, the total tax revenue curve is

- *monotonically increasing if $\eta > 1$ and $(1 + \lambda)\tau^n - \lambda \leq \eta$;*

- *U shaped if $\eta > 1$ and $(1 + \lambda)\tau^n - \lambda > \eta$;*

- *monotonically decreasing if $\eta = 1$ and $(1 + \lambda)\tau^n - \lambda > \eta$;*

- *monotonically increasing if $\eta = 1$ and $(1 + \lambda)\tau^n - \lambda < \eta$;*

- *flat if $\eta = 1$ and $(1 + \lambda)\tau^n - \lambda = \eta$; and*

- *monotonically decreasing if $\eta < 1$ and $(1 + \lambda)\tau^n - \lambda \geq \eta$.*

The total tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded. Otherwise, the total tax revenue curve for consumption tax is monotonically increasing. The total tax revenue is bounded if and only if $\eta + \lambda \leq 1$. Otherwise, it is unbounded.

Proof. The optimization condition for the consumption–labor choice,

$$\kappa(1 + \lambda)c^\eta n^\lambda = \frac{1 - \tau^n}{1 + \tau^c}w,$$

and $n = (1 + \tau^c)/(1 - \tau^n)c$ indicate that

$$c = [\kappa(1 + \lambda)]^{-\frac{1}{\eta+\lambda}} (1 - \tau^n)^{\frac{1+\lambda}{\eta+\lambda}}(1 + \tau^c)^{-\frac{1+\lambda}{\eta+\lambda}}.$$

The total tax revenue is

$$T = \tau^c c + \tau^n wn$$

$$= [\kappa(1 + \lambda)]^{-\frac{1}{\eta+\lambda}} (1 - \tau^n)^{\frac{1+\lambda}{\eta+\lambda}}(1 + \tau^c)^{-\frac{1+\lambda}{\eta+\lambda}} \left[\frac{\tau^c + \tau^n}{1 - \tau^n} \right]$$

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and thus,

$$\frac{dT}{d\tau^c} = [\kappa(1+\lambda)]^{-\frac{1}{\eta+\lambda}} (1-\tau^n)^{\frac{1+\lambda}{\eta+\lambda}} (1+\tau^c)^{-\frac{1+\lambda}{\eta+\lambda}-1} \left[\frac{1}{1-\tau^n}\right] \times$$

$$\left\{\left[\frac{\eta-1}{\eta+\lambda}\right]\tau^c - \frac{1+\lambda}{\eta+\lambda}\tau^n + 1\right\}$$

Suppose that $\eta = 1$.

If $\tau^n < \frac{\eta+\lambda}{1+\lambda}$, then $dT/d\tau^c > 0$.

If $\tau^n > \frac{\eta+\lambda}{1+\lambda}$, then $dT/d\tau^c < 0$.

If $\tau^n = \frac{\eta+\lambda}{1+\lambda}$, then $dT/d\tau^c = 0$.

Suppose that $\eta \neq 1$. The following is obtained:

$$\frac{dT}{d\tau^c} = [\kappa(1+\lambda)]^{-\frac{1}{\eta+\lambda}} (1-\tau^n)^{\frac{1+\lambda}{\eta+\lambda}} (1+\tau^c)^{-\frac{1+\lambda}{\eta+\lambda}-1} \left[\frac{1}{1-\tau^n}\right] \times$$

$$\left[\frac{\eta-1}{\eta+\lambda}\right]\left\{\tau^c - \frac{(1+\lambda)\tau^n - (\eta+\lambda)}{\eta-1}\right\}.$$

Suppose that $\eta > 1$.

If $\tau^n < \frac{\eta+\lambda}{1+\lambda}$, then $\left|\frac{dT}{d\tau^c}\right| > 0$.

If $\tau^n > \frac{\eta+\lambda}{1+\lambda}$, then $\frac{dT}{d\tau^c}|_{\tau^c=0} < 0$ and $\frac{dT}{d\tau^c} > 0$ for $\tau^c > \frac{(1+\lambda)\tau^n-(\eta+\lambda)}{\eta-1}$.

Suppose $\eta < 1$.

If $\tau^n < \frac{\eta+\lambda}{1+\lambda}$, then $\frac{dT}{d\tau^c}|_{\tau^c=0} > 0$ and $\frac{dT}{d\tau^c} < 0$ for $\tau^c > \frac{(1+\lambda)\tau^n-(\eta+\lambda)}{\eta-1}0$.

If $\tau^n \geq \frac{\eta+\lambda}{1+\lambda}$, then $\frac{dT}{d\tau^c}$.

Finally, the condition $\tau^n < \frac{\eta+\lambda}{1+\lambda}$ is rewritten as

$$\tau^n < \frac{\eta+\lambda}{1+\lambda}$$

$$\Longleftrightarrow \quad \eta > (1+\lambda)\tau^n - \lambda.$$

Since the consumption tax revenue is bounded if and only if $\eta \leq 1$, as in Proposition 3 in the main text, the total tax revenue is also bounded if and only if $\eta \leq 1$. $\qquad\square$

Proposition A. 6. *Suppose that the utility function is additively separable, U^{GHH}. The total tax revenue curve for consumption tax under Scheme (2′) is hump shaped if and only if*

$$\lambda > \frac{\tau^n}{1 - \tau^n}$$

and the revenue is maximized at $\tau^c = \lambda - (1 + \lambda)\tau^n$. Otherwise, the total tax revenue curve is monotonically decreasing. The total tax revenue is bounded.

Proof. The optimization condition for the consumption–labor choice,

$$\kappa(1 + \lambda)n^\lambda = \frac{1 - \tau^n}{1 + \tau^c}w,$$

and $n = (1 + \tau^c)/(1 - \tau^n)c$ indicate that

$$c = [\kappa(1 + \lambda)]^{-\frac{1}{\lambda}} (1 - \tau^n)^{\frac{1+\lambda}{\lambda}} (1 + \tau^c)^{-\frac{1+\lambda}{\lambda}}.$$

The total tax revenue is

$$T = \tau^c c + \tau^n wn$$

$$= [\kappa(1 + \lambda)]^{-\frac{1}{\lambda}} (1 - \tau^n)^{\frac{1+\lambda}{\lambda}} (1 + \tau^c)^{-\frac{1+\lambda}{\lambda}} \left[\frac{\tau^c + \tau^n}{1 - \tau^n}\right]$$

and thus,

$$\frac{dT}{d\tau^c} = -[\kappa(1 + \lambda)]^{-\frac{1}{\lambda}} (1 - \tau^n)^{\frac{1+\lambda}{\lambda}} (1 + \tau^c)^{-\frac{1+\lambda}{\lambda}-1} \left[\frac{1}{1 - \tau^n}\right] \times \left\{\left[\frac{1}{\lambda}\right]\tau^c - \left(1 - \frac{1 + \lambda}{\lambda}\tau^n\right)\right\}$$

$$= -[\kappa(1 + \lambda)]^{-\frac{1}{\lambda}} (1 - \tau^n)^{\frac{1+\lambda}{\lambda}} (1 + \tau^c)^{-\frac{1+\lambda}{\lambda}-1} \left[\frac{1}{1 - \tau^n}\right]\left(\frac{1}{\lambda}\right)(\tau^c - [\lambda - (1 + \lambda)\tau^n]).$$

If $\lambda - (1 + \lambda)\tau^n \leq 0$, then $\frac{dT}{d\tau^c} < 0$.

If $\lambda - (1 + \lambda)\tau^n > 0$, then

$\frac{dT}{d\tau^c} > 0$ for $\tau^c < \lambda - (1 + \lambda)\tau^n$,

$\frac{dT}{d\tau^c} = 0$ for $\tau^c = \lambda - (1 + \lambda)\tau^n$, and

$\frac{dT}{d\tau^c} < 0$ for $\tau^c > \lambda - (1 + \lambda)\tau^n$.

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Finally, the condition $\lambda - (1 + \lambda)\tau^n >$ is rewritten as

$$\lambda > \frac{\tau^n}{1 - \tau^n}.$$

The total tax revenue is also bounded. $\hspace{1cm}\square$

A.3 Alternative fiscal policy schemes

Schemes (1) and (2) consider that all tax revenue is used as a lump-sum transfer or government consumption. Here, some relaxed versions are investigated. In this sub-section, for the simplicity of analysis, the labor income tax rate is set to zero, and the consumption tax revenue curve is the point of focus.

The following two schemes are one of the analogues of Schemes (1) and (2).

Definition A. 3. *Scheme (1*): The ratio of government consumption to output, g/y, is constant and positive. The rest of tax revenue is used as a lump-sum transfer to households.*

$$s = \tau^c c - g, \qquad g/y = \bar{\phi}_{gy}$$

Definition A. 4. *Scheme (2*): The ratio of lump-sum transfer to output, s/y, is constant and positive. The rest of tax revenue is used as government consumption.*

$$g = \tau^c c - s, \qquad s/y = \bar{\phi}_{sy}$$

The following two schemes are other options, and are based on similar assumptions employed by Trabandt and Uhlig (2011).

Definition A. 5. *Scheme (1**): Government consumption, g, is constant and positive. The rest of tax revenue is used as a lump-sum transfer to households.*

$$s = \tau^c c - g, \qquad g = \bar{g}$$

Definition A. 6. *Scheme (2**): The lump-sum transfer, s, is constant and positive. The rest of tax revenue is used as government consumption.*

$$g = \tau^c c - s, \qquad s = \bar{s}$$

A.3.1 Scheme (1*): g/y is constant and changes in tax revenue are adjusted by a lump-sum transfer

By the resource constraint,

$$\frac{c}{y} + \frac{g}{y} = 1.$$

Then, c/y is constant (and independent from τ^c) under Scheme (1*). The following holds.

Remark A. 1. *The elasticity of consumption with respect to the consumption tax rate equals that of output:*

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \left| \frac{dy/y}{d\tau^c/\tau^c} \right|.$$

Proposition A.7 is about the case of KPR utility.

Proposition A. 7. *Suppose that the utility function is KPR, U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (1*) is monotonically increasing. The consumption tax revenue is unbounded except for $\lambda = 0$.*

Proof. The optimization condition for the consumption–labor choice,

$$\eta (1 + \lambda) \left\{ \frac{\kappa c n^\lambda}{1 - \kappa(1 - \eta)n^{1+\lambda}} \right\} = \frac{1}{1 + \tau^c},$$

yields

$$y = (\kappa)^{-1/(1+\lambda)} \left[(1 - \eta) + \left(\frac{c}{y} \right) \eta(1 + \lambda)(1 + \tau^c) \right]^{-1/(1+\lambda)}.$$

Since $\tilde{y} > 0$ for $\tau^c \geq 0$,

$$(1 - \eta) + \left(\frac{c}{y}\right) \eta (1 + \lambda) > 0.$$

By Remark A.1, it follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = \frac{dy/y}{d\tau^c/\tau^c} = -\frac{\left(\frac{c}{y}\right)\eta\tau^c}{(1 - \eta) + \left(\frac{c}{y}\right)\eta(1 + \tau^c)(1 + \lambda)}.$$

Letting

$$\Psi = (1 - \eta) + \left(\frac{c}{y}\right)\eta(1 + \tau^c)(1 + \lambda) > 0,$$

it follows that

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = -\frac{1}{\Psi}\left\{(1 - \eta) + \left(\frac{c}{y}\right)\eta(1 + \lambda) + \left(\frac{c}{y}\right)\eta\tau^c\lambda\right\} \leq 0.$$

The consumption tax revenue is unbounded if $\lambda > 0$, since $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| > 0$. In the case of $\lambda = 0$, the consumption tax revenue is given by

$$\tau^c c = \phi\tau^c y$$

$$= (\kappa)^{-1/(1+\lambda)}\left[(1 - \eta) + \left(\frac{c}{y}\right)\eta(1 + \lambda)\right]^{-1/(1+\lambda)}\frac{\tau^c}{1 + \tau^c},$$

where $\phi = c/y$. This converges to $(\kappa)^{-1/(1+\lambda)}\left[(1 - \eta) + \left(\frac{c}{y}\right)\eta(1 + \lambda)\right]^{-1/(1+\lambda)}$. □

Proposition A.8 is about the case of additively separable utility.

Proposition A. 8. *Suppose that the utility function is additively separable, U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (1^*) is hump shaped if and only if $\eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta + \lambda \leq 1$. Otherwise, it is unbounded.*

Proof. In the case of additively separable utility, the consumption–labor choice condition is

$$\kappa(1 + \lambda)c^{\eta}n^{\lambda} = \frac{1}{1 + \tau^c}w$$

$$\iff \quad \kappa(1 + \lambda)\left(\frac{c}{y}\right)^{\eta}\left(\frac{n}{y}\right)^{\lambda} y^{\eta+\lambda} = \frac{1}{1 + \tau^c},$$

then

$$y = (1 + \tau^c)^{-1/(\eta+\lambda)}\left[\frac{1}{\kappa(1 + \lambda)}\left(\frac{c}{y}\right)^{-\eta}\right]^{1/(\eta+\lambda)}.$$

By Remark A.1, it follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = \frac{dy/y}{d\tau^c/\tau^c} = -\frac{1}{\eta + \lambda} \cdot \frac{\tau^c}{1 + \tau^c}.$$

Then,

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\eta + \lambda} \cdot \frac{1}{1 + \tau^c}\left\{(1 - \eta - \lambda)\tau^c - (\eta + \lambda)\right\}.$$

Suppose $\eta + \lambda = 1$. In this case, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 < 0$.

Suppose $\eta + \lambda \neq 1$. In this case,

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1 - \eta - \lambda}{\eta + \lambda} \cdot \frac{1}{1 + \tau^c}\left\{\tau^c - \frac{\eta + \lambda}{1 - \eta - \lambda}\right\}.$$

If $\eta + \lambda \geq 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \geq 0$.

If $\eta + \lambda < 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq (\eta + \lambda)/(1 - \eta - \lambda)$, and $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > (\eta + \lambda)/(1 - \eta - \lambda)$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\eta + \lambda < 1$ and unbounded if $\eta + \lambda > 1$. In the case of $\eta + \lambda = 1$, the consumption tax revenue is

$$\tau^c c = \phi\tau^c y$$

$$= \phi\frac{\tau^c}{1 + \tau^c}\left[\frac{1}{\kappa(1 + \lambda)}\left(\frac{c}{y}\right)^{-\eta}\right]^{1/(\eta+\lambda)}.$$

where $\phi = c/y$. This converges to $\phi\left[\frac{1}{\kappa(1+\lambda)}\left(\frac{c}{y}\right)^{-\eta}\right]^{1/(\eta+\lambda)}$ as $\tau^c \to \infty$. $\qquad \square$

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Proposition A.9 is about the case of GHH utility.

Proposition A. 9. *Suppose that the utility function is additively separable, U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (1^*) is hump shaped if and only if $\lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\lambda}{1-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\lambda \leq 1$. Otherwise, it is unbounded.*

Proof. In the case of additively separable utility, the consumption–labor choice condition is

$$\kappa(1 + \lambda)n^\lambda = \frac{1}{1 + \tau^c}w$$

$$\iff \quad \kappa(1 + \lambda)\left(\frac{n}{y}\right)^\lambda y^\lambda = \frac{1}{1 + \tau^c},$$

then

$$y = (1 + \tau^c)^{-1/\lambda}\left[\frac{1}{\kappa(1 + \lambda)}\right]^{1/\lambda}.$$

By Remark A.1, it follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = \frac{dy/y}{d\tau^c/\tau^c} = -\frac{1}{\lambda} \cdot \frac{\tau^c}{1 + \tau^c}.$$

Then,

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1}{\lambda} \cdot \frac{1}{1 + \tau^c}\left\{(1 - \lambda)\tau^c - \lambda\right\}.$$

Suppose $\lambda = 1$. In this case, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 < 0$.

Suppose $\lambda \neq 1$. In this case,

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| - 1 = \frac{1 - \lambda}{\lambda} \cdot \frac{1}{1 + \tau^c}\left\{\tau^c - \frac{\lambda}{1 - \lambda}\right\}.$$

If $\lambda \geq 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \geq 0$.

If $\lambda < 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq \lambda/(1 - \lambda)$, and $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > \lambda/(1 - \lambda)$.

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By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\lambda < 1$ and unbounded if $\lambda > 1$. In the case of $\lambda = 1$, the consumption tax revenue is

$$\tau^c c = \phi \tau^c y$$

$$= \phi \frac{\tau^c}{1 + \tau^c} \left[\frac{1}{\kappa(1 + \lambda)} \right]^{1/+\lambda}.$$

where $\phi = c/y$. This converges to $\phi \left[\frac{1}{\kappa(1+\lambda)} \right]^{1/\lambda}$ as $\tau^c \to \infty$. $\qquad\square$

Note that Propositions A.7, A.8, and A.9 are the exactly same as Propositions 1, 2, and 3 in the main text. Therefore, Scheme (1^*) is a natural extension of Scheme (1).

A.3.2 Scheme (2^*): s/y is constant and changes in tax revenue are adjusted by government consumption

By the government budget constraint, it follows that

$$\frac{g}{y} + \frac{s}{y} = \tau^c \frac{c}{y}.$$

Since s/y is constant,

$$\frac{g}{y} = \tau^c \frac{c}{y} - constant.$$

The resource constraint is

$$\frac{c}{y} + \frac{g}{y} = 1$$

$$\Longleftrightarrow \quad \frac{c}{y} + \tau^c \frac{c}{y} - constant = 1,$$

and then,

$$(1 + \tau^c) \frac{c}{y} = constant.$$

Therefore, the following remark holds.

Remark A. 2. *The elasticity of consumption with respect to consumption tax rate equals that of $y/(1 + \tau^c)$:*

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \left| \frac{d(1 + \tau^c)^{-1}y/(1 + \tau^c)^{-1}y}{d\tau^c/\tau^c} \right|$$

Proposition A. 10. *Suppose that the utility function is KPR, U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (2^*) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. By the consumption–labor choice condition, y is obtained

$$\eta(1 + \lambda) \left\{ \frac{\kappa c n^\lambda}{1 - \kappa(1 - \eta)n^{1+\lambda}} \right\} = \frac{1}{1 + \tau^c}$$

$$\iff \quad \eta(1 + \lambda) \left\{ \frac{\kappa \left(\frac{c}{y}\right)\left(\frac{n}{\tilde{y}}\right)^\lambda}{\tilde{y}^{-1-\lambda} - \kappa(1 - \eta)\left(\frac{n}{y}\right)^{1+\lambda}} \right\} = \frac{1}{1 + \tau^c}$$

$$\iff \quad y = (\kappa)^{-1/(1+\lambda)} \left[(1 - \eta) + \left((1 + \tau^c)\frac{c}{y}\right)\eta(1 + \lambda) \right]^{-1/(1+\lambda)}.$$

Then, y is independent from τ^c.

By Remark A.2, the elasticity of $(1 + \tau^c)^{-1}y$ is considered. Since

$$(1 + \tau^c)^{-1}y = (1 + \tau^c)^{-1}(\kappa)^{-1/(1+\lambda)} \left[(1 - \eta) + \left((1 + \tau^c)\frac{c}{y}\right)\eta(1 + \lambda)(1 + \tau^c) \right]^{-1/(1+\lambda)},$$

it follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = \frac{d(1 + \tau^c)^{-1}y/(1 + \tau^c)^{-1}y}{d\tau^c/\tau^c}$$

$$= -\frac{\tau^c}{1 + \tau^c}.$$

$\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = 0$. As $\tau^c \to \infty$, $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \to 1$. Therefore, the consumption tax revenue curve is monotonically increasing.

The boundedness is shown as follows. Letting $(1 + \tau^c)c/y = \phi$ yields

$$\tau^c c = \tau^c \phi y(1 + \tau^c)^{-1}$$

$$= \phi \frac{\tau^c}{1 + \tau^c}(\kappa)^{-1/(1+\lambda)} \left[(1 - \eta) + \left((1 + \tau^c)\frac{c}{y}\right)\eta(1 + \lambda)(1 + \tau^c) \right]^{-1/(1+\lambda)}.$$

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As $\tau^c \to \infty$, it converges to $\phi(\kappa)^{-1/(1+\lambda)}\left[(1-\eta) + \left((1+\tau^c)\frac{c}{y}\right)\eta(1+\lambda)(1+\tau^c)\right]^{-1/(1+\lambda)}$.

$\square$

Proposition A. 11. *Suppose that the utility function is additively separable, U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (2^*) is hump shaped if and only if $\eta < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is unbounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. By the consumption–labor choice condition,

$$\kappa(1+\lambda)c^\eta n^\lambda = \frac{1}{1+\tau^c}$$

$$\Longleftrightarrow \quad \kappa(1+\lambda)y^{\eta+\lambda}\left(\frac{c}{y}\right)^\eta\left(\frac{n}{y}\right)^\lambda = \frac{1}{1+\tau^c}$$

$$\Longleftrightarrow \quad y = (1+\tau^c)^{-(1-\eta)/(\eta+\lambda)}\left[\frac{1}{\kappa(1+\lambda)}\left((1+\tau^c)\frac{c}{y}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.$$

By Remark A.2, the elasticity of $(1+\tau^c)y$ is considered. Since

$$(1+\tau^c)^{-1}y = (1+\tau^c)^{-(1+\lambda)/(\eta+\lambda)}\left[\frac{1}{\kappa(1+\lambda)}\left((1+\tau^c)\frac{c}{y}\right)^{-\eta}\right]^{1/(\eta+\lambda)},$$

it follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = \frac{d(1+\tau^c)^{-1}y/(1+\tau^c)^{-1}y}{d\tau^c/\tau^c}$$

$$= -\frac{1+\lambda}{\eta+\lambda}\cdot\frac{\tau^c}{1+\tau^c}.$$

$\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = 0$. If $\tau^c \to \infty$, then $\left|\frac{dc/c}{d\tau^c/\tau^c}\right| \to (1+\lambda)/(\eta+\lambda)$. Therefore, a necessary and sufficient condition for a hump-shaped consumption tax revenue curve is $\eta < 1$. The peak tax rate is

$$\tau^c_{max} = \frac{\eta+\lambda}{1-\eta}.$$

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By the elasticity of consumption, it is obvious that the tax revenue is bounded if $\eta < 1$ and unbounded if $\eta > 1$. Suppose $\eta = 1$. Letting $(1 + \tau^c)c/y = \phi$ yields

$$\tau^c c = \tau^c \phi y(1 + \tau^c)^{-1}$$

$$= \phi \frac{\tau^c}{1 + \tau^c} \left[\frac{1}{\kappa(1 + \lambda)} \left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} \right)^{-1} \right]^{1/(1+\lambda)}.$$

As $\tau^c \to \infty$, it converges to $\phi \left[\frac{1}{\kappa(1+\lambda)} \left((1 + \tau^c)\frac{c}{y} \right)^{-1} \right]^{1/(1+\lambda)}$.

$\square$

Proposition A. 12. *Suppose that the utility function is GHH, U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (2^*) is hump shaped if and only if $\eta < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is unbounded if and only if $\eta \le 1$. Otherwise, it is unbounded.*

Proof. By the consumption–labor choice condition,

$$\kappa(1 + \lambda)n^\lambda = \frac{1}{1 + \tau^c}$$

$$\iff \quad \kappa(1 + \lambda)y^\lambda \left(\frac{n}{y} \right)^\lambda = \frac{1}{1 + \tau^c}$$

$$\iff \quad y = (1 + \tau^c)^{-1/\lambda} \left[\frac{1}{\kappa(1 + \lambda)} \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/\lambda}.$$

By Remark A.2, the elasticity of $(1 + \tau^c)y$ is considered. Since

$$(1 + \tau^c)^{-1}y = (1 + \tau^c)^{-(1+\lambda)/\lambda} \left[\frac{1}{\kappa(1 + \lambda)} \right]^{1/\lambda},$$

it follows that

$$\frac{dc/c}{d\tau^c/\tau^c} = \frac{d(1 + \tau^c)^{-1}y/(1 + \tau^c)^{-1}y}{d\tau^c/\tau^c}$$

$$= -\frac{1 + \lambda}{\lambda} \cdot \frac{\tau^c}{1 + \tau^c}.$$

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$\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = 0$. If $\tau^c \to \infty$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \to (1 + \lambda)/\lambda$. Therefore, the tax revenue curve is hump shaped. The peak tax rate is

$$\tau^c_{max} = \lambda.$$

The tax revenue is bounded.

$\square$

A.3.3 Scheme (1**): government consumption g is constant and changes in tax revenue are adjusted by a lump-sum transfer

In the case of KPR utility, the consumption tax revenue curve is monotonically increasing, but tax revenue is bounded under Scheme (1**), as in the following proposition.

Proposition A. 13. *Suppose that the utility function is KPR, U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (1**) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. By the production function and the resource constraint, it follows that

$$y = n = c + g.$$

Then, the consumption–labor choice condition is

$$\eta(1 + \lambda)\frac{\kappa c n^\lambda}{1 - \kappa(1 - \eta)n^{1+\lambda}} = \frac{1}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \eta(1 + \lambda)\frac{\kappa c(c + g)^\lambda}{1 - \kappa(1 - \eta)(c + g)^{1+\lambda}} = \frac{1}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \eta(1 + \lambda)\kappa c(c + g)^\lambda = \frac{1 - \kappa(1 - \eta)(c + g)^{1+\lambda}}{1 + \tau^c}$$

Taking the total derivatives yields

$$\eta(1 + \lambda)\kappa \left[(c + g)^\lambda \frac{dc}{d\tau^c} + \lambda c(c + g)^{\lambda-1} \frac{dc}{d\tau^c} \right]$$

$$= -\frac{(1 + \tau^c)(1 + \lambda)\kappa(1 - \eta)(c + g)^\lambda \frac{dc}{d\tau^c} + [1 - \kappa(1 - \eta)(c + g)^{1+\lambda}]}{(1 + \tau^c)^2}.$$

$$\iff \quad (1 + \tau^c)^2 \eta(1 + \lambda)\kappa(c + g)^{\lambda-1} \left[(c + g) + \lambda c\right] \frac{dc}{d\tau^c}$$

$$= -(1 + \tau^c)(1 + \lambda)\kappa(1 - \eta)(c + g)^\lambda \frac{dc}{d\tau^c} - [1 - \kappa(1 - \eta)(c + g)^{1+\lambda}].$$

By the consumption–labor choice condition, it follows that

$$1 - \kappa(1 - \eta)(c + g)^{1+\lambda} = (1 + \tau^c)\eta(1 + \lambda)\kappa c(c + g)^\lambda.$$

Then, the following holds

$$(1 + \tau^c)^2 \eta(1 + \lambda)\kappa(c + g)^{\lambda-1} \left[(c + g) + \lambda c\right] \frac{dc}{d\tau^c}$$

$$= -(1 + \tau^c)(1 + \lambda)\kappa(1 - \eta)(c + g)^\lambda \frac{dc}{d\tau^c} - (1 + \tau^c)\eta(1 + \lambda)\kappa c(c + g)^\lambda$$

$$\iff \quad (1 + \tau^c)\eta \left[(c + g) + \lambda c\right] \frac{dc}{d\tau^c} = -(1 - \eta)(c + g)\frac{dc}{d\tau^c} - \eta c(c + g)$$

$$\iff \quad \{(1 + \tau^c)\eta \left[(c + g) + \lambda c\right] + (1 - \eta)(c + g)\} \frac{dc}{d\tau^c} = -\eta c(c + g)$$

$$\iff \quad \frac{dc}{d\tau^c} = -\frac{\eta c(c + g)}{(1 + \tau^c)\eta \left[(c + g) + \lambda c\right] + (1 - \eta)(c + g)}.$$

The elasticity of consumption is given by

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\eta\tau^c(c + g)}{(1 + \tau^c)\eta \left[(c + g) + \lambda c\right] + (1 - \eta)(c + g)}$$

$$= \frac{\eta\tau^c(c + g)}{\eta(c + g) + \tau^c\eta(c + g) + (1 + \tau^c)\eta\lambda c + (c + g) - \eta(c + g)}$$

$$= \frac{\eta\tau^c(c + g)}{\tau^c\eta(c + g) + (1 + \tau^c)\eta\lambda c + (c + g)}$$

$$= \left[1 + \frac{1}{\eta\tau^c} + \frac{1 + \tau^c}{\tau^c} \cdot \frac{\lambda c}{c + g} \right]^{-1}$$

$$= \left[1 + \frac{1}{\eta\tau^c} + \frac{1 + \tau^c}{\tau^c} \cdot \frac{\lambda}{1 + g/c} \right]^{-1}.$$

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$\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is increasing in τ^c. If $\tau^c = 0$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = 0$. If $\tau^c \to \infty$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \to 1$, since $c \to 0$.

Therefore, the consumption tax revenue curve is monotonically increasing.

$c \to 0$ as $\tau^c \to \infty$, which is proved as follows. Since the elasticity of consumption is negative, as $\tau^c \to \infty$, c converges to a non-negative value $a \geq 0$. By the consumption–labor choice condition, it follows that

$$\eta(1 + \lambda)\kappa c(c + g)^\lambda = \frac{1 - \kappa(1 - \eta)(c + g)^{1+\lambda}}{1 + \tau^c}$$

$$\Longleftrightarrow \quad c = \frac{1 - \kappa(1 - \eta)(c + g)^{1+\lambda}}{(1 + \tau^c)\eta(1 + \lambda)\kappa(c + g)^\lambda}.$$

The left-hand side of this equation converges to a as $\tau^c \to \infty$, and the right-hand side converges to zero. Therefore, $c \to 0$.

The consumption tax revenue is given by

$$\tau^c c = \frac{1 - \kappa(1 - \eta)(c + g)^{1+\lambda}}{\eta(1 + \lambda)\kappa(c + g)^\lambda} \frac{\tau^c}{1 + \tau^c}.$$

As $\tau^c \to \infty$, consumption tax revenue converges to $\frac{1 - \kappa(1-\eta)g^{1+\lambda}}{\eta(1+\lambda)\kappa g^\lambda}$.

$\square$

The result of Proposition A.13 is consistent with the result of Trabandt and Uhlig (2011). They report that the slope of the tax revenue curve for consumption tax converges to zero as $\tau^c \to \infty$ by numerical simulation under a similar fiscal policy scheme (although their model is dynamic).

Proposition A. 14. *Suppose that the utility function is additively separable, U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (1^{**}) is hump shaped if and only if $\eta < 1$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. By the resource constraint and the production function, it follows that

$$y = n = c + g.$$

In the case of additively separable utility, the consumption–labor choice condition is

$$\kappa(1 + \lambda)c^{\eta}n^{\lambda} = \frac{1}{1 + \tau^c}w$$

$$\Longleftrightarrow \quad \kappa(1 + \lambda)c^{\eta}(c + g)^{\lambda} = \frac{1}{1 + \tau^c}$$

Taking the total derivatives yields

$$\frac{dc}{d\tau^c} = -\kappa^{-1}(1 + \lambda)^{-1}c^{1-\eta}(c + g)^{1-\lambda}\left[\eta(c + g) + \lambda c\right]^{-1}\frac{1}{(1 + \tau^c)^2}.$$

By the consumption–labor choice condition, it follows that

$$c^{-\eta} = \kappa(1 + \lambda)(c + g)^{\lambda}(1 + \tau^c),$$

and then,

$$\frac{dc}{d\tau^c} = -c(c + g)(1 + \tau^c)\left[\eta(c + g) + \lambda c\right]^{-1} \cdot \frac{1}{(1 + \tau^c)^2}$$

$$= -c\frac{c + g}{\eta(c + g) + \lambda c} \cdot \frac{1}{1 + \tau^c}.$$

The elasticity of consumption is

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{c + g}{\eta(c + g) + \lambda c} \times \frac{\tau^c}{1 + \tau^c}$$

$$= \frac{1}{\eta + \lambda\frac{c}{c+g}} \times \frac{\tau^c}{1 + \tau^c}$$

$$= \frac{1}{\eta + \lambda\frac{1}{1+g/c}} \times \frac{\tau^c}{1 + \tau^c}.$$

$|\frac{dc/c}{d\tau^c/\tau^c}|$ is increasing in τ^c. If $\tau^c = 0$, then $|\frac{dc/c}{d\tau^c/\tau^c}| = 0$. If $\tau^c \to \infty$, then $|\frac{dc/c}{d\tau^c/\tau^c}| = 1/\eta$, since $c \to 0$. Therefore, $\eta < 1$ is a necessary and sufficient condition for a hump-shaped consumption tax revenue curve.

$c \to 0$ as $\tau^c \to \infty$, which is proved as follows. Since $\frac{dc}{d\tau^c} < 0$, c converges to a non-negative value $a \geq 0$. By the consumption–labor choice condition, it follows that

$$c = [\kappa(1 + \lambda)]^{-1/\eta}(c + g)^{-\lambda/\eta}(1 + \tau^c)^{-1/\eta}.$$

The left-hand side converges to a, and the right-hand side converges to zero. Therefore, $c \to 0$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\eta < 1$, and unbounded if $\eta > 1$. Suppose $\eta = 1$. The consumption tax revenue is given by

$$\tau^c c = [\kappa(1 + \lambda)]^{-1}(c + g)^{-\lambda}\frac{\tau^c}{1 + \tau^c}.$$

This converges to $[\kappa(1 + \lambda)]^{-1}g^{-\lambda}$ as $\tau^c \to \infty$.

□

Note that the condition for a hump-shaped consumption tax revenue curve is $\eta < 1$, not $\eta + \lambda < 1$, under Scheme (1^{**}). This is because g/y increases as τ^c increases, and represents downward pressure on consumption (negative income effect).

Proposition A. 15. *Suppose that the utility function is additively separable, U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (1^{**}) is hump shaped. The consumption tax revenue is bounded.*

Proof. By the resource constraint and the production function, it follows that

$$y = n = c + g.$$

In the case of additively separable utility, the consumption–labor choice condition is

$$\kappa(1 + \lambda)n^{\lambda} = \frac{1}{1 + \tau^c}w$$

$$\Longleftrightarrow \quad \kappa(1 + \lambda)(c + g)^{\lambda} = \frac{1}{1 + \tau^c}$$

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Taking the total derivatives yields

$$\frac{dc}{d\tau^c} = -\kappa^{-1}(1 + \lambda)^{-1}\lambda(c + g)^{1-\lambda}\frac{1}{(1 + \tau^c)^2} < 0.$$

By the consumption–labor choice condition, it follows that

$$\kappa(1 + \lambda)(c + g)^{\lambda}(1 + \tau^c) = 1,$$

and then,

$$\frac{dc}{d\tau^c} = -\frac{\lambda(c + g)}{1 + \tau^c}.$$

The elasticity of consumption is

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{\lambda(c + g)}{c} \times \frac{\tau^c}{1 + \tau^c}$$

$$= \lambda(1 + g/c) \times \frac{\tau^c}{1 + \tau^c}.$$

$|\frac{dc/c}{d\tau^c/\tau^c}|$ is increasing in τ^c, since $\frac{dc}{d\tau^c} < 0$. If $\tau^c = 0$, then $|\frac{dc/c}{d\tau^c/\tau^c}| = 0$. Because $g/c > 0$, it is obvious that $|\frac{dc/c}{d\tau^c/\tau^c}| > 1$ for sufficiently high τ^c. Therefore, the consumption tax revenue curve is hump shaped.

□

A.3.4 Scheme (2**): Transfer s is constant and changes in tax revenue are adjusted by government consumption

Proposition A. 16. *Suppose that the utility function is KPR, U^{KPR}. The consumption tax revenue curve for consumption tax under Scheme (2**) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. By the government budget constraint, it follows that

$$y = n = c + g = (1 + \tau^c)c - s.$$

By the consumption–labor choice condition, the following is obtained:

$$\eta(1 + \lambda)\frac{\kappa c n^\lambda}{1 - \kappa(1 - \eta)n^{1+\lambda}} = \frac{1}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \eta(1 + \lambda)\frac{\kappa c[(1 + \tau^c)c - s]^\lambda}{1 - \kappa(1 - \eta)[(1 + \tau^c)c - s]^{1+\lambda}} = \frac{1}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \eta(1 + \lambda)\kappa c[(1 + \tau^c)c - s]^\lambda = \frac{1 - \kappa(1 - \eta)[(1 + \tau^c)c - s]^{1+\lambda}}{1 + \tau^c}.$$

Taking the total derivatives yields

$$\eta(1 + \lambda)\kappa\left[\frac{dc}{d\tau^c}[(1 + \tau^c)c - s]^\lambda + c\lambda[(1 + \tau^c)c - s]^{\lambda-1}[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right]$$

$$= \frac{-(1 + \tau^c)\kappa(1 - \eta)(1 + \lambda)[(1 + \tau^c)c - s]^\lambda[c + (1 + \tau^c)\frac{dc}{d\tau^c}] - \left\{1 - \kappa(1 - \eta)[1 + \tau^c)c - s)]^{1+\lambda}\right\}}{(1 + \tau^c)^2}$$

$$\Longleftrightarrow \quad (1 + \tau^c)^2\eta(1 + \lambda)\kappa\left[\frac{dc}{d\tau^c}[(1 + \tau^c)c - s]^\lambda + c\lambda[(1 + \tau^c)c - s]^{\lambda-1}[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right]$$

$$= -(1 + \tau^c)\kappa(1 - \eta)(1 + \lambda)[(1 + \tau^c)c - s]^\lambda[c + (1 + \tau^c)\frac{dc}{d\tau^c}]$$

$$- \left\{1 - \kappa(1 - \eta)[(1 + \tau^c)c - s)]^{1+\lambda}\right\}$$

$$\Longleftrightarrow \quad (1 + \tau^c)^2\eta(1 + \lambda)\kappa[(1 + \tau^c)c - s]^{\lambda-1}\left[\frac{dc}{d\tau^c}[(1 + \tau^c)c - s] + c\lambda[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right]$$

$$= -(1 + \tau^c)\kappa(1 - \eta)(1 + \lambda)[(1 + \tau^c)c - s]^\lambda[c + (1 + \tau^c)\frac{dc}{d\tau^c}]$$

$$- \left\{1 - \kappa(1 - \eta)[(1 + \tau^c)c - s)]^{1+\lambda}\right\}.$$

By the consumption–labor choice condition, it follows that

$$1 - \kappa(1 - \eta)[(1 + \tau^c)c - s)]^{1+\lambda} = \eta(1 + \lambda)\kappa c[(1 + \tau^c)c - s]^\lambda(1 + \tau^c),$$

and then,

$$\Longleftrightarrow \quad (1 + \tau^c)^2 \eta(1 + \lambda)\kappa[(1 + \tau^c)c - s]^{\lambda-1}\left[\frac{dc}{d\tau^c}[(1 + \tau^c)c - s] + c\lambda[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right]$$

$$= -(1 + \tau^c)\kappa(1 - \eta)(1 + \lambda)[(1 + \tau^c)c - s]^{\lambda}[c + (1 + \tau^c)\frac{dc}{d\tau^c}]$$

$$- \eta(1 + \lambda)\kappa c[(1 + \tau^c)c - s]^{\lambda}(1 + \tau^c)$$

$$\Longleftrightarrow \quad (1 + \tau^c)\eta[(1 + \tau^c)c - s]^{-1}\left[\frac{dc}{d\tau^c}[(1 + \tau^c)c - s] + c\lambda[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right]$$

$$= -(1 - \eta)[c + (1 + \tau^c)\frac{dc}{d\tau^c}] - \eta c$$

$$\Longleftrightarrow \quad \eta\frac{dc}{d\tau^c} + \eta\lambda[(1 + \tau^c)c - s]^{-1}c^2 + \eta\lambda c(1 + \tau^c)[(1 + \tau^c)c - s]^{-1}\frac{dc}{d\tau^c}$$

$$= -\frac{1}{1 + \tau^c}\left[(1 - \eta)[c + (1 + \tau^c)\frac{dc}{d\tau^c}] + \eta c\right]$$

$$\Longleftrightarrow \quad \eta\frac{dc}{d\tau^c} + \eta\lambda[(1 + \tau^c)c - s]^{-1}c^2 + \eta\lambda c(1 + \tau^c)[(1 + \tau^c)c - s]^{-1}\frac{dc}{d\tau^c}$$

$$= -\frac{1}{1 + \tau^c}\left[(1 - \eta)(1 + \tau^c)\frac{dc}{d\tau^c} + c\right].$$

$$\Longleftrightarrow \quad \frac{dc}{d\tau^c}\left\{\eta + \eta\lambda c(1 + \tau^c)[(1 + \tau^c)c - s]^{-1} + (1 - \eta)\right\} = -\eta\lambda[(1 + \tau^c)c - s]^{-1}c^2 - \frac{c}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \frac{dc}{d\tau^c}\left\{1 + \eta\lambda c(1 + \tau^c)[(1 + \tau^c)c - s]^{-1}\right\} = -\eta\lambda[(1 + \tau^c)c - s]^{-1}c^2 - \frac{c}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \frac{dc}{d\tau^c} = -\frac{\eta\lambda[(1 + \tau^c)c - s]^{-1}c^2 + \frac{c}{1+\tau^c}}{1 + \eta\lambda c(1 + \tau^c)[(1 + \tau^c)c - s]^{-1}}.$$

The elasticity of consumption is given by

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{\eta\lambda c[(1 + \tau^c)c - s]^{-1} + \frac{1}{1+\tau^c}}{1 + \eta\lambda c(1 + \tau^c)[(1 + \tau^c)c - s]^{-1}} \times \tau^c$$

$$= \frac{\tau^c}{1 + \tau^c} \times \frac{\eta\lambda c(1 + \tau^c)[(1 + \tau^c)c - s]^{-1} + 1}{1 + \eta\lambda c(1 + \tau^c)[(1 + \tau^c)c - s]^{-1}}$$

$$= \frac{\tau^c}{1 + \tau^c}.$$

Therefore, the consumption tax revenue curve is monotonically increasing.

The following results are obtained for boundedness. First, $c \to 0$ as $\tau^c \to \infty$. This is

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proved as follows. By the consumption–labor choice condition, it follows that

$$1 - \kappa(1 - \eta)[(1 + \tau^c)c - s)]^{1+\lambda} = \eta(1 + \lambda)\kappa c[(1 + \tau^c)c - s]^{\lambda}(1 + \tau^c)$$

$$\Longleftrightarrow \quad c = \frac{1 - \kappa(1 - \eta)[(1 + \tau^c)c - s)]^{1+\lambda}}{\eta(1 + \lambda)\kappa[(1 + \tau^c)c - s]^{\lambda}} \times \frac{1}{1 + \tau^c}.$$

Suppose that $c \to a$ as $\tau^c \to \infty$. The left-hand side converges to a, and the right-hand side converges to zero. Therefore, $c \to 0$.

The consumption tax revenue is given by

$$\tau^c c = \frac{1 - \kappa(1 - \eta)[(1 + \tau^c)c - s)]^{1+\lambda}}{\eta(1 + \lambda)\kappa[(1 + \tau^c)c - s]^{\lambda}} \times \frac{\tau^c}{1 + \tau^c}.$$

Suppose that $\tau^c c \to z$ as $\tau^c \to \infty$. This implies

$$z = \frac{1 - \kappa(1 - \eta)[z - s)]^{1+\lambda}}{\eta(1 + \lambda)\kappa[z - s]^{\lambda}}.$$

The limit z must satisfy this equation. It is obvious that z is finite (otherwise, the above equation does not hold). □

Proposition A. 17. *Suppose that the utility function is additively separable, U^{AS}. The consumption tax revenue curve for consumption tax under Scheme (2^{**}) is hump shaped if and only if $\eta < 1$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta \le 1$. Otherwise, it is unbounded.*

Proof. By the government budget constraint, it follows that

$$g = \tau^c c - s.$$

By the production function and the resource constraint, it follows that

$$y = n = c + g = (1 + \tau^c)c - s.$$

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In the case of additively separable utility U^{AS}, the consumption–labor choice condition is

$$\kappa(1 + \lambda)c^{\eta}n^{\lambda} = \frac{1}{1 + \tau^c}w$$

$$\Longleftrightarrow \quad \kappa(1 + \lambda)c^{\eta}[(1 + \tau^c)c - s]^{\lambda} = \frac{1}{1 + \tau^c}.$$

Taking the total derivatives yields

$$\kappa(1 + \lambda)\left[\eta c^{\eta-1}[(1 + \tau^c)c - s]^{\lambda}\frac{dc}{d\tau^c} + \lambda c^{\eta}[(1 + \tau^c)c - s]^{\lambda-1}[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right] = -\frac{1}{(1 + \tau^c)^2}$$

$$\Longleftrightarrow \quad \kappa(1 + \lambda)c^{\eta-1}[(1 + \tau^c)c - s]^{\lambda-1}\left[\eta[(1 + \tau^c)c - s]\frac{dc}{d\tau^c} + \lambda c[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right] = -\frac{1}{(1 + \tau^c)^2}.$$

By the consumption–labor choice condition, it follows that

$$c^{\eta} = (1 + \tau^c)^{-1}\kappa^{-1}(1 + \lambda)^{-1}[(1 + \tau^c)c - s]^{-\lambda},$$

and then,

$$\Longleftrightarrow \quad c^{-1}[(1 + \tau^c)c - s]^{-1}\left[\eta[(1 + \tau^c)c - s]\frac{dc}{d\tau^c} + \lambda c[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right] = -\frac{1}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \left[\eta[(1 + \tau^c)c - s]\frac{dc}{d\tau^c} + \lambda c[c + (1 + \tau^c)\frac{dc}{d\tau^c}]\right] = -\frac{c[(1 + \tau^c)c - s]}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \left[\eta[(1 + \tau^c)c - s]\frac{dc}{d\tau^c} + \lambda c(1 + \tau^c)\frac{dc}{d\tau^c} + \lambda c^2\right] = -\frac{c[(1 + \tau^c)c - s]}{1 + \tau^c}$$

$$\Longleftrightarrow \quad [\eta[(1 + \tau^c)c - s] + \lambda c(1 + \tau^c)]\frac{dc}{d\tau^c} = -\frac{c[(1 + \tau^c)c - s]}{1 + \tau^c} - \lambda c^2$$

$$\Longleftrightarrow \quad \frac{dc}{d\tau^c} = -[\eta[(1 + \tau^c)c - s] + \lambda c(1 + \tau^c)]^{-1}\left[\frac{c[(1 + \tau^c)c - s]}{1 + \tau^c} + \lambda c^2\right].$$

The elasticity of consumption with respect to consumption tax rate is given by

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{\tau^c}{c}[\eta[(1 + \tau^c)c - s] + \lambda c(1 + \tau^c)]^{-1}\left[\frac{c[(1 + \tau^c)c - s]}{1 + \tau^c} + \lambda c^2\right]$$

$$= \frac{\tau^c}{c}[\eta[(1 + \tau^c)c - s] + \lambda c(1 + \tau^c)]^{-1}\left[\frac{c[(1 + \tau^c)c - s] + \lambda c^2(1 + \tau^c)}{1 + \tau^c}\right]$$

$$= \frac{\tau^c}{1 + \tau^c} \times \frac{[(1 + \tau^c)c - s] + \lambda c(1 + \tau^c)}{\eta[(1 + \tau^c)c - s] + \lambda c(1 + \tau^c)}$$

$$= \frac{\tau^c}{1 + \tau^c} \times \frac{[(1 + \tau^c) - \frac{s}{c}] + \lambda(1 + \tau^c)}{\eta[(1 + \tau^c) - \frac{s}{c}] + \lambda(1 + \tau^c)}$$

If $\eta > 1$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| < 1$.

If $\eta < 1$, then the limit of $\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is greater than $1/\eta$, since $c \to 0$. Therefore, the condition $\eta < 1$ is a necessary and sufficient condition for a hump-shaped consumption tax revenue curve.

$c \to 0$ as $\tau^c \to \infty$, which is proved as follows. By the consumption–labor choice condition, it follows that

$$c = (1 + \tau^c)^{-1/\eta} \kappa^{-1/\eta} (1 + \lambda)^{-1/\eta} [(1 + \tau^c)c - s]^{-\lambda/\eta}.$$

Suppose that $c \to a$ as $\tau^c \to \infty$. The left-hand side of this equation converges to a, while the right-hand side converges to zero. Therefore, $c \to 0$.

By the elasticity of consumption, it is obvious that tax revenue is unbounded if $\eta > 1$, and it is bounded if $\eta < 1$. Suppose $\eta = 1$. The consumption tax revenue is given by

$$\tau^c c = \frac{\tau^c}{1 + \tau^c} \kappa^{-1} (1 + \lambda)^{-1} [(1 + \tau^c)c - s]^{-\lambda}$$

Let $\tau^c c \to z$. The left-hand side converges to z, while the right-hand side converges to $\kappa^{-1}(1 + \lambda)^{-1}[z - s]^{-\lambda}$. Therefore, z must satisfies the following:

$$z = \kappa^{-1}(1 + \lambda)^{-1}[z - s]^{-\lambda}.$$

It is obvious that z is finite. (Otherwise, this equality does not hold.)

□

Proposition A. 18. *Suppose that the utility function is additively separable, U^{GHH}. The consumption tax revenue curve for consumption tax under Scheme (2^{**}) is hump shaped The consumption tax revenue is bounded.*

Proof. By the government budget constraint, it follows that

$$g = \tau^c c - s.$$

By the production function and the resource constraint, it follows that

$$y = n = c + g = (1 + \tau^c)c - s.$$

In the case of additively separable utility U^{AS}, the consumption–labor choice condition is

$$\kappa(1 + \lambda)n^\lambda = \frac{1}{1 + \tau^c}w$$

$$\Longleftrightarrow \quad \kappa(1 + \lambda)[(1 + \tau^c)c - s]^\lambda = \frac{1}{1 + \tau^c}.$$

Taking the total derivatives yields

$$\kappa(1 + \lambda)\lambda[(1 + \tau^c)c - s]^{\lambda-1}\left(c + (1 + \tau^c)\frac{dc}{d\tau^c}\right) = -\frac{1}{(1 + \tau^c)^2}.$$

By the consumption–labor choice condition, it follows that

$$\kappa(1 + \lambda)[(1 + \tau^c)c - s]^\lambda(1 + \tau^c) = 1,$$

and then,

$$\lambda[(1 + \tau^c)c - s]^{-1}\left(c + (1 + \tau^c)\frac{dc}{d\tau^c}\right) = -\frac{1}{1 + \tau^c}$$

$$\Longleftrightarrow \quad \frac{dc}{d\tau^c} = -\frac{1}{(1 + \tau^c)^2}\lambda^{-1}[(1 + \tau^c)c - s] - \frac{c}{1 + \tau^c} < 0.$$

The elasticity of consumption is given by

$$\left|\frac{dc/c}{d\tau^c/\tau^c}\right| = \frac{\tau^c}{1 + \tau^c} \times \frac{1}{\lambda} \times \frac{(1 + \tau^c)c - s}{(1 + \tau^c)c} + \frac{\tau^c}{1 + \tau^c}$$

$$= \frac{\tau^c}{1 + \tau^c}\left[\frac{1}{\lambda}\left(1 - \frac{s}{(1 + \tau^c)c}\right) + 1\right].$$

$\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ if $\tau^c = 0$. $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is greater than one for sufficiently high τ^c because $(1 + \tau^c)c - s = n > 0$. $\left|\frac{dc/c}{d\tau^c/\tau^c}\right|$ is increasing in τ^c because

$$\frac{d\left|\frac{dc/c}{d\tau^c/\tau^c}\right|}{d\tau^c} = \frac{1}{(1 + \tau^c)^2}\left[\frac{1}{\lambda}\left(1 - \frac{s}{(1 + \tau^c)c}\right) + 1\right] + \frac{\tau^c}{1 + \tau^c}\left(\frac{1}{\lambda}\right)\frac{s(c + 1 + \tau^c)}{(1 + \tau^c)^2 c^2} > 0.$$

Therefore, the consumption tax revenue curve is hump shaped.

$\square$

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B Dynamic economy à la Trabandt and Uhlig (2011)

B.1 Model

Representative households hold capital stock k_{t-1} and debt b_{t-1} as assets at the beginning of the period. They supply labor n_t and capital stock k_{t-1} to firms, and earn wage rate w_t, rental rate of capital d_t, and interest rate on debt R_t^b. They also receive government transfers s_t and transfers from abroad m_t. Let τ_t^c, τ_t^n, and τ_t^k denote the consumption tax, labor tax, and capital tax rates, respectively. The budget constraint of households is

$$(1 + \tau_t^c)c_t + x_t + b_t \leq (1 - \tau_t^n)w_t n_t + (1 - \tau_t^k)(d_t - \delta)k_{t-1} + \delta k_{t-1} + R_t^b b_t + s_t + m_t,$$

where c_t denotes consumption, δ the depreciation rate of capital, and x_t investment. The capital stock evolves according to the following equation.

$$k_t = (1 - \delta)k_{t-1} + x_t.$$

The firms are perfectly competitive. Their production function is

$$y_t = \xi^t k_{t-1}^\theta n_t^{1-\theta},$$

where ξ denotes the technology growth rate and θ the capital share of production. The profit maximization problem implies

$$w_t = (1 - \theta)\frac{y_t}{n_t} \qquad \text{and}$$

$$d_t = \theta\frac{y_t}{k_{t-1}}.$$

The government budget constraint is

$$g_t + s_t + R_t^b b_{t-1} \leq b_t + T_t,$$

where g_t denotes government consumption. The total tax revenue T_t is defined as

$$T_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k(d_t - \delta)k_{t-1}.$$

The resource constraint of this economy is

$$y_t = c_t + x_t + g_t - m_t.$$

The KPR utility function for this dynamic economy is

$$\mathbb{U}^{KPR} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\eta} \left\{ c_t^{1-\eta} \left[1 - \kappa(1-\eta)n_t^{1+\lambda} \right]^{\eta} - 1 \right\} + v(g_t) \right],$$

where $v(\cdot)$ is an increasing function. The additively separable utility function is

$$\mathbb{U}^{AS} = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\eta} - 1}{1-\eta} - \kappa \psi^{t(1-\eta)} n_t^{1+\lambda} + v(g_t) \right].$$

The preference over labor supply shifts with the level of technology, $\psi^{t(1-\eta)}$, to guarantee the existence of a balanced growth path, as utilized by Erceg, Guerrieri, and Gust (2006). The GHH utility function is

$$\mathbb{U}^{GHH} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\bar{\eta}} \left\{ \left(c_t - \kappa \psi^t n_t^{1+\lambda} \right)^{1-\bar{\eta}} - 1 \right\} + v(g_t) \right].$$

The preference shift parameter for the balanced growth path is $\psi^{t(1-\eta)}$ in this case.

The tax revenue curve for consumption tax is given by the relationship between the tax revenue and The tax rate on the balanced growth path. With regard to the use of tax revenue, the following two schemes are considered.

Definition A. 7. *Scheme (3): Government bond, b, grows at the balanced growth rate and g/y is constant. The other changes in tax revenue are adjusted by a lump-sum transfer to households.*

$$s = T - (R^b - 1)b - g, \qquad g/y = \phi_{gy}$$

Definition A. 8. *Scheme (4): b/y and s/y are constant. The other changes in tax revenue are adjusted by government consumption.*

$$g = T - (R^b - 1)b - s, \qquad s/y = \phi_{sy}, \qquad b/y = \phi_{by}$$

Assumption A. 1. *The ratio of net imports to GDP, m/y, is constant.*

B.2   Equilibrium system of the dynamic model

The equilibrium system of the dynamic model is

$$(1 + \tau_t^c)\lambda_t = u_1(c_t, n_t),$$

$$\lambda_t(1 - \tau_t^n)w_t = -u_2(c_t, n_t),$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[(1 - \delta) + (1 - \tau_{t+1}^k)(d_{t+1} - \delta) + \delta \right] \right\},$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} R_{t+1}^b \right],$$

$$k_t = (1 - \delta)k_{t-1} + x_t,$$

$$y_t = \xi^t [k_{t-1}]^\theta n_t^{1-\theta},$$

$$w_t = (1 - \theta)\frac{y_t}{n_t},$$

$$d_t = \theta\frac{y_t}{k_{t-1}},$$

$$y_t = c_t + x_t + g_t - m_t,$$

$$T_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k (d_t - \delta)k_{t-1},$$

where, if the utility function is KPR $\mathbb{U}^{KPR}$, marginal utility is defined as

$$U_c(c_t, n_t) \equiv (c_t)^{-\eta} \left[1 - \kappa(1 - \eta)n_t^{1+\lambda} \right]^\eta,$$

$$U_n(c_t, n_t) \equiv -\eta(1 + \lambda) \left\{ (c_t)^{1-\eta} \left[1 - \kappa(1 - \eta)n_t^{1+\lambda} \right]^{\eta-1} \kappa n_t^\lambda \right\}$$

if the utility function is additively separable $\mathbb{U}^{AS}$, by

$$U_c(c_t, n_t) \equiv (c_t)^{-\eta},$$

$$U_n(c_t, n_t) \equiv -\kappa\psi^{t(1-\eta)}(1 + \lambda)n_t^\lambda$$

and if the utility function is GHH $\mathbb{U}^{GHH}$, by

$$U_c(c_t, n_t) \equiv (c_t - \kappa\psi^t n_t^{1+\lambda})^{-\bar\eta},$$

$$U_n(c_t, n_t) \equiv -\kappa\psi^t(1 + \lambda)n_t^\lambda (c_t - \psi^t \kappa n_t^{1+\lambda})^{-\bar\eta}.$$

The detrended equilibrium system is

$$(1 + \tau_t^c)\tilde{\lambda}_t = u_1(\tilde{c}_t, n_t),$$

$$\tilde{\lambda}_t(1 - \tau_t^n)\tilde{w}_t = -u_2(\tilde{c}_t, n_t),$$

$$\tilde{\lambda}_t = \beta\psi^{-\eta}E_t\left\{\tilde{\lambda}_{t+1}\left[(1 - \delta) + (1 - \tau_{t+1}^k)(d_{t+1} - \delta) + \delta\right]\right\},$$

$$\tilde{\lambda}_t = \beta\psi^{-\eta}E_t\left[\tilde{\lambda}_{t+1}R_{t+1}^b\right],$$

$$\psi\tilde{k}_t = (1 - \delta)\tilde{k}_{t-1} + \tilde{x}_t,$$

$$\tilde{y}_t = \left[\tilde{k}_{t-1}\right]^\theta n_t^{1-\theta},$$

$$\tilde{w}_t = (1 - \theta)\frac{\tilde{y}_t}{n_t},$$

$$d_t = \theta\frac{\tilde{y}_t}{\tilde{k}_{t-1}}.$$

$$\tilde{y}_t = \tilde{c}_t + \tilde{x}_t + \tilde{g}_t - \tilde{m}_t,$$

$$\tilde{T}_t = \tau_t^c\tilde{c}_t + \tau_t^n\tilde{w}_t n_t + \tau_t^k(d_t - \delta)\tilde{k}_{t-1}.$$

On the balanced growth path, the system becomes

$$(1 + \tau^c)\tilde{\lambda} = u_1(\tilde{c}, n),$$

$$\tilde{\lambda}(1 - \tau^n)\tilde{w} = -u_2(\tilde{c}, n),$$

$$1 = \beta\psi^{-\eta}\left[(1 - \delta) + (1 - \tau^k)(d - \delta) + \delta\right],$$

$$1 = \beta\psi^{-\eta}R^b,$$

$$\psi\tilde{k} = (1 - \delta)\tilde{k} + \tilde{x},$$

$$\tilde{y} = \left[\tilde{k}\right]^\theta n^{1-\theta},$$

$$\tilde{w} = (1 - \theta)\frac{\tilde{y}}{n},$$

$$d = \theta\frac{\tilde{y}}{\tilde{k}}.$$

$$\tilde{y} = \tilde{c} + \tilde{x} + \tilde{g} - \tilde{m}_t,$$

$$\tilde{T} = \tau_t^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d - \delta)\tilde{k}.$$

Scheme (3): Changes in tax revenue are adjusted by a lump-sum transfer: Under Scheme (3), $\tilde{g}/\tilde{y} = \phi_g$ and $\tilde{m}/\tilde{y} = \phi_m$ are constant. Then, the balanced growth path values are obtained by

$$R^b = \frac{\psi^\eta}{\beta},$$

$$d = \frac{1}{1 - \tau^k}\left[R^b - 1 + \delta\right],$$

$$\frac{\tilde{k}}{\tilde{y}} = \frac{\theta}{d},$$

$$\frac{\tilde{x}}{\tilde{y}} = [\psi - (1 - \delta)]\frac{\tilde{k}}{\tilde{y}},$$

$$\frac{\tilde{c}}{\tilde{y}} = 1 - \frac{\tilde{x}}{\tilde{y}} - \frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{m}}{\tilde{y}},$$

$$\frac{n}{\tilde{y}} = \left[\frac{\tilde{y}}{\tilde{k}}\right]^{\theta/(1-\theta)},$$

$$\tilde{w} = (1 - \theta)\frac{\tilde{y}}{\tilde{n}},$$

From this system, the following lemma and corollary are obtained from the balanced growth path equilibrium system.

Lemma A. 1. *On the balanced growth path, the dividend (d), capital–output ratio ($k/y = \tilde{k}/\tilde{y}$), investment–output ratio ($x/y = \tilde{x}/\tilde{y}$), consumption–output ratio ($c/y = \tilde{c}/\tilde{y}$), and labor–output ratio ($n/\tilde{y}$) are independent from the consumption tax rate (τ^c).*

Remark A. 3. *The elasticity of consumption with respect to the consumption tax rate equals that of output:*

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = \left|\frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c}\right|.$$

Scheme (4): Changes in tax revenue are adjusted by government consumption: By the budget constraint, the following is obtained:

$$\tilde{T} = \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d - \delta)\tilde{k}.$$

Dividing by $\tilde{y}$ yields

$$\frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{s}}{\tilde{y}} + (R^b - 1)\frac{\tilde{b}}{\tilde{y}} = \tau^c\frac{\tilde{c}}{\tilde{y}} + \tau^n\tilde{w}\frac{\tilde{n}}{\tilde{y}} + \tau^k(d - \delta)\frac{\tilde{k}}{\tilde{y}}$$

Since $n/\tilde{y}$ and $\tilde{k}/\tilde{y}$ are independent from τ^c,

$$\frac{\tilde{g}}{\tilde{y}} = \tau^c\frac{\tilde{c}}{\tilde{y}} + constant.$$

The resource constraint can be rewritten as

$$c + i + g - m = y$$

$$\Longleftrightarrow \quad \frac{\tilde{c}}{\tilde{y}} + \frac{\tilde{i}}{\tilde{y}} + \tau^c\frac{\tilde{c}}{\tilde{y}} + const - \frac{\tilde{m}}{\tilde{y}} = 1.$$

Since $\tilde{i}/\tilde{y}$ and $\tilde{m}/\tilde{y}$ is independent from τ^c, it follows that

$$(1 + \tau^c)\frac{\tilde{c}}{\tilde{y}} = constant.$$

Therefore, the following lemma holds.

Lemma A. 2. *$(1 + \tau^c)\tilde{c}/\tilde{y}$ is independent from τ^c*

By Lemma A.2, the following is obtained.

Remark A. 4. *The elasticity of consumption with respect to the consumption tax rate equals that of $y/(1 + \tau^c)$:*

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = \left|\frac{d(1 + \tau^c)^{-1}\tilde{y}/(1 + \tau^c)^{-1}\tilde{y}}{d\tau^c/\tau^c}\right|$$

B.3 Consumption tax revenue curve in the dynamic model

In the main text, the total tax revenue curve is investigated as the tax revenue curve. Here, the results for the consumption tax revenue curve are shown.

B.3.1 Scheme (3): Changes in tax revenue are adjusted by a lump-sum transfer

Propositions A.19, A.20, and A.21 refer to the consumption tax revenue curve under Scheme (3) in the dynamic economy.

Proposition A. 19. *Suppose that the utility function is KPR, $\mathbb{U}^{KPR}$. The consumption tax revenue curve for consumption tax under Scheme (3) is monotonically increasing. The consumption tax revenue is unbounded except for $\lambda = 0$.*

Proof. The optimization condition for the consumption–labor choice,

$$\eta(1+\lambda)\left\{\frac{\kappa\tilde{c}n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right\} = \frac{1-\tau^{n}}{1+\tau^{c}}(1-\theta)\frac{\tilde{y}}{h},$$

yields

$$\tilde{y} = \left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1+\tau^{c}}{1-\tau^{n}}\right]^{-1/(1+\lambda)}.$$

Since $\tilde{y} > 0$ for $\tau^{c} \geq 0$,

$$(1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}} > 0.$$

By Remark A.3, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}} = \frac{d\tilde{y}/\tilde{y}}{d\tau^{c}/\tau^{c}} = -\frac{\frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{\tau^{c}}{1-\tau^{n}}}{(1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\left(\frac{1+\tau^{c}}{1-\tau^{n}}\right)(1+\lambda)}.$$

Letting

$$\Psi = (1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\left(\frac{1+\tau^{c}}{1-\tau^{n}}\right)(1+\lambda) > 0,$$

it follows that

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}}\right| - 1 = -\frac{1}{\Psi}\left\{(1-\eta) + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}} + \frac{1}{1-\theta}\left(\frac{\tilde{c}}{\tilde{y}}\right)\eta\frac{\tau^{c}}{1-\tau^{n}}\lambda\right\} \leq 0.$$

The consumption tax revenue is unbounded if $\lambda > 0$, since $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| > 0$. In the case of $\lambda = 0$, the consumption tax revenue is given by

$$
\tau^c \tilde{c} = \phi \tau^c \tilde{y}
$$

$$
= \left(\frac{\tilde{y}}{n} \right) (\kappa)^{-1} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta \frac{1 + \tau^c}{1 - \tau^n} \right]^{-1} \tau^c
$$

$$
= \left(\frac{\tilde{y}}{n} \right) (\kappa)^{-1} \left[\frac{1 - \eta}{\tau^c} + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta \frac{1 + \tau^c}{\tau^c} \frac{1}{1 - \tau^n} \right]^{-1},
$$

where $\phi = c/y$. This converges to $\left(\frac{\tilde{y}}{n} \right) (\kappa)^{-1} \left[\frac{1}{1-\theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta \frac{1}{1-\tau^n} \right]^{-1}$. $\square$

Proposition A. 20. *Suppose that the utility function is additively separable; $\mathbb{U}^{AS}$. The consumption tax revenue curve for consumption tax under Scheme (3) is hump shaped if and only if $\eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta + \lambda \le 1$. Otherwise, it is unbounded.*

Proof. By the optimization condition for the consumption–labor choice,

$$
\kappa(1 + \lambda)\tilde{c}^\eta n^\lambda = \frac{1 - \tau^n}{1 + \tau^c}\tilde{w},
$$

it follows that

$$
\tilde{y} = (1 + \tau^c)^{-1/(\eta+\lambda)} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{\tilde{c}}{\tilde{y}} \right)^{-\eta} \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/(\eta+\lambda)}.
$$

By Remark A.3, it follows that

$$
\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c} = -\frac{1}{\eta + \lambda} \cdot \frac{\tau^c}{1 + \tau^c}.
$$

Then,

$$
\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| - 1 = \frac{1}{\eta + \lambda} \cdot \frac{1}{1 + \tau^c} \left\{ (1 - \eta - \lambda)\tau^c - (\eta + \lambda) \right\}.
$$

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Suppose $\eta + \lambda = 1$. In this case, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 < 0$.

Suppose $\eta + \lambda \neq 1$. In this case,

$$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| - 1 = \frac{1-\eta-\lambda}{\eta+\lambda} \cdot \frac{1}{1+\tau^c}\left\{\tau^c - \frac{\eta+\lambda}{1-\eta-\lambda}\right\}.$$

If $\eta + \lambda \geq 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \geq 0$.

If $\eta + \lambda < 1$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \leq 1$ for $\tau^c \leq (\eta + \lambda)/(1 - \eta - \lambda)$, and $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| > 1$ for $\tau^c > (\eta + \lambda)/(1 - \eta - \lambda)$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\eta + \lambda < 1$ and unbounded if $\eta + \lambda > 1$. In the case of $\eta + \lambda = 1$, the consumption tax revenue is

$$\tau^c\tilde{c} = \phi\tau^c\tilde{y}$$

$$= \phi\frac{\tau^c}{1+\tau^c}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right].$$

where $\phi = c/y$. This converges to $\phi\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]$ as $\tau^c \to \infty$. $\qquad \square$

Proposition A. 21. *Suppose that the utility function is additively separable, $\mathbb{U}^{GHH}$. The consumption tax revenue curve for consumption tax under Scheme (3) is hump shaped if and only if $\lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\lambda}{1-\lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\lambda \leq 1$. Otherwise, it is unbounded.*

Proof. By the optimization condition for the consumption–labor choice,

$$\kappa(1+\lambda)n^\lambda = \frac{1-\tau^n}{1+\tau^c}\tilde{w},$$

it follows that

$$\tilde{y} = (1+\tau^c)^{-1/\lambda}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}.$$

By Remark A.3, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c} = -\frac{1}{\lambda} \cdot \frac{\tau^c}{1 + \tau^c}.$$

Then,

$$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| - 1 = \frac{1}{\lambda} \cdot \frac{1}{1 + \tau^c} \left\{ (1 - \lambda)\tau^c - \lambda \right\}.$$

Suppose $\lambda = 1$. In this case, $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| - 1 < 0$.

Suppose $\lambda \neq 1$. In this case,

$$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| - 1 = \frac{1 - \lambda}{\lambda} \cdot \frac{1}{1 + \tau^c} \left\{ \tau^c - \frac{\lambda}{1 - \lambda} \right\}.$$

If $\lambda \geq 1$, then $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| \leq 1$ for $\tau^c \geq 0$.

If $\lambda < 1$, then $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| < 1$ for $\tau^c \leq \lambda/(1 - \lambda)$,

$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| = 1$ for $\tau^c = \lambda/(1 - \lambda)$, and

$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| > 1$ for $\tau^c > \lambda/(1 - \lambda)$.

By the elasticity of consumption, it is obvious that the consumption tax revenue is bounded if $\eta + \lambda < 1$ and unbounded if $\lambda > 1$. In the case of *lambda* $= 1$, the consumption tax revenue is

$$\tau^c \tilde{c} = \phi \tau^c \tilde{y}$$

$$= \phi \frac{\tau^c}{1 + \tau^c} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/\lambda}.$$

where $\phi = c/y$. This converges to $\phi \left[\frac{1-\theta}{\kappa(1+\lambda)} (1 - \tau^n) \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/\lambda}$ as $\tau^c \to \infty$. □

Note that these propositions are the same as Propositions 1, 2, and 3 in the main text, while the dynamic economy has a far richer structure (capital, investment, debt evolution, etc.) than the static economy does.

B.3.2 Scheme (4): Changes in tax revenue are adjusted by government consumption

Proposition A. 22. *Suppose that the utility function is KPR, $\mathbb{U}^{KPR}$. The consumption tax revenue curve for consumption tax under Scheme (4) is monotonically increasing. The consumption tax revenue is bounded.*

Proof. By the consumption–labor choice condition, $\tilde{y}$ is obtained

$$\eta(1+\lambda)\left\{\frac{\kappa\tilde{c}n^{\lambda}}{1-\kappa(1-\eta)n^{1+\lambda}}\right\} = \frac{1-\tau^{n}}{1+\tau^{c}}(1-\theta)\frac{\tilde{y}}{h}$$

$$\iff \eta(1+\lambda)\left\{\frac{\kappa\left(\frac{\tilde{c}}{\tilde{y}}\right)\left(\frac{n}{\tilde{y}}\right)^{\lambda}}{\tilde{y}^{-1-\lambda}-\kappa(1-\eta)\left(\frac{n}{\tilde{y}}\right)^{1+\lambda}}\right\} = \frac{1-\tau^{n}}{1+\tau^{c}}(1-\theta)\frac{\tilde{y}}{n}$$

$$\iff \tilde{y} = \left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left((1+\tau^{c})\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}}\right]^{-1/(1+\lambda)}.$$

Then, $\tilde{y}$ is independent from τ^{c}.

By Remark A.4, the elasticity of $(1+\tau^{c})^{-1}\tilde{y}$ is considered. Since

$$(1+\tau^{c})^{-1}\tilde{y} = (1+\tau^{c})^{-1}\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left((1+\tau^{c})\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}}\right]^{-1/(1+\lambda)},$$

it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}} = \frac{d(1+\tau^{c})^{-1}\tilde{y}/(1+\tau^{c})^{-1}\tilde{y}}{d\tau^{c}/\tau^{c}}$$

$$= -\frac{\tau^{c}}{1+\tau^{c}}.$$

$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}}\right|$ is monotonically increasing in τ^{c}. If $\tau^{c}=0$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}}\right| = 0$. As $\tau^{c}\to\infty$, $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^{c}/\tau^{c}}\right| \to 1$. Therefore, the consumption tax revenue curve is monotonically increasing.

The boundedness is shown as follows. Let $(1+\tau^{c})\tilde{c}/\tilde{y} = \phi$; then,

$$\tau^{c}\tilde{c} = \tau^{c}\phi\tilde{y}(1+\tau^{c})^{-1}$$

$$= \phi\frac{\tau^{c}}{1+\tau^{c}}\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta)+\frac{1}{1-\theta}\left((1+\tau^{c})\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^{n}}\right]^{-1/(1+\lambda)}.$$

As $\tau^c \to \infty$, it converges to $\phi\left(\frac{\tilde{y}}{n}\right)(\kappa)^{-1/(1+\lambda)}\left[(1-\eta) + \frac{1}{1-\theta}\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)\eta(1+\lambda)\frac{1}{1-\tau^n}\right]^{-1/(1+\lambda)}$.

$\square$

Proposition A. 23. *Suppose that the utility function is additively separable, $\mathbb{U}^{AS}$. The consumption tax revenue curve for consumption tax under Scheme (4) is hump shaped if and only if $\eta < 1$, and the revenue is maximized at $\tau^c = \frac{\eta+\lambda}{1-\eta}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing. The consumption tax revenue is bounded if and only if $\eta \leq 1$. Otherwise, it is unbounded.*

Proof. By the consumption–labor choice condition,

$$\kappa(1+\lambda)\tilde{c}^\eta n^\lambda = \frac{1-\tau^n}{1+\tau^c}\tilde{w}$$

$$\Longleftrightarrow \quad \kappa(1+\lambda)\tilde{y}^{\eta+\lambda}\left(\frac{\tilde{c}}{\tilde{y}}\right)^\eta\left(\frac{n}{\tilde{y}}\right)^\lambda = \frac{1-\tau^n}{1+\tau^c}(1-\theta)\frac{\tilde{y}}{n}$$

$$\Longleftrightarrow \quad \tilde{y} = (1+\tau^c)^{-(1-\eta)/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)}.$$

By Remark A.4, the elasticity of $(1+\tau^c)\tilde{y}$ is considered. Since

$$(1+\tau^c)^{-1}\tilde{y} = (1+\tau^c)^{-(1+\lambda)/(\eta+\lambda)}\left[\frac{1-\theta}{\kappa(1+\lambda)}(1-\tau^n)\left((1+\tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(\eta+\lambda)},$$

it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d(1+\tau^c)^{-1}\tilde{y}/(1+\tau^c)^{-1}\tilde{y}}{d\tau^c/\tau^c}$$

$$= -\frac{1+\lambda}{\eta+\lambda}\cdot\frac{\tau^c}{1+\tau^c}.$$

$\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| = 0$. If $\tau^c \to \infty$, then $\left|\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c}\right| \to (1+\lambda)/(\eta+\lambda)$. Therefore, a necessary and sufficient condition for a hump-shaped consumption tax revenue curve is $\eta < 1$. The peak tax rate is

$$\tau^c_{max} = \frac{\eta+\lambda}{1-\eta}.$$

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By the elasticity of consumption, it is obvious that the tax revenue is bounded if $\eta < 1$ and unbounded if $\eta > 1$. Suppose $\eta = 1$. Letting $(1 + \tau^c)\tilde{c}/\tilde{y} = \phi$, the following is obtained:

$$\tau^c \tilde{c} = \tau^c \phi \tilde{y}(1 + \tau^c)^{-1}$$

$$= \phi \frac{\tau^c}{1 + \tau^c} \left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-1}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(1+\lambda)}.$$

As $\tau^c \to \infty$, it converges to $\phi \left[\frac{1-\theta}{\kappa(1+\lambda)}(1 - \tau^n)\left((1 + \tau^c)\frac{\tilde{c}}{\tilde{y}}\right)^{-1}\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/(1+\lambda)}.$

$\square$

Proposition A. 24. *Suppose that the utility function is additively separable, $\mathbb{U}^{GHH}$. The consumption tax revenue curve for consumption tax under Scheme (4) is hump shaped, and the revenue is maximized at $\tau^c = \lambda$. The consumption tax revenue is bounded.*

Proof. By the consumption–labor choice condition,

$$\kappa(1 + \lambda)n^\lambda = \frac{1 - \tau^n}{1 + \tau^c}\tilde{w}$$

$$\iff \quad \kappa(1 + \lambda)\tilde{y}^\lambda\left(\frac{n}{\tilde{y}}\right)^\lambda = \frac{1 - \tau^n}{1 + \tau^c}(1 - \theta)\frac{\tilde{y}}{n}$$

$$\iff \quad \tilde{y} = (1 + \tau^c)^{-1/\lambda}\left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda}.$$

By Remark A.4, the elasticity of $(1 + \tau^c)\tilde{y}$ is considered. Since

$$(1 + \tau^c)^{-1}\tilde{y} = (1 + \tau^c)^{-(1+\lambda)/\lambda}\left[\frac{1 - \theta}{\kappa(1 + \lambda)}(1 - \tau^n)\left(\frac{n}{\tilde{y}}\right)^{-1-\lambda}\right]^{1/\lambda},$$

it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d(1 + \tau^c)^{-1}\tilde{y}/(1 + \tau^c)^{-1}\tilde{y}}{d\tau^c/\tau^c}$$

$$= -\frac{1 + \lambda}{\lambda} \cdot \frac{\tau^c}{1 + \tau^c}.$$

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$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right|$ is monotonically increasing in τ^c. If $\tau^c = 0$, then $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| = 0$. If $\tau^c \to \infty$, then $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| \to (1 + \lambda)/\lambda > 1$. Therefore, the consumption tax revenue curve is hump shaped. The peak tax rate is

$$\tau^c_{max} = \lambda.$$

It is obvious that the tax revenue is bounded.

$\square$

These propositions are the same as Propositions 4, 5, and 6 in the main text, which concern the consumption tax revenue curve in the static economy.