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A benefit of monetary policy response to inequality*

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Abstract

The main objective of this paper is to investigate a monetary policy response to inequality in a Two-Agent New Keynesian (TANK) model with hand-to-mouth households. I derive the analytical condition for equilibrium determinacy and show that a monetary policy response to inequality is helpful in achieving equilibrium determinacy. On the other hand, the impulse responses to structural shocks show that a monetary policy response to inequality does not necessarily reduce the volatilities of both inflation and output although it mitigates the volatility of inequality.

Keywords: Inequality; monetary policy; TANK; hand-to-mouth; equilibrium indeterminacy

JEL classifications: E25; E31; E32; E52; E58

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1 Introduction

In recent years, much attention has been paid to the relationship between monetary policy and inequality, both in academia and in practice. The central bank conducts monetary policy mainly by controlling the nominal interest rate. Economic agents are heterogeneous in many aspects including age, gender, wealth, productivity, and employment status. Therefore, a change in the nominal interest rate can have different effects on different economic agents. In other words, monetary policy has a redistribution effect on the economy. From this viewpoint, there is an argument to be made that the central bank should manage monetary policy considering the level of inequality in the economy.

Existing studies have analyzed the effects of monetary policy on inequality, both empirically and theoretically. If we were to go a step further and suppose that inequality is made the mandate of the central bank, what is the macroeconomic consequence of this type of monetary policy?

The main objective of this paper is to examine the effect of a monetary policy response to inequality in a Two-Agent New Keynesian (TANK) model. There are standard optimizing households and hand-to-mouth (HtM) households in the model. The inequality measure used in this paper is the gap of income between optimizing and HtM households. In this model, monetary easing reduces inequality, and the central bank lowers the nominal interest rate if inequality rises. Existing studies find that the Taylor principle no longer guarantees equilibrium determinacy in TANK models by numerical analyses. I derive the analytical condition for equilibrium determinacy and show that the monetary policy response to inequality is helpful in achieving equilibrium determinacy. This determinacy result is robust for a model with capital and a model with an alternative inequality measure. The impulse responses to some structural shocks are also investigated. The results show that a monetary policy response to inequality does not necessarily reduce both inflation and output volatilities while it mitigates the volatility of inequality.

The key to understanding the determinacy results is the relationship between inflation and inequality. In a standard sticky price model, the central bank needs to fight inflation to achieve equilibrium determinacy, which is known as the Taylor principle. While the Taylor principle no longer guarantees equilibrium determinacy in TANK models, in this paper, I analytically find that equilibrium determinacy can be still achieved if the central bank increases the nominal interest rate by a sufficiently large amount. In the model, a permanent increase in inflation reduces inequality. Therefore, the monetary policy response to inequality strengthens the overall reaction of the central bank to inflation, which ensures that equilibrium determinacy can be achieved.

Note that these results do not justify the central bank's response to inequality. The impulse response analyses show that the monetary policy response to inequality does not necessarily reduce both inflation and output volatility. According to the paper by Debirtoli and Galí (2017), price stability is nearly optimal even in their TANK model although consumption heterogeneity is included in their welfare function. Then, inequality itself would not be the mandate of the central bank from the viewpoint of welfare. However, the main results of this paper do imply that if the central bank responds to inequality, it is beneficial in the sense that it enlarges the determinacy region of equilibrium.

Related literature: The effect of monetary policy on inequality has been analyzed by many researchers. For example, the empirical effects of monetary policy on inequality are investigated by Carpenter and Rodgers (2004), Coibion et al. (2017), Andersen et al. (2021), and Bartscher et al. (2021). On the other hand, Kaplan, Moll, and Violante (2018), Auckert (2019), Gornemann, Kuester, and Nakajima (2021), Bayer, Born, and Luetticke (2021), Eskelinen (2021), and Nakajima (2021) employ heterogeneous agent New Keynesian (HANK) models that include many aspects of heterogeneity among households. While the effects of monetary policy on inequality are considered in these papers, this paper makes an original contribution to the literature by investigating the effects of a monetary policy responses to inequality.

In this paper, a TANK model is employed for simplicity of analysis. TANK models have been employed by many researchers, including Amato and Laubach (2003), Galí, Lopez-Salido, and Valles (2004, 2007), Bilbiie and Ragot (2017), and Debirtoli and Galí (2017). The optimal monetary policy is investigated by Amato and Laubach (2003), Bilbiie and Ragot (2017) and Debirtoli and Gali (2017). On the other hand, I focus on equilibrium determinacy in a TANK model. In this sense, this paper is closely related to the work done by Galí, Lopez-Salido, and Valles (2004). They use numerical simulations to demonstrate that the Taylor principle no longer guarantees equilibrium determinacy in a TANK model.[1] I derive the analytical condition for equilibrium determinacy in my TANK model and show that the Taylor principle no longer guarantees equilibrium determinacy. In addition, I also find that the monetary policy response to inequality can be a solution to this indeterminacy problem.

The method used in this paper is closely related to that in the papers by Bullard and Schaling (2002), Carlstrom and Fuerst (2007), and Nutahara (2014, 2015), who have analyzed the monetary policy response to asset price fluctuations. These studies introduce a term that responds to asset price in the Taylor-type monetary policy rule, and analyze its effects on equilibrium determinacy. In this paper, I introduce a similar term that responds to inequality in the monetary policy rule.

Organization of the paper: The remainder of the paper is organized as follows. In Section 2, I introduce the baseline model and derive the key equations for analyses. Section 3 presents the main results and their interpretation. Section 4 presents the impulse responses to some structural shocks of the model and the effect of the monetary policy response to inequality on these impulse responses. The robustness of the main results is also shown in a case with capital and one with an alternative inequality measure. Section

[1] Ravn and Sterk (2021) also show that the Taylor principle no longer guarantees equilibrium determinacy in a HANK model.

5 presents the conclusion.

2 The Model

2.1 Model

In this model, there are two types of households: optimizing and HtM households. The total population of households is normalized to be one. The number of optimizing households is $n \in (0, 1)$, while that of HtM is $1 - n$. There are two types of firms: final good firms and intermediate good firms. The final good market is perfectly competitive while the intermediate good market is monopolistically competitive.

Optimizing Households: An optimizing household consumes C_t^O, holds safe asset B_t^O and supplies labor service L_t^O.

The utility function of the optimizing household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^O)^{1-\sigma} - 1}{1 - \sigma} - \mu \frac{(L_t^O)^{1+\gamma}}{1 + \gamma} \right], \tag{1}$$

where $\beta \in (0, 1)$ denotes the discount factor; $\sigma > 0$, the relative risk aversion; and $\chi > 0$, the inverse of the labor supply elasticity.

The budget constraint is

$$P_t C_t^O + B_{t+1}^O \leq R_{t-1} B_t^O + P_t W_t L_t^O + P_t D_t, \tag{2}$$

where P_t denotes the price level; C_t^O is consumption; B_t^O is nominal bond holding; R_{t-1} is the (risk-free) gross nominal interest rate from bond holding; W_t is the real wage; and D_t is the dividend from firms. The optimizing households are the owners of firms and receive firms' profits as dividends.

The first order conditions are given by

$$\Lambda_t^O = (C_t^O)^{-\sigma}, \tag{3}$$

$$\mu(L_t^O)^\gamma = \Lambda_t^O W_t, \tag{4}$$

$$\Lambda_t^O = \beta E_t \left[\Lambda_{t+1}^O \cdot \frac{R_t}{\pi_{t+1}} \right], \tag{5}$$

where Λ_t^O is the marginal utility of consumption; and $\pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate.

Hand-to-Mouth (HtM) Households: HtM households cannot access to the asset market and do not smooth their consumption. In each period, all labor income is consumed; thus, the budget constraint is given by

$$P_t C_t^H \leq P_t W_t L_t^H, \tag{6}$$

where C_t^H denotes consumption and L_t^H denotes labor supply.

The utility function of HtM households is the same as that of the optimizing household:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^H)^{1-\sigma} - 1}{1-\sigma} - \mu \frac{(L_t^H)^{1+\gamma}}{1+\gamma} \right]. \tag{7}$$

Then, the first order condition is given by

$$\mu(C_t^H)^{-\sigma}(L_t^H)^\gamma = W_t. \tag{8}$$

Final good firms: The final good market is perfectly competitive. The final good firm produces a final good Y_t using intermediate good $Y_t(j)$. The production function is given by

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta_P - 1}{\theta_P}} dj \right]^{\frac{\theta_P}{\theta_P - 1}}, \tag{9}$$

where θ_P is the elasticity of substitution among intermediate goods. Letting $P_t(j)$ denote the price of intermediate good $Y_t(j)$, the first order condition of the profit maximization problem is

$$Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\theta_P} Y_t. \tag{10}$$

Intermediate good firms: The intermediate good market is monopolistically competitive. The intermediate good firm indexed by j produces a differentiated intermediate good $Y_t(j)$ using labor input $L_t(j)$. The production function is given by

$$Y_t(j) = L_t(j). \tag{11}$$

Letting W_t denote the real wage rate, the first order condition of the cost minimization problem is

$$W_t = MC_t, \tag{12}$$

where MC_t is the Lagrange multiplier and it can be interpreted as the real marginal cost of the intermediate good firm.

Sticky prices are introduced as in Calvo (1983). At every period, a fraction $1 - \delta_P \in [0, 1]$ of the intermediate good firms can reoptimize their prices. The remainder of the firms does not change their prices. The objective function of the intermediate good firms that do reoptimize their prices at period t is given by

$$E_t \sum_{s=0}^{\infty} (\beta\delta_P)^s \left(\frac{\Lambda_{t+s}^O}{\Lambda_t^O}\right)\left[\left(\frac{P_t(j)}{P_{t+s}}\right) Y_{t+s}(j) - TC(Y_{t+s}(j))\right], \tag{13}$$

where Λ_t^O is the marginal utility of consumption of the optimizing households, $TC(\cdot)$ is the total cost function, and $\beta^s \frac{\Lambda_{t+s}^O}{\Lambda_t^O}$ is the stochastic discount factor. The demand function for $Y_{t+s}(j)$ is given by the equation (10).

The reoptimized price P_t^o is the same for all intermediate good firms. The first order condition for the reoptimized price P_t^o is

$$1 = \frac{E_t \sum_{s=0}^{\infty} (\beta\delta_P)^s \theta_P MC_{t+s} \Lambda_{t+s} Y_{t+s} \left[\frac{P_t^0}{P_{t+s}}\right]^{-\theta_P}}{E_t \sum_{s=0}^{\infty} (\beta\delta_P)^s (\theta_P - 1) \Lambda_{t+s} Y_{t+s} \left[\frac{P_t^0}{P_{t+s}}\right]^{1-\theta_P}}. \tag{14}$$

Market clearing conditions: The goods market clearing condition is given by

$$Y_t = nC_t^O + (1-n)C_t^H. \tag{15}$$

The labor market clearing condition is given by

$$L_t = nL_t^O + (1-n)L_t^H. \tag{16}$$

The government does not issue bonds in this economy. Thus, the optimizing households are the only participants of the nominal bond market, and there is no heterogeneity among them. Finally, the market clearing condition for the bond market is given by

$$B_t^O = 0. \tag{17}$$

Inequality measure: The definition of the inequality measure Q_t is given by

$$Q_t = \frac{\text{(income of optimizing households)}}{\text{(income of HtM households)}}. \tag{18}$$

I focus on the income inequality between the optimizing and HtM households.

In this model, income inequality is the same as consumption inequality. HtM household's income is consumed as shown in (6), while the income of the optimizing households becomes their consumption by (2) and (17). Thus, the following is obtained at equilibrium:

$$Q_t = \frac{C_t^O}{C_t^H}. \tag{19}$$

Because the optimizing households receive the dividend from firms, consumption of the optimizing households is greater than that of HtM households. Appendix A shows it.

Monetary Policy: The central bank controls the nominal interest rate R_t following an extended Taylor rule as given below:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Q_t}{Q}\right)^{-\phi_Q}, \tag{20}$$

where $\phi_\pi > 1$ and $\phi_Q \geq 0$ denote the central bank's stances on inflation and inequality, respectively. R, π and Q denote the steady-state values of R_t, π_t and Q_t, respectively. In this paper, I focus on the case with $\phi_\pi > 1$, in which the Taylor principle is satisfied. If $\phi_Q > 0$, the central bank is meant to ease monetary policy if inequality rises.[2] As I show in Section 4.1, monetary easing reduces inequality in this model.

2.2 Key equations

For simplicity of analysis, I focus on a steady state where the trend inflation is zero and the aggregate labor supply L is one.[3] Let $\hat{A}_t$ denote the log-deviation from its steady-state value. The log-linearized equilibrium system is given in Appendix B.

The intertemporal equations in the log-linearized equilibrium system are (i) the Euler equation of the optimizing households:

$$-\sigma\hat{C}_t^O = -\sigma\hat{C}_{t+1}^O - \hat{\pi}_{t+1} + \hat{R}_t,$$

and (ii) the New Keynesian Phillips curve:

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \lambda\widehat{MC}_t,$$

where $\lambda = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa}$. The nominal interest rate is given by the monetary policy rule:

$$\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_Q\hat{Q}_t.$$

[2]For simplicity of analysis, the output term is omitted from the monetary policy rule. Even in the case where monetary policy responds to output, the main result does not change qualitatively.

[3]This implies that the value of the weight parameter of disutility from labor supply ϕ is calibrated to be consistent with $L = 1$.

The keys are consumptions of the optimizing and HtM households. They are given by

$$\hat{C}_t^H = \chi \widehat{MC}_t, \tag{21}$$

$$\hat{C}_t^O = \delta \widehat{MC}_t, \tag{22}$$

where

$$\chi = \frac{1+\gamma}{\sigma+\gamma} > 0,$$
$$\delta = \frac{\omega_L(1+\gamma) + \gamma\chi(\omega_C - \omega_L) - \gamma}{\sigma\omega_L + \gamma\omega_C}.$$

The parameters $\omega_L \in [0,1]$ and $\omega_C \in [0,1]$ are given in Appendix A. The coefficient δ is increasing in n, and it is negative if and only if

$$n < \bar{n} = 1 - \frac{1}{(1+\gamma)MC + \gamma\chi(1-MC)\left(\frac{MC^{1-\sigma}}{\mu}\right)^{1/(\sigma+\gamma)}}, \tag{23}$$

and $\delta \geq 0$ otherwise. In other words, consumption of the optimizing household is a negative function of the real marginal cost if and only if the number of HtM households is sufficiently large.

Then, the inequality measure is given by

$$\begin{aligned} \hat{Q}_t &= \hat{C}_t^O - \hat{C}_t^H \\ &= (\delta - \chi)\widehat{MC}_t, \end{aligned} \tag{24}$$

where

$$\delta - \chi = -\frac{\gamma}{\sigma\omega_L + \gamma\omega_C} < 0.$$

Therefore, inequality is decreasing in the real marginal cost.

Finally, the equilibrium system can be summarized as the following two-variable system:

$$\begin{bmatrix} 1 & \sigma\delta \\ \beta & 0 \end{bmatrix} \begin{bmatrix} \hat{\pi}_{t+1} \\ \widehat{MC}_{t+1} \end{bmatrix} = \begin{bmatrix} \phi_\pi & \sigma\delta - \phi_Q(\delta-\chi) \\ 1 & -\lambda \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \widehat{MC}_t \end{bmatrix}.$$

3 Main Results

3.1 Main proposition

The main result is given by the following Proposition 1.

Proposition 1. *Assume that $\phi_\pi > 1$.*

(1) If $n \geq \bar{n}$, equilibrium is determinate.

(2) If $n < \bar{n}$, a necessary and sufficient condition for equilibrium determinacy is given by

$$\lambda(\phi_\pi + 1) + (1+\beta)(\chi - \delta)\phi_q + 2(1+\beta)\sigma\delta > 0.$$

Otherwise, equilibrium indeterminacy arises.

Proof. See Appendix C. □

Proposition 1 implies that equilibrium is determinate if $\phi_\pi > 1$ and the number of HtM households is below the threshold. Proposition 1 also implies that the Taylor principle ($\phi_\pi > 1$) no longer guarantees equilibrium determinacy if the number of HtM households is above the threshold.

In the case with $\phi_q = 0$ and $n < \bar{n}$, a necessary and sufficient condition for equilibrium determinacy in Proposition 1 becomes

$$\phi_\pi > \bar{\phi}_\pi = -\frac{2(1+\beta)\sigma\delta}{\lambda} - 1. \tag{25}$$

The threshold value $\bar{\phi}_\pi$ is increasing in n because δ is increasing in the ratio of optimizing households n. In other words, if the number of HtM households increases, $\bar{\phi}_\pi$ increases, and then, equilibrium indeterminacy is more likely to arise. The threshold value $\bar{\phi}_\pi$ also depends on λ, which is decreasing in the price stickiness. In other words, if price stickiness increases, $\bar{\phi}_\pi$ increases. Then, equilibrium indeterminacy is more likely to arise. These analytical results are consistent with the numerical results of Galí, Lopez-Salido, and Valles (2004).

In the case with $\phi_q > 0$ and $n < \bar{n}$, equilibrium is determinate if and only if

$$\phi_q > \bar{\phi}_q = -\frac{2(1+\beta)\sigma\delta}{(1+\beta)(\chi-\delta)} - \frac{\lambda}{(1+\beta)(\chi-\delta)}(\phi_\pi + 1). \tag{26}$$

Then, if ϕ_q is high, equilibrium can be determinate. In other words, if the central bank is sufficiently sensitive to inequality, equilibrium determinacy can be achieved.

3.2 Numerical examples

Based on the analytical condition for equilibrium determinacy derived in the previous section, I show some numerical examples here.

The parameter values are those taken as standard in the literature. The model period is one quarter. The discount factor β is set as 0.99 so that the annual real interest rate is 4%. The relative risk aversion σ is set as 2. The inverse of the Frisch elasticity of labor supply γ is set as 2. The elasticity of substitution among intermediate goods θ_P is 6. Under this value, the steady-state markup rate is 20%. This markup rate is consistent with the micro evidence from De Loecker and Warzynski (2012). The reset price probability $1 - \delta_P$ is 0.34, following Khan, Phaneuf, and Victor (2020). This value is consistent with the estimates by Smets and Wouters (2007) and the micro evidence from Nakamura and Steinsson (2008).

Figure 1 shows the determinacy region of equilibrium in the case of $\phi_q = 0$. The horizontal axis is the central bank's stance on inflation $\phi_\pi \in (1, 10)$, and the vertical axis is the ratio of HtM households $1 - n \in (0.1, 0.9)$. I have discretized the $(\phi_\pi, 1-n)$ plane and checked the Blanchard-Kahn condition for each point. The region with diamonds indicates equilibrium determinacy, while the other region indicates equilibrium indeterminacy. The real line is the threshold value $\bar{\phi}_\pi$ for equilibrium determinacy, given by (25).

[Figure 1]

Figure 1 implies that an increase in the ratio of HtM households shrinks the determinacy region of ϕ_π, as shown analytically in Proposition 1. Even if the Taylor principle is satisfied ($\phi_\pi > 1$), equilibrium indeterminacy arises in the high $1-n$ region.

Figure 2 shows the determinacy region of equilibrium in the case where $\phi_\pi = 1.5$. The horizontal axis is the central bank's stance on inequality $\phi_q \in [0,4]$, and the vertical axis is the ratio of HtM households $1-n \in (0.1, 0.9)$. As before, the region with diamonds indicates equilibrium determinacy, while the other region indicates equilibrium indeterminacy. The real line is the threshold value $\bar{\phi}_q$ for equilibrium determinacy, given by (26).

[Figure 2]

Figure 2 implies that an increase in ϕ_q enlarges the determinacy region of equilibrium, as in Proposition 1. Note that the threshold value $\bar{\phi}_q$ for equilibrium determinacy is much smaller than the value of $\bar{\phi}_\pi$ in Figure 1. For example, if $1-n = 0.5$ and $\phi_q = 0$, $\bar{\phi}_\pi$ is approximately 10 as shown in Figure 1. By contrast, if $1-n = 0.5$ and $\phi_\pi = 1.5$, then $\bar{\phi}_q$ is approximately 1 as shown in Figure 2. Thus, the relatively low sensitivity of the central bank to inequality can achieve equilibrium determinacy.

3.3 Interpretation

It is useful to consider the effects of a permanent increase in inflation. In a simple standard New Keynesian model, the central bank needs to increase the nominal interest rate more than the increase in inflation to achieve equilibrium determinacy. This is known as the Taylor principle. If some households are HtM, the Taylor principle no longer guarantees equilibrium determinacy, as shown numerically by Galí, Lopez-Salido, and Valles (2004, 2007). Proposition 1 and (25) analytically show the same result if the number of HtM households is above the threshold. Nevertheless, even in such a case, Proposition 1 and (25) also show that equilibrium determinacy can be achieved if the central bank increases the nominal interest rate by a sufficiently large amount.

The key factor here is the effect of a permanent increase in inflation on inequality. If a permanent increase in inflation reduces inequality, then the monetary policy response to inequality is beneficial from the viewpoint of equilibrium determinacy. This is because the monetary policy response to inequality strengthens the overall reaction of the central bank to inflation through the monetary policy rule.

Suppose that a 1% permanent increase in inflation occurs. As $\hat{\pi}_t$ and $\hat{\pi}_{t+1}$ increase by 1%, the New Keynesian Phillips curve implies that the real marginal cost $\widehat{MC}_t$ also increases. This is because a fraction of firms cannot change their prices due to price stickiness. Through (24), this increase in the real marginal cost reduces inequality $\hat{Q}_t$. Finally, a permanent increase in inflation reduces inequality, and then, the monetary policy response to inequality strengthens the overall reaction of the central bank to inflation, and it is helpful in achieving equilibrium determinacy.

Why does an increase in the real marginal cost reduce inequality as in (24)? An increase in the real marginal cost implies an increase in the real wage rate. It should have positive effects on the incomes of both optimizing and HtM households. HtM households increase consumption as shown in (21). By contrast, optimizing households have another source of income, which is the dividend from firms D_t as in (2). This dividend is the monopolistic profit of intermediate good firms and is given by

$$D_t = \frac{1}{n}\left[Y_t - W_t L_t\right]. \tag{27}$$

The coefficient $1/n$ comes from the population of optimizing households, which is n.[4]

[4]The dividend (27) can be obtained as follows. At equilibrium, consumptions of HtM and optimizing households are given by

$$C_t^H = W_t L_t^H,$$
$$C_t^O = W_t L_t^O + D_t.$$

Taking the sum of the first equation multiplied by $1-n$ and the second equation multiplied by n, and using

Thus, the dividend can be rewritten as

$$D_t = \frac{1}{n}\left(1 - MC_t\right) Y_t.$$

The term of $1 - MC_t$ can be interpreted as the markup of firms and an increase in the real marginal cost can decrease the dividend. Carlstrom and Fuerst (2007) show that an increase in the real marginal cost decreases the dividend in a simple sticky price model. In this TANK model, this mechanism does work and a decrease in the dividend has negative effects on the income and consumption of optimizing households. As a result, an increase in the real marginal cost reduces inequality.

4 Discussions

4.1 Impulse responses and the effects of the monetary policy response to inequality

In the main results, I focus on equilibrium determinacy. In this section, I investigate the impulse responses to some structural shocks and how the monetary policy response to inequality affects these impulse responses. I consider five structural shocks, that are popularly used in the business cycle literature: productivity, monetary easing, price markup, discount factor, and labor disutility shocks.

To add these structural shocks, the baseline model is modified as follows. The productivity term A_t is introduced to the production function of an intermediate good firm:

$$Y_t(j) = e^{A_t} L_t(j).$$

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the market clearing conditions of the goods and the labor markets result in the following equation:

$$Y_t = W_t L_t + n D_t.$$

The monetary easing measure MP_t is introduced to the monetary policy rule:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Q_t}{Q}\right)^{-\phi_Q} e^{-MP_t}.$$

The price markup measure MU_t shifts the parameter for the elasticity of substitution among intermediate goods θ_P. It is introduced to the New Keynesian Phillips curve:

$$\pi_t = \beta\hat{\pi}_{t+1} + \lambda\widehat{MC}_t + MU_t.$$

The discount factor shifter DF_t and the labor disutility shifter LD_t are introduced to the utility function of both optimizing and HtM households:

$$E_0 \sum_{t=0}^{\infty} e^{DF_t}\beta^t \left[\frac{(C_t^k)^{1-\sigma} - 1}{1-\sigma} - e^{LD_t}\mu\frac{(L_t^k)^{1+\gamma}}{1+\gamma}\right],$$

for $k = O$ and H. These additional exogenous variables follow the AR(1) processes:

$$\begin{aligned}
A_t &= \rho_A A_{t-1} + \varepsilon_t^A, \\
MP_t &= \rho_{MP} MP_{t-1} + \varepsilon_t^{MP}, \\
MU_t &= \rho_{MU} MU_{t-1} + \varepsilon_t^{MU}, \\
DF_t &= \rho_{DF} DF_{t-1} + \varepsilon_t^{DF}, \\
LD_t &= \rho_{LD} LD_{t-1} + \varepsilon_t^{LD}.
\end{aligned}$$

where ε_t^A, ε_t^{MP}, ε_t^{MU}, ε_t^{DF}, and ε_t^{LD} denote technology, monetary easing, price markup, discount factor, and labor disutility shocks, respectively.

To compute the impulse responses to these shocks, I set the persistence parameters $\rho_A = \rho_{MP} = \rho_{MU} = \rho_{DF} = \rho_{LD} = 0.8$. The ratio of HtM households $1 - n$ is 0.2. The monetary policy sensitivity to inflation ϕ_π is 1.5. The remaining parameters are the same as those in Section 3.2. Under these parameter values, equilibrium determinacy is guaranteed for $\phi_q \geq 0$. I, then consider three cases for monetary policy sensitivity to inequality: $\phi_q = 0$, 0.5, and 1.5.

Figure 3 shows the impulse responses to a 1% monetary easing shock. A monetary easing shock reduces inequality in the economy, which is consistent with the empirical result of Coibion et al. (2017). On the contrary, monetary easing increases inflation. Due to price stickiness, this increase in inflation increases the real marginal cost, which consequently reduces inequality as shown in Section 2.2. Figure 3 also shows that the monetary policy response to inequality stabilizes both the responses of inflation and output to the shock.

[Figure 3]

Figure 4 shows the impulse responses to a 1% positive productivity shock. Contrary to the monetary easing shock, the positive productivity shock raises inequality in the economy. This is because an increase in productivity implies a decrease in the real marginal cost. The monetary policy response to inequality stabilizes the responses of inequality, marginal cost, inflation, total labor supply, and labor supply of optimizing household, whereas it amplifies the responses of output, consumptions, and labor supply of HtM. Thus, the monetary policy responses to inequality do not necessarily reduce the volatilities of output.

[Figure 4]

Figures 5, 6, and 7 are the impulse responses to 1% price markup, discount factor, and labor disutility shocks, respectively. They are contractionary for the economy and raise inequality. The monetary policy response to inequality stabilizes the volatility of inequality. However, the effects on output and inflation are different. In the case of the price markup shock, the monetary policy response to inequality stabilizes output, while inflation becomes more volatile. In the case of the discount factor shock, the monetary policy response to inequality stabilizes both output and inflation. In the case of the labor disutility shock, the monetary policy response to inequality stabilizes inflation, while output becomes more volatile. Therefore, the monetary policy response to inequality does not necessarily reduce both inflation and output volatilities, and it is not clear whether the monetary policy response to inequality improves economic welfare.

[Figures 5, 6, and 7]

These results are closely related to the findings of Debirtoli and Galí (2017). They show that price stability is nearly optimal even in TANK models although consumption heterogeneity is included in their welfare function. Therefore, inequality itself would not be the mandate of the central bank from the viewpoint of welfare.

4.2 Return of capital as an alternative source of income

In the baseline model, there is no capital stock. In a model with capital, the return of capital is included into the income of optimizing households. However, the main result here is qualitatively robust to this extension.

For simplicity of analysis, the supply of capital stock is fixed to be one: $K = 1$. Thus, the budget constraint of the optimizing households (2) becomes

$$P_t C_t^O + B_{t+1}^O \leq R_t B_t^O + P_t W_t L_t^O + P_t R_t^K K + D_t,$$

where R_t^K denotes the return rate of capital.

The production function of the intermediate good firm (11) becomes

$$Y_t(j) = K_t(j)^\alpha L_t(j)^{1-\alpha},$$

where $\alpha \in (0, 1)$ is the cost share of capital. The equilibrium condition of this extended model is given in Appendix D.

In this model, consumptions of optimizing and HtM households are given by

$$\hat{C}_t^H = \chi_K \widehat{MC}_t,$$
$$\hat{C}_t^O = \delta_K \widehat{MC}_t,$$

where

$$
\begin{aligned}
\chi_K &= \chi\phi, \\
\delta_K &= \tilde{\delta}\phi, \\
\phi &= \left[1 + \alpha\omega_L \frac{1-\sigma\tilde{\delta}}{\gamma} + \alpha(1-\omega_L)(\chi - 1)\right]^{-1}, \\
\tilde{\delta} &= \frac{(1-\alpha)\omega_L(1+\gamma) + (1-\alpha)\gamma\chi(\omega_C - \omega_L) - \gamma - \alpha\gamma(\chi - 1)}{(1-\alpha)\sigma\omega_L + \gamma\omega_C}.
\end{aligned}
$$

If the cost share of capital α is zero, then $\chi_K = \chi$ and $\delta_K = \delta$ and the model is reduced to the baseline model.

The coefficient $\tilde{\delta}$ is still increasing in n and there is a threshold value $\bar{n}^K$ such that if $n < \bar{n}^K$, then $\tilde{\delta} < 0$, otherwise $\tilde{\delta} \geq 0$. The sign of ϕ is not clear, but if it is supposed that $\phi > 0$, Proposition 1 still holds in this extended model if $\bar{n}$, χ, and δ are replaced by $\bar{n}^K$, χ_K, and δ_K.

Figures 8 and 9 show the determinacy regions of equilibrium in this extended model and they are the analogues of Figures 1 and 2. The cost share of capital α is set to be 0.3 and the values of the remaining parameters are the same as those in Section 3.2. The real lines are the threshold values for equilibrium determinacy. The dotted lines are those of the baseline model as shown in Figures 1 and 2. Figures 8 and 9 imply that the determinacy results in Section 3 are qualitatively robust to this model with capital.

[Figures 8 and 9]

4.3 Alternative inequality measure: the Gini coefficient

In the baseline analysis, the inequality measure is the income (consumption) gap between optimizing and HtM households. Here, I consider the Gini coefficient, a popular measure of inequality used in empirical analyses.

Figure 10 shows the Lorenz curve of the baseline model.

[Figure 10]

The Gini coefficient is twice the area of the red-shaded portion. Then, the Gini coefficient Q_t^{Gini} is calculated by

$$
\begin{aligned}
Q_t^{Gini} &= 1 - (n+1)(1-n)\frac{C_t^H}{C_t} - n^2 \frac{C_t^O}{C_t} \\
&= \frac{n(1-n)\left[C_t^O - C_t^H\right]}{C_t}.
\end{aligned}
$$

The log-linearized equation at equilibrium is given by

$$
\begin{aligned}
\hat{Q}_t^{Gini} &= (1-v_H)\hat{C}_t^O - v_H \hat{C}_t^H - \hat{C}_t \\
&= \left[(1 - v_H - \omega_C)\delta - (1 + v_H - \omega_C)\chi\right] \widehat{MC}_t,
\end{aligned}
$$

where $v_H \in (0,1)$ is given by

$$
v_H = \frac{C^H}{C^O - C^H}.
$$

I employ numerical simulations to investigate the determinacy region. Figure 11 is the analogue of Figure 2. The parameter values are the same as those in Section 3.2.

[Figure 11]

Figure 11 implies that an increase in ϕ_q enlarges the determinacy region of equilibrium, as in the baseline inequality measure. There are two points that differ from the baseline. First, the value of $\bar{\phi}_q$ for equilibrium determinacy is higher than that in the baseline. In the baseline case, equilibrium determinacy is guaranteed if $\phi_q \geq 3.5$, as in Figure 2. Second, the effect of the ratio of HtM $1-n$ is not monotonic. In the baseline, the larger the number of HtM is, the higher is the value of $\bar{\phi}_q$. In contrast, in the case of the Gini coefficient, equilibrium determinacy is most unlikely to be achieved in the case where $1-n$ is around 0.8.

5 Concluding Remarks

In this paper, I have investigated the effect of the monetary policy response to inequality using a TANK model. It is known that the Taylor principle no longer guarantees equilibrium determinacy in TANK models by numerical analyses. I have derived the analytical conditions for equilibrium determinacy and have shown that the monetary policy response to inequality is helpful in achieving equilibrium determinacy. Thus, these results imply a benefit from the monetary policy response to inequality. On the other hand, the impulse responses to structural shocks show that the monetary policy response to inequality does not necessarily reduce the volatilities of both inflation and output while it mitigates the volatility of inequality.

These results do not justify the central bank's response to inequality. The impulse response analyses have shown that the monetary policy response to inequality does not necessarily reduce both inflation and output volatility. According to the paper by Debirtoli and Galí (2017), price stability is nearly optimal even in their TANK model although consumption heterogeneity is included in their welfare function. Then, inequality itself would not be the mandate of the central bank from the viewpoint of welfare. However, the main results of this paper do imply that if the central bank responds to inequality, it is beneficial in the sense that it enlarges the determinacy region of equilibrium.

There are some limitations to this paper. For example, I have employed a simple TANK model here for simplicity of analysis. Therefore, an extension to HANK models would be one of the future tasks. However, I believe that the findings of this paper still contribute to the study of the relationship between monetary policy and inequality.

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Appendix A Consumptions of two types of households

This Appendix shows that $C^O > C^H$.

By the budget constraint of HtM (6), the first-order condition (8), and the cost minimization condition (12), consumption and labor supply of HtM households at a steady state are given by

$$C^H = MC^{\chi}\mu^{-\frac{1}{\sigma+\gamma}},$$
$$L^H = MC^{\chi-1}\mu^{-\frac{1}{\sigma+\gamma}}.$$

By the market clearing conditions of the goods and the labor markets (15) and (16) and $C = L = 1$ at a steady state, consumption and labor supply of the optimizing households are given by

$$C^O = \frac{1}{n}\left[1 - (1-n)MC^{\chi}\mu^{-\frac{1}{\sigma+\gamma}}\right],$$
$$L^O = \frac{1}{n}\left[1 - (1-n)MC^{\chi-1}\mu^{-\frac{1}{\sigma+\gamma}}\right].$$

The difference between consumptions of optimizing and HtM households is given by

$$C^O - C^H = \frac{1 - MC^{\chi}\mu^{-\frac{1}{\sigma+\gamma}}}{n}. \tag{A.1}$$

Then, the sign of the numerator of (A.1) is focused.

Here, the labor supply curve of the optimizing households (4) is rewritten as

$$\mu\left[1 - (1-n)MC^{\chi}\mu^{-\frac{1}{\sigma+\gamma}}\right]^{\sigma}\left[1 - (1-n)MC^{\chi-1}\mu^{-\frac{1}{\sigma+\gamma}}\right]^{\gamma} = MCn^{\sigma+\gamma}. \tag{A.2}$$

By taking the limit of (A.2) as n goes to zero, it is obtained that

$$1 - MC^{\chi}\mu^{-\frac{1}{\sigma+\gamma}} > 1 - MC^{\chi-1}\mu^{-\frac{1}{\sigma+\gamma}} = 0,$$

because $MC < 1$. Then, $C^O > C^H$.

In this paper, $\gamma > 0$. On the other hand, if γ is zero, then $C^O = C^H$. This is because taking the limit of (A.2) as n goes to zero implies that

$$1 - MC^{\chi}\mu^{-\frac{1}{\sigma+\gamma}} = 0.$$

Appendix B Linearized equilibrium condition

The log-linearized equilibrium system is given by

$$\sigma\hat{C}_t^O + \gamma\hat{L}_t^O = \hat{W}_t, \tag{B.1}$$

$$-\sigma\hat{C}_t^O = -\sigma\hat{C}_{t+1}^O - \hat{\pi}_{t+1} + \hat{R}_t, \tag{B.2}$$

$$\sigma\hat{C}_t^H + \gamma\hat{L}_t^H = \hat{W}_t, \tag{B.3}$$

$$\hat{C}_t^H = \hat{L}_t^H + \hat{W}_t, \tag{B.4}$$

$$\hat{C}_t = \hat{Y}_t, \tag{B.5}$$

$$\hat{W}_t = \widehat{MC}_t, \tag{B.6}$$

$$\hat{Y}_t = \hat{L}_t, \tag{B.7}$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \lambda\widehat{MC}_t, \tag{B.8}$$

$$\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_Q\hat{Q}_t, \tag{B.9}$$

$$\hat{Q}_t = \hat{C}_t^O - \hat{C}_t^H, \tag{B.10}$$

$$\omega_C\hat{C}_t^O + (1-\omega_C)\hat{C}_t^O = \hat{C}_t, \tag{B.11}$$

$$\omega_L\hat{L}_t^O + (1-\omega_L)\hat{L}_t^O = \hat{L}_t, \tag{B.12}$$

The steady-state value of the real marginal cost MC is given by

$$MC = \frac{\theta_P - 1}{\theta_P}.$$

The weight parameters ω_C and ω_L are the steady-state ratio of the optimizing households' consumption and labor supply:

$$\omega_C = \frac{nC^O}{C},$$
$$\omega_L = \frac{nL^O}{L},$$

where the steady-state values are given by

$$C = L = 1,$$
$$nC^O = C - (1-n)MC\left(\frac{MC^{1-\sigma}}{\mu}\right)^{\frac{1}{\sigma+\gamma}},$$
$$nL^O = L - (1-n)\left(\frac{MC^{1-\sigma}}{\mu}\right)^{\frac{1}{\sigma+\gamma}}.$$

By definition, the steady-state value of the real marginal cost MC is less than one, and then

$$\omega_C - \omega_L = (1-n)(1-MC)\left(\frac{MC^{1-\sigma}}{\mu}\right)^{\frac{1}{\sigma+\gamma}} > 0.$$

By (B.3), (B.4) and (B.6), it is obtained that

$$\hat{C}_t^H = \chi\widehat{MC}_t, \tag{B.13}$$
$$\hat{L}_t^H = (\chi - 1)\widehat{MC}_t, \tag{B.14}$$

where

$$\chi = \frac{1+\gamma}{\sigma+\gamma} > 0.$$

By (B.1), (B.11), (B.12), (B.13) and (B.14), it is obtained that

$$\hat{C}_t^O = \delta\widehat{MC}_t, \tag{B.15}$$

where

$$\delta = \frac{\omega_L(1+\gamma) + \gamma\chi(\omega_C - \omega_L) - \gamma}{\sigma\omega_L + \gamma\omega_C}.$$

Appendix C Proof of Proposition 1

Proof. The characteristic equation of the equilibrium system is

$$F(x) = x^2 - Tx + D,$$

where

$$T = \frac{\lambda + (1+\beta)\sigma\delta + \beta(\chi - \delta)\phi_q}{\beta\sigma\delta},$$
$$D = \frac{\lambda\phi_\pi + (\chi - \delta)\phi_q + \sigma\delta}{\beta\sigma\delta}.$$

Equilibrium is determinate if and only if two roots of this equation are outside the unit circle.

(1) Suppose that $n \geq \bar{n}$, then $\delta \geq 0$. It is obvious that $F(0) = D > 0$, and $F'(0) = -T < 0$. Then, equilibrium is determinate if and only if

$$F(1) = \frac{\lambda(\phi_\pi - 1) + (1-\beta)(\chi - \delta)\phi_q}{\beta\sigma\delta} > 0.$$

It can be rewritten as

$$\lambda(\phi_\pi - 1) + (1-\beta)(\chi - \delta)\phi_q > 0.$$

If $\phi_\pi > 1$, this condition is satisfied.

(2) Suppose that $n < \bar{n}$, then $\delta < 0$. Because $\phi_\pi > 1$, it is obtained that

$$F(1) = \frac{\lambda(\phi_\pi - 1) + (1-\beta)(\chi - \delta)\phi_q}{\beta\sigma\delta} < 0.$$

Then, equilibrium is determinate if and only if

$$F(-1) = \frac{\lambda(1 + \phi_\pi) + (1+\beta)(\chi - \delta)\phi_q + 2(1+\beta)\sigma\delta}{\beta\sigma\delta} < 0.$$

It can be rewritten as

$$\lambda(1 + \phi_\pi) + (1+\beta)(\chi - \delta)\phi_q + 2(1+\beta)\sigma\delta > 0.$$

□

Appendix D Linearized equilibrium condition of the model with capital in Section 4.2

The log-linearized equilibrium system of the model with capital in Section 4.2 is given by

$$\sigma \hat{C}_t^O + \gamma \hat{L}_t^O = \hat{W}_t, \tag{D.1}$$

$$-\sigma \hat{C}_t^O = -\sigma \hat{C}_{t+1}^O - \hat{\pi}_{t+1} + \hat{R}_t, \tag{D.2}$$

$$\sigma \hat{C}_t^H + \gamma \hat{L}_t^H = \hat{W}_t, \tag{D.3}$$

$$\hat{C}_t^H = \hat{L}_t^H + \hat{W}_t, \tag{D.4}$$

$$\hat{C}_t = \hat{Y}_t, \tag{D.5}$$

$$\hat{W}_t = \widehat{MC}_t + \hat{Y}_t - \hat{H}_t, \tag{D.6}$$

$$\hat{Y}_t = (1-\alpha)\hat{L}_t, \tag{D.7}$$

$$\hat{\pi}_t = \beta \hat{\pi}_{t+1} + \lambda \widehat{MC}_t, \tag{D.8}$$

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \phi_Q \hat{Q}_t, \tag{D.9}$$

$$\hat{Q}_t = \hat{C}_t^O - \hat{C}_t^H, \tag{D.10}$$

$$\omega_C \hat{C}_t^O + (1-\omega_C)\hat{C}_t^O = \hat{C}_t, \tag{D.11}$$

$$\omega_L \hat{L}_t^O + (1-\omega_L)\hat{L}_t^O = \hat{L}_t, \tag{D.12}$$

By (D.3) and (D.4), it is obtained that

$$\hat{C}_t^H = \chi \widehat{W}_t, \tag{D.13}$$

$$\hat{L}_t^H = (\chi - 1)\widehat{W}_t. \tag{D.14}$$

By (D.1), (D.11), (D.12), (D.13) and (D.14), it is obtained that

$$\hat{C}_t^O = \tilde{\delta} \widehat{W}_t, \tag{D.15}$$

$$\hat{L}_t^O = \frac{1 - \sigma \tilde{\delta}}{\gamma} \widehat{W}_t, \tag{D.16}$$

where

$$\tilde{\delta} = \frac{(1-\alpha)\omega_L(1+\gamma) + (1-\alpha)\gamma\chi(\omega_C - \omega_L) - \gamma - \alpha\gamma(\chi - 1)}{(1-\alpha)\sigma\omega_L + \gamma\omega_C}.$$

By (D.6), (D.7), (D.1), (D.14) and (D.16), it is obtained that

$$\hat{L}_t = \phi\widehat{MC}_t,$$

where

$$\phi = \left[1 + \alpha\omega_L \frac{1 - \sigma\tilde{\delta}}{\gamma} + \alpha(1-\omega_L)(\chi - 1)\right]^{-1}.$$

Thus, consumptions of HtM and optimizing households are given by

$$\hat{C}_t^H = \chi_K\widehat{MC}_t,$$
$$\hat{C}_t^O = \delta_K\widehat{MC}_t,$$

where

$$\chi_K = \chi\phi,$$
$$\delta_K = \tilde{\delta}\phi.$$

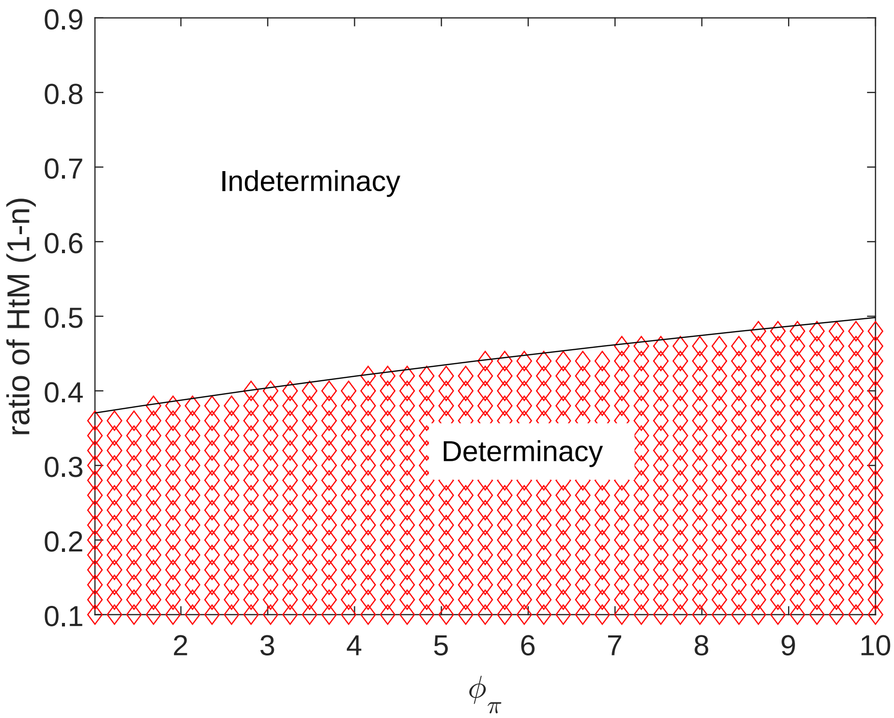
Finally, the inequality measure is given by

$$\begin{aligned}\hat{Q}_t &= \hat{C}_t^O - \hat{C}_t^H \\ &= (\delta_K - \chi_K)\widehat{MC}_t.\end{aligned}$$

Here,

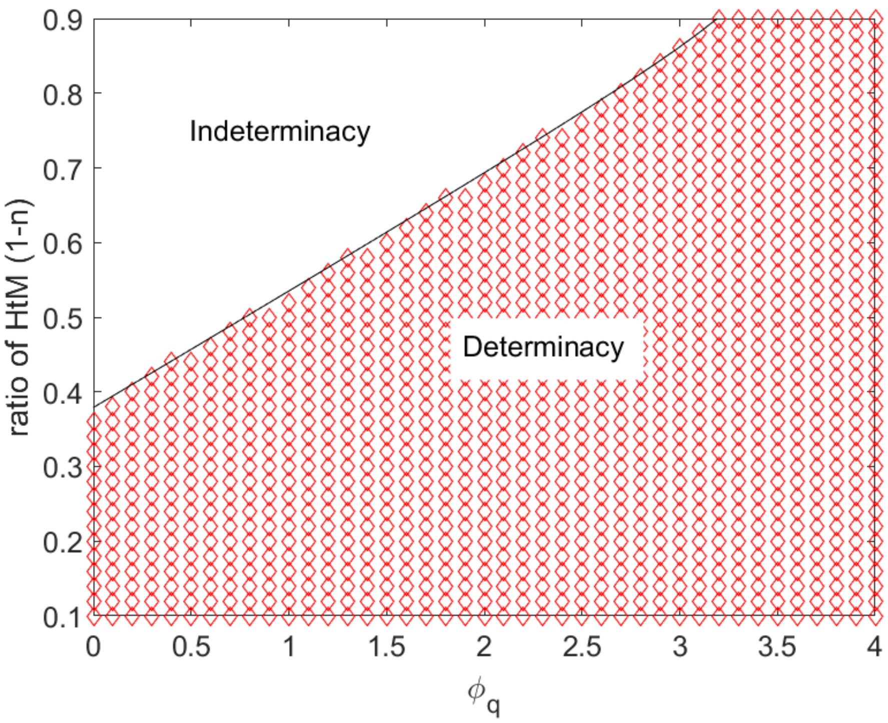
$$\delta_K - \chi_K = -\frac{\gamma[1 + \alpha(\chi - 1)]}{(1-\alpha)\sigma\omega_L + \gamma\omega_C}\phi.$$

Figure 1: Determinacy region (1): $\phi_q = 0$



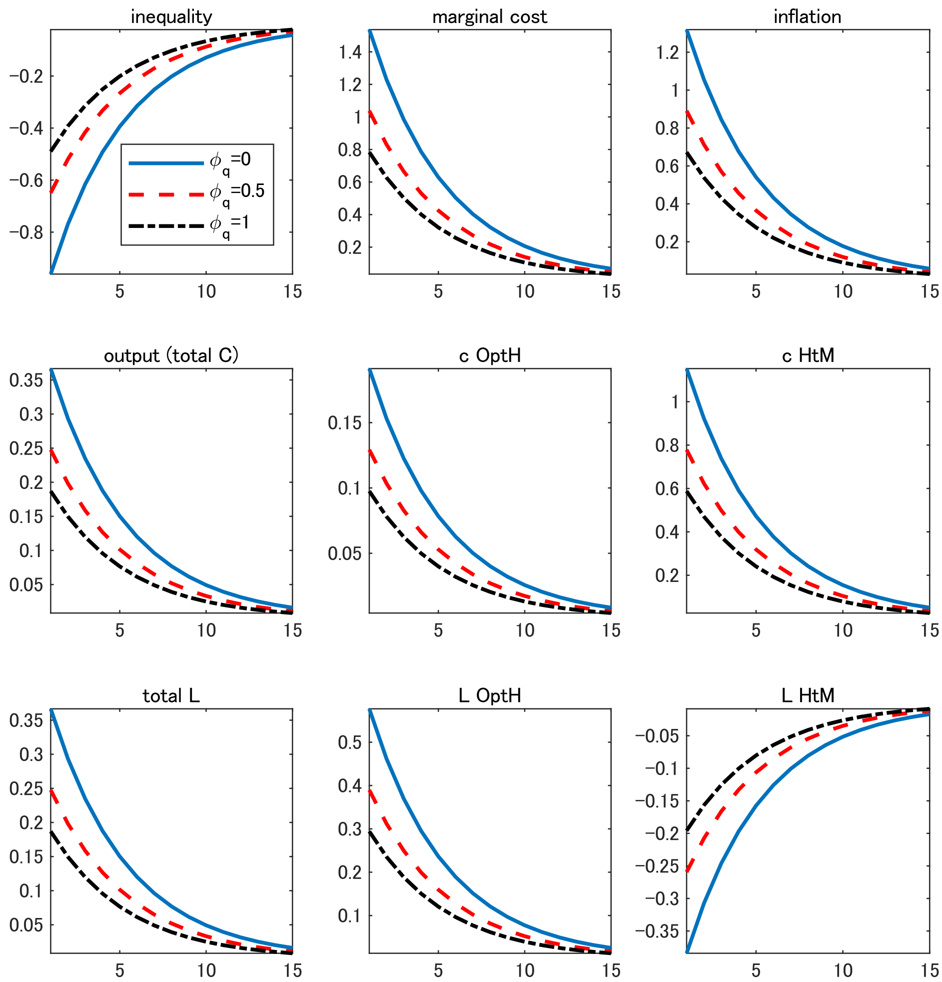
NOTE: The horizontal axis is the central bank's stance on inflation ϕ_π, and the vertical axis is the ratio of HtM households $1 - n$. The region with diamonds indicates equilibrium determinacy, and the other region indicates equilibrium indeterminacy. The real line is the threshold value of $\bar{\phi}_\pi$ for equilibrium determinacy.

Figure 2: Determinacy region (2): $\phi_\pi = 1.5$



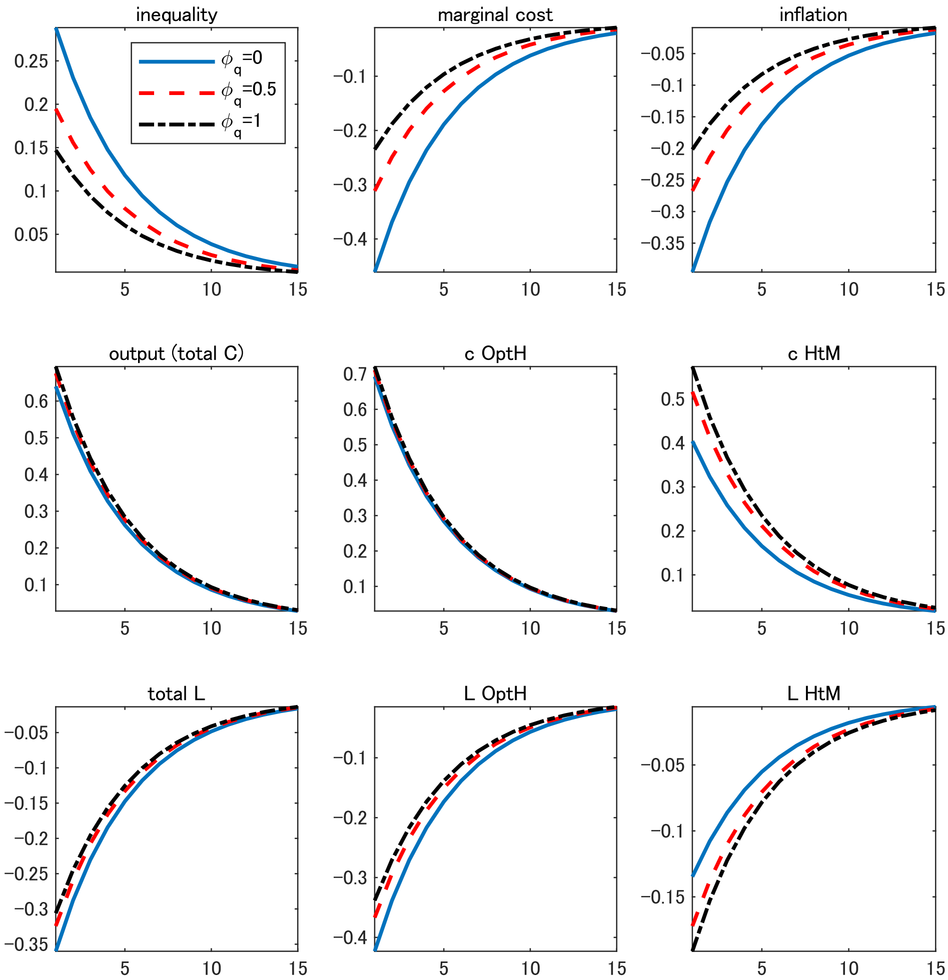
NOTE: The horizontal axis is the central bank's stance on inequality ϕ_q, and the vertical axis is the ratio of HtM households $1 - n$. The region with diamonds indicates equilibrium determinacy, and the other region indicates equilibrium indeterminacy. The real line is the threshold value of $\bar{\phi}_q$ for equilibrium determinacy.

Figure 3: Impulse responses to a 1% monetary easing shock



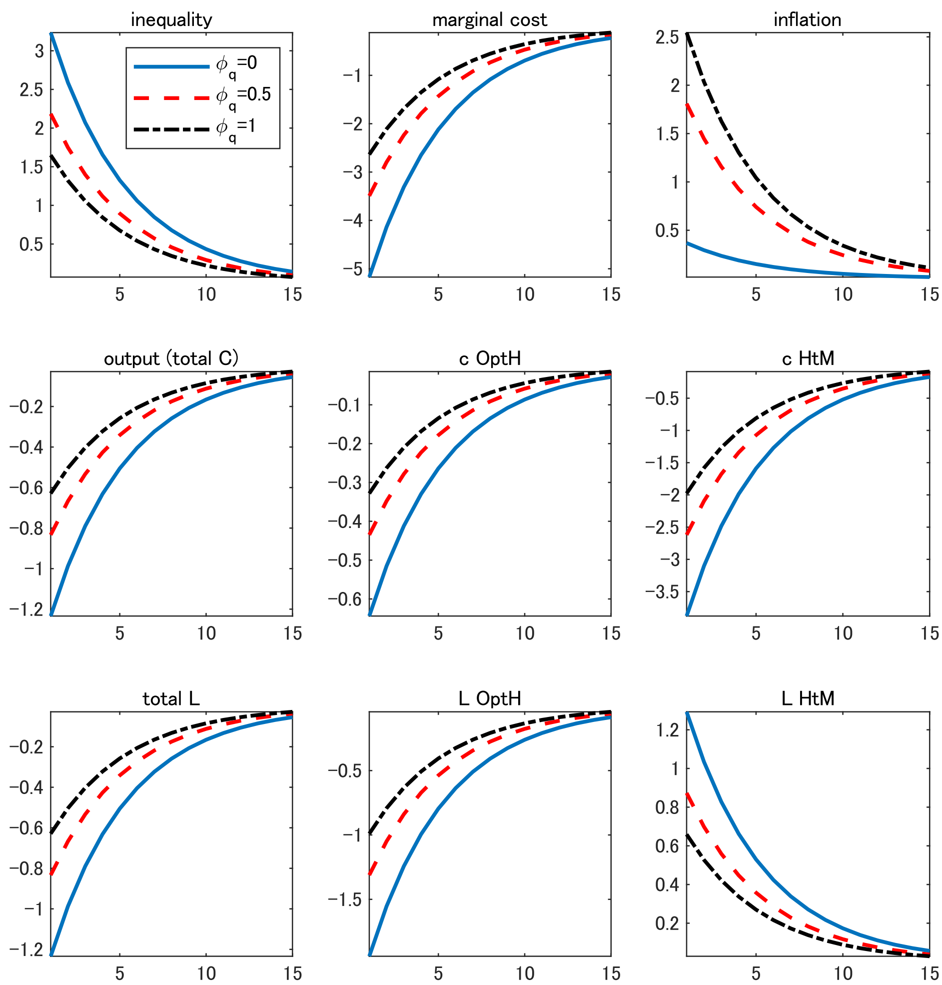
NOTE: The vertical axis is the percentage deviation from its steady-state value. In the first period, a shock hits the economy.

Figure 4: Impulse responses to a 1% positive productivity shock



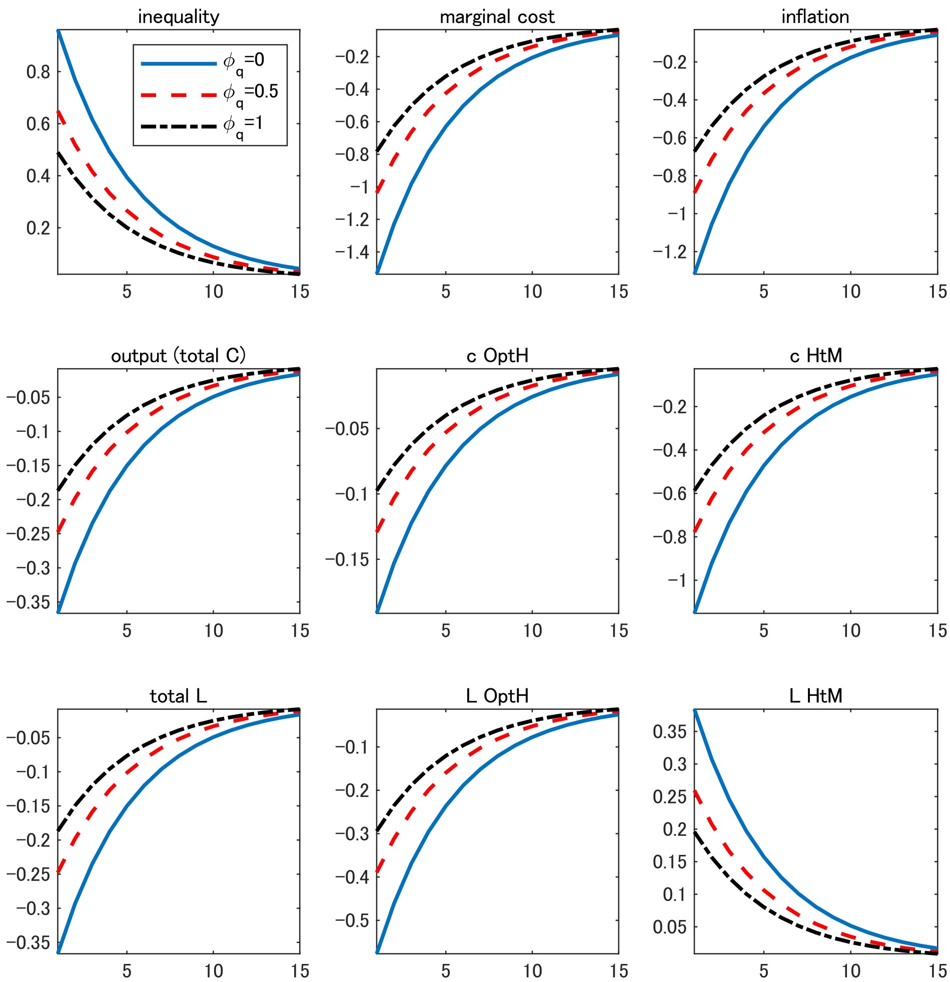
NOTE: The vertical axis is the percentage deviation from its steady-state value. In the first period, a shock hits the economy.

Figure 5: Impulse responses to a 1% price markup shock



NOTE: The vertical axis is the percentage deviation from its steady-state value. In the first period, a shock hits the economy.

Figure 6: Impulse responses to a 1% discount factor shock



NOTE: The vertical axis is the percentage deviation from its steady-state value. In the first period, a shock hits the economy.

Figure 7: Impulse responses to a 1% labor disutility shock

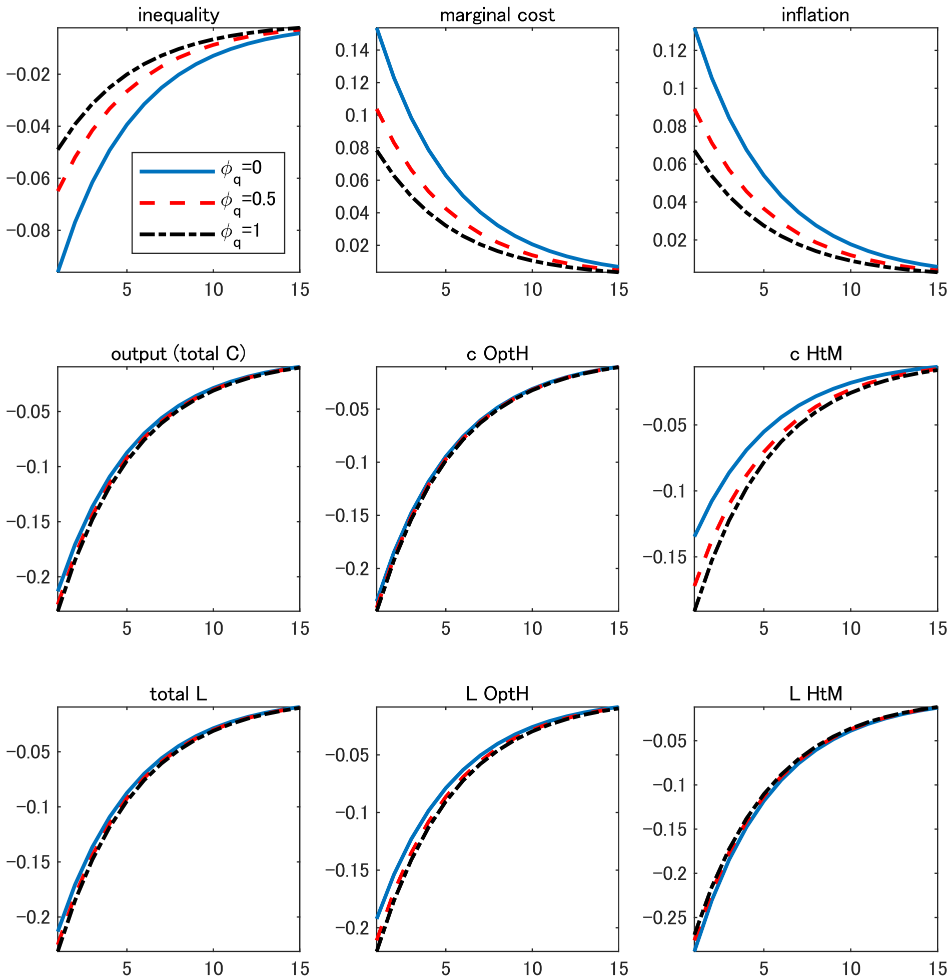
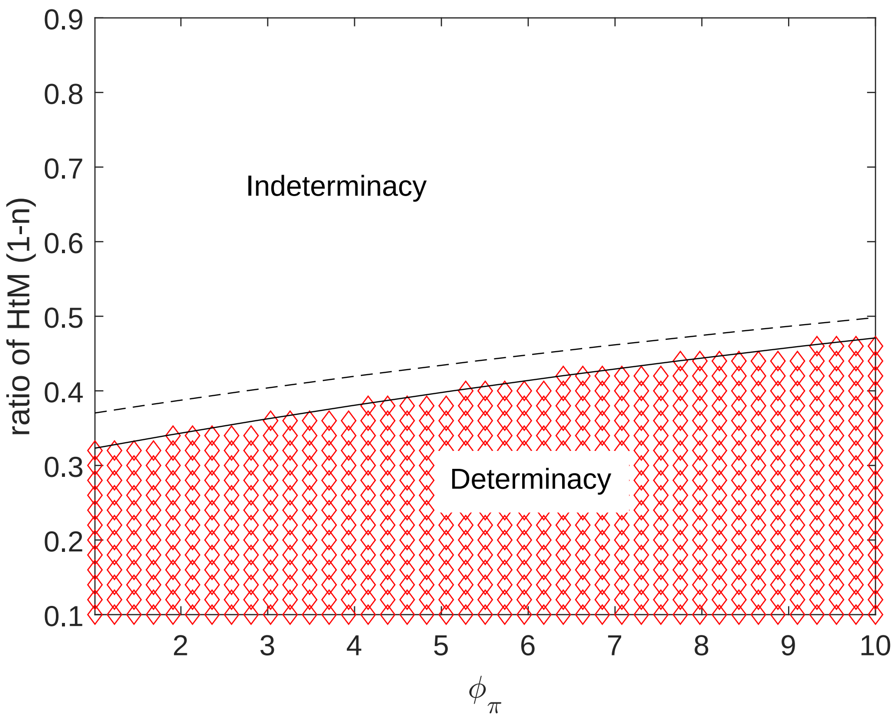
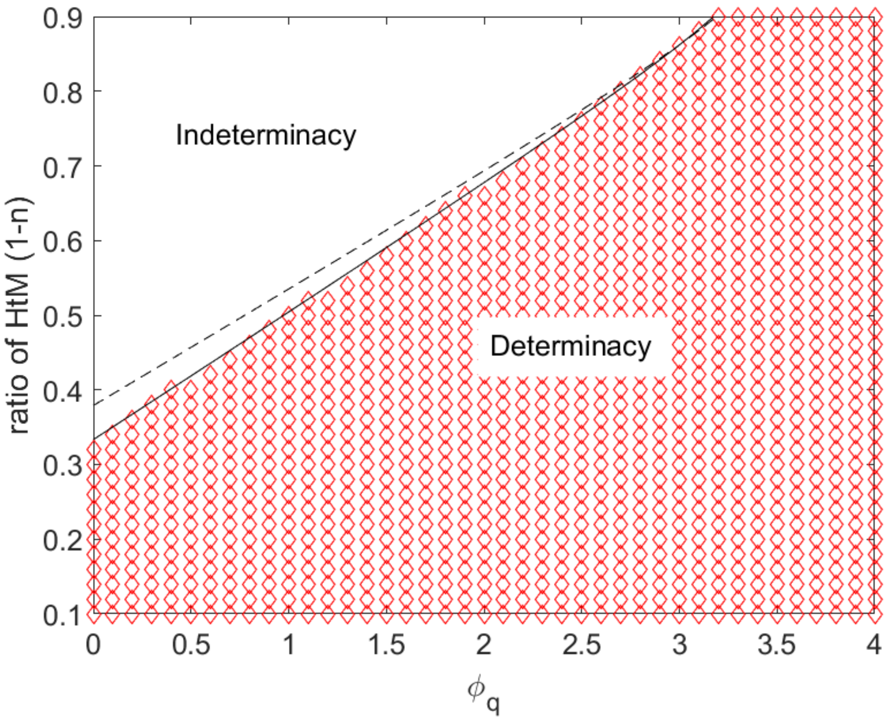


Figure 8: Determinacy region (3): Model with capital: $\phi_q = 0$



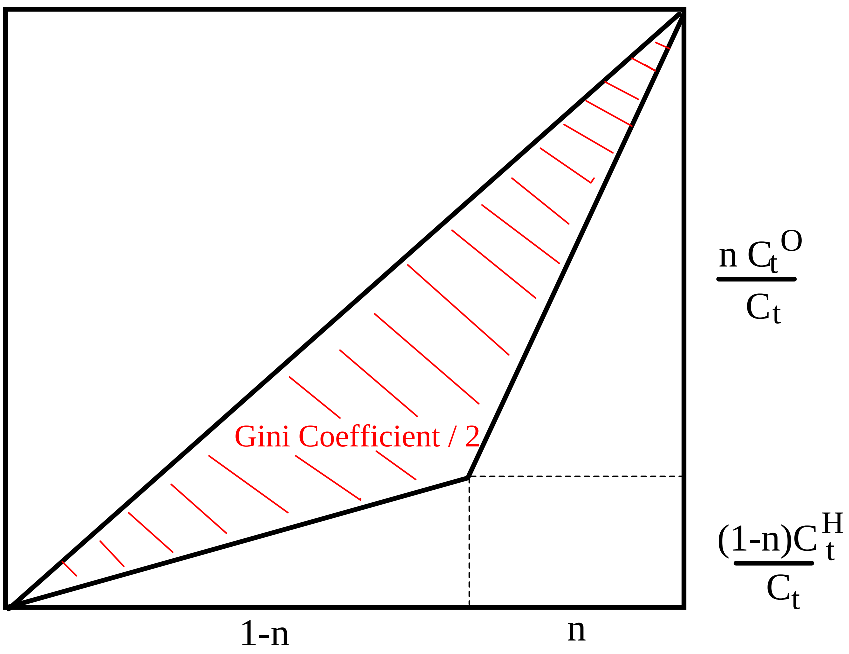
NOTE: The horizontal axis is the central bank's stance on inflation ϕ_π, and the vertical axis is the ratio of HtM households $1 - n$. The region with diamonds indicates equilibrium determinacy, and the other region indicates equilibrium indeterminacy. The real line is the threshold value of $\bar{\phi}_\pi$ for equilibrium determinacy, while the dotted line is that in the baseline model.

Figure 9: Determinacy region (4): Model with capital: $\phi_\pi = 1.5$



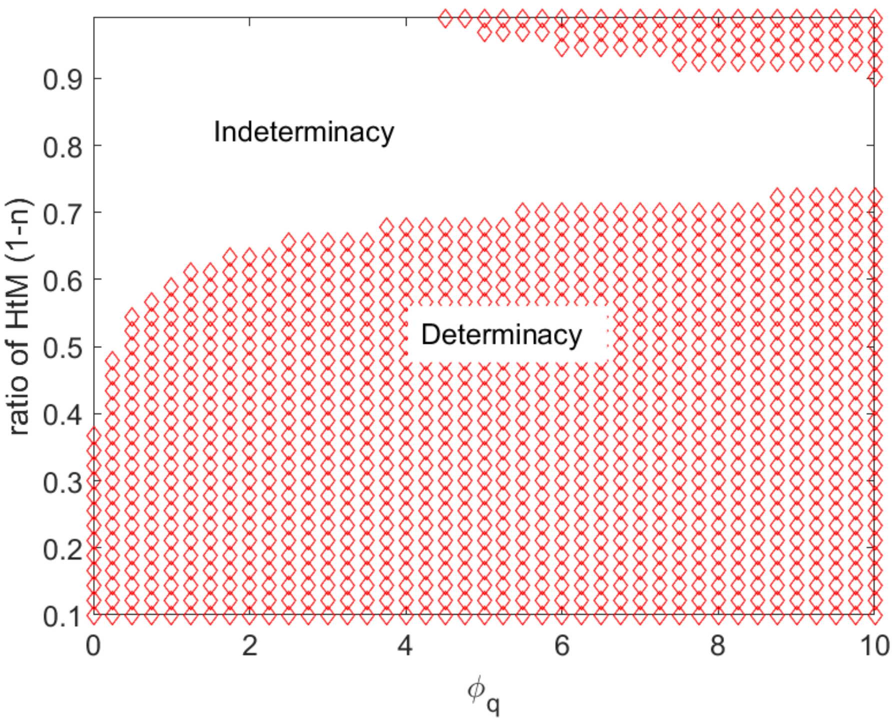
NOTE: The horizontal axis is the central bank's stance on inequality ϕ_q, and the vertical axis is the ratio of HtM households $1 - n$. The region with diamonds indicates equilibrium determinacy, and the other region indicates equilibrium indeterminacy. The real line is the threshold value of $\bar{\phi}_q$ for equilibrium determinacy, while the dotted line is that in the baseline model.

Figure 10: Lorenz curve



NOTE: The Gini coefficient is twice the area of the red-shaded portion.

Figure 11: Determinacy region (5): Case of the Gini Coefficient: $\phi_\pi = 1.5$



NOTE: The horizontal axis is the central bank's stance on inequality ϕ_q, and the vertical axis is the ratio of HtM households $1 - n$. The region with diamonds indicates equilibrium determinacy, and the other region indicates equilibrium indeterminacy.